Competition and Regulations in Telecommunications with Asymmetric Firms

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Abstract

This thesis examines competition among asymmetric firms in three different theoretical frameworks. The first study investigates mobile telecommunications, with a strong focus on access charge on off-net calls. Compared to the symmetric cost-based access charge regulation, whilst the asymmetric cost-based access charge regulation facilitates entry, it dampens social welfare if, relative to the incumbent, the new firm is significantly inefficient in cost, distinctly inferior in reputation, and incapable of clearly differentiating its service from the one provided by the established firm.

The second study sheds light on a broader framework of infrastructure sharing among telecommunications firms with asymmetric cost structures. Compared to stand-alone investment, co-investment is deemed to be collusive in quality upgrade, and consequently decreases industry output and consumer surplus when infrastructure sharing does not yield a sufficient amount of cost saving. Even though the fullydistributed-cost regulation can stimulate investment in quality upgrade, it undermines incentives to expand consumer bases, leads to price increases, and eventually dampens consumer welfare.

The last study captures the competition beyond only one product market where a multi-product firm competes with its single-product rivals by using a variety of bundling strategies that impact on firms' incentives for quality enhancement in different ways. The pure-bundling strategy can encourage the multi-product firm to invest in quality enhancement when the associated costs are comparatively low and the additional utility from quality enhancement is relatively high, but it certainly discourages the single-product firms from improving quality. In the mixed-bundling case, this outcome inevitably occurs in the more competitive market, and it is likely to be found in the less competitive market when the markets are not too different in competition intensity. Therefore, both bundling strategies threaten consumer welfare when the two markets are significantly different in competition intensity due to the negative influence of the market distortions after tying the two markets.

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I was accidentally inspired by a Thai movie two years ago. I would like to thank the cast and production team of Thai film, *Kid Tueng Wittaya (Teacher's Diary)* for giving me an illustration of how people live without telecommunications services as a consequence of lack of investment in wireless telecommunications and internet services in remote areas. Chapter 3 has been dedicated to practical suggestions for some optimal solutions to this problem.

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Declaration

I hereby declare that this thesis is my original work. Otherwise, other sources of information used in this thesis have been acknowledged in the text and as references. None of the work has been submitted for another degree at this or any other institution.

Several early versions of Chapter 2 was presented at the $15th$ Centre for Competition and Regulatory Policy Workshop at City University, London in January 2013, at the $2nd$ White Rose Economics PhD Conference at the University of York in March 2013, and at the $8th$ National Conference of Economists at Chulalongkorn University, Thailand in October 2013, as published in the proceedings of the $8th$ National Conference of Economists. An early version of Chapter 3 was also presented at the $5th$ NIE Doctoral Student Colloquium at the University of Nottingham in June 2015.

Chapter 1 Introduction

Telecommunications services were first provided by government agencies because of the requirement of high investment in infrastructure and the obligation to the public, like other public services. After the privatisation of telecommunications services and telecommunications reforms, the industry has still been in need of being monitored and regulated by national regulatory authorities (NRAs) to ensure an acceptable degree of competition and social benefit. In form of commissions or quasi-judicial boards, the regulators have been entitled to make judgments on telecommunications affairs and disputes between telecommunications service providers. There was a rapidly growing trend in the foundation of NRAs in many countries around the world.

In competition with the incumbents, some entrants are granted licenses to operate as telecommunications service providers in anticipation of a boost in the degree of competition and consumer welfare. When telecommunications service providers are symmetric in terms of cost efficiency, financial constraints and market power, the regulators can easily achieve their goals of finance, efficiency and equity. However, in the real business world, firms are normally asymmetric. According to the main characteristics of incumbents with market dominance, the dominant firms are likely to seize this opportunity to corner the markets by employing predatory strategies and creating barriers to entry, which can hinder the improvement of market efficiency and social welfare. Similar to other industries, firm asymmetry between incumbents and entrants can lead to several crucial issues about market inefficiency such as predatory pricing, vertical integration, margin squeezing, collusion, tying and bundling. These problems seem much more complicated for the telecommunications regulators because the nature of the industry is oligopolistic. Consequently, in a period of transition between infant markets and mature markets, the regulators continue to closely monitor the market efficiency in this oligopolistic situation. The regulators may impose behavioural interventions and/or structural interventions in order to promote social welfare in the long run. An example of behavioural interventions is price regulation, while structural interventions involve service providers' business modules and organisational structures. This thesis aims to discuss some crucially important issues of competition and regulations in telecommunications, including interconnection, infrastructure sharing and service bundling in the context of firm asymmetry.

1.1 Firm asymmetry in telecommunications

Telecommunications markets are distinct from other product markets in that both competition and cooperation between competitive firms are likely to occur simultaneously. In general, firms compete for consumers to maximise their own profit. However, telecommunications service providers compete in the retail market while they also cooperate with one another to complete their services that run on not only their own networks, but also their rivals' networks. Theoretically, the relationship among these firms is based on the concept of one-way access or two-way access. Competitive firms compete for end users, whereas they have an agreement in the wholesaler-retailer relationship. This can be seen in several telecommunications sectors such as mobile telephony, fixed line and the Internet. They have to agree on interconnection in order to allow transmission from one network to another network. In the mobile telephony market, they impose access charges (interconnection charges/ mobile termination rates) for providing call termination. This is the reason why they are highly interdependent.

Moreover, after subscribing to a network, a consumer may have difficulty in switching to another network and accordingly become locked in the network. Consumers may experience high technical switching costs and a complicated procedure for switching networks. Similar to other experience goods, if consumers are satisfied with their current networks, they tend to believe that their current networks are more reliable than other networks which they have never used before. This type of brand loyalty can influence consumers' decisions, especially when an entrant struggles to introduce its service in the market where an incumbent has long been operating. In the mobile telephony market, the regulators in several counties try to lower switching cost by implementing Mobile Number Portability (MNP). This policy allows consumers to change their mobile networks more easily while they can also keep their current mobile numbers. However, there is some evidence to support that firms tend to set high access charges in order to soften competition. High access charge is also considered to be a predatory strategy in some situations. Most regulators recognise these problems and adopt cost-based access charge regulation to mitigate the distortion in retail prices.

When the market is mature, all firms are capable of competing in strong competition. As a result, social welfare is enhanced and consumers enjoy the benefits of the competitive market. However, in the infant market, entrants are likely to be at a disadvantage. Incumbents are more likely to foreclose the market. The competitive market is impeded by the predatory behaviour of the incumbents. In addition, it is admitted that technology in the telecommunications industry rapidly changes. Therefore, the issues of asymmetry become inevitably involved in regulatory regimes. For instance, in the mobile market, cost asymmetry occurs when mobile network operators (MNOs) have different experiences and/or technologies. Advanced technology is mostly designed to reduce cost and bring service providers an advantage to compete in the market. Nevertheless, investment in more advanced technology may be suppressed by its prohibitively high fixed cost. Big companies that hold significant market power have a strong tendency to dominate with more advanced services. This situation may be beneficial to consumers when the big firms develop their expertise in telecommunications. Conversely, it may be too risky for an entrant to enter the market. Consequently, the regulators should facilitate entry in order to attain their long-run goal of fair and efficient facility-based competition.

For example, Thai telecommunications regulator, NBTC, implements costbased access charge in the mobile telephony market. New network operators have been encouraged to compete with the incumbents in anticipation of an increase in degree of competition and welfare. Mobile network operators in Thailand are legally required to report their termination costs to the regulator. These reports are used to support their proposals to collect cost-based access charges. The dominant established networks, which earn comparatively large market shares, have proposed significantly lower access charge than small networks. In 2011, AIS with the largest market share (43.66 %) has proposed access charge at 1.07 baht per minute. Likewise, DTAC and TrueMove with 30.04 % and 23.73 % of market shares have proposed access charge at 1 and 1.07 baht per minute respectively. On the other hand, a new mobile network operator, TOT, with a negligible market share of 0.15 %, has proposed significantly higher access charge at 1.25 baht per minute (NBTC, 2012). It is implied that the new network may incur higher termination cost. The new network is less likely to penetrate the market. The regulators should consider an effective policy to intensify the competition among asymmetric networks and promote social welfare.

In addition to facility-based competition, telecommunications firms may use other modes of entry. Co-investment is an approach telecommunications firms adopt to facilitate the launch of advanced services with a lighter burden of investment in infrastructure. According to a business model, capital expense (CAPEX) and operation expense (OPEX) are claimed to significantly decrease after the costs are shared under co-investment. The models of infrastructure sharing vary according to degrees of sharing. Some firms may agree on partial sharing of some common facilities. Others may agree on full sharing of network and frequency pooling/trading. For instance, in the wireless telephony sector, both passive RAN sharing¹ and active RAN sharing² have been admissible in Austria, Germany, Sweden, Switzerland and UK. Meanwhile, frequency pooling/trading has not yet been admissible in Austria, Germany, UK and Switzerland (Frisanco, Tafertshofer, Lurin, and Ang, 2008). This concept of infrastructure sharing is also applied to the broadband internet market in an effort to hasten the next generation network deployment. However, there is some concern about incentive to quality enhancement under co-investment. In addition,

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¹ passive radio access network sharing

² active radio access network sharing

service-based entry is another approach of the infrastructure-sharing concept. Entrants use the incumbent's facilities by virtue of local loop unbundling. The entrants benefit from the service-based entry, especially when they are concerned about the uncertainty of technological change and demand. The competition between a facility-based operator and a service-based operator may affect competition intensity and the market outcomes under this asymmetry situation.

Furthermore, a telecommunications firm may provide a wide variety of services and also strategically offer service bundles in order to become dominant in the industry. As seen in the fixed broadband market, consumers have a tendency to subscribe to broadband internet providers that supply other telecommunications services such as landline, multichannel TV and mobile telephony. As seen in Figure 1.1, in the UK, the consumption of bundles of services served by the same service providers has been increasing since 2005. According to Figure 1.2, as broadband internet subscribers, the majority of respondents choose other telecommunications services which are included in a bundle offered by the same provider as broadband service. This pattern of consumption can be seen in many countries.

Figure 1.1 Take-up of bundled services over time

Figure 1.2 Proportion of consumers buying their fixed broadband service in conjunction with other communications services

The firms that offer a wider range of telecommunications services benefit from their favourable position to extract rents by tying their services in the related markets. There is some concern about the survival of the single-product firms that are incapable of offering attractive bundles. The competition intensity, quality and prices in the context of bundling are highly debatable. For example, in Canada, bundling in telecommunications has been closely monitored by the regulator, CRTC. The firms which offer bundles have been obliged to set reasonable bundle prices on the grounds of a cost-oriented basis in order to prevent predatory pricing behaviour. Additionally, the integrated firms or the upstream firms with the bundling strategy have also been monitored. The regulator aims to ensure that the downstream rivals are not threatened by the bundling strategy when they compete for end users in the retail markets.

Fair and efficient regulation is a challenge for the telecommunications regulators. It is ambiguous to point out clear-cut policies and regulations in the context of firm asymmetry. However, interventions by the regulators are still necessary to facilitate the market outcome to approach the ideal state of efficiency in terms of social welfare. Suggestions for optimal regulations vary according to circumstances as seen in the existing literature reviewed in the next section.

1.2 Literature review

This thesis focuses on the competition in the environment of firm asymmetry in order to investigate the competitive behaviours of asymmetric firms, especially in telecommunications markets. This thesis reviews the existing related literature on firm asymmetry issues, which have recently been brought into focus, including (1) interconnection in telecommunications, (2) infrastructure sharing, and (3) bundling.

1.2.1 Interconnection in telecommunications

Telecommunications regulators' goal is to promote market competition among telecommunications service providers. Mobile telephony is one of the examples of these complicated market structures. Due to a small number of mobile network operators, they may tacitly collude and not fiercely compete with each other. Basically, an on-net call is delivered within the same network and an off-net call is originated and terminated by different networks. As a wholesaler, a network supplies its rivals with terminating service for off-net calls despite the fact that they still compete in the downstream (retail) market. Each network's revenue is divided into two main components based on the Calling Party Pays (CPP) principle. The first one is revenue from call origination. This revenue is collected from its own subscribers. The second one is access revenue/deficit from call termination. When a network originates off-net calls, it has to pay access charges to other networks which terminate these off-net calls. Conversely, the network generates revenue from terminating the incoming off-net calls from other networks. This two-way access shapes the telephony markets in a complicated way. In other words, an access charge is deemed to be a wholesale price that an originating network has to pay for terminating service provided by a terminating network. Consequently, firms can influence their rivals' retail prices by charging high access charge in the upstream (wholesale) market. The recent literature related to mobile and fixed networks reveals that network operators can soften competition or even foreclose the markets by means of access charge and retail price setting.

Theoretical and empirical studies in this topic proposed various models underlain by different assumptions about the market structures. They concluded different findings and made alternative suggestions. Most policy implications from the literature give a strong focus on enhancing competition and social welfare.

Uniform Pricing

Under the uniform pricing scheme, on-net calls and off-net calls are charged at the same price. Under this pricing scheme with linear tariffs, access charge is claimed to be an instrument of tacit collusion (Armstrong, 1998; Laffont, Rey and Tirole, 1998a). Symmetric networks set high reciprocal access charge in order to prevent price-cutting. Under the condition that access charge is not too high and networks' services are not closely substitutable, networks finally agree to choose above-cost reciprocal access charge and their retail prices increase with the access charge markup. An equilibrium retail price is greater than perceived cost because of the effect of double marginalization. It is set to cover a mark-up on associated access charge (Laffont, Rey and Tirole, 1998a). However, in accordance with double marginalization, socially optimal access charge should be set below cost in order to pull the corresponding retail price down to the welfare-maximising level (Armstrong, 1998; Laffont, Rey and Tirole, 1998a).

Furthermore, under uniform pricing with two-part tariffs, networks can generate profit from fixed fees. The profit-maximising retail price is set at the weighted-average associated cost of on-net marginal cost and off-net perceived cost (Carter and Wright, 2003; Laffont, Rey and Tirole, 1998a; Lopez and Rey, 2009; Peitz, 2005). Most of the related literature assumes the balanced calling pattern. According to this assumption, all subscribers have an equal probability to be called and they are not classified as heavy or light users. Thus, the numbers of on-net calls and off-net calls made by a subscriber correspond to his network's market share. Due to the balanced calling pattern assumption, when the reciprocal access charge is set above the marginal cost, a larger network can charge a lower retail price than a smaller network. This is because the large firm originates a greater number of on-net calls than off-net calls and then its weighted-average associated cost is lower than that of the small firm (Carter and Wright, 2003; Laffont, Rey and Tirole, 1998a). However, the profits of both networks are independent of access charge when the networks reach an agreement on reciprocal access charge. At any possible level of access charge, the networks' profits are not affected because they will set their retail prices to cover their perceived costs and reap profits from fixed fees. As a result, costbased reciprocal access charge tends to occur if the negotiation on reciprocal access charge is successful. Nevertheless, a network may reap more profit if it can set higher access charge than its competitor. Therefore, the networks have an incentive to unilaterally increase access charge and cost-based reciprocal access charge may not occur in equilibrium (Laffont, Rey and Tirole, 1998a).

Network-based price discrimination

As seen in the mobile market, a network may set different tariffs for its on-net and off-net calls. An access mark-up directly affects a corresponding off-net price. In other words, off-net prices increase with associated access charges, which are set by rival networks. This price discrimination may ameliorate the effect of double marginalization because the mark-up on access charge no longer has a direct effect on the corresponding on-net price. In addition, the price discrimination may intensify competition because the average price of on-net and off-net calls is lower than the retail price under uniform pricing (Laffont, Rey and Tirole, 1998b).

Under two-part tariffs, networks collect fixed fees in addition to usage fees. Networks choose on-net and off-net prices equal to perceived costs, and they consequently gain no profit from usage fees. However, the networks still make profits from fixed fees. In other words, networks set retail prices as low as perceived costs and extract profits from consumer surplus by charging fixed fees (Gans and King, 2001; Laffont, Rey and Tirole, 1998b; Lopez and Rey, 2009; Peitz, 2005). Laffont, Rey and Tirole (1998b) concluded that networks may agree to set cost-based access charge in order to earn the maximum profits. If networks set any mark-up on access charge, their profits will decrease from the maximum levels. On the contrary, Gans and King (2001) argued that if below-cost access charge is allowed, networks may agree on below-cost reciprocal access charge in order to maximise their own profits. As a result, each network charges its on-net price higher than its off-net price instead. According to the balanced calling pattern assumption, the small network is more attractive. This is because if a consumer chooses to join the small network, he has a higher probability to make off-net calls than on-net calls and accordingly benefits from relatively low payment. Subsequently, the networks do not have an incentive to expand their market shares because the big firm will experience access deficit and its profit will dwindle in this setting. The networks will not attract a marginal consumer by reducing their fixed fees. Therefore, under price discrimination and non-linear tariffs, setting below-cost access charge may soften price competition in this manner. However, below-cost access charge is unlikely to be practical in the business world. It is difficult to draw up a contract for below-cost access charge.

As seen in the previous section, it is claimed that an access mark-up can be used as tacit collusion to avoid price war under uniform pricing with linear tariffs (Armstrong, 1998; Laffont, Rey and Tirole, 1998a). Likewise, Gans and King (2001) concluded that bill-and-keep, which can be interpreted as no access charge (access charge at a rate of zero), may also increase networks' profits and decrease social welfare. In addition to above-cost access charge, networks may adopt bill-and-keep and use this scheme as tacit collusion under price discrimination with non-linear tariffs. Similar to Gans and King (2001), Calzada and Valletti (2008) showed that even in the model of several identical firms, networks agree to choose below-cost reciprocal access charge and dampen competition.

Asymmetric networks

The aforementioned literature concerns symmetric networks which can be seen only in the mature market. Networks can compete with each other without any entry barrier. This situation occurs at the mature stage of the industry. However, at the early stage of the market, an entrant enters later than an incumbent. As a result, the incumbent is likely to be more attractive to consumers than the entrant. Consumers do not carefully consider only competitive retail prices but also networks' reputations. The incumbent may occupy a dominant position and take advantage of its good reputation and brand loyalty. Carter and Wright (2003) set up a model of asymmetric networks having the same cost structure but different levels of brand loyalty. In their model, networks compete under uniform pricing with two-part tariffs. From consumers' perspectives, the incumbent is more reliable than the entrant. Thus, the entrant has to give extra benefit to consumers by offering a lower retail price in order to persuade them to choose its service instead of joining the incumbent. This asymmetry can be interpreted as one type of switching cost, which can cause consumers to hesitate to join the new network. Carter and Wright (2003) introduced a parameter representing brand loyalty which equally affects all consumers on the preference line in the standard Hotelling model. They found that if the large network and the small network can negotiate reciprocal access charge, the profit-maximising access charge is equal to termination cost which is a benchmark for the socially optimal access charge. On the other hand, if the networks can unilaterally set their access charges, both firms will choose above-cost access charges and the large firm's access charge is higher than that of the small firm.

Peitz (2005) assumed different levels of fixed utilities which consumers receive from different networks under two-part tariffs with network-based discriminatory pricing. These fixed utilities vary from network to network. If a consumer subscribes to the incumbent, he will gain higher fixed utility than that from joining the entrant because of reliability, goodwill and/or special service obtained only from the incumbent. The study suggests that the regulator should regulate the incumbent to set cost-based access charge. Accordingly, the entrant will impose an access mark-up. This asymmetric access charge regulation is more felicitous than the symmetric costbased access charge because it can increase consumer surplus and the entrant's profit. Despite a reduction in the incumbent's profit, the asymmetric regulation is designed to achieve the regulator's goals of improving consumer welfare, encouraging entry and promoting market competition.

Baranes and Vuong (2012) further examined the asymmetric mobile market where the incumbent deploys a new technology with full market coverage but the entrant offers the new technology with partial market coverage and competes for the rest of the market with the old technology. They omitted an issue of differences in marginal cost of call origination and termination but emphasised the degree of asymmetry between networks according to the discrepancy in technology deployment. They asserted that the asymmetric regulation on termination charges can increase the entrant's market share and intensify market competition, which benefits consumers. However, the asymmetric regulation reduces the incumbent's profit. Therefore, this regulation may increase social welfare under certain circumstances when the positive impact on market competition prevails.

Without regulatory intervention, Hoernig (2007) pointed out that the on-net/ off-net differential set by the large network can act as predatory pricing. Due to the assumption of the utility from receiving calls, the large network can reduce the utility of its rival's subscribers by raising access price and accordingly pushing up its rival's off-net price. In addition, the volume of outgoing calls from the large network to its rival network decreases. Therefore, the large network may set high access charge to discourage entry.

To assess the predatory behaviour of the incumbent, Lopez and Rey (2009) examined the asymmetric networks regarding switching cost under network-based price discrimination and non-linear tariffs. Consumers will confront switching cost when they decide to switch from the incumbent to the entrant. The incumbent has an advantage from consumer inertia when switching cost is large. The incumbent tends to set high reciprocal access charge and a prohibitive off-net price to drive the entrant or the small network out of the market. They also pointed out that in the absence of switching cost, the incumbent still have control over the market under the reciprocal access charge agreement. When network-based price discrimination is allowed, the incumbent or the large network probably corners the market.

The incumbent has larger market share as a result of consumers' perception of extra benefits from joining the incumbent, even though both the incumbent and the entrant are symmetric in cost structure and quality. The incumbent may take this opportunity to harm the entrant by setting high access charge. This predatory behaviour is reported by recent studies in the setting of two-part tariffs under not only network-based price discrimination, but also uniform pricing. Under uniform pricing, the incumbent may set such high access charge that the entrant cannot make profit when the networks are allowed to unilaterally set access charges (Carter and Wright, 2003). Likewise, under network-based price discrimination, the incumbent uses high access charge and the price differential between on-net and off-net calls to deter entry. When the incumbent set high access charge to limit entry, the entrant's off-net price soars and finally the entrant is not attractive (Calzada and Valletti, 2008; Hoernig, 2007; Lopez and Rey, 2009). Thus, access charge should be rigorously monitored by regulators.

There has been empirical evidence confirming that access charge plays an important role in retail price setting by mobile network operators. In the early 2000s, the volume of mobile calls was significantly lower than that of fixed-line, which has served the market with full coverage long before the emergence of mobile service. In this situation, it was asserted that an increase in mobile termination rate $(MTR)^3$ could lead to a significant reduction in mobile retail price (the waterbed effect) in both theoretical and empirical studies. An allowance of mark-up on MTR could encourage mobile market expansion through subsidising cost of providing mobile service to mobile subscribers (Armstrong and Wright, 2009; Genakos and Valletti, 2011; Harbord and Pagnozzi, 2010). Genakos and Valletti (2015) added more recent data in their

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³ Similar to *access charge* in the present study and some theoretical work, the term *mobile termination rate* (MTR) refers to a fee for the provision of call termination by a mobile network operator.

empirical work to revisit the issue of the waterbed effect that had been found in Genakos and Valletti (2011). They found that the effect becomes less significant than the competition effect between mobile network operators because mobile has been growing in importance with greater outgoing-call volume than fixed-line according to a report by Ofcom (2014). They supported that mobile retail price decreases after a reduction in MTR (Genakos and Valletti, 2015) in line with the aforementioned findings in the relevant theoretical work (Armstrong and Wright, 2009; Calzada and Valletti, 2008; Hoernig, 2007; Lopez and Rey, 2009). From a theoretical perspective, in the situation where an entrant enters the mobile market in which an incumbent has already established dominance, the unregulated small network unilaterally increases its MTR under the asymmetric regulation (Baranes and Vuong, 2012; Lee, Lee and Jung, 2010; Peitz, 2005). Empirical evidence from European countries also shows that a smaller mobile network sets higher access charge because its access charge has a less significant effect on the average price and demand with regard to consumer ignorance (Dewenter and Haucap, 2005). However, when small firms can penetrate into the market and the firm asymmetry is less substantial in the mature phase of mobile market, the regulators in many countries, for example, Sweden, Denmark, Poland and Portugal, have adopted the symmetric regulation instead of the asymmetric regulation in expectation of welfare enhancement (Lee, Lee and Jung, 2010)

In addition to the issue of access charge in the firm-asymmetry environment, Mobile Number Portability can reduce switching cost associated with the difficulty of changing telephone numbers. Under this scheme, consumers can change their networks without changing their current mobile numbers (Maicas, Polo and Sese, 2009). This policy supports the entrant to be viable in the market by alleviating the asymmetry between the incumbent and the entrant. Gabrielsen and Vagstad (2008) also studied the effect of calling club, which locks consumers into the same network as their friends and families. When networks are allowed to discriminate price between on-net and off-net calls, the presence of calling club can be interpreted as switching cost, which discourages subscribers from changing their networks.

Some studies further investigated asymmetric networks in different ways. Cambini and Valletti (2003) argued that networks with asymmetry in quality, but identical cost, set above-cost access charge, while socially optimal access charge should be set below cost. They also supported that bill-and-keep should be implemented instead of LRIC, 4 because the bill-and-keep agreement would stimulate investment in quality.

The aforementioned studies investigated asymmetric networks in the context of reputation and incumbency of established firms. In order to emphasise the effects of this aspect of firm asymmetry, those studies generally assume that networks have the same cost structure, i.e. both origination and termination costs are identical among competing firms.

Call externality

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Several studies assume that a caller enjoys utility from making a call but a receiver has no utility from answering the call (Gans and King, 2001; Laffont, Rey and Tirole, 1998a). Under the assumption of call externality, both callers and receivers have utility from making and answering calls respectively. Under networkbased price discrimination, call externality plays an important role in firms' competitive behaviours and consumers' decisions. In the absence of call externality, under networkbased price discrimination and two-part tariffs, it is claimed that networks can make profit from fixed fees and offer both on-net and off-net prices at perceived costs (Cambini and Valletti, 2003; Gans and King, 2001; Laffont, Rey and Tirole, 1998b; Lopez and Rey; 2009; Peitz, 2005). When call externality is taken into account, networks set retail prices below associated costs (Berger, 2005; Hoernig, 2007).

Hoernig (2007) set up a model incorporating call externality and additional utility, which consumers receive only from the incumbent. One example of additional utility is the benefit from the incumbent's reputation. It is found that networks charge their equilibrium on-net prices below their associated costs. Additionally, the large network may offer a higher off-net price than the small firm. Even though the large

 4 LRIC is one of the methodologies which the regulators implement in order to oblige mobile network providers to set their access charges on the grounds of marginal termination costs.

network sets higher off-net price, most consumers still choose its service. This is because the utility from receiving on-net calls within the large network predominates when the volume of on-net calls within the large network is greater than that of offnet. Moreover, the large network's high off-net price can decrease the volume of corresponding off-net calls and then reduce the utility that its rival's subscribers receive from answering these off-net calls. As a result, most consumers consider the rival network less attractive.

A network has incentive to widen the gap between on-net and off-net prices in order to put its rival at a disadvantage. According to call externality, a network may set a low on-net price to attract consumers, whereas it may set a high off-net price to decrease the number of outgoing off-net calls. Subsequently, the subscribers of its rival receive less incoming off-net calls, and the rival network becomes less attractive because of offering lower utility (Hoernig, 2007; Jeon, Laffont and Tirole, 2004). The incumbent with a larger network has an advantage from call externality. It can impose a large on-net/off-net differential and a high off-net price as a barrier to entry (Hoernig, 2007).

In addition to facility-based entry, an entrant may choose service-based entry and other approaches of infrastructure sharing, especially when facility-based entry is not financially feasible. There are other forms of relationships among competitive firms, which are usually asymmetric in some aspects. The next section reviews the literature on various approaches of infrastructure sharing in the telecommunications industry.

1.2.2 Infrastructure sharing

In telecommunications markets, facility-based firms have a great financial burden of investment in their own network facilities, whereas they incur only comparatively negligible marginal cost. An entrant with financial constraints may seek for access network to enter the market as a service-based firm. Moreover, co-investment in infrastructure is also an alternative to conventional investment, especially in new advanced technology in association with incredibly high investment cost.

In the mobile telephony market, there are several approaches of infrastructure sharing from different perspectives of business and engineering. In a business model, firms may agree on infrastructure sharing with contractual obligations. In a geographic model, a firm may share other firms' infrastructures in the regions that it does not directly invest in. Lastly, in a technology model, infrastructure sharing raises several issues of technical engineering (Frisanco, Tafertshofer, Lurin, and Ang, 2008). From a technical viewpoint, mobile network operators may share their facilities at different levels of infrastructure sharing. The intensity of sharing ranges from a small degree of infrastructure sharing (firms co-invest in some common facilities but separately invest in other equipment) to full sharing, roaming and service-based entry by a mobile virtual network operator (MVNO). A higher degree of infrastructure sharing leads to a bigger portion of cost saving that firms are more likely to achieve (Beckman and Smith, 2005; Song, Zo and Lee, 2012). From an economic viewpoint, the consideration of infrastructure sharing is associated with not only cost saving but also market competition, retail price and social welfare as a whole.

Infrastructure sharing in a single-area framework

In the broadband internet market, the trade-off between static efficiency and dynamic efficiency of next generation access network has been a debatable issue for over a decade. An obvious example is an issue of access networks which a facilitybased incumbent provides for a service-based entrant. This can be seen as an extreme level of infrastructure sharing, e.g. local loop unbundling, where the entrant pays an access price as a wholesale price for using the incumbent's access network. From a static perspective, high access price can increase the incumbent's profit in the wholesale market. Moreover, high access price may dampen market competition because the service-based entrant incurs a high wholesale price and its retail price is pushed up accordingly (Bourreau, Cambini and Hoernig, 2015). In contrast, low access price encourages the service-based entrant to enter the retail market, and it then intensifies competition in the short run because the incumbent is regulated to set reasonable

access price. Consequently, access regulation is necessary if the regulator pays more attention to retail market competition from a static viewpoint. Some studies support cost-related access price in an attempt to promote competition and protect consumer welfare.

However, from a dynamic viewpoint, the regulator should consider the tradeoff between the positive effects of access regulation on competition in the short run and the negative effects on investment and innovation in the long run. According to a literature review by Cambini and Jiang (2009), most of the theoretical studies support that cost-oriented access price, which encourages service-based entrants for the purpose of increasing static efficiency, can undermine incentives to investment and innovation. With low access price or cost-based access price, the facility-based incumbent has less incentive to develop and invest in advanced-technology infrastructure. On the other hand, high access price can stimulate the facility-based incumbent to invest more heavily in new technology. Additionally, high access price may influence the entrant's strategic decision on mode of entry. With high access price, the entrant may find it more profitable to invest in its own infrastructure instead of leasing access network from the incumbent. Then, network duplication benefits consumers with a wider variety of services (Bourreau, Cambini and Hoernig, 2015). In summary, access regulation may have a negative effect on incentive to invest in advanced technology (Bourreau, Cambini and Hoernig, 2015; Cambini and Silvestri, 2013; Cambini and Valletti, 2003; Godinho de Matos and Ferreira, 2011; Kotakorpi, 2006; Nitsche and Wiethaus, 2011; Vareda, 2010). From a static view point, the competition in the existing market may be less intense because of high retail price corresponding directly to high access price in the wholesale market (Bourreau, Cambini and Doğan, 2012; Bourreau, Cambini and Hoernig, 2015).

Vereda (2010) further investigated the competition between a facility-based incumbent and a service-based entrant in the broadband market. In the study, the incumbent decides on further investment in quality upgrade and cost reduction. It is commonly found that the incumbent increases a level of quality if it can charge high access price. Conversely, high access price undermines incentive to reduce cost if a process of cost reduction requires a substantial level of further investment.

Nitsche and Wiethaus (2011) studied an incumbent's decision on investment in next generation network (NGN). Their model assumed only one incumbent and one entrant competing à la Cournot. The entrant is only a service-based operator and it has to pay access price to get access to the incumbent's technology. They concluded that LRIC (an approach of cost-based access price) yields the lowest investment and the poorest consumer welfare amongst other access price regimes. The study shows that risk-sharing by co-investment yields the highest consumer welfare amongst other access price regimes, including the fully-distributed-cost approach and the regulatory holiday.

In contrast to the majority of the relevant literature, a few studies argue that high access price can have a negative impact on investment in innovation under certain circumstances. Gayle and Weisman (2007) examined an incumbent's incentive to invest in cost-reducing innovation when an entrant makes a make-or-buy decision to use the incumbent's network under mandatory unbundling. In line with the standard result, they supported that mandatory unbundling, which encourages service-based competition, can undermine the incumbent's incentive to invest. Nevertheless, similar to Vareda (2010), they also pointed out that the incumbent decreases investment in innovation when access price is raised in the access price range that preserves the efficient make-or-buy decision. This is because raising access price leads to an increase in retail price and a corresponding decrease in output, which finally discourages investment in innovation.

The recent empirical evidence also supports that the access price regulation and mandatory local loop unbundling do not stimulate the next generation access network deployment (Bacache, Bourreau, and Gaudin, 2014; Bouckaert, van Dijk and Verboven, 2010; Briglauer, Ecker and Gugler, 2013; Briglauer, Gugler and Haxhimusa, 2015; Crandall, Eisenach, and Ingraham, 2013; Grajek and Röller, 2012). Briglauer, Ecker and Gugler (2013), using data from the EU27 member states for the years from 2005 to 2011, revealed that service-based competition under mandatory local loop unbundling has an adverse impact on the deployment of fibre optic as highspeed broadband internet service. This finding is also confirmed by evidence from OECD countries. Bouckaert, van Dijk and Verboven (2010), using data from 20 OECD countries in the period from December 2003 to March 2008, found that broadband penetration is promoted by inter-platform competition such as cable and wireless, but it is impeded by service-based intra-platform competition. Therefore, it is suggested that the promotion of inter-platform competition is the efficient way to expand broadband penetration. Briglauer, Gugler and Haxhimusa (2015) employed firm-level data of incumbents and entrants from European countries in the period from 2003 to 2012. They provided significant evidence that facility-based competition has a positive effect on investment by both incumbents and entrants. The empirical evidence does not link the NGN deployment to the concept of *stepping stone* or *ladder of investment* (Cave, 2006; Cave and Vogelsang, 2003) that supports servicebased entry in an early phase of liberalisation in expectation of the gradual development of the entrants' infrastructure along the ladder of investment after they can build up their consumer bases and expertise.

The regulators take on the challenge of new regulatory regimes for NGN in the broadband internet market when their consideration is beyond a static framework. Although cost-based access price may dampen investment in NGN from a dynamic perspective, other alternatives still require support from a substantial number of related studies. Deregulation on access price or the regulatory holiday may be suitable under some circumstances; however, it is necessary for the regulators to carefully monitor the competition in NGN (Kirsch and von Hirschhausen, 2008). In the US, deregulation has been implemented in the broadband market for the purpose of facilitating the NGN deployment (Cambini and Jiang, 2009).

Infrastructure sharing with coverage concern

In addition to the competition in one single area, some studies extend their models to firms' decisions on coverage of telecommunications services when firms enter in multiple areas. In the presence of local loop unbundling, access price still plays a crucial role not only in firms' decisions on investment in a single area, but also in the strategic coverage decision. In a single area, there is evidence to support that high access charge stimulates the investment of the facility-based incumbent. In the setting of multi-area competition, an increase in access price also gives the facility-based firms an incentive to invest in the coverage expansion (Bourreau, Cambini and Doğan, 2012; Bourreau, Cambini and Hoernig, 2015).

Valletti, Hoernig and Barros (2002) examined the coverage of two facilitybased networks under different regulatory policies. They focused on only one technology and omitted the issue of investment in service enhancement. This static framework boils down to a two-stage game where the two firms decide on their own coverage and then choose their retail prices later. On the assumption that the incumbent invests in larger coverage, it operates in both monopoly areas and duopoly areas in competition with the entrant. Like Armstrong and Vickers (1993) and Armstrong (2001), the study reported a very low price in duopoly areas but a very high price in monopoly areas. However, when price discrimination across areas is not allowed, the uniform price is set between the high monopoly price and the low duopoly price. The uniform price decreases with the entrant's coverage relative to the incumbent's coverage, i.e. the number of duopoly areas relative to that of monopoly areas. Moreover, a price cap imposed by the regulator also plays a role in the firms' decisions on coverage. If the price cap is strict, the incumbent with monopoly areas is the first to be affected. The incumbent will reduce its strategic coverage in response to the price cap. In addition to the incumbent's reaction, the entrant will also shrink their strategic coverage if the price cap is very strict. Additionally, when the incumbent is regulated under the coverage constraint with uniform pricing, an increase in the incumbent's coverage encourages the entrant to expand its coverage according to strategic complement in coverage. However, an increase in the incumbent's coverage is larger than that of the entrant. Consequently, the incumbent with its extended monopoly areas can raise its uniform price and soften the price competition in duopoly areas. The entrant can increase profit, while the incumbent's total profit decreases due to the coverage obligation. Even though the coverage constraint with uniform pricing causes an increase in price, it brings telecommunications service to greenfield areas. Therefore, consumer welfare varies with different areas. If expanding coverage does not involve a too substantial increase in fixed cost, it seems to be worthwhile to impose the coverage constraint under uniform pricing across multiple areas.

Godinho de Matos and Ferreira (2011) studied the competition among facilitybased providers and virtual providers in the NGN deployment. They pointed out that a high access price reduces the number of virtual providers and forces the entrants to choose to invest in their own facilities instead of leasing access networks. On the other hand, a low access price encourages virtual providers, but it undermines the facility-based providers by reducing incentive to invest and lowering their profits. As suggested by some of the aforementioned studies, access prices should not be too high because they bring about a reduction in social welfare. It is also claimed that deregulated access price (high access price) in a high potential market (a densely populated or low-cost area) makes virtual operators reluctant to enter the market. Accordingly, it may eventually lead to a monopoly in an adjacent market, which has less potential (a sparsely populated or high-cost area).

Bourreau, Cambini and Doğan (2012) captured the competition where firms decide to switch to newer technology while the older technology network still exists. In this model, the incumbent already built the old generation network (OGN) with full coverage in all areas. The study focuses on the new generation network (NGN) roll-out. If the entrant chooses to serve OGN in a particular area, it seeks OGN access from the incumbent on payment of access price. Like the incumbent, if the entrant decides to deploy NGN in a specific area, the entrant has to invest on its own NGN infrastructure. Similar to Bourreau, Cambini and Hoernig (2015), they concluded that the entrant has an incentive to expand its own NGN coverage if access charge on OGN infrastructure increases.

Bourreau, Cambini and Hoernig (2015) extended their model to different geographic areas. In their model, two facility-based incumbents decide upon their investment coverage, whereas one service-based entrant enters the market by seeking access network. There exist two different types of competitive areas; (1) singleinfrastructure areas with only one incumbent rolling out a network, and (2) duplicateinfrastructure areas with both incumbents racing for infrastructure investment. Under the assumption of increasing marginal investment cost in remote areas, firms have a tendency to duplicate network and yield a wider range of service variety in the cheapinvestment areas, but they leave the costly-investment areas with either one network provider or no network roll-out at all. In the wholesale market, firms can set different access prices according to different competition situations in the areas. It is found that coverage in both area types increases with access prices. Like Bourreau, Cambini and Doğan (2012) and Bourreau, Cambini and Hoernig (2013), even though high access price gives firms an incentive to duplicate network and to expand its coverage, it may cause an increase in retail price, dampen competition and finally reduce per-area social welfare. Thus, the regulators should consider optimal access price in compliance with a trade-off between dynamic efficiency (benefit from coverage extension) and static efficiency (service variety and competition enhancement).

They also mentioned that the access price should not be so high that the incumbent corners the market. In addition, high access price may lead to unnecessary duplication of facilities. Further, they suggested that uniform access price among these two types of areas is not the optimal solution. To maximise social welfare, access price in single-infrastructure areas should be lower than that in duplicateinfrastructure areas. Therefore, regulatory intervention is still necessary. However, the regulator can impose these discriminatory access prices only under full commitment. The problem of commitment is one of the regulator's concerns. The regulator may use duplication-based remedies, i.e. only access price in single-infrastructure areas is regulated, but that in duplicate-infrastructure areas is monitored in compliance with the dispute resolution access price. Nevertheless, this partial deregulation policy is claimed to be another possible solution only under the restricted circumstances such as the situation where services are sufficiently differentiated. It is not a universal solution in the presence of partial commitment.

Infrastructure sharing by co-investment

Due to high investment cost, facility-based firms probably encounter financial difficulties in the deployment of advanced technology infrastructure. Co-investment is one funding approach by which firms agree to share infrastructure and financial risks. When firms compete in a single area and they can vary their quality enhancement levels, the investment level under an infrastructure sharing agreement is higher than that with no cooperation. An infrastructure sharing agreement can stimulate investment in quality-enhanced/value-added services (Cambini and Silvestri, 2013; Nitsche and Wiethaus, 2011). Overall, social welfare increases after an infrastructure sharing agreement (Cambini and Silvestri, 2013; Foros, Hansen and Sand, 2002; Nitsche and Wiethaus, 2011). In a setting of multiple areas, it is found that coinvestment enables the deployment in further greenfield areas and expands the total coverage under these circumstances (Bourreau, Cambini and Hoernig, 2013).

Cambini and Silvestri (2013) investigated the effect of investment sharing in the broadband market. They assumed two rival firms competing à la Cournot in a single area. In this model, the incumbent is deciding on rolling out NGN technology. It has three options of investment; (1) investment without any sharing agreement (the incumbent collects cost-based access price from the service-based entrant, who seeks for NGN access), (2) basic investment sharing with the entrant without side payment, and (3) the joint-venture agreement. At a given level of investment, the number of subscribers under the basic investment sharing is higher than those under no sharing agreement and the joint-venture agreement respectively. However, among these options, the joint-venture agreement is the most effective way to stimulate investment. In addition to maximising their joint profit, the joint-venture firms will set the abovecost reciprocal side payment in order to soften competition. Thus, the joint-venture approach is likely to yield larger profit and a higher level of investment than other options. On the other hand, similar to the results reported by Bourreau, Cambini and Hoernig (2015), Cambini and Valletti (2003), Godinho de Matos and Ferreira (2011), Kotakorpi (2006), Nitsche and Wiethaus (2011) and Vareda (2010), no sharing agreement with regulated access price creates the lowest incentive to invest because the incumbent cannot make wholesale profit from cost-based access price. Due to the tacit-collusion effect, the joint-venture agreement yields lower consumer surplus and social welfare than the basic sharing agreement. Moreover, less incentive to invest in the case of no sharing agreement causes consumer surplus and social welfare to be lower than those under the basic sharing agreement. This outcome still occurs when another service-based entrant enters the market late.

The previous studies assume that co-investment has no effect on the total investment cost in a particular area. Bourreau, Cambini and Hoernig (2013) further
extended this assumption and found the interesting linkage between co-investment and the coverage expansion in the situation where co-investment is another feasible approach of investment in multiple areas. In the study, co-investment can reduce the total investment cost in a particular area if the co-investment reduces financial risk by raising funds from co-investors instead of from outside loans. In contrast, coinvestment may require additional equipment or transaction to operate services for multiple network operators. Consequently, co-investment may increase the total investment cost. It is found that co-investment can increase the total coverage if rival services are differentiated and cost reduction from co-investment is sufficiently significant. Additionally, access price is also associated with the coverage in this environment. When the facility-based incumbent increases access price, the servicebased rivals incur higher opportunity cost of leasing access network. The rivals may find it more profitable to co-invest with the facility-based incumbent instead. For this reason, the total coverage can expand. High access price can encourage the incumbent and the co-investors to extend their network coverage. Retail prices are pushed up and the competition is less intense from the static viewpoint. As seen in the aforementioned studies, the regulators must consider this trade-off.

The tension between the static and dynamic perspectives on co-investment is also confirmed by experimental evidence. In the experimental study by Krämer and Vogelsang (2014), it is asserted that co-investment generates cost-reduction in the deployment of network infrastructure and has a tendency to push down retail prices. Communication about co-investment induces coverage expansion. However, tacit price collusion, which jeopardises consumer welfare, may occur in the retail market when firms can negotiate on co-investment at the earlier stage. According to these opposing effects of co-investment on consumer welfare, it is suggested that regulators should carefully consider the implementation of co-investment. This experimental study underlines the concern that co-investment may soften competition, as highlighted in the related theoretical literature.

In the model of Foros, Hansen and Sand (2002) without the coverage issues, two facility-based networks agree to share each other's networks under a roaming agreement in à la Cournot competition. They choose a reciprocal degree of roaming quality to spill over each other under the bilateral agreement. Then, they separately invest in quality enhancement on their own networks. The quality enhancement gives additional utility to representative consumers whose utility levels of basic service are uniformly distributed on a certain interval. These assumptions on the demand side have also been adopted and developed in the related studies such as Brito, Pereira and Vareda (2010), Cambini and Silvestri (2012), Cambini and Silvestri (2013), Foros (2004), Kotakorpi (2006) and Nitsche and Wiethaus (2011). It is supported that the firms should be allowed to collude in the stage of investment decision because the collusion leads to the socially optimum level of quality spillover under the roaming agreement. Conversely, if the collusion is not permitted, firms' decisions will end with an exceptionally higher level of quality spillover than the social optimum.

In addition to a specific telecommunications market, the advent of digital convergence facilitates firms entering multiple telecommunications sectors by offering various types of telecommunications services. However, this multi-service firm requires an enormous amount of investment. A big company with adequate financial support may take advantage of its dominant position by means of offering service bundles. Meanwhile, a small firm that provides only one single service is likely to be threatened in this situation. Asymmetry in scope of products/services plays such a key role in the markets, especially when digital convergence is growing in accordance with consumers' daily life. The existing literature on product bundling is discussed in the next section.

1.2.3 Bundling

Recent literature points out that a multi-product firm may put its singleproduct rivals at a disadvantage through a bundling strategy. However, the market outcomes vary according to different competitive environments.

Bundling in a monopoly framework

When a monopolist invests in several product markets, it can implement bundling as an effective tool of price discrimination. Consumers may have different valuations of goods. For example, a consumer's reservation price of the first good is high but that of the second good is low. In contrast, another consumer may have a low valuation of the first good but a high valuation of the second good. This is evidence of a negative correlation of valuations. In this case, the monopolist may offer product bundles in order to increase profit (Adams and Yellen, 1976). The consumers' valuations of the whole bundle become subtly different. Bundling can encourage consumers to be more homogeneous. This strategy acts as price discrimination because the monopolist can extract consumers' rent through the sorting effect of bundling. Conversely, consumer welfare decreases due to distortion of the allocation of goods after bundling.

Bundling in a symmetric duopoly framework

The effect of bundling in a duopoly framework differs from that in the monopoly setting. Reisinger (2006) set up a model of two firms that compete in two duopolistic markets based on the Salop model of circular spatial competition. Each consumer is assumed to buy only one unit of product from each market. The key variable is consumers' valuations of the two products. Under the negative correlation of valuations, the firms find that bundling decreases profit. However, in the absence of collusion, both firms still choose to offer bundles despite a reduction in profit. This outcome is similar to a prisoner's dilemma. In this environment, some consumers prefer the first product from one firm but prefer the second product from the other firm. Therefore, bundling can persuade these consumers to be more homogeneous, and the competition becomes more intense. The business-stealing effect outweighs the sorting effect, so the firms' profits are lower than those in the case of no bundling. Despite a bundle discount, the prices of individual products increase after bundling. On the other hand, under the positive correlation of valuations, firms can increase profit by bundling. In this case, each consumer prefers to buy a pair of products from the same firm. Each firm has monopoly power over its customer base. As a result, the firms can set higher prices than those under the negative correlation. Bundling brings about the sorting effect which dominates the business-stealing effect. Thus, the firms can increase their profits because they can extract consumers' rent by employing the bundling strategy.

Following Reisinger (2006), Granier and Podesta (2010) extended their model to capture an issue of mergers and bundling. Unlike Reisinger (2006), they allowed firms to decide whether or not to merge and which firm to be merged. They found that in the circular model, each firm finally choose to merge with a homogeneous firm at the same location if the correlation of reservation prices is positive. On the other hand, each firm merges with a heterogeneous firm at the opposite location if the correlation is negative. In other words, the decision about merging depends on the correlation of reservation prices. After the merger, the firms can offer bundles and increase profits without a prisoner's dilemma because they can choose which type of firms to merge with. In this situation, the sorting effect dominates the business-stealing effect (competition effect). The merger occurs when the merging firms are allowed to offer bundles. If they cannot employ the bundling strategy, there is no incentive to merge.

Rennhoff and Serfes (2009) also set up their model based on the Salop model. They focused on the upstream-downstream competition in the cable TV industry. Upstream firms are content providers and downstream firms are system operators. Consumer preference for content variety is set as a parameter. This parameter acts as incremental utility from bundle consumption, which affects every consumer in the same way. It is concluded that all downstream firms will offer mixed bundles (pure bundles) if consumers have moderately (extremely) strong preference for content variety. This is because the bundling strategy can raise profit while a prisoner's dilemma does not occur.

In a framework of symmetric firms, merging firms have an incentive to offer a bundle discount (Granier and Podesta, 2010; Reisinger, 2006; Rennhoff and Serfes, 2009; Thanassoulis, 2011). However, consumer surplus and social welfare may be threatened by the bundling strategy. After bundling, some consumers are persuaded to buy bundles containing some products they do not prefer. Moreover, individual prices are likely to be pushed up despite a substantial discount on bundles (Reisinger, 2006). It is found that this distributive inefficiency may finally decrease consumer surplus (Gans and King, 2006; Granier and Podesta, 2010; Reisinger, 2006; Rennhoff and Serfes, 2009). For this reason, some studies support forbidding bundling (Granier and Podesta, 2010; Rennhoff and Serfes, 2009).

Bundling in an asymmetric duopoly framework

Bundling is considered to be an instrument of entry deterrence. Nalebuff (2004) investigated the competition between an incumbent and an entrant. The incumbent operates in two product markets, whereas the entrant decides to enter one of the two markets. Thus, the incumbent still has a monopoly in the other market. If the incumbent cannot deter entry in the market, it will go into duopolistic competition. Bundling is a credible strategy for the incumbent. This is because the incumbent earns higher profit from bundling, regardless of the entry decision of the entrant. In other words, bundling can transfer market power in the monopolistic market to the competitive market (the leverage theory). The bundling strategy significantly diminishes the entrant's profit. In some situations, it can deter entry. By contrast with limit-pricing, the bundling strategy becomes more acceptable because it does not require the incumbent to sacrifice its profit for entry deterrence.

Gans and King (2006) examined product bundling in two duopolistic markets. Each consumer purchases one unit of each product. Based on the Hotelling model, the consumers are assumed to be uniformly distributed in a unit square. To increase their own profits, the allied independent firms in different markets have an incentive to make an agreement on a bundle discount. By contrast, their rivals, which separately offer their products, lose profits. However, their rivals will copy the bundling strategy, and then all firms will finally offer bundle discounts in equilibrium. The firms' profits will be pushed down to the same level as the outcome of no bundling. Nevertheless, social welfare is lower than that in the no-bundling case, because bundling distorts the allocation of consumers by persuading consumers not to buy their preferred products.

Gans and King (2006) also extended their model to the case of horizontal integration. An integrated firm aims to maximise their joint profit, while its rivals in both markets still operate independently. After offering bundles, the integrated firm can expand its market share and earn higher profit than its independent rivals. In response to the integrated firm's bundling strategy, the independent firms will not offer bundles by means of alliance because this may intensify the competition and finally decrease their own profits. Even though the individual prices and bundle discount in this integration model are lower than those under unilateral bundling by the allied independent firms in the basic model, social welfare is still threatened by bundling. The market share of the integrated firm expands, and the allocation of consumers is more inefficient. If all firms are eligible to merge in this integration game, they all decide to merge but not to bundle their products. This is because bundling will trigger more aggressive competition between the two integrated firms and then all the prices and profits finally decrease. As a result, there is no incentive for the merging firms to bundle their products. Therefore, social welfare in the case of the allied independent firms is lower than that in the case of bilateral integration due to the distributive inefficiency of the bundling outcome.

Thanassoulis (2007) introduced single-product consumers in addition to multiproduct consumers. The number of consumers in each group is fixed. In this model of two duopolistic markets, the Hotelling model is applied to the uniformly distributed consumers in the single-product groups in the form of unit lines and in the multiproduct group in the form of a unit square. With firm-specific preferences, the multiproduct consumers enjoy the benefit of economies of scope, i.e. saving shopping cost from one-stop shopping. Thus, the multi-product consumers have more elastic demand than the single-product consumers. In equilibrium, both firms offer bundles in order to reap higher profit. The individual prices are pushed up but the bundle price is set lower than the sum of the individual prices in the no-bundling case. However, bundling decreases consumer surplus. The multi-product consumers may benefit from a bundle discount, but they cannot choose products from different firms. Conversely, with product-specific preferences, all combinations of products are available to the multi-product consumers. In this case, a prisoner's dilemma occurs. The profits of the two firms decrease after bundling. The individual prices and the bundle price are lower than those in the no-bundling situation. Consequently, consumer surplus increases.

Thanassoulis (2011) further assumed that one market is more intense because the products in this market are less differentiated than the other market in a merger game with firm-specific preferences. Under partial convergence, the merging firm can reap higher profit without offering a bundle discount. The single-product consumers in the more competitive market are worse off because of an increase in price, while those in the less competitive market are better off due to a price reduction. Thus, the multi-product consumers experience an increase in price. The consumer surplus of the multi-product consumers is still ambiguous. It can either increase or decrease depending on the competitive natures of the two markets. However, under full convergence, both merging firms definitely offer bundle discounts to increase profits. A multi-product consumer can reduce taste cost due to economies of scope from whichever bundle he chooses. Therefore, the bundles are less differentiated because taste cost (transportation cost in the Hotelling model) decreases. As a result, the merging firms have to offer bundle discounts in this stronger market.

It is also found that under full convergence, the two merging firms compete for the multi-product consumers and the single-product consumers separately. The individual prices are equal to the no-convergence benchmark. Compared to the noconvergence situation, the single-product consumers gain nothing from the full convergence, whereas the multi-product consumers are better off. Thus, aggregate consumer surplus increases after the full convergence. Nevertheless, a merging firm's profit in the partial convergence is higher than that in the full convergence. Consequently, in the two-stage merger game, partial convergence is the pure strategy outcome. A pair of firms that merge first will take a better position. If its rivals have already merged, the firms will remain independent in order to mitigate the negative effect on their own profits. This result contrasts with the merger-wave outcome shown by Granier and Podesta (2010) and Reisinger (2006), and it deviates from the nobundling outcome under bilateral integration supported by Gans and King (2006).

In a framework of firm asymmetry, a firm which can unilaterally offer a bundle is more likely to achieve a dominant position. A firm can boost its profit by employing a bundling strategy. If its rivals cannot implement the bundling strategy in response, they may lose huge profit or become on the verge of exit (Gans and King, 2006; Nalebuff, 2004; Thanassoulis, 2011). To survive in the market, independent firms may decide to lower their individual prices (Gans and King, 2006; Thanassoulis, 2011). However, there is some concern over distributive inefficiency as an adverse effect of bundling, which undermines consumer welfare (Gans and King, 2006; Thanassoulis, 2007; Thanassoulis, 2011).

Bundling in various types of product markets

A bundling strategy is a profit-enhancing tool for multi-product firms not only in the markets of independent and complementary products, but also in the markets of substitute products. After a consumer buys a substitute good, his incremental utility from consuming another substitute good inevitably decreases. However, firms may offer a discount on the second product (as a bundle) to extract consumers' rent.

Venkatesh and Kamakura (2003) examined bundling in a monopoly setting of independent, complementary and substitutable products. The study incorporates a consumer's degree of contingency, which increases (decreases) the sum of stand-alone reservation prices for the two products when they are complements (substitutes). The degree of contingency is constant across all consumers. Under the assumption of relatively low cost, the monopolist chooses the mixed-bundling strategy when the products are independent, weak substitutes or weak complements. Conversely, if the products are strong substitutes, it chooses not to bundle its products.

In addition, Armstrong (2011) also showed that the competitive firms with substitute goods cooperate in offering a bundle discount rather than compete in price in order to extract consumer surplus. Bundling can act as an instrument of collusion. The individual price under this cooperation is higher than that under the nocooperation benchmark, while bundle price is lower due to bundle discount.

Bundling and R&D

Choi (2004) introduced R&D in cost reduction in a bundling framework based on the Hotelling model. Firms compete in price and they also invest in cost reduction. It is found that a multi-product firm with the pure-bundling strategy invest more heavily in R&D in cost reduction, whereas a single-product firm reduces investment in cost reduction. In this framework, social welfare decreases. Heeb (2003) also supported that the integrated firm, which can offer bundles, increases the level of innovation but the single-product firm reduces investment in innovation.

Bundling and vertical differentiation

The aforementioned literature focuses on horizontal differentiation, where consumers have different taste costs associated with the difference between their most preferred products and the products available in the market. In the setting of vertical differentiation, consumers with varying degrees of quality concern have different willingness to pay. Basically, in a vertical differentiation model, firms try to maximise their differentiation in order to avoid intense competition. The firms will choose different levels of product quality. Generally speaking, the firms target different groups of consumers. The high-quality firm attracts consumers who prefer high quality. Meanwhile, the low-quality firm tempts consumers whose valuation of low quality is subtly different from that of high quality (Tirole, 1988; Wauthy, 1996).

Krӓmer (2009) incorporated quality decision in a model of the competition between a multi-product firm and a single-product firm. The multi-product firm has market power in one monopolistic market. It has just entered in a duopolistic market in which the single-product firm is an incumbent. According to vertical differentiation, firms differentiate themselves by choosing different levels of quality to serve different groups of consumers. In the absence of bundling, as an incumbent, the single-product firm chooses to first offer high quality because the high-quality firm can reap higher profit than the low-quality firm. However, the multi-product firm can transfer its market power from the monopolistic market to the duopolistic market by means of pure bundling. After bundling, the multi-product firm chooses to offer the highquality product. Instead, the single-product rival is forced to offer the low-quality product in compliance with product differentiation, and as a consequence it earns lower profit than the multi-product firm.

Avenali, D'Annunzio and Reverberi (2013) investigated the effects of bundling by a multi-product firm, which offers a monopoly component to be consumed together with another component served in the duopolistic market. Similar to Krämer (2009), it is argued that bundling is the multi-product firm's dominant strategy that reduces the single-product firm's incentive to invest in quality. The bundling strategy forces the single-product firm to reconsider about the efficiency of its quality investment in terms of sunk cost of investment and consumers' incremental willingness to pay for quality. As a result, bundling can prevent undesirable investment by the single-product firm, and it has a tendency to improve social welfare in spite of a relatively small decrease in consumer surplus and the single-product firm's profit under certain circumstances. Therefore, a price test to monitor a distortion of monopoly component price after bundling should be implemented instead of a ban on bundling.

1.3 Structure of the thesis

In the business world, dominant firms have a tendency to take full advantage of their dominance through their predatory behaviour, whereas cooperation among competing firms may lead to negotiation and collusion. As mentioned in the previous sections, the majority of related literature captures the competition in the setting of symmetric firms. Despite firm asymmetry's crucial influence on equilibrium market outcomes, a few existing studies emphasise this issue. Therefore, this thesis sheds light on asymmetry among firms in different vulnerable situations.

Chapter 2, Access Charge Regulations with Asymmetric Mobile Network Operators, suggests the optimal access charge when facility-based firms with asymmetry in cost and reputation agree to interconnect each other in order to complete off-net calls. In the unregulated market, the low-cost firm is more likely to be dominant with higher access charge when its reputation is not too worse than the high-cost firm. The symmetric cost-based access charge regulation can eliminate the firms' abilities to forcibly increase their rivals' retail prices through setting high access mark-ups. Meanwhile, the asymmetric cost-based access charge regulation facilitates the entrant's dominance. Thus, the asymmetric regulation may be more appropriate than the symmetric regulation in order to encourage the underdog entrant to enter the market and promote competition and social welfare on condition that the cost differential and the discrepancy in reputation are not too substantial and the two networks are differentiated enough.

Chapter 3, Infrastructure Sharing in Telecommunications, investigates the effects of various approaches of infrastructure sharing, especially co-investment, on incentives to upgrade quality and consumer welfare. Under stand-alone investment, the low-cost firm is dominant with a higher level of quality upgrade than the high-cost firm, due to its cost saving. Co-investment can be employed to enhance both firms' profits, even though the high-cost firm with lower bargaining power agrees to invest in a larger proportion of total investment and has lower profit than the low-cost firm. Compared to stand-alone investment, co-investment undermines incentive for quality upgrade, decreases industry output and threatens consumer welfare when infrastructure sharing does not yield a sufficient amount of cost saving. From the high-cost firm's static perspective, access to the low-cost incumbent's infrastructure under the fully-distributed-cost regulation is the most profitable mode of entry because of an equal burden of total investment cost. The low-cost firm may also prefer the fully-distributed-cost regulation to co-investment if the difference in cost structures of both firms is negligible. Compared to co-investment, this approach can stimulate quality upgrade but lower output levels and consumer surplus. The regulator may support co-investment, especially when infrastructure sharing can yield the sufficiently great benefit of cost reduction. Otherwise, the welfare-dampening collusion in quality upgrade is likely to occur.

From a broader perspective, Chapter 4, Bundling and Incentives for Quality Enhancement, examines the competition in multiple product markets where one multi-product firm can employ different types of bundling, which can adversely affect its single-product rivals. In contrast to the symmetric market outcomes in the nobundling case, both pure bundling and mixed bundling, which increase the multiproduct firm's profit but decrease those of the single-product firms in most situations, lead to the asymmetric market outcomes. Investment in quality enhancement made by the multi-product firm can be stimulated by pure bundling when the associated costs are comparatively low and the additional utility from quality enhancement is relatively high, and similarly by mixed bundling in the more competitive market. Meanwhile, the single-product firms react by decreasing their quality enhancement levels to focus mainly on saving costs instead. This outcome is likely to be found in the less competitive market with mixed bundling when the two markets are not too different in competition intensity. Therefore, both bundling strategies may diminish consumer welfare when the two markets are significantly different in competition intensity because of the adverse impacts of the market distortions after tying the two markets. However, in some situations, the regulators should take into account the positive impacts of bundling on boosting quality enhancement.

Finally, Chapter 5 concludes the significant results and policy implications in the presence of various types of firm asymmetry. Additionally, there is a discussion of limitations of this thesis and suggestions for further research.

Chapter 2

Access Charge Regulations with Asymmetric Mobile Network Operators

2.1 Introduction

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In the mobile market, incumbents and entrants may have different cost structures due to differences in experience, scope of services, frequency spectrum and/or technology deployment. An entrant is more likely to be dominated due to late entry. This is a major aspect of firm asymmetry, which becomes a debatable issue about antitrust and predatory practices by dominant firms. Accordingly, there has been intense debate about whether asymmetric networks should be treated in different way (NBTC, 2012) in order to remove the inequalities caused by exogenous factors (Goral and Karacaer, 2011). The new firm may incur higher cost, and it may require support from the regulator in order to achieve market penetration and become financially viable. To encourage entry and promote competition especially in the short run, the regulators may impose *the asymmetric regulation on access charge*. 5 This

⁵ The asymmetric regulation on access charge has been considered to be acceptable in an immature market in many countries such as some European countries, South Korea, Taiwan and Thailand. Under this regulation, an incumbent with significant market power is regulated to set cost-based access charge while an entrant with small market share should set reasonable access charge. This regulation may promote entry of small networks and subsequently intensify competition.

problem is less complicated when firm asymmetry becomes subtle after a transition period of asymmetry. Therefore, the US and most of the European countries have implemented a glide path toward the symmetric regulation on access charge (Goral and Karacaer, 2011; Lee, Lee and Jung, 2010; Ofcom, 2011). The asymmetric regulation has been abandoned and the symmetric regulation is imposed instead, for example, in Sweden, Denmark, Poland, and Portugal (Lee, Lee, and Jung, 2010). However, in some countries, especially in developing countries, there is substantial evidence about network asymmetry, which still plays a significant role in market concentration.⁶

Most of the existing literature, however, focuses only on symmetric networks or asymmetric networks from a conventional viewpoint on firm asymmetry. The present study compares the effects of the asymmetric regulation on social welfare in the competition between two mobile network operators with asymmetry in both cost structure and reputation. In addition to the incumbent's superior reputation as assumed by Carter and Wright (2003), from consumers' perspective, the incumbent may be as good as or inferior to the entrant in terms of reputation, especially when technology has changed rapidly and legacy networks give way to newcomers. In this model, both firms offer their services under network-based price discrimination and two-part tariffs. Social welfare, consumer surplus and producer surplus are closely examined under three different regulatory policies; (1) no regulation, (2) the symmetric cost-based access charge regulation, and (3) the asymmetric cost-based access charge

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⁶ For example, in Thailand, a mobile entrant sent Thai telecommunications regulator (NBTC) a petition about the situation where the entrant has been charged unfair interconnection charge from an incumbent. The entrant provided some evidence supporting that the incumbent with larger network had lower marginal cost than the entrant. It is evident that when the incumbent provides a wider variety of services than the entrant, its joint and common cost allocated to the voice service sector is lower than that of the entrant, which has smaller market share (NBTC, 2012).

In South Korea, there has been the imposition of asymmetric regulations such as a retail price cap for the incumbent, asymmetric prices of access services and an implementation schedule for number portability in favour of entrants (Lee, Lee and Jung, 2010).

regulation. The asymmetry in cost leads to the asymmetric market outcomes. In contrast to on-net prices, off-net prices diverge from marginal cost pricing because firms charge access mark-ups. The regulations on access charge are still necessary to eliminate the distortion in order to promote consumer surplus in this firm-asymmetry setting. However, the regulator should consider these regulations very carefully when asymmetric networks are significantly different in efficiency. It may be reasonable to implement the asymmetric regulation instead of the symmetric regulation when the cost differential, the discrepancy in reputation and/or the degree of substitutability between the networks are not too large. Otherwise, the asymmetric regulation threatens consumer welfare and the symmetric regulation is more appropriate in the circumstances.

Related literature

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The telecommunications regulators in many countries monitor networks' behaviours and impose some regulations on access charge in order to discourage the large network's predatory behaviour regarding access charge and enhance competition in the mobile market.⁷ A majority of the related literature supported access charge regulations on two-way access between facility-based firms. In the symmetric setting, mobile networks may abuse reciprocal access charge to soften the competition as an instrument of tacit collusion (Armstrong, 1998; Laffont, Rey and Tirole, 1998a). When they agree on an access mark-up, the retail prices are kept high to cover the access charge and they will not trigger price war. Regarding a linear pricing scheme, it is shown that retail prices are high because of a mark-up on access charge (Laffont, Rey and Tirole, 1998a). Similarly, under two-part tariffs, networks have profits from fixed fees and set retail prices at weighted-average associated cost

 7 For example, the UK telecommunications regulator (Ofcom) has applied Long Run Incremental Cost (LRIC) to policy on access charge. LRIC is one of the approaches on the basis of costbased access charge. Ofcom (2011) has announced proposed mobile termination rates (MTRs) for the four large mobile networks, including EE, H3G, Telefonica and Vodafone, and it has allowed other designated mobile communications providers to set access charges on the basis of being fair and reasonable.

of on-net marginal cost and off-net perceived cost (Carter and Wright, 2003; Laffont, Rey and Tirole, 1998a; Lopez and Rey, 2009). In addition, off-net price is still higher than actual cost even under network-based price discrimination with a linear pricing scheme because networks still charge access mark-ups (Laffont, Rey and Tirole, 1998b). These results indicate that double marginalisation occurs and social welfare may be threatened. However, regardless of call externality, networks reap profits from fixed fees and set on-net and off-net prices equal to their perceived costs under twopart tariffs and network-based price discrimination. Under certain circumstances, networks might set access charge at cost to maximise their profit (Laffont, Rey and Tirole, 1998b). Moreover, Calzada and Valletti (2008) and Gans and King (2001) argued that networks might agree to set below-cost access charge to soften competition and then social welfare might decrease.

Most relevant literature also mentions that networks may choose above-cost access charge to soften the competition or undermine their rivals when the regulator does not intervene. This behaviour is a threat to social welfare. Even though some studies support below-cost access charge or Bill and Keep (Cambini and Valletti, 2003) to enhance social welfare, most regulators still choose cost-based access charge regulations. However, in the presence of firm asymmetry, the asymmetric regulation on access charge, especially in the immature market, is very important for the regulators to consider. This is because there is some concern that a large network with dominant market power has a tendency to put its rivals at a disadvantage or foreclose the market by setting high access charge and widening the gap between on-net and off-net prices (Hoernig, 2007; Lopez and Rey, 2009) in line with the findings revealed in this study.

Carter and Wright (2003) introduced competition between asymmetric networks in terms of brand loyalty, but the networks still have the same cost structure under uniform pricing. Consumers receive extra benefit only from the incumbent because of brand loyalty. As a result, the incumbent and the entrant are asymmetric from consumers' perspective. They pointed out that the larger network prefers costbased reciprocal access charge. Similar to Carter and Wright (2003), Peitz (2005) investigated asymmetric networks under network-based price discrimination with two-part tariffs. He concluded that the asymmetric regulation on access charge is more appropriate than the symmetric regulation because it boosts consumer surplus and the entrant's profit. Stühmeier (2013) also supported the standard result that the asymmetric regulation in favour of the entrant can promote the entrant's profit in the setting of cost asymmetry. However, it is found that there is no effect on equilibrium market shares as a result of network-based discriminatory pricing. Baranes and Vuong (2012) pointed out that the asymmetric regulation can intensify market competition and may enhance social welfare in some situations. However, Lee, Lee and Jung (2010) argued that a decrease in the asymmetry of access charges can enhance consumer surplus when the entrant's cost is low and the degree of substitutability between services is high.

In the recent empirical literature, retail prices significantly decline after access charges decrease under the cost-based access charge regulation. Nevertheless, there is no evidence to support the adverse effect of the regulation on mobile network operators' profits (Genakos and Valletti, 2015). Additionally, Dewenter and Haucap (2005) employed data from some European mobile network operators to confirm that the small networks have incentive to increase their access charges under the asymmetric regulation. This empirical evidence supports the result of the present study along with the relevant theoretical literature. However, the conclusions about the asymmetric regulation may vary according to different concerns and perspectives.

In the next section, the model of asymmetric networks under two-part tariffs and network-based discriminatory pricing is illustrated. The different cost structures and the parameter of asymmetry in reputation are introduced. In Section 2.3, the market outcomes and the effects of three regulatory policies on the equilibrium outcomes are discussed. Policy implications are also mentioned. Finally, Section 2.4 concludes the findings and suggestions for the regulators.

2.2 Model

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There are only two mobile network operators providing one type of service ⁸ with the same quality in the market. Network 1 has lower cost than network 2. From a static point of view, both networks have already invested in network facilities and entered the price competition. A consumer can subscribe to only one network. According to Mobile Number Portability, it is assumed that subscribers can change their mobile networks without switching cost. Consequently, there is no locked-in consumer. The networks compete for subscribers who are uniformly differentiated based on the standard Hotelling model. Consumers are located on the interval [0,1]. Both networks are available with full coverage, and they situated at both ends of the Hotelling line. Network 1 is at point 0 and network 2 is at point 1. Consumers have different preferences. A consumer's preference for a particular network can be interpreted as the distance between the consumer's location and the network's location. The closer the consumer and the network are located, the stronger preference for the network the consumer has. This is because the consumer has disutility known as transportation cost in the Hotelling model. The disutility corresponds to the discrepancy between the consumer's ideal network and the network which is available for him to join. In the model of two mobile networks, t represents disutility per unit of distance on the interval [0,1]. The disutility is assumed to be a linear function of distance.

According to the assumption of full coverage, following Laffont, Rey and Tirole (1998a), fixed surplus of connecting to network $i(\mu_i)$ presumably outweighs the disutility from not connecting to an ideal network.⁹ As a result, no one rejects joining a network. μ_i is constant and derived from network i 's reputation related to its popularity, reliability of service or brand image. The network with a better

⁸ This study can be applied to voice service, texts and other types of service.

⁹ Laffont, Rey and Tirole (1998a) assumed that in the setting of symmetric networks, both networks offer consumers equal fixed surplus from being connected to either network. However, in this study, asymmetric networks provide different levels of fixed surplus.

reputation gives a higher level of fixed surplus to consumers. β is the parameter of asymmetry in reputation, which represents the difference between the fixed surplus levels of the two networks. $\beta = \mu_1 - \mu_2$. In addition to the model of Carter and Wright (2003), which only assumed $\beta > 0$, this study adopts β which can be positive, negative or zero. $\beta > 0$ if network 1 (the low-cost firm) provides larger fixed surplus than network 2 (the high-cost firm), e.g. network 1 may be well-known or more reliable than network 2, whereas β < 0 if network 1 offers smaller fixed surplus than network 2 (network 2 earns a better reputation than network 1). Lastly, $\beta = 0$ if the two networks offer the same amount of fixed surplus to consumers. Therefore, consumers perceive that the two networks are symmetric in reputation.¹⁰

In the absence of switching cost, consumers can alternate between the two networks unconditionally. A consumer will choose the network which offers him higher net utility. A subscriber choosing network *i* will obtain net utility (w_i) . Both networks compete in two-part tariffs. Network i offers on-net price (p_i) , off-net price (\hat{p}_i) and fixed fee (F_i) . After an agreement on interconnection, each network completes its incoming off-net calls and accordingly collects access charge from its rival. Network *i* sets access charge (a_i) unilaterally.

Due to the balanced calling pattern assumption, every consumer has the same probability to be called. Thus, the number of receivers who are in the same network as a caller and that in the opposite network are indicated by market shares. α_1 is the market share of network 1; accordingly, $\alpha_2 = 1 - \alpha_1$ is the market share of network 2. Utility of receiving calls is dropped out to simplify the model. A consumer's net utility (w_i) is detailed below.

 $w_i(p_i, \hat{p}_i, F_i)$ = net surplus of making on-net calls + net surplus of making off-net calls – *fixed fee*

¹⁰ In this study, the networks' reputations are exogenous and assumed to be constant. However, from a dynamic perspective, reputation may depend on endogenous variables such as market shares, service quality and networks' strategies. For this reason, one may enable networks to vary their reputation levels in a model.

$$
w_1(p_1, \hat{p}_1, F_1) = \alpha_1 v(p_1) + \alpha_2 v(\hat{p}_1) - F_1
$$
\n(2.1)

$$
w_2(p_2, \hat{p}_2, F_2) = \alpha_2 v(p_2) + \alpha_1 v(\hat{p}_2) - F_2
$$
\n(2.2)

 $v(p)$ is a consumer's net surplus from making a call as given below.

$$
\nu(p) = \max_{q} \{U(q) - pq\} = \frac{1}{2} - p + \frac{p^2}{2}
$$
 (2.3)

p is usage price. $p \in \{p_1, p_2, \hat{p}_1, \hat{p}_2\}$. Consumers have the identical demand function. However, each subscriber may generate different traffic flow of calls because he experiences different levels of retail prices according to the network he subscribes to. In this model, without the assumption of call externality, a caller has to pay for making a call (Calling Party Pays) and a receiver pays no charge to answer that call. A consumer's gross surplus of making a call is given by $U(q) = q - \frac{1}{3}$ $\frac{1}{2}q^2$, where q is call duration in terms of minutes, corresponding to associated usage price (p) ; $q \in \{q_1, q_2, \hat{q}_1, \hat{q}_2\}.$ ¹¹

In equilibrium, a marginal consumer who is indifferent to both networks obtains the same utility from joining network 1 as network 2. This utility consists of the net utility (w_i), fixed surplus from reputation (μ_i) and disutility from not connecting to his ideal network. The marginal consumer is situated at point α_1 on the interval [0,1]. The position of the marginal consumer indicates the market share of network 1. Thus,

$$
U'(q) = p
$$

$$
1 - q = p
$$

¹¹ For simplicity, every subscriber is assumed to have an identical utility function of making a call. As a result, demand function of making a call is shown below.

Consequently, the demand function is linear; $q(p) = 1 - p$ where $p \in \{p_1, p_2, \hat{p}_1, \hat{p}_2\}$ and $q \in \{q_1, q_2, \hat{q}_1, \hat{q}_2\}$ respectively.

However, some studies assumed other forms of utility function and demand function. For example, Hoernig (2007) and Laffont, Rey and Tirole (1998a, 1998b) employed a constant elasticity demand function.

from the marginal consumer's viewpoint, the utility from network 1 is equal to that from network 2 as shown below.

$$
w_1 + \mu_1 - t\alpha_1 = w_2 + \mu_2 - t(1 - \alpha_1)
$$

$$
w_1 + \beta - t\alpha_1 = w_2 - t(1 - \alpha_1)
$$

 $\sigma \equiv \frac{1}{2}$ $\frac{1}{2t}$ is the degree of substitutability between the two networks.¹²

By substituting $t = \frac{1}{2}$ $\frac{1}{2\sigma}$ in the above equation and rearranging it, market share of network 1 is

$$
\alpha_1(p_1, \hat{p}_1, p_2, \hat{p}_2, F_1, F_2) = \frac{1}{2} + \sigma \beta + \sigma [w_1(p_1, \hat{p}_1, F_1) - w_2(p_2, \hat{p}_2, F_2)] \tag{2.4}
$$

The two networks have different cost structures. This asymmetry may result from sequential entry, asymmetric allocation in frequency spectrum resources, differences in frequency bands and/or discrepancies in technology deployment, which directly affect marginal costs. It is assumed that the networks incur different marginal costs of originating and terminating a call, but they have the same cost of connecting a subscriber. f_i is the cost from connecting a subscriber such as network connection cost, billing cost and administration cost to deal with a customer as lump sum of firm *i*; *i* \in {1,2}. For simplicity, assume $f_1 = f_2$.¹³ To deliver a call, the relevant networks provide call origination, call transit and call termination. Call transit occurs when signals or data is transferred from the originating facilities to the terminating facilities. Firm *i* has marginal cost of originating a call (c_{0i}) and marginal cost of terminating a

¹² Following Laffont, Rey and Tirole (1998a), the degree of substitutability (σ) is positive. If σ approaches zero, the two networks are extremely differentiated. Conversely, if σ is high, the two networks are closely substitutable.

 13 The purpose of this study is to investigate on-net prices and off-net prices offered by the two networks which have different cost structures. This model assumes that the networks have asymmetric origination and termination costs because these costs play an important role in price setting. The cost of connecting a subscriber may vary among competing firms according to administration and management, which are not the main focus of this study. For simplicity, both networks' fixed costs of connecting a subscriber (f_i) are assumed to be identical.

call (c_{ti}) in terms of per-minute expense; $i \in \{1,2\}$. Cost of call transit is assumed to be zero for simplicity. Therefore, total marginal cost for an on-net call is $c_{0i} + c_{ti}$. It is assumed that marginal origination cost and marginal termination cost of one network are identical because they involve the same facilities to originate an outgoing call and terminate an incoming call.¹⁴ Moreover, fixed cost such as joint cost and common cost on facilities is assumed to be zero.

In this study, it is assumed that the two networks do not enter the market at the same time. The new firm enters the market when the established firm has already operated in the market. The established firm may have experience or economies of scale in the monopolistic period, and accordingly its marginal cost is lower than that of the new firm. In contrast, it is also possible that the established firm has higher cost. The new firm may invest in cost-reduction technology or have the advantages of frequency spectrum allocation, while the established firm still sticks to the sunk cost of its old facilities and cannot upgrade its facilities immediately. As a result, the new firm may incur lower cost than the established firm.¹⁵ In this model, origination and termination costs of network 1 are assumed to be lower than network 2. Origination cost (c_{0i}) and termination cost (c_{ti}) can be compared in the following way.

$$
c_{0i} = c_{ti} = c_i
$$
; $i \in \{1,2\}$
 $c_1 < c_2$ and $f_1 = f_2$

In order to assess the asymmetric access charge regulation which tends to be implemented to stimulate market entry in the situation where the new firm has the disadvantage of its higher cost, this study focuses mainly on the case that the established firm incurs lower cost.

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¹⁴ For instance, some Thai mobile network operators revealed that their origination cost and termination cost are equal because the technical process of call origination and termination normally deploys the same network facilities (NBTC, 2012).

 15 An example of the different technologies is 3G technology which is more advanced and expected to yield lower marginal cost than 2G technology (Ofcom, 2011).

The structure of two-part tariffs (post-paid scheme)

Both networks impose two-part tariffs (post-paid pricing scheme) under network-based price discrimination. A subscriber first spends on fixed fee (F_i) to connect to a network. Then, he pays usage fees (p_i, \hat{p}_i) when he makes calls.¹⁶ Profit of network is comprised of the revenue from providing service to its subscribers and the access revenue. The revenue collected from its subscribers consists of profit from providing on-net calls, originating off-net calls, and charging fixed fee. In addition, network *i* collects per-minute access charge (a_i) from network *j* when it terminates incoming calls originated by network *i*; $i, j \in \{1,2\}$ and $i \neq j$. Profit function can be written as

$$
\pi_i = \alpha_i [\alpha_i (p_i q_i - 2c_i q_i) + \alpha_j (\hat{p}_i \hat{q}_i - (c_i + a_j) \hat{q}_i) + F_i - f_i]
$$

+
$$
\alpha_j [\alpha_i (a_i - c_i) \hat{q}_j].
$$
\n(2.5)

Timing of the game

l

To find the equilibrium outcome, a two-stage game is set up and solved by backward induction.¹⁷

Stage 1 The two networks agree on interconnection and set their own access charges simultaneously.

Stage 2 The networks compete in price competition by setting fixed fees, on-net prices and off-net prices simultaneously.

¹⁶ In several countries such as the UK, networks recently offer a wide variety of mobile contracts instead of typical two-part tariffs (monthly subscription and usage fees). However, the model follows the conventional models of two-part tariffs. In this setting, the cost of connecting a customer (f_i) may include cost of free allowance of service, which is offered in a recent pricing scheme. If a consumer uses up his free allowance, then he has to pay usage fees for additional calls.

 17 In this model, the entrant has already entered in the market and hence a dynamic viewpoint of entry and exit is beyond this framework.

Scenarios

The market outcomes are investigated in the three following scenarios. The first two scenarios are observed as benchmarks. The market outcome, aggregate consumer surplus and aggregate producer surplus in the last scenario will be compared with the two benchmarks.

I. Benchmark 1: the unregulated market

The first benchmark concerns the equilibrium market outcome without any regulatory intervention. Both networks can set their own access charges unilaterally.

II. Benchmark 2: the symmetric cost-based access charge regulation

The second benchmark introduces the symmetric regulation on access charge. Each network is regulated to set access charge at termination cost. Profit function of network i is

$$
\pi_i = \alpha_i \big[\alpha_i (p_i q_i - 2c_i q_i) + \alpha_j (\hat{p}_i \hat{q}_i - (c_i + c_j) \hat{q}_i) + F_i - f_i \big] \tag{2.6}
$$

where $i, j \in \{1,2\}$ and $i \neq j$.

III. The asymmetric cost-based access charge regulation

To assess the asymmetric regulation, which is usually adopted for the purpose of facilitating entry of a vulnerable entrant, this scenario focuses only on the situation where the new firm has higher cost and smaller market share. The established firm (network 1 with low cost) is regulated to impose cost-based access charge, $a_1 = c_1$. On the other hand, the new firm (network 2 with high cost) is allowed to set a_2 . Thus, profit functions of network 1 and network 2 are as follows.

$$
\pi_1 = \alpha_1 [\alpha_1 (p_1 q_1 - 2c_1 q_1) + \alpha_2 (\hat{p}_1 \hat{q}_1 - (c_1 + a_2) \hat{q}_1) + F_1 - f_1]
$$
(2.7)

$$
\pi_2 = \alpha_2 [\alpha_2 (p_2 q_2 - 2c_2 q_2) + \alpha_1 (\hat{p}_2 \hat{q}_2 - (c_2 + c_1) \hat{q}_2) + F_2 - f_2]
$$

$$
+ \alpha_1 [\alpha_2 (a_2 - c_2) \hat{q}_1]
$$
 (2.8)

Social welfare

Aggregate consumer surplus (CS) is the sum of all consumers' utility. Each consumer's utility consists of the net utility from making calls, the differential of the two networks' fixed surplus levels from reputation (if he chooses network 1) and the disutility from choosing a network which is not his ideal networks.

$$
CS = \left[\alpha_1 (w_1 + \beta) - \int_0^{\alpha_1} \alpha t \, d\alpha \right] + \left[\alpha_2 w_2 - \int_{\alpha_1}^1 (1 - \alpha) t \, d\alpha \right]
$$

$$
CS = \alpha_1 (w_1 + \beta) + (1 - \alpha_1) w_2 - \left[\frac{[\alpha_1^2 + (1 - \alpha_1)^2]}{4\sigma} \right]
$$
 (2.9)

 $\alpha_1(w_1 + \beta)$ is the sum of the net utility of the consumers who subscribe to network 1 (the low-cost firm), including the differential of fixed surplus levels.

 $(1 - \alpha_1)w_2$ is the sum of the net utility of the consumers who subscribe to network 2 (the high-cost firm).

 $\left[\frac{[\alpha_1^2 + (1-\alpha_1)^2]}{1} \right]$ $\left(\frac{(1-a_1)^{3}}{4\sigma}\right)$ is the total of all consumers' disutility from connecting to either network 1 or network 2 which is not their ideal networks.

Aggregate producer surplus (PS) is the sum of both networks' profits.

$$
PS = \pi_1 + \pi_2 \tag{2.10}
$$

Social welfare is the total of consumer surplus and producer surplus.

2.3 Analysis

The market outcomes of the three scenarios are compared in the following sections. The equilibrium outcome of the unregulated market and that of the symmetric cost-based access charge regulation are benchmark 1 and benchmark 2 respectively.

2.3.1 Benchmark 1: the unregulated market

In this scenario, firms are allowed to set their own access charges unilaterally.

The behaviour of mobile network operators in the unregulated market

Firms have three instruments (on-net price (p_i) , off-net price (\hat{p}_i) and fixed fee (F_i)) to compete for subscribers in the second stage of the game. For simplicity, fixed fee (F_i) is rearranged as a function of market share α_i . If a network varies its usage prices and intends to keep its market share constant, it has to change its fixed fee in order to balance its net utility. In the second stage of the game, after differentiating profit function with respect to on-net price, off-net price, and market share, the profit maximising outcome is as follows.

$$
p_i^* = 2c_i \text{ and } \hat{p}_i^* = c_i + a_j \; ; \; i, j \in \{1, 2\} \text{ and } i \neq j. \tag{2.11}
$$

Proposition 2.1

 $\overline{}$

When asymmetric networks compete under network-based price discrimination and two-part tariffs, both the high-cost firm and the low-cost firm set usage prices equal to their perceived costs 18 (marginal cost pricing). The high-cost firm offers higher on-net price than the low-cost firm, i.e. $p_2^* > p_1^*$. Proof See the appendix (Section 2.5.1).

Both equilibrium on-net prices and off-net prices are based on marginal cost pricing. This outcome is not different from the relevant literature about two-part tariffs in the setting of identical cost among firms (Cambini and Valletti, 2003; Gans and King, 2001; Laffont, Rey and Tirole, 1998b; Lopez and Rey, 2009; Peitz, 2005). When networks charge two-part tariffs, they set usage prices at perceived costs and

¹⁸ Perceived costs vary according to call termination. The perceived cost of an on-net call is the sum of actual marginal costs of call origination and call termination in the same network. The perceived cost of an off-net call is the total of actual marginal origination cost and access charge imposed by the terminating network.

cannot make profit from originating calls. Therefore, the profit function depends only on the revenue from fixed fee and the revenue from terminating incoming off-net calls (access revenue). In addition, when a firm increases fixed fee, two effects on profit occur. On one hand, the direct effect of an increase in fixed fee leads to an increase in profit from subscription. On the other hand, the indirect effect may reduce its market share because its corresponding net utility drops and the marginal consumer may switch to the other network instead. When the firm loses its market share, there is a reduction in the total fixed fee revenue and access revenue may also decrease. As a result, the profit-maximising fixed fee should balance these two effects in order that the network's profit reaches the maximum.

Remark 2.1

In the unregulated market, the critical value of asymmetry parameter $(\hat{\beta})$ indicating the dominant network is

$$
\hat{\beta} = -\frac{1}{2} [\nu(c_1 + a_2^*) - \nu(2c_2) - \nu(c_2 + a_1^*) + \nu(2c_1)] < 0.
$$

The low-cost firm (network 1) holds larger market share than the high-cost firm if $\beta > \hat{\beta}$.

Proof See the appendix (Section 2.5.2).

The critical value of asymmetry parameter $(\hat{\beta})$ is determined by costs of the two networks. $\hat{\beta}$ is negative. Even though the low-cost has a worse reputation than the high-cost firm (but the asymmetry parameter is still greater than the critical level $\hat{\beta}$), the low-cost firm still serves a majority of consumers. This is because the low-cost firm is more efficient in cost and offers lower usage prices to attract customers. From a static point of view, marginal cost is exogenous, so firms cannot adjust their cost structure in the short run. To take over the dominant position, the high-cost firm has to build up its reputation to be much better than the low-cost firm until the asymmetry parameter is less than the critical level. In addition, when the cost differential decreases, the critical value of asymmetry parameter increases and the gap between the two networks' market shares is narrowed. A network's market share is determined by the asymmetry parameter and both networks' cost structures.

Proposition 2.2

The network with larger market share sets higher fixed fee and earns higher profit than the network with smaller market share. Proof See the appendix (Section 2.5.3).

According to marginal cost pricing for usage fees, the networks have profit only from fixed fee revenue and access revenue. Assuming both firms have the same per-subscriber connecting cost $(f_1 = f_2)$, when the two networks have different sizes of market shares in the asymmetric equilibrium, the large firm (the firm with larger market share) will charge higher fixed fee than the small firm. Nevertheless, the large firm still holds larger market share despite its higher fixed fee because it offers more attractive utility from lower on-net price and/or higher fixed surplus from reputation. Consequently, the majority of the consumers still perceive that the large firm's benefit outweighs the disadvantage of its higher fixed fee.

Proposition 2.3

Both networks unilaterally choose above-cost access charges, i.e. $a_1^* > c_1$ and $a_2^* > c_2$. The network with larger market share sets higher mark-up on access charge than the network with smaller market share.

Proof See the appendix (Section 2.5.4).

In the absence of access charge regulations, both firms unilaterally charge mark-ups on access charges in the first stage of the game. Accordingly, the off-net prices, which are equal to their perceived costs, are higher than their actual marginal costs because the prices are set to cover the associated access mark-ups. Similar to the findings under uniform pricing reported by Armstrong (1998) and Laffont, Rey and Tirole (1998a), this result strongly indicates the appearance of tacit collusion in this environment of cost asymmetry. They will not trigger price war through off-net call pricing. In contrast to some findings supporting that firms with identical costs would agree on a cost-based reciprocal access charge (Laffont, Rey and Tirole, 1998b) or even a below-cost reciprocal access charge (Calzada and Valletti, 2008; Gans and King, 2001), this study reveals that when the networks have different costs, the dominant firm will take advantage of the cost difference and a gap in above-cost

access charges. The above-cost off-net prices do not approach the socially optimal prices, which should equal the actual marginal costs. Regardless of the access markups, on-net prices are intact. Both networks still charge their on-net prices at the actual marginal costs.

It is also found that the large network sets not only higher fixed fee, but also higher access charge than the small network. When a network increases access charge, there are both positive and negative effects on its own profit. First, when its access charge increases, its rival's off-net price is pushed up. As a result, its own market share may expand, and its profit increases accordingly. Second, when its access charge increases, its rival adjusts by lowering its fixed fee. In order to tackle its rival's reaction, the network has to lower its fixed fee, which has a negative impact on its own profit. For the large network, the first effect more significantly outweighs the second effect, compared to the small network. Therefore, the large network can set higher access charge and earn higher profit than the small network.

In the absence of asymmetry in reputation ($\beta = 0$), the low-cost firm certainly has larger market share because of its cost efficiency. Further, when the two networks are different in reputation, the low-cost firm is more likely to be the large network. If its reputation is not too bad, compared to that of the high-cost firm, the low-cost firm has larger market share and gains higher profit than the high-cost firm. However, the differences in the firms' market shares and profits become subtle when the discrepancy in reputation decreases. When the low-cost firm's reputation declines $(\beta$ decreases), the low-cost firm loses its power to set a comparatively high fixed fee and a relatively high access charge (to forcibly increase its rival's off-net price) 19 . As a result, the firm loses profit. Moreover, when its reputation is comparatively extremely bad, the low-cost firm cannot preserve its dominant position in the market. The lowcost firm becomes the small network and earns lower profit than the high-cost firm, which turns to be the large network instead.

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 $\frac{dm_1}{d\beta} > 0$ where m_1 is a mark-up on access charge of network 1; $m_1 = a_1 - c_1$. See proof in the appendix (Section 2.5.5).

■ Comparative statics analysis

In the unregulated market, the effects of the asymmetry parameter (β) , the degree of substitutability (σ) and the networks' costs on the equilibrium outcomes are as follows.

Observation 2.1

(i) When the asymmetry parameter increases, the low-cost firm can expand its market share, i.e. $\frac{\partial \alpha_1^*}{\partial \beta} > 0$.

(ii) When the degree of substitutability between the networks is higher, the market share of the large network expands. As a result, the gap between the market shares of the two networks is wider, i.e. when $\alpha_i^* > \alpha_j^*$, $\frac{\partial \alpha_i^*}{\partial \sigma} > 0$ and $\frac{\partial (\alpha_i^* - \alpha_j^*)}{\partial \sigma} > 0$; $i \in \{1,2\}, i \neq j.$

(iii) When a network's cost increases, its market share decreases but its rival's market share increases, i.e. $\frac{\partial a_i^*}{\partial x_i}$ $\frac{\partial a_i^*}{\partial c_i} < 0, \frac{\partial a_j^*}{\partial c_i}$ $\frac{\partial a_j}{\partial c_i} > 0$ where $i \in \{1,2\}, i \neq j$.

(iv) When a network decides to increase its access charge, equilibrium fixed fees of both networks decrease, i.e. $\frac{\partial F_i^*}{\partial x_i}$ $\frac{\partial F_i^*}{\partial a_i} < 0, \frac{\partial F_j^*}{\partial a_i}$ $\frac{\partial f_j}{\partial a_i}$ < 0 where $i \in \{1,2\}, i \neq j$.

Proof See the appendix (Section 2.5.6).

Market share is affected by several exogenous factors. Firstly, when a network's reputation grows, the network can increase the number of its subscribers and finally expand its market share.

Secondly, if the two networks are slightly substitutable, some consumers decide not to switch to join the large network because they still have some benefit from the small network which is closer to their ideal networks than the large network. On the other hand, if the two networks are more substitutable, some of these consumers may decide to join the large network instead. Therefore, if the networks are closer competitors in terms of substitutability, the large network can expand its market share more easily. Subsequently, the small network is more likely to leave the market. As a result, differentiating service can be one of the small firm's strategies to

penetrate the market in this setting. In other words, when the degree of substitutability is higher or the horizontal differentiation (represented by t) is closer to zero, the competition approaches the conventional Bertrand price competition. However, this study focuses on the shared-market equilibrium. Thus, the degree of substitutability should not be too high when compared to the cost differential.²⁰

Thirdly, the difference in cost is a key factor in the asymmetric outcomes. When a network's cost increases, its on-net price certainly goes up. From consumers' perspective, the network's service is less attractive. Some of its subscribers switch to its rival, and consequently its market share decreases. Accordingly, its rival's market share increases.

In addition to the three exogenous factors, in equilibrium, a network's decision on access charge affects fixed fee pricing in the later stage. When a network increases its access charge, it is likely to gain more profit from access revenue. Nevertheless, its rival will react by reducing fixed fee. The network has to decrease its own fixed fee to discourage some customers from switching to its rival. Consequently, if a network decides to push up its access charge, the profit-maximising fixed fees of both firms will decrease.

2.3.2 Benchmark 2: the symmetric cost-based access charge regulation

The high-cost firm and the low-cost firm are regulated to set their access charges at termination costs.

The behaviour of mobile network operators under the symmetric cost-based access charge regulation.

When $a_1 = c_1$ and $a_2 = c_2$ in the first stage of the game, according to (2.11), the on-net and off-net prices are shown below.

²⁰ See proof in the appendix (Section 2.5.7).

$$
p_i^* = 2c_i
$$
 and $\hat{p}_i^* = c_i + c_j$; $i, j \in \{1, 2\}$ and $i \neq j$

According to the assumption that the marginal costs of network 1 are less than those of network 2, the on-net price of the high-cost firm is higher than off-net prices of both networks. $p_2^* > \hat{p_2}^* = \hat{p_1}^* > p_1^*$.

Proposition 2.4

Under the symmetric cost-based access charge regulation, both firms set their on-net prices at actual marginal cost. The high-cost firm offers higher on-net price than the low-cost firm. However, both firms offer the same level of off-net prices which equals actual marginal cost.

Proof

Both networks still set the profit-maximising usage fee in accordance with marginal cost pricing. From (2.11), $p_1^* = 2c_1$, $p_2^* = 2c_2$, $\hat{p}_1^* = \hat{p}_2^* = c_1 + c_2$. \Box

Under the symmetric regulation, on-net and off-net prices of the two networks are set at cost because no access mark-up is allowed. The low-cost firm can set on-net price lower than the high-cost firm due to its cost efficiency.

Remark 2.2

Under the symmetric cost-based access charge regulation, the critical value of asymmetry parameter $(\hat{\hat{\beta}})$ is

$$
\hat{\hat{\beta}} = -\frac{1}{2} [\nu(2c_1) - \nu(2c_2)] < 0.
$$

The low-cost firm has larger market share if $\beta > \hat{\beta}$. Moreover, the critical value under the symmetric regulation $(\hat{\beta})$ is greater than that in the unregulated market $(\hat{\beta})$, i.e. $\hat{\hat{\beta}} > \hat{\beta}$.

Proof See the appendix (Section 2.5.8).

The symmetric regulation eliminates the effect of access mark-up that is strategically imposed by the low-cost firm. As a result, the critical value of asymmetric parameter is higher than that in the unregulated market because it depends only on the cost differential. The high-cost firm has a greater tendency to take over the dominant

position under the symmetric regulation than in the unregulated market²¹, especially when the high-cost firm has a better reputation.

Proposition 2.5

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Under the symmetric cost-based access charge regulation, the network with larger market share sets higher fixed fee and earns higher profit than the network with smaller market share.

Proof See the appendix (Section 2.5.9).

Under the symmetric regulation, both on-net prices and off-net prices are equal to their actual marginal costs. Firms generate profit only from fixed fees due to marginal cost pricing and the absence of access mark-up. Similar to the outcome in the unregulated market, the large network can charge higher fixed fee and still serve the majority of consumers because of its comparatively attractive benefit in terms of cost efficiency and/or reputation.

2.3.3 The asymmetric cost-based access charge regulation

The new firm may be a potential competitor that can increase the degree of competition in the market if it is cost-efficient and/or if its reputation is excellent enough. For example, its cost may be lower than the established firm. As a close rival, the new firm is capable of competing fiercely, and social welfare will be improved accordingly. In this case, the asymmetric regulation is unnecessary. Nevertheless, in the initial stage of entry, the new firm is more likely to be inefficient in terms of cost and reputation, and as a consequence it is disadvantaged and at a risk to shut down. Despite the entrant's inefficiency, the regulator may impose this regulation for the purpose of intensifying competition rather than let the established firm seize control of the whole market as a monopolist.

²¹ For instance, suppose the asymmetry parameter is *b* where $\hat{\beta} < b < \hat{\beta} < 0$, the high-cost firm earns smaller market share than the low-cost firm in the unregulated market but it becomes dominant with larger market share under the symmetric regulation.

This section focuses on the situation where the new firm has higher cost and earns smaller market share in order to assess the asymmetric regulation as an entryfacilitating policy. Under this regulation, the established firm (the low-cost firm) is regulated to set access charge at cost. In contrast, the new firm (the high-cost firm) is allowed to choose its access charge in this setting.

■ The behaviour of mobile network operators under the asymmetric **cost-based access charge regulation**

Network 1 (the established firm with low cost) sets its access charge equal to its marginal cost, and network 2 (the new firm with high cost) can set its own access charge. According to (2.11), on-net and off-net prices in equilibrium are as follows.

$$
p_i^* = 2c_i
$$
; $i, j \in \{1, 2\}$ and $i \neq j$
 $\hat{p}_1^* = c_1 + a_2$
 $\hat{p}_2^* = c_2 + c_1$

 p_2^* > $\hat{p_2}^*$ > p_1^* and $\hat{p_1}^*$ > $\hat{p_2}^*$ > p_1^* because network 2 will choose a mark-up on access charge.

Proposition 2.6

Under the asymmetric cost-based access charge regulation, the new firm with high cost sets a mark-up on access charge. When the asymmetric regulation is imposed instead of the symmetric regulation, its effects on the equilibrium outcomes are as follows.

(i) The market share of the low-cost established firm decreases, but that of the new firm increases.

(ii) Equilibrium fixed fees of both firms decrease.

(iii) Overall, the profit of the new firm increases, but that of the established firm decreases.

Proof See the appendix (Section 2.5.10).

While the access charge of the low-cost established firm is fixed at cost by regulation, the new firm with high cost will choose above-cost access charge to maximise its profit. The asymmetric regulation has significant effects on the market outcome. Firstly, the access mark-up of the new firm can expand the firm's market share and shrink that of the established firm. This is because the new firm can raise the established firm's off-net price through its access mark-up.

Secondly, the access mark-up also causes the established firm to react by decreasing its fixed fee. Accordingly, the equilibrium fixed fee of the new firm is affected in two different ways. When the established firm's off-net price increases according to the new firm's access mark-up, the established firm's service seems less attractive. Thus, the market share of the new firm may increase. The new firm will receive more profit if it decides to raise its fixed fee. However, when it charges an access mark-up, the established firm decides to reduce its equilibrium fixed fee $\frac{\partial F_1^*}{\partial x}$ $\frac{\partial r_1}{\partial a_2}$ $a_2 = c_2$ $<$ 0) as a response. If the new firm does not reduce its fixed fee, it will lose market share and it cannot take full advantage of its access mark-up. Therefore, the new firm finally maximises its profit by lowering its fixed fee. In summary, when the new firm sets a mark-up on access charge, both equilibrium fixed fees decrease. This finding is similar to the result of Peitz (2005), studying asymmetric networks with different fixed utilities but identical cost.

Thirdly, the fixed fee of the established firm changes more drastically than that of the new firm, and its off-net price is higher. This leads to a reduction in the established firm's market share. Consequently, the established firm's profit decreases and the new firm's profit certainly increases from the symmetric regulation benchmark.

Remark 2.3

Under the asymmetric cost-based access charge regulation, the critical value of asymmetry parameter $(\hat{\hat{\beta}})$ is

$$
\hat{\hat{\beta}} = -\frac{1}{2} [\nu(c_1 + a_2^*) - \nu(2c_2) - \nu(c_1 + c_2) + \nu(2c_1)].
$$

The low-cost firm has larger market share if $\beta > \hat{\hat{\beta}}$. The critical value under the asymmetric regulation is the greatest, followed by those under the symmetric regulation $(\hat{\hat{\beta}})$ and in the unregulated market $(\hat{\beta})$ respectively, i.e. $\hat{\hat{\hat{\beta}}} > \hat{\hat{\beta}} > \hat{\beta}$. Proof See the appendix (Section 2.5.11).

Under the asymmetric regulation, the critical value of asymmetry parameter is greater than those under the symmetric regulation and in the unregulated market respectively. Regarding benchmark 1 and benchmark 2, the high-cost firm has to offer significantly higher fixed surplus than the low-cost firm in order to occupy larger market share. The huge discrepancy in reputation is necessary for the high-cost firm to achieve dominance in the market. However, to become the large network under the asymmetric regulation, the high-cost firm requires only a smaller discrepancy in reputation than those under other regulatory regimes.

Proposition 2.7a

When the established firm has lower cost and larger market share than the new firm, the comparison of market shares in the unregulated market ($unreg$), under the symmetric cost-based access charge regulation (sym) and under the asymmetric costbased access charge regulation $(asym)$ is shown below.

(i) The comparison of the established firm's market share is as follows.

$$
\alpha_{1}^*_{unreg} > \alpha_{1}^*_{sym} > \alpha_{1}^*_{asym}
$$

(ii) The comparison of the new firm's market share is as follows.

$$
\alpha_{2}^*_{unreg} < \alpha_{2}^*_{sym} < \alpha_{2}^*_{asym}
$$

Proof See the appendix (Section 2.5.12).

The asymmetric regulation is most effective to facilitate entry of the new firm with high cost. Conversely, the low-cost established firm may prefer no regulatory intervention. This is because its market share decreases after the regulator adopts either the symmetric regulation or the asymmetric regulation.
Proposition 2.7b

When the established firm has lower cost and larger market share than the new firm, the comparison of the new firm's profits in the unregulated market (ω *nreg*), under the symmetric cost-based access charge regulation (sym) and under the asymmetric cost-based access charge regulation $(asym)$ is as follows.

$$
\pi_{2}^*_{unreg} < \pi_{2}^*_{sym} < \pi_{2}^*_{asym}.
$$

Proof See the appendix (Section 2.5.13).

The asymmetric regulation is beneficial only to the new firm (the high-cost firm in this scenario). In comparison with the other regulatory regimes, the new firm can achieve market penetration and reap the highest profit under the asymmetric regulation. Conversely, the low-cost established firm inevitably loses profit as a consequence of the asymmetric regulation.²²

2.3.4 Social welfare analysis

The effects of the three regulatory regimes on social welfare are discussed as follows.

■ The unregulated market

As stated in Proposition 2.3, both networks set mark-ups on access charges, and consequently off-net prices are pushed up. The price distortions have adverse effects on consumers. Additionally, the large network takes full advantage of its dominance in order to threaten the small network by means of aggressive pricing on access charge. Therefore, deregulation is not an appropriate policy in this setting.

■ The symmetric cost-based access charge regulation

Compared to the market outcome in the unregulated market, off-net prices dramatically decrease because access mark-ups are not allowed under this regulation.

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²² See proof in the appendix (Section 2.5.13).

Moreover, the large network can no longer use its access mark-up as a predatory tool for undermining its small rival. As a result, the distortions of usage fees are avoidable. Due to the cost-based access charges and usage prices, consumers and the underdog firm benefit greatly from this regulation, as stated in Proposition 2.7b.

The asymmetric cost-based access charge regulation

In the unregulated market (benchmark 1), both networks unilaterally set their access charges above cost and generate some profits from mark-ups on access charges. The on-net prices are set at actual marginal costs, but the off-net prices are distorted by the access mark-ups. This may reduce aggregate consumer surplus. The symmetric regulation and the asymmetric regulation may be pragmatic approaches to alleviate the distortion in this situation. When both networks are efficient in terms of costs and reputation, it is appropriate that the symmetric regulation should be imposed to curb access mark-ups and retail prices.

However, when the issues of facilitating entry and promoting the long-run competition are taken into consideration, the asymmetric regulation should be considered in comparison with the symmetric regulation.

The next section is dedicated to the case in which network 1 is the low-cost established firm with larger market share than network 2, which is the new firm with high cost. The effects of the asymmetric regulation on aggregate consumer surplus and aggregate producer surplus are compared with the effect of the symmetric regulation (benchmark 2).

Aggregate consumer surplus

According to (2.9), the asymmetric regulation, which allows the new firm to set access mark-up $(a_2 > c_2)$, has an ambiguous effect on consumer surplus. The asymmetric regulation may not guarantee an increase in the total of net utility of each consumer group. However, the asymmetric regulation certainly reduces the total

disutility.²³ The difference between the two firms' market shares becomes subtle when the new firm is allowed to set above-cost access charge. Compared with the outcome of the symmetric regulation, the new firm can steal some of market share from the low-cost established firm. Thus, the asymmetric regulation narrows the gap between the market shares of the two networks. Therefore, it is ambiguous to conclude that the asymmetric regulation can raise aggregate consumer surplus as a whole. The derivative of aggregate consumer surplus with respect to the new firm's access charge is as follows.

$$
\frac{\partial CS}{\partial a_2} = 2\alpha_1 v(p_1) \frac{\partial \alpha_1}{\partial a_2} + (\alpha_1 - \alpha_1^2) v'(\hat{p}_1) + (1 - 2\alpha_1) v(\hat{p}_1) \frac{\partial \alpha_1}{\partial a_2} - \alpha_1 \frac{\partial F_1}{\partial a_2}
$$

$$
-F_1 \frac{\partial \alpha_1}{\partial a_2} + \beta \frac{\partial \alpha_1}{\partial a_2} - 2(1 - \alpha_1) v(p_2) \frac{\partial \alpha_1}{\partial a_2} + (1 - 2\alpha_1) v(\hat{p}_2) \frac{\partial \alpha_1}{\partial a_2}
$$

$$
-(1 - \alpha_1) \frac{\partial F_2}{\partial a_2} + \frac{\partial \alpha_1}{\partial a_2} F_2 + \frac{1}{2\sigma} (1 - 2\alpha_1) \frac{\partial \alpha_1}{\partial a_2}
$$

The asymmetric regulation may boost aggregate consumer surplus on condition that the parameter of asymmetry in reputation (β) is not too high, and the cost differential is not too large when compared with the degree of substitutability (σ) .²⁴ When the asymmetric regulation is imposed, the new firm's market share and profit increase. On the other hand, the low-cost established firm loses market share and profit. Thus, the difference in market shares becomes subtle under the asymmetric regulation.

The asymmetric regulation makes two different effects on aggregate consumer surplus. First, a negative effect on consumer surplus occurs when the market share of the established network diminishes. A distinct example is the case in which the established firm offers greater fixed-surplus from reputation $(\beta > 0)$ but some consumers decide to switch to the new network. These consumers have to give up the extra fixed-surplus (β) from joining the established firm. As a result, the asymmetric regulation causes a reduction in the consumers' benefit from fixed surplus. Second, if

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²³ See proof in the appendix (Section 2.5.14).

²⁴ See proof in the appendix (Section 2.5.14).

the cost differential is not too large by comparison with the degree of substitutability (σ) , there is a positive effect of the asymmetric regulation on consumer surplus. When some consumers switch to the new network, the total disutility from not connecting to consumers' ideal networks decreases (the disutility is lowest when the two networks share the market equally). Meanwhile, the consumers of the new network have to incur higher average usage prices as a result of cost inefficiency. In summary, the benefit from the asymmetric regulation may outweigh its disadvantage if these conditions are satisfied: (1) the asymmetry parameter (β) is not too high, (2) the degree of substitutability is low enough, and (3) the costs of both networks are not much different.

Aggregate producer surplus

Compared to the outcome under the symmetric regulation (benchmark 2), the low-cost established firm loses some profit whereas the new firm with high cost gains more profit under the asymmetric regulation. In the neighbourhood of cost-based access charges, when the new firm decides to charge an access mark-up, the magnitude of profit which the established firm loses outweighs that of profit which the new firm gains. Overall, aggregate producer surplus decreases.²⁵

■ The optimal regulations

If the newcomer is efficient enough to compete with the established firm, the regulator may choose the symmetric regulation to vanish the distortions of access charges and off-net prices, which inevitably occur in the unregulated market. However, it is questionable whether the asymmetric regulation should be imposed in an effort to encourage entry and foster market competition in the situation where the new firm has higher cost and smaller market share. Similar to the standard result of Baranes and Vuong (2012) and Peitz (2005), this study provides the clear evidence that the asymmetric regulation is an efficient measure to promote the new firm's profit and market penetration. While the result of Peitz (2005) suggests that the asymmetric

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²⁵ See proof in the appendix (Section 2.5.15).

regulation can increase consumer surplus, the present study finds that the effect of the asymmetric regulation on consumer surplus and social welfare are not clear-cut, as stated in the preceding section. It can enhance social welfare in the situation where the cost differential and the discrepancy in reputation are not significant and both networks' services are differentiated enough.

The main focus of this discussion is the efficiency of the entrant and its differentiated service. This study supports that the firm asymmetry caused by the entrant's inherent inefficiency should not be the justification for imposing the asymmetric regulation. Additionally, the asymmetric regulation is socially acceptable when the entrant is launching an alternative to the legacy telecommunications services such as higher generation wireless telecommunications with an increasing trend in demand for mobile broadband. In the period of product introduction, the entrant may incur higher cost and the allowance of its access charge under the asymmetric regulation can preserve the viability of the entrant. From a social perspective, the asymmetric regulation is necessary to intensify market competition through the sufficiently differentiated service in this situation. In addition, the asymmetric regulation should be implemented as a temporary measure in the early phase of the new advanced service. When the firm asymmetry becomes subtle in the more mature phase, it is pointless to implement the asymmetric regulation.

On the other hand, the asymmetric regulation dampens social welfare when the new firm has considerably higher cost, significantly worse reputation, and high substitutability with the established firm. This is because the new firm is obviously inefficient in terms of cost and reputation. Moreover, from consumers' viewpoint, its product is not differentiated enough from the existing product, and it cannot fill in the gap in the market to satisfy consumers. Thus, any regulatory support for the new firm may distort the market and reduce social welfare. Meanwhile, on the supply side, the asymmetric regulation causes a reduction in the profit of the established firm and aggregate producer surplus. This may spark controversy by the established firm when an issue of the asymmetric regulation is publicly debated.

2.4 Conclusion

This study investigates competition between two asymmetric networks under two-part tariffs and network-based discriminatory pricing. The networks are different in cost and reputation. To maximise their own profits, both networks apply marginal cost pricing to usage fees. As a result, they have profits only from fixed fees and access revenues. When the market is unregulated, they unilaterally choose above-cost access charges. The low-cost firm has a greater tendency to have larger market share if its reputation is not too worse than that of the high-cost firm. When the discrepancy in reputation is smaller, the gap between the market shares of both networks is narrower. In addition, the network with larger market share can charge higher access charge, higher fixed fee and earn higher profit than the network with smaller market share.

Off-net prices of both networks are pushed up by each other because they choose above-cost access charges in the unregulated market. This is a threat to consumers. The regulator may impose the symmetric cost-based access charge regulation which can push down the two networks' off-net prices to their actual marginal costs. A network can no longer make its rival's off-net price less attractive by means of setting an access mark-up. Thus, the asymmetric market outcome directly reflects the cost differential and/or the discrepancy in reputation, and it is not distorted by the strategic access mark-ups. Compared with the outcome in the unregulated market, the symmetric regulation can narrow the gap between the two networks' market shares. However, there is some concern about the viability of the new firm in the situation where the new firm has high cost and smaller market share. The regulator may choose the asymmetric cost-based access charge regulation in order to encourage the new firm to enter the market and force the established firm to relinquish its monopoly power. Consumer welfare may increase if the cost differential and the discrepancy in reputation are not substantial and the two networks' services are differentiated enough. On the other hand, the profit and market share of the established firm decrease. Consequently, the regulator should implement the asymmetric regulation instead of the symmetric regulation when the underdog highcost firm with differentiated service is not too inefficient in terms of cost and reputation. Otherwise, the asymmetric regulation is more likely to cause a welfareundermining distortion of the market outcome.

Compared to other regulatory approaches, these cost-based access charge regulations are more practical because the regulators require only the declaration of cost structure from mobile network operators to set cost-based access charge. Additionally, in a static framework, the asymmetric regulation can increase the profit of the new firm with high cost and may enhance consumer welfare in some situations. It is acceptable for the regulator to allow the new firm to set an access mark-up in order to facilitate market entry and the launch of the new advanced service in the initial market. Nevertheless, in the long run, both networks should compete strongly and increase efficiency such as cost reduction and quality improvement. The asymmetric regulation should act as an incentive for the new firm to enter the market, but it should not last long. In addition, the regulator should encourage networks to develop in the stronger competition. Further research is still needed to suggest optimal regulations in the long run. Moreover, this study assumes the networks provide only one service, so it is interesting to investigate the competition in multiple service markets.

2.5 Appendix

2.5.1 Proof of Proposition 2.1

Substituting (2.1)-(2.3) in (2.4) and rearranging the equation, the explicit function of α_1 is

$$
\alpha_1 = \frac{\frac{1}{2} + \sigma \beta + \sigma [\nu(\hat{p}_1) - \nu(p_2) - F_1 + F_2]}{1 + \sigma [\nu(\hat{p}_1) - \nu(p_2) + \nu(\hat{p}_2) - \nu(p_1)]}.
$$
\n(2. A1)

 $\mathbb{D} = 1 + \sigma[\nu(\hat{p}_1) - \nu(p_2) + \nu(\hat{p}_2) - \nu(p_1)]$

 $\mathbb{D} > 0$ when there exists a stable shared-market equilibrium.

Keeping the market share (α_i) constant and treating F_i as a function of p_i , \hat{p}_i and α_i from (2. A1), one can differentiate (2.5) with respect to p_i and \hat{p}_i as follows.

$$
\frac{\partial \pi_i}{\partial p_i} = \alpha_i^2 q_i + \alpha_i^2 q_i' (p_i - 2c_i) + \alpha_i \frac{\partial F_i}{\partial p_i} = 0
$$

Then, substituting $\frac{\partial F_i}{\partial p_i} = \alpha_i v'(p_i)$ and $v'(p_i) = -q_i$ in the above equation yields the following.

$$
p_i - 2c_i = 0
$$

\n
$$
p_i^* = 2c_i
$$
 (2.A2)

Therefore, the profit-maximising on-net price is equal to actual marginal cost.²⁶

Next, determine the profit-maximising off-net price.

$$
\frac{\partial \pi_i}{\partial \hat{p}_i} = \alpha_i \alpha_j \hat{q}_i + \alpha_i \alpha_j (\hat{p}_i - c_i - a_j) \hat{q}'_i + \alpha_i \frac{\partial F_i}{\partial \hat{p}_i} = 0
$$

26 $\frac{\partial^2 \pi_i}{\partial x^2}$ $\frac{\partial^2 \pi_i}{\partial p_i^2} = -\alpha_i^2 < 0, \frac{\partial^2 \pi_i}{\partial p_i \partial i}$ $\frac{\partial^2 \pi_i}{\partial p_i \partial \hat{p}_i} = \frac{\partial^2 \pi_i}{\partial \hat{p}_i \partial p_i}$ $\frac{\partial^2 \pi_i}{\partial \hat{p}_i \partial p_i} = 0$ and $\frac{\partial^2 \pi_i}{\partial \hat{p}_i^2}$ $\frac{\partial^{i} n_i}{\partial \hat{p}_i^2} = -\alpha_i \alpha_j < 0$ to satisfy the second-order

condition (SOC) for profit maximisation.

 \overline{a}

Substituting $\frac{\partial F_i}{\partial \hat{z}}$ $\frac{\partial F_i}{\partial \hat{p}_i} = \alpha_j v'(\hat{p}_i)$ and $v'(\hat{p}_i) = -\hat{q}_i$ in the above first-order condition (FOC) and the profit-maximising off-net price is

$$
\hat{p}_i^* = c_i + a_j. \tag{2.A3}
$$

Thus, the profit-maximising off-net price is equal to perceived cost.²⁷

After substituting (2. A2) and (2. A3) in (2.5), the profit function is

$$
\pi_i = \alpha_i F_i - \alpha_i f_i + \alpha_j \alpha_i (a_i - c_i) \hat{q}_j. \tag{2. A4}
$$

The derivative of the profit function (2. A4) with respect to fixed fee (F_i) is

$$
\frac{\partial \pi_i}{\partial F_i} = \alpha_i + F_i \frac{\partial \alpha_i}{\partial F_i} - f_i \frac{\partial \alpha_i}{\partial F_i} + (\alpha_j - \alpha_i)(a_i - c_i)\hat{q}_j \frac{\partial \alpha_i}{\partial F_i} = 0.
$$

Substituting $\frac{\partial a_i}{\partial F_i} = -\frac{\sigma}{\mathbb{D}}$ $\frac{6}{5}$ in the above equation yields

$$
\alpha_i\left(-\frac{\mathbb{D}}{\sigma}\right)+F_i-f_i+(\alpha_j-\alpha_i)(a_i-c_i)\hat{q}_j=0.
$$

After rearranging the equation, one finds that

$$
F_i^* = f_i + \alpha_i \frac{\mathbb{D}}{\sigma} + (\alpha_i - \alpha_j)(a_i - c_i)\hat{q}_j
$$
 (2. A5)

where $\mathbb{D} = 1 + \sigma[\nu(\hat{p}_1) - \nu(p_2) + \nu(\hat{p}_2) - \nu(p_1)], \sigma > 0, \mathbb{D} > 0$ and $\frac{\mathbb{D}}{\sigma} > 0$.

The second-order condition (SOC) should be negative for the maximum profit.

$$
\frac{\partial^2 \pi_i}{\partial F_i^2} = 2 \frac{\partial \alpha_i}{\partial F_i} - 2 \left(\frac{\partial \alpha_i}{\partial F_i} \right)^2 (a_i - c_i) \hat{q}_j < 0
$$

where $\frac{\partial \alpha_i}{\partial F_i} = -\frac{\sigma}{\mathbb{D}}$ $\frac{\sigma}{\mathbb{D}}$.

 \overline{a}

²⁷ $\frac{\partial^2 \pi_i}{\partial x^2}$ $\frac{\partial^2 \pi_i}{\partial \hat{p}_i^2} = -\alpha_i \alpha_j < 0, \frac{\partial^2 \pi_i}{\partial p_i \partial i}$ $\frac{\partial^2 \pi_i}{\partial p_i \partial \hat{p}_i} = \frac{\partial^2 \pi_i}{\partial \hat{p}_i \partial p_i}$ $\frac{\partial^2 \pi_i}{\partial \hat{p}_i \partial p_i} = 0$ and $\frac{\partial^2 \pi_i}{\partial p_i^2}$ $\frac{\partial^2 \pi_i}{\partial p_i^2} = -\alpha_i^2 < 0$ to satisfy the SOC for profit maximisation.

$$
\frac{\partial^2 \pi_i}{\partial F_i^2} = -2 \frac{\sigma}{\mathbb{D}} \Big[1 + \frac{\sigma}{\mathbb{D}} (a_i - c_i) \hat{q}_j \Big] < 0
$$

The above expression holds when $\left[1+\frac{\sigma}{n}\right]$ $\frac{\partial}{\partial \rho}(a_i - c_i)\hat{q}_j > 0$. Consequently,

$$
(a_i - c_i)\hat{q}_j > -\frac{\mathbb{D}}{\sigma}.
$$
 (2. A6)

In other words, the access mark-up is assumed to be nonnegative or slightly less than termination cost in accordance with the concavity of profit function. The above assumption (2. A6) will be referred in the following analysis.

2.5.2 Proof of Remark 2.1

After substituting $(2.A2)$, $(2.A3)$ and $(2.A5)$ in $(2.A1)$, one can obtain the following equation.

$$
\alpha_1^* = \frac{1}{2} + \frac{\sigma \beta + \frac{\sigma}{2} [\nu(\hat{p}_1^*) - \nu(p_2^*) - \nu(\hat{p}_2^*) + \nu(p_1^*)]}{\mathbb{D}^*}
$$
\n
$$
\mathbb{D}^* = 3\mathbb{D} + 2\sigma [(a_1^* - c_{t1})\hat{q}_2 + (a_2^* - c_{t2})\hat{q}_1] > 0
$$
\n
$$
\mathbb{D} = 1 + \sigma [\nu(\hat{p}_1^*) - \nu(p_2^*) + \nu(\hat{p}_2^*) - \nu(p_1^*)]
$$
\n(2. A7)

According to (2. A7), $\alpha_1^* \geq \frac{1}{2}$ $\frac{1}{2}$ if $\beta \geq -\frac{1}{2}$ $\frac{1}{2}[\nu(\hat{p}_1^*) - \nu(p_2^*) - \nu(\hat{p}_2^*) + \nu(p_1^*)].$ Conversely, $\alpha_1^* < \frac{1}{2}$ $\frac{1}{2}$ if $\beta < -\frac{1}{2}$ $\frac{1}{2} [\nu(\hat{p}_1^*) - \nu({p_2}^*) - \nu(\hat{p}_2^*) + \nu({p_1}^*)] < 0$. Therefore, $\hat{\beta} = -\frac{1}{2}$ $\frac{1}{2}[\nu(c_1 + a_2^*) - \nu(2c_2) - \nu(c_2 + a_1^*) + \nu(2c_1)].$

2.5.3 Proof of Proposition 2.2

In the unregulated market, both networks charge access mark-ups. Suppose network *i* has larger market share than network $j \ (\alpha_i > \alpha_j; i, j \in \{1,2\} \text{ and } i \neq j),$ a comparison can be made to compare the equilibrium fixed fees expressed in (2. A5). One may apply the condition for the profit-maximising access charge and find that

$$
F_i^* - F_j^* = f_i - f_j + (\alpha_i - \alpha_j) \left[\frac{\mathbb{D}}{\sigma} + (\alpha_i - c_i) \hat{q}_j + (\alpha_j - c_j) \hat{q}_i \right].
$$

Since $f_i = f_j$, $\alpha_i - \alpha_j > 0$, $(a_i - c_i)\hat{q}_j > 0$ and $(a_j - c_j)\hat{q}_i > 0$, $F_i^* - F_j^* > 0$. Thus, the large network charges higher fixed fee than the small network.

Suppose network 1 is the large firm $(\alpha_1 > \alpha_2)$, under the assumption of linear demand ($\hat{q}_i = 1 - c_1 - c_2 - m_j$ and $m_i = a_i - c_i$; $i, j \in \{1, 2\}$ and $i \neq j$), The equilibrium profits of both networks in (2. A8) are

$$
\pi_1^* = \alpha_1^2 \left[\frac{\mathbb{D}}{\sigma} + m_1^* \hat{q}_2 \right]
$$

$$
\pi_2^* = \alpha_2^2 \left[\frac{\mathbb{D}}{\sigma} + m_2^* \hat{q}_1 \right].
$$

$$
\pi_1^* - \pi_2^* = (\alpha_1^2 - \alpha_2^2) \frac{\mathbb{D}}{\sigma} + \alpha_1^2 m_1^* (1 - c_1 - c_2 - m_1^*) - \alpha_2^2 m_2^* (1 - c_1 - c_2 - m_2^*)
$$

Let $\theta = 1 - c_1 - c_2 > 0$. Assume $\theta - m_1^* - m_2^* > 0$ (demand is positive). According to Proposition 2.3, when $\alpha_1 > \alpha_2$, $m_1^* - m_2^* > 0$.

$$
m_1^*(\theta - m_1^*) - m_2^*(\theta - m_2^*) = (m_1^* - m_2^*)(\theta - m_1^* - m_2^*) > 0
$$

and $\alpha_1^2 > \alpha_2^2$. Thus, $\pi_1^* - \pi_2^* > 0$.

2.5.4 Proof of Proposition 2.3

After substituting (2. A2), (2. A3), (2. A5) in (2.5), the profit function in stage 1 of the game is

$$
\pi_i^*(a_i) = \alpha_i^2 \left[\frac{\mathbb{D}}{\sigma} + (a_i - c_i)\hat{q}_j \right]
$$
 (2. A8)

In stage 1, network i 's problem is

$$
\max_{a_i} \pi_i^*(a_i)
$$

where $i, j \in \{1, 2\}$.

Network 1 The FOC is

$$
\frac{\partial \pi_1^*}{\partial a_1} = \alpha_1^2 (a_1 - c_1) \hat{q}_2' + 2 \alpha_1 \frac{\partial \alpha_1}{\partial a_1} \left[\frac{\mathbb{D}}{\sigma} + (a_1 - c_1) \hat{q}_2 \right] = 0 \tag{2.49}
$$

 a_1^* cannot be expressed in closed form, but a_1^* can be indicated by considering the profit function in the neighbourhood of $a_1 = c_1$. At $a_1 = c_1$,

$$
\left. \frac{\partial \pi_1^*}{\partial a_1} \right|_{a_1 = c_1} = 2\alpha_1 \frac{\partial \alpha_1}{\partial a_1} \left[\frac{\mathbb{D}}{\sigma} \right] > 0,
$$

where $\frac{\partial \alpha_1}{\partial a_1}$ $a_1 = c_1$ $> 0^{28}$ and $\mathbb{D} > 0$ according to the existence of a stable shared-market equilibrium. In the neighbourhood of $a_1 = c_1$, if network 1 increases its access charge above cost, its market share expands. This is because the significant impact of its access mark-up can push up its rival's off-net price. Consequently, its rival is less attractive and its own market share tends to increase. Thus, $\frac{\partial \pi_1^*}{\partial z_1}$ $\frac{\partial u_1}{\partial a_1}$ $a_1 = c_1$ > 0. Firm 1 will not set access charge at cost, but it has an incentive to increase access charge in order to earn more profit. In addition, if the access charge is set below termination cost $(a_1 < c_1)$, the FOC in (2.49) will hold only when the second term is negative. This is because the first term $(\alpha_1^2(\alpha_1 - c_1)\hat{q}_2)$ is strongly positive. According to (2.A6), $\partial \alpha_1$ $\frac{\partial a_1}{\partial a_1}$ must be negative to make the equation (2.A9) hold in this case. However, when access charge is below cost, the derivative of network 1's market share with respect to its access charge is positive $\left(\frac{\partial a_1}{\partial x_1}, a_2\right)$ $\frac{\partial u_1}{\partial a_1}$ $a_1 < c_1$ > 0 .²⁹ As a result, below-cost access charge cannot yield maximum profit. Thus, in the unregulated market, network 1 chooses above-cost access charge.

²⁸ See proof of $\frac{\partial \alpha_1}{\partial a_1}$ $a_1 = c_1$ > 0 in the following section in the appendix. ²⁹ See proof of $\frac{\partial \alpha_1}{\partial a_1}$ $a_1 < c_1$ > 0 in the following section in the appendix.

 \overline{a}

Network 2

$$
\frac{\partial \pi_2^*}{\partial a_2} = \alpha_2^2 (a_2 - c_2) \hat{q}'_1 - 2\alpha_2 \frac{\partial \alpha_1}{\partial a_2} \left[\frac{\mathbb{D}}{\sigma} + (a_2 - c_2) \hat{q}_1 \right] = 0 \tag{2. A10}
$$

At $a_2 = c_2$,

$$
\left. \frac{\partial \pi_2^*}{\partial a_2} \right|_{a_2 = c_2} = -2\alpha_2 \frac{\partial \alpha_1}{\partial a_2} \left[\frac{\mathbb{D}}{\sigma} \right] > 0.
$$

This is because at $a_2 = c_2$, $\frac{\partial a_1}{\partial a_2}$ $\frac{\partial u_1}{\partial a_2}$ $a_2 = c_2$ $< 0^{30}$ and $\mathbb{D} > 0$ according to the existence of a stable shared-market equilibrium. If network 2 unilaterally raises its access charge above cost, its market share increases. Accordingly, network 1's market share decreases. As a result of the access mark-up of network 2, network 1's off-net price increases and some subscribers of network 1 may switch to network 2. Therefore, network 2 also has incentive to charge its access mark-up. Furthermore, at $a_2 < c_2$, the first term of (2. A10) is positive. Therefore, the profit-maximising access charge of network 2 should make the second term of (2. A10) negative. According to (2. A6), \mathbb{D} $\frac{D}{\sigma} + (a_2 - c_2)\hat{q}_1 > 0$. To make (2.410) hold, it is necessary that $\frac{\partial a_1}{\partial a_2}$ $a_2 < c_2$ > 0 . In contrast, it is found that $\frac{\partial \alpha_1}{\partial a_2}$ $a_2 < c_2$ < 0 .³¹ It is a contradiction. Thus, network 2 will not choose a below-cost access charge. Similar to network 1, network 2 will set a mark-up on access charge.

Proof that
$$
\left. \frac{\partial a_1}{\partial a_1} \right|_{a_1 = c_1} > 0
$$

 \overline{a}

After substituting (2. A2), (2. A3) and (2. A5) in (2. A1) and rearranging the equation, one can obtain

$$
{\alpha_1}^* = \frac{\frac{1}{2} + \sigma \beta + \sigma [\nu(\hat{p}_1) - \nu(p_2)] + \mathbb{D} + \sigma [(a_1 - c_1)\hat{q}_2 + (a_2 - c_2)\hat{q}_1]}{3\mathbb{D} + 2\sigma [(a_1 - c_1)\hat{q}_2 + (a_2 - c_2)\hat{q}_1]}.
$$
 (2. A11)

³⁰ See proof of $\frac{\partial \alpha_1}{\partial a_2}$ $a_2 = c_2$ < 0 in the following section in the appendix. ³¹ See proof of $\frac{\partial \alpha_1}{\partial a_2}$ $a_2 < c_2$ < 0 in the following section in the appendix.

$$
\mathbb{T}^* = \frac{1}{2} + \sigma \beta + \sigma [\nu(\hat{p}_1) - \nu(p_2)] + \mathbb{D} + \sigma [(a_1 - c_1)\hat{q}_2 + (a_2 - c_2)\hat{q}_1]
$$

$$
\mathbb{D}^* = 3\mathbb{D} + 2\sigma [(a_1 - c_{t1})\hat{q}_2 + (a_2 - c_{t2})\hat{q}_1]
$$

Taking differentiation of α_1 with respect to α_1 yields

$$
\frac{\partial a_1^*}{\partial a_1} = \frac{\mathbb{D}^* \sigma(a_1 - c_1) \hat{q}_2' - \mathbb{T}^* [(-\sigma) \hat{q}_2 + 2\sigma(a_1 - c_1) \hat{q}_2']}{\mathbb{D}^{*^2}}.
$$

At $a_1 = c_1$,

$$
\left. \frac{\partial \alpha_1}{\partial a_1}^* \right|_{a_1 = c_1} = \frac{\mathbb{T}^* [\sigma \hat{q}_2]}{\mathbb{D}^*^2}
$$

where $\mathbb{D}^* > 0$ and $\mathbb{T}^* > 0$. Thus, $\frac{\partial a_1}{\partial a_1}$ ∗ | $a_1 = c_1$ > 0 .

Proof that
$$
\left. \frac{\partial a_1}{\partial a_1} \right|_{a_1 < c_1} > 0
$$

Differentiating (2. A11) with respect to a_1 and rearranging it yields

$$
\frac{\partial a_1^*}{\partial a_1} = \frac{-\sigma \mathbb{D}^* (a_1 - c_1) + \sigma \mathbb{T}^* [\hat{q}_2 + 2(a_1 - c_1)]}{\mathbb{D}^{*^2}}
$$

where

$$
\mathbb{T}^* = \frac{1}{2} + \sigma \beta + \sigma [\nu(\hat{p}_1) - \nu(p_2)] + \mathbb{D} + \sigma [(a_1 - c_1)\hat{q}_2 + (a_2 - c_2)\hat{q}_1] > 0
$$

$$
\mathbb{D}^* = 3\mathbb{D} + 2\sigma [(a_1 - c_{t1})\hat{q}_2 + (a_2 - c_{t2})\hat{q}_1] > 0.
$$

Suppose the demand is high enough, $\hat{q}_2 > |2(a_1 - c_1)|$ and $\hat{q}_2 + 2(a_1 - c_1) > 0$. Therefore, when $a_1 < c_1$, $\frac{\partial a_1^*}{\partial a_1}$ $\frac{\partial u_1}{\partial a_1}$ $a_1 < c_1$ $> 0.$

Proof that
$$
\left. \frac{\partial a_1}{\partial a_2} \right|_{a_2 = c_2} < 0
$$

Differentiating (2. A11) with respect to a_2 yields

$$
\frac{\partial a_1^*}{\partial a_2} = \frac{\mathbb{D}^* \left[-\sigma \hat{q}_1 + \sigma (a_2 - c_2) \hat{q}_1' \right] - \mathbb{T}^* \left[(-\sigma) \hat{q}_1 + 2\sigma (a_2 - c_2) \hat{q}_1' \right]}{\mathbb{D}^{*^2}}.
$$

At $a_2 = c_2$,

$$
\left. \frac{\partial \alpha_1}{\partial a_2}^* \right|_{a_2 = c_2} = \frac{\sigma \hat{q}_1 (\mathbb{T}^* - \mathbb{D}^*)}{\mathbb{D}^*^2}
$$

where

$$
\mathbb{T}^* = \frac{1}{2} + \sigma \beta + \sigma [\nu(\hat{p}_1) - \nu(p_2)] + \mathbb{D} + \sigma [(a_1 - c_1)\hat{q}_2 + (a_2 - c_2)\hat{q}_1] > 0
$$

$$
\mathbb{D}^* = 3\mathbb{D} + 2\sigma [(a_1 - c_{t1})\hat{q}_2 + (a_2 - c_{t2})\hat{q}_1] > 0.
$$

 $\mathbb{T}^* < \mathbb{D}^*$ in a shared-market equilibrium $(0 < \alpha_1^* < 1)$. Thus, $\frac{\partial \alpha_1}{\partial \alpha_1}$ ∂a_2 ∗ | $a_2 = c_2$ $< 0.$

Proof that
$$
\left. \frac{\partial a_1}{\partial a_2} \right|_{a_2 < c_2} < 0
$$

After differentiating (2.A11) with respect to a_2 and rearranging it, one can find that

$$
\frac{\partial a_1^*}{\partial a_2} = \frac{-\sigma \mathbb{D}^* \left[\hat{q}_1 + (a_2 - c_2) \right] + \sigma \mathbb{T}^* \left[\hat{q}_1 + 2(a_2 - c_2) \right]}{\mathbb{D}^{*^2}}
$$

where

$$
\mathbb{T}^* = \frac{1}{2} + \sigma \beta + \sigma [\nu(\hat{p}_1) - \nu(p_2)] + \mathbb{D} + \sigma [(a_1 - c_1)\hat{q}_2 + (a_2 - c_2)\hat{q}_1] > 0
$$

$$
\mathbb{D}^* = 3\mathbb{D} + 2\sigma [(a_1 - c_{t1})\hat{q}_2 + (a_2 - c_{t2})\hat{q}_1] > 0.
$$

Suppose the demand is high enough, $\hat{q}_1 > |2(a_2 - c_2)| > |a_2 - c_2|$. Therefore, $\hat{q}_1 + 2(a_2 - c_2) > 0$ and $\hat{q}_1 + (a_2 - c_2) > 0$. In a shared-market equilibrium where $0 < \alpha_1^* < 1$, $\mathbb{T}^* < \mathbb{D}^*$. Moreover, $\hat{q}_1 + 2(a_2 - c_2) < \hat{q}_1 + (a_2 - c_2)$ when $a_2 < c_2$. Therefore, when $a_2 < c_2$, $\frac{\partial a_1}{\partial a_2}$ $\frac{\partial u_1}{\partial a_2}$ $a_2 < c_2$ < 0*.*

To compare the access charges of the two networks, replacing access mark-up $m_i = a_i - c_i$ in (2.411) gives the market shares as shown below.

$$
{\alpha_1}^* = \frac{\frac{1}{2} + \sigma\beta + \sigma[\nu(\hat{p}_1) - \nu(p_2)] + \mathbb{D} + \sigma[m_1^*\hat{q}_2 + m_2^*\hat{q}_1]}{3\mathbb{D} + 2\sigma[(a_1 - c_1)\hat{q}_2 + (a_2 - c_2)\hat{q}_1]} = \frac{\mathbb{T}_1^*}{\mathbb{D}^*}
$$

$$
{\alpha_2}^* = \frac{\frac{1}{2} - \sigma\beta + \sigma[\nu(\hat{p}_2) - \nu(p_1)] + \mathbb{D} + \sigma[m_1^*\hat{q}_2 + m_2^*\hat{q}_1]}{3\mathbb{D} + 2\sigma[(a_1 - c_1)\hat{q}_2 + (a_2 - c_2)\hat{q}_1]} = \frac{\mathbb{T}_2^*}{\mathbb{D}^*}
$$

Suppose $\alpha_1^* > \alpha_2^*$, $\mathbb{T}_1^* > \mathbb{T}_2^*$. Analogously, replacing $m_i = a_i - c_i$ in (2.49) and (2. A10), the FOC for the profit-maximising access charge of network 1 can be rearranged as

$$
-m_1^* + 2\sigma \left[\frac{-m_1^*}{\mathbb{T}_1^*} + \frac{(\theta + m_1^*)}{\mathbb{D}^*} \right] \left[\frac{\mathbb{D}}{\sigma} + m_1^*(\theta - m_1^*) \right] = 0. \tag{2. A12}
$$

The FOC for the profit-maximising access charge of network 2 can be rearranged as

$$
-m_2^* + 2\sigma \left[\frac{-m_2^*}{\mathbb{T}_2^*} + \frac{(\theta + m_2^*)}{\mathbb{D}^*} \right] \left[\frac{\mathbb{D}}{\sigma} + m_2^*(\theta - m_2^*) \right] = 0.
$$

$$
m_2^* = 2\sigma \left[\frac{-m_2^*}{\mathbb{T}_2^*} + \frac{(\theta + m_2^*)}{\mathbb{D}^*} \right] \left[\frac{\mathbb{D}}{\sigma} + m_2^*(\theta - m_2^*) \right]
$$
(2. A13)

Then, one can replace m_1^* in (2. A12) with m_2^* and obtain (2. A14). If (2. A14) equals zero, one may conclude that $m_2^* = m_1^*$. However, (2.414) cannot be guaranteed to equal zero yet.

$$
-m_2^* + 2\sigma \left[\frac{-m_2^*}{\mathbb{T}_1^*} + \frac{(\theta + m_2^*)}{\mathbb{D}^*} \right] \left[\frac{\mathbb{D}}{\sigma} + m_2^*(\theta - m_2^*) \right]
$$
(2. A14)

To compare m_2^* with m_1^* , substituting (2.A13) in (2.A14) yields the following expression.

$$
-2\sigma\left[\frac{-m_2^*}{\mathbb{T}_2^*}+\frac{(\theta+m_2^*)}{\mathbb{D}^*}\right]\left[\frac{\mathbb{D}}{\sigma}+m_2^*(\theta-m_2^*)\right]+2\sigma\left[\frac{-m_2^*}{\mathbb{T}_1^*}+\frac{(\theta+m_2^*)}{\mathbb{D}^*}\right]\left[\frac{\mathbb{D}}{\sigma}+m_2^*(\theta-m_2^*)\right]
$$

The above expression is positive because $\mathbb{T}_1^* > \mathbb{T}_2^*$. Therefore, $m_2^* \neq m_1^*$. If network 1 chooses its access mark-up equal to m_2^* rather than m_1^* , $\frac{\partial n_1^*}{\partial q_1}$ $\frac{\partial n_1}{\partial a_1} > 0$. Consequently, when $\alpha_1^* > \alpha_2^*$, the access mark-up of network 1 exceeds that of network 2 ($m_1^* > m_2^*$).

2.5.5 Proof that $\frac{dm_1}{d\beta}>0$

Substituting $m_1 = a_1 - c_1$ and $\hat{q}_2 = 1 - c_1 - c_2 - m_1 = \theta - m_1$ in (2.A9) gives the following equation in equilibrium.

$$
\frac{\partial \pi_1^*}{\partial a_1} = -\alpha_1^2(m_1) + 2\alpha_1 \frac{\partial \alpha_1}{\partial a_1} \left[\frac{\mathbb{D}}{\sigma} + m_1(\theta - m_1) \right] = 0
$$

Total differential of the above equation with respect to m_1 and β is shown below.

$$
\frac{\partial^2 \pi_1^*}{\partial a_1 \partial m_1} \cdot dm_1 + \frac{\partial^2 \pi_1^*}{\partial a_1 \partial \beta} \cdot d\beta = 0
$$

$$
\frac{dm_1}{d\beta} = \frac{-\frac{\partial^2 \pi_1^*}{\partial a_1 \partial \beta}}{\frac{\partial^2 \pi_1^*}{\partial a_1 \partial m_1}} = \frac{A}{B}
$$

$$
\frac{dm_1}{d\beta} = \frac{2m_1\alpha_1\frac{\partial\alpha_1}{\partial\beta} - 2\left[\frac{\mathbb{D}}{\sigma} + m_1(\theta - m_1)\right]\left(\alpha_1\frac{\partial^2\alpha_1}{\partial a_1\partial\beta} + \frac{\partial\alpha_1}{\partial\beta}\cdot\frac{\partial\alpha_1}{\partial a_1}\right)}{-\alpha_1^2 - 4m_1\alpha_1\frac{\partial\alpha_1}{\partial a_1} + 2\left[\alpha_1\frac{\partial^2\alpha_1}{\partial a_1^2} + \left(\frac{\partial\alpha_1}{\partial a_1}\right)^2\right]\left[\frac{\mathbb{D}}{\sigma} + m_1(\theta - m_1)\right]}
$$

Denotation of A and B is as follows.

$$
\begin{split}\n\mathbb{A} &= -\frac{\partial^2 \pi_1^*}{\partial a_1 \partial \beta} \\
&= 2m_1 \alpha_1 \left(\frac{\sigma}{\mathbb{D}^*}\right) - 2 \left[\frac{\mathbb{D}}{\sigma} + m_1 (\theta - m_1) \right] \left(\frac{\alpha_1 \sigma^2 (\theta + m_1)}{\mathbb{D}^*} + \frac{\sigma}{\mathbb{D}^*} \cdot \frac{\partial \alpha_1}{\partial a_1}\right) \\
&= \frac{2\sigma}{\mathbb{D}^*} \left\{ m_1 \alpha_1 - \left[\frac{\mathbb{D}}{\sigma} + m_1 (\theta - m_1) \right] \left(\frac{\alpha_1 \sigma (\theta + m_1)}{\mathbb{D}^*} + \frac{\partial \alpha_1}{\partial a_1}\right) \right\} \\
\mathbb{B} &= \frac{\partial^2 \pi_1^*}{\partial a_1 \partial m_1} = -\alpha_1^2 - 4m_1 \alpha_1 \frac{\partial \alpha_1}{\partial a_1} + 2 \left[\alpha_1 \frac{\partial^2 \alpha_1}{\partial a_1^2} + \left(\frac{\partial \alpha_1}{\partial a_1}\right)^2 \right] \left[\frac{\mathbb{D}}{\sigma} + m_1 (\theta - m_1)\right]\n\end{split}
$$

At the profit-maximising access mark-up, (2. A9) can be rewritten as

$$
m_1\alpha_1 = 2\frac{\partial \alpha_1}{\partial a_1} \left[\frac{\mathbb{D}}{\sigma} + m_1(\theta - m_1) \right].
$$

Substituting the above expression into A yields

$$
\begin{split} \mathbb{A} &= \frac{2\sigma}{\mathbb{D}^*} \bigg\{ 2\frac{\partial \alpha_1}{\partial a_1} \bigg[\frac{\mathbb{D}}{\sigma} + m_1(\theta - m_1) \bigg] - \bigg[\frac{\mathbb{D}}{\sigma} + m_1(\theta - m_1) \bigg] \bigg(\frac{\alpha_1 \sigma(\theta + m_1)}{\mathbb{D}^*} + \frac{\partial \alpha_1}{\partial a_1} \bigg) \bigg\} \\ &= \frac{2\sigma}{\mathbb{D}^*} \bigg\{ \bigg[\frac{\mathbb{D}}{\sigma} + m_1(\theta - m_1) \bigg] \bigg[2\frac{\partial \alpha_1}{\partial a_1} - \bigg(\frac{\alpha_1 \sigma(\theta + m_1)}{\mathbb{D}^*} + \frac{\partial \alpha_1}{\partial a_1} \bigg) \bigg] \bigg\}. \end{split}
$$

One can find that

$$
2\frac{\partial \alpha_1}{\partial a_1} - \left(\frac{\alpha_1 \sigma(\theta + m_1)}{\mathbb{D}^*} + \frac{\partial \alpha_1}{\partial a_1}\right) = \frac{\partial \alpha_1}{\partial a_1} - \frac{\alpha_1 \sigma(\theta + m_1)}{\mathbb{D}^*}
$$

$$
= \frac{-\sigma \mathbb{D}^* m_1 + \sigma \mathbb{T}^*(\theta + m_1)}{\mathbb{D}^*^2} - \frac{\sigma \mathbb{T}^*(\theta + m_1)}{\mathbb{D}^*} < 0.
$$

Thus, $A < 0$. Similarly, $B = \frac{\partial^2 \pi_1^*}{\partial \xi_1 \partial \eta_2}$ $\frac{\partial^2 \pi_1^*}{\partial a_1 \partial m_1} = \frac{\partial^2 \pi_1^*}{\partial a_1^2}$ $\frac{\partial^{1} n_1}{\partial a_1^2}$ < 0 since the SOC should be negative due to the concavity of profit function. In summary, $\frac{dm_1}{d\beta} > 0$.

2.5.6 Proof of Observation 2.1

(i)
$$
\frac{\partial a_1^*}{\partial \beta} = \frac{\sigma}{3\mathbb{D} + 2\sigma[(a_1 - c_1)\hat{q}_2 + (a_2 - c_2)\hat{q}_1]} > 0
$$

(ii) Rearranging (2. A11) gives

$$
{\alpha_1}^* = \frac{1}{2} + \frac{\sigma \beta + \frac{\sigma}{2} [\nu(p_1) - \nu(p_2) + \nu(\hat{p}_1) - \nu(\hat{p}_2)]}{\mathbb{D}^*}.
$$
 (2. A15)

$$
\mathbb{D}^* = 3\mathbb{D} + 2\sigma[(a_1 - c_1)\hat{q}_2 + (a_2 - c_2)\hat{q}_1], m_1 = a_1 - c_1 \text{ and } m_2 = a_2 - c_2.
$$

The derivative of α_1^* with respect to σ is

$$
\frac{\partial \alpha_1^*}{\partial \sigma} = \left(\frac{1}{\mathbb{D}^*} \right) \left\{ \begin{aligned} &\mathbb{D}^* \left[\beta + \frac{1}{2} [\nu(p_1) - \nu(p_2) + \nu(\hat{p}_1) - \nu(\hat{p}_2)] \right] \\ &- \left[\begin{aligned} &\left(\sigma \beta + \frac{\sigma}{2} [\nu(p_1) - \nu(p_2) + \nu(\hat{p}_1) - \nu(\hat{p}_2)] \right) \times \\ &[3[\nu(\hat{p}_1) - \nu(p_2) + \nu(\hat{p}_2) - \nu(p_1)] + 2m_1 \hat{q}_2 + 2m_2 \hat{q}_1] \end{aligned} \right] \end{aligned} \right\}.
$$

When $\alpha_1^* > \alpha_2^*$, $\frac{\partial \alpha_1^*}{\partial \sigma} > 0$. Conversely, when $\alpha_1^* < \alpha_2^*$, $\frac{\partial \alpha_1^*}{\partial \sigma} < 0$.

(iii) From (2. A11), differentiating α_1^* with respect to c_1 yields the following.

$$
\frac{\partial \alpha_1^*}{\partial c_1} = \left(\frac{1}{\mathbb{D}^*} \right) \left\{\begin{matrix} \begin{bmatrix} \mathbb{D}^* \left(\frac{\sigma}{2}\right) [2 \nu'(p_1) + \nu'(\hat{p}_1)] \end{bmatrix} \\ -\left[\left(\sigma \beta + \frac{\sigma}{2} \begin{bmatrix} \nu(p_1) - \nu(p_2) \\ + \nu(\hat{p}_1) - \nu(\hat{p}_2) \end{bmatrix} \right) \end{bmatrix} \begin{Bmatrix} 3 \sigma [-2 \nu'(p_1) + \nu'(\hat{p}_1)] \\ -2 \sigma (\hat{q}_2 + a_1 - c_1) \end{Bmatrix} \right\} < 0
$$

One can conclude $\frac{\partial a_1^*}{\partial x_2^*}$ $\frac{\partial a_1^*}{\partial c_1}$ < 0 and $\frac{\partial a_2^*}{\partial c_1}$ $\frac{\partial u_2}{\partial c_1} > 0.$

(iv) After inserting (2. A8) into (2. A5), the profit-maximising fixed fee can be rewritten as

$$
F_i^* = f_i + \frac{\pi_i}{\alpha_i} - \alpha_j (a_i - c_i) \hat{q}_j.
$$

After considering the neighbourhood of a_i^* , one may find that

$$
\frac{\partial F_i^*}{\partial a_i} = -\frac{\partial \alpha_i}{\partial a_i} \left(\frac{\mathbb{D}}{\sigma}\right) - \alpha_j \left(\hat{q}_j - (a_i - c_i)\right) < 0
$$

because $\frac{\partial a_i}{\partial a_i}$ > 0 . When network *i* decides to increase its access charge in stage 1 $a_i = a_i^*$

of the game, it also decreases its fixed fee in the later stage.

Analogously, In the neighbourhood of a_i^* , one can conclude that

$$
\frac{\partial F_j^*}{\partial a_i} = -\alpha_j \hat{q}_j + \frac{\partial \alpha_j}{\partial a_i} \left(\frac{\mathbb{D}}{\sigma}\right) + 2 \frac{\partial \alpha_j}{\partial a_i} (a_j - c_j) \hat{q}_i < 0
$$

since $\frac{\partial a_j}{\partial a_i}$ < 0. When network *i* increases its access charge, network *j* decides c_i <a≀≤ a_i^*

to reduce its own fixed fee in response.

2.5.7 Proof of the degree of substitutability in a shared-market equilibrium

To focus on a shared-market equilibrium, the denominator of (2. A1) should be positive.

$$
1 + \sigma[\nu(\hat{p}_1) - \nu(p_2) + \nu(\hat{p}_2) - \nu(p_1)] > 0
$$

$$
\frac{1}{\sigma} > -[\nu(\hat{p}_1) - \nu(p_1) + \nu(\hat{p}_2) - \nu(p_2)]
$$

 $p \in \{p_1, p_2, \hat{p}_1, \hat{p}_2\}$ whereas $p_1^* = 2c_1, p_2^* = 2c_2, \hat{p}_1^* = c_1 + a_2, \hat{p}_2^* = c_2 + a_1$ and $m_i = a_i - c_i.$

Substituting $v(p) = \frac{1}{2}$ $rac{1}{2} - p + \frac{p^2}{2}$ $\frac{y}{2}$ in the above expression yields

$$
\frac{1}{\sigma} > (c_2 - c_1)^2 + (m_1 + m_2)(1 - c_1 - c_2) - \frac{1}{2}(m_1^2 + m_2^2).
$$

Therefore, the degree of substitutability (σ) must not be too high when compared with cost differential in order that a shared-market equilibrium exists.

2.5.8 Proof of Remark 2.2

Both networks are regulated to set their access charges at cost, $a_1 = c_1$, $a_2 = c_2$. According to (2.A7), network 1 gains larger or equal market share when $\beta \geq -\frac{1}{2}$ $\frac{1}{2} \left[\frac{\nu(\hat{p}_1^*) - \nu(p_2^*)}{-\nu(\hat{p}_2^*) + \nu(p_1^*)} \right]$ $(\nu(\rho_1)^{\nu} - \nu(\rho_2)^{\nu})$. In this scenario, the equilibrium prices are $p_1^* = 2c_1$, $(\nu(\rho_2^*) + \nu(\rho_1^*)$. $p_2^* = 2c_1$ and $\hat{p}_1^* = \hat{p}_2^* = c_1 + c_2$. Thus, $\hat{\hat{\beta}} = -\frac{1}{2}$ $\frac{1}{2} [\nu(2c_1) - \nu(2c_2)]; \hat{\beta} < \hat{\beta} < 0.$

2.5.9 Proof of Proposition 2.5

Under the symmetric regulation, $a_1 = c_1$, $a_2 = c_2$, $p_2^* > \hat{p}_2^*$, $\hat{p}_1^* > p_1^*$ and $\hat{p}_1^* = \hat{p}_2^*$. From (2.45), the fixed fee of the low-cost firm (network 1) is

$$
F_1^* = f_1 + \alpha_1 \frac{\mathbb{D}}{\sigma}.
$$

The fixed fee of the high-cost firm (network 2) is

$$
F_2^* = f_2 + \alpha_2 \frac{\mathbb{D}}{\sigma}.
$$

Suppose $\alpha_1 > \alpha_2$ and $f_1 = f_2$ with no access mark-up, it is found that $F_1^* > F_2^*$.

According to (2. A8) , when the symmetric regulation is imposed, access revenue vanishes and each firm earns profit only from its fixed fee revenue. The equilibrium profit is shown below.

$$
\pi_i^* = \alpha_i^2 \left[\frac{\mathbb{D}}{\sigma}\right]
$$

If $\alpha_1 > \alpha_2$, $\pi_1^* - \pi_2^* = (\alpha_1^2 - \alpha_2^2) \Big|_{\pi}^{\mathbb{D}}$ $\frac{1}{\sigma}$ > 0. As a result, network 1 with larger market share has higher profit than network 2.

2.5.10 Proof of Proposition 2.6

This scenario focuses on the situation where the established firm with low cost (network 1) is dominant in the market $(\alpha_1^* > \frac{1}{2})$ $\frac{1}{2}$).

(i) When the two networks' access charges are set at cost by regulation ($a_1 = c_1$) and $a_2 = c_2$),

$$
\left. \frac{\partial \pi_2^*}{\partial a_2} \right|_{a_2 = c_2} = -2a_2 \frac{\partial a_1}{\partial a_2} \left(\frac{\mathbb{D}}{\sigma} \right) > 0
$$

since $\frac{\partial a_1}{\partial a_2}$ ∗ | $a_2 = c_2$ $< 0.$

After considering the above derivative in the neighbourhood of $a_2 = c_2$, network 2 increases its access charge to gain more profit. As a result, the market share of the low-cost established firm decreases while that of the new firm increases.

(ii) The new firm can set a_2 arbitrarily under the asymmetric regulation. By considering only at point $a_1 = c_1$ and $a_2 = c_2$, one may rewrite (2.45) as

$$
F_1^* = f_1 + \alpha_1 \frac{\mathbb{D}}{\sigma}.
$$

Taking differentiation of the profit-maximising fixed fee with respect to the new firm's access charge shows

$$
\frac{\partial F_1^*}{\partial a_2} = \alpha_1^*(-\hat{q}_1) + \frac{\partial \alpha_1^*}{\partial a_2} \left(\frac{\mathbb{D}}{\sigma}\right).
$$

When the regulator sets $a_1 = c_1$, at $a_2 = c_2$,

$$
\frac{\partial F_1^*}{\partial a_2}\Big|_{\substack{a_1 = c_1 \\ a_2 = c_2}} = -\alpha_1 \mathbf{a}_1 + \left(\frac{\partial \alpha_1^*}{\partial a_2}\Big|_{\substack{a_1 = c_1 \\ a_2 = c_2}}\right) \left(\frac{\mathbb{D}}{\sigma}\right)
$$

where

$$
\left.\frac{\partial \alpha_1^*}{\partial a_2}\right|_{\substack{a_1=c_1\\a_2=c_2}}=\frac{\sigma \widehat{q}_1(\mathbb{T}^*-\mathbb{D}^*)}{\mathbb{D}^*^2}.
$$

Substituting $\mathbb{T}^*|_{\substack{a_1 = c_1 \\ a_2 = c_2}} = \frac{1}{2}$ $\frac{1}{2} + \sigma \beta + \sigma [\nu(\hat{p}_1) - \nu(p_2)] + \mathbb{D}$ and $\mathbb{D}^*|_{\substack{a_1 = c_1 \\ a_2 = c_2}} = 3\mathbb{D}$ in the above derivative of fixed fee yields the following.

$$
\frac{\partial F_1^*}{\partial a_2}\Big|_{\substack{a_1 = c_1 \\ a_2 = c_2}} = -\alpha_1^* \hat{q}_1 + \left(\frac{\sigma \hat{q}_1(\mathbb{T}^* - \mathbb{D}^*)}{\mathbb{D}^*} \right) \left(\frac{\mathbb{D}}{\sigma}\right)
$$

\n
$$
= -\alpha_1^* \hat{q}_1 - \left(\frac{\mathbb{D}^* - \mathbb{T}^*}{\mathbb{D}^*}\right) \left(\frac{\mathbb{D}}{\mathbb{D}^*}\right) \hat{q}_1
$$

\n
$$
= -\alpha_1^* \hat{q}_1 - (1 - \alpha_1^*) \left(\frac{\mathbb{D}}{3\mathbb{D}}\right) \hat{q}_1
$$

\n
$$
= -\alpha_1^* \hat{q}_1 - (1 - \alpha_1^*) \left(\frac{1}{3}\right) \hat{q}_1
$$

\n
$$
= \left(-\frac{1}{3} - \frac{2}{3}\alpha_1^*\right) \hat{q}_1
$$

Since $\alpha_1^*|_{\substack{a_1 = c_1 \\ a_2 = c_2}} > \frac{1}{2}$ $\frac{1}{2}$, $\frac{\partial F_1^*}{\partial a_2}$ $\frac{\partial r_1}{\partial a_2}\Big|_{\substack{a_1 = c_1 \\ a_2 = c_2}}$ < 0 .

The new firm has incentive to set a mark-up on access charge. When the new firm sets its above-cost access charge, the market share of the low-cost established firm decreases. Then, the established firm adjusts to the new equilibrium by reducing its fixed fee in order to compete with the new firm.

One may consider the effect of the new firm's access mark-up on its own fixed fee from (2. A5) and find that

$$
F_2^* = f_2 + (1 - {\alpha_1}^*) \frac{\mathbb{D}}{\sigma} + (1 - 2{\alpha_1}^*)(\alpha_2 - c_2)\hat{q}_1.
$$

$$
\frac{\partial F_2^*}{\partial a_2} = (1 - a_1^*)[-\hat{q}_1] - \frac{\partial a_1^*}{\partial a_2} \frac{D}{\sigma} + (1 - 2a_1^*)[(a_2 - c_2)\hat{q}_1' + \hat{q}_1]
$$

At $a_2 = c_2$,

$$
\frac{\partial F_2^*}{\partial a_2}\Big|_{\substack{a_1=c_1\\a_2=c_2}} = (1 - a_1^*)(-\hat{q}_1) - \left[\left(\frac{\partial a_1^*}{\partial a_2}\Big|_{\substack{a_1=c_1\\a_2=c_2}} \right) \left(\frac{\mathbb{D}}{\sigma} \right) \right] + (1 - 2a_1^*)\hat{q}_1
$$
\n
$$
= -a_1^*\hat{q}_1 - \left[\left(\frac{\partial a_1^*}{\partial a_2}\Big|_{\substack{a_1=c_1\\a_2=c_2}} \right) \left(\frac{\mathbb{D}}{\sigma} \right) \right]
$$
\n
$$
\frac{\partial a_1^*}{\partial a_2}\Big|_{\substack{a_1=c_1\\a_2=c_2}} = \frac{\sigma \hat{q}_1(\mathbb{T}^* - \mathbb{D}^*)}{\mathbb{D}^*^2}
$$

Substituting $\mathbb{T}^* \left| \begin{array}{cc} a_1 = c_1 \\ a_2 = c_2 \end{array} \right|$ $\frac{a_1 = c_1}{a_2 = c_2} = \frac{1}{2}$ $\frac{1}{2} + \sigma \beta + \sigma [\nu(\hat{p}_1) - \nu(p_2)] + \mathbb{D}$ and $\mathbb{D}^* \mid_{a_2 = c_2}^{a_1 = c_1}$ $a_1 = c_1 = 3D$ in the derivative of the new firm's fixed fee gives the following.

$$
\frac{\partial F_2^*}{\partial a_2}\Big|_{\substack{a_1=c_1\\a_2=c_2}} = -\alpha_1^* \hat{q}_1 - \left[\frac{\sigma \hat{q}_1(\mathbb{T}^* - \mathbb{D}^*)}{\mathbb{D}^*} \left(\frac{\mathbb{D}}{\sigma}\right)\right]
$$

$$
= -\alpha_1^* \hat{q}_1 + \left(\frac{\mathbb{D}^* - \mathbb{T}^*}{\mathbb{D}^*}\right) \left(\frac{\mathbb{D}}{\mathbb{D}^*}\right) \hat{q}_1
$$

$$
= -\alpha_1^* \hat{q}_1 + (1 - \alpha_1^*) \left(\frac{\mathbb{D}}{3\mathbb{D}}\right) \hat{q}_1
$$

$$
= -\alpha_1^* \hat{q}_1 + (1 - \alpha_1^*) \left(\frac{1}{3}\right) \hat{q}_1
$$

$$
=\left(-\frac{4}{3}\alpha_1{}^*+\frac{1}{3}\right)\widehat{q}_1
$$

Since $\alpha_1^*|_{\substack{a_1 = c_1 \\ a_2 = c_2}} > \frac{1}{2}$ $\frac{1}{2}$, $\frac{\partial F_2^*}{\partial a_2}$ $\frac{\partial r_2}{\partial a_2}\Big|_{\substack{a_1 = c_1 \\ a_2 = c_2}}$ < 0. When the new firm's access charge increases, its equilibrium fixed fee decreases.

The effect of the new firm's access mark-up on its own fixed fee is

$$
\left.\frac{\partial F_2^*}{\partial a_2}\right|_{\substack{a_1=c_1\\ a_2=c_2}}=\left(-\frac{4}{3}{\alpha_1}^*+\frac{1}{3}\right)\hat{q}_1.
$$

The effect of the new firm's access mark-up on the low-cost established firm's fixed fee is

$$
\frac{\partial F_1^*}{\partial a_2}\bigg|_{\substack{a_1=c_1\\a_2=c_2}}=\left(-\frac{1}{3}-\frac{2}{3}{\,\alpha_1}^*\right)\hat q_1.
$$

Comparing these two effects gives

$$
\left(-\frac{1}{3} - \frac{2}{3} \alpha_1{}^*\right)\hat{q}_1 < \left(-\frac{4}{3} \alpha_1{}^* + \frac{1}{3}\right)\hat{q}_1 < 0.
$$

Hence, the effect of the asymmetric regulation on the low-cost established firm's fixed fee outweighs the effect on the new firm's fixed fee.

 (iii) From $(2.A8)$, the new firm's profit is

$$
\pi_2^* = \alpha_2^2 \left[\frac{\mathbb{D}}{\sigma} + (a_2 - c_2)\hat{q}_1 \right]
$$

$$
\frac{\partial \pi_2^*}{\partial a_2} = \alpha_2^2 (a_2 - c_{t2}) \hat{q}'_1 - 2\alpha_2 \frac{\partial \alpha_1}{\partial a_2} \left[\frac{\mathbb{D}}{\sigma} + (a_2 - c_{t2})\hat{q}_1 \right] = 0
$$

When the low-cost established firm sets access charge at cost by regulation, at $a_2 = c_2$,

$$
\left. \frac{\partial \pi_2^*}{\partial a_2} \right|_{\substack{a_1 = c_1 \\ a_2 = c_2}} = -2\alpha_2 \frac{\partial \alpha_1}{\partial a_2} \left(\frac{\mathbb{D}}{\sigma} \right) > 0.
$$

This is because at $a_2 = c_2$, $\frac{\partial a_1}{\partial a_2}$ $\frac{\partial u_1}{\partial a_2}$ < 0 and $\mathbb{D} > 0$. Cost-based access charge is not the profit-maximising outcome. When Network 2 can set its access charge, it increases access charge with access mark-up. Accordingly, its profit is higher than that under the symmetric regulation.

From (2. A8), when the low-cost established firm is regulated, it has no profit from access revenue. Its equilibrium profit is shown below.

$$
\pi_1^* = \alpha_1^2 \left(\frac{\mathbb{D}}{\sigma}\right)
$$

$$
\frac{\partial \pi_1^*}{\partial a_2}\Big|_{\substack{a_1 = c_1 \\ a_2 = c_2}} = \alpha_1^2 \nu'(\hat{p}_1) + 2\alpha_1 \frac{\partial \alpha_1}{\partial a_2} \left(\frac{\mathbb{D}}{\sigma}\right)
$$

$$
\frac{\partial \pi_1^*}{\partial a_2}\Big|_{\substack{a_1 = c_1 \\ a_2 = c_2}} = \alpha_1^2(-\hat{q}_1) + 2\alpha_1 \frac{\partial \alpha_1}{\partial a_2} \left(\frac{\mathbb{D}}{\sigma}\right)
$$

$$
< 0, \left.\frac{\partial \pi_1^*}{\partial a_2}\right|_{a_2 = c_2} < 0.
$$

 $a_2 = c_2$ $a_2 = c_2$

Since $\frac{\partial \alpha_1^*}{\partial x_1^*}$

 $\frac{\partial u_1}{\partial a_2}$

2.5.11 Proof of Remark 2.3

Under the asymmetric cost-based access charge regulation, $a_1 = c_1, a_2^* > c_2$. According to (2. A7), the low-cost established firm has larger market share when $\beta > -\frac{1}{2}$ $\frac{1}{2}[\nu(\hat{p}_1^*) - \nu(p_2^*) - \nu(\hat{p}_2^*) + \nu(p_1^*)]$. In equilibrium, $p_1^* = 2c_1$, $p_2^* = 2c_2$, $\hat{p}_1^* = c_1 + a_2^*$ and $\hat{p}_2^* = c_1 + c_2$. After substituting the equilibrium prices in the above expression, $\hat{\hat{\beta}} = -\frac{1}{3}$ $\frac{1}{2} [v(c_1 + a_2^*) - v(2c_2) - v(c_1 + c_2) + v(2c_1)] > \hat{\beta} > \hat{\beta}.$

2.5.12 Proof of Proposition 2.7a

This section focuses only on the case in which the low-cost established firm (network 1) is dominant with larger market share.

Rearranging (2.A15) by replacing $\alpha_2^* = 1 - \alpha_1^*$ in the equation, one can obtain

$$
{\alpha_2}^* = \frac{1}{2} + \frac{(-\sigma\beta) + \frac{\sigma}{2}[\nu(p_2) - \nu(p_1) + \nu(\hat{p}_2) - \nu(\hat{p}_1)]}{\mathbb{D}^*}.
$$
 (2. A16)

 $M_2 = (-\sigma \beta) + \frac{\sigma}{2}$ $\frac{\partial}{\partial z} [\nu(p_2) - \nu(p_1) + \nu(\hat{p}_2) - \nu(\hat{p}_1)]$ and inserting it in (2.416) yields the following expressions.

$$
\alpha_{2^{*}_{unreg}} = \frac{1}{2} + \frac{M_{2^{unreg}}}{D^{*}_{unreg}}
$$

 \mathbb{M}_{2 _{unreg} = $(-\sigma\beta)$ + σ $\frac{6}{2}[\nu(2c_2)-\nu(2c_1)+\nu(c_2+a_1^*)-\nu(c_1+a_2^*)]$

$$
\mathbb{D}^*_{\text{unreg}} = 3 + 3\sigma \begin{bmatrix} -\nu(2c_2) - \nu(2c_1) \\ + \nu(c_2 + a_1^*) + \nu(c_1 + a_2^*) \end{bmatrix} + 2\sigma m_1^* \hat{q}_2 + 2\sigma m_2^* \hat{q}_1
$$

$$
\alpha_{2\,sym} = \frac{1}{2} + \frac{M_{2\,sym}}{D^*_{sym}}
$$

$$
M_{2sym} = -\sigma\beta + \frac{\sigma}{2} [\nu(2c_2) - \nu(2c_1) + \nu(c_1 + c_2) - \nu(c_1 + c_2)]
$$

$$
\mathbb{D}^*_{sym} = 3 + 3\sigma [-\nu(2c_2) - \nu(2c_1) + 2\nu(c_1 + c_2)]
$$

In the unregulated market, both networks choose $p_1^* = 2c_1$, $\hat{p}_1^* = c_1 + a_2^*$, $p_2^* = 2c_2$ and $\hat{p}_2^* = c_2 + a_1^*$. Under the symmetric regulation, the market outcome is $p_1^* = 2c_1$, $\hat{p}_1^* = c_1 + c_2$, $p_2^* = 2c_2$ and $\hat{p}_2^* = c_1 + c_2$. After substituting these market outcomes in the above expressions, one can find that \mathbb{M}_{2 unreg < 0 certainly and M_{2sym} < 0 in most situations where β is not too strongly negative. A comparison between α_{2sym}^* and α_{2inner}^* is shown below.

$$
\alpha_{2\,sym}^{*} - \alpha_{2\,unreg}^{*} = \frac{M_{2\,sym}}{D^{*}\,sym} - \frac{M_{2\,unreg}}{D^{*}\,unreg}
$$

 m_i is access mark-up of network *i*. Suppose network 1 is the dominant established firm, $m_1^* > m_2^*$ in the unregulated market,³² one may write the following expressions.

$$
M_{2sym} - M_{2unreg} = \frac{\sigma}{2} \Big[(m_1^* - m_2^*) \Big(\theta - \frac{(m_1^* + m_2^*)}{2} \Big) \Big] > 0
$$

$$
D^*_{sym} - D^*_{unreg} = \sigma \Big(\theta m_1^* + \frac{m_1^{*2}}{2} + \theta m_2^* + m_2^{*2} \Big) > 0
$$

It is implied that, $\mathbb{D}^*_{sym} > \mathbb{D}^*_{unreg} > 0 > M_{2}^{*} > M_{2}^{*}$, accordingly, $\alpha_{2sym}^* - \alpha_{2unreg}^* > 0$ and $\alpha_{2sym}^* > \alpha_{2unreg}^*$.

Due to $\frac{\partial \alpha_1}{\partial a_2}$ $a_2 = c_2$ < 0, network 1's market share decreases when network 2 raises its access charge. In other words, under the asymmetric regulation, network 2 sets a mark-up on access charge and expands its market share. Thus, $\alpha_{2sym}^* < \alpha_{2*sym}^*$. In summary, $\alpha_{2,n+1}^* < \alpha_{2,n}^* < \alpha_{2,n}^*$ and $\alpha_{1,n+1}^* > \alpha_{1,n+1}^* > \alpha_{1,n+1}^*$.

2.5.13 Proof of Proposition 2.7b

One can rewrite (2. A8) as the equation shown below.

$$
\pi_{2_{unreg}}^{*} = \left(\alpha_{2_{unreg}}^{*}\right)^{2} \left[\frac{\mathbb{D}_{unreg}}{\sigma} + m_{2_{unreg}}^{*}\hat{q}_{1}\right]
$$

$$
\pi_{2_{sym}}^{*} = \left(\alpha_{2_{sym}}^{*}\right)^{2} \left[\frac{\mathbb{D}_{sym}}{\sigma}\right]
$$

$$
\pi_{2_{asym}}^{*} = \left(\alpha_{2_{asym}}^{*}\right)^{2} \left[\frac{\mathbb{D}_{asym}}{\sigma} + m_{2_{asym}}^{*}\hat{q}_{1}\right]
$$

where

 $\overline{}$

$$
\mathbb{D}_{unreg} = 1 + \sigma \left[\begin{array}{c} \nu \left(c_1 + c_2 + m_{2unreg}^* \right) - \nu(2c_2) \\ + \nu \left(c_1 + c_2 + m_{1unreg}^* \right) - \nu(2c_1) \end{array} \right]
$$

 32 See proof in the appendix (Section 2.5.4).

$$
\mathbb{D}_{sym} = 1 + \sigma[\nu(c_1 + c_2) - \nu(2c_2) + \nu(c_1 + c_2) - \nu(2c_1)]
$$

$$
\mathbb{D}_{asym} = 1 + \sigma[\nu(c_1 + c_2 + m_{2asym}^*) - \nu(2c_2) + \nu(c_1 + c_2) - \nu(2c_1)].
$$

Since $\alpha_2^*_{asym} > \alpha_2^*_{sym}$, $\frac{\partial \pi_2^*}{\partial a_2}$ $\frac{\partial u_2}{\partial a_2}\Big|_{\substack{a_1 = c_1 \\ a_2 = c_2}}$ > 0 and $a_2^* > c_2(m_2^*)$ $_{asym} > 0$), one can

conclude that $\pi_{2,asym}$ > $\pi_{2,sym}$.

$$
\left[\frac{\mathbb{D}_{unreg}}{\sigma} + m_{2unreg}^* \hat{q}_1\right] - \left[\frac{\mathbb{D}_{sym}}{\sigma}\right]
$$

=
$$
-\frac{m_{2unreg}^*}{2} + \left[-m_{1unreg}^* + (c_1 + c_2)m_{1unreg}^* + \frac{(m_{1unreg}^*)^2}{2}\right]
$$

It is concluded that the expression $\vert -m_1^* \vert$ $_{unreg} + (c_1 + c_2)m_1^*$ $_{unreg} + \frac{(m_{1unreg}^*)^2}{2}$ $\left|\frac{nreg}{2}\right|$ < 0

since $v'(p) < 0$ and $v(c_1 + c_2 + m_{1$ _{unreg} $) < v(c_1 + c_2)$. The first term is also negative because m_2^* $_{unreg} > 0$. Thus, $\left[\frac{\mathbb{D}_{unreg}}{\sigma}\right]$ $rac{nreg}{\sigma} + m_2^*$ $\left[\frac{w_{sym}}{\sigma}\hat{q}_1\right] < \left[\frac{w_{sym}}{\sigma}\right]$ $\left[\frac{sym}{\sigma}\right]$. The previous section shows that $\alpha_{2^{*}_{unreg}} < \alpha_{2^{*}_{sym}}$. Therefore, $\pi_{2^{*}_{unreg}} < \pi_{2^{*}_{sym}}$. In conclusion, $\pi_{2,nreg}^* < \pi_{2,nre}^* < \pi_{2,nre}^*$

Differentiating (2. A8) with respect to a_2 yields the following expression.

$$
\frac{\partial \pi_1^*}{\partial a_2}\Big|_{\substack{a_1=c_1\\a_2=c_2}} = -\alpha_1^2 \hat{q}_1 + 2\alpha_1 \frac{\partial \alpha_1}{\partial a_2} \left[\frac{\mathbb{D}}{\sigma}\right] < 0
$$

where $\frac{\partial a_1}{\partial a_2}\Big|_{a_1=c_1}$ $a_2 = c_2$ < 0 .

At $a_1 = c_1$, if network 2 increases its access mark-up $(a_2 > c_2)$, network 1 inevitably loses its profit due to the asymmetric regulation, i.e. $\pi_{1sym}^* > \pi_{1asym}^*$.

2.5.14 Proof of the effect of the asymmetric regulation on aggregate consumer surplus

According to (2.9) , differentiating the sum of net utility of network 1's subscribers with respect to a_2 yields the following equation.

$$
\frac{\partial \alpha_1(w_1 + \beta)}{\partial a_2} = 2\alpha_1 v(p_1) \frac{\partial \alpha_1}{\partial a_2} + (\alpha_1 - \alpha_1^2) v'(p_1) + (1 - 2\alpha_1) v(p_1) \frac{\partial \alpha_1}{\partial a_2}
$$

$$
- \alpha_1 \frac{\partial F_1}{\partial a_2} - F_1 \frac{\partial \alpha_1}{\partial a_2} + \beta \frac{\partial \alpha_1}{\partial a_2}
$$

$$
= \frac{\partial \alpha_1}{\partial a_2} [2\alpha_1 v(p_1) + (1 - 2\alpha_1) v(p_1) - F_1 + \beta]
$$

$$
+ (\alpha_1 - \alpha_1^2) v'(p_1) - \alpha_1 \frac{\partial F_1}{\partial a_2}
$$

The sign is ambiguous. The asymmetric regulation $(a_2 > c_2)$ may not guarantee an increase in the total of net utility among the consumers of network 1.

The effect of a_2 on the sum of the net utility of network 2's subscribers is

$$
\frac{\partial (1 - \alpha_1)w_2}{\partial a_2} = -2(1 - \alpha_1)\nu(p_2)\frac{\partial \alpha_1}{\partial a_2} + (1 - 2\alpha_1)\nu(\hat{p}_2)\frac{\partial \alpha_1}{\partial a_2}
$$

$$
-(1 - \alpha_1)\frac{\partial F_2}{\partial a_2} + \frac{\partial \alpha_1}{\partial a_2}F_2
$$

$$
= \frac{\partial \alpha_1}{\partial a_2}[-2(1 - \alpha_1)\nu(p_2) + (1 - 2\alpha_1)\nu(\hat{p}_2) + F_2]
$$

$$
-(1 - \alpha_1)\frac{\partial F_2}{\partial a_2}
$$

At $a_2 = c_2$,

$$
\left. \frac{\partial (1 - \alpha_1) w_2}{\partial a_2} \right|_{a_2 = c_2} = \frac{\partial \alpha_1}{\partial a_2} [-2(1 - \alpha_1)v(p_2) + (1 - 2\alpha_1)v(p_2) + F_2] - (1 - \alpha_1) \frac{\partial F_2}{\partial a_2}.
$$

Note that $\frac{\partial \alpha_1}{\partial x_1}$ $\frac{\partial u_1}{\partial a_2}$ $a_2 = c_2$ $< 0, \frac{\partial F_2^*}{\partial x}$ $\frac{\partial r_2}{\partial a_2}$ $a_2 = c_2$ < 0 .

The sign is also ambiguous. The asymmetric regulation may not guarantee an increase in the total of net utility among the consumers of network 2 either.

The effect of the asymmetric regulation on the total disutility is shown below.

$$
\frac{\partial \left[\frac{[\alpha_1^2 + (1 - \alpha_1)^2]}{4\sigma} \right]}{\partial a_2} = \frac{\partial \left[\frac{2\alpha_1^2 - 2\alpha_1 + 1}{4\sigma} \right]}{\partial a_2}
$$

$$
= \left(\frac{1}{2\sigma} \right) (2\alpha_1 - 1) \frac{\partial \alpha_1}{\partial a_2}
$$

At $a_2 = c_2$,

$$
\frac{\partial \left[\frac{\left[\alpha_1{}^2 + (1 - \alpha_1)^2 \right]}{4\sigma} \right]}{\partial a_2} \Bigg|_{a_2 = c_2} = \left(\frac{1}{2\sigma} \right) (2\alpha_1 - 1) \left(\frac{\partial \alpha_1}{\partial a_2} \Big|_{a_2 = c_2} \right) < 0
$$

where $\frac{\partial a_1}{\partial a_2}\Big|_{\substack{a_1 = c_1 \\ a_2 = c_2}}$ < 0 and $\alpha_1^*|_{\substack{a_1 = c_1 \\ a_2 = c_2}} > \frac{1}{2}$ $\frac{1}{2}$ in case network 1 is the dominant established

firm. The asymmetric regulation can decrease the total disutility.

Consequently, the derivative of consumer surplus with respect to network 2's access charge is as follows.

$$
\frac{\partial CS}{\partial a_2} = 2\alpha_1 v(p_1) \frac{\partial \alpha_1}{\partial a_2} + (\alpha_1 - \alpha_1^2) v'(p_1) + (1 - 2\alpha_1) v(p_1) \frac{\partial \alpha_1}{\partial a_2} - \alpha_1 \frac{\partial F_1}{\partial a_2}
$$

$$
-F_1 \frac{\partial \alpha_1}{\partial a_2} + \beta \frac{\partial \alpha_1}{\partial a_2} - 2(1 - \alpha_1) v(p_2) \frac{\partial \alpha_1}{\partial a_2} + (1 - 2\alpha_1) v(p_2) \frac{\partial \alpha_1}{\partial a_2}
$$

$$
-(1 - \alpha_1) \frac{\partial F_2}{\partial a_2} + \frac{\partial \alpha_1}{\partial a_2} F_2 + \frac{1}{2\sigma} (1 - 2\alpha_1) \frac{\partial \alpha_1}{\partial a_2}
$$

Substituting F_1^* and F_2^* from (2.A5) in the above equation and rearranging it yield the following.

$$
\frac{\partial CS}{\partial a_2} = \frac{\partial \alpha_1}{\partial a_2} \left\{ \begin{array}{c} 3(1 - 2\alpha_1) \left[\frac{1}{\sigma} + v(\hat{p}_1) + v(\hat{p}_2) - v(p_1) - v(p_2) \right] + \beta \\ - (1 - \alpha_1)v(p_2) + v(p_1) - \frac{1}{2\sigma}(1 - 2\alpha_1) - \frac{1}{\sigma} \left(\frac{\mathbb{T}^{*2}}{\mathbb{D}^* - \mathbb{T}^*} \right) \end{array} \right\}
$$

$$
= \frac{\partial \alpha_1}{\partial a_2} \left\{ \begin{array}{c} 3(1 - 2\alpha_1) \left[\frac{1}{\sigma} + v(\hat{p}_1) + v(\hat{p}_2) - v(p_1) - v(p_2) \right] + \beta \\ + [v(p_1) - v(p_2)] + \alpha_1 v(p_2) + \frac{1}{\sigma} \left[\left(\frac{1}{2} \right) \left(\frac{2\mathbb{T}^* - \mathbb{D}^*}{\mathbb{D}^*} \right) - \left(\frac{\mathbb{T}^{*2}}{\mathbb{D}^* - \mathbb{T}^*} \right) \right] \end{array} \right\}
$$

Note that $\frac{\partial \alpha_1}{\partial a_2} < 0, \frac{1}{\sigma}$ $\frac{1}{\sigma} + v(\hat{p}_1) + v(\hat{p}_2) - v(p_1) - v(p_2) > 0$. After considering only the case of a shared-market equilibrium where network 1 is dominant, one can find out that $\left(\frac{1}{2}\right)$ $\frac{1}{2}$ $\left(\frac{2\mathbb{T}^*-\mathbb{D}^*}{\mathbb{D}^*}\right)$ $(\mathbb{D}^*)^*$ - $(\mathbb{D}^*)^*$ - $(\mathbb{D}^*)^*$ $\frac{1}{\mathbb{D}^* - \mathbb{T}^*}$ < 0. If β and σ is not too high and cost differential is not too significant, it is likely that an increase in network 2's access mark-up enhances consumer surplus $\left(\frac{\partial \mathcal{C} S}{\partial x}\right)$ $\frac{\partial c_3}{\partial a_2} > 0$.

2.5.15 Proof of the effect of the asymmetric regulation on aggregate producer surplus

Differentiating (2.10) with respect to a_2 yields the following expressions.

$$
\frac{\partial PS}{\partial a_2}\Big|_{\substack{a_1=c_1 \\ a_2=c_2}} = \frac{\partial \pi_1^*}{\partial a_2}\Big|_{\substack{a_1=c_1 \\ a_2=c_2}} + \frac{\partial \pi_2^*}{\partial a_2}\Big|_{\substack{a_1=c_1 \\ a_2=c_2}} = \frac{\partial \pi_1^*}{\partial a_2}\Big|_{\substack{a_2=c_2 \\ a_2=c_2}} = -\alpha_1^2 \hat{q}_1 + 2\alpha_1 \frac{\partial \alpha_1}{\partial a_2} \Big|_{\substack{a_1=b_1 \\ a_2=b_2}}^{\text{max}} \ge 0
$$
\n
$$
\frac{\partial \pi_2^*}{\partial a_2}\Big|_{\substack{a_1=c_1 \\ a_2=c_2}} = -2(1-\alpha_1) \frac{\partial \alpha_1}{\partial a_2} \Big|_{\substack{a_1 \\ a_2}}^{\text{max}} \Big|_{\substack{a_1 = c_1 \\ a_2 = c_2}} < 0.
$$
\nwhere $\frac{\partial \alpha_1}{\partial a_2}\Big|_{\substack{a_1 = c_1 \\ a_2 = c_2}} < 0$.

$$
\frac{\partial PS}{\partial a_2}\Big|_{\substack{a_1 = c_1 \\ a_2 = c_2}} = -\alpha_1^2 \hat{q}_1 + 2\alpha_1 \frac{\partial \alpha_1}{\partial a_2} \left[\frac{\mathbb{D}}{\sigma}\right] - 2(1 - \alpha_1) \frac{\partial \alpha_1}{\partial a_2} \left[\frac{\mathbb{D}}{\sigma}\right]
$$

$$
= -\alpha_1^2 \hat{q}_1 - 2(1 - 2\alpha_1) \frac{\partial \alpha_1}{\partial a_2} \left[\frac{\mathbb{D}}{\sigma}\right]
$$

where $\alpha_1^*|_{\substack{a_1 = c_1 \\ a_2 = c_2}} > \frac{1}{2}$ $\frac{1}{2}$, $\frac{\partial \alpha_1}{\partial a_2}$ $\frac{\partial a_1}{\partial a_2}\Big|_{a_1=c_1}$ $a_2 = c_2$ $< 0.$

Therefore,

$$
\frac{\partial PS}{\partial a_2}\Big|_{\substack{a_1=c_1\\a_2=c_2}}=-\alpha_1^2\hat{q}_1-2(1-2\alpha_1)\frac{\partial\alpha_1}{\partial a_2}\left[\frac{\mathbb{D}}{\sigma}\right]<0.
$$

Chapter 3

Infrastructure Sharing in Telecommunications

3.1 Introduction

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In the broadband internet market, the upward trend is towards next generation access network, e.g. fibre optic network, with high capacity to satisfy the increasing demand for high-speed internet.³³ In addition to high quality of service, network operators may gain advantage from cost reduction and product differentiation in consequence of the advanced technology. However, investment in new technology for quality-enhanced or value-added service involves a large amount of funding. A firm must incur high investment cost in building its own infrastructure if it chooses facility-based entry (a conventional mode of market entry). In accordance with entryfacilitating policies by regulators, an entrant may choose service-based entry and pay a fee (access price) to the incumbent in return for leasing access network from the incumbent's facilities. In line with the ladder of investment (Cave, 2006; Cave and Vogelsang, 2003), this mode of entry can stimulate competition in the retail market when facility-based entry is not a feasible option for an entrant because of a lack of funding and the unprofitable duplication of facility. Additionally, co-investment is an alternative to conventional investment to hasten the advanced-technology deployment,

 33 Likewise, in the mobile telecommunications market, 2G wireless technology has gradually been replaced by 3G and most recently 4G with the prospect of higher technology which enables consumers to enjoy not only voice service, but also value-added non-voice services such as data transfer and the Internet according to technological convergence.

especially in the areas with low demand and high investment cost. The co-investors may separately build their own infrastructure in the high-demand areas and agree on co-investment in the low-demand areas in compliance with the coverage obligation. Moreover, from the engineering aspect, some co-investors have contracts to share only a small portion of selected facilities, while other co-investors agree to share the whole infrastructure.³⁴ It is asserted that cost saving is proportional to the intensity of infrastructure sharing (Beckman and Smith, 2005; Song, Zo and Lee, 2012).

The optimal approach to stimulate investment in quality upgrade, especially the next generation network deployment, has been a debatable issue for over a decade under a tension between static and dynamic efficiency, as discussed in both theoretical and empirical literature. The concept of local loop unbundling has long been implemented to encourage service-based competition, but it is claimed that this approach cannot promote investment and innovation in the long run (Bacache, Bourreau and Gaudin, 2014; Bouckaert, van Dijk and Verboven, 2010; Briglauer, Ecker and Gugler, 2013; Briglauer, Gugler and Haxhimusa, 2015; Crandall, Eisenach and Ingraham, 2013; Grajek and Röller, 2012). Other infrastructure-sharing approaches have been critically discussed in the context of dynamic efficiency. The related studies examine the effects of co-investment in several aspects. Most of them focus only on how co-investment induces the extent of investment in the absence of firm asymmetry (Bourreau, Cambini and Hoernig, 2013; Cambini and Silvestri, 2013; Nitsche and Wiethaus, 2011). Nevertheless, telecommunications firms are usually asymmetric, especially in the immature market with a rapid change in technology. This leads to differences in cost structure and demand, discrepancies in service quality, predatory behaviours by the dominant firms and collusion that have significant effects on social welfare and the advanced-technology deployment in the future.

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 34 In mobile telecommunications, due to a large amount of investment in base station, mobile network operators (MNOs) may outsource these facilities from a third-party company, e.g. TowerCo, which provides only costly facilities such as base station, tower and mast, feeder cables, antenna system and other equipment. The agreements vary from full sharing, roaming and mobile virtual network operator (MVNO) in order to satisfy the coverage obligation, especially in the rural areas with low business potential.

This study aims to investigate firms' decisions on the extent of quality upgrade for advanced-technology services when other approaches of infrastructure sharing, especially co-investment, are alternative modes to launch new advanced services. On top of the various agreements, this model captures an issue of firm asymmetry, where firms' cost structures vary. This study reveals that co-investment promotes higher consumer welfare than the fully-distributed-cost regulation in spite of lower investment in quality upgrade. However, compared to stand-alone investment, coinvestment brings about a welfare-undermining compromise over quality upgrade when infrastructure sharing induces a considerable amount of incremental cost of sharing facilities and operation. Therefore, in contrast to what telecommunications firms usually claim, co-investment may not be an appropriate way to stimulate the advanced-technology deployment in some situations. The regulator should closely monitor the co-investment negotiation on the grounds of tacit collusion, especially when firms have cost asymmetry in the advanced-technology deployment.

Relevant literature

The related literature emphasises that access price has a crucial effect on the degree of competition under infrastructure sharing in the presence of service-based entry. The facility-based incumbent may promote its wholesale profit and place the service-based entrant at a disadvantage by setting high access price. An increase in wholesale price causes the retail price to go up and finally dampens the competition (Bourreau, Cambini and Doğan, 2012; Bourreau, Cambini and Hoernig, 2015). Meanwhile, low access price can promote market competition by facilitating servicebased entrants to enter the market feasibly. Thus, for the purpose of static efficiency, cost-related access price regulation tends to be necessary.

However, from a dynamic perspective, access price has a significant effect on investment in advanced technology. Low access price may discourage the facilitybased firm from investing in higher technology because it cannot extract substantial profit under the regulated infrastructure sharing with low access price (Bourreau, Cambini and Hoernig, 2015; Cambini and Silvestri, 2013; Cambini and Valletti, 2003; Godinho de Matos and Ferreira, 2011; Kotakorpi, 2006; Nitsche and Wiethaus, 2011; Vareda, 2010). On the other hand, high access price can encourage investment in the newer technology roll-out (Vereda, 2010), and it also stimulates entrants to duplicate infrastructure instead of choosing service-based entry. Nitsche and Wiethaus (2011) compared the outcome of four different approaches of infrastructure sharing, including the regulatory holiday, the fully-distributed-cost regime, risk-sharing by coinvestment and long run incremental costs regulation (LRIC). They supported that service-based entry under the cost-based access price regulation (LRIC) leads to the smallest extent of next generation network (NGN) deployment and consumer welfare. In addition, risk-sharing by co-investment boosts the highest consumer welfare.

Compared to stand-alone investment, co-investment is claimed to stimulate investment in quality-enhanced services (Cambini and Silvestri, 2013; Nitsche and Wiethaus, 2011) and finally promote social welfare (Cambini and Silvestri, 2013; Foros, Hansen and Sand, 2002; Nitsche and Wiethaus, 2011). Moreover, co-investment gives an incentive to co-investors to expand their coverage under some conditions in the context of multiple-area competition (Bourreau, Cambini and Hoernig, 2013; Krämer and Vogelsang, 2014). Cambini and Silvestri (2013) examined the broadband market in the setting of à la Cournot in a single area under three different alternatives of investment; (1) no investment sharing with a cost-based access price charged to a service-based firm, (2) basic investment sharing without side payment and (3) jointventure agreement with side payment. It is found that the joint-venture agreement yields the greatest incentive to invest in new technology. Despite the highest level of advanced technology, the joint-venture agreement may soften competition because the joint-venture firms will collude to set the above-cost reciprocal side payment. As a result, the joint-venture agreement with the side-payment collusion and the cost-based access price regime with less investment incentive both yield lower consumer surplus and social welfare than the basic investment sharing.

Furthermore, Bourreau, Cambini and Hoernig (2013) captured the effect of coinvestment on firms' decisions about coverage expansion. In that setting, it is assumed that investment cost structure may change after the co-investment agreement according to benefits or additional costs from sharing infrastructure. Post-sharing investment cost may decline due to a reduction in financial risk. The investment cost probably
increases because of incremental cost of additional technical equipment for operating two networks on the shared infrastructure. Similar to the present study, they concluded that co-investment promotes coverage expansion when service differentiation and the benefit from cost reduction are sufficiently significant. Additionally, it is also found that substantial access price encourages an entrant to enter into the co-investment negotiation rather than choose service-based entry. The positive effect of co-investment on coverage expansion is also asserted by the more recent experimental evidence (Krämer and Vogelsang, 2014). The existing literature shows support for co-investment in expectation of the advanced-technology deployment. However, the present study cautions that co-investment may not be an appropriate approach to hasten investment in quality upgrade in some situations.

This study is organised as follows. The model is introduced in Section 3.2. Section 3.3 and Section 3.4 reveal the analysis of the equilibrium outcomes and the comparison of stand-alone investment and other approaches of infrastructure sharing respectively. Finally, the conclusion is in Section 3.5.

3.2 Model

 \overline{a}

This study concentrates on telecommunications services, especially broadband internet service with massive investment in advanced technology, which is an onerous burden for network providers and entrants. There are two firms investing in next generation technology in one region. The quality of new technology varies according to the level of quality upgrade that the two firms choose. 35

 35 This study bases the model on the competition in the broadband internet market. The firms have been racing for higher quality through the next generation network (NGN) such as fibreto-the-home (FTTH), and the issue about co-investment in NGN has been hotly debated. However, this model can be applied to the next generation mobile network which is heading to 4G or a more advanced generation with technology convergence. Another application is quality enhancement on the existing technology. The speed of data transfer, a reduction in data traffic congestion, sustainable connectivity and customer service may be taken into consideration for the purpose of strategic planning.

Cost

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It is assumed that the investment cost of new technology deployment 36 when firm *i* chooses to roll out its advanced service at the level of quality upgrade s_i is

$$
c_i(s_i) = \frac{\alpha_i}{2} s_i^2.
$$

 $c'_i(s_i) > 0$ and $c''_i(s_i) > 0$. α_i is cost parameter; $\alpha_i > 0$ and $s_i \ge 0$, where $i \in \{1, 2\}^{37}$ Therefore, investment cost strictly increases with s. For simplicity, constant marginal production costs of services are normalised to zero for both firms.³⁸ Instead, this study emphasises cost asymmetry in investment cost structure where firm 1 has lower cost than firm 2.³⁹ $\alpha_1 < \alpha_2$ and Δ_{α} denotes a cost-asymmetry parameter; $\Delta_{\alpha} = \frac{\alpha_2}{\alpha_1}$ $\frac{u_2}{\alpha_1} > 1.$

³⁶ The investment cost is the fixed cost involved with the advanced service rollout such as the fixed cost of facilities and operation according to a selected level of quality upgrade.

 37 To upgrade basic service to advanced service, firms need to invest further in infrastructure to facilitate the newer technology. In addition, investment cost of advanced technology deployment may vary among areas with different geographic landscape, nature of demand and population density. However, this model assumes only one region and omits the coverage issue. 38 In the telecommunications markets, the quality-enhanced infrastructure involves an enormous amount of fixed cost in comparison with a negligible amount of variable production cost corresponding to unit of demand or subscriber. Thus, for simplicity, it is assumed that marginal production cost is constant regardless of the quality-upgrade level. Nevertheless, the firms' decisions on the quality-upgrade extent may have an impact on the structure of variable production cost and then marginal production cost. For instance, the advanced technology may be developed to serve a higher quality of telecommunications services, and it may also reduce the associated variable production cost.

³⁹ This model is set up on the assumption of perfect information on firms' cost structures. Both firms have complete information on each other's cost structure. The incumbent's cost may be observed through its strategic behaviour and its obligation to reveal its cost structure to the regulator. Likewise, the reputation of the entrant as an international corporation or as an existing service provider in other regions may signal the entrant's cost structure in this region.

Demand

Following Foros (2004) and Kotakorpi (2006), representative consumers are uniformly distributed with different levels of willingness to pay for basic service on a continuum of $(-\infty, a]$. They will gain additional utility from quality upgrade. Some consumers who have too low valuation of service will not subscribe. Under the assumption of unit demand, 40 a representative consumer's valuation for firm *i*'s service is

$$
v+S(s_i).
$$

v is willingness to pay for basic service, varying on the interval ($-\infty$, a) among consumers. $S(s_i)$ is additional utility from the quality upgrade of firm i's service where firm i chooses the extent of quality upgrade at s_i ; $s_i \ge 0$ and $S(s_i) \ge 0$; $i \in \{1,2\}$

If firm *i* decides not to upgrade its basic service, $s_i = 0$ and $S(s_i) = 0$. To simplify the model, all consumers have homogeneous additional utility from the quality upgrade in the following form: 41

$$
S(s_i) = \omega s_i
$$

with parameter $\omega > 0.^{42}$

 $\overline{}$

 40 The unit demand assumption is reasonable in the telecommunications sector. For example, in the broadband internet market, one household normally chooses to set up an internet connection to one internet provider in purchase of unlimited internet usage with a monthly fee. It is also seen in the mobile market where mobile network operators usually offer a monthly package of mobile usage including calls, texts and internet connection.

 41 ⁴¹ This model assumes the additive form of utility from service quality upgrade that affects all representative consumers in exactly the same way (Foros, 2004; Foros, Hansen and Sand, 2002; Foros, Kind and Sand, 2005; Kotakorpi, 2006). Additionally, the discrete choice theory of product differentiation incorporates the product characteristics such as quality into the utility function in the additive form (Anderson, de Palma, and Thisse, 1992, ch. 6 and 9).

⁴² As assumed by Mussa and Rosen (1978) and Valletti (2000) in the light of vertical differentiation, a consumer's utility for a particular product is simply derived from the multiplication of his taste parameter and the product quality level. In other words, a representative θ -taste-type consumer has constant marginal utility of quality equal to θ .

This model is based on Cournot competition. Firm i offers the quality-upgrade extent (s_i) ; $i \in \{1,2\}$. Thus, the firms probably choose different levels of quality upgrade in the competition. A representative consumer evaluates the two services. He will subscribe to the firm that offers higher net utility whenever his net utility is still positive. For example, the consumer will buy firm *i*'s service instead of firm *j*'s service under the following condition.

$$
v + \omega s_i - p_i > v + \omega s_j - p_j
$$

where p_i (respectively p_j) is retail price of advanced service of firm *i* (respectively firm *j*) with the quality-upgrade extent of s_i (respectively s_j); $i, j \in \{1,2\}$ and $i \neq j$.

Quality-adjusted price of firm i is

 $\overline{}$

$$
P_i = p_i - \omega s_i \; ; \; i \in \{1,2\}.
$$

In this competition, firm j will be cornered if its quality-adjusted price (P_j) is still higher than firm i 's (P_i) . This is because every consumer will give higher valuation to firm *i*'s service than firm *j*'s service. This study aims to shed light on the sharedmarket equilibrium where both firms are still active in the market. Therefore, the quality-adjusted prices of both active firms' services must be equal, otherwise the firm

 Moreover, in the telecommunications industry, the majority of consumers are likely to be concerned with high quality. They easily adopt advanced technology with higher quality and capacity. Ofcom (2015) revealed that in the UK, smartphones have been rapidly adopted, especially among respondents aged 16-24. It also reported that 61 per cent of mobile users in 2014, compared to 26 per cent in 2010, claim to possess smartphones.

 Additionally, in the real business world, firms investing in higher quality technology can also operate low-quality or standard service on the same platform in order to compete in the niche market, targeting consumers who choose a lower quality service. Therefore, this study narrows down the various consumer types to just one consumer group with the identical taste parameter ω , and the additional utility follows the linear functional form corresponding to constant marginal utility of quality. This study excludes the issue about classical vertical differentiation.

with higher quality-adjusted price would be driven out of the market. The qualityadjusted price satisfying the shared-market equilibrium (P) is shown below.

$$
P = P_i = P_j = p_i - \omega s_i = p_j - \omega s_j
$$

In this context, the industry output level (Q) is the number of consumers who subscribe to either service. A consumer whose utility is higher than or equal to quality-adjusted price (P) will participate ($v - P \ge 0$). According to the assumption of uniformly distributed population and unit demand, the total number of subscribers is $a - P$ at quality-adjusted price P. It is implied that the aggregate demand function is $Q = a - P$ and $Q = q_1 + q_2$, where q_1 and q_2 are the quantity levels that firm 1 and firm 2 choose to serve, respectively.⁴³ Thus, the inverse demand function faced by firm i is derived below.

$$
p_i = a + \omega s_i - q_1 - q_2 \text{ where } i \in \{1, 2\}. \tag{3.1}
$$

Firm *i*'s price (p_i) corresponds directly to its choices of quality upgrade (s_i) and output level (q_i) .⁴⁴

⁴⁴ The aggregate demand function in this given area is $Q = a - P$. After replacing Q with $q_1 + q_2$ and P with $p_i - \omega s_i$,

Thus,

$$
q_1 + q_2 = a - p_i + \omega s_i.
$$

$$
p_i = a + \omega s_i - q_1 - q_2.
$$

 \overline{a}

 43 To invest in new technology infrastructure in telecommunications, the facility capacity should be planned together with business strategy. For instance, firms decide on access network's location and capacity to serve the targeted households with their broadband internet services. Another example is that mobile network operators plan for the allocation of mobile numbers by the regulator.

As seen in the studies of Brito, Pereira and Vareda (2010), Cambini and Silvestri (2012), Cambini and Silvestri (2013), Foros (2004), Foros, Hansen and Sand (2002), Kotakorpi (2006) and Nitsche and Wiethaus (2011), setting up Cournot competition instead of price competition is reasonable in these settings. However, most relevant studies show that the outcomes of various competition models are not significantly different and they draw the main conclusions in a similar way.

This study examines the effects of infrastructure sharing on the firms' decisions on quality-upgrade levels of advanced services. The model is designed to capture the competition regarding different approaches of infrastructure deployment. In this study, there are three practical approaches, introduced in the following cases.

Case 1: The two facility-based firms separately invest in quality upgrade (stand-alone investment).

The two firms are assumed to be asymmetric in investment cost structure of advanced service. They decide to invest separately with no infrastructure sharing and maximise their own profit. Profit functions are

$$
\pi_{N1} = (a + \omega s_{N1} - q_{N1} - q_{N2})q_{N1} - \frac{\alpha_1}{2} s_{N1}^2
$$
\n(3.2)

$$
\pi_{N2} = (a + \omega s_{N2} - q_{N1} - q_{N2})q_{N2} - \frac{\alpha_2}{2} s_{N2}^2
$$
\n(3.3)

where $Q_N = q_{N1} + q_{N2}$.

l

Timing of the game

This model is set up in a static framework. Each firm makes a one-shot decision on the extent of quality upgrade for advanced-technology service (next generation network) in the following two-stage game, solved by backward induction.

Stage 1 The firms choose the extent of quality upgrade simultaneously $(s_{N1},$ S_{N2}).

Stage 2 The firms simultaneously choose their service quantity levels (q_{N1}, q_{N2}) .

 In this setting of Cournot competition, firms decide on quality upgrade in the investment stage, and they later choose their output levels (the number of their subscribers to be served). Suppose the investment stage is exogenous, at a given pair of firms' quality-upgrade levels, firms choose their own output levels which lead to the market-clearing quality-adjusted price (P) and then their retail prices (subscription fees) must correspond to the quality-adjusted price in the shared-market equilibrium. When the investment stage is endogenous, firms strategically plan their quality upgrade. Their retail prices must correspond to the firms' decisions on the extent of quality upgrade and output levels in the shared-market equilibrium.

Case 2: The two firms agree on co-investment.

After the agreement, both firms choose the extent of mutual quality upgrade (s_c) .⁴⁵ The low-cost firm shares its facilities and technology with its rival. Cost structure may change after accommodating the operations of the two firms instead of only one firm. $c_{sharing}$ is the investment cost after the agreement of infrastructure sharing, which facilitates both firms' services based on the low-cost firm's facilities.

$$
c_{sharing}(s_c) = \frac{\phi \alpha_1}{2} s_c^2.
$$

 ϕ is a positive cost-adjustment parameter of infrastructure sharing. $\phi = 1$ when the infrastructure can be shared without any additional cost. $\phi > 1$ when firms incur additional cost of equipment and operation in order to enable the two services to run on the shared infrastructure. ϕ < 1 when cost saving from infrastructure sharing outweighs the additional cost.⁴⁶ The two firms negotiate on co-investment to reach a bargain on the basis of Nash Bargaining Solution. After a successful negotiation, they mutually choose the extent of quality upgrade (s_c) and agree to portion the total investment cost. β is the proportion of the total investment cost that firm 1 pays and firm 2 takes on the rest of the total investment cost burden, $1 - \beta$.

 \overline{a}

⁴⁵ It is assumed that consumers' preferences for quality-enhanced services offered by both firms are not different because both services run on the shared infrastructure at the mutual quality-upgrade level on the assumption of no firm-specific preference.

⁴⁶ To guarantee the existence of a successful negotiation on co-investment, $\phi > 0$ and ϕ should not be too large in order that both asymmetric firms agree to share investment cost; i.e. the equilibrium proportion of total investment cost of the low-cost firm is positive, $\beta^* > 0$. When infrastructure sharing causes a large amount of additional investment cost (ϕ is too large), it is more likely that $\beta^* \le 0$ because $\frac{\partial \beta^*}{\partial \phi} < 0$. If $\beta^* \le 0$, the low-cost firm does not embark on setting up facilities at all and the infrastructure-sharing agreement on using the low-cost firm's technology cannot occur. Consequently, the equilibrium under co-investment does not exist in this case. See proof in the appendix (Section 3.6.2).

Profit functions are

$$
\pi_{C1} = (a + \omega s_C - q_{C1} - q_{C2})q_{C1} - \beta \frac{\phi \alpha_1}{2} s_C^2 \tag{3.4}
$$

$$
\pi_{C2} = (a + \omega s_C - q_{C1} - q_{C2})q_{C2} - (1 - \beta) \frac{\phi \alpha_1}{2} s_C^2 \tag{3.5}
$$

where $Q_c = q_{c1} + q_{c2}$.

 $\overline{}$

Timing of the game

The following two-stage game is solved by backward induction in a static framework. Both firms successfully reach an infrastructure-sharing agreement on the extent of mutual quality upgrade (s_c) and the allocation of the total investment cost (β) following Nash Bargaining Solution in stage 1. Then, they enter Cournot competition in the retail market in stage 2.

Stage 1 The firms negotiate on the extent of mutual quality upgrade (s_c) and (β) on the basis of Nash Bargaining Solution.

Stage 2 The firms simultaneously choose their own service quantity levels (q_{c1}, q_{c2}) .

Case 3: Access to infrastructure under a fully-distributed-cost regime

The entrant with higher cost decides to use the incumbent's facilities to deliver its service in this region instead of building its own infrastructure. The incumbent is regulated to give access network to the entrant in accordance with cost-based access price. The payment is calculated on a fully-distributed-cost basis, and consequently the investment cost of quality upgrade is shared on the basis of usage proportions. In this case, the incumbent with lower cost (firm 1) unilaterally decides on the extent of quality upgrade (s_F) in order to maximise its own profit in response to this regulatory regime. 47

⁴⁷ To simplify the model, consumers perceive that the quality-enhanced service of the servicebased entrant does not differ from that of the incumbent which owns the infrastructure because they run on the shared infrastructure with the same extent of quality. In other words, consumers evaluate the services without firm-specific preference.

Profit functions are

$$
\pi_{F1} = (a + \omega s_F - q_{F1} - q_{F2})q_{F1} - \frac{\phi \alpha_1}{2} s_F^2 + \left(\frac{\phi \alpha_1}{2} s_F^2\right) \left(\frac{q_{F2}}{Q_F}\right) \tag{3.6}
$$

$$
\pi_{F2} = (a + \omega s_F - q_{F1} - q_{F2})q_{F2} - \left(\frac{\phi \alpha_1}{2} s_F^2\right) \left(\frac{q_{F2}}{Q_F}\right)
$$
(3.7)

where $Q_F = q_{F1} + q_{F2}$.

The service-based entrant pays the total access charge of $\left(\frac{\phi a_1}{2}\right)$ $\left(\frac{\alpha_1}{2} s_F^2\right) \left(\frac{q_{F2}}{Q_F}\right)$ $\frac{q_{F2}}{Q_F}$ to the facility-based incumbent according to the proportion of usage.

Timing of the game

As the low-cost incumbent, firm 1 gives access network to the service-based entrant in the following two-stage game, solved by backward induction.

Stage 1 Firm 1 decides on the extent of quality upgrade (s_F) to maximise its own profit.

Stage 2 The two firms simultaneously choose their own service quantity levels $(q_{F1}, q_{F2}).$

Consumer surplus

Consumer surplus can be calculated in the following way.

Consumer surplus is

$$
CS_k = \left(\frac{1}{2}\right)(q_{k1} + q_{k2})^2. \tag{3.8}
$$

denotes the outcome of the three cases.

 $k \in \{N, C, F\}$ where N denotes Stand-alone investment, C denotes Co-investment and F denotes Fully-distributed-cost regulation.

3.3 Analysis

The market outcomes are different regarding the various approaches of the infrastructure deployment in the three cases.

3.3.1 Case 1: The two facility-based firms invest in quality upgrade separately (stand-alone investment).

In stage 2 of the game, firm 1 and firm 2 have their profit-maximisation problems, max_{q_{N1}} π_{N1} and max_{q_{N2} π_{N2} respectively. From (3.2) and (3.3), the first-} order conditions are

$$
\frac{\partial \pi_{N1}}{\partial q_{N1}} = a + \omega s_{N1} - 2q_{N1} - q_{N2} = 0 \tag{3.9}
$$

$$
\frac{\partial \pi_{N2}}{\partial q_{N2}} = a + \omega s_{N2} - q_{N1} - 2q_{N2} = 0. \tag{3.10}
$$

Solving the above FOCs gives the equilibrium quantity levels as below.

$$
q_{Ni}^{*}(s_{Ni}, s_{Nj}) = \frac{a}{3} + \frac{2}{3}\omega s_{Ni} - \frac{1}{3}\omega s_{Nj} \text{ where } i, j \in \{1, 2\}, i \neq j.
$$
 (3.11)

Substituting q_{Ni}^* in profit function (3.2) and (3.3) yields the following profit functions in stage 1.

$$
\pi_{Ni}(s_{Ni}, s_{Nj}) = \left(\frac{a}{3} + \frac{2}{3}\omega s_{Ni} - \frac{1}{3}\omega s_{Nj}\right)^2 - \frac{\alpha_i}{2}s_{Ni}^2 \text{ where } i, j \in \{1, 2\}, i \neq j. \tag{3.12}
$$

Firm *i*'s problem is max_{*s_{Ni}* π_{Ni} . Differentiating the reduced-form profit} functions with respect to associated quality-upgrade levels gives the following:

$$
\frac{\partial \pi_{Ni}}{\partial s_{Ni}} = \frac{4}{9}a\omega + \frac{8}{9}\omega^2 s_{Ni} - \frac{4}{9}\omega^2 s_{Nj} - \alpha_i s_{Ni} = 0
$$
\n(3.13)

where $i, j \in \{1, 2\}, i \neq j$.

In equilibrium, the quality-upgrade levels of the two firms are

$$
s_{N1}^{*} = \frac{12a\omega\alpha_{2} - 16a\omega^{3}}{27\alpha_{1}\alpha_{2} - 24\omega^{2}\alpha_{1} - 24\omega^{2}\alpha_{2} + 16\omega^{4}}
$$
(3.14)

$$
s_{N2}^{*} = \frac{12a\omega\alpha_1 - 16a\omega^3}{27\alpha_1\alpha_2 - 24\omega^2\alpha_2 - 24\omega^2\alpha_1 + 16\omega^4}
$$
(3.15)

Proposition 3.1

In the absence of infrastructure sharing, the low-cost firm chooses higher quality-upgrade level and offers a greater number of subscribers with higher price than the high-cost firm, i.e. $s_{N1}^* > s_{N2}^*$, $q_{N1}^* > q_{N2}^*$, $p_{N1}^* > p_{N2}^*$. Finally, the low-cost firm gains higher profit than the high-cost firm, i.e. $\pi_{N1}^* > \pi_{N2}^*$. ProofSee the appendix (Section 3.6.1).

The low-cost firm takes the dominant position because of its lower investment cost in quality upgrade. Due to cost efficiency, it can invest more heavily in quality upgrade to attract more consumers and gain larger market share and then make higher profit than the high-cost firm whose quality upgrade is costly.

3.3.2 Case 2: The two firms agree on co-investment.

The firms agree to share the whole infrastructure for advanced service. In stage 2, firm 1 and firm 2 have their profit-maximisation problems, $\max_{q_{C_1}} \pi_{C_1}$ and $\max_{q_{c2}} \pi_{c2}$ respectively. From (3.4) and (3.5), the FOCs are shown below.

$$
\frac{\partial \pi_{C1}}{\partial q_{C1}} = a + \omega s_C - 2q_{C1} - q_{C2} = 0 \tag{3.16}
$$

$$
\frac{\partial \pi_{C2}}{\partial q_{C2}} = a + \omega s_C - q_{C1} - 2q_{C2} = 0 \tag{3.17}
$$

From (3.16) and (3.17), one can solve the FOCs and obtain the equilibrium levels of service quantity as below.

$$
q_{Ci}^*(s_c) = \frac{a}{3} + \frac{1}{3}\omega s_c \text{ where } i, j \in \{1, 2\}, i \neq j. \tag{3.18}
$$

Back to stage 1, substituting (3.18) in the profit functions (3.4) and (3.5) gives

$$
\pi_{C1}(s_c) = \left(\frac{a}{3} + \frac{1}{3}\omega s_c\right)^2 - \beta \frac{\phi \alpha_1}{2} s_c^2 \tag{3.19}
$$

$$
\pi_{C2}(s_c) = \left(\frac{a}{3} + \frac{1}{3}\omega s_c\right)^2 - (1 - \beta)\frac{\phi \alpha_1}{2} s_c^2.
$$
 (3.20)

The firms enter into a negotiation on the mutual quality-upgrade level (s_c) and the allocation of total investment cost (β) . The outcome is based on Nash Bargaining Solution. The outcome of the negotiation solves the following problem.

$$
\max_{\beta,s_C} (\pi_{C1} - \pi_{N1}^*)(\pi_{C2} - \pi_{N2}^*)
$$

Ψ denotes $(\pi_{C1} - \pi_{N1}^*)(\pi_{C2} - \pi_{N2}^*)$.

$$
\frac{\partial \Psi}{\partial \beta} = (\pi_{C1} - \pi_{N1}^*) \frac{\phi \alpha_1}{2} s_C^2 + (\pi_{C2} - \pi_{N2}^*) \left(-\frac{\phi \alpha_1}{2} s_C^2 \right) = 0
$$

$$
\pi_{C1} - \pi_{N1}^* = \pi_{C2} - \pi_{N2}^*
$$
(3.21)

$$
\frac{\partial \Psi}{\partial s_C} = 4a\omega + 4\omega^2 s_C - 9\phi a_1 s_C = 0
$$
\n(3.22)

From (3.22), the level of mutual quality upgrade is

$$
s_C^* = \frac{4a\omega}{9\phi\alpha_1 - 4\omega^2} \,. \tag{3.23}
$$

Proposition 3.2

By means of co-investment, the low-cost firm reaches a successful agreement to pay a smaller proportion of total investment cost of shared infrastructure, and then earns higher profit than the high-cost firm, despite the same equilibrium output level,

i.e. $\beta^* < \frac{1}{2}$ $\frac{1}{2}$, $q_{C1}^* = q_{C2}^*, \pi_{C1}^* > \pi_{C2}^*$.

ProofSee the appendix (Section 3.6.2).

In the light of stand-alone investment, the low-cost firm is in the dominant position with higher profit and higher quality upgrade. After co-investment, they have equal market shares because both services run on exactly the same shared infrastructure with the same quality-upgrade level. In stage 2, both firms offer the symmetric equilibrium outputs. However, in the co-investment negotiation in stage 1,

the low-cost firm has higher bargaining power to negotiate for the smaller proportion of total investment cost. The high-cost firm has to accept the larger proportion of total investment cost because the successful negotiation on co-investment finally brings it higher net profit than stand-alone investment. As a result, under co-investment, the low-cost firm has higher profit than the high-cost firm, even though their services are not different in terms of quality upgrade and service quantity from consumers' perspective in the retail market.

To elaborate on the outcome, this result can be analysed in accordance with Split-The-Difference Rule.⁴⁸ Each firm first agrees to extract the same amount of its own profit as it would earn under stand-alone investment. Further, both firms reach a consensus to equally split the incremental aggregate profit that they both receive from choosing co-investment instead of stand-alone investment. Even though they earn equal shares of the incremental aggregate profit, the profits of the two firms are still different. This is because the low-cost firm has higher bargaining power to extract a higher amount of profit as a consequence of its opportunity cost from agreeing on coinvestment instead of entering the stand-alone investment competition.⁴⁹

Remark 3.1

In the light of co-investment, the equilibrium quality-upgrade level is independent of the allocation of investment cost (β) . Nevertheless, the equilibrium quality-upgrade level decreases with the cost-adjustment parameter of infrastructure sharing (ϕ) .

Proof

 $\overline{}$

Differentiating (3.23) with respect to β and ϕ gives

$$
\frac{\partial s_C^*}{\partial \beta} = 0 \text{ and } \frac{\partial s_C^*}{\partial \phi} = \frac{-36a\omega \alpha_1}{(9\phi \alpha_1 - 4\omega^2)^2} < 0.
$$

⁴⁸ Muthoo (1999, p. 15) illustrates an example of Split-The-Difference Rule in which case two players bargain over a portion of a cake where the utility functions of the players are simply their shares of the cake. The utility of one player can be rewritten as a linear function of the utility of the other. After bargaining, they agree to obtain their first portions equal to what they would receive from the disagreement, and they further agree to split the remaining cake equally. 49 See proof in the appendix (Section 3.6.2).

The co-investors choose their mutual quality-upgrade level regardless of the allocation of investment cost. The quality-upgrade level directly affects the amount of total investment cost. However, they decide on the quality-upgrade level only on the grounds of the magnitude of total investment cost after co-investment and consumers' reservation price. The allocation of the total investment cost among the co-investors depends on their bargaining power. They have less incentive to upgrade quality when infrastructure sharing causes more substantial additional cost. Thus, the equilibrium quality-upgrade level decreases when the cost-adjustment parameter of infrastructure sharing increases.

3.3.3 Case 3: Access to infrastructure under a fully-distributedcost regime

In stage 2, firm 1 and firm 2 choose their output levels to maximise their own profits, max_{q_{F1}} π_{F1} and max $_{q_{F2}}$ π_{F2} respectively. From (3.6) and (3.7), the FOCs are

$$
\frac{\partial \pi_{F1}}{\partial q_{F1}} = a + \omega s_F - 2q_{F1} - q_{F2} - \left(\frac{\phi \alpha_1}{2} s_F^2\right) \left(\frac{q_{F2}}{Q_F^2}\right) = 0 \tag{3.24}
$$

$$
\frac{\partial \pi_{F2}}{\partial q_{F2}} = a + \omega s_F - q_{F1} - 2q_{F2} - \left(\frac{\phi \alpha_1}{2} s_F^2\right) \left(\frac{q_{F1}}{Q_F^2}\right) = 0. \tag{3.25}
$$

To satisfy (3.24) and (3.25), the symmetric equilibrium at this stage is

$$
q_F^*(s_F) = q_{F1}^*(s_F) = q_{F2}^*(s_F) = \frac{a + \omega s_F}{6} + \sqrt{\frac{(a + \omega s_F)^2}{36} - \frac{\phi \alpha_1 s_F^2}{24}}.
$$
 (3.26)

Under the fully-distributed-cost regulatory regime, firm 1 chooses the extent of quality upgrade in stage 1 to solve its problem, $\max_{S_F} \pi_{F_1}$. After substituting (3.26) in (3.6), one can differentiate the reduced-form profit function of firm 1 and obtain the following equation.

$$
\frac{\partial \pi_{F1}}{\partial s_F} = (a + \omega s_F - 4q_{F1}^*) \frac{\partial q_{F1}^*}{\partial s_F} + \omega q_{F1}^* - \frac{\phi \alpha_1 s_F}{2} = 0 \tag{3.27}
$$

The equilibrium level of quality upgrade (s_F^*) cannot be explicitly expressed in closed form. However, it can be shown in comparison with the outcomes of other cases, as seen in the next section.

Proposition 3.3

 $\overline{}$

Under the fully-distributed-cost regulatory regime, the market share and the profit of the low-cost facility-based firm equal those of the high-cost service-based rival that asks the low-cost firm for access network, i.e. $q_{F1}^* = q_{F2}^*$ and $\pi_{F1}^* = \pi_{F2}^*$. Proof

Substituting (3.26) in (3.6) and (3.7) shows $\pi_{F1}^* = \pi_{F2}^*$.

$$
\pi_{F1}^* = \pi_{F2}^* = (a + \omega s_F^* - 2q_F^*)q_F^* - \left(\frac{\phi \alpha_1}{4} s_F^{*2}\right) \square
$$

Under the fully-distributed-cost regulation, the low-cost firm is regulated to allow its service-based rival to share its infrastructure. In the retail market, they compete with each other by offering exactly the same level of quality upgrade run on the shared infrastructure.⁵⁰ Thus, the two firms are equal in market share and profit.

Access to infrastructure by the unregulated access price approach

When the low-cost facility-based firm is allowed to set access price without any intervention by the regulator, it will reject the infrastructure-sharing proposal or set a prohibitive access price until the service-based rival has no market share and consequently exits the market. Finally, the low-cost firm becomes a monopolist in the region and certainly imposes a limitation on the number of subscribers that induces an increase in retail price. Therefore, this approach is not optimal in terms of consumer welfare, even though it can stimulate investment by the facility-based incumbent.⁵¹

⁵⁰ In this model, it is assumed that there is no horizontal product differentiation among service providers (no firm-specific preference). Consumers consider only price and quality of service without brand preference.

 $⁵¹$ See proof and further details in the appendix (Section 3.6.3).</sup>

3.4 The optimal regulatory regimes

Table 3.1 shows comparisons of the equilibrium market outcomes under different regulatory regimes to find out the most effective regulation/deregulation on the advanced-technology deployment in several facets of quality enhancement, the supply of service and overall consumer welfare.

Scenario	The extent of quality upgrade
Case 1: Stand-alone investment	$s_{N1}^{*} = \frac{12a\omega\alpha_{2} - 16a\omega^{3}}{27\alpha_{1}\alpha_{2} - 24\omega^{2}\alpha_{1} - 24\omega^{2}\alpha_{2} + 16\omega^{4}}$
	$s_{N2}^* = \frac{12a\omega\alpha_1 - 16a\omega^3}{27\alpha_1\alpha_2 - 24\omega^2\alpha_2 - 24\omega^2\alpha_1 + 16\omega^4}$
Case 2: Infrastructure sharing by co-investment	$s_c^* = \frac{4a\omega}{9\phi\alpha_1 - 4\omega^2}$
Case 3: Access to infrastructure under the fully-distributed-cost regulation	n/a
Scenario	Industry output level
Case 1: Stand-alone investment	$Q_N^* = \frac{2a}{2} + \frac{\omega s_{N1}^*}{2} + \frac{\omega s_{N2}^*}{2}$
Case 2: Infrastructure sharing by co-investment	$Q_C^* = \frac{2(a + \omega s_C^*)}{2}$
Case 3: Access to infrastructure under the fully-distributed-cost regulation	$Q_F^* = \frac{(a + \omega s_F^*)}{3} + 2 \sqrt{\frac{(a + \omega s_F^*)^2}{36} - \frac{\phi \alpha_1 s_F^{*^2}}{24}}$

Table 3.1 Equilibrium market outcomes in comparison

Scenario	Consumer Surplus
Case 1: Stand-alone investment	$CS_N = \left(\frac{1}{2}\right)\left(\frac{2a}{3} + \frac{\omega s_{N1}^*}{2} + \frac{\omega s_{N2}^*}{2}\right)^2$
Case 2: Infrastructure sharing by co-investment	$CS_c = \left(\frac{1}{2}\right) \left(\frac{2(a + \omega s_c^*)}{2}\right)^2$
Case 3: Access to infrastructure under the fully-distributed-cost regulation	$CS_F = \left(\frac{1}{2}\right) \left(\frac{(a + \omega s_F^*)}{3} + 2 \sqrt{\frac{(a + \omega s_F^*)^2}{36} - \frac{\phi \alpha_1 s_F^{*2}}{24}} \right)$

Table 3.1 Equilibrium market outcomes in comparison (continued)

Note: n/a is the equilibrium outcome that cannot be expressed in closed form.

The equilibrium outcomes vary according to circumstances. This study aims to bring co-investment to light, so the outcome of co-investment will be compared with other cases as a benchmark.

■ **Co-investment VS Stand-alone investment**

This section highlights comparisons between the outcomes of infrastructure sharing by co-investment and stand-alone investment.

Proposition 3.4

Compared to stand-alone investment without infrastructure sharing, the coinvestors agree to mutually change the level of quality upgrade according to their adjusted cost structure after infrastructure sharing.

(i) If the cost-adjustment parameter is very low, $0 < \phi < \frac{9\alpha_1\Delta_\alpha - 8\omega^2 - 4\omega^2\Delta_\alpha}{2(1-\Delta_\alpha + 4\Delta_\alpha)^2}$ $\frac{\Delta_{\alpha}-\delta\omega-4\omega\Delta_{\alpha}}{9\alpha_1\Delta_{\alpha}-12\omega^2},$ the co-investors agree to increase the mutual quality-upgrade level, i.e. $s_{N2}^* < s_{N1}^* < s_c^*$.

(ii) If the cost-adjustment parameter is moderate,

 $\frac{9\alpha_1\Delta_\alpha-8\omega^2-4\omega^2\Delta_\alpha}{9\alpha_1\Delta_\alpha-12\omega^2} \leq \phi \leq \frac{9\alpha_1\Delta_\alpha-8\omega^2\Delta_\alpha-4\omega^2}{9\alpha_1-12\omega^2}$ $\frac{\alpha^{100}}{9\alpha_1-12\omega^2}$, the mutual quality-upgrade level ranges between that of the high-cost firm and that of the low-cost firm with stand-alone investment, i.e. $s_{N2}^* \leq s_C^* \leq s_{N1}^*$.

(iii) Finally, if the cost-adjustment parameter is sufficiently high,

 $\phi > \frac{9\alpha_1\Delta_{\alpha}-8\omega^2\Delta_{\alpha}-4\omega^2}{2\omega^2}$ $\frac{\alpha^{-600} \Delta \alpha^{-400}}{9 \alpha_1 - 12 \omega^2}$, the co-investors decrease the level of mutual quality upgrade, i.e. $s_c^* < s_{N2}^* < s_{N1}^*$.

ProofSee the appendix (Section 3.6.4).

When the benefit from cost reduction of infrastructure sharing is sufficiently significant, the co-investors can offer a higher level of mutual quality upgrade than the equilibrium outcome of stand-alone investment. The considerable cost-saving from infrastructure sharing can stimulate their mutual quality upgrade. However, if infrastructure sharing requires additional equipment and special operation that generate substantial incremental cost, the co-investors agree to choose a lower level of mutual quality upgrade than the outcome of stand-alone investment. This is because the co-investors incur substantial investment cost after the agreement. Hence, they agree to soften competition in quality upgrade by lowering the extent of mutual quality upgrade to earn more profit than that of stand-alone investment.

The influence of the cost-adjustment parameter of infrastructure sharing (ϕ) in **the negotiation between the co-investors**

To highlight the effect of the cost-adjustment parameter of infrastructure sharing (ϕ) on the firms' incentive to upgrade quality, the two firms are assumed to be symmetric in investment cost structure $(\alpha_1 = \alpha_2)$. In this situation, the costadjustment parameter (ϕ) has a significant effect on the negotiation over the extent of quality upgrade under co-investment as follows.

Corollary 3.1

When the two firms are symmetric in investment cost structure, the symmetric equilibrium outcome occurs, i.e. $s_{N1}^* = s_{N2}^*$. When $0 < \phi < 1$, $\phi = 1$, or $\phi > 1$, the mutual quality-upgrade level under the co-investment agreement is higher than, equal

to, or lower than those under the stand-alone investment respectively. Each co-investor agrees to pay half of the total investment cost of infrastructure sharing, i.e. $\beta = \frac{1}{2}$ $\frac{1}{2}$. Proof See the appendix (Section 3.6.5).

In the special case of symmetric cost structure, the cost-adjustment parameter (ϕ) plays a crucial role in firms' decisions on quality upgrade. In the absence of infrastructure sharing, both stand-alone firms choose the same level of quality upgrade and earn equal profit. They both have equal bargaining power when entering into the negotiation over co-investment. As a result, the total investment cost is divided equally among the co-investors. When infrastructure sharing does not induce an increase in cost $(\phi = 1)$, the co-investors pay exactly the equal amount of investment cost and the co-investors' decision problem is identical to the stand-alone investors' decision problems. Therefore**,** after co-investment, firms do not change their equilibrium level of quality upgrade. However, when infrastructure sharing involves additional cost ($\phi > 1$), the co-investors agree to reduce the quality-upgrade extent in response to the additional cost. By contrast, when infrastructure sharing yields cost saving $(0 < \phi < 1)$, the cost of quality upgrade becomes a lighter burden after co-investment. The co-investors find it more profitable to enhance service quality to attract more consumers. Thus, in this setting of symmetric cost structure, even though firms can lighten their own burden of cost investment in quality upgrade through co-investment, it is not guaranteed that firms will have incentive to increase their mutual quality-upgrade level. When co-investment incurs additional cost to facilitate infrastructure sharing, the co-investors tacitly collude to lower the mutual quality-upgrade level through the co-investment negotiation.

Similarly, back to the main model in the setting of asymmetric cost structures as seen in Proposition 3.4, the tacit collusion to decrease the mutual quality-upgrade level is likely to occur when co-investment incurs a considerable amount of additional investment cost to implement infrastructure sharing.

In addition, the cost-adjustment parameter of infrastructure sharing (ϕ) also plays a crucial role in the equilibrium industry output and consumer welfare under coinvestment, as stated in the following proposition.

Proposition 3.5

The equilibrium industry output level and consumer surplus change according to the co-investors' cost structure.

If $0 < \phi \leq \frac{6\alpha_1\Delta_{\alpha}-4\omega^2\Delta_{\alpha}-4\omega^2}{2\alpha_1+2\alpha_2\Delta_{\alpha}-2\omega^2}$ $\frac{3a_1+a_2-a_0}{a_1+a_1+a_1+a_2-a_2}$, the equilibrium industry output level and consumer surplus are higher than or equal to those of stand-alone investment, i.e. $Q_C^* \geq Q_N^*$ and $CS_C \geq CS_N$. Conversely, if $\phi > \frac{6a_1\Delta_{\alpha}-4\omega^2\Delta_{\alpha}-4\omega^2}{3\alpha+3\alpha\Delta_{\alpha}-8\omega^2}$ $\frac{\alpha_1 \alpha_2 - \alpha_0}{\alpha_1 + 3\alpha_1 \alpha_0 - 8\omega^2}$, the equilibrium industry output level and consumer surplus are lower than those of stand-alone investment, i.e. $Q_C^* < Q_N^*$ and $CS_C < CS_N$. Proof See the appendix (Section 3.6.6).

When the co-investors find that co-investment leads to considerable incremental cost of infrastructure sharing $(\phi > \frac{6\alpha_1\Delta_{\alpha}-4\omega^2\Delta_{\alpha}-4\omega^2}{3\alpha_1+3\alpha_1\Delta_{\alpha}-8\omega^2} > 1)$, they decide to decrease the total number of subscribers in the area. According to Proposition 3.4, compared to stand-alone investment, if the cost-adjustment parameter is very high, the co-investors focus on offering a low level of quality upgrade to save cost. Even though this strategy forces the co-investors to give up some subscribers, it can increase the co-investors' profit by dampening competition in quality upgrade. Consequently, when the coinvestors incur high investment cost after sharing infrastructure, they will collude to soften competition by providing services with a lower quality-upgrade level. This causes a reduction in consumer surplus. In contrast, if co-investment yields cost saving or only a small amount of additional cost, $0 < \phi < \frac{6\alpha_1\Delta_{\alpha}-4\omega^2\Delta_{\alpha}-4\omega^2}{2\alpha_1+2\alpha_2\Delta_{\alpha}-2\omega^2}$ $\frac{\alpha_1 \alpha_2 - 4\omega}{\alpha_1 + 3\alpha_1 \alpha_2 - 8\omega^2}$, the co-investors have an incentive to increase the total number of subscribers in the market. Compared to stand-alone investment, all of the active consumers in case of the sufficiently low cost-adjustment parameter and most of the active consumers in case of the moderate cost-adjustment parameter both benefit from higher levels of quality upgrade. Thus, the consumer base is expanded and consumer surplus increases accordingly.

■ Co-investment VS Fully-distributed-cost regulation

Comparison of the two approaches to infrastructure sharing is made in terms of quality upgrade, the supply of service, price and consumer surplus.

Proposition 3.6

Infrastructure sharing under the fully-distributed-cost regulation causes a higher quality-upgrade level but a lower industry output level with higher retail price than the outcomes of co-investment, i.e. $s_F^* > s_C^*$, $Q_F^* < Q_C^*$ and $p_F^* > p_C^*$. Overall, the regulation generates lower consumer surplus than co-investment, i.e. $CS_F < CS_C$. ProofSee the appendix (Section 3.6.7).

In the absence of horizontal differentiation in brand preference, the competition between the facility-based firm and its service-based rival in the retail market finally ends up with equal market shares. Then, they share half of the total investment cost under the fully-distributed-cost regulation. However, the low-cost facility-based firm invests in the whole facilities and has the right to unilaterally decide on the level of quality upgrade. Compared to co-investment, the facility-based firm finds it more profitable to attract consumers by raising the quality-upgrade level which allows it to charge higher retail price under this regulation. Despite more attractive advanced service, consumer surplus under the fully-distributed-cost regulation is less than that under co-investment. This is because the facility-based firm aims to limit the number of industry output (then the corresponding retail price increases) in order to maximise its own profit.

The effects of infrastructure sharing on firms' profitability

This section discusses the effects of the different approaches of infrastructure sharing on firms' profitability.

Proposition 3.7a

Both facility-based firms tend to reach a successful co-investment agreement because their profits of co-investment are higher than those of stand-alone investment according to the benefit from the cost-adjustment parameter of infrastructure sharing (ϕ), i.e. $\pi_{c1}^* > \pi_{N1}^*$ and $\pi_{c2}^* > \pi_{N2}^*$. Proof See the appendix (Section 3.6.8).

Under the assumption that the total investment cost of stand-alone investment is greater than that of infrastructure sharing, the low-cost facility-based firms find it more profitable to agree on co-investment instead of building their own infrastructure separately. Similarly, the high-cost firm also benefits from the low-cost facilities provided by the low-cost firm. Hence, the two firms reach a successful agreement on co-investment instead of separate investment.

Proposition 3.7b

(i) The comparison between the low-cost firm's profits under co-investment and under the fully-distributed-cost regulation is ambiguous.

If the cost-asymmetry parameter is very low (Δ_{α} approaches one), the low-cost firm's profit under the fully-distributed-cost regulation is higher than that under coinvestment, i.e. $\pi_{F1}^* > \pi_{C1}^*$.

Conversely, if the cost-asymmetry parameter (Δ_{α}) is sufficiently high, the low-cost firm's profit under the fully-distributed-cost regulation has a tendency to be lower than that under co-investment, i.e. $\pi_{F_1}^* < \pi_{C_1}^*$.

(ii) The low-cost facility-based firm prefers infrastructure sharing by the unregulated access price approach to co-investment, i.e. $\pi_{A1}^* > \pi_{C1}^*$.

Proof See the appendix (Section 3.6.9).

When the regulator intervenes with the fully-distributed-cost regulation, the low-cost facility-based firm may gain or lose profit due to this regulation. When the cost asymmetry is less significant, the bargaining power of the low-cost firm is less dominant. Its equilibrium profit under co-investment is less than that under the fullydistributed-cost regulation. This is because under the fully-distributed-cost regulation, the low-cost firm has the right to unilaterally choose the quality-upgrade level to maximise its own profit. Meanwhile, under co-investment, the low-cost firm has to negotiate with the other co-investor to choose the mutual quality-upgrade level by taking its rival's interest into account. On the other hand, when the cost asymmetry is sufficiently significant, the low-cost firm's profit under co-investment is greater than that under the fully-distributed-cost regulation. The low-cost firm has clear dominance in the co-investment negotiation. It pays a considerably smaller proportion of total investment cost under co-investment, whereas it has to share half of the total

investment cost under the fully-distributed-cost regulation according to the equal output levels in equilibrium. As a result, its profit under co-investment is likely to be larger than that under the fully-distributed-cost regulation.

When the low-cost facility-based firm is allowed to set access price, it can corner the market by charging extremely high access price. The monopolistic profit of the low-cost firm under this regime is definitely higher than that of co-investment in a duopoly.

Proposition 3.7c

In the presence of service-based entry, the high-cost firm prefers service-based entry under the fully-distributed-cost regulation to co-investment, stand-alone investment and service-based entry by the unregulated access price respectively because its profit under the fully-distributed-cost regulation is the highest, i.e. π_{F2}^* > $\pi_{C2}^* > \pi_{N2}^* > \pi_{A2}^*.$

Proof See the appendix (Section 3.6.10).

When the regulator intervenes with the fully-distributed-cost regulation, the high-cost firm chooses service-based entry instead of co-investment in order to increase profit. However, when the low-cost facility-based firm is allowed to set access price, the high-cost firm will be driven out of the market if it asks the low-cost firm for access network. Therefore, the high-cost firm prefers the fully-distributedcost regulation.

3.5 Conclusion

This study examines the equilibrium outcomes in the presence of infrastructure sharing by various approaches. The two asymmetric firms have different cost structures of investment in quality. They choose the extent of quality upgrade that can enhance consumers' utility and then choose the scale of consumer base to be served. Under stand-alone investment, the low-cost firm offer higher levels of quality upgrade, firm output and corresponding retail price than the high-cost firm. The low-cost firm is in the better position with greater profit than the high-cost firm due to its advantage of lower cost.

The two firms agree to co-invest in infrastructure sharing instead of standalone investment because they can raise their own profits under cooperation. In accordance with Nash Bargaining Solution, the low-cost firm with dominant bargaining power will carry a lighter burden of investment cost than the high-cost firm. As a result, the low-cost firm earns higher profit. However, the comparison between the outcomes of co-investment and stand-alone investment is ambiguous. When infrastructure sharing yields substantial benefit of cost reduction (respectively infrastructure sharing leads to a considerable amount of incremental cost of infrastructure sharing), the equilibrium quality-upgrade level, firm output, industry output and consumer surplus under co-investment are greater than (respectively less than) those under stand-alone investment. Therefore, co-investment may soften competition if infrastructure sharing does not generate a sufficient amount of cost saving. In this situation, the negotiation on co-investment becomes collusion rather than promotes quality upgrades and consumer surplus.

After the intervention of the fully-distributed-cost regulation, the high-cost service-based firm has an incentive not to co-invest in the shared infrastructure but seeks access network from the low-cost facility-based firm. Thus, they have equal market shares and profits because they both operate on the same network facility and offer identical quality-upgraded services. Compared to co-investment, the high-cost firm can raise its profit under this regulation because it bears a lighter burden of investment cost. When the cost structures of both firms are slightly different (respectively significantly different), the low-cost firm's profit increases (respectively has a tendency to decrease). The low-cost firm decides to offer a higher qualityupgrade level but serve a smaller group of subscribers than it would under coinvestment. As a result, the retail price under this regulation is higher than that under co-investment. Overall, the fully-distributed-cost regulation reduces consumer surplus.

The telecommunications regulator should monitor a co-investment agreement, especially when infrastructure sharing does not yield the sufficiently substantial benefit of cost saving. In this situation, firms may co-invest to dampen competition in quality upgrade, and consumer surplus finally decreases from the stand-alone investment outcome. In addition, the fully-distributed-cost regulation is more effective to promote service-based entry than co-investment and the unregulated access price approach. This regulation may also support the low-cost facility-based firm if the two firms are slightly different in cost structure. Conversely, if the cost structures of the two firms are significantly different, this regulation is likely to threaten the low-cost firm. This is because its bargaining power over cost allocation is much less dominant under the fully-distributed-cost regime than under co-investment. The fully-distributedcost regulation can stimulate the low-cost firm's quality upgrade and accommodates the high-cost service-based entrant, but it causes a reduction in the size of consumer base and consumer surplus. The regulator should take this into consideration if it plans to impose this regulation in order to facilitate service-based entry of the high-cost firm instead of promoting the co-investment negotiation. The optimal regulation depends on what the regulator deems the highest priority.

This study examines the competition of two firms with complete information on their investment cost structures. However, firms may have incomplete information on their rivals' costs. Additionally, this study is based on the assumption of unit demand. Firms decide on the quality upgrade of only one service in the product line which has no line depth. Further research may be conducted in the light of the incomplete information on firms' cost types. Further studies may be also extended to the case of multiple firms, variable demand and firms' decisions on quality of a wider range of product line to serve different consumer groups. Moreover, other approaches of infrastructure sharing and the issue of telecommunications service coverage may be of interest.

3.6 Appendix

3.6.1 Proof of Proposition 3.1

From (3.14) and (3.15),

$$
s_{N1}^* - s_{N2}^* = \frac{12a\omega(\alpha_2 - \alpha_1)}{27\alpha_1\alpha_2 - 24\omega^2\alpha_2 - 24\omega^2\alpha_1 + 16\omega^4}
$$

Firm 1 has lower cost than firm 2, $\alpha_2 - \alpha_1 > 0$, thus $s_{N1}^* > s_{N2}^*$.

The inverse demand function is $p_{Ni} = a + \omega s_{Ni} - q_{N1} - q_{N2}$; $i \in \{1, 2\}$. When $s_{N1}^* > s_{N2}^*$, $p_{N1}^* > p_{N2}^*$.

From (3.11), the differential between the two equilibrium levels of service quantity is $q_{N1}^* - q_{N2}^* = \omega (s_{N1}^* - s_{N2}^*)$. $s_{N1}^* > s_{N2}^*$ so $q_{N1}^* - q_{N2}^* > 0$. Therefore, $q_{N1}^* > q_{N2}^*.$

From (3.12),

$$
\pi_{N1}^* - \pi_{N2}^* = \left[\left(\frac{a}{3} + \frac{2}{3} \omega s_{N1} - \frac{1}{3} \omega s_{N2} \right)^2 - \left(\frac{a}{3} + \frac{2}{3} \omega s_{N2} - \frac{1}{3} \omega s_{N1} \right)^2 \right]
$$

$$
- \left(\frac{\alpha_1}{2} s_{N1}^2 - \frac{\alpha_2}{2} s_{N2}^2 \right)
$$

$$
= \left[\left(\frac{2a}{3} + \frac{1}{3} \omega s_{N1} + \frac{1}{3} \omega s_{N2} \right) \left(\omega s_{N1} - \omega s_{N2} \right) \right] - \left(\frac{\alpha_1}{2} s_{N1}^2 - \frac{\alpha_2}{2} s_{N2}^2 \right)
$$

 $s_{N1}^* > s_{N2}^*$, $\alpha_2 > \alpha_1$ so one can conclude $\left(\frac{2a}{3}\right)$ $rac{2a}{3} + \frac{1}{3}$ $\frac{1}{3}\omega s_{N1} + \frac{1}{3}$ $\frac{1}{3}\omega s_{N2}$ $\bigg(\omega s_{N1} - \omega s_{N2} \bigg) > 0$ and $\frac{\alpha_1}{2} s_{N1}^2 - \frac{\alpha_2}{2}$ $\frac{x_2}{2} s_{N2}^2 < 0$. Thus, $\pi_{N1}^* - \pi_{N2}^* > 0$.

To satisfy the requirement that s_{N1}^* and s_{N2}^* are non-negative in the sharedmarket equilibrium, it is assumed that $\alpha_1 > \frac{4}{3}$ $\frac{4}{3}\omega^2$ and $\alpha_2 > \frac{4}{3}$ $\frac{4}{3}\omega^2$ where the investment cost in telecommunications is enormous relative to additional utility a consumer obtains

from quality upgrade. These assumptions comply with the second-order conditions; $\frac{\partial^2 \pi_{Ni}}{\partial s_{Ni}^2} = \frac{8}{9}$ $\frac{8}{9}\omega^2 - \alpha_i < 0$ or $\alpha_i > \frac{8}{9}$ $\frac{8}{9}\omega^2$; $i \in \{1, 2\}$.

3.6.2 Proof of Proposition 3.2

Substituting (3.12), (3.19) and (3.20) in (3.21) yields the following equation.

$$
\left(\frac{a}{3} + \frac{1}{3}\omega s_c\right)^2 - \beta^* \frac{\phi \alpha_1}{2} s_c^2 - \left(\frac{a}{3} + \frac{2}{3}\omega s_{N1}^* - \frac{1}{3}\omega s_{N2}^*\right)^2 + \frac{\alpha_1}{2} s_{N1}^2
$$

$$
= \left(\frac{a}{3} + \frac{1}{3}\omega s_c\right)^2 - (1 - \beta^*) \frac{\phi \alpha_1}{2} s_c^2 - \left(\frac{a}{3} + \frac{2}{3}\omega s_{N2}^* - \frac{1}{3}\omega s_{N1}^*\right)^2 + \frac{\alpha_2}{2} s_{N2}^*^2
$$

$$
\beta^* \phi a_1 s_c^2 = \frac{\phi a_1 s_c^2}{2} - \frac{2}{3} a \omega s_{N1}^* + \frac{2}{3} a \omega s_{N2}^* - \frac{1}{3} \omega^2 s_{N1}^* + \frac{1}{3} \omega^2 s_{N2}^* + \frac{a_1}{2} s_{N1}^* - \frac{a_2}{2} s_{N2}^*.
$$

$$
\beta^* = \frac{1}{2} + \frac{-\frac{2}{3}a\omega(s_{N1}^* - s_{N2}^*) - \frac{1}{3}\omega^2(s_{N1}^{*2} - s_{N2}^{*2}) + \frac{\alpha_1}{2}(s_{N1}^{*2} - \Delta_{\alpha}s_{N2}^{*2})}{\phi\alpha_1s_c^2}
$$
(3. A1)

One may consider (3.19), (3.20) and (3.21).

If $\beta^* = \frac{1}{2}$ $\frac{1}{2}$, $\pi_{C1} = \pi_{C2}$ and $\pi_{N1}^* = \pi_{N2}^*$ in compliance with (3.21). It contradicts the result of Case 1 that $\pi_{N1}^* > \pi_{N2}^*$.

If $\beta^* > \frac{1}{2}$ $\frac{1}{2}$, $\pi_{c1} < \pi_{c2}$ along with $\pi_{N1}^* < \pi_{N2}^*$ to make (3.21) hold. It also contradicts the result of Case 1 that $\pi_{N1}^* > \pi_{N2}^*$.

If $\beta^* < \frac{1}{2}$ $\frac{1}{2}$, $\pi_{C1} > \pi_{C2}$ and $\pi_{N1}^* > \pi_{N2}^*$. It is possible to make (3.21) hold with $\beta^* < \frac{1}{2}$ $\frac{1}{2}$. Therefore, $\beta^* < \frac{1}{2}$ $\frac{1}{2}$.

Substituting $\beta^* < \frac{1}{2}$ $\frac{1}{2}$ and (3.23) in (3.18) - (3.20) reveals that $q_{C1}^* = q_{C2}^*$ and $\pi_{c1}^* > \pi_{c2}^*.$

In accordance with Split-The-Difference Rule, one may rewrite (3.19) and (3.20) as

$$
\pi_{C1} = \mathbf{a} - \beta \mathbf{b} \tag{3.A2}
$$

$$
\pi_{C2} = \mathbf{a} - (1 - \beta)\mathbf{b} \tag{3.43}
$$

where $a = \left(\frac{a}{2}\right)$ $\frac{a}{3} + \frac{1}{3}$ $\left(\frac{1}{3}\omega s_c^*\right)^2$, $\mathbb{b} = \frac{\phi a_1}{2}$ $\frac{a_1}{2} s_c^{\ast 2}$ and $s_c^{\ast} = \frac{4a\omega}{9\phi\alpha_1 - 4a}$ $\frac{4aw}{9\phi\alpha_1-4\omega^2}$.

Substituting (3.A2) and (3.A3) in (3.21) gives the following result. (π_{N1}^*, π_{N2}^*) is the disagreement point.

$$
a - \beta b - \pi_{N1}^* = a - (1 - \beta)b - \pi_{N2}^* \tag{3. A4}
$$

$$
\beta^* = \frac{1}{2} + \frac{1}{2\mathbb{b}} (\pi_{N2}^* - \pi_{N1}^*)
$$
 (3. A5)

Substituting (3. A5) into (3. A2) and (3. A3) yields

$$
\pi_{C1}^* = \pi_{N1}^* + \frac{1}{2} (2a - b - \pi_{N1}^* - \pi_{N2}^*), \qquad (3. A6)
$$

$$
\pi_{C2}^* = \pi_{N2}^* + \frac{1}{2} (2a - b - \pi_{N1}^* - \pi_{N2}^*).
$$
 (3. A7)

The term $(2a - b - \pi_{N1}^* - \pi_{N2}^*)$ in (3.46) and (3.47) is exactly the difference between the sum of (3. A2) and (3. A3) (the aggregate profit under co-investment) and the sum of the two firms' profits under stand-alone investment.

According to the existence of a successful co-investment agreement, β^* should be positive in this setting. Differentiating (3. A1) with respect to ϕ yields

$$
\frac{\partial \beta^*}{\partial \phi} = \frac{\left\{ \frac{\left[(18\alpha_1)(96a^2\omega^2\phi\alpha_1)(9\phi\alpha_1 - 4\omega^2)\mathbb{GS}^2 \right]}{-(96a^2\omega^2\alpha_1)(9\phi\alpha_1 - 4\omega^2)^2\mathbb{GS}^2} \right\}}{\left(96a^2\omega^2\phi\alpha_1 \right)^2 \mathbb{S}^4}
$$
\n
$$
= (96a^2\omega^2\alpha_1)(9\phi\alpha_1 - 4\omega^2)\mathbb{GS}^2 \frac{\left[18\phi\alpha_1 - (9\phi\alpha_1 - 4\omega^2) \right]}{(96a^2\omega^2\phi\alpha_1)^2 \mathbb{S}^4}
$$

$$
= (9\phi\alpha_1 - 4\omega^2) \mathbb{G} \frac{\left[18\phi\alpha_1 - (9\phi\alpha_1 - 4\omega^2)\right]}{(96a^2\omega^2\alpha_1)(\phi)^2 \mathbb{S}^2}
$$

$$
= (9\phi\alpha_1 - 4\omega^2) \mathbb{G} \frac{\left[9\phi\alpha_1 + 4\omega^2\right]}{(96a^2\omega^2\alpha_1)(\phi)^2 \mathbb{S}^2}
$$

where $\mathbb{S} = 27\alpha_1^2 \Delta_\alpha - 24\omega^2 \alpha_1 - 24\omega^2 \alpha_1 \Delta_\alpha + 16\omega^4$,

$$
\mathbb{G} = a^2 \omega^2 \alpha_1 (\Delta_\alpha - 1) [-864(\alpha_1 - \omega^2)(\alpha_1 \Delta_\alpha - \omega^2) + 96\omega^4].
$$

According to (3.23), $\phi a_1 > \frac{4\omega^2}{9}$ $\frac{w}{9}$ to ensure that the equilibrium qualityupgrade level is non-negative. Therefore, $(9\phi\alpha_1 - 4\omega^2) > 0$. Due to the existence of the shared-market equilibrium under stand-alone investment, $\alpha_1 > \frac{4}{3}$ $\frac{4}{3}\omega^2$. $\Delta_{\alpha} > 1$. Thus, \mathbb{G} < 0. Consequently, $\frac{\partial \beta^*}{\partial \phi}$ < 0.

3.6.3 Proof of the outcome of the unregulated access price approach

In contrast to Case 3, without regulation, the incumbent sets access price (m) which is collected from the entrant in return for serving one subscriber (one unit of output) under the assumption of unit demand. 52

Profit functions are

$$
\pi_{A1} = (a + \omega s_A - q_{A1} - q_{A2})q_{A1} - \frac{\phi \alpha_1}{2} s_A^2 + m q_{A2}
$$
 (3. A8)

$$
\pi_{A2} = (a + \omega s_A - q_{A1} - q_{A2})q_{A2} - m q_{A2}
$$
\n(3.49)

where $Q_A = q_{A1} + q_{A2}$.

 $\overline{}$

 52 For simplicity, there is no firm-specific preference. Thus, from consumers' perspective, the quality-enhanced service of the service-based entrant is not different from that of the incumbent that owns the infrastructure because they both run on the same shared infrastructure at an identical quality-upgrade level.

Timing of the game

The following three-stage game is solved by backward induction.

Stage 1 Firm 1 chooses the extent of quality upgrade (s_A) to maximise its own profit.

Stage 2 Firm 1 sets access price (m) .

Stage 3 The two firms simultaneously choose their own service quantity levels $(q_{A1}, q_{A2}).$

In stage 3, the FOCs are shown below.

$$
\frac{\partial \pi_{A1}}{\partial q_{A1}} = a + \omega s_A - 2q_{A1} - q_{A2} = 0 \tag{3. A10}
$$

$$
\frac{\partial \pi_{A2}}{\partial q_{A2}} = a + \omega s_A - q_{A1} - 2q_{A2} - m = 0 \tag{3. A11}
$$

From (3. A10) and (3. A11), the equilibrium quantity levels are

$$
q_{A1}^*(m, s_A) = \frac{a}{3} + \frac{1}{3}\omega s_A + \frac{m}{3}
$$
 (3. A12)

$$
q_{A2}^*(m, s_A) = \frac{a}{3} + \frac{1}{3}\omega s_A - \frac{2}{3}m
$$
 (3. A13)

Substituting (3. A12) and (3. A13) into (3. A8) yields the following.

$$
\pi_{A1}(m, s_A) = \left(\frac{a}{3} + \frac{\omega s_A}{3} + \frac{m}{3}\right)^2 + m\left(\frac{a}{3} + \frac{\omega s_A}{3} - \frac{2m}{3}\right) - \frac{\phi a_1}{2} s_A^2 \tag{3. A14}
$$

Back to stage 2, differentiating the reduced-form profit function in (3. A14) with respect to m gives the profit-maximising access price as shown below.

$$
m^*(s_A) = \frac{a + \omega s_A}{2} \tag{3. A15}
$$

One may substitute (3. A15) into (3. A14) and obtain the reduced-form profit function of firms 1 as shown below.

$$
\pi_{A1}(s_A) = \left(\frac{a}{3} + \frac{\omega s_A}{3} + \frac{a + \omega s_A}{6}\right)^2 + \left(\frac{a + \omega s_A}{2}\right)\left(\frac{a}{3} + \frac{\omega s_A}{3} - \frac{2}{3}\left(\frac{a + \omega s_A}{2}\right)^2\right) - \frac{\phi \alpha_1}{2} s_A^2 \tag{3. A16}
$$

In stage 1, from (3. A16), firm 1 chooses the extent of quality upgrade which satisfies the following FOC.

$$
\frac{\partial \pi_{A1}}{\partial s_A} = \frac{a\omega}{2} + \frac{\omega^2 s_A}{2} - \phi \alpha_1 s_A = 0 \tag{3. A17}
$$

$$
s_A^* = \frac{a\omega}{(2\phi\alpha_1 - \omega^2)}\tag{3. A18}
$$

Substituting (3.A15) and (3.A18) in (3.A13) and (3.A9) yields $q_{A2}^* = 0$ and $\pi_{A2}^* = 0$ respectively.

One may find that the unregulated access price approach undermines consumer welfare by comparing this approach with co-investment.

Comparing (3.23) with (3. A18) yields

$$
s_c^* - s_A^* = \frac{4a\omega}{9\phi\alpha_1 - 4\omega^2} - \frac{a\omega}{2\phi\alpha_1 - \omega^2}
$$

$$
= \frac{-a\omega\phi\alpha_1}{(9\phi\alpha_1 - 4\omega^2)(2\phi\alpha_1 - \omega^2)} < 0
$$

Therefore, $s_A^* > s_C^*$.

From (3. A12), (3. A13), (3. A15) and (3. A18),

$$
Q_A^* = q_{A1}^* + q_{A2}^* = \frac{a}{2} + \frac{\omega}{2} \left(\frac{a\omega}{2\phi\alpha_1 - \omega^2} \right) + 0
$$

$$
= \frac{2a\phi\alpha_1}{4\phi\alpha_1 - 2\omega^2} \tag{3. A19}
$$

From (3.18) and (3.23),

$$
Q_C^* = 2q_C^* = \frac{2a}{3} + \frac{2\omega}{3} \left(\frac{4a\omega}{9\phi\alpha_1 - 4\omega^2}\right)
$$

$$
=\frac{6a\phi\alpha_1}{9\phi\alpha_1 - 4\omega^2} \tag{3. A20}
$$

Under the assumption that $\phi a_1 > \frac{16}{9}$ $\frac{16}{9}\omega^2$, comparing (3.A19) with (3.A20) gives the following.

$$
Q_{c}^{*}-Q_{A}^{*}=\frac{6a\phi^{2}\alpha_{1}^{2}-4a\omega^{2}\phi\alpha_{1}}{(4\phi\alpha_{1}-2\omega^{2})(9\phi\alpha_{1}-4\omega^{2})}>0
$$

Thus, $Q_A^* < Q_C^*$ and $CS_A < CS_C$.

From (3.1), co-investment leads to the symmetric retail price in the retail market, i.e. $p_{c1}^* = p_{c2}^* = p_c^*$ and $p_{A1}^* = p_A^*$. To compare p_A^* with p_c^* , one can write

$$
p_C^* - p_A^* = (a + \omega s_C^* - Q_C^*) - (a + \omega s_A^* - Q_A^*)
$$

= $\omega (s_C^* - s_A^*) + (Q_A^* - Q_C^*).$

 $(s_c^* - s_A^*) < 0$ and $(Q_A^* - Q_C^*) < 0$, thus $p_A^* > p_C^*$.

3.6.4 Proof of Proposition 3.4

From (3.14), (3.15) and (3.23), one can write the following.

$$
s_{N1}^{*} - s_{C}^{*} = \frac{12a\omega\alpha_{1} (9\phi\alpha_{1}\Delta_{\alpha} - 12\omega^{2}\phi - 9\alpha_{1}\Delta_{\alpha} + 4\omega^{2}\Delta_{\alpha} + 8\omega^{2})}{(9\phi\alpha_{1} - 4\omega^{2})(27\alpha_{1}^{2}\Delta_{\alpha} - 24\omega^{2}\alpha_{1} - 24\omega^{2}\alpha_{1}\Delta_{\alpha} + 16\omega^{4})}
$$
(3. A21)

 $(9\phi\alpha_1 - 4\omega^2) > 0$ and $(27\alpha_1^2\Delta_\alpha - 24\omega^2\alpha_1 - 24\omega^2\alpha_1\Delta_\alpha + 16\omega^4) > 0$

The sign of $(s_{N1}^* - s_c^*)$ corresponds to the sign of $\left($ $9\phi\alpha_1\Delta_\alpha - 12\omega^2\phi - 9\alpha_1\Delta_\alpha$ $+4\omega^2\Delta_\alpha+8\omega^2$ \cdot

$$
(9\phi\alpha_1\Delta_\alpha - 12\omega^2\phi - 9\alpha_1\Delta_\alpha + 4\omega^2\Delta_\alpha + 8\omega^2) > 0 \text{ when } \phi > \frac{9\alpha_1\Delta_\alpha - 8\omega^2 - 4\omega^2\Delta_\alpha}{9\alpha_1\Delta_\alpha - 12\omega^2}.
$$

Therefore, if $0 < \phi < \frac{9\alpha_1 \Delta_{\alpha} - 8\omega^2 - 4\omega^2 \Delta_{\alpha}}{2\omega_1 \Delta_{\alpha} + 3\omega_1^2}$ $\frac{\Delta_{\alpha}-8\omega^2-4\omega^2\Delta_{\alpha}}{9\alpha_1\Delta_{\alpha}-12\omega^2}$, $s_{N1}^* < s_c^*$. If $\frac{9\alpha_1\Delta_{\alpha}-8\omega^2-4\omega^2\Delta_{\alpha}}{9\alpha_1\Delta_{\alpha}-12\omega^2} \leq \phi < 2$, $s_{N1}^* \geq s_C^*$.

$$
s_{N2}^* - s_C^* = \frac{12a\omega\alpha_1 (9\phi\alpha_1 - 12\omega^2\phi - 9\alpha_1\Delta_\alpha + 4\omega^2 + 8\omega^2\Delta_\alpha)}{(9\phi\alpha_1 - 4\omega^2)(27\alpha_1^2\Delta_\alpha - 24\omega^2\alpha_1 - 24\omega^2\alpha_1\Delta_\alpha + 16\omega^4)}
$$
(3. A22)

The sign of $(s_{N2}^* - s_c^*)$ corresponds to the sign of $\left($ $9\phi\alpha_1 - 12\omega^2\phi - 9\alpha_1\Delta_\alpha$ $+4\omega^2+8\omega^2\Delta_\alpha$).

 $9\phi\alpha_1 - 12\omega^2\phi - 9\alpha_1\Delta_\alpha + 4\omega^2 + 8\omega^2\Delta_\alpha > 0$ when $\phi > \frac{9\alpha_1\Delta_\alpha - 8\omega^2\Delta_\alpha - 4\omega^2}{9\alpha_1 - 12\omega^2}$ $\frac{\alpha_0 - \alpha_0}{9\alpha_1 - 12\omega^2}$. Thus,

if
$$
0 < \phi \le \frac{9\alpha_1 \Delta_{\alpha} - 8\omega^2 \Delta_{\alpha} - 4\omega^2}{9\alpha_1 - 12\omega^2}
$$
, $s_{N2}^* \le s_C^*$. If $\phi > \frac{9\alpha_1 \Delta_{\alpha} - 8\omega^2 \Delta_{\alpha} - 4\omega^2}{9\alpha_1 - 12\omega^2}$, $s_{N2}^* > s_C^*$.

$$
\phi^a = \frac{9\alpha_1\Delta_\alpha - 8\omega^2 - 4\omega^2\Delta_\alpha}{9\alpha_1\Delta_\alpha - 12\omega^2}
$$
 and
$$
\phi^b = \frac{9\alpha_1\Delta_\alpha - 8\omega^2\Delta_\alpha - 4\omega^2}{9\alpha_1 - 12\omega^2}
$$
. To compare ϕ^a with ϕ^b ,

one may rewrite these expressions as

$$
\phi^{a} = \frac{9\alpha_{1}\Delta_{\alpha} - 8\omega^{2} - 4\omega^{2}\Delta_{\alpha}}{9\alpha_{1}\Delta_{\alpha} - 12\omega^{2}} = 1 - \frac{4\omega^{2}(\Delta_{\alpha} - 1)}{9\alpha_{1}\Delta_{\alpha} - 12\omega^{2}} < 1
$$

$$
\phi^{b} = \frac{9\alpha_{1}\Delta_{\alpha} - 8\omega^{2}\Delta_{\alpha} - 4\omega^{2}}{9\alpha_{1} - 12\omega^{2}} = 1 + \frac{(9\alpha_{1} - 8\omega^{2})(\Delta_{\alpha} - 1)}{9\alpha_{1} - 12\omega^{2}} > 1
$$

where $\Delta_{\alpha} > 1$ and $\alpha_1 > \frac{4}{3}$ $\frac{4}{3}\omega^2$ to satisfy the requirements of shared equilibrium in the stand-alone investment case.

They can be illustrated in the following number line.

 0 1

Therefore, if $0 < \phi < \frac{9\alpha_1 \Delta_{\alpha} - 8\omega^2 - 4\omega^2 \Delta_{\alpha}}{2\omega_1 \Delta_{\alpha} + 3\omega_1^2}$ $\frac{\Delta_{\alpha}-8\omega^2-4\omega^2\Delta_{\alpha}}{9\alpha_1\Delta_{\alpha}-12\omega^2}$, $S_{N2}^* < S_{N1}^* < S_{C}^*$.

If $\frac{9\alpha_1\Delta_\alpha - 8\omega^2 - 4\omega^2\Delta_\alpha}{9\alpha_1\Delta_\alpha - 12\omega^2} \leq \phi \leq \frac{9\alpha_1\Delta_\alpha - 8\omega^2\Delta_\alpha - 4\omega^2}{9\alpha_1 - 12\omega^2}$ $\frac{\alpha^{-8\omega^2\Delta_{\alpha}-4\omega^2}}{9\alpha_1-12\omega^2}$, $S_{N2}^* \leq S_C^* \leq S_{N1}^*$.

Finally, if $\phi > \frac{9\alpha_1\Delta_{\alpha}-8\omega^2\Delta_{\alpha}-4\omega^2}{2(1-\alpha^2)^2}$ $\frac{\alpha^{-8\omega^2\Delta_{\alpha}-4\omega^2}}{9\alpha_1-12\omega^2}$, $s_C^* < s_{N2}^* < s_{N1}^*$.

3.6.5 Proof of Corollary 3.1

If $\alpha_2 = \alpha_1$, (3.14) and (3.15) can be rewritten as

$$
s_{N1}^* = s_{N2}^* = \frac{4a\omega}{9\alpha_1 - 4\omega^2}.
$$
 (3. A23)

Comparing (3. A23) with (3.23) yields the following results which vary with the magnitude of cost saving/ cost increment from infrastructure sharing.

If $0 < \phi < 1$, $9\phi \alpha_1 - 4\omega^2 < 9\alpha_1 - 4\omega^2$ and $s_c^* > s_{N1}^* = s_{N2}^*$. If $\phi = 1$, $s_c^* = s_{N1}^* = s_{N2}^*$. Finally, if $\phi > 1$, $9\phi\alpha_1 - 4\omega^2 > 9\alpha_1 - 4\omega^2$ and consequently $s_c^* < s_{N1}^* = s_{N2}^*$.

3.6.6 Proof of Proposition 3.5

From (3.8), (3.11) and (3.18),

$$
Q_N^* = \frac{2}{3}a + \frac{\omega}{3}(s_{N1}^* + s_{N2}^*) \text{ and } Q_C^* = \frac{2}{3}a + \frac{2}{3}\omega s_C^*
$$

$$
CS_N = \frac{1}{2}(Q_N^*)^2 \text{ and } CS_C = \frac{1}{2}(Q_C^*)^2.
$$

$$
Q_N^* - Q_C^* = \frac{\omega}{3}(s_{N1}^* + s_{N2}^* - 2\omega s_C^*)
$$

 $Q_N^* - Q_C^* > 0$ when $s_{N1}^* + s_{N2}^* - 2s_C^* > 0$.

$$
s_{N1}^{*} + s_{N2}^{*} - 2\omega s_{C}^{*}
$$

=
$$
\frac{36a\omega\alpha_{1} [3\phi\alpha_{1} + 3\phi\alpha_{1}\Delta_{\alpha} - 8\omega^{2}\phi - 6\alpha_{1}\Delta_{\alpha} + 4\omega^{2}\Delta_{\alpha} + 4\omega^{2}]}{(9\phi\alpha_{1} - 4\omega^{2})(27\alpha_{1}^{2}\Delta_{\alpha} - 24\omega^{2}\alpha_{1} - 24\omega^{2}\alpha_{1}\Delta_{\alpha} + 16\omega^{4})}
$$
(3. A24)

 $(9\phi\alpha_1 - 4\omega^2) > 0$ and $(27\alpha_1^2\Delta_\alpha - 24\omega^2\alpha_1 - 24\omega^2\alpha_1\Delta_\alpha + 16\omega^4) > 0$

The sign of
$$
(s_{N1}^* + s_{N2}^* - 2\omega s_C^*)
$$
 is the sign of $\begin{pmatrix} 3\phi\alpha_1 + 3\phi\alpha_1\Delta_\alpha - 8\omega^2\phi \\ -6\alpha_1\Delta_\alpha + 4\omega^2\Delta_\alpha + 4\omega^2 \end{pmatrix}$.

 $3\phi\alpha_1 + 3\phi\alpha_1\Delta_\alpha - 8\omega^2\phi - 6\alpha_1\Delta_\alpha + 4\omega^2\Delta_\alpha + 4\omega^2 > 0$ when the cost-adjustment parameter is sufficiently high, i.e. $\phi > \frac{6\alpha_1\Delta_{\alpha}-4\omega^2\Delta_{\alpha}-4\omega^2}{2\omega_1\Delta_{\alpha}+2\omega_1\Delta_{\alpha}-2\omega^2}$ $\frac{\alpha_1 \Delta_{\alpha} - 4\omega \Delta_{\alpha} - 4\omega}{3\alpha_1 \Delta_{\alpha} + 3\alpha_1 - 8\omega^2}.$

$$
\phi^{c} = \frac{6\alpha_{1}\Delta_{\alpha} - 4\omega^{2}\Delta_{\alpha} - 4\omega^{2}}{3\alpha_{1}\Delta_{\alpha} + 3\alpha_{1} - 8\omega^{2}} = 1 + \frac{3\alpha_{1}\Delta_{\alpha} - 3\alpha_{1} - 4\omega^{2}\Delta_{\alpha} + 4\omega^{2}}{3\alpha_{1}\Delta_{\alpha} + 3\alpha_{1} - 8\omega^{2}}
$$

$$
= 1 + \frac{(3\alpha_{1} - 4\omega^{2})(\Delta_{\alpha} - 1)}{3\alpha_{1}\Delta_{\alpha} + 3\alpha_{1} - 8\omega^{2}}
$$

Since $\Delta_{\alpha} > 1$ and $\alpha_1 > \frac{4}{3}$ $rac{4}{3}\omega^2$, $(3\alpha_1\Delta_\alpha - 3\alpha_1 - 4\omega^2\Delta_\alpha + 4\omega^2) - (3\alpha_1\Delta_\alpha + 3\alpha_1 - 8\omega^2) < 0$. Thus, $1 < \phi^c < 2$.

In conclusion, if $0 < \phi \leq \frac{6a_1\Delta_{\alpha}-4\omega^2\Delta_{\alpha}-4\omega^2}{2(1-4a_1\Delta_{\alpha}-4a_2\Delta_{\alpha}-4a_3\Delta_{\alpha}-4a_4\Delta_{\alpha}-4a_5\Delta_{\alpha}-4a_6\Delta_{\alpha}-4a_6\Delta_{\alpha}-4a_6\Delta_{\alpha}-4a_6\Delta_{\alpha}-4a_6\Delta_{\alpha}-4a_6\Delta_{\alpha}-4a_6\Delta_{\alpha}-4a_6\Delta_{\alpha}-4a_6\Delta_{\alpha}-4a_6\Delta_{\alpha}-4a_6\Delta_{\alpha}$ $\frac{\alpha_1 \Delta_\alpha - 4\omega^2 \Delta_\alpha - 4\omega^2}{3\alpha_1 + 3\alpha_1 \Delta_\alpha - 8\omega^2}$, $Q_c^* \geq Q_N^*$ and $CS_C \geq CS_N$. On the contrary, if $\phi > \frac{6\alpha_1\Delta_{\alpha}-4\omega^2\Delta_{\alpha}-4\omega^2}{2\alpha_1+2\omega^2\Delta_{\alpha}-2\omega^2}$ $\frac{\alpha_1\Delta_\alpha-4\omega^2\Delta_\alpha-4\omega^2}{3\alpha_1+3\alpha_1\Delta_\alpha-8\omega^2}$, $Q_c^* < Q_N^*$ and $CS_C < CS_N$.

3.6.7 Proof of Proposition 3.6

 s_F^* must satisfy the first-order condition (3.27). From (3.23) and (3.27), to compare s_F^* with s_C^* , one may substitute s_C^* in (3.27).

$$
\frac{\partial \pi_{F1}}{\partial s_F}\Big|_{s=s_C^*} = \left(a + \omega s_C^* - 4(q_{F1}^*|_{s=s_C^*})\right) \frac{\partial q_{F1}^*}{\partial s_F}\Big|_{s=s_C^*} + \omega(q_{F1}^*|_{s=s_C^*}) - \frac{\phi \alpha_1 s_C^*}{2}
$$

$$
= \frac{a^2 \omega \phi \alpha_1}{36(\mathbb{H}|_{s=s_C^*})(9\phi \alpha_1 - 4\omega^2)^2} \Bigg[-12\left(\frac{9}{4}(\phi \alpha_1)^2 - \frac{2}{3}\omega^2 \phi \alpha_1\right)^{1/2} \Bigg] (3. A25)
$$

where

$$
\mathbb{H}|_{s=s_C^*} = \sqrt{\frac{(a + \omega s_C^*)^2}{36} - \frac{\phi \alpha_1 s_C^*^2}{24}}.
$$

Assuming that $\phi \alpha_1 > \frac{16}{9}$ $rac{16}{9}\omega^2$, $rac{\partial \pi_{F1}}{\partial s_F}$ $\frac{n_{F1}}{\partial s_F}$ $s=s_C^* > 0$. Thus, $s_F^* > s_C^*$.

From (3.18) and (3.26), $q_F^*(s)$ and $q_C^*(s)$ as output functions of qualityupgrade levels (s) will never intersect at any given $s > 0$. The proof is as follows.

$$
q_C^*(s) = q_F^*(s)
$$

$$
\frac{a}{3} + \frac{1}{3}\omega s = \frac{a + \omega s}{6} + \sqrt{\frac{(a + \omega s)^2}{36} - \frac{\phi \alpha_1 s^2}{24}}
$$

It can be rearranged as

$$
\frac{a + \omega s}{6} - \sqrt{\left[\frac{(a + \omega s)}{6}\right]^2 - \frac{\phi \alpha_1 s^2}{24}} = 0.
$$

The above expression will never hold when $s > 0$. Therefore, there is no qualityupgrade level yielding the same output of these two cases. At $s = 0$, $q_c^*(0) = \frac{a}{3}$ $\frac{a}{3}$ and $q_F^*(0) = \frac{a}{6}$ $\frac{a}{6}$. The slope of $q_C^*(s)$ is $\frac{\partial q_C^*}{\partial s} = \frac{\omega}{3}$ $\frac{\omega}{3}$. Meanwhile, the slope of $q_F^*(s)$ is

$$
\frac{\partial q_{F}^{*}}{\partial s} = \frac{\omega}{6} + \frac{\left(\frac{(a + \omega s)\omega}{18} - \frac{\phi a_{1} s}{12}\right)}{2\sqrt{\frac{(a + \omega s)^{2}}{36} - \frac{\phi a_{1} s^{2}}{24}}}.
$$
(3. A26)

The second-order derivative of $q_F^*(s)$ is

$$
\frac{\partial^2 q_F^*}{\partial s^2} = -\frac{\left(\frac{(a+\omega s)\omega}{18} - \frac{\phi \alpha_1 s}{12}\right)^2}{4\left(\frac{(a+\omega s)^2}{36} - \frac{\phi \alpha_1 s^2}{24}\right)\sqrt{\frac{(a+\omega s)^2}{36} - \frac{\phi \alpha_1 s^2}{24}} - \frac{\left(\frac{\omega^2}{18} - \frac{\phi \alpha_1}{12}\right)}{2\sqrt{\frac{(a+\omega s)^2}{36} - \frac{\phi \alpha_1 s^2}{24}}}
$$
(3.A27)
From (3.A26), at $s = s_c^*$,

$$
\frac{\partial q_{F}^{*}}{\partial s}\Big|_{s=s_{C}^{*}} = \frac{\omega}{6} + \frac{\frac{(a+\omega s_{C}^{*})\omega}{18} - \frac{\phi \alpha_{1} s_{C}^{*}}{12}}{2\sqrt{\frac{(a+\omega s_{C}^{*})^{2}}{36} - \frac{\phi \alpha_{1} s_{C}^{*}}{24}}}
$$
(3. A28)

Due to the fact that $\sqrt{\frac{(a+\omega s_c^*)^2}{c}}$ $\frac{\omega s_{C}^{*}}{36} - \frac{\phi \alpha_{1} s_{C}^{*2}}{24}$ $\frac{x_1 s_c^*^2}{24} > 0$, the sign of $\frac{\partial q_F^*}{\partial s}$ $\frac{q_F}{\partial s}$ corresponds to the $s = s_C^*$ sign of $\left(\frac{(a+\omega s_c^*)\omega}{4\Omega}\right)$ $\frac{\omega s_{\mathcal{C}}^*\right)\omega}{18}-\frac{\phi\alpha_1 s_{\mathcal{C}}^*}{12}\Big).$

Under the assumption that $\phi a_1 > \frac{16}{9}$ $\frac{16}{9}$ ω^2 , one can rewrite the expression as

$$
\frac{(a+\omega s_{\mathcal{C}}^*)\omega}{18}-\frac{\phi\alpha_1 s_{\mathcal{C}}^*}{12}=\frac{6a\omega\phi\alpha_1}{36(9\phi\alpha_1-4\omega^2)}>0.
$$

Therefore, $\frac{\partial q_F^*}{\partial q_F^*}$ $\frac{q_F}{\partial s}$ $_{s=s_C^*} > 0$. From (3. A27), since $\phi \alpha_1 > \frac{16}{9}$ $rac{16}{9} \omega^2$, $rac{\partial^2 q_F^*}{\partial s^2}$ $\frac{q_F}{\partial s^2} < 0.$

 s_F^U denotes an upper limit of s_F^* in association with a negative value of the second-order condition for profit maximisation in Case 3 (Access to infrastructure under the fully-distributed-cost regulatory regime), i.e. $s_c^* < s_F^* < s_F^U$. From (3.24),

$$
\frac{\partial^2 \pi_{F1}}{\partial q_{F1}^2} = -2 + \frac{\phi \alpha_1 s_F^2 q_{F2}}{Q_F^3} < 0.
$$

In equilibrium, $q_{F1}^* = q_{F2}^*$ in stage 2 of the game. Thus,

$$
\frac{\partial^2 \pi_{F_1}}{\partial q_{F_1}^2} = -2 + \frac{\phi \alpha_1 s_F^2}{8(q_{F_1}^*)^2} < 0
$$

$$
q_{F_1}^* > \frac{\sqrt{\phi \alpha_1 s_F^2}}{4}
$$
 (3.A29)

From (3.26), the equilibrium output level (q_{F1}^*) should comply with (3.A29).

$$
\frac{a + \omega s_F}{6} + \sqrt{\left[\frac{(a + \omega s_F)}{6}\right]^2 - \frac{\phi \alpha_1 s_F^2}{24}} > \frac{\sqrt{\phi \alpha_1 s_F^2}}{4}
$$

After rearranging the above expression, s_F^U is an upper limit of s_F^* as shown below.

$$
s_F^* < s_F^U = \frac{4a}{5\phi\alpha_1 - 4\omega} \tag{3. A30}
$$

From (3.A26), under the assumption that $\phi a_1 > \frac{16}{9}$ $rac{16}{9} \omega^2$,

$$
\frac{\partial q_F^*}{\partial s}\Big|_{s=s_F^U} = \frac{a + \omega s_F^U}{6} + \sqrt{\frac{(a + \omega s_F^U)^2}{36} - \frac{\phi \alpha_1 s_F^U^2}{24}} \qquad (3. A31)
$$

$$
= \frac{108a\omega\phi\alpha_1 - 48a\omega^2\sqrt{\phi \alpha_1} - 60a\sqrt{\phi \alpha_1}\phi\alpha_1}{12a\sqrt{\phi \alpha_1}(5\sqrt{\phi \alpha_1} - 4\omega)} > 0
$$

Therefore, $\frac{\partial q_F^*}{\partial q}$ $\frac{q_F}{\partial s}$ $_{s=s_F^U} > 0.$

From (3.18), (3.23), (3.26) and (3.430), one can compare $q_F^*|_{s=s_F^U}$ with $q_c^*|_{s=s_c^*}$ as follows.

$$
q_{C}^{*}|_{s=s_{C}^{*}} - q_{F}^{*}|_{s=s_{F}^{U}} = \frac{a}{6} + \left(\frac{4a\omega^{2}}{3(9\phi\alpha_{1} - 4\omega^{2})} - \frac{4a\omega}{6(5\sqrt{\phi\alpha_{1}} - 4\omega)}\right) - \frac{a\sqrt{\phi\alpha_{1}}}{6(9\phi\alpha_{1} - 4\omega)} = \frac{3\phi\alpha_{1} - 6\omega\sqrt{\phi\alpha_{1}} + 2\omega^{2}}{(54\phi\alpha_{1} - 24\omega^{2})(30\omega\sqrt{\phi\alpha_{1}} - 24\omega^{2})} > 0
$$
(3. A32)

When $\phi \alpha_1 > \frac{16}{9}$ $\frac{16}{9}\omega^2$, $q_c^*|_{s=s_c^*}-q_F^*|_{s=s_F^U}>0$.

> According to $s_c^* < s_F^* < s_F^U$ and the conclusion that $\frac{\partial q_F^*}{\partial s}$ $\frac{q_F}{\partial s}$ $s = s_c^*$, $\frac{\partial q_F^*}{\partial s}$ $\frac{\partial q_F}{\partial s}$ $\sum_{s=s_F^U} > 0$

and $\frac{\partial^2 q_F^*}{\partial z^2}$ $\frac{q_i}{q_s^2}$ < 0, one can conclude that $\left| \frac{q_i}{q_f} \right|_{s=s_F^*} < \left| \frac{q_i}{s-s_F^*} \right|$. Further, one may employ transitivity to make a comparison of the equilibrium output levels. From (3. A32), $q_F^*|_{s=s_F^y} < q_C^*|_{s=s_C^*}$. Therefore, in the absence of the intersection of $q_C^*(s)$ and $q_F^*(s)$, it can be concluded that $q_F^*|_{s=s_F^*} < q_C^*|_{s=s_C^*}$ and $Q_F^* < Q_C^*$. From (3.8), $CS_F < CS_C$.

According to (3.1), infrastructure sharing by co-investment and by the fullydistributed-cost approach lead to the symmetric retail prices in the retail market, i.e.

 $p_{c1}^* = p_{c2}^* = p_c^*$ and $p_{F1}^* = p_{F2}^* = p_F^*$. To compare p_F^* with p_c^* , one can write the following equation.

$$
p_C^* - p_F^* = (a + \omega s_C^* - Q_C^*) - (a + \omega s_F^* - Q_F^*)
$$

= $\omega (s_C^* - s_F^*) + (Q_F^* - Q_C^*).$

Since $(s_c^* - s_F^*) < 0$ and $(Q_F^* - Q_C^*) < 0$, $p_F^* > p_C^*$.

3.6.8 Proof of Proposition 3.7a

Consider (3.12), (3.14) and (3.15). To satisfy the FOCs (3.21) and (3.22), $(\pi_{c1}^* - \pi_{N1}^*)$ and $(\pi_{c2}^* - \pi_{N2}^*)$ should be positive simultaneously. Thus, $\pi_{c1}^* > \pi_{N1}^*$ and $\pi_{C2}^* > \pi_{N2}^*$.

3.6.9 Proof of Proposition 3.7b

(i) The comparison between π_{F1}^* and π_{C1}^* is ambiguous according to the cost-asymmetry parameter (Δ_{α}) .

From (3.6), (3.19), (3.23) and (3.26),

$$
\pi_{F1}^* - \pi_{C1}^* = \left[\frac{(a + \omega s_F^*)^2}{9} - \frac{(a + \omega s_C^*)^2}{9} \right] + \frac{(a + \omega s_F^*)}{3} \sqrt{\frac{(a + \omega s_F^*)^2}{36}} - \frac{\phi \alpha_1 s_F^{*2}}{24}
$$

$$
-2 \left[\frac{(a + \omega s_F^*)^2}{36} - \frac{\phi \alpha_1 s_F^{*2}}{24} \right] + \phi \alpha_1 \left[\frac{\beta^* s_C^{*2}}{2} - \frac{s_F^{*2}}{4} \right]
$$

$$
\frac{\partial (\pi_{F_1}^* - \pi_{c_1}^*)}{\partial \Delta_{\alpha}} = \frac{\phi \alpha_1 s_c^{*2}}{2} \frac{\partial \beta^*}{\partial \Delta_{\alpha}}
$$
\n
$$
= \left(\frac{1}{6S^2}\right)\n\begin{cases}\n864a^2 \omega^4 \alpha_1^2 \Delta_{\alpha} - 432a^2 \omega^2 \alpha_1^3 - 432a^2 \omega^2 \alpha_1^3 (\Delta_{\alpha} - 1) \\
-384a^2 \omega^2 \alpha_1 - 384a^2 \omega^2 (\Delta_{\alpha} - 1) \\
(24\omega^2 \alpha_1 + 27\alpha_1^2) \left(\frac{864a^2 \omega^4 \alpha_1^2 (\Delta_{\alpha}^2 - 1)}{6} + \frac{864a^2 \omega^2 \alpha_1^3 \Delta_{\alpha} + 768a^2 \omega^2 \alpha_1 (\Delta_{\alpha} - 1)}{\mathbb{S}}\right)\n\end{cases}
$$

where $\mathbb{S} = 27\alpha_1^2 \Delta_\alpha - 24\omega^2 \alpha_1 \Delta_\alpha - 24\omega^2 \alpha_1 + 16\omega^4$.

In this section, it is assumed that the investment cost of firm 1 is not too low, $\alpha_1 \geq 1.5\omega^2$. At $\Delta_{\alpha} = 1$, $\frac{\partial (\pi_{F_1}^* - \pi_{C_1}^*)}{\partial \Delta_{\alpha}}$ $\frac{F_1 - \mu_{C_1}}{\partial \Delta_{\alpha}}$ $\Delta_{\alpha}=1$ < 0 and $(\pi_{F1}^* - \pi_{C1}^*)|_{\Delta_{\alpha}=1} > 0$. One may conclude that when Δ_{α} is very low, $\pi_{F_1}^* > \pi_{C_1}^*$. When Δ_{α} increases from 1, the term $\pi_{F1}^* - \pi_{C1}^*$ decreases in value. Hence, it can be concluded that when Δ_{α} is sufficiently high, there is a tendency that $\pi_{F1}^* < \pi_{C1}^*$.

(ii) $\pi_{A1}^* > \pi_{C1}^*$ From (3.19), (3.23), (3. A14), (3. A15) and (3. A18), $\pi_{A1}^*|_{m=m^*}$ $n=m^*_{s=s_A^*}$ $16a^2\phi^2\alpha_1^2 - 8a^2\omega^2\phi\alpha_1$ $(8\phi\alpha_1 - 4\omega^2)^2$ $\pi_{c1}^*|_{s=s_c^*} =$ $9a^2\phi^2\alpha_1^2 - 8\beta^*a^2\omega^2\phi\alpha_1$ $(9\phi\alpha_1 - 4\omega^2)^2$ $\pi_{A1}^*|_{m=m*}$ $\lim_{s=s_A^*} -\pi_{C1}^*|_{s=s_C^*} = \frac{\sqrt{2}}{2}$ L \mathbf{I} L $90a^2\phi^4\alpha_1^4 - 153a^2\omega^2\phi^3\alpha_1^3 + 64\beta^*a^2\omega^2\phi^3\alpha_1^3$ +86 $a^2 \omega^4 \phi^2 \alpha_1^2 - 64 \beta^* a^2 \omega^4 \phi^2 \alpha_1^2$ $-16a^2\omega^6\phi\alpha_1 + 16\beta^*a^2\omega^6\phi\alpha_1$ $\overline{}$ $\overline{}$ $\overline{}$ $\overline{}$ $\sqrt{(8\phi\alpha_1 - 4\omega^2)^2(9\phi\alpha_1 - 4\omega^2)^2} > 0$

Under the assumption that $\phi \alpha_1 > \frac{16}{9}$ $\frac{16}{9}\omega^2$, $\pi_{A1}^*|_{m=m*}$ $\lim_{s=s_A^*} -\pi_{C1}^*|_{s=s_C^*} > 0.$

3.6.10 Proof of Proposition 3.7c

From (3.7), (3.20), (3.23), and (3.26),

$$
\pi_{C2}^*|_{s=s_C^*} = \frac{(a + \omega s_C^*)^2}{9} - (1 - \beta^*) \frac{\phi \alpha_1}{2} s_C^{*2}
$$

$$
\pi_{F2}^*|_{s=s_C^*} = \left[\frac{2(a + \omega s_C^*)}{3} - 2\sqrt{\frac{(a + \omega s_C^*)^2}{36} - \frac{\phi \alpha_1 s_C^{*2}}{24}}\right] \left(\frac{\frac{(a + \omega s_C^*)}{6}}{4} - \frac{\phi \alpha_1 s_C^{*2}}{24}\right)
$$

$$
-\frac{\phi \alpha_1}{4} s_C^{*2}
$$

 $\pi_{F2}^*|_{s=s_C^*} - \pi_{C2}^*|_{s=s_C^*}$

$$
= 2 \sqrt{\frac{(a + \omega s_{C}^{*})^{2}}{36} - \frac{\phi \alpha_{1} s_{C}^{*2}}{24}} \left(\frac{(a + \omega s_{C}^{*})}{6} - \sqrt{\frac{(a + \omega s_{C}^{*})^{2}}{36} - \frac{\phi \alpha_{1} s_{C}^{*2}}{24}} \right)
$$

$$
+ \left[\frac{(1 - \beta^{*})}{2} - \frac{1}{4} \right] \phi \alpha_{1} s_{C}^{*2}
$$

$$
\beta^* < \frac{1}{2} \quad \text{so} \quad \left[\frac{(1-\beta^*)}{2} - \frac{1}{4} \right] > 0. \quad \text{When} \quad \frac{\phi a_1 s_c^{*2}}{24} > 0, \quad \frac{(a + \omega s_c^*)}{6} - \sqrt{\frac{(a + \omega s_c^*)^2}{36} - \frac{\phi a_1 s_c^{*2}}{24}} > 0.
$$

Thus, $\pi_{F2}^*|_{s=s_C^*} - \pi_{C2}^*|_{s=s_C^*} > 0$. In other words, $\pi_{C2}^*|_{s=s_C^*} < \pi_{F2}^*|_{s=s_C^*}$.

From (3.26) and proof of Proposition 3.3, both firms have identical reducedform profit functions of s, i.e. $\pi_{F_1}^*(s) = \pi_{F_2}^*(s)$. Consequently, firm 1 maximises its own profit by choosing the level of mutual quality upgrade (s_F^*) that also generates firm 2's maximum profit, i.e. arg max $_{s} \pi_{F2}(s) = s_{F}^{*}$. According to transitivity, $\pi_{c2}^*|_{s=s_c^*} < \pi_{F2}^*|_{s=s_c^*}$ and $\pi_{F2}^*|_{s=s_c^*} < \pi_{F2}^*|_{s=s_F^*}$, thus $\pi_{F2}^*|_{s=s_F^*} > \pi_{c2}^*|_{s=s_c^*}$.

Under the unregulated access price approach, access price is set so high that the service-based firm becomes cornered. As a result, $\pi_{A2}^* = 0 < \pi_{C2}^*$. According to Proposition 3.7a, $\pi_{c2}^* > \pi_{N2}^*$. After comparing these equilibrium market outcomes, one can conclude that $\pi_{F_2}^* > \pi_{C_2}^* > \pi_{N_2}^* > \pi_{A_2}^*$.

Chapter 4

Bundling and Incentives for Quality Enhancement

4.1 Introduction

Digital convergence encourages telecommunications firms to expand their product lines and provide multiple services under the same brands. For example, in the UK, TalkTalk offers triple play including cable TV, broadband and phone. It exploits brand loyalty and offers a tempting bundle discount. Multi-product firms are likely to be dominant with more weapons to compete with single-product firms. In price competition, a multi-product firm can use the bundling strategy to enhance profit and expand its market share, while a single-product firm is likely to lose its profit and customers. If the bundling strategy is extremely aggressive, the small firm is probably on the verge of exit. On the demand side, even though some consumers who purchase the bundles benefit from a bundle discount, the distorted individual prices tend to threaten other consumers, especially single-product users (Armstrong, 2011; Reisinger, 2006) and undermine aggregate consumer surplus (Gans and King, 2006; Granier and Podesta, 2010; Reisinger, 2006; Rennhoff and Serfes, 2009). In the context of telecommunications with the rapid growth of advanced technology, the main focus of the bundling issue is not only price distortion but also incentive for quality enhancement. However, most of the existing literature emphasises the effects of bundling in the setting of the competition in price. Only some studies extend to firms' decisions on other aspects in the context of bundling, e.g. quality choices and R&D in cost reduction (Avenali, D'Annunzio and Reverberi, 2013; Choi, 2004; Heeb, 2003; Krӓmer, 2009).

From a broader viewpoint, this study extends to a model in which competing firms have not only price choices, but also further investment decision on distinct product features of quality.⁵³ Firms in the two duopolistic markets can choose their own quality enhancement levels and then prices.⁵⁴ The situation alters because a singleproduct firm has additional tools to respond to the aggressive bundling strategies. With pure bundling, compared to the no-bundling benchmark, the single-product firms' incentives for quality enhancement are undermined. Meanwhile, due in part to its concern about spillover, the multi-product firm increases its quality enhancement levels if the quality enhancement process is cost-efficient enough in terms of generating additional utility. The single-product firm's price in the less competitive market certainly decreases. Similar to the multi-product firm, the single-product firm in the more competitive market can raise its price when it benefits significantly from a substantial reduction in competition intensity with a limited number of product choices after pure bundling. With mixed bundling, the multi-product firm offers a bundle discount in line with the standard results about individual price setting (Armstrong, 2011; Reisinger, 2006). In the more competitive market, the multiproduct firm's quality enhancement is promoted in contrast to the deterioration of the single-product firm's counterpart. The similar result can be found in the less competitive market when the two markets are not too different in terms of competition intensity. Both bundling strategies are likely to threaten consumer surplus when the two markets are significantly different in competition intensity. In this case, the inefficient distortion of the consumer allocation predominates. The sectoral regulators should monitor the implementation of bundling, especially when a more competitive market is tied with a much less competitive market. However, bundling strategies may be a boost for quality enhancement in some situations.

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 53 For example, a broadband internet company may improve its connection stability or customer service to make its service comparatively more attractive than its rivals' service.

⁵⁴ In this context, this issue of quality does not concern vertical differentiation in quality, where consumers have different preferences for product quality. Instead, this model is based on horizontal differentiation where consumers have different tastes for a particular product offered by a particular firm.

Relevant literature

The existing literature examines the market outcomes of bundling strategy in various situations according to the degree of competition and the correlation of valuations of products. In the setting of symmetric duopoly, the correlation of valuations has a crucial role. Under the negative correlation of valuations, consumers become more homogeneous when they evaluate bundles instead of individual products. The business-stealing effect dominates the sorting effect of bundling. Firms' profits decline due to a prisoner's dilemma when both firms choose to sell bundles rather than individual products. On the other hand, the opposite result is found under the positive correlation of valuations (Reisinger, 2006). This can apply to the situation of a merger among homogeneous single-product firms under the positive correlation of valuations in a circular model (Granier and Podesta, 2010) and the situation where competing firms collaborate in offering product bundles to extract consumers' rent instead of compete in price such as tourist attractions (Armstrong, 2011). However, after bundling, individual prices increase despite a substantial bundle discount (Armstrong, 2011; Reisinger, 2006). Moreover, consumers are persuaded to buy bundles instead of a combination of preferred individual products. As a profitenhancing tool causing distributive inefficiency, the bundling strategy may threaten consumer welfare (Gans and King, 2006; Granier and Podesta, 2010; Reisinger, 2006; Rennhoff and Serfes, 2009) and even social welfare (Granier and Podesta, 2010; Reisinger, 2006; Rennhoff and Serfes, 2009).

In addition, in the setting of asymmetric duopolistic competition, a multiproduct firm is in a better position than a single-product firm because of its bundling strategies. Nalebuff (2004) pointed out that bundling strategy is a credible strategy for a multi-product incumbent to increase profit regardless of entry decision of an entrant. This result conforms to Choi (1996, 2004), supporting the leverage theory where the incumbent transfers its monopoly power in the monopolistic market to the duopolistic market. The bundling strategy is more reasonable than the limit-pricing strategy which forces the incumbent to sacrifice some profit. Gans and King (2006) found that in their extended model of one integrated firm and two independent firms, social welfare decreases because of inefficiency in the consumer allocation in spite of no bundling strategy of the independent single-product firms. In contrast, if the two independent firms are allowed to merge, they will agree to merge but finally the two integrated firms will not employ bundling in order to soften competition. This yields higher social welfare than the case of the allied independent firms with bundling.

In the introduction of the single-product consumer group with the assumption of cost saving from one-stop shopping by the multi-product consumers, Thanassoulis (2007) concluded that under firm-specific preference, firms can increase profit by charging higher individual prices than the no-bundling outcome and offer a bundle discount to attract the multi-product consumers whose demand is more elastic than the single-product consumers. Under product-specific preferences (hybrid-bundles are available), a prisoner's dilemma occurs when firms decide to sell bundles despite a decrease in profit. In addition, the individual prices are lower than those in the nobundling case. As a result, consumer surplus under firm-specific preferences decreases but consumer surplus under product-specific preferences increases after bundling.

In the extended model of different degrees of product differentiation in the two markets, Thanassoulis (2011) found that under partial convergence, the merged firm can increase profit even though it does not offer a bundle discount. Individual prices in the more competitive market increase but those in the less competitive market decrease due to the bundling strategy which adversely impacts on the single-product consumers. The multi-product consumers are charged higher price, but consumer surplus of this consumer group is ambiguous according to the degree of competition of the markets. Under full convergence, both merged firms bilaterally decide to sell bundles so product bundles are less differentiated because of a taste cost reduction. As a result of more intense competition, both merged firms decide to offer bundle discounts. Compared to the no-convergence environment, the individual prices remain unchanged but the multi-product consumers enjoy competitive bundle discounts. Overall, aggregate consumer surplus is enhanced under the full convergence. The bilateral merged firm in the full convergence has lower profit than the unilateral merged firms in the partial convergence. In contrast to the merger-wave outcome of Granier and Podesta (2010) and Reisinger (2006) in the two-stage games where the firms can decide on whether to offer a bundle discount, the first merger is at a distinct advantage but its rivals decide to remain independent in order to soften competition.

Bundling also influences firms' incentive to research and development. The multi-product firms with a bundling strategy invest more heavily in R&D in cost reduction and innovation while their single-product rivals strategically decrease their investment (Choi, 2004; Heeb, 2003). In a broader setting of vertical differentiation, firms offer different quality levels to target different groups of quality-preference type consumers (Tirole, 1988; Wauthy, 1996). In the presence of bundling, a multi-product firm can transfer its monopoly power in one market to another duopolistic market by the pure-bundling strategy. It becomes dominant and serves the high-quality type consumers. Its profit is higher than that of its single-product rival which avoids intense competition by choosing to serve the low-quality type consumers instead (Avenali, D'Annunzio and Reverberi, 2013; Krӓmer, 2009). From a broader perspective on quality investment, bundling may have a beneficial effect on social welfare in certain circumstances (Avenali, D'Annunzio and Reverberi, 2013). In the business world, firms attract consumers by various types of offers when market competition becomes more intense. It is necessary to further examine the vexed question of the bundling strategies' effects on welfare in various aspects other than pricing in order that the regulator can make a careful and comprehensive assessment of bundling strategies.

This study is organised as follows. The model of the two duopolistic markets is detailed in Section 4.2. The analysis of the market outcomes after bundling is in Section 4.3. Finally, Section 4.4 contains conclusion and some suggestions for the regulators.

4.2 Model

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There are one multi-product firm and two single-product firms in two duopolistic product markets. In market A, the multi-product firm competes with one single-product firm. In market B, it competes with the other single-product firm.

Figure 4.1 The two duopolistic markets

In each market, there are two horizontally differentiated products from each pair of competing firms (differentiation in products, not in firms). Firm 1 has entered both markets. As seen in Figure 4.1, firm 1 offers product A1 and product B1 to compete with firm A2 and firm B2 in market A and market B respectively. Product A and product B can be independently consumed.⁵⁵ Regarding an assumption of unit demand, each consumer can purchase only one unit of product A and one unit of product B.⁵⁶ Firm 1 may implement a bundling strategy to attract consumers in both markets.

⁵⁵ This model focuses on telecommunications services such as cable TV and broadband internet. Unlike system goods, e.g. computers and software, these services can be consumed separately.

⁵⁶ This assumption can be applied to most telecommunications services. For instance, one household normally monthly subscribes to only one service provider for a particular telecommunications service such as internet, cable TV, mobile and fixed line telephony.

Firms

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Firms have already entered the markets. Their products meet at least the standard-quality requirements by the regulator.⁵⁷ Thus, the fixed costs of standard products are sunk so they are assumed to be zero. For simplicity, constant marginal cost of standard product is assumed to be zero and identical across firms.⁵⁸ Basically, firms compete in price. P_{ki} is unit price of product ki ; $k \in \{A, B\}$ and $i \in \{1,2\}$.

In addition to price competition, firms are allowed to differentiate their products by improving their product quality. This model endogenises quality enhancement. ⁵⁹ Every consumer perceives this quality enhancement and has additional utility from the improved product quality. β_{ki} is a level of quality enhancement which a firm adds in its standard product ki ; $k \in \{A, B\}$ and $i \in \{1,2\}$. This model has its focus only on $\beta_{ki} > 0$. Reducing quality from quality standard is not practical, especially in the markets that are closely monitored by the regulators. Firms will incur additional investment costs associated with the level of quality enhancement they choose. $I_k(\beta_{ki})$ is investment cost function of investing in quality enhancement at level β_{ki} in market k ; $k \in \{A, B\}$ and $i \in \{1, 2\}$. Investment cost function is convex in quality enhancement level.

$$
I_k(\beta_{ki}) = b_k \beta_{ki}^2
$$

 b_k is constant and positive. In addition to investment cost, a firm's quality enhancement may lead to a change in its marginal production cost. It is assumed that firms have constant marginal production cost (mc) . Due to zero marginal production cost of standard product, the marginal cost directly derives from quality enhancement,

 57 Telecommunications regulators keep monitoring the quality of telecommunications services regularly.

⁵⁸ Most telecommunications services incur huge fixed costs of platform facilities, whereas their marginal costs are comparatively negligible. Moreover, in the business world, marginal cost is mostly calculated in constant term.

 59 This issue is not vertical differentiation in quality.

i.e. $mc = 0 + C_k(\beta_{ki}) = C_k(\beta_{ki})$. $C_k(\beta_{ki})$ is assumed to be a linear function of additional marginal cost at quality enhancement level β_{ki} of product ki.

$$
C_k(\beta_{ki})=c_k\beta_{ki}
$$

 c_k is constant and positive.⁶⁰ For example, if a firm chooses a high level of quality enhancement to add in its product, every unit sold has high marginal production cost due to high additional marginal production cost of quality enhancement. For simplicity, the investment cost function and the relationship between quality enhancement and marginal production cost are identical across firms in the same market.⁶¹

Consumers

 $\overline{}$

Each consumer purchases one unit of product A and one unit of product B (unit demand).⁶² Based on the Hotelling model, consumers are uniformly distributed on a unit square $[0,1]^2$. The pure combination of [A1B1] and [A2,B2] are located at (0,0) and (1,1) respectively. Meanwhile, the hybrid combination of [A1,B2] and $[A2,B1]$ are located at $(0,1)$ and $(1,0)$ respectively. The number of consumers is normalised to 1.

A consumer has a preference for a particular product according to the distance between his location and the location of the product. A consumer incurs disutility when the product he has chosen is not exactly his ideal product, which should be

 60 For example, a broadband company may add free wi-fi service in more public places for its consumers who subscribe its broadband service at home. The broadband company has to invest in additional investment cost for the public wi-fi service and incurs higher marginal cost.

 $⁶¹$ Firms may have different cost function of quality improvement because they own different</sup> technologies, experiences and know-how. Moreover, to attract consumers, firms possibly use different methods to increase their product values. However, at the first stage of this study, the process of quality enhancement is assumed to be common knowledge so firms have the same cost functions of quality enhancement.

 62 Some studies endogenised light users who buy either product A or product B. In this study, only heavy users who purchase both products are focused because they play a crucial role in the setting of product bundling.

located at his own location. t_k is taste cost⁶³ referring to the disutility per distance unit in market k ; $k \in \{A, B\}$. It is assumed that market A is more competitive than market B. $t_A = \theta t$; $t_B = t$; $\theta \in (0,1)$.⁶⁴ V_k is gross utility from standard product k, which is identical across consumers. V_k is large enough to ensure that every consumer participates and the markets are covered. When a firm chooses quality enhancement of product ki at level β_{ki} , consumers perceive the quality enhancement and have additional utility $v_k(\beta_{ki})$ from product ki.

$$
v_k(\beta_{ki}) = a_k \beta_{ki}
$$

 a_k is constant and positive. $k \in \{A, B\}$ and $i \in \{1, 2\}$. Assume $a_k > c_k$.⁶⁵

 \overline{a}

Consumers purchase both product A and product B.⁶⁶ In this model, even though a consumer decides to buy a pair of products from firm 1, there is no economy of scope from the pure-combination consumption. Benefits from payment in a single bill or one-stop service are trivial and omitted for simplicity.

 63 As mentioned by Thanassoulis (2007), the term "taste cost" in this model is tantamount to the transportation cost in the Hotelling model, which inversely represents the degree of substitutability between the two rival products in the market.

⁶⁴ When taste cost decreases, the two differentiated products in the market are more substitutable. Thus, the market is more competitive in the substitutability aspect.

⁶⁵ With this assumption, a positive equilibrium level of quality enhancement exists.

⁶⁶ In the initial model, it is assumed that consuming a combination of product A and product B yields additive gross utility, $V_A + V_B$. However, due to technological support, different kinds of telecommunications services are (imperfectly) partially substitutable from consumers' viewpoint. Thus, the gross utility from consuming a product combination may be sub-additive (Armstrong, 2011; Venkatesh and Kamakura, 2003).

For instance, even though mobile voice service is obviously distinct from communicating via the Internet, advanced technology allows internet users to use communicating software such as Skype and to chat on Facebook. Therefore, mobile service and internet broadband may be partially substitutable especially because of the digital convergence. Nevertheless, for simplicity, this study assumes additive gross utility of a product combination.

A consumer, who is located at (x, y) on a unit square $[0,1]^2$, has the four following choices as illustrated in Figure 4.2.

Figure 4.2 Consumers in the two-market horizontal product differentiation model

(I) [A1B1] from firm 1

His net utility from consuming product A1 and B1 is

$$
V_A + V_B + a_A \beta_{A1} + a_B \beta_{B1} - x\theta t - yt - P_{A1B1} \,. \tag{1}
$$

(II) [A1,B2] from separate firms

His net utility from consuming product A1 and B2 is

$$
V_A + V_B + a_A \beta_{A1} + a_B \beta_{B2} - x \theta t - (1 - y)t - P_{A1} - P_{B2}. \qquad (II)
$$

(III) [A2,B1] from separate firms

His net utility from consuming product A2 and B1 is

$$
V_A + V_B + a_A \beta_{A2} + a_B \beta_{B1} - (1 - x)\theta t - yt - P_{A2} - P_{B1}.
$$
 (III)

(IV) [A2,B2] from separate firms

His net utility from consuming product A2 and B2 is

$$
V_A + V_B + a_A \beta_{A2} + a_B \beta_{B2} - (1 - x)\theta t - (1 - y)t - P_{A2} - P_{B2}.
$$
 (IV)

The gross utility of a product combination is assumed to be additive $(V_A + V_B)$.

 P_{A1B1} is the total price of combination [A1B1]. Firm 1 offers P_{A1B1} according to its bundling strategy.

- In the no-bundling case, $P_{A1B1} = P_{A1} + P_{B1}$.
- In the pure-bundling case, $P_{A1B1} = \tilde{P}$ (the price of pure bundle [A1B1]).

In the mixed-bundling case, $P_{A1B1} = P_{A1} + P_{B1} - \delta$, where δ is bundle discount.

As seen in Figure 4.2, the consumer at (x, y) compares net utility from available choices of (I) - (IV) and finally chooses the product combination which yields the highest net utility. In this model, the multi-product firm have three different options of pricing; (1) no bundling, (2) the pure-bundling strategy, and (3) the mixedbundling strategy.

(1) No bundling

Firm 1 offers products separately at price P_{A1} and P_{B1} . All four choices (I) - (IV) are available. Thus, market share is determined by the marginal consumer in each market. \hat{x} and \hat{y} denote the locations of the marginal consumers in market A and market B respectively. The marginal consumers in market A and market B solve the following conditions, accordingly.

$$
V_A + a_A \beta_{A1} - \hat{x} \theta t - P_{A1} = V_A + a_A \beta_{A2} - (1 - \hat{x}) \theta t - P_{A2}
$$

$$
\hat{x} = \frac{1}{2} + \frac{1}{2\theta t} (P_{A2} - P_{A1} + a_A \beta_{A1} - a_A \beta_{A2})
$$
(4.1)

$$
V_B + a_B \beta_{B1} - \hat{y}t - P_{B1} = V_B + a_B \beta_{B2} - (1 - \hat{y})t - P_{B2}
$$

$$
\hat{y} = \frac{1}{2} + \frac{1}{2t} (P_{B2} - P_{B1} + a_B \beta_{B1} - a_B \beta_{B2})
$$
(4.2)

The market shares are shown in Figure 4.3.

Figure 4.3 Market shares with no bundling

According to the unit demand assumption, aggregate demand for each product directly derives from market share.

Profit functions

Profit function includes revenue, less investment cost of quality enhancement.

$$
\frac{\text{Firm 1}}{\pi_1 \big(P_{A1}, P_{B1}, \beta_{A1}, \beta_{B1} \big)} = \hat{x} \big[P_{A1} - c_A \beta_{A1} \big] + \hat{y} \big[P_{B1} - c_B \beta_{B1} \big] - b_A \beta_{A1}^2 - b_B \beta_{B1}^2 \tag{4.3}
$$

Firm A2

$$
\pi_{A2}(P_{A2}, \beta_{A2}) = (1 - \hat{x})[P_{A2} - c_A \beta_{A2}] - b_A \beta_{A2}^2 \tag{4.4}
$$

Firm B2

$$
\pi_{B2}(P_{B2}, \beta_{B2}) = (1 - \hat{y})[P_{B2} - c_B \beta_{B2}] - b_B \beta_{B2}^2 \tag{4.5}
$$

Consumer surplus

Consumers evaluate product A and product B separately.

$$
CS = \int_0^{\hat{x}} \left[V_A + a_A \beta_{A1} - x \theta t - P_{A1} \right] dx + \int_{\hat{x}}^1 \left[V_A + a_A \beta_{A2} - (1 - x) \theta t - P_{A2} \right] dx
$$

$$
+\int_{0}^{\hat{y}} \left[V_B + a_B \beta_{B1} - yt - P_{B1} \right] dy
$$

+
$$
\int_{\hat{y}}^{1} \left[V_B + a_B \beta_{B2} - (1 - y)t - P_{B2} \right] dy,
$$
 (4.6)

where

 \hat{x} is market share of firm 1 in market A.

 \hat{v} is market share of firm 1 in market B.

(2) The pure-bundling strategy

Firm 1 sells only a bundle of product A and product B at price \tilde{P} . Consumers have only two choices, (I) and (IV). The consumers who are indifferent between [A1B1] and [A2,B2] have the following condition.

$$
V_A + V_B + a_A \beta_{A1} + a_B \beta_{B1} - x\theta t - yt - \tilde{P}
$$

= $V_A + V_B + a_A \beta_{A2} + a_B \beta_{B2} - (1 - x)\theta t - (1 - y)t - P_{A2} - P_{B2}$

Rearranging the above equation yields the following.

$$
\theta x + y = \frac{1+\theta}{2} + \left(\frac{1}{2t}\right)[P_{A2} + P_{B2} - \tilde{P} + a_A\beta_{A1} - a_A\beta_{A2} + a_B\beta_{B1} - a_B\beta_{B2}]
$$

$$
Y(x) = y = \frac{1+\theta}{2} - \theta x + \left(\frac{1}{2t}\right)[P_{A2} + P_{B2} - \tilde{P} + a_A\beta_{A1} - a_A\beta_{A2} + a_B\beta_{B1} - a_B\beta_{B2}]
$$

We can write y as a function of x which is denoted by $Y(x)$ and represented by the dotted line in Figure 4.4. The marginal consumers are located on this line. At a given x , the marginal consumer who is indifferent between product [A1B1] and [A2,B2] is located at $(x, Y(x))$. Consumers in area *I* buy bundle [A1B1]. The rest in area H buy [A2,B2]. Due to the unit demand assumption, demand for product A1 and B1 derive from area I and demand for product A2 and B2 derive from area II .

Figure 4.4 Market shares with the pure-bundling strategy

Profit functions

Profit function includes revenue, less investment cost of quality improvement.⁶⁷

$$
\frac{\text{Firm 1}}{\pi_1(\tilde{P}, \beta_{A1}, \beta_{B1})} = \frac{1}{2} [Y(0) + Y(1)] [\tilde{P} - c_A \beta_{A1} - c_B \beta_{B1}] - b_A \beta_{A1}^2 - b_B \beta_{B1}^2 \quad (4.7)
$$

Firm A2

$$
\pi_{A2}(P_{A2}, \beta_{A2}) = \left\{1 - \frac{1}{2}[Y(0) + Y(1)]\right\} [P_{A2} - c_A \beta_{A2}] - b_A \beta_{A2}^2 \tag{4.8}
$$

Firm B2

 \overline{a}

$$
\pi_{B2}(P_{B2}, \beta_{B2}) = \left\{1 - \frac{1}{2}[Y(0) + Y(1)]\right\}[P_{B2} - c_B \beta_{B2}] - b_B \beta_{B2}^2 \quad (4.9)
$$

 67 This section emphasises a range of θ that the pure-bundling strategy is credible. In this analysis of the pure-bundling case, θ should not be too extremely high in order to ensure that the benefit from tying the markets is not trivial. See the appendix (Section 4.5.4) for details.

Consumer surplus

$$
CS = \int_0^1 \int_0^{Y(x)} \left[V_A + V_B + a_A \beta_{A1} + a_B \beta_{B1} - x\theta t - yt - \tilde{P} \right] dy dx
$$

+
$$
\int_0^1 \int_{Y(x)}^1 \left[-\frac{V_A + V_B + a_A \beta_{A2} + a_B \beta_{B2}}{-(1-x)\theta t - (1-y)t - P_{A2} - P_{B2}} \right] dy dx, \tag{4.10}
$$

where \tilde{P} is price of pure bundle [A1B1]; $P_{A1B1} = \tilde{P}$.

(3) The mixed-bundling strategy

 \overline{a}

Firm 1 offers products separately at price P_{A1} and P_{B1} . Additionally, firm 1 also offers discount δ on a bundle [A1B1]. Consumers still have all four choices, (I)-(IV). In Figure 4.5, at a given set of individual prices and quality enhancement levels, \hat{x} and \hat{y} are the locations of the marginal consumers in market A and market B respectively if bundle discount is zero. When firm 1 offers the bundle discount, its market shares in both markets expands according to δ .⁶⁸

Figure 4.5 Market shares with the mixed-bundling strategy

⁶⁸ This model is set up in the two-market framework, similar to those of Gans and King (2006) and Matutes and Regibeau (1992).

$$
\hat{x} = \frac{1}{2} + \frac{1}{2\theta t} (P_{A2} - P_{A1} + a_A \beta_{A1} - a_A \beta_{A2})
$$
(4.11)

$$
\hat{y} = \frac{1}{2} + \frac{1}{2t} (P_{B2} - P_{B1} + a_B \beta_{B1} - a_B \beta_{B2})
$$
(4.12)

$$
\hat{\hat{x}} = \hat{x} + \frac{\delta}{2\theta t} \tag{4.13}
$$

$$
\hat{\hat{y}} = \hat{y} + \frac{\delta}{2t} \tag{4.14}
$$

Consumers in area I choose a bundle of $[A1B1]$ from firm 1 at discounted price. Consumers in area II , III and IV purchase product [A1,B2], [A2,B1] and [A2,B2] from separate firms respectively.

Profit functions

Profit function includes revenue, less investment cost of quality improvement.

Firm 1

$$
\pi_1(P_{A1}, P_{B1}, \delta, \beta_{A1}, \beta_{B1}) = D_{A1}[P_{A1} - c_A \beta_{A1}] + D_{B1}[P_{B1} - c_B \beta_{B1}] -D_{A1B1}\delta - b_A \beta_{A1}^2 - b_B \beta_{B1}^2
$$
(4.15)

Firm A2

$$
\pi_{A2}(P_{A2}, \beta_{A2}) = D_{A2}[P_{A2} - c_A \beta_{A2}] - b_A \beta_{A2}^2 \tag{4.16}
$$

Firm B2

$$
\pi_{B2}(P_{B2}, \beta_{B2}) = D_{B2}[P_{B2} - c_B \beta_{B2}] - b_B \beta_{B2}^2 \tag{4.17}
$$

 D_{A1} is demand for product A1 from consumers who buy bundle [A1B1] or combination [A1,B2]. Thus, D_{A1} is equal to the total of area *I* and area *II*.

$$
D_{A1} = \hat{x} + \frac{\delta}{2\theta t} \hat{y} + \frac{1}{2} \left(\frac{\delta}{2\theta t}\right) \left(\frac{\delta}{2t}\right)
$$

 D_{B1} is demand for product B1 from consumers who buy bundle [A1B1] or combination [A2,B1]. Thus, D_{B1} is equal to the total of area *I* and area *III*.

$$
D_{B1} = \hat{y} + \frac{\delta}{2t}\hat{x} + \frac{1}{2}\left(\frac{\delta}{2\theta t}\right)\left(\frac{\delta}{2t}\right)
$$

 D_{A1B1} is demand for bundle [A1B1] equal to area *l*. These consumers get a special discount (δ) deducted from the original price of bundle [A1B1], $(P_{A1} + P_{B1})$. Firm 1 sacrifices some of its revenue to attract this consumer group.

$$
D_{A1B1} = \hat{x}\hat{y} + \hat{x}\left(\frac{\delta}{2t}\right) + \hat{y}\left(\frac{\delta}{2\theta t}\right) + \frac{1}{2}\left(\frac{\delta}{2\theta t}\right)\left(\frac{\delta}{2t}\right)
$$

 D_{A2} is demand for product A2 from consumers who buy combination [A2,B1] or [A2,B2]. Thus, D_{A2} is equal to the total of area *III* and area *IV*.

$$
D_{A2} = 1 - \left\{\hat{x} + \frac{\delta}{2\theta t}\hat{y} + \frac{1}{2}\left(\frac{\delta}{2\theta t}\right)\left(\frac{\delta}{2t}\right)\right\}
$$

 D_{B2} is demand for product B2 from consumers who buy combination [A1,B2] or [A2,B2]. Thus, D_{B2} is equal to the total of area *II* and area *IV*.

$$
D_{B2} = 1 - \left\{\hat{y} + \frac{\delta}{2t}\hat{x} + \frac{1}{2}\left(\frac{\delta}{2\theta t}\right)\left(\frac{\delta}{2t}\right)\right\}
$$

Consumer surplus

Consumer surplus after mixed bundling is the sum of consumer surplus from the four areas. $Z(x)$ denotes a function of x as represented by the thick solid linear line in Figure 4.5, $x \in [\hat{x}, \hat{\hat{x}}], Z(x) \in [\hat{y}, \hat{\hat{y}}]. Z(x) = \hat{y} + \theta \hat{x} + \frac{\delta}{\hat{x}}$ $\frac{\partial}{\partial t} - \theta x$

$$
CS = \int_0^{\hat{x}} \int_0^{\hat{y}} [V_A + V_B + a_A \beta_{A1} + a_B \beta_{B1} - x\theta t - yt - P_{A1} - P_{B1} - \delta] dy dx
$$

+
$$
\int_{\hat{x}}^{\hat{x}} \int_0^{Z(x)} [V_A + V_B + a_A \beta_{A1} + a_B \beta_{B1} - x\theta t - yt - P_{A1} - P_{B1} - \delta] dy dx
$$

+
$$
\int_0^{\hat{x}} \int_0^1 [V_A + V_B + a_A \beta_{A1} + a_B \beta_{B2} - x\theta t - (1 - y)t - P_{A1} - P_{B2}] dy dx
$$

+
$$
\int_{\hat{x}}^1 \int_0^{\hat{y}} [V_A + V_B + a_A \beta_{A2} + a_B \beta_{B1} - (1 - x)\theta t - yt - P_{A2} - P_{B1}] dy dx
$$

$$
+\int_{\hat{x}}^{\hat{x}} \int_{Z(x)}^{1} \left[\frac{V_A + V_B + a_A \beta_{A2} + a_B \beta_{B2}}{-(1-x)\theta t - (1-y)t - P_{A2} - P_{B2}} \right] dy dx
$$

+
$$
\int_{\hat{x}}^{1} \int_{\hat{y}}^{1} \left[\frac{V_A + V_B + a_A \beta_{A2} + a_B \beta_{B2}}{-(1-x)\theta t - (1-y)t - P_{A2} - P_{B2}} \right] dy dx
$$
(4.18)

Timing of the game

This model boils down to a three-stage game solved by backward induction.

Stage 1 Firm 1 decides whether to implement the bundling strategies (pure bundling and mixed bundling) instead of selling products A1 and product B1 separately (no bundling). 69

Stage 2 Firms simultaneously choose their quality enhancement levels Stage 3 Firms choose their prices simultaneously.

4.3 Analysis

The outcome with the no-bundling strategy is considered a benchmark to be compared with the pure-bundling and the mixed-bundling outcomes.

4.3.1 No bundling

 $\overline{}$

In the absence of bundling, firm 1 competes with firm A2 and firm B2 in market A and market B separately. The equilibrium outcomes are as follows.

Equilibrium market outcomes

When firm 1 is not allowed to bundle, the markets are independent of each other. Consumers give consideration to choices in each market independently.

Stage 3 Firm 1 solves the following profit-maximisation problem.

max $\max_{P_{A1}, P_{B1}} \pi_1$

 69 All firms have already entered the market. The issue of market entry is omitted.

Differentiating profit function (4.3) with respect to associated prices yields the following.

$$
\frac{\partial \pi_1}{\partial P_{A1}} = \left\{ \frac{1}{2} + \left(\frac{1}{2\theta t} \right) \left[P_{A2} - P_{A1} + a_A \beta_{A1} - a_A \beta_{A2} \right] \right\} - \frac{1}{2\theta t} \left(P_{A1} - c_A \beta_{A1} \right) = 0 \quad (4.19)
$$

$$
\frac{\partial \pi_1}{\partial P_{B1}} = \left\{ \frac{1}{2} + \left(\frac{1}{2t} \right) \left[P_{B2} - P_{B1} + a_B \beta_{B1} - a_B \beta_{B2} \right] \right\} - \frac{1}{2t} (P_{B1} - c_B \beta_{B1}) = 0 \quad (4.20)
$$

Analogously, the single-product firms' problems are shown below.

max $\max_{P_{k2}} \pi_{k2}$

 $k \in \{A, B\}$. Differentiating profit functions (4.4) and (4.5) with respect to associated prices gives

$$
\frac{\partial \pi_{A2}}{\partial P_{A2}} = \left\{ \frac{1}{2} - \left(\frac{1}{2\theta t} \right) [P_{A2} - P_{A1} + a_A \beta_{A1} - a_A \beta_{A2}] \right\} - \frac{1}{2\theta t} (P_{A2} - c_A \beta_{A2}) = 0 \quad (4.21)
$$

$$
\frac{\partial \pi_{B2}}{\partial P_{B2}} = \left\{ \frac{1}{2} - \left(\frac{1}{2t} \right) \left[P_{B2} - P_{B1} + a_B \beta_{B1} - a_B \beta_{B2} \right] \right\} - \frac{1}{2t} \left(P_{B2} - c_B \beta_{B2} \right) = 0. \quad (4.22)
$$

Solving (4.19) - (4.22) reveals the price functions in stage 3 at a given set of quality enhancement levels.

$$
P_{A1}^*(\beta_{A1}, \beta_{A2}) = \theta t + \frac{2}{3} c_A \beta_{A1} + \frac{1}{3} c_A \beta_{A2} + \frac{1}{3} (a_A \beta_{A1} - a_A \beta_{A2})
$$
(4.23)

$$
P_{A2}^*(\beta_{A1}, \beta_{A2}) = \theta t + \frac{1}{3} c_A \beta_{A1} + \frac{2}{3} c_A \beta_{A2} - \frac{1}{3} (a_A \beta_{A1} - a_A \beta_{A2})
$$
(4.24)

$$
P_{B1}^*(\beta_{B1}, \beta_{B2}) = t + \frac{2}{3}c_B\beta_{B1} + \frac{1}{3}c_B\beta_{B2} + \frac{1}{3}(a_B\beta_{B1} - a_B\beta_{B2})
$$
(4.25)

$$
P_{B2}^*(\beta_{B1}, \beta_{B2}) = t + \frac{1}{3}c_B\beta_{B1} + \frac{2}{3}c_B\beta_{B2} - \frac{1}{3}(a_B\beta_{B1} - a_B\beta_{B2})
$$
(4.26)

After substituting (4.23) - (4.26) in (4.3) - (4.5) , the firm profit functions in reduced form are shown below.

$$
\pi_1(\beta_{A1}, \beta_{A2}, \beta_{B1}, \beta_{B2})
$$
\n
$$
= \left[\frac{1}{2} - \frac{c_A \beta_{A1}}{6\theta t} + \frac{c_A \beta_{A2}}{6\theta t} + \frac{a_A \beta_{A1} - a_A \beta_{A2}}{6\theta t}\right] \left[\begin{array}{l} \theta t - \frac{1}{3} c_A \beta_{A1} + \frac{1}{3} c_A \beta_{A2} \\ + \frac{1}{3} (a_A \beta_{A1} - a_A \beta_{A2}) \end{array}\right]
$$
\n
$$
+ \left[\frac{1}{2} - \frac{c_B \beta_{B1}}{6t} + \frac{c_B \beta_{B2}}{6t} + \frac{a_A \beta_{A1} - a_A \beta_{A2}}{6t}\right] \left[\begin{array}{l} t - \frac{1}{3} c_B \beta_{B1} + \frac{1}{3} c_B \beta_{B2} \\ + \frac{1}{3} (a_B \beta_{B1} - a_B \beta_{B2}) \end{array}\right]
$$
\n
$$
-b_A \beta_{A1}^2 - b_B \beta_{B1}^2
$$
\n(4.27)

$$
\pi_{A2}(\beta_{A1}, \beta_{A2}, \beta_{B1}, \beta_{B2})
$$
\n
$$
= \left[\frac{1}{2} + \frac{c_A \beta_{A1}}{6\theta t} - \frac{c_A \beta_{A2}}{6\theta t} - \frac{a_A \beta_{A1} - a_A \beta_{A2}}{6\theta t}\right] \left[\frac{\theta t + \frac{1}{3} c_A \beta_{A1} - \frac{1}{3} c_A \beta_{A2}}{-\frac{1}{3} (a_A \beta_{A1} - a_A \beta_{A2})}\right]
$$
\n
$$
-b_A \beta_{A2}^2
$$
\n(4.28)

$$
\pi_{B2}(\beta_{A1}, \beta_{A2}, \beta_{B1}, \beta_{B2})
$$
\n
$$
= \left[\frac{1}{2} + \frac{c_B \beta_{B1}}{6t} - \frac{c_B \beta_{B2}}{6t} - \frac{a_A \beta_{A1} - a_A \beta_{A2}}{6t}\right] \left[\frac{t + \frac{1}{3}c_B \beta_{B1} - \frac{1}{3}c_B \beta_{B2}}{-\frac{1}{3}(a_B \beta_{B1} - a_B \beta_{B2})}\right]
$$
\n
$$
-b_B \beta_{B2}^2
$$
\n(4.29)

Stage 2 The multi-product firm and the single-product firms maximise their own profits as shown below.

$$
\max_{\beta_{A1},\beta_{B1}} \pi_1
$$

$$
\max_{\beta_{k_2}} \pi_{k_2}
$$

One may differentiate (4.27) - (4.29) with respect to associated quality enhancement levels (β_{ki}) ; $k \in \{A, B\}$, $i \in \{1, 2\}$. After solving the derivatives, the equilibrium quality enhancement levels are shown below.

$$
\beta_{A1}^* = \beta_{A2}^* = \frac{a_A - c_A}{6b_A} \tag{4.30}
$$

$$
\beta_{B1}^* = \beta_{B2}^* = \frac{a_B - c_B}{6b_B} \tag{4.31}
$$

Substituting (4.30) and (4.31) in the price functions (4.23) - (4.26) yields the equilibrium prices below.

$$
P_{A1}^* = P_{A2}^* = \theta t + \frac{c_A (a_A - c_A)}{6b_A} \tag{4.32}
$$

$$
P_{B1}^* = P_{B2}^* = t + \frac{c_B(a_B - c_B)}{6b_B}.
$$
\n(4.33)

Proposition 4.1

When bundling is not allowed, firms choose symmetric prices and quality enhancement levels. Therefore, firms have equal market shares and profits in each market.

Proof See the appendix (Section 4.5.1).

The result is consistent with the standard Hotelling model; firms choose symmetric prices and quality enhancement levels in each market because firm 1 competes with its rivals in each market separately. The equilibrium prices are determined by taste cost, costs of quality enhancement and additional utility from the quality enhancement within the same market. As a result of the symmetric outcome in each separate market, the multi-product firm and the single-product rivals have equal market shares and profits in each market.

Observation 4.1

Taste cost has positive effects on the equilibrium prices but no effect on the equilibrium levels of quality enhancement.

Proof

From (4.30) - (4.33),

$$
\frac{\partial P_{A1}^*}{\partial t} = \frac{\partial P_{A2}^*}{\partial t} = \theta > 0; \quad \frac{\partial P_{B1}^*}{\partial t} = \frac{\partial P_{B2}^*}{\partial t} = 1 > 0;
$$

$$
\frac{\partial \beta_{A1}^*}{\partial t} = \frac{\partial \beta_{A2}^*}{\partial t} = 0; \quad \frac{\partial \beta_{B1}^*}{\partial t} = \frac{\partial \beta_{B2}^*}{\partial t} = 0.
$$

In each market, firms choose their quality enhancement levels depending only on relevant costs and additional utility from quality enhancement, but independent of taste cost. In general, firms set prices to cover additional marginal costs of quality enhancement. In addition, they can raise prices according to an increase in taste cost because substitutability between rival products deceases. If cost structure and additional utility from quality enhancement in the two markets are identical, the price in the less competitive market is certainly higher.

Consumer surplus with no bundling

After substituting the equilibrium outcomes (4.30) - (4.33) in (4.6), consumer surplus in the no-bundling case is

$$
CS = V_A + V_B + \frac{(a_A - c_A)^2}{6b_A} + \frac{(a_B - c_B)^2}{6b_B} - \frac{5\theta t}{4} - \frac{5}{4}t.
$$
 (4.34)

■ Comparative statics analysis

In the no-bundling case, the equilibrium quality enhancement level (β_{ki}) and prices (P_{ki}) are determined by associated costs and additional utility in the market k; $k \in \{A, B\}$ and $i \in \{1, 2\}$.

Quality enhancement

According to (4.30) - (4.31), the equilibrium quality enhancement levels are affected by the costs and addition utility in the following way.

1)
$$
\frac{\partial \beta_{ki}^*}{\partial b_k} = -\frac{(a_k - c_k)}{6b_k^2} < 0
$$

When the investment cost increases, firms find it less profitable to compete fiercely on quality enhancement. Thus, the equilibrium quality enhancement levels drop.

$$
2) \frac{\partial \beta_{ki}^*}{\partial c_k} = -\frac{1}{6b_k} < 0
$$

Likewise, if the additional marginal cost increases, firms have less incentive to invest in quality enhancement.

$$
3) \frac{\partial \beta_{ki}^*}{\partial a_k} = \frac{1}{6b_k} > 0
$$

By contrast with the costs, if the additional utility from quality enhancement is perceived to be greater from consumers' perspective, firms have a tendency to invest in higher quality enhancement level to attract consumers.

Price

Consider (4.32) - (4.33), the equilibrium prices are clearly affected by the costs and addition utility as follows.

1)
$$
\frac{\partial P_{ki}^*}{\partial b_k} = -\frac{c_k(a_k - c_k)}{6b_k^2} < 0
$$

Prices decrease with investment cost. When investment cost decreases, firms compete by increasing the equilibrium quality enhancement levels and then raise prices to cover the costs of quality enhancement.

$$
2) \frac{\partial P_{ki}^*}{\partial c_k} = \frac{a_k - 2c_k}{6b_k}
$$

The effect of additional marginal cost on prices is ambiguous. Prices may decrease with additional marginal cost if the cost is comparatively high $(c_k > \frac{a_k}{2})$ $\frac{\mu_k}{2}$). Under this assumption, when the cost increases, the quality enhancement levels decrease and the marginal costs drop noticeably, and consequently the prices go down. On the other hand, when the cost is relatively low $(c_k < \frac{a_k}{2})$ $\frac{u_k}{2}$, prices may increase with the cost. For example, when the per-quality-unit cost of additional marginal cost (c_k) decreases, firms decide to increase their quality enhancement levels. However, the per-quality-unit cost is so low that the additional marginal cost (C_k) decreases. Overall, the prices go down.

$$
3) \frac{\partial P_{ki}^*}{\partial a_k} = \frac{c_k}{6b_k} > 0
$$

Prices increase with additional utility from quality enhancement. This is because this factor precisely determines the equilibrium levels of quality enhancement which firms take into account in their price decision.

4.3.2 The pure-bundling strategy

When pure bundling is allowed, firm 1 implements the strategy which affects the market outcomes as follows.

Equilibrium market outcomes

Stage 3 Firm 1 only sells its products as a product bundle, so consumers have only two choices, bundle [A1B1] and combination [A2,B2]. In this stage, firm 1 solves the following problem.

$$
\max_{\tilde{P}} \pi_1
$$

From (4.7), the first-order condition (FOC) is

$$
\frac{\partial \pi_1}{\partial \tilde{P}} = -\frac{1}{2t} (\tilde{P} - c_A \beta_{A1} - c_B \beta_{B1}) + \frac{1}{2}
$$

+ $\left(\frac{1}{2t}\right) [P_{A2} + P_{B2} - \tilde{P} + a_A \beta_{A1} - a_A \beta_{A2} + a_B \beta_{B1} - a_B \beta_{B2}] = 0.$ (4.35)

Analogously, the single-products firms' problems are shown below.

$$
\max_{P_{k2}} \pi_{k2}
$$

where $k \in \{A, B\}$. From (4.8) and (4.9), the FOCs are as follows.

$$
\frac{\partial \pi_{A2}}{\partial P_{A2}} = -\frac{1}{2t} (P_{A2} - c_A \beta_{A2}) + \frac{1}{2}
$$

$$
-\left(\frac{1}{2t}\right) [P_{A2} + P_{B2} - \tilde{P} + a_A \beta_{A1} - a_A \beta_{A2} + a_B \beta_{B1} - a_B \beta_{B2}] = 0 \quad (4.36)
$$

$$
\frac{\partial \pi_{B2}}{\partial P_{B2}} = -\frac{1}{2t} (P_{B2} - c_B \beta_{B2}) + \frac{1}{2}
$$

$$
-\left(\frac{1}{2t}\right) [P_{A2} + P_{B2} - \tilde{P} + a_A \beta_{A1} - a_A \beta_{A2} + a_B \beta_{B1} - a_B \beta_{B2}] = 0 \quad (4.37)
$$

At any given set of quality enhancement levels, the equilibrium prices are

$$
\tilde{P}^*(\beta_{A1}, \beta_{A2}, \beta_{B1}, \beta_{B2}) = \frac{5}{4}t + \frac{3}{4}c_A\beta_{A1} + \frac{1}{4}c_A\beta_{A2} + \frac{3}{4}c_B\beta_{B1} + \frac{1}{4}c_B\beta_{B2} + \frac{1}{4}[a_A\beta_{A1} - a_A\beta_{A2} + a_B\beta_{B1} - a_B\beta_{B2}] \tag{4.38}
$$

$$
P_{A2}^*(\beta_{A1}, \beta_{A2}, \beta_{B1}, \beta_{B2}) = \frac{3}{4}t + \frac{1}{4}c_A\beta_{A1} + \frac{3}{4}c_A\beta_{A2} + \frac{1}{4}c_B\beta_{B1} - \frac{1}{4}c_B\beta_{B2} - \frac{1}{4}[a_A\beta_{A1} - a_A\beta_{A2} + a_B\beta_{B1} - a_B\beta_{B2}] \tag{4.39}
$$

$$
P_{B2}^*(\beta_{A1}, \beta_{A2}, \beta_{B1}, \beta_{B2}) = \frac{3}{4}t + \frac{1}{4}c_A\beta_{A1} - \frac{1}{4}c_A\beta_{A2} + \frac{1}{4}c_B\beta_{B1} + \frac{3}{4}c_B\beta_{B2} - \frac{1}{4}[a_A\beta_{A1} - a_A\beta_{A2} + a_B\beta_{B1} - a_B\beta_{B2}].
$$
 (4.40)

Substituting (4.38) - (4.40) in (4.7) - (4.9), one obtains the following.

$$
\pi_{1}(\beta_{A1}, \beta_{A2}, \beta_{B1}, \beta_{B2})
$$
\n
$$
= \begin{cases}\n\left[\frac{5}{8} + \frac{1}{2t} \left(\frac{1}{4} c_{A} \beta_{A2} + \frac{1}{4} c_{B} \beta_{B2} - \frac{1}{4} c_{A} \beta_{A1} - \frac{1}{4} c_{B} \beta_{B1} \right) \right] \\
+ \frac{1}{4} \left[a_{A} \beta_{A1} - a_{A} \beta_{A2} + a_{B} \beta_{B1} - a_{B} \beta_{B2} \right] \right] \\
\times \left[\frac{5}{4} t - \frac{1}{4} c_{A} \beta_{A1} + \frac{1}{4} c_{A} \beta_{A2} - \frac{1}{4} c_{B} \beta_{B1} + \frac{1}{4} c_{B} \beta_{B2} \right] \\
+ \frac{1}{4} \left[a_{A} \beta_{A1} - a_{A} \beta_{A2} + a_{B} \beta_{B1} - a_{B} \beta_{B2} \right]\n\end{cases}
$$
\n
$$
-b_{A} \beta_{A1}^{2} - b_{B} \beta_{B1}^{2}
$$
\n(4.41)

$$
\pi_{A2}(\beta_{A1}, \beta_{A2}, \beta_{B1}, \beta_{B2})
$$
\n
$$
= \begin{cases}\n\left[\frac{3}{8} - \frac{1}{2t} \left(\frac{1}{4} c_A \beta_{A2} + \frac{1}{4} c_B \beta_{B2} - \frac{1}{4} c_A \beta_{A1} - \frac{1}{4} c_B \beta_{B1} \right) \right] \\
+ \frac{1}{4} \left[a_A \beta_{A1} - a_A \beta_{A2} + a_B \beta_{B1} - a_B \beta_{B2} \right] \right] \\
\times \left[\frac{3}{4} t + \frac{1}{4} c_A \beta_{A1} - \frac{1}{4} c_A \beta_{A2} + \frac{1}{4} c_B \beta_{B1} - \frac{1}{4} c_B \beta_{B2} \right] \\
- \frac{1}{4} \left[a_A \beta_{A1} - a_A \beta_{A2} + a_B \beta_{B1} - a_B \beta_{B2} \right]\n\end{cases}
$$
\n(A.42)

$$
\pi_{B2}(\beta_{A1}, \beta_{A2}, \beta_{B1}, \beta_{B2})
$$
\n
$$
= \begin{cases}\n\left[\frac{3}{8} - \frac{1}{2t} \left(\frac{1}{4} c_A \beta_{A2} + \frac{1}{4} c_B \beta_{B2} - \frac{1}{4} c_A \beta_{A1} - \frac{1}{4} c_B \beta_{B1} \right) \right] \\
+ \frac{1}{4} \left[a_A \beta_{A1} - a_A \beta_{A2} + a_B \beta_{B1} - a_B \beta_{B2} \right] \right] \\
\times \left[\frac{3}{4} t + \frac{1}{4} c_A \beta_{A1} - \frac{1}{4} c_A \beta_{A2} + \frac{1}{4} c_B \beta_{B1} - \frac{1}{4} c_B \beta_{B2} \right] \\
- \frac{1}{4} \left[a_A \beta_{A1} - a_A \beta_{A2} + a_B \beta_{B1} - a_B \beta_{B2} \right] \right]\n\end{cases} (4.43)
$$

Stage 2 The multi-product firm and the single-product firms' problems are defined below.

$$
\max_{\beta_{A1},\beta_{B1}} \pi_1
$$

$$
\max_{\beta_{k2}} \pi_{k2}
$$

One may differentiate (4.41) - (4.43) with respect to associated quality enhancement levels (β_{ki}) ; $k \in \{A, B\}$, $i \in \{1, 2\}$. The following outcomes satisfy the FOCs.

$$
\beta_{A1}^{*} = \frac{(a_A - c_A)[20tb_A b_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2]}{8b_A[16tb_A b_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2]}
$$
(4.44)

$$
\beta_{A2}^* = \frac{(a_A - c_A)[12tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2]}{8b_A[16tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2]}
$$
(4.45)

$$
\beta_{B1}^* = \frac{(a_B - c_B)[20tb_A b_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2]}{8b_B[16tb_A b_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2]}
$$
(4.46)

$$
\beta_{B2}^* = \frac{(a_B - c_B)[12tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2]}{8b_B[16tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2]}
$$
(4.47)

Substituting (4.44) - (4.47) in (4.38) - (4.40) gives the equilibrium prices of the game as follows.

$$
\tilde{P}^* = \frac{5}{4}t
$$
\n
$$
+ \left(\frac{1}{16}\right) \left[+ \left(\frac{20tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2}{16tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2}\right) \left(\frac{a_A^2 - c_A^2}{b_A} + \frac{a_B^2 - c_B^2}{b_B}\right) \right] (4.48)
$$

$$
P_{A2}^{*} = \frac{3}{4} t
$$
\n
$$
+\left(\frac{1}{16}\right) \left[\begin{array}{c} \frac{3c_A(a_A - c_A)}{b_A} \\ -\frac{c_A(a_A - c_A)}{b_A} \left(\frac{20tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2}{16tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2}\right) \\ -\frac{4a_A(a_A - c_A)tb_B + 4(a_B - c_B)^2tb_A}{16tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2} \end{array}\right] (4.49)
$$

$$
P_{B2}^{*} = \frac{3}{4} t
$$
\n
$$
+\left(\frac{1}{16}\right)\begin{bmatrix}\n\frac{3c_B(a_B - c_B)}{b_B} \\
-\frac{c_B(a_B - c_B)}{b_B}\left(\frac{20tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2}{16tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2}\right) \\
-\frac{4a_B(a_B - c_B)tb_A + 4(a_A - c_A)^2tb_B}{16tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2}\n\end{bmatrix}
$$
\n(4.50)

Proposition 4.2

(i) When the multi-product firm employs the pure-bundling strategy, it chooses higher quality enhancement levels than the single-product firms, i.e. $\beta_{A1}^* > \beta_{A2}^*$ and $\beta_{B1}^* > \beta_{B2}^*.$

(ii) The multi-product firm sets its bundle price lower than the total price of the single-product firms' product combination when quality enhancement involves relatively high costs but yields comparatively low additional utility:

$$
\tilde{P}^* < (\ge) P_{A2}^* + P_{B2}^* \quad \text{iff} \quad \begin{bmatrix} b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 \\ + b_Bc_A(a_A - c_A) + b_Ac_B(a_B - c_B) \end{bmatrix} < (\ge) 4tb_Ab_B.
$$

Proof See the appendix (Section 4.5.2).

After pure bundling, consumers have only two choices of products, bundle [A1B1] and combination [A2,B2]. The differentiation between these two choices becomes based on the taste cost of the less competitive market. Costs and additional utility from quality improvement in one market have some effect on the equilibrium prices and quality enhancement levels in the other market. The multi-product firm certainly chooses higher levels of quality enhancement than its single-product rivals. This is because it incorporates the spillover between the two markets into its profitmaximisation problem, but the single-product firms still maximise their own profit separately (without cooperation). Choi (2004) and Krämer (2009) also reported similar results in their settings of pure bundling where the multi-product firm ties its monopoly product with its competitive product. Even though the multi-product firm faces duopolistic competition in both markets in the present study, it still employs pure bundling to achieve dominance through its superior quality.

The price comparison is ambiguous. The multi-product firm's bundle price is lower than the total price of the single-product firms' product combination when the costs of quality enhancement, compared to the additional utility, are extremely high. In this case, the process of quality enhancement involves high investment costs and/or results in a substantial increase in marginal production costs, so the multi-product firm gives a stronger focus on price than quality enhancement because of the inefficiency in cost of quality enhancement. It offers slightly higher quality enhancement levels, and it aggressively sets lower price than its rivals in order to expand its consumer base. In this model, consumers evaluate the two product choices with regard to perfect information on quality and price. Consumers can perceive the lower price of bundle as an indirect discount. This is similar to the result of Nalebuff (2004), where firms are not allowed to vary quality of product in the setting of pure bundling. However, when the costs of quality enhancement, compared to the additional utility, are not too high, the multi-product firm concentrates on its investment in quality enhancement and accordingly sets its bundle price higher than the single-product firms.

Observation 4.2

After pure bundling, the strong competition in the more competitive market (θ) no longer affects the equilibrium levels of quality enhancement and prices.

Proof

According to (4.44) - (4.50),

$$
\frac{\partial \beta_{ki}^*}{\partial \theta} = 0, \frac{\partial \tilde{P}^*}{\partial \theta} = 0, \frac{\partial P_{A2}^*}{\partial \theta} = 0, \frac{\partial P_{B2}^*}{\partial \theta} = 0; \ k \in \{A, B\}, i \in \{1, 2\}
$$

After pure bundling, only [A1B1] and [A2,B2] remain available to consumers. From consumers' viewpoint, they have to choose either [A1B1] or [A2,B2], and the products in the more competitive market become more clearly differentiated after being tied with those in the less competitive market. Hence, consumers evaluate the two available product choices mainly based on the taste cost of the less competitive market (t) .

Proposition 4.3

After pure bundling, the multi-product firm has higher profit with larger market share than the single-product firms, i.e. $\pi_1^* > \pi_{A2}^* + \pi_{B2}^*$ and $MS_1 > \frac{1}{2}$ $\frac{1}{2}$ > MS₂. Proof See the appendix (Section 4.5.3).

After internalising the spillover between the two tied markets, the multiproduct firm finds it profitable to offer higher quality enhancement levels to attract consumers and then becomes dominant with larger market share and higher profit than its single-product rivals.

Consumer surplus with pure bundling

Consumer surplus after pure bundling is

$$
CS = V_A + V_B - \frac{\theta^2 t}{6} - \frac{t}{2} + tY^0(1 - Y^0) - \theta t (1 - Y^0) + a_A \beta_{A2}^* + a_B \beta_{B2}^*
$$

$$
-(P_{A2}^* + P_{B2}^*) + \left(\frac{\theta}{2} - Y^0\right) \begin{bmatrix} \tilde{P}^* - (P_{A2}^* + P_{B2}^*) \\ -a_A \beta_{A1}^* + a_A \beta_{A2}^* - a_B \beta_{B1}^* + a_B \beta_{B2}^* \end{bmatrix},
$$
(4.51)

where $Y^0 = Y(0) = \left(\frac{1+\theta}{2}\right)$ $\left(\frac{1+b}{2}\right) + \frac{2tb_Ab_B}{16tb_Ab_B-b_B(a_A-c_A))}$ $\frac{2\tau b_A b_B}{16\tau b_A b_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2}$.

The pure-bundling strategy VS the no-bundling strategy

In stage 1, to emphasise the effect of pure bundling, the pure-bundling market outcomes are compared with the no-bundling benchmark in this section.

Denote
$$
\Omega = \frac{20tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2}{16tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2}
$$
, $A = a_A - c_A > 0$, $\mathbb{B} = a_B - c_B > 0$.

Proposition 4.4a

 \overline{a}

When the pure-bundling strategy is implemented instead of the no-bundling strategy, the outcomes of the multi-product firm change as follows.

(i) In both markets, the multi-product firm decreases its quality enhancement levels when quality enhancement involves relatively high costs but yields comparatively low additional utility:

 $\beta_{A1\,Pure}^* < \beta_{A1\, No}^*$, $\beta_{B1\, Pure}^* < \beta_{B1\, No}^*$ iff $b_B \mathbb{A}^2 + b_A \mathbb{B}^2 < 4tb_A b_B$.

Conversely, it increases the levels when the process of quality enhancement is efficient enough in terms of cost and additional utility:

 $\beta_{A1\,Pure}^* > \beta_{A1\, No}^*$, $\beta_{B1\, Pure}^* > \beta_{B1\, No}^*$ iff $4tb_Ab_B < b_B\mathbb{A}^2 + b_A\mathbb{B}^2 < 12tb_Ab_B$.

(ii) The multi-product firm raises its bundle price when the two tied markets are significantly different in competition intensity:

$$
\tilde{P}_{pure}^* > (\le) P_{A1No}^* + P_{B1No}^* \text{ iff } \theta < (\ge) \frac{1}{4} + \frac{1}{48tb_Ab_B} \left\{ \begin{aligned} & (3\Omega - 3) \left[b_B a_A \mathbb{A} + b_A a_B \mathbb{B} \right] \\ & + (3\Omega - 5) \left[b_B c_A \mathbb{A} + b_A c_B \mathbb{B} \right] \end{aligned} \right\}.
$$

(iii) The multi-product firm's market shares increase in both markets, i.e. $MS_{1 \, pure} > MS_{A1 \, No} = MS_{B1 \, No}.$

(iv) The multi-product firm's profit increases, i.e. π_{1}^{*} $_{pure} > \pi_{1}^{*}$ $_{No}$. The purebundling strategy dominates the no-bundling strategy for the multi-product firm in most situations.⁷⁰

⁷⁰ In this section, it is assumed that θ is not too extremely high in order that the strategy is still credible in the analysis of pure bundling. See the appendix (Section 4.5.4) for details.

ProofSee the appendix (Section 4.5.4).

Compared to the no-bundling benchmark, the pure-bundling strategy can increase the multi-product firm's profit in most situations. Consequently, at stage 1 of the game, firm 1 will use the pure-bundling strategy instead of no bundling. When the two markets are tied, the product choices are limited. Firm 1 takes this opportunity to reconsider about quality enhancement and pricing by incorporating the spillover concern into its profit-maximisation problem. It will spend less on quality enhancement to save cost if the investment cost and additional marginal cost of quality enhancement are very high in associated with a comparatively negligible amount of additional utility.⁷¹ It may not be worthwhile to attract consumers by heavy investment in quality enhancement with this inefficient technology. This is because the markets seem less competitive when only two options of product combinations remain available after pure bundling. Otherwise, due to the significant effect of the spillover, the pure-bundling strategy may stimulate the multi-product firm's investment if quality enhancement does not involve too huge investment cost and additional marginal cost but yields relatively substantial additional-utility. This study points out that the multi-product firm's incentive to invest in quality depends on the efficiency of the quality enhancement approach in terms of cost and additional utility. This contrasts with the close paper by Choi (2004) that argued that the multi-product firm certainly increases its investment in cost reduction after pure bundling.

The effects of the pure-bundling strategy on prices are ambiguous. Two effects of pure bundling occur. First, pure bundling reduces the number of product choices and obscures the strong competition in market A, thus firms may charge higher prices. Second, in response to pure bundling, firms may change their quality enhancement levels, which directly affect marginal costs and finally their pricing. When the two markets' taste costs are significantly different (θ is sufficiently low), the price of bundle [A1B1] is higher than the no-bundling benchmark. When the more competitive market is tied with the far less competitive market, the reduction in competition intensity after pure bundling benefits the multi-product firm so significantly that it can

 $\overline{}$

 71 See more details in the appendix (Section 4.5.7).
raise its bundle price. Otherwise, it will lower its bundle price in the process of profit maximisation, which internalises the spillover after pure bundling. With more attractive quality and reasonable price, the multi-product firm can expand its market share.

Proposition 4.4b

When the multi-product firm implements the pure-bundling strategy instead of the no-bundling strategy, the single-product firms are affected as follows.

(i) The single-product firms decrease their quality enhancement levels in both markets, i.e. $\beta_{A2\,Pure}^* < \beta_{A2\,No}^*$ and $\beta_{B2\, Pure}^* < \beta_{B2\,No}^*$.

(ii) The single-product firm in the less competitive market decreases price, i.e. $P_{B2\,pure}^* < P_{B2\,No}^*$. Ambiguously, the single-product firm in the more competitive market raises its price when the two tied markets are significantly different in competition intensity:

$$
P_{A2~Pure}^* > (\leq) P_{A2~No}^* \text{ iff}
$$

$$
\theta < (\geq) \frac{3}{4} - \frac{(3\Omega - 1)c_A \mathbb{A} (16tb_A b_B - b_B \mathbb{A}^2 - b_A \mathbb{B}^2) + 12 a_A \mathbb{A} tb_A b_B + 12 \mathbb{B}^2 t b_A^2}{48tb_A (16tb_A b_B - b_B \mathbb{A}^2 - b_A \mathbb{B}^2)}.
$$

(iii) The market shares of the single-product firms shrink in both markets, i.e. $MS_{A2\,Pure} < MS_{A2\,No}$ and $MS_{B2\, Pure} < MS_{B2\, No}$.

(iv) In the more competitive market, the single-product firm's profit increases when the two tied markets are significantly different in competition intensity:

 $\pi_{A2\,Pure}^* > (\leq) \pi_{A2\,No}^*$ iff

$$
\theta < (\ge) \left\{ \begin{array}{c} 9 \\ 16 + \frac{A^2}{18tb_A} \left[1 - \frac{1}{2} \left[\frac{\left(\begin{matrix} 12tb_Ab_B \\ -b_B\mathbf{A}^2 - b_A\mathbf{B}^2 \end{matrix} \right)}{\left(\begin{matrix} 16tb_Ab_B \\ -b_B\mathbf{A}^2 - b_A\mathbf{B}^2 \end{matrix} \right)} \right]^2 \right] \\ - \frac{6}{16} \left[\frac{b_B\mathbf{A}^2 + b_A\mathbf{B}^2}{\left(\begin{matrix} 16tb_Ab_B \\ -b_B\mathbf{A}^2 - b_A\mathbf{B}^2 \end{matrix} \right)} \right] + \frac{1}{16} \left[\frac{b_B\mathbf{A}^2 + b_A\mathbf{B}^2}{\left(\begin{matrix} 16tb_Ab_B \\ -b_B\mathbf{A}^2 - b_A\mathbf{B}^2 \end{matrix} \right)} \right]^2 \right\} . \end{array}
$$

In the less competitive market, the single-product firm's profit decreases, i.e. π_{B2}^* pure $<$ π_{B2No}^* .

Proof See the appendix (Section 4.5.5).

When the multi-product firm implements pure bundling, the single-product firms decide to save cost and react by decreasing their quality enhancement levels, which are finally lower than those of firm 1. This is because they separately consider their own profit-maximisation problems without the spillover concern. This finding confirms the result of Choi (2004) and Krӓmer (2009) that pure bundling reduces the single-product rivals' incentive to invest, even in the duopolistic environment with more intense competition.

According to Proposition 4.2 and Proposition 4.4a (i), with pure bundling, firm 1 becomes stronger and acts aggressively in investing in quality enhancement. The single-product firm in the less competitive market is adversely affected by this aggressive bundling strategy. As a result, firm B2 responds by reducing price to maximise its profit in this difficult situation. Finally, it certainly loses its market share and profit. However, the outcomes of the single-product firm in the more competitive market ambiguously change. When the two markets' taste costs are very different (θ is very low), firm A2 indirectly benefits from pure bundling by firm 1. When the more competitive market is tied with the much less competitive market, the substitutability between product choices drastically diminishes according to a limited number of available product choices. For this reason, firm A2 may increase its price and its profit grows despite a reduction in market share. Otherwise, tying the markets leads to only a small drop in the degree of competition of market A, and consequently firm A2 has to reduce price according to a decrease in its quality enhancement level. In this case, firm A2, similar to firm B2, earns smaller market share and lower profit than the no-bundling benchmark.

Proposition 4.4c

Compared to the no-bundling case, the effect of the pure-bundling strategy on consumer surplus is ambiguous. Consumer surplus decreases when the two tied markets are significantly different in competition intensity:

 $CS_{pure} \leq (\geq) CS_{No}$ iff

$$
\theta(\theta+9) < (\geq) 9 - 9\Omega + 3\Omega^2 - \frac{1}{t} \left[\frac{\mathbb{A}^2}{b_a} + \frac{\mathbb{B}^2}{b_B} \right] \left[\frac{-2 + \frac{15}{8}\Omega - \frac{3}{8}\Omega^2}{16tb_Ab_B - b_B\mathbb{A}^2 - b_A\mathbb{B}^2} \right].
$$

ProofSee the appendix (Section 4.5.6).

A change in consumer surplus is ambiguous. Pure bundling reduces consumer surplus when the taste costs of the two markets are significantly different (θ is low). The limitation of product choices causes distortion of the allocation of consumers, which leads to the disutility from the unavailability of consumers' ideal product combinations. In addition, due to a reduction in competition intensity after pure bundling, price distortion occurs, especially in the more competitive market. The distortions of price and the allocation of consumers adversely affect consumer welfare. This dominates the benefit from firm 1's decision on quality enhancement levels and bundle price by virtue of firm 1's spillover concern. On the other hand, consumer surplus may increase when the two markets' taste costs are not significantly different (θ is sufficiently high). Tying the two markets does not lead to severe distortion. Therefore, the benefit of the spillover concern outweighs the disutility and the negative effect of price distortion.

4.3.3 The mixed-bundling strategy

In addition to a product bundle, firm 1 also offers its products separately in accordance with the mixed-bundling strategy, which has effects on the market outcomes as follows.

Equilibrium market outcomes

Stage 3 The multi-product firm solves the following problem.

$$
\max_{P_{A1},P_{B1},\delta} \pi_1
$$

Differentiating (4.15) with respect to associated prices and bundle discount (δ) yields the following FOCs.

$$
\frac{\partial \pi_1}{\partial P_{A1}} = -\frac{1}{2\theta t} (P_{A1} - c_A \beta_{A1}) - \frac{\delta}{4\theta t^2} (P_{B1} - c_B \beta_{B1}) + \hat{x} + \frac{\delta \hat{y}}{\theta t} + \frac{3\delta^2}{8\theta t^2} = 0 \quad (4.52)
$$

$$
\frac{\partial \pi_1}{\partial P_{B1}} = -\frac{\delta}{4\theta t^2} (P_{A1} - c_A \beta_{A1}) - \frac{1}{2t} (P_{B1} - c_B \beta_{B1}) + \hat{y} + \frac{\delta \hat{x}}{t} + \frac{3\delta^2}{8\theta t^2} = 0 \tag{4.53}
$$

$$
\frac{\partial \pi_1}{\partial \delta} = \begin{cases} \left(\frac{\hat{y}}{2\theta t} + \frac{\delta}{4\theta t^2} \right) (P_{A1} - c_A \beta_{A1}) + \left(\frac{\hat{x}}{2t} + \frac{\delta}{4\theta t^2} \right) (P_{B1} - c_B \beta_{B1}) \\ -\hat{x}\hat{y} - \frac{\delta \hat{x}}{t} - \frac{\delta \hat{y}}{\theta t} - \frac{3\delta^2}{8\theta t^2} \end{cases} = 0 \quad (4.54)
$$

Analogously, the single-product firms' problems are shown below.

$$
\max_{\beta_{k2}}\pi_{k2}
$$

 $k \in \{A, B\}$. The FOCs obtained from differentiating (4.16), (4.17) with respect to associated prices are shown below.

$$
\frac{\partial \pi_{A2}}{\partial P_{A2}} = -\frac{1}{2\theta t} (P_{A2} - c_A \beta_{A2}) + 1 - \hat{x} - \frac{\delta \hat{y}}{2\theta t} - \frac{\delta^2}{8\theta t^2} = 0
$$
 (4.55)

$$
\frac{\partial \pi_{B2}}{\partial P_{B2}} = -\frac{1}{2t} (P_{B2} - c_B \beta_{B2}) + 1 - \hat{y} - \frac{\delta \hat{x}}{2t} - \frac{\delta^2}{8\theta t^2} = 0
$$
\n(4.56)

where

$$
\hat{x} = \frac{1}{2} + \frac{1}{2\theta t} (P_{A2} - P_{A1} + a_A \beta_{A1} - a_A \beta_{A2})
$$

$$
\hat{y} = \frac{1}{2} + \frac{1}{2t} (P_{B2} - P_{B1} + a_B \beta_{B1} - a_B \beta_{B2}).
$$

The function of the equilibrium discount by firm 1 cannot be expressed explicitly as a closed-form expression. However, after considering (4.54) in the neighbourhood of $\delta = 0$, it is found that

$$
\left. \frac{\partial \pi_1}{\partial \delta} \right|_{\delta=0} = \left(\frac{\hat{y}}{2\theta t} \right) (P_{A1} - c_A \beta_{A1}) + \left(\frac{\hat{x}}{2t} \right) (P_{B1} - c_B \beta_{B1}) - \hat{x} \hat{y}.
$$

If $\delta = 0$, the mixed-bundling competition becomes the no-bundling case in which firms choose symmetric quality enhancement levels and prices in equilibrium. Therefore,

$$
\left. \frac{\partial \pi_1}{\partial \delta} \right|_{\delta=0} = \frac{1}{4} > 0. \tag{4.57}
$$

As a result, if mixed bundling is allowed, firm 1 has incentive to offer a bundle discount in order to increase its profit. Thus, in equilibrium, $\delta^* > 0$.

Due to the fact that equilibrium bundle discount cannot be expressed in closed form, one may write the equilibrium prices as functions of quality enhancement levels and equilibrium bundle discount (δ^*) . After solving (4.52) - (4.56), the equilibrium prices at any given set of quality enhancement levels are as follows.

$$
P_{A1}^{*}(\beta_{A1}, \beta_{A2}, \beta_{B1}, \beta_{B2}) =
$$
\n
$$
\begin{bmatrix}\n\theta \left(\theta t^{3} - \frac{\delta^{*2}t}{3}\right) + \frac{\delta^{*}\theta t^{2}}{6} + \frac{\delta^{*5}}{2880t^{2}} - \frac{2}{9}\delta^{*3} \\
+\left(\frac{96\theta^{2}t^{4} - 40\theta t^{2}\delta^{*2} + \delta^{*4}}{144\theta t^{2}}\right)c_{A}\beta_{A1} \\
+\left(\frac{4\theta t^{2} - 3\delta^{*2}}{12}\right)c_{A}\beta_{A2} \\
+\left(\frac{4\theta t^{2} - 3\delta^{*2}}{36\theta t}\right)c_{B}\beta_{B1} \\
+\theta \left(\frac{28\theta t^{2} + \delta^{*3}}{36\theta t}\right)c_{B}\beta_{B2} \\
+\left(\frac{4\theta t^{2} - 3\delta^{*2}}{12}\right)(\alpha_{A}\beta_{A1} - \alpha_{A}\beta_{A2}) \\
+\theta \left(\frac{28\theta t^{2} + \delta^{*3}}{36\theta t}\right)(\alpha_{B}\beta_{B1} - \alpha_{B}\beta_{B2})\n\end{bmatrix}
$$
\n(4.58)

$$
P_{B1}^{*}(\beta_{A1}, \beta_{A2}, \beta_{B1}, \beta_{B2}) =
$$
\n
$$
\left\{\left(\theta t^{3} - \frac{\delta^{*2}t}{3}\right) + \frac{\delta^{*}\theta t^{2}}{6} + \frac{\delta^{*5}}{288\theta t^{2}} - \frac{2}{9}\delta^{*3}\right\}
$$
\n
$$
-\left(\frac{2\delta^{*}\theta t^{2} + \delta^{*3}}{36\theta t}\right)c_{A}\beta_{A1}
$$
\n
$$
+\left(\frac{2\delta\theta t^{2} + \delta^{*3}}{36\theta t}\right)c_{A}\beta_{A2}
$$
\n
$$
\left(\frac{144\theta^{2}t^{4} - 76\theta t^{2}\delta^{*2} + \delta^{*4}}{144\theta t^{2}}\right) + \left(\frac{96\theta^{2}t^{4} - 40\theta t^{2}\delta^{*2} + \delta^{*4}}{144\theta t^{2}}\right)c_{B}\beta_{B1}
$$
\n
$$
+\left(\frac{4\theta t^{2} - 3\delta^{*2}}{12}\right)c_{B}\beta_{B2}
$$
\n
$$
+\left(\frac{28\theta t^{2} + \delta^{*3}}{36\theta t}\right)(\alpha_{A}\beta_{A1} - \alpha_{A}\beta_{A2}) + \left(\frac{4\theta t^{2} - 3\delta^{*2}}{12}\right)(\alpha_{B}\beta_{B1} - \alpha_{B}\beta_{B2})
$$
\n
$$
\left(\frac{4\theta t^{2} - 3\delta^{*2}}{12}\right)(\alpha_{B}\beta_{B1} - \alpha_{B}\beta_{B2})
$$

$$
P_{A2}^{*}(\beta_{A1}, \beta_{A2}, \beta_{B1}, \beta_{B2}) =
$$
\n
$$
\begin{bmatrix}\n\left(\theta t^{3} - \frac{\delta^{*2}t}{3}\right) + \frac{\delta^{*}\theta t^{2}}{6} + \frac{\delta^{*5}}{2880t^{2}} - \frac{2}{9}\delta^{*3} \\
-\left(\frac{2\delta^{*}\theta t^{2} + \delta^{*3}}{36\theta t}\right)c_{A}\beta_{A1} \\
+\left(\frac{2\delta\theta t^{2} + \delta^{*3}}{36\theta t}\right)c_{A}\beta_{A2} \\
-\frac{\delta^{*}\left(\frac{144\theta t^{2}}{144\theta^{2}t^{4} - 76\theta t^{2}\delta^{*2} + \delta^{*4}}{6t^{2} + \delta^{*4}}\right)\n\end{bmatrix} + \left(\frac{96\theta^{2}t^{4} - 40\theta t^{2}\delta^{*2} + \delta^{*4}}{144\theta t^{2}}\right)c_{B}\beta_{B1} + \left(\frac{4\theta t^{2} - 3\delta^{*2}}{12}\right)c_{B}\beta_{B2} + \left(\frac{2\delta\theta t^{2} + \delta^{*3}}{12}\right)(a_{A}\beta_{A1} - a_{A}\beta_{A2}) + \theta t + \frac{\delta^{*2}}{12t} + \frac{1}{3}c_{A}\beta_{A1} + \frac{2}{3}c_{A}\beta_{A2} + \frac{\delta^{*}}{6t}c_{B}\beta_{B1} - \frac{1}{3}(a_{A}\beta_{A1} - a_{A}\beta_{A2})\n\end{bmatrix} \tag{4.60}
$$

$$
P_{B2}^{*}(\beta_{A1}, \beta_{A2}, \beta_{B1}, \beta_{B2}) =
$$
\n
$$
\begin{bmatrix}\n\theta \left(\theta t^{3} - \frac{\delta^{*2}t}{3}\right) + \frac{\delta^{*}\theta t^{2}}{6} + \frac{\delta^{*5}}{2880t^{2}} - \frac{2}{9}\delta^{*3} \\
+\left(\frac{96\theta^{2}t^{4} - 40\theta t^{2}\delta^{*2} + \delta^{*4}}{144\theta t^{2}}\right)c_{A}\beta_{A1} \\
+\left(\frac{4\theta t^{2} - 3\delta^{*2}}{12}\right)c_{A}\beta_{A2} \\
+\left(\frac{4\theta t^{2} - 3\delta^{*2}}{12}\right)c_{A}\beta_{A2} \\
+\left(\frac{4\theta t^{2} - 3\delta^{*2}}{12}\right)c_{B}\beta_{B1} \\
+\theta \left(\frac{2\delta^{*}\theta t^{2} + \delta^{*3}}{36\theta t}\right)c_{B}\beta_{B2} \\
+\left(\frac{4\theta t^{2} - 3\delta^{*2}}{12}\right)(a_{A}\beta_{A1} - a_{A}\beta_{A2}) \\
+\theta \left(\frac{2\delta^{*}\theta t^{2} + \delta^{*3}}{36\theta t}\right)(a_{B}\beta_{B1} - a_{B}\beta_{B2})\n\end{bmatrix}
$$
\n
$$
+ t + \frac{\delta^{*2}}{12\theta t} + \frac{\delta^{*}}{6\theta t}c_{A}\beta_{A1} + \frac{1}{3}c_{B}\beta_{B1} + \frac{2}{3}c_{B}\beta_{B2} - \frac{1}{3}(a_{B}\beta_{B1} - a_{B}\beta_{B2})
$$
\n
$$
(4.61)
$$

Stage 2 Firms choose quality enhancement levels simultaneously. The multiproduct firm and the single-product firms' problems are as follows.

$$
\max_{\beta_{A1},\beta_{B1}} \pi_1
$$

$$
\max_{\beta_{k_2}} \pi_{k_2}
$$

One may differentiate the reduced-form profit functions with respect to associated quality enhancement levels (β_{ki}) ; $k \in \{A, B\}$, $i \in \{1, 2\}$. Further, one can substitute (4.55) and (4.56) into the derivatives and then obtain the following.

$$
\frac{\partial \pi_1}{\partial \beta_{A1}} = \left[1 - \frac{1}{2\theta t} (P_{A2} - c_A \beta_{A2}) \right] \left(\frac{\partial P_{A2}}{\partial \beta_{A1}} + a_A - c_A \right) \n+ \left[1 - \frac{1}{2t} (P_{B2} - c_B \beta_{B2}) \right] \frac{\partial P_{B2}}{\partial \beta_{A1}} - 2b_A \beta_{A1} = 0
$$
\n(4.62)

$$
\frac{\partial \pi_1}{\partial \beta_{B1}} = \left[1 - \frac{1}{2\theta t} (P_{A2} - c_A \beta_{A2}) \right] \frac{\partial P_{A2}}{\partial \beta_{B1}} + \left[1 - \frac{1}{2t} (P_{B2} - c_B \beta_{B2}) \right] \left(\frac{\partial P_{B2}}{\partial \beta_{B1}} + a_B - c_B \right) - 2b_B \beta_{B1} = 0 \tag{4.63}
$$

$$
\frac{\partial \pi_{A2}}{\partial \beta_{A2}} = \frac{1}{2\theta t} (P_{A2} - c_A \beta_{A2}) \left[\frac{\partial P_{A1}}{\partial \beta_{A2}} + a_A - c_A - \frac{\delta}{2t} \frac{\partial P_{B2}}{\partial \beta_{A2}} + \frac{\delta}{2t} \frac{\partial P_{B1}}{\partial \beta_{A2}} - \left(\hat{y} + \frac{\delta}{2t} \right) \frac{\partial \delta}{\partial \beta_{A2}} \right]
$$

$$
-2b_A \beta_{A2}
$$

= 0 (4.64)

$$
\frac{\partial \pi_{B2}}{\partial \beta_{B2}} = \frac{1}{2t} (P_{B2} - c_B \beta_{B2}) \left[\begin{array}{c} \frac{\partial P_{B1}}{\partial \beta_{B2}} + a_B - c_B - \frac{\delta}{2\theta t} \frac{\partial P_{A2}}{\partial \beta_{B2}} + \frac{\delta}{2\theta t} \frac{\partial P_{A1}}{\partial \beta_{B2}}\\ -\left(\hat{x} + \frac{\delta}{2\theta t}\right) \frac{\partial \delta}{\partial \beta_{B2}} \end{array} \right] - 2b_B \beta_{B2}
$$
\n
$$
= 0 \tag{4.65}
$$

Proposition 4.5

(i) The multi-product firm offers a bundle discount $(\delta^* > 0)$ with the mixedbundling strategy and sets higher individual prices than those of the single-product rivals, i.e. $P_{A1}^* > P_{A2}^*$ and $P_{B1}^* > P_{B2}^*$.

(ii) In the more competitive market, the multi-product firm's quality enhancement level is higher than the single-product firm's counterpart.

The quality enhancement outcome in the less competitive market is ambiguous according to θ . However, if the two markets are identical in cost structure and additional utility of quality enhancement and taste costs are not too different (θ is sufficiently high), the outcome is similar to that in the more competitive market. ProofSee the appendix (Section 4.5.8).

The no-bundling case can be seen as the mixed-bundling case with $\delta = 0$. The multi-product firm offers a bundle discount ($\delta^* > 0$). Even though the multi-product firm's prices are higher than its rivals' prices, its product are still attractive because it can offer a bundle discount as a price-discriminatory tool under the mixed-bundling strategy. It rewards the consumers who purchase its bundles at a discount. Moreover, in the more competitive market, it invests more heavily in quality enhancement and then attracts consumers, especially those who do not buy its bundle but purchase either product A1 or product B1. The single-product firms cannot discriminate consumers by a bundle discount. In response to the multi-product firm's discounted prices, the single-product firms (without cooperation) have to attract consumers by setting lower prices, which affect all consumers in the same way. This reaction is commonly found as strategic complements in a price game. Thus, in the more competitive market, the single-product firm certainly decreases its quality enhancement level to save cost. These quality enhancement outcomes is also found in the less competitive market if the taste costs of the two markets are not too different (θ is sufficiently high). Even though the multi-product firm gives up some revenue due to the bundle discount, it can promote its profit after all. This is partly because of its higher individual prices than those of the single-product firms.

Quality discrimination is not practical in this setting. The multi-product firm offers the same levels of quality enhancement to all consumers. Suppose the taste costs of the two markets are not too different (θ is sufficiently high), the multiproduct firm decides to set higher levels of quality enhancement than the singleproduct firms in order to attract the consumers who confront its higher individual prices. Similarly, with the pure-bundling strategy, as stated in Proposition 4.2, the multi-product firm also chooses to invest in quality enhancement more heavily than its single-product rivals.

Proposition 4.6

(i) After mixed bundling, the multi-product firm has higher profit than the single-product firms, i.e. $\pi_1^* > \pi_{A2}^* + \pi_{B2}^*$.

(ii) The equilibrium market shares are ambiguous.

In the more competitive market, it is more likely that the multi-product firm has larger market share than the single-product firm.

The outcome in the less competitive market is ambiguous according to θ . However, if the two markets are identical in cost structure and additional utility of quality enhancement and taste costs in the two markets are not too different (θ is sufficiently high), the outcome is similar to that in the more competitive market. ProofSee the appendix (Section 4.5.9).

The multi-product firm may persuade some consumers to switch to its product bundle by means of mixed bundling. Compared to the less competitive market, the multi-product firm is more likely to steal some market share from its single-product rival and become dominant in the more competitive market with lower taste cost. The outcome in the less competitive market is ambiguous. However, if the two markets are identical in cost structure and additional utility of quality enhancement and taste costs in the two markets are not too different (θ is sufficiently high), this market share outcome is also found in the less competitive market. Additionally, it is certain that the sum of the single-product firms' profits is less than the multi-product firm's profit. The mixed-bundling strategy puts the single-product firms at a disadvantage.

In addition to the pure-bundling strategy, the multi-product firm may earn larger market shares than its rivals after adopting the mixed-bundling strategy. Both pure bundling and mixed bundling can increase the multi-product firm's profit, and it certainly has higher profit than its rivals.

■ The mixed-bundling strategy VS the no-bundling strategy

The effects of the mixed-bundling strategy on market outcomes are discussed in form of a comparison between the mixed-bundling outcomes and the no-bundling benchmark.

Proposition 4.7a

After the implementation of the mixed-bundling strategy instead of no bundling, the multi-product firm's profit increases whereas the two single-product firms' profits decrease in both markets, i.e. $\pi_{1 \text{ Mixed}}^* > \pi_{1 \text{ No}}^*$, $\pi_{A2 \text{ Mixed}}^* < \pi_{A2 \text{ No}}^*$ and $\pi_{B2\;Mixed}^* < \pi_{B2\;No}^*$. The mixed-bundling strategy dominates the no-bundling strategy for the multi-product firm.

ProofSee the appendix (Section 4.5.10).

In stage 1 of the game, the multi-product firm has incentive to employ the mixed-bundling strategy instead of no bundling because it can increase profit after mixed bundling. The bundle discount can be seen as a tool for the multi-product firm to persuade the consumers who originally buy other product combinations to choose its bundle [A1B1] instead.

Proposition 4.7b

When the multi-product firm employs the mixed-bundling strategy instead of the no-bundling strategy, the prices and quality enhancement change as follows.

(i) The multi-product firm raises its individual prices, whereas the single-product firms reduce their prices in both markets, i.e. $P_{A1 \text{ Mixed}}^* > P_{A1 \text{ No}}^*$, $P_{B1 \text{ Mixed}}^* > P_{B1 \text{ No}}^*$, $P_{A2 \text{ Mixed}}^* < P_{A2 \text{ No}}^*$ and $P_{B2 \text{ Mixed}}^* < P_{B2 \text{ No}}^*$.

(ii) In the more competitive market, the multi-product firm increases its quality enhancement level, but the single-product firm decreases its quality enhancement level, i.e. $\beta_{A1 \text{ Mixed}}^* > \beta_{A1 \text{ No}}^*$ and $\beta_{A2 \text{ Mixed}}^* < \beta_{A2 \text{ No}}^*$.

A change in quality enhancement in the less competitive market is ambiguous. However, if the two markets are identical in cost structure and additional utility of quality enhancement and the taste costs of the two markets are not too different (θ is sufficiently high), the multi-product firm increases its quality enhancement level but the single-product firm decreases its quality enhancement level in the less competitive market.

Proof See the appendix (Section 4.5.11).

Similar to Gans and King (2006), Reisinger (2006) and Avenali, D'Annunzio and Reverberi (2013), this study finds that the multi-product firms strategically increase its individual prices, but its bundle is still attractive because of a bundle discount. The single-product firms' reaction is to reduce their prices. The consumers who intend to buy other product combinations exclusive of bundle [A1B1] will perceive that the multi-product firm's individual prices are higher than the single-product firms' prices. Consequently, in the more competitive market, the multi-product firm finds it profitable to increase investment in quality enhancement from the no-bundling benchmark in order to attract some of these consumers. Meanwhile, the singleproduct firm decreases its quality enhancement level in order to reduce associated cost and accordingly offers lower price. This key finding is also found by Avenali, D'Annunzio and Reverberi (2013). Even though the multi-product firm's monopoly power is removed from the present study, mixed bundling still undermines the singleproduct firms' incentive to invest in the setting of more intense competition. However, the result in the less competitive market is ambiguous. Provided that the two markets are identical in cost structure and additional utility of quality enhancement and the two markets are not too different in competition intensity (θ is sufficiently high), the quality enhancement outcome is similar to the more competitive market's counterpart.

Proposition 4.7c

After the implementation of the mixed-bundling strategy instead of the nobundling strategy, a change in the market share outcome is ambiguous.

In the more competitive market, it is more likely that the multi-product firm can increase its market share, whereas the single-product firm's market share decreases.

The outcome in the less competitive market is also ambiguous. However, if the two markets are identical in cost structure and additional utility of quality enhancement and the taste costs of the two markets are not too different (θ is sufficiently high), the outcome is similar to that in the more competitive market. ProofSee the appendix (Section 4.5.12).

A comparison of market shares is ambiguous. If the two markets are identical in cost structure and additional utility of quality enhancement and the two markets are not too different in terms of competition intensity, the mixed-bundling strategy does not only threaten the single-product firms' profit, but also decreases their market shares.

In summary, the multi-product firm takes advantage of a wider range of product lines by means of mixed bundling in order to reap more profit. It can offer a bundle discount as an extra price-discriminatory tool. Meanwhile, the single-product firms (without cooperation) cannot implement this discriminatory pricing and still carry out only one pricing scheme which applies to all consumers.

Proposition 4.7d

After the implementation of the mixed-bundling strategy instead of the nobundling strategy, a change in consumer surplus is ambiguous.

However, if the two markets are identical in cost structure and additional utility of quality enhancement and the taste costs of the two markets are significantly different (θ is sufficiently low), mixed bundling certainly decreases consumer surplus. Proof See the appendix (Section 4.5.13).

In the extreme case analysed by Avenali, D'Annunzio and Reverberi (2013) where the multi-product firm ties its monopoly component with its competitive component, the mixed-bundling strategy jeopardises consumer surplus. However, the present study relaxes the assumption about competition intensity and finds that the effect of mixed bundling on consumer welfare is ambiguous. The mixed-bundling strategy, similar to the pure-bundling strategy, is more likely to undermine consumer welfare when the two tied markets are significantly different in competition intensity. This result is clearly illustrated in Corollary 4.1 as mentioned below.

Corollary 4.1

Suppose both markets are identical in taste cost, cost structure and additional utility of quality enhancement, consumer surplus certainly decreases after mixed bundling.

ProofSee the appendix (Section 4.5.14).

According to Proposition 4.7d, suppose the two markets are significantly different in taste cost (θ is sufficiently low), the markets are noticeably distorted after the less competitive market is tied with the more competitive market. Even though mixed bundling may stimulate the firms to offer higher quality enhancement levels of product A1 and B2 in this case, the changes in quality enhancement of the other products have a considerably negative impact on consumer surplus. As a result, consumer surplus decreases after all. In addition, suppose the two markets are not too different in taste cost (θ is sufficiently high), the multi-product firm's quality enhancement levels slightly increase while those of the single-product firms substantially decrease from the symmetric no-bundling benchmark. The effect of the mixed-bundling strategy on consumer surplus is ambiguous in this case. However, a substantial drop in the single-product firms' quality enhancement levels is more

noticeable than a slight increase in those of the multi-product firm. Thus, it is likely that consumer surplus decreases after mixed bundling.⁷²

As stated in Corollary 4.1, if the two markets are identical in taste cost, cost structure and additional utility of quality enhancement, i.e. $\theta = 1$, $b_A = b_B$, $c_A = c_B$, $a_A = a_B$, the equilibrium outcome in market A is identical to that in market B. It is found that the deterioration of the single-product firms' quality enhancement has more significant effects on consumer surplus than a boost in that of the multi-product firm. In this setting of identical parameters, the mixed-bundling strategy reduces consumer surplus.

4.4 Conclusion

 \overline{a}

This study examines the effect of the pure-bundling and the mixed-bundling strategy on the market outcomes and consumer surplus. In this model, one multiproduct firm and two single-product rivals compete in two duopolistic markets, which have different degrees of competition (different taste costs). They choose quality enhancement levels and then prices to attract consumers. Without bundling, firms compete with each other in each market separately. This leads to the symmetric equilibrium in each market. However, both bundling strategies dominate the nobundling strategy because the multi-product firm can increase profit after bundling, while the single-product firms lose profits in most situations.

With pure bundling, the multi-product firm considers the spillover between the two tied markets, whereas its single-product rivals (without cooperation) neglect it. As a result, the multi-product firm can offer higher quality enhancement levels than its rivals. The bundle price of the multi-product firm is lower (higher) than the total

⁷² The effect of mixed bundling on consumer surplus is ambiguous according to θ . The result is illustrated in the three cases of θ (θ is low, θ is moderate, and θ is high). However, this study focuses its analysis of consumer welfare on the case of low θ . This attracts the regulators' attention and relates closely to leverage theory in the context of bundling. See proof in the appendix (Section 4.5.13).

price of the product combination of the single-product firms if the process of quality enhancement is inefficient (efficient enough) in terms of cost and additional utility.

However, compared to the no-bundling benchmark, pure bundling undermines the single-product firm's incentive to invest in quality in both markets, but the multiproduct firm's decision on quality enhancement is ambiguous. If the process of quality enhancement is inefficient (efficient enough) in terms of cost and additional utility, the multi-product firm decreases (increases) its quality enhancement levels. The comparison in prices is also ambiguous. The bundle price of the multi-product firm and the price of the single-product firm in the more competitive market are higher than the no-bundling benchmarks if the two markets' degrees of competition are so different that these firms benefit from a reduction in competition intensity after pure bundling. Meanwhile, the single-product firm in the less competitive market reduces its price because its multi-product rival is more aggressive with the purebundling strategy. The multi-product firm's market share expands. The single-product firm in the less competitive market certainly loses profit. Despite a reduction in market share, the single-product firm in the more competitive market may enjoy an increase in profit if the benefit from tying the more competitive market with the much less competitive market prevails.

Mixed bundling is a price-discriminatory strategy. Compared to the no-bundling benchmark, the multi-product firm increases its individual prices in both markets in spite of its bundle discount. Meanwhile, the single-product firms decrease their prices. In the more competitive market, the multi-product firm raises its quality enhancement level in order to attract the consumers who are charged high individual prices. The single-product firm decreases its quality enhancement level corresponding to its lower price. In the less competitive market, a change in quality enhancement is ambiguous. The outcome is similar to that in the more competitive market if the two markets are not too different in competition intensity. A change in market shares is also ambiguous. The multi-product firm's market shares are likely to expand in both markets when the two markets are not too different in competition intensity. In addition to pure bundling, the multi-product firm can also implement the mix-bundling strategy to achieve dominance under these circumstances.

Compared to the no-bundling benchmark, the pure-bundling strategy reduces consumer surplus when the two markets are significantly different in competition intensity. In this case, the negative effect of the less intense competition with a limited number of product choices outweighs the benefit from the multi-product firm's spillover concern. Similarly, the effect of the mixed-bundling strategy on consumer surplus is ambiguous. Consumer welfare is threatened if the two markets' degrees of competition are significantly different. A decrease in quality enhancement levels of the single-product firms is likely to be more substantial than an increase in quality enhancement levels of the multi-product firm. In this case, the mixed-bundling strategy is likely to have a negative impact on consumer surplus. Similar to pure bundling, mixed bundling tends to be employed for the purpose of enlarging the product differentiation in the more competitive market rather than promoting strong competition in price and quality enhancement, especially when a more competitive market is tied with a much less competitive market.

Bundling seems unacceptable when consumer welfare is main concern in some situations. However, the sectoral regulators should take into account some positive effects of bundling. For instance, pure bundling can promote the multi-product firm's quality enhancement when the quality enhancement does not involve relatively huge costs. Therefore, in the remote areas with low demand and prohibitive investment cost in telecommunications, pure bundling is not an appropriate approach to stimulate quality enhancement. Pure bundling should not be allowed in this situation in order to prevent the predatory practice by the multi-product firm and protect consumer welfare. On the contrary, in the more competitive market, mixed bundling can at least stimulate quality enhancement by the multi-product firm, which is likely to serve the majority of consumers. The bundling strategy may pave the way for higher standard of product quality in the future. In addition, the effect of mixed bundling on consumer surplus is still ambiguous, so it is possible that consumer surplus may increase under some conditions. A ban on mixed bundling in this situation is a short-sighted intervention. The single-product rivals may submit a petition against bundling in an attempt to halt a decline in their own profits. However, it is not necessary for the regulators to intervene when all of the tied markets are highly competitive.

Further research may extend to a group of light consumers, who intend to consume either product A or product B. Additionally, the issues about vertical differentiation in quality and market segmentation are also interesting in the context of bundling.

4.5 Appendix

4.5.1 Proof of Proposition 4.1

Substituting the equilibrium outcomes (4.30) - (4.31) in the profit functions (4.27) - (4.29), the equilibrium profits are as follows.

$$
\pi_1^* = \frac{\theta t}{2} + \frac{t}{2} - \frac{(a_A - c_A)^2}{36b_A} - \frac{(a_B - c_B)^2}{36b_B} \tag{4. A1}
$$

$$
\pi_{A2}^* = \frac{\theta t}{2} - \frac{(a_A - c_A)^2}{36b_A} \tag{4. A2}
$$

$$
\pi_{B2}^* = \frac{t}{2} - \frac{(a_B - c_B)^2}{36b_B} \tag{4. A3}
$$

It is certain that $\pi_1^* = \pi_{A2}^* + \pi_{B2}^*$. Substituting (4.30) - (4.33) in (4.1) and (4.2), the locations of marginal consumers show symmetric market shares in both markets.

$$
MS_{A1\,No} = MS_{A2\,No} = MS_{B1\,No} = MS_{B2\,No} = \frac{1}{2}
$$
 (4. A4)

 $MS_{ki\,No}$ is market share of product ki in market k in the no-bundling case; $k \in \{A, B\}$, $i \in \{1,2\}.$

4.5.2 Proof of Proposition 4.2

Proof that $b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 < 12tb_Ab_B$ to guarantee the **existence of interior equilibrium**

This study casts light on the telecommunications markets which are involved with extremely high cost structure, especially investment cost of infrastructure. It is assumed that all the quality enhancement levels are non-negative. In the business world, firms cannot or hardly decrease their quality from the standard level. From (4.44) - (4.47), β_{A1}^* , β_{A2}^* , β_{B2}^* and β_{B2}^* are non-negative when the condition that $b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 < 12tb_Ab_B$ is satisfied.

(i) Quality enhancement

From (4.44) - (4.47),

$$
\beta_{A1}^* - \beta_{A2}^* = \frac{(a_A - c_A)[8tb_A b_B]}{8b_A[16tb_A b_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2]} > 0
$$

$$
\beta_{B1}^* - \beta_{B2}^* = \frac{(a_B - c_B)[8tb_A b_B]}{8b_B[16tb_A b_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2]} > 0.
$$

This is because $16tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2 > 0$ under the condition for an interior equilibrium mentioned above, $b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 < 12tb_Ab_B$. Therefore, $\beta_{A1}^* - \beta_{A2}^* > 0$, $\beta_{B1}^* - \beta_{B2}^* > 0$.

(ii) Prices

From (4.48) - (4.50),

$$
P_{A2}^{*} + P_{B2}^{*} = \frac{3}{2}t
$$
\n
$$
+ \left(\frac{1}{16}\right) \left[-\left(\frac{20tb_{A}b_{B} - b_{B}(a_{A} - c_{A})^{2} - b_{A}(a_{B} - c_{B})^{2}}{16tb_{A}b_{B} - b_{B}(a_{A} - c_{A})^{2} - b_{A}(a_{B} - c_{B})^{2}} \right) \left(\frac{(a_{A} - c_{A})^{2}}{b_{A}} + \frac{(a_{B} - c_{B})^{2}}{b_{B}} \right) \right] + \left(\frac{12tb_{A}b_{B} - b_{B}(a_{A} - c_{A})^{2} - b_{A}(a_{B} - c_{B})^{2}}{16tb_{A}b_{B} - b_{B}(a_{A} - c_{A})^{2} - b_{A}(a_{B} - c_{B})^{2}} \right) \left(\frac{a_{A}^{2} - c_{A}^{2}}{b_{A}} + \frac{a_{B}^{2} - c_{B}^{2}}{b_{B}} \right)
$$

$$
P_{A2}^{*} + P_{B2}^{*} - \tilde{P}^{*}
$$
\n
$$
= \frac{1}{4}t + \frac{1}{16} \left\{ \left(\frac{(a_{A} - c_{A})^{2}}{b_{A}} + \frac{(a_{B} - c_{B})^{2}}{b_{B}} \right) \left[1 - \left(\frac{20tb_{A}b_{B} - b_{B}\mathbf{A}^{2} - b_{A}\mathbf{B}^{2}}{16tb_{A}b_{B} - b_{B}\mathbf{A}^{2} - b_{A}\mathbf{B}^{2}} \right) \right] \right\}
$$
\n
$$
= -\frac{t}{(16tb_{A}b_{B} - b_{B}\mathbf{A}^{2} - b_{A}\mathbf{B}^{2})} \left[b_{B}\mathbf{A}^{2} + b_{A}\mathbf{B}^{2} + b_{B}c_{A}\mathbf{A} + b_{A}c_{B}\mathbf{B} - 4tb_{A}b_{B} \right],
$$

where $\mathbb{A} = a_A - c_A > 0$ and $\mathbb{B} = a_B - c_B > 0$.

An interior equilibrium exists when $b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 < 12tb_Ab_B$. Under this condition, $16tb_Ab_B-b_B\mathbb{A}^2 - b_A\mathbb{B}^2 > 0$. Accordingly, $P_{A2}^* + P_{B2}^* - \tilde{P}^* > 0$ when $b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 + b_Bc_A(a_A - c_A) + b_Ac_B(a_B - c_B) < 4tb_Ab_B$.

4.5.3 Proof of Proposition 4.3

Substituting the equilibrium quality enhancement levels (4.44) - (4.47) and the equilibrium prices (4.48) - (4.50) in (4.7) - (4.9) yields the following equilibrium profits.

$$
\pi_1^* = \frac{25t}{32} + \frac{5t}{16} \left(\frac{b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2}{16tb_A b_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2} \right)
$$

+
$$
\frac{t}{32} \left(\frac{b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2}{16tb_A b_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2} \right)^2
$$

-
$$
\frac{1}{64} \left(\frac{(a_A - c_A)^2}{b_A} + \frac{(a_B - c_B)^2}{b_B} \right) \left(\frac{-b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2}{16tb_A b_B} \right)^2
$$

-
$$
\frac{1}{64} \left(\frac{(a_A - c_A)^2}{b_A} + \frac{(a_B - c_B)^2}{b_B} \right) \left(\frac{-b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2}{16tb_A b_B} \right)^2
$$
(4. A5)

$$
\pi_{A2}^{*} = \frac{9t}{32} - \frac{3t}{16} \left(\frac{b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2}{16tb_b \, b_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2} \right)
$$

+
$$
\frac{t}{32} \left(\frac{b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2}{16tb_b \, b_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2} \right)^2
$$

$$
-\frac{1}{64} \frac{(a_A - c_A)^2}{b_A} \left(\frac{12tb_A b_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2}{16tb_b \, b_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2} \right)^2
$$
(4. A6)

$$
\pi_{B2}^{*} = \frac{9t}{32} - \frac{3t}{16} \left(\frac{b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2}{16tb_b\phi - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2} \right)
$$

+
$$
\frac{t}{32} \left(\frac{b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2}{16tb_b\phi - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2} \right)^2
$$

$$
-\frac{1}{64} \frac{(a_B - c_B)^2}{b_B} \left(\frac{12tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2}{16tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2} \right)^2
$$
(4. A7)

Due to the condition $b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 < 12tb_Ab_B$, firm 1 has higher profit than the total of its rivals' profits, i.e. $\pi_1^* > \pi_{A2}^* + \pi_{B2}^*$.

As seen in Figure 4.4, market share of firm 1 in the tied markets is as follows when $b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 < 12tb_Ab_B$ in the existence of interior equilibrium. $MS₁ pure = ($ 1 2 $[Y(0) + Y(1)]$ = 5 8 + (1 8 $\int \left(\frac{b_B (a_A - c_A)^2 + b_A (a_B - c_B)^2}{\frac{1}{2} (a_B - a_A)^2 + \frac{1}{2} (a_B - a_B)^2} \right)$ $\frac{B_{B}(x_{A} - c_{A}) + B_{A}(x_{B} - c_{B})}{16tb_{A}b_{B} - b_{B}(a_{A} - c_{A})^{2} - b_{A}(a_{B} - c_{B})^{2}}$ 1 2 (4. A8)

$$
MS_{A2\,Pure} = MS_{B2\, Pure}
$$

= $\frac{3}{8} - \left(\frac{1}{8}\right) \left(\frac{b_B (a_A - c_A)^2 + b_A (a_B - c_B)^2}{16tb_A b_B - b_B (a_A - c_A)^2 - b_A (a_B - c_B)^2}\right) < \frac{1}{2}$ (4. A9)

4.5.4 Proof of Proposition 4.4a

(i)
$$
\beta_{A1 \text{ Pure}}^* \leq \beta_{A1 \text{ No}}^*, \beta_{B1 \text{ Pure}}^* \leq \beta_{B1 \text{ No}}^*
$$

One can compare (4.30) with (4.44) , and (4.31) with (4.46) to show that $\beta_{A1\,Pure}^* - \beta_{A1\,No}^* \leq 0$ and $\beta_{B1\,Pure}^* - \beta_{B1\,No}^* \leq 0$ respectively.

The condition $b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 < 12tb_Ab_B$ should be satisfied to guarantee the existence of interior equilibrium. $A = a_A - c_A > 0$, $B = a_B - c_B > 0$.

$$
\beta_{A1\,Pure}^* - \beta_{A1\, No}^* = \frac{3A(20tb_Ab_B - b_BA^2 - b_AB^2) - 4A(16tb_Ab_B - b_BA^2 - b_AB^2)}{24b_A(16tb_Ab_B - b_BA^2 - b_AB^2)}
$$

 $\beta_{A1\,Pure}^* - \beta_{A1\,No}^* < 0$ when $b_B \mathbb{A}^2 + b_A \mathbb{B}^2 < 4tb_A b_B$. $\beta_{A1\,Pure}^* - \beta_{A1\,No}^* > 0$ when $4tb_A b_B < b_B \mathbb{A}^2 + b_A \mathbb{B}^2 < 12tb_A b_B$.

$$
\beta_{B1\,Pure}^* - \beta_{B1\, No}^* = \frac{3\mathbb{B}(20tb_Ab_B - b_B\mathbb{A}^2 - b_A\mathbb{B}^2) - 4\mathbb{B}(16tb_Ab_B - b_B\mathbb{A}^2 - b_A\mathbb{B}^2)}{24b_A(16tb_Ab_B - b_B\mathbb{A}^2 - b_A\mathbb{B}^2)}
$$

 $\beta_{B1\,Pure}^* - \beta_{B1\,No}^* < 0$ when $b_B \mathbb{A}^2 + b_A \mathbb{B}^2 < 4tb_A b_B$. $\beta_{B1\,Pure}^* - \beta_{B1\,No}^* > 0$ when $4tb_A b_B < b_B \mathbb{A}^2 + b_A \mathbb{B}^2 < 12tb_A b_B$.

(ii) $\widetilde{P}_{pure}^* \gtrless P_{A1No}^* + P_{B1No}^*$

From (4.32), (4.33), and (4.48), $\tilde{P}_{1~Pure}^* - (P_{A1~No}^* + P_{B1~No}^*) > 0$ when

$$
\theta < \frac{1}{4} + \frac{1}{48tb_Ab_B} \left\{ \begin{array}{l} (3\Omega - 3) \left[b_B a_A \mathbb{A} + b_A a_B \mathbb{B} \right] \\ + (3\Omega - 5) \left[b_B c_A \mathbb{A} + b_A c_B \mathbb{B} \right] \end{array} \right\}
$$

where $\mathbb{A} = a_A - c_A$, $\mathbb{B} = a_B - c_B$.

Thus, $\tilde{P}_{Pure}^* \ge P_{A1No}^* + P_{B1No}^*$ if $\theta \le \frac{1}{4}$ $\frac{1}{4} + \frac{1}{48tb}$ $\frac{1}{48tb_Ab_B}\left\{ \begin{array}{l} (3\Omega-3) \left[b_Ba_A\mathbb{A}+b_Aa_B\mathbb{B} \right] \\ + (3\Omega-5) \left[b_Bc_A\mathbb{A}+b_Ac_B\mathbb{B} \right] \end{array} \right.$ $+(3\Omega-5)\left[b_B c_A A + b_A c_B B\right]$ respectively.

(iii)
$$
MS_{1\,Pure} > MS_{A1\,No} = MS_{B1\,No}
$$

From $(4. A4)$ and $(4. A8)$, firm 1's market share increases.

(iv) π_{1}^{*} *pure* $> \pi_{1}^{*}$ *No*

Comparing (4.A1) with (4.A5), π_{1}^{*} $_{Pure} - \pi_{1}^{*}$ $_{N0} > 0$ in the range of θ where the pure-bundling strategy of firm 1 is still credible.

Proof of the range of θ **in the analysis of pure bundling**

If θ is too extremely high, the pure-bundling strategy of firm 1 will not be credible and then pure bundling is pointless to be analysed because firm 1 will not deviate from the no-bundling strategy to the pure-bundling strategy. Hence, in the pure-bundling section, θ is assumed to be within the range of θ that varies according to the costs and the addition utility from quality enhancement as in Figure 4.6 with regard to the condition for an interior equilibrium, $b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 < 12tb_Ab_B$.

Figure 4.6 The maximum value of θ in the analysis of the pure-bundling strategy

The maximum value of θ decreases according to a considerable increase in costs and a substantial decrease in additional utility. When the costs are not too high and the additional utility is not too low, the pure-bundling strategy of firm 1 is credible, regardless of value of θ .

4.5.5 Proof of Proposition 4.4b

(i) $\beta_{A2~Pure}^* < \beta_{A2~No}^*, \beta_{B2~Pure}^* < \beta_{B2~No}^*$

Comparing (4.30) with (4.45), and (4.31) with (4.47) gives the following.

$$
\beta_{A2\,Pure}^* - \beta_{A2\, No}^* = \frac{3A(12tb_Ab_B - b_BA^2 - b_AB^2) - 4A(16tb_Ab_B - b_BA^2 - b_AB^2)}{24b_A(16tb_Ab_B - b_BA^2 - b_AB^2)} < 0
$$

$$
\beta_{B2\,pure}^* - \beta_{B2\,No}^*
$$
\n
$$
= \frac{3\mathbb{B}(12tb_Ab_B - b_B\mathbb{A}^2 - b_A\mathbb{B}^2) - 4\mathbb{B}(16tb_Ab_B - b_B\mathbb{A}^2 - b_A\mathbb{B}^2)}{24b_A(16tb_Ab_B - b_B\mathbb{A}^2 - b_A\mathbb{B}^2)} < 0
$$

In regard to the condition that $b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 < 12tb_Ab_B$ for an interior equilibrium, $16tb_Ab_B - b_B\mathbb{A}^2 - b_A\mathbb{B}^2 > 0$ and $12tb_Ab_B - b_B\mathbb{A}^2 - b_A\mathbb{B}^2 > 0$.

(ii)
$$
P_{A2\,pure}^* \gtrless P_{A2\,No}^*
$$
, $P_{B2\,Pure}^* < P_{B2\,No}^*$
From (4.32) and (4.49), $P_{A2\,Pure}^* - P_{A2\,No}^* > 0$ when

$$
\theta < \frac{3}{4} - \frac{(3\Omega - 1)c_A \mathbb{A}(16tb_A b_B - b_B \mathbb{A}^2 - b_A \mathbb{B}^2) + 12a_A \mathbb{A}tb_A b_B + 12 \mathbb{B}^2 t b_A^2}{48tb_A(16tb_A b_B - b_B \mathbb{A}^2 - b_A \mathbb{B}^2)},
$$

where $\mathbb{A} = a_A - c_A$, $\mathbb{B} = a_B - c_B$.

 $P_{A2~Pure}^* \ge P_{A2~No}^*$ when $\theta \le \frac{3}{4}$ $\frac{3}{4}-\frac{(3\Omega-1)c_A\mathbb{A}(16tb_Ab_B-b_B\mathbb{A}^2-b_A\mathbb{B}^2)+12a_A\mathbb{A}tb_Ab_B+12\mathbb{B}^2tb_A^2}{48tb_A(16tb_Ab_B-b_B\mathbb{A}^2-b_A\mathbb{B}^2)}$ $48tb_A(16tb_Ab_B-b_B\mathbb{A}^2-b_A\mathbb{B}^2)$ respectively.

From (4.33) and (4.50), after rearranging the expression, $P_{B2\,Pure}^* - P_{B2\,No}^* > 0$ if $\Omega < \frac{1}{2}$ $\frac{1}{3}-\frac{4tb_Ab_B(c_B\mathbb{B}+16tb_B)}{c_B\mathbb{B}\left(16tb_Ab_B-b_B\mathbb{A}^2-b_A\right)}$ $\frac{4tb_{A}b_{B}(c_{B} \mathbb{B}+16tb_{B})}{c_{B} \mathbb{B} (16tb_{A}b_{B}-b_{B} \mathbb{A}^{2}-b_{A} \mathbb{B}^{2})};$ $\Omega = \frac{20tb_{A}b_{B}-b_{B}(a_{A}-c_{A})^{2}-b_{A}(a_{B}-c_{B})^{2}}{16tb_{A}b_{B}-b_{B}(a_{A}-c_{A})^{2}-b_{A}(a_{B}-c_{B})^{2}}$ $\frac{201D_AD_B-D_B(u_A-c_A)-D_A(u_B-c_B)}{16tb_Ab_B-b_B(a_A-c_A)^2-b_A(a_B-c_B)^2}$. However, $\Omega > 1 > \frac{1}{2}$ $\frac{1}{3} - \frac{4tb_Ab_B(c_B \mathbb{B} + 16tb_B)}{c_B \mathbb{B} (16tb_Ab_B - b_B\mathbb{A}^2 - b_A)}$ $\frac{4\pi\sigma_{ADB}(c_B\omega+16\pi\sigma_B)}{c_B\mathbb{E}(16t_{A}b_B-b_B\mathbb{A}^2-b_A\mathbb{B}^2)}$ when a_k , b_k , c_k are in the range yielding an interior equilibrium; $k \in \{A, B\}$. Therefore, $P_{B2\, Pure}^* < P_{B2\, No}^*$.

(iii) $MS_{A2~Pure} < MS_{A2~No}$, $MS_{B2~Pure} < MS_{B2~No}$

From (4. A4) and (4. A9), firm A2 and firm B2's market shares decrease after pure bundling.

(iv)
$$
\pi_{A2\,Pure}^* \geq \pi_{A2\,No}^*
$$
, $\pi_{B2\,Pure}^* < \pi_{B2\,No}^*$

Comparing (4. A2) with (4. A6), and analogously (4. A3) with (4. A7) reveals that $\pi_{A2\,Pure}^* - \pi_{A2\,No}^* \gtrless 0$ and $\pi_{B2\,Pure}^* - \pi_{B2\,No}^* < 0$.

$$
\pi_{A2\,Pure}^{*} - \pi_{A2\, No}^{*}
$$
\n
$$
= \frac{t}{576} \left\{\n\begin{array}{l}\n162 - 288\theta + \frac{16A^{2}}{tb_{A}} - 108\left(\frac{b_{B}A^{2} + b_{A}\mathbb{B}^{2}}{16tb_{A}b_{B} - b_{B}A^{2} - b_{A}\mathbb{B}^{2}}\right) \\
+ 18\left(\frac{b_{B}A^{2} + b_{A}\mathbb{B}^{2}}{16tb_{A}b_{B} - b_{B}A^{2} - b_{A}\mathbb{B}^{2}}\right)^{2} - \frac{9A^{2}}{tb_{A}}\left(\frac{12tb_{A}b_{B} - b_{B}A^{2} - b_{A}\mathbb{B}^{2}}{16tb_{A}b_{B} - b_{B}A^{2} - b_{A}\mathbb{B}^{2}}\right)^{2}\n\end{array}\n\right\}
$$

 $\pi_{A2\,Pure}^* - \pi_{A2\,No}^* > 0$ when

$$
\theta < \left\{\begin{array}{c} 9 \\ 16 + \frac{A^2}{18tb_A} \left[1 - \frac{1}{2} \left(\frac{12tb_Ab_B}{16tb_Ab_B} \right)^2 \right] \\ - \frac{6}{16} \left(\frac{b_BA^2 + b_A\mathbb{B}^2}{16tb_Ab_B} \right) + \frac{1}{16} \left(\frac{b_BA^2 + b_A\mathbb{B}^2}{16tb_Ab_B} \right)^2 \right\} \\ - \frac{6}{16} \left(\frac{b_BA^2 + b_A\mathbb{B}^2}{16tb_Ab_B} \right) + \frac{1}{16} \left(\frac{b_BA^2 + b_A\mathbb{B}^2}{16tb_Ab_B} \right)^2 \right\} \end{array}\right\}
$$

where $A = a_A - c_A$ and $B = a_B - c_B$.

4.5.6 Proof of Proposition 4.4c

From (4.34) and (4.51), one can rearrange $CS_{Pure} - CS_{No} < 0$ as

$$
\theta(\theta+9) < 9-9\Omega + 3\Omega^2 - \frac{1}{t}\left[\frac{\mathbb{A}^2}{b_a} + \frac{\mathbb{B}^2}{b_B}\right]\left[\begin{array}{c} -2 + \dfrac{15}{8}\Omega - \dfrac{3}{8}\Omega^2 \\[0.4em] + \dfrac{(3\Omega - 6)t b_A b_B}{16tb_A b_B - b_B\mathbb{A}^2 - b_A\mathbb{B}^2} \end{array}\right]
$$

where
$$
\Omega = \frac{20tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2}{16tb_Ab_B - b_B(a_A - c_A)^2 - b_A(a_B - c_B)^2}, \mathbb{A} = a_A - c_A > 0, \mathbb{B} = a_B - c_B > 0.
$$

$$
CS_{Pure} \lesseqgtr C S_{No} \text{ if } \theta(\theta + \theta) \lesseqgtr 9 - 9\Omega + 3\Omega^2 - \frac{1}{t} \Big[\frac{\mathbb{A}^2}{b_a} + \frac{\mathbb{B}^2}{b_B} \Big] \Bigg[\frac{-2 + \frac{15}{8} \Omega - \frac{3}{8} \Omega^2}{+ \frac{(3\Omega - 6)t b_A b_B}{16tb_A b_B - b_B \mathbb{A}^2 - b_A \mathbb{B}^2}} \Bigg].
$$

4.5.7 Proof of the conditions for the multi-product firm's equilibrium quality enhancement levels in the pure-bundling case

It is more likely that $b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 - 4tb_Ab_B < 0$ when b_k and c_k are comparatively high and a_k is relatively low in accordance with the following; $k \in \{A, B\}$.

$$
\frac{\partial [b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 - 4tb_A b_B]}{\partial b_A} = (a_B - c_B)^2 - 4tb_B < 0 \text{ if } (a_B - c_B)^2 < 4tb_B.
$$
\n
$$
\frac{\partial [b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 - 4tb_A b_B]}{\partial b_B} = (a_A - c_A)^2 - 4tb_A < 0 \text{ if } (a_A - c_A)^2 < 4tb_A.
$$
\n
$$
\frac{\partial [b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 - 4tb_A b_B]}{\partial c_A} = -2b_B(a_A - c_A) < 0
$$
\n
$$
\frac{\partial [b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 - 4tb_A b_B]}{\partial c_B} = -2b_A(a_B - c_B) < 0
$$
\n
$$
\frac{\partial [b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 - 4tb_A b_B]}{\partial a_A} = 2b_B(a_A - c_A) > 0
$$
\n
$$
\frac{\partial [b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 - 4tb_A b_B]}{\partial a_A} = 2b_B(a_A - c_A) > 0
$$

$$
\frac{\partial [b_B(a_A - c_A)^2 + b_A(a_B - c_B)^2 - 4tb_Ab_B]}{\partial a_B} = 2b_A(a_B - c_B) > 0
$$

It is assumed that $a_k > c_k$ and b_k is relatively high according to the nature of telecommunications cost structure and the assumption of the second-order conditions for the existence of equilibrium. $\frac{\partial^2 \pi_1}{\partial \theta_1}$ $\frac{\partial^2 \pi_1}{\partial \beta_{A_1}^2}$ < 0 and $\frac{\partial^2 \pi_{A_2}}{\partial \beta_{A_2}^2}$ $\frac{\partial^2 \pi_{A2}}{\partial \beta_{A2}^2}$ < 0 when $(a_A - c_A)^2$ < 32tb_A. $\partial^2 \pi_1$ $\frac{\partial^2 \pi_1}{\partial \beta_{B_1}^2}$ < 0 and $\frac{\partial^2 \pi_{B_2}}{\partial \beta_{B_2}^2}$ $\frac{\partial^2 \pi_{B_2}}{\partial \beta_{B_2}^2}$ < 0 when $(a_B - c_B)^2$ < 32tb_B; $k \in \{A, B\}.$

4.5.8 Proof of Proposition 4.5

(i) Prices

Proof that $\delta^* > 0$

From (4.54), the equilibrium discount cannot be expressed explicitly. One may consider the derivative at $\delta = 0$, which is similar to the no-bundling situation. At $\delta = 0$, the equilibrium prices and quality enhancement levels are symmetric, i.e. $P_{A1}^* = P_{A2}^*$, $P_{B1}^* = P_{B2}^*$, $\beta_{A1}^* = \beta_{A2}^*$, $\beta_{B1}^* = \beta_{B2}^*$. At $\delta = 0$, firm 1 has incentive to increase δ (offer a bundle discount) in order to reap more profit.

$$
\frac{\partial \pi_1}{\partial \delta} \bigg|_{\delta=0} = \begin{cases} \left(\frac{1}{2\theta t} \right) \left(\frac{1}{2} \right) [\theta t + c_A(\beta_{A1}^* |_{\delta=0}) - c_A(\beta_{A1}^* |_{\delta=0})] \\ + \left(\frac{1}{2t} \right) \left(\frac{1}{2} \right) [t + c_B(\beta_{B1}^* |_{\delta=0}) - c_B(\beta_{B1}^* |_{\delta=0})] \\ - \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \end{cases} = \frac{1}{4} > 0 \quad (4. A10)
$$

Proof that $P_{A1}^* > P_{A2}^*$ **and** $P_{B1}^* > P_{B2}^*$ **in the mixed-bundling case**

Considering the case of no bundling ($\delta = 0$), firms choose the symmetric equilibrium outcomes of prices and quality enhancement levels in both markets. However, when firm 1 is allowed to vary δ in the mixed-bundling case, it definitely chooses $\delta^* > 0$ to maximise profit. Starting from the point $\delta = 0$, the derivatives of prices in market A (4.58) and (4.60) are shown below.

$$
\frac{\partial P_{A1}^*}{\partial \delta}\Big|_{\delta=0} = \frac{1}{6} + \frac{1}{18t} (a_B - c_B)(\beta_{B1}^*|_{\delta=0} - \beta_{B2}^*|_{\delta=0}) = \frac{1}{6} > 0 \tag{4. A11}
$$

$$
\frac{\partial P_{A2}^*}{\partial \delta} \Big|_{\delta=0} = \frac{1}{6t} \left(-P_{B1}^* \big|_{\delta=0} + c_B \beta_{B1}^* \big|_{\delta=0} \right) = \frac{1}{6t} \left(-t - c_B \beta_{B1}^* \big|_{\delta=0} + c_B \beta_{B1}^* \big|_{\delta=0} \right)
$$
\n
$$
= -\frac{1}{6} < 0 \tag{4. A12}
$$

As a result of $\delta^* > 0$, P_{A1}^* increases but P_{A2}^* decreases from the symmetric nobundling benchmark. Therefore, after mixed bundling, $P_{A1}^* > P_{A2}^*$.

Similarly, at $\delta = 0$, the derivatives of prices in market B (4.59) and (4.61) are shown below.

$$
\frac{\partial P_{B1}^*}{\partial \delta}\Big|_{\delta=0} = \frac{1}{6} + \frac{1}{18\theta t} (a_A - c_A) (\beta_{A1}^*|_{\delta=0} - \beta_{A2}^*|_{\delta=0}) = \frac{1}{6} > 0 \tag{4. A13}
$$

$$
\frac{\partial P_{B2}^*}{\partial \delta} \Big|_{\delta=0} = \frac{1}{6\theta t} \left(-P_{A1}^* \big|_{\delta=0} + c_A \beta_{A1}^* \big|_{\delta=0} \right) = \frac{1}{6\theta t} \left(-\theta t - c_A \beta_{A1}^* \big|_{\delta=0} + c_A \beta_{A1}^* \big|_{\delta=0} \right)
$$
\n
$$
= -\frac{1}{6} < 0 \tag{4. A14}
$$

When firm 1 chooses $\delta^* > 0$, P_{B1}^* increases but P_{B2}^* decreases from the symmetric nobundling benchmark. Thus, in the mixed-bundling equilibrium, $P_{B1}^* > P_{B2}^*$.

(ii) Quality enhancement

Proof of the comparison of $\beta^*_{A1}, \beta^*_{A2}, \beta^*_{B1}, \beta^*_{B2}$ in the mixed-bundling case

According to (4.A10), firm 1 will offer a bundle discount ($\delta^* > 0$) but the equilibrium discount function cannot be expressed explicitly. Instead, one may consider at a fixed positive value of discount δ^* to compare the equilibrium quality enhancement levels with the no-bundling case in which the discount value is fixed at zero. However, it is found that the equilibrium levels of quality enhancement are ambiguous according to θ . To illustrate the equilibrium outcomes, it is assumed that the two markets are identical in cost structure and additional utility of quality enhancement, but different in taste cost, i.e. $\theta \in (0,1)$.

Parameter	value
t	$\mathbf{1}$
θ	(0,1)
a_A	$\overline{2}$
\mathcal{C}_A	$\mathbf{1}$
b_A	100
a_B	$\overline{2}$
\mathcal{C}_B	$\mathbf{1}$
b_B	100
V_A	20
V_B	20

Table 4.1 Parameter assumptions

Consider the derivatives of profits with respect to quality enhancement levels at a fixed discount in the neighbourhood of $\delta = 0$. First, when θ is sufficiently high, the multi-product firm has incentive to increase quality enhancement levels and the single-product firms decrease quality enhancement levels from the symmetric nobundling benchmark in both markets. Thus, $\beta_{A1}^* > \beta_{A2}^*$ and $\beta_{B1}^* > \beta_{B2}^*$.

Second, when θ is moderate, the multi-product firm has incentive to increase the quality enhancement level in the more competitive market but decrease the level in the less competitive market. Meanwhile, the single-product firms still decrease the quality enhancement levels in both markets. Therefore, it is certain that $\beta_{A1}^* > \beta_{A2}^*$ but the comparison in the less competitive market is ambiguous.

Third, when θ is sufficiently low, the multi-product firm still has incentive to increase the quality enhancement level in the more competitive market but decrease the level in the less competitive market. Meanwhile, the single-product firm in the more competitive market decreases the quality enhancement level but the singleproduct firm in the less competitive market decides to increase the level. Therefore, $\beta_{A1}^* > \beta_{A2}^*, \beta_{B1}^* < \beta_{B2}^*$. These results are shown in Figure 4.7a with $\delta = 0.1$, Figure 4.7b with $\delta = 0.2$ and Figure 4.7c with $\delta = 0.3$.

Figure 4.7a The derivatives of profits with respect to quality enhancement levels at the symmetric no-bundling benchmark $(\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6h_A})$ $\frac{A^{-c}A}{6b_A}$, $\beta_{B1} = \beta_{B2} = \frac{a_B - c_B}{6b_B}$ $\frac{B-\epsilon_B}{6b_B}$ when δ is fixed at 0.1.

the symmetric no-bundling benchmark $(\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6h_A})$ $\frac{A^{-c}A}{6b_A}$, $\beta_{B1} = \beta_{B2} = \frac{a_B - c_B}{6b_B}$ $\frac{B-c_B}{6b_B}$ when δ is fixed at 0.2.

Figure 4.7c The derivatives of profits with respect to quality enhancement levels at the symmetric no-bundling benchmark $(\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6h_A})$ $\frac{A^{-c}A}{6b_A}$, $\beta_{B1} = \beta_{B2} = \frac{a_B - c_B}{6b_B}$ $\frac{B-c_B}{6b_B}$ when δ is fixed at 0.3.

When δ is fixed at a given positive value around zero, the levels of quality enhancement in (4.30) - (4.31), which are in equilibrium in the no-bundling case, can no longer maximise the corresponding profit in the mixed-bundling case. In the more competitive market, regardless of θ , the multi-product firm increases the quality enhancement level $(\beta_{A1}^* > \frac{a_A - c_A}{6h})$ $\frac{A^{-1}c_A}{6b_A}$ and the single-product firm decreases the level $\left(\beta_{A2}^* < \frac{a_A - c_A}{6h}\right)$ $\frac{A-c_A}{6b_A}$) certainly.

However, it is ambiguous in the less competitive market according to θ . First, when θ is sufficiently high, the multi-product firm increases the level $(\beta_{B1}^*) \frac{a_B - c_B}{6h_B}$ $\frac{B-c_B}{6b_B}$ but the single-product firm decreases the level $(\beta_{B2}^* < \frac{a_B - c_B}{6h_B})$ $\left(\frac{B-C}{6b_B}\right)$. Second, when θ is moderate, the multi-product firm and the single-product firm decrease the levels $\left(\beta_{B1}^*, \beta_{B2}^* < \frac{a_B - c_B}{6h_B}\right)$ $\frac{B - c_B}{6b_B}$). Lastly, when θ is sufficiently low, the multi-product firm decreases the level $\left(\beta_{B1}^* < \frac{a_B - c_B}{6h_B}\right)$ $\frac{B - c_B}{6b_B}$) but the single-product firm increases the level instead $\left(\beta_{B2}^* \geq \frac{a_B - c_B}{6h_B}\right)$ $\frac{B-C}{6b_B}$).

4.5.9 Proof of Proposition 4.6

(i) Proof that $\pi_1^* > \pi_{A2}^* + \pi_{B2}^*$ in the mixed-bundling case

According to (4.16), (4.17) and the equilibrium outcomes in (4.58) - (4.61),

$$
\frac{\partial \pi_{A2}^*}{\partial \delta} = (P_{A2}^* - c_A \beta_{A2}^*) \left(-\frac{\hat{y}}{2\theta t} - \frac{2\delta}{8\theta t^2} \right) < 0 \tag{4. A15}
$$

$$
\frac{\partial \pi_{B2}^*}{\partial \delta} = (P_{B2}^* - c_B \beta_{B2}^*) \left(-\frac{\hat{x}}{2t} - \frac{2\delta}{8\theta t^2} \right) < 0 \tag{4. A16}
$$

After mixed bundling, firm 1 offers a bundle discount so δ increases from zero. Profits of firm A2 and firm B2 decrease whereas firm 1's profit increases from the symmetric outcomes of the no-bundling case. Therefore, π_1^* > $\pi_{A2}^* + \pi_{B2}^*$ in the mixed-bundling case.

(ii) Proof of the equilibrium market shares in the mixed-bundling case

As seen in Figure 4.5 with the unit demand assumption, demand for product A1 is clearly interpreted as market share of product A1,

$$
MS_{A1\,Mixed} = \hat{x} + \frac{\delta}{2\theta t} \hat{y} + \frac{1}{2} \left(\frac{\delta}{2\theta t}\right) \left(\frac{\delta}{2t}\right).
$$
 (4. A17)

Demand for product A2 indicates market share of product A2,

$$
MS_{A2\;Mixed} = 1 - MS_{A1\;Mixed} = 1 - \left\{\hat{x} + \frac{\delta}{2\theta t}\hat{y} + \frac{1}{2}\left(\frac{\delta}{2\theta t}\right)\left(\frac{\delta}{2t}\right)\right\}.
$$
 (4. A18)

Likewise, market shares in market B are the following.

$$
MS_{B1\,Mixed} = \hat{y} + \frac{\delta}{2t}\hat{x} + \frac{1}{2}\left(\frac{\delta}{2\theta t}\right)\left(\frac{\delta}{2t}\right).
$$
 (4. A19)

$$
MS_{B2\,Mixed} = 1 - MS_{B1\,Mixed} = 1 - \left\{\hat{y} + \frac{\delta}{2t}\hat{x} + \frac{1}{2}\left(\frac{\delta}{2\theta t}\right)\left(\frac{\delta}{2t}\right)\right\}.
$$
 (4. A20)

Henceforth, one may employ the total differential of market share as an approximation of the sensitivity of market shares in response to a change of equilibrium quality enhancement levels from the no-bundling benchmark.

$$
dMS_{A1} = \frac{\partial MS_{A1}}{\partial \beta_{A1}} d\beta_{A1} + \frac{\partial MS_{A1}}{\partial \beta_{B1}} d\beta_{B1} + \frac{\partial MS_{A1}}{\partial \beta_{A2}} d\beta_{A2} + \frac{\partial MS_{A1}}{\partial \beta_{B2}} d\beta_{B2}
$$

$$
dMS_{B1} = \frac{\partial MS_{B1}}{\partial \beta_{A1}} d\beta_{A1} + \frac{\partial MS_{B1}}{\partial \beta_{B1}} d\beta_{B1} + \frac{\partial MS_{B1}}{\partial \beta_{A2}} d\beta_{A2} + \frac{\partial MS_{B1}}{\partial \beta_{B2}} d\beta_{B2}
$$

The derivatives of (4. A17) and (4. A19) with respect to each quality enhancement level in the neighbourhood of $\delta = 0$ are shown in Figure 4.8a and Figure 4.8b according to a range of θ . The parameter values are assumed in Table 4.1.

Figure 4.8a The derivatives of market share of product A1 with respect to quality enhancement levels in the neighbourhood of $\delta = 0$

quality enhancement levels in the neighbourhood of $\delta = 0$

To illustrate the effect of mixed bundling on market share, one may follow (4.62) - (4.65) and manipulate the derivatives to obtain the differentials of equilibrium quality enhancement levels below. The parameters are assumed in Table 4.1 and bundle discount (δ) is considered as a fixed positive value in the neighbourhood of $\delta = 0$.

 $d\beta_{A1} = \beta_{A1\,Mixed}^* - \beta_{A1\,No}^*$

Substituting $\beta_{A1\,Mixed}^*$ and $\beta_{A1\,No}^*$ in (4.62) and rearranging the equations gives (4. A21) and (4. A22) respectively.

$$
\beta_{A1\,Mixed}^{*} = \left(\frac{1}{2b_{A}}\right) \begin{Bmatrix} Z_{1}\mathbb{A} \left[1 - \frac{1}{2\theta t} (P_{A2\,Mixed}^{*} - C_{A}\beta_{A2\,Mixed}^{*})\right] \\ + Z_{2} \left[1 - \frac{1}{2t} (P_{B2\,Mixed}^{*} - C_{B}\beta_{B2\,Mixed}^{*})\right] \end{Bmatrix}
$$
\n
$$
\beta_{A1\,No}^{*} = \left(\frac{1}{2b_{A}}\right) \begin{Bmatrix} Z_{1}\mathbb{A} \left[1 - \frac{1}{2\theta t} (P_{A2\,No}^{*} - C_{A}\beta_{A2\,No}^{*})\right] \\ + Z_{2} \left[1 - \frac{1}{2t} (P_{B2\,No}^{*} - C_{B}\beta_{B2\,No}^{*})\right] \\ - \left(\frac{\partial \pi_{1}}{\partial \beta_{A1}}\Big|_{\beta_{A1} = \beta_{A2} = \frac{a_{A} - c_{A}}{6b_{A}} \end{Bmatrix} \right)
$$
\n(4.A22)

Therefore,

 $d\beta_{A1}$

$$
= \left(\frac{1}{2b_{A}}\right)\left\{\begin{aligned} Z_{1}\mathbb{A}\left[\frac{1}{2\theta t}\left[(P_{A2\,No}^{*} - C_{A}\beta_{A2\,No}^{*}\right) - (P_{A2\,Mixed}^{*} - C_{A}\beta_{A2\,Mixed}^{*})\right] \Big| \\ + Z_{2}\left[\frac{1}{2t}\left[(P_{B2\,No}^{*} - C_{B}\beta_{B2\,No}^{*}) - (P_{B2\,Mixed}^{*} - C_{B}\beta_{B2\,Mixed}^{*})\right] \Big| \\ + \left(\frac{\partial \pi_{1}}{\partial \beta_{A1}}\Big|_{\beta_{A1} = \beta_{A2} = \frac{a_{A} - c_{A}}{6b_{A}}}{\beta_{B1} = \beta_{B2} = \frac{a_{B} - c_{B}}{6b_{B}}}\right) \end{aligned}\right.\tag{4.A23}
$$

 $d\beta_{B1} = \beta^*_{B1\,Mixed} - \beta^*_{B1\,No}$

Substituting $\beta_{B1\,Mixed}^*$ and $\beta_{B1\,No}^*$ in (4.63) yields the following.

$$
\beta_{B1\,Mixed}^{*} = \left(\frac{1}{2b_{B}}\right) \left\{\n\begin{aligned}\nZ_{3} \left[1 - \frac{1}{2\theta t} (P_{A2\,Mixed}^{*} - C_{A} \beta_{A2\,Mixed}^{*})\right] \\
+ Z_{1} \mathbb{B} \left[1 - \frac{1}{2t} (P_{B2\,Mixed}^{*} - C_{B} \beta_{B2\,Mixed}^{*})\right]\n\end{aligned}\n\right\} (4. A24)
$$

$$
\beta_{B1\,No}^{*} = \left(\frac{1}{2b_{B}}\right) \left\{\n\begin{aligned}\nZ_{3} \left[1 - \frac{1}{2\theta t} (P_{A2\,No}^{*} - C_{A}\beta_{A2\,No}^{*})\right] \\
+ Z_{1} \mathbb{B} \left[1 - \frac{1}{2t} (P_{B2\,No}^{*} - C_{B}\beta_{B2\,No}^{*})\right] \\
- \left(\frac{\partial \pi_{1}}{\partial \beta_{B1}}\Big|_{\beta_{A1} = \beta_{A2} = \frac{a_{A} - c_{A}}{6b_{A}} \\
\beta_{B1} = \beta_{B2} = \frac{a_{B} - c_{B}}{6b_{B}}\n\end{aligned}\n\right\} (4. A25)
$$

Therefore,

$$
d\beta_{B1} = \left(\frac{1}{2b_B}\right) \left\{ \begin{aligned} Z_3 \left[\frac{\frac{1}{2\theta t} (P_{A2\ N0}^* - C_A \beta_{A2\ N0}^*)}{\frac{1}{2\theta t} (P_{A2\ Nixed}^* - C_A \beta_{A2\ Nixed}^*)} \right] \\ + Z_1 \mathbb{B} \left[\frac{\frac{1}{2t} (P_{B2\ N0}^* - C_B \beta_{B2\ N0}^*)}{\frac{1}{2t} (P_{B2\ Nixed}^* - C_B \beta_{B2\ Nixed}^*)} \right] \\ + \left(\frac{\partial \pi_1}{\partial \beta_{B1}} \Big|_{\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6b_A}} \right) \\ \beta_{B1} = \beta_{B2} = \frac{a_B - c_B}{6b_B} \end{aligned} \right\}.
$$
(4.A26)

 $d\beta_{A2} = \beta_{A2\;Mixed}^* - \beta_{A2\;No}^*$

Substituting $\beta_{A2 \text{ Mixed}}^*$ and $\beta_{A2 \text{ No}}^*$ in (4.64) gives the following.

$$
\beta_{A2\;Mixed}^* = \left(\frac{1}{2b_A}\right) \left\{ Z_4 \mathbb{A} \left[\frac{1}{2\theta t} \left(P_{A2\;Mixed}^* - C_A \beta_{A2\;Mixed}^* \right) \right] \right\} \tag{4. A27}
$$

$$
\beta_{A2\,No}^{*} = \left(\frac{1}{2b_{A}}\right) \left\{ Z_{4} \mathbb{A} \left[\frac{1}{2\theta t} \left(P_{A2\,No}^{*} - C_{A} \beta_{A2\,No}^{*} \right) \right] - \frac{\partial \pi_{A2}}{\partial \beta_{A2}} \middle|_{\beta_{A1} = \beta_{A2} = \frac{a_{A} - c_{A}}{6b_{A}}} \right\} (4. A28)
$$

Thus,

$$
d\beta_{A2} = \left(\frac{1}{2b_A}\right) \left\{\n\begin{aligned}\n&= Z_4 A \left[-\frac{1}{2\theta t} (P_{A2\ N0}^* - C_A \beta_{A2\ N0}^*)\right] \\
&= \frac{1}{2\theta t} (P_{A2\ Nixed}^* - C_A \beta_{A2\ Nixed}^*)\n\end{aligned}\n\right\} + \frac{\partial \pi_{A2}}{\partial \beta_{A2}} \bigg|_{\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6b_A}} \left.\n\right\}.
$$
\n(4. A29)

$$
d\beta_{B2} = \beta_{B2\;Mixed}^* - \beta_{B2\;No}^*
$$

Substituting $\beta_{B2 \text{ Mixed}}^*$ and $\beta_{B2 \text{ No}}^*$ in (4.65) yields the following.

$$
\beta_{B2\;Mixed}^{*} = \left(\frac{1}{2b_{B}}\right) \left\{ Z_{4} \mathbb{B} \left[\frac{1}{2t} (P_{B2\;Mixed}^{*} - C_{B} \beta_{B2\;Mixed}^{*}) \right] \right\}
$$
(4. A30)

$$
\beta_{B2\,No}^{*} = \left(\frac{1}{2b_B}\right) \begin{Bmatrix} Z_4 \mathbb{B} \left[\frac{1}{2t} (P_{B2\,No}^{*} - C_B \beta_{B2\,No}^{*}) \right] \\ -\frac{\partial \pi_{B2}}{\partial \beta_{B2}} \Big|_{\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6b_A} \\ \beta_{B1} = \beta_{B2} = \frac{a_B - c_B}{6b_B} \end{Bmatrix}
$$
(4. A31)

Therefore,

$$
d\beta_{B2} = \left(\frac{1}{2b_B}\right) \left\{\n\begin{aligned}\n& -Z_4 \mathbb{B} \left[\frac{\frac{1}{2t} (P_{B2No}^* - C_B \beta_{B2No}^*)}{-\frac{1}{2t} (P_{B2 Mixd}^* - C_B \beta_{B2 Mixd}) \right] \\
& + \frac{\partial \pi_{B2}}{\partial \beta_{B2}} \bigg|_{\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6b_A} \\
& \beta_{B1} = \beta_{B2} = \frac{a_B - c_B}{6b_B}\n\end{aligned}\n\right\},\n\tag{4.A32}
$$
where

 $\beta_{ki\,No}^*$ is the equilibrium level of quality enhancement of product ki in the no-bundling case.

 $\beta_{ki\,Mixed}^*$ is the equilibrium level of quality enhancement of product ki in the mixedbundling case.

$$
A = a_A - c_A > 0
$$

\n
$$
B = a_B - c_B > 0
$$

\n
$$
k \in \{A, B\}
$$

\n
$$
i \in \{1, 2\}
$$

\n
$$
Z_1 = -\frac{\delta}{6t} \left(\frac{144\theta t^2}{144\theta^2 t^4 - 76\theta t^2 \delta^2 + \delta^4} \right) \left(\frac{2\delta\theta t^2 + \delta^3}{36\theta t} \right) + \frac{2}{3}
$$

\n
$$
Z_2 = -\frac{\delta}{6\theta t} \left(\frac{144\theta t^2}{144\theta^2 t^4 - 76\theta t^2 \delta^2 + \delta^4} \right) \left[c_A \left(\frac{96\theta^2 t^4 - 40\theta t^2 \delta^2 + \delta^4}{144\theta t^2} \right) + \frac{\delta}{6\theta t} c_A \right]
$$

$$
Z_3 = -\frac{\delta}{6t} \left(\frac{144\theta t^2}{144\theta^2 t^4 - 76\theta t^2 \delta^2 + \delta^4} \right) \left[c_B \left(\frac{96\theta^2 t^4 - 40\theta t^2 \delta^2 + \delta^4}{144\theta t^2} \right) + \frac{\delta}{6t} c_B \right]
$$

$$
Z_4 = 1 - \left(\frac{144\theta t^2}{144\theta^2 t^4 - 76\theta t^2 \delta^2 + \delta^4}\right) \left(\frac{4\theta t^2 - 3\delta^2}{12}\right) - \frac{\delta^2}{12\theta t^2} \left(\frac{144\theta t^2}{144\theta^2 t^4 - 76\theta t^2 \delta^2 + \delta^4}\right) \left(\frac{4\theta t^2 - 3\delta^2}{12}\right) - \frac{\delta}{2t} \left(\frac{144\theta t^2}{144\theta^2 t^4 - 76\theta t^2 \delta^2 + \delta^4}\right) \left(\frac{2\delta\theta t^2 + \delta^3}{36\theta t}\right).
$$

At a fixed value of $\delta \ge 0$, $Z_1 > 0$, $Z_2 \le 0$, $Z_3 \le 0$, $Z_4 > 0$ and $Z_4 > Z_1$.

 $d\beta_{A2} > 0$ and $d\beta_{A1} < 0$ for all values of θ . Comparing (4.A23) with (4.A29) reveals the following.

$$
|d\beta_{A2}| - |d\beta_{A1}|
$$
\n
$$
= \left(\frac{1}{2b_{A}}\right)
$$
\n
$$
= \left(\frac{1}{2b_{A}}\right)
$$
\n
$$
- Z_{2} \left(\frac{1}{2t}\right) \left[\begin{array}{c} (P_{A2\ N0}^{*} - C_{A}\beta_{A2\ N0}^{*}) \\ -(P_{A2\ Nixed}^{*} - C_{A}\beta_{A2\ Nixed}^{*}) \end{array}\right]
$$
\n
$$
- Z_{2} \left(\frac{1}{2t}\right) \left[\begin{array}{c} (P_{B2\ N0}^{*} - C_{B}\beta_{B2\ N0}^{*}) \\ -(P_{B2\ Nixed}^{*} - C_{B}\beta_{B2\ Nixed}^{*}) \end{array}\right]
$$
\n
$$
- \left(\frac{\partial \pi_{A2}}{\partial \beta_{A2}}\Big|_{\beta_{A1} = \beta_{A2} = \frac{a_{A} - c_{A}}{6b_{A}}} - \left(\frac{\partial \pi_{1}}{\partial \beta_{A1}}\Big|_{\beta_{A1} = \beta_{A2} = \frac{a_{A} - c_{A}}{6b_{A}}} - \frac{a_{B} - c_{B}}{6b_{B}}\right)\right]
$$
\n(4. A33)

However, the signs of $d\beta_{B1}$ and $d\beta_{B2}$ depend on value of θ . Compare (4. A26) with (4. A32) yields the three following cases of θ .

First, when θ is high, $d\beta_{B1} > 0$ and $d\beta_{B2} < 0$.

$$
|d\beta_{B2}| - |d\beta_{B1}|
$$
\n
$$
= \left(\frac{1}{2b_B}\right)
$$
\n
$$
- Z_3 \left(\frac{1}{2\theta t}\right) \left[\begin{array}{c} (P_{B2\ N0}^* - C_B \beta_{B2\ N0}^*) \\ -(P_{B2\ Nixed}^* - C_B \beta_{B2\ Nixed}^*) \end{array}\right]
$$
\n
$$
- Z_3 \left(\frac{1}{2\theta t}\right) \left[\begin{array}{c} (P_{A2\ N0}^* - C_A \beta_{A2\ N0}^*) \\ -(P_{A2\ Nixed}^* - C_A \beta_{A2\ Nized}^*) \end{array}\right]
$$
\n
$$
- \left(\frac{\partial \pi_{B2}}{\partial \beta_{B2}}\Big|_{\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6b_A}} - \left(\frac{\partial \pi_1}{\partial \beta_{B1}}\Big|_{\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6b_A}} - \frac{a_A - c_A}{6b_A}\right) \right]
$$
\n
$$
\beta_{B1} = \beta_{B2} = \frac{a_B - c_B}{6b_B}
$$

Second, when θ is moderate, $d\beta_{B1}$, $d\beta_{B2} < 0$. $|d\beta_{B2}| - |d\beta_{B1}|$

$$
= \left(\frac{1}{2b_B}\right) \left\{\n\begin{array}{c}\n(Z_1 + Z_4) \mathbb{B} \left(\frac{1}{2t}\right) \left[\begin{array}{c}\n(P_{B2\ N0}^* - C_B \beta_{B2\ N0}^*) \\
-(P_{B2\ Mixed}^* - C_B \beta_{B2\ Mixed}^*)\n\end{array}\right] \\
+ Z_3 \left(\frac{1}{2\theta t}\right) \left[\begin{array}{c}\n(P_{A2\ N0}^* - C_A \beta_{A2\ N0}^*) \\
-(P_{A2\ Mixed}^* - C_A \beta_{A2\ Mixed}^*)\n\end{array}\right] \\
-\left(\n\frac{\partial \pi_{B2}}{\partial \beta_{B2}}\bigg|_{\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6B_A}}\n\right) + \left(\n\frac{\partial \pi_1}{\partial \beta_{B1}}\bigg|_{\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6B_A}}\n\end{array}\n\right)\n\tag{4. A35}
$$

Third, when θ is low, $d\beta_{B1} < 0$ and $d\beta_{B2} > 0$. $|d\beta_{B2}| - |d\beta_{B1}|$

$$
= \left(\frac{1}{2b_{B}}\right) \left\{\n\begin{array}{c}\n(Z_{1} - Z_{4}) \mathbb{B} \left(\frac{1}{2t}\right) \left[\n-\left(P_{B2 \text{ Mixed}}^{*} - C_{B} \beta_{B2 \text{ Mixed}}^{*}\right)\n\right] \\
+ Z_{3} \left(\frac{1}{2\theta t}\right) \left[\n-\left(P_{A2 \text{ Mixed}}^{*} - C_{A} \beta_{A2 \text{ No}}^{*}\right)\n\right] \\
+ Z_{3} \left(\frac{1}{2\theta t}\right) \left[\n-\left(P_{A2 \text{ Mixed}}^{*} - C_{A} \beta_{A2 \text{ Mixed}}^{*}\right)\n\right] \\
+\left(\frac{\partial \pi_{B2}}{\partial \beta_{B2}}\bigg|_{\beta_{A1} = \beta_{A2} = \frac{a_{A} - c_{A}}{6b_{A}}} + \left(\frac{\partial \pi_{1}}{\partial \beta_{B1}}\bigg|_{\beta_{A1} = \beta_{A2} = \frac{a_{A} - c_{A}}{6b_{A}}} \beta_{B1} = \beta_{B2} = \frac{a_{B} - c_{B}}{6b_{B}}\n\end{array}\n\right\} (4. A36)
$$

It is also found that

$$
\frac{\partial (P_{A2\text{ Mixed}}^* - C_A \beta_{A2\text{ Mixed}}^*)}{\partial \beta_{A2}} = \frac{\delta}{6t} \mathbb{A} \left[\frac{144\theta t^2}{144\theta^2 t^4 - 76\theta t^2 \delta^2} \right] \left(\frac{28\theta t^2 + \delta^3}{36\theta t} \right) + \frac{\mathbb{A}}{3} > 0
$$

$$
\frac{\partial (P_{B2\,Mixed}^* - C_A \beta_{B2\,Mixed}^*)}{\partial \beta_{B2}} = \frac{\delta}{6t} \mathbb{B} \left[\frac{144\theta t^2}{144\theta^2 t^4 - 76\theta t^2 \delta^2} \right] \left(\frac{2\delta\theta t^2 + \delta^3}{36\theta t} \right) + \frac{\mathbb{B}}{3} > 0.
$$

In conclusion, when θ is high, $\beta_{A2 \text{ Mixed}}^* < \beta_{A2 \text{ No}}^*$ and $\beta_{B2 \text{ Mixed}}^* < \beta_{B2 \text{ No}}^*$. One can conclude that $\left[\frac{1}{2}a\right]$ $\frac{1}{2\theta t}(P_{A2\,No}^* - C_A\beta_{A2\,No}^*) - \frac{1}{2\theta}$ $\frac{1}{2\theta t}(P_{A2\;Mixed}^* - C_A\beta_{A2\;Mixed}^*)$ > 0, and analogously, $\left[\frac{1}{2}\right]$ $\frac{1}{2t}(P_{B2\,No}^* - C_B\beta_{B2\,No}^*) - \frac{1}{2t}$ $\frac{1}{2t}(P_{B2\,Mixed}^* - C_B \beta_{B2\,Mixed}^*) > 0.$ From (4. A33) and (4. A34), $|d\beta_{A2}| - |d\beta_{A1}| > 0$ and $|d\beta_{B2}| - |d\beta_{B1}| > 0$.

When θ is moderate, $\beta_{A2 \text{ Mixed}}^* < \beta_{A2 \text{ No}}^*$ and $\beta_{B2 \text{ Mixed}}^* < \beta_{B2 \text{ No}}^*$. According to (4. A33) and (4. A35), it is concluded that $|d\beta_{A2}| - |d\beta_{A1}| > 0$ but the sign of $|d\beta_{B2}| - |d\beta_{B1}|$ is still ambiguous.

When θ is low, $\beta_{A2 \text{ Mixed}}^* < \beta_{A2 \text{ No}}^*$ and $\beta_{B2 \text{ Mixed}}^* > \beta_{B2 \text{ No}}^*$. Therefore, one may conclude that $\left[\frac{1}{2\pi}\right]$ $\frac{1}{2\theta t}(P_{A2\,No}^* - C_A\beta_{A2\,No}^*) - \frac{1}{2\theta}$ $\frac{1}{2\theta t}(P_{A2 \text{ Mixed}}^* - C_A \beta_{A2 \text{ Mixed}}^*)$ > 0 and $\left[\frac{1}{2}\right]$ $\frac{1}{2t}(P_{B2\,No}^* - C_B\beta_{B2\,No}^*) - \frac{1}{2t}$ $\frac{1}{2t}(P_{B2\,Mixed}^* - C_B \beta_{B2\,Mixed}^*)$ < 0. According to (4.A33) and (4. A36), the signs of $|d\beta_{A2}| - |d\beta_{A1}|$ and $|d\beta_{B2}| - |d\beta_{B1}|$ are ambiguous.

As shown in Figure 4.8a and 4.8b, $\frac{\partial MS_{A1} Mixed}{\partial \beta_{A1}}$ and $\frac{\partial MS_{A1} Mixed}{\partial \beta_{B1}}$ are positive, but $\partial MS_{A1~Mixed}$ $\frac{\delta_{A1} Nixed}{\delta \beta_{A2}}$ and $\frac{\partial MS_{A1} Nixed}{\delta \beta_{B2}}$ are negative for all values of θ . $\frac{\partial MS_{A1} Nixed}{\partial \beta_{A2}}$ $\frac{\delta_{A1 \text{ Mixed}}}{\partial \beta_{A2}} = -\frac{\partial \text{MS}_{A1 \text{ Mixed}}}{\partial \beta_{A1}}$ $\partial \beta_{A1}$ and $\frac{\partial MS_{A1~Mixed}}{\partial \beta_{B2}} = -\frac{\partial MS_{A1~Mixed}}{\partial \beta_{B1}}$ $\frac{\delta_{A1}$ Mixed. Likewise, $\frac{\partial MS_{B1}$ Mixed and $\frac{\partial MS_{B1}$ Mixed are positive, but $\partial \beta_{B1}$ дМS $_{B1\,Mixed}$ $\frac{\delta_{B1}$ Mixed and $\frac{\partial MS_{B1}$ Mixed are negative for all values of θ . $\frac{\partial MS_{B1}$ Mixed $\partial \beta_{A2}}{\partial \beta_{A2}}$ $\frac{\delta_{B1\,Mixed}}{\partial \beta_{A2}} = -\frac{\partial MS_{B1\,Mixed}}{\partial \beta_{A1}}$ $\partial \beta_{A1}$ and analogously $\frac{\partial MS_{B1 \text{ Mixed}}}{\partial \beta_{B2}} = -\frac{\partial MS_{B1 \text{ Mixed}}}{\partial \beta_{B1}}$ $\frac{\delta_{B1\,Mixed}}{\partial \beta_{B1}}$. Moreover, $\frac{\partial MS_{A1\,Mixed}}{\partial \beta_{A1}} > \frac{\partial MS_{A1\,Mixed}}{\partial \beta_{B1}}$ $\frac{\partial A1~Mixed}{\partial \beta_{B1}}>0$ and $\frac{\partial MS_{B1\,Mixed}}{\partial \beta_{B1}} > \frac{\partial MS_{B1\,Mixed}}{\partial \beta_{A1}}$ $\frac{\partial B1\,Mixed}{\partial \beta_{A1}}>0.$

In accordance with proof of Proposition 4.5, first, when θ is sufficiently high, $d\beta_{A1}$, $d\beta_{B1} > 0$ and $d\beta_{A2}$, $d\beta_{B2} < 0$. It is certain that $MS_{A1\,Mixed} > MS_{A2\, Mixed}$ and $MS_{B1\,Mixed} > MS_{B2\, Mixed}$ in this case.

Second, when θ is moderate, $d\beta_{A1} > 0$ and $d\beta_{A2}$, $d\beta_{B1}$, $d\beta_{B2} < 0$. Thus, if $d\beta_{A1} - d\beta_{A2} + d\beta_{B1} - d\beta_{B2} > 0$, firm 1's market shares strongly increase after mixed bundling. Assuming parameter values in Table 4.1, firm 1 can expand its market shares certainly. Therefore, it is likely that firm 1's market shares

increase from the symmetric benchmark after all, i.e. $MS_{A1\,Mixed} > MS_{A2\, Mixed}$ and $MS_{B1\;Mixed} > MS_{B2\; Mixed}$.

Third, when θ is sufficiently low, $d\beta_{A1}$, $d\beta_{B2} > 0$ and $d\beta_{A2}$, $d\beta_{B1} < 0$. Additionally, in the more competitive market, $\frac{\partial MS_{A1} Mixed}{\partial \beta_{A1}} > \frac{\partial MS_{A1} Mixed}{\partial \beta_{B1}}$ $\frac{\partial A_1 Mixed}{\partial \beta_{B_1}} > 0$. The positive effect of $d\beta_{A1}$ and $d\beta_{A2}$ is likely to outweigh the negative effect of $d\beta_{B1}$ and $d\beta_{B2}$ on firm 1's market share. Mixed bundling is likely to expand firm 1's market share in the more competitive market. However, in the less competitive market, $\frac{\partial MS_{B1\,Mixed}}{\partial \beta_{B1}} > \frac{\partial MS_{B1\,Mixed}}{\partial \beta_{A1}}$ $\frac{\partial B_1 Mixed}{\partial \beta_{A_1}} > 0$. The negative effect of $d\beta_{B_1}$ and $d\beta_{B_2}$ has a tendency to outweigh the positive effect of $d\beta_{A1}$ and $d\beta_{A2}$ on firm 1's market share. Consequently, it is likely that firm 1's market share in the less competitive market shrinks.

In addition, as clearly shown in Figure 4.8a in the more competitive market, the effects of quality enhancement levels on their own markets are softened by taste cost. When θ increases (taste cost increases whereas the degree of competition decreases), the market is less competitive. In other words, firms are more powerful to maintain their market shares, and it is more difficult for firms to approach their rivals' customers by offering attractive quality enhancement.

4.5.10 Proof of Proposition 4.7a

 $\pi_{1~Mixed}^* > \pi_{1~No}^*$, $\pi_{A2~Mixed}^* < \pi_{A2~No}^*$ and $\pi_{B2~Mixed}^* < \pi_{B2~No}^*$ At $\delta = 0$, (4. A10), (4. A15) and (4. A16) show that $\frac{\partial \pi_1}{\partial \delta}\Big|_{\delta=0} > 0$, $\frac{\partial \pi_{A2}^*}{\partial \delta} < 0$ and

 $\frac{\partial \pi_{B2}^*}{\partial \delta}$ < 0 respectively. Thus, compared to the no-bundling case, firm 1 chooses $\delta^* > 0$ and accordingly reaps more profit, while firm A2 and firm B2 lose profits.

4.5.11 Proof of Proposition 4.7b

(i) Prices

 $P_{A1 \text{ Mixed}}^* > P_{A1 \text{ No}}^*$ and $P_{B1 \text{ Mixed}}^* > P_{B1 \text{ No}}^*$ $P_{A2 \text{ Mixed}}^* < P_{A2 \text{ No}}^*$ and $P_{B2 \text{ Mixed}}^* < P_{B2 \text{ No}}^*$

From (4. A11) - (4. A14), at $\delta = 0$, $\frac{\partial P_{A1}^*}{\partial \delta}\Big|_{\delta=0} > 0$, $\frac{\partial P_{A2}^*}{\partial \delta}\Big|_{\delta=0} < 0$, $\frac{\partial P_{B1}^*}{\partial \delta}\Big|_{\delta=0} > 0$ and $\frac{\partial P_{B2}^*}{\partial \delta}\Big|_{\delta=0} < 0$. The bundle discount (δ) will deviate from zero and finally $\delta^* > 0$. Therefore, firm 1's prices increase but the prices of firm A2 and firm B2 decrease from the symmetric price benchmark in the no-bundling case in which δ is fixed at zero.

(ii) Quality enhancement

In the more competitive market, $\beta_{A1\;Mixed}^* > \beta_{A1\;No}^*$ and $\beta_{A2\;Mixed}^* < \beta_{A2\;No}^*$. However, in the less competitive market, a change in quality enhancement levels is ambiguous. According to the proof of Proposition 4.5 (ii) under the parameter assumptions in Table 4.1, there are different results in the three cases of θ .

First, when θ is sufficiently high, firm 1 increases its quality enhancement levels while firm A2 and firm B2 decrease their quality enhancement levels in market A and market B respectively. Therefore, $\beta_{A1\,Mixed}^* > \beta_{A1\,No}^*$, $\beta_{B1\,Mixed}^* > \beta_{B1\,No}^*$ and $\beta_{A2\;Mixed}^* < \beta_{A2\;No}^*, \beta_{B2\;Mixed}^* < \beta_{B2\;No}^*.$

Second, when θ is moderate, firm 1 increases the quality enhancement level in market A but it decreases the level in market B. Meanwhile, firm A2 and B2 decrease the levels. $\beta_{A1 \text{ Mixed}}^* > \beta_{A1 \text{ No}}^*$, $\beta_{B1 \text{ Mixed}}^* < \beta_{B1 \text{ No}}^*$ and $\beta_{A2 \text{ Mixed}}^* < \beta_{A2 \text{ No}}^*$, $\beta_{B2\;Mixed}^* < \beta_{B2\;No}^*$.

Third, when θ is sufficiently low, firm 1 increases the quality enhancement level in market A but it decreases the level in market B. Meanwhile, firm A2 decreases the level in market A but firm B2 increases the level in market B. $\beta_{A1\;Mixed}^* > \beta_{A1\;No}^*$, $\beta_{B1\;Mixed}^* < \beta_{B1\;No}^*$ and $\beta_{A2\;Mixed}^* < \beta_{A2\;No}^*$, $\beta_{B2\;Mixed}^* > \beta_{B2\;No}^*$.

4.5.12 Proof of Proposition 4.7c

A change in equilibrium market shares is ambiguous. Under the assumptions in Table 4.1, the results depend on θ as stated in proof of Proposition 4.6 (ii).

Firstly, when θ is sufficiently high, it is certain that $MS_{A1 Mixed} > MS_{A1 No}$, $MS_{A2\;Mixed} < MS_{A2\;No}$ and $MS_{B1\;Mixed} > MS_{B1\;No}$, $MS_{B2\;Mixed} < MS_{B2\;No}$.

Secondly, when θ is moderate, the market share outcomes are ambiguous. If $d\beta_{A1} - d\beta_{A2} + d\beta_{B1} - d\beta_{B2} > 0$, $MS_{A1 Mixed} > MS_{A1 No}$, $MS_{A2 Mixed} < MS_{A2 No}$ and similarly $MS_{B1\,Mixed} > MS_{B1\, No}$, $MS_{B2\, Mixed} < MS_{B2\, No}$.

Thirdly, when θ is sufficiently low, the outcomes are ambiguous. In the more competitive market, it is likely that $MS_{A1\,Mixed} > MS_{A1\,No}$, $MS_{A2\, Mixed} < MS_{A2\,No}$. Conversely, in the less competitive market, it is likely that $MS_{B1\,Mixed} < MS_{B1\, No}$, $MS_{B2\,Mixed} > MS_{B2\,No}$.

4.5.13 Proof of Proposition 4.7d

A change in consumer surplus is ambiguous. According to (4.30) and (4.31), the equilibrium outcomes in the no-bundling case ($\delta = 0$) are symmetric and also contribute to consumer surplus in (4.34) . After mixed bundling, δ increases from zero to a certain amount of positive value. The levels of equilibrium quality enhancement cannot be expressed explicitly. Instead, the total differential of consumer surplus can be used as an approximation of a change in consumer surplus to compare consumer surplus in the mixed-bundling case with that in the no-bundling case.

$$
dCS = \frac{\partial CS}{\partial \beta_{A1}} d\beta_{A1} + \frac{\partial CS}{\partial \beta_{B1}} d\beta_{B1} + \frac{\partial CS}{\partial \beta_{A2}} d\beta_{A2} + \frac{\partial CS}{\partial \beta_{B2}} d\beta_{B2}
$$

In the neighbourhood of $\delta = 0$, the derivatives of consumer surplus with respect to quality enhancement levels when θ varies in a range of $(0,1)$ are shown in Figure 4.9.

Substituting (4. A23), (4. A26), (4. A29), and (4. A32) in the above differential (dCS) yields the following.

 200.5

$$
dCS = \frac{1}{2\theta t} \left[\begin{array}{c} (P_{A2\ N0}^* - C_A \beta_{A2\ N0}^*) \\ -(P_{A2\ Mixed}^* - C_A \beta_{A2\ Mixed}^*) \end{array} \right] \left\{ \begin{array}{c} Z_1 A \left(\frac{\partial CS}{\partial \beta_{A1}} \right) - Z_4 A \left(\frac{\partial CS}{\partial \beta_{A2}} \right) \\ + Z_3 \left(\frac{\partial CS}{\partial \beta_{B1}} \right) \\ + Z_3 \left(\frac{\partial CS}{\partial \beta_{B1}} \right) \end{array} \right\}
$$

$$
+ \frac{1}{2t} \left[\begin{array}{c} (P_{B2\ N0}^* - C_B \beta_{B2\ N0}^*) \\ -(P_{B2\ Mixed}^* - C_B \beta_{B2\ Nized}^*) \end{array} \right] \left\{ \begin{array}{c} Z_2 \left(\frac{\partial CS}{\partial \beta_{A1}} \right) + Z_1 B \left(\frac{\partial CS}{\partial \beta_{B1}} \right) \\ - Z_4 B \left(\frac{\partial CS}{\partial \beta_{B2}} \right) \\ - Z_4 B \left(\frac{\partial CS}{\partial \beta_{B2}} \right) \end{array} \right\}
$$

$$
+ \frac{\left(\frac{\partial CS}{\partial \beta_{A1}} \right)}{2b_A} \cdot \left(\frac{\partial \pi_1}{\partial \beta_{A1}} \Big|_{\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6b_A}} \right) + \frac{\left(\frac{\partial CS}{\partial \beta_{B1}} \right)}{2b_B} \cdot \left(\frac{\partial \pi_1}{\partial \beta_{B1}} \Big|_{\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6b_A}} \right)
$$

$$
+\frac{\left(\frac{\partial CS}{\partial \beta_{A2}}\right)}{2b_{A}} \cdot \left(\frac{\partial \pi_{A2}}{\partial \beta_{A2}}\Big|_{\beta_{A1}=\beta_{A2}=\frac{a_{A}-c_{A}}{6b_{A}}}\right) + \frac{\left(\frac{\partial CS}{\partial \beta_{B2}}\right)}{2b_{B}} \cdot \left(\frac{\partial \pi_{B2}}{\partial \beta_{B2}}\Big|_{\beta_{A1}=\beta_{A2}=\frac{a_{A}-c_{A}}{6b_{A}}}\right) (4. A37)
$$

According to proofs of Proposition 4.5 and Proposition 4.6, the result is divided into three cases of θ . First, when θ is sufficiently high, $d\beta_{A1}$, $d\beta_{B1} > 0$ and $d\beta_{A2}$, $d\beta_{B2}$ < 0. One may consider (4. A37) in the neighbourhood of $\delta = 0$ by rewriting (4. A37) with regard to the derivatives and the parameters assumed in Table 4.1. The sign of dCS is ambiguous.

Second, when θ is moderate, $d\beta_{A1} > 0$ and $d\beta_{B1}$, $d\beta_{A2}$, $d\beta_{B2} < 0$. From (4. A37) in the neighbourhood of $\delta = 0$, the sign of dCS is still ambiguous.

Third, when θ is sufficiently low, $d\beta_{A1}$, $d\beta_{B2} > 0$ and $d\beta_{A2}$, $d\beta_{B1} < 0$. $(P_{A2\ N0}^* - C_A\beta_{A2\ N0}^*) - (P_{A2\ Mixed}^* - C_A\beta_{A2\ Mixed}^*) > 0$ $(P_{B2\ N0}^* - C_B \beta_{B2\ N0}^*) - (P_{B2\ Mixed}^* - C_B \beta_{B2\ Mixed}^*) < 0.$

In addition to the expressions, it is also found that

$$
\frac{1}{2\theta t} \left\{ Z_1 \mathbb{A} \frac{\left(\frac{\partial CS}{\partial \beta_{A1}}\right)}{2b_A} - Z_4 \mathbb{A} \frac{\left(\frac{\partial CS}{\partial \beta_{A2}}\right)}{2b_A} + Z_3 \frac{\left(\frac{\partial CS}{\partial \beta_{B1}}\right)}{2b_B} \right\} < 0,
$$

and

$$
\frac{1}{2t} \left\{ Z_2 \frac{\left(\frac{\partial CS}{\partial \beta_{A1}}\right)}{2b_A} + Z_1 \mathbb{B} \frac{\left(\frac{\partial CS}{\partial \beta_{B1}}\right)}{2b_B} - Z_4 \mathbb{B} \frac{\left(\frac{\partial CS}{\partial \beta_{B2}}\right)}{2b_B} \right\} > 0
$$

Furthermore,

$$
\left[\begin{matrix}\frac{\partial CS}{\partial\beta_{A1}}\\\frac{\partial T_1}{\partial A_1}\end{matrix}\right]\cdot\left(\begin{matrix}\frac{\partial \pi_1}{\partial\beta_{A1}}\end{matrix}\Bigg|_{\beta_{A1}=\beta_{A2}=\frac{a_A-c_A}{6b_A}}+\frac{\left(\frac{\partial CS}{\partial\beta_{B1}}\right)}{2b_B}\cdot\left(\begin{matrix}\frac{\partial \pi_1}{\partial\beta_{B1}}\end{matrix}\Bigg|_{\beta_{A1}=\beta_{A2}=\frac{a_A-c_A}{6b_A}}\frac{\partial T_1}{\partial\beta_{B1}}\Bigg|_{\beta_{A1}=\beta_{A2}=\frac{a_A-c_A}{6b_B}}\end{matrix}\right)\right]+ \left(\begin{matrix}\frac{\partial CS}{\partial\beta_{A2}}\end{matrix}\right)\cdot\left(\begin{matrix}\frac{\partial T_1}{\partial\beta_{A2}}\end{matrix}\Bigg|_{\beta_{A1}=\beta_{A2}=\frac{a_A-c_A}{6b_A}}+\frac{\left(\frac{\partial CS}{\partial\beta_{B2}}\right)}{2b_B}\left(\begin{matrix}\frac{\partial T_1}{\partial\beta_{B2}}\end{matrix}\Bigg|_{\beta_{A1}=\beta_{A2}=\frac{a_A-c_A}{6b_A}}\frac{\partial T_2}{\partial\beta_{B1}}\Bigg|_{\beta_{A1}=\beta_{A2}=\frac{a_A-c_A}{6b_A}}\end{matrix}\right)<0.
$$

Therefore, one can conclude that $dCS < 0$ certainly in this case.

4.5.14 Proof of Corollary 4.1

To consider the sign of dCS , one may start with the assumptions of the markets with the identical degrees of competition and the parameter values in Table 4.1, i.e. $\theta = 1$, $b_A = b_B$, $c_A = c_B$, $a_A = a_B$. Firm 1 will set product A1's price equal to product B1's price and choose product A1's quality enhancement level equal to product B1's level. Likewise, firm A2 will choose the same price and level of quality enhancement as firm B2. This can be seen in both the no-bundling case and the mixed-bundling case, i.e. $P_{A2\ N0}^* = P_{B2\ N0}^*$, $\beta_{A2\ N0}^* = \beta_{B2\ N0}^*$, $P_{A2\ Mixed}^* = P_{B2\ Mixed}^*$, $\beta_{A2 \text{ Mixed}}^* = \beta_{B2 \text{ Mixed}}^*$. One may rearrange (4.837) with these assumptions and obtain the following.

$$
dCS < 0 \text{ if}
$$

\n
$$
(P_{A2\ N0}^* - C_A \beta_{A2\ N0}^*) - (P_{A2\ Nixed}^* - C_A \beta_{A2\ Nixed}^*) > -\frac{T_3}{T_1 + T_2}
$$
 (4. A38)

where

$$
T_1 = \frac{1}{2\theta t} \left\{ Z_1 A \frac{\left(\frac{\partial CS}{\partial \beta_{A1}}\right)}{2b_A} - Z_4 A \frac{\left(\frac{\partial CS}{\partial \beta_{A2}}\right)}{2b_A} + Z_3 \frac{\left(\frac{\partial CS}{\partial \beta_{B1}}\right)}{2b_B} \right\},\,
$$

$$
T_2 = \frac{1}{2t} \left\{ Z_2 \frac{\left(\frac{\partial CS}{\partial \beta_{A1}}\right)}{2b_A} + Z_1 B \frac{\left(\frac{\partial CS}{\partial \beta_{B1}}\right)}{2b_B} - Z_4 B \frac{\left(\frac{\partial CS}{\partial \beta_{B2}}\right)}{2b_B} \right\},\,
$$

$$
T_3 = \frac{\left(\frac{\partial CS}{\partial \beta_{A1}}\right)}{2b_A} \cdot \left(\frac{\partial \pi_1}{\partial \beta_{A1}} \middle|_{\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6b_A}}\right) + \frac{\left(\frac{\partial CS}{\partial \beta_{B1}}\right)}{2b_B} \cdot \left(\frac{\partial \pi_1}{\partial \beta_{B1}} \middle|_{\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6b_A}}\right)
$$

$$
+ \frac{\left(\frac{\partial CS}{\partial \beta_{A2}}\right)}{2b_A} \cdot \left(\frac{\partial \pi_{A2}}{\partial \beta_{A2}} \middle|_{\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6b_A}}\right) + \frac{\left(\frac{\partial CS}{\partial \beta_{B2}}\right)}{2b_B} \cdot \left(\frac{\partial \pi_{B2}}{\partial \beta_{B2}} \middle|_{\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6b_A}}\right)
$$

$$
\frac{\left(\frac{\partial CS}{\partial \beta_{A2}}\right)}{\beta_{B1} = \beta_{B2} = \frac{a_B - c_B}{6b_B}} \cdot \left(\frac{\partial \pi_{B2}}{\partial \beta_{B2}} \middle|_{\beta_{A1} = \beta_{A2} = \frac{a_A - c_A}{6b_A}}\right)
$$

(4. A38) is true because $(P_{A2 N0}^* - C_A \beta_{A2 N0}^*) - (P_{A2 Mixed}^* - C_A \beta_{A2 Mixed}^*) > 0$ and $-\frac{T_3}{T_3}$ $\frac{1}{T_1+T_2}$ < 0 in the neighbourhood of $\delta = 0$. Therefore, in this setting of identical parameters, the mixed-bundling strategy decreases consumer surplus from the nobundling benchmark.

δ	T_3 $T_1 + T_2$
0.1	-0.03752
0.2	-0.05548
0.3	-0.06758
0.4	-0.07617

Table 4.2 The value of $-\frac{T_3}{T_1}$ $\frac{T_3}{T_1+T_2}$ when δ is fixed around zero⁷³

l

⁷³ This table shows the value of $-\frac{T_3}{T_1}$ $\frac{1}{T_1+T_2}$ when δ is fixed around zero under the assumptions of the markets with identical taste cost ($\theta = 1$) and the parameter values in Table 4.1.

Chapter 5

Conclusion

The telecommunications sector has been growing rapidly as a result of the revolution in telecommunications technology and consumer trends. However, entry into this industry is naturally suppressed by a requirement of enormous amounts of investment in infrastructure and operation, in addition to the spectrum allocation managed by the telecommunications regulator. Theoretically speaking, in the absence of collusion, symmetric firms are expected to compete strongly. It would be easier for the regulator to monitor the competition among symmetric firms. However, it can be seen in the real business world that firms are more likely to be different in terms of cost structure, reputation, consumer bases, the advantages of incumbency, and the scope of services. All of these aspects in the environment of asymmetry greatly influence the equilibrium market outcomes. A firm in a dominant position has a great tendency to impose an aggressive strategy to extract rent and to corner its underdog rivals by predatory practices.

The main objective of this thesis is to investigate the competition in the telecommunications sector in the context of asymmetric firms in order to point out the effects of firm asymmetry on the market outcomes and social welfare under various critical situations. Chapter 2 has studied the competition between two facility-based mobile network operators under different access charge regulations in the presence of firm asymmetry in cost and reputation. Further studying cooperation in the investment stage, Chapter 3 has examined the quality-enhancing effects of various approaches of infrastructure sharing, ranging from co-investment amongst competing firms to service-based entry, in a duopoly setting based on Cournot competition. In addition to competition in a single product market, Chapter 4 has developed the bigger picture to capture the competition between a multi-product firm and its single-product rivals in two duopolistic product markets with different degrees of competition intensity or product differentiation. The effects of the multi-product firm's bundling strategies on quality enhancement incentives and consumer welfare were explored in this chapter.

5.1 Significant findings and policy implications

As seen in the aforementioned theoretical studies and empirical evidence, interconnection among mobile network operators is deemed to be a critical aspect in market inefficiency, especially if firms are asymmetric. To suggest an appropriate regulatory regime in the setting of asymmetry in cost and reputation, Chapter 2 has drawn comparisons of three different pragmatic regulations on interconnection; (1) no regulation on access charge, (2) the symmetric cost-based access charge regulation and (3) the asymmetric cost-based access charge regulation. In the unregulated market, the low-cost firm is more likely to secure a dominant position and serve the majority of subscribers, despite earning an inferior reputation to the highcost firm, only if its reputation is not significantly worse. This is because the low-cost firm can offer a lower on-net price to attract consumers in accordance with marginal cost pricing. Additionally, similar to the main findings reported by Hoernig (2007), Lopez and Rey (2009), in the wholesale market, the dominant low-cost firm can set a higher mark-up on access charge to widen a gap of firm asymmetry and finally reap higher profit than its high-cost rival. Therefore, intervention by the regulator is necessary to protect consumer welfare and promote competition in this environment of asymmetry.

The symmetric regulation has an immediate effect on off-net prices, which are pushed down to actual marginal costs. After both firms' access mark-ups are eliminated, the low-cost firm can no longer undermine its high-cost rival by means of access charge pricing, but it still takes advantage of its low cost and/or its reputation to hold the dominant position in the market. Similarly, the asymmetric regulation is a highly effective way of facilitating entry, especially for a small entrant with high cost. In other words, there is an increased probability that the high-cost entrant can take over the dominant position from the low-cost established firm if it has a sufficiently better reputation than the established firm. However, the optimal regulation is ambiguous. The asymmetric regulation can generate higher social welfare if the difference in cost and the discrepancy in reputation are not too substantial and the two networks are differentiated enough. Otherwise, it damages social welfare because the new firm lacks efficiency and the established firm loses its profit more significantly. Thus, the implementation of the asymmetric regulation should be assessed carefully. It may be needed for the launch of a new mobile service to stimulate entry of the new network that may have higher cost and/or a worse reputation (unknown service) than the established firm. Under the asymmetric regulation, the high-cost firm should be a potential competitor in terms of cost, reputation and service differentiation to ensure a boost in social welfare. When the new network is gradually adopted by consumers and/or improves efficiency in cost, the regulator may reconsider imposing the symmetric regulation instead because the asymmetry between the two firms becomes subtle.

In addition to the complex relationship amongst competing firms in the pricing stage, Chapter 3 has shed light on the earlier stage, where firms make their decisions on investment. Infrastructure sharing in telecommunications was investigated under the key assumption that firms have different cost structures. This chapter has compared the impacts of various infrastructure-sharing approaches on incentives for quality enhancement in next generation network, including (1) stand-alone investment without infrastructure sharing, (2) co-investment, and (3) the fully-distributed-cost regulatory regime. Under stand-alone investment, the low-cost firm can offer higher levels of quality upgrade, firm output and retail price and earn higher profit than the high-cost firm due to its cost saving. However, the high-cost firm can employ other approaches of infrastructure sharing to enter the market instead of network duplication. Coinvestment can boost the profits of both firms, even though the high-cost firm with lower bargaining power agrees to invest in a larger proportion of total investment, earning lower profit than the low-cost firm. Compared to stand-alone investment, coinvestment may be considered to be collusive in quality upgrade and potentially results in a shrinkage of consumer bases and a decline in consumer welfare, when infrastructure sharing does not yield a sufficient amount of cost saving.

Access to infrastructure under the fully-distributed-cost regulation may be an alternative solution to the problem of little incentive to upgrade quality. However, compared to co-investment, this approach causes firms to shrink their consumer bases and raise corresponding prices instead. Overall, the fully-distributed-cost approach threatens consumer surplus despite the quality upgrade incentive.

Among these approaches in accordance with infrastructure sharing, the optimal approach depends on what is the highest priority from the regulator's perspective. Coinvestment causes lower incentive to upgrade quality, but it can expand the size of the aggregate subscriber group and yield higher aggregate consumer surplus than the fullydistributed-cost regulation. In this chapter, the comparison between quality upgrade and consumer surplus under co-investment and those under stand-alone investment is still ambiguous according to the degree of benefit from cost saving through infrastructure sharing. If the cost-saving benefit is substantial, service quality will be significantly enhanced with the efficient cost structure and consumer welfare will finally be improved. Therefore, it is suggested that the regulator should support negotiation on co-investment, only if infrastructure sharing can yield the substantial benefit of cost reduction. Otherwise, collusion in quality upgrade is likely to occur and stand-alone investment seems more appropriate in terms of consumer welfare.

In contrast to Chapter 2 and Chapter 3, where firm asymmetry was introduced in only one product market, Chapter 4 has extended to the competition in the broader context of asymmetry, involved with multiple product markets with different degrees of competition intensity. When bundling is not allowed, the multi-product firm is forced to compete with its single-product rivals in the two separate markets, resulting in the symmetric equilibrium outcomes of quality enhancement, price, market share and profit in each market.

However, the multi-product firm has strong incentives to employ some bundling strategies in order to reap higher profit. These strategies threaten its singleproduct rivals in almost all situations. With the pure-bundling strategy, the multiproduct firm offers more attractive quality enhancement, has larger market share, and accordingly earns greater profit than its rivals because it takes into consideration the spillover between the two tied markets, which its single-product rivals omit. Compared to the no-bundling case, the pure-bundling strategy dampens the singleproduct firms' incentives for quality enhancement in both markets. Meanwhile, due to the spillover, pure bundling can stimulate the multi-product firm's investment in quality enhancement, if the associated costs are comparatively low and the additional utility from quality enhancement is relatively high. When the two markets are significantly different in competition intensity, the outcome is surprising in that, in addition to the multi-product firm, the single-product firm in the more competitive market can also raise its price and increase profit as a result of a sharp decrease in competition intensity after pure bundling. Nevertheless, the single-product firm in the less competitive market undoubtedly decreases its price in response and loses profit because of the aggressive strategy.

With the mixed-bundling strategy, similar to the findings reported by Armstrong (2011) and Reisinger (2006), the multi-product firm can discriminate consumers by offering a bundle discount and charging higher individual prices than its single-product rivals, and it subsequently earns greater profit. Clearly, this strategy greatly influences the outcome in the more competitive market. The multi-product firm can expand market share by attracting the consumers who encounter its high individual prices by its superior product quality to its single-product rival. These findings have also been observed in the less competitive market under the condition that the two markets are not too different in competition intensity. Compared to the no-bundling case, in the more competitive market, this strategy certainly encourages the multi-product firm to improve quality and to expand its market share, but it undermines the single-product rival's incentive for quality enhancement. This outcome also unfolds in the less competitive market when the two markets are not too different in degree of competition.

The pure-bundling strategy is likely to threaten consumer welfare when the two tied markets are significantly different in terms of competition intensity. The regulators should not allow pure bundling if a tying market and a tied market are significantly different in competition intensity. However, the regulators may take into consideration a boost in quality enhancement through the pure-bundling strategy in the situation where the quality enhancement is cost efficient and the two markets are not too different in competition intensity. Additionally, the regulators should carefully consider the mixed-bundling strategy because its effect on consumer surplus is also ambiguous. Mixed bundling may harm consumer welfare if there is a sharp distinction in degrees of competition in the two markets. In this situation, mixed bundling distorts the more competitive market so significantly that consumer surplus is destroyed by the increasing disutility as a consequence of deviating from the originally preferred products. Otherwise, in addition to an increase in consumer welfare, it is interesting to note that mixed bundling may stimulate quality enhancement by the multi-product firm, especially in the more competitive market.

In conclusion, as observed in the three different frameworks of firm asymmetry, a dominant firm may employ a predatory strategy that puts its rivals at a disadvantage. The dominance usually results from lower cost structure, better reputation and/or wider scope of services. In addition to market foreclosure, incentive for quality enhancement is one of the major concerns. Most predatory strategies adopted by a dominant firm are more likely to damage consumer welfare. The regulator should carefully monitor the dominant firm's behaviour under certain circumstances, which inevitably involves making trade-offs between static efficiency and dynamic efficiency. From a dynamic viewpoint, this includes a boost in competition intensity and quality enhancement.

In the context of interconnection in mobile telecommunications, regulations on access charge are still necessary to eliminate the aforementioned price distortions. Compared to the symmetric cost-based access charge regulation, the asymmetric regulation is more effective in encouraging facility-based entry, but it may not be optimal in the situation in which the high-cost entrant is too inefficient in terms of cost, reputation and service differentiation. Moreover, with concern over infrastructure sharing, this thesis has strongly supported that co-investment yields higher consumer surplus than the other infrastructure-sharing approaches, including the fully-distributedcost regulation and the unregulated access price approach. However, it is argued that compared to stand-alone investment, co-investment is likely to provoke collusion to suppress quality enhancement, consequently dampening consumer welfare, when cost saving from infrastructure sharing is negligible. Finally, in the setting of multiple product markets, the pure-bundling strategy and the mixed-bundling strategy probably reduce consumer surplus when the less competitive market is very distinct from the more competitive market in terms of competition intensity or product differentiation. Thus, the regulator should keep a close eye on these strategies in various competitive situations because their far-reaching effects on welfare vary depending on firm asymmetry in cost structure, reputation and the competitive natures of the markets.

5.2 Limitations and further research

To systematically analyse the asymmetric firms' behaviour, it is necessary to model the competition based on some reasonable assumptions and limitations. These assumptions are made to highlight the far-reaching effects of firm asymmetry in critical situations.

Firstly, on the demand side, this thesis assumes specific functional forms of demand. In order to simplify the models, it is acceptable to set up a specific demand function and a utility function to capture how consumers perceive products or services in a particular way. Further research may attain a higher level of generality by assuming general functions. In addition, in Chapter 3 and Chapter 4, the assumption of unit demand simplifies the models for telecommunications services in that a consumer normally subscribes to only one service provider. This assumption may be reconsidered in further research when it plays a pivotal part in adapting models for other product markets with different consumption patterns.

Secondly, on the supply side, this study investigates the market outcomes in the setting of firm asymmetry based on perfect information on the types of the firms. For example, it is assumed that the firms' cost structures are not private information. The regulator can distinguish the low-cost firm from the high-cost firm. Likewise, the competitors and consumers can correctly perceive cost structures and reputation. In the telecommunications markets, the assumption about complete information is reasonable. Service providers in several countries are legally obliged to report their cost structures or reveal some relevant information in order that the regulator, the public and other service providers can deduce information about cost. However, it is interesting to conduct research on the assumption that information on cost structure is private and incomplete.

Thirdly, the timing of the games in this thesis is mainly based on multi-stage games where firms choose their strategies of prices and/or quality enhancement simultaneously in each stage. It is difficult to ensure that the competition in the real world follows a one-shot game. Firms may adjust their prices or levels of quality enhancement frequently. Additionally, firms may cooperate in price setting. From a dynamic perspective, entry into the market may occur in the form of a repeated game. Nevertheless, these advanced versions of extensive-form games are too complex for the purpose of this study. To clearly interpret the main findings, these assumptions are still necessary for this thesis.

Lastly, the models are designed on the basis of duopolistic competition. There are only two asymmetric firms in each market. Future research may extend to the competition among multiple firms. However, the duopolistic models are reasonable in the telecommunications markets because it can be seen in the real business world that only a few service providers operate as a result of a fundamental requirement of largescale investment.

Despite these simplistic assumptions and other limitations, this study is useful in that it can point out the results and the suggestions for the second-best solutions when the first-best practice is impossible. It is still necessary to base these studies on some assumptions in order to focus on the key variables and their impacts on the equilibrium market outcomes. The regulators should monitor the competition in the telecommunications sector with regard to the concerns mentioned in this thesis. The regulators can derive the optimal policies and the appropriate regulations in certain circumstances from the suggestions in this thesis.

Further research may investigate the competition among asymmetric firms regarding other interesting issues. For instance, telecommunications spectrum allocation and licensing is one of the major concerns in telecommunications. Underdog firms are

likely to be unsuccessful in auction. Incumbents or big firms are more likely to win the auction than other bidders. In other words, they can transfer their dominance to the next generation telecommunications markets. The auction may be a process of allocating spectrum to the most efficient players. Nevertheless, there are some concerns about collusion and auction design. Future research should support the regulators in designing appropriate mechanisms and imposing optimal regulations/deregulations in the telecommunications industry, which has changed in line with the advancement of technology and consumer trends.

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