

A computer model of drafting effects on collective behavior in elite 10,000 m runners

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Title: A computer model of drafting effects on collective behavior in elite 10,000 m runners Submission Type: Original Investigation Authors and Affiliations: Hugh Trenchard¹, Andrew Renfree², Derek M. Peters^{2,3} ¹805 647 Michigan Street, Victoria, British Columbia, Canada V8V 1S9 Tel: +1 250 472 0718 ²Institute of Sport & Exercise Science, University of Worcester, Henwick Grove, Worcester, United Kingdom, WR2 6AJ ³Faculty of Health & Sport Sciences, University of Agder, Kristiansand, Norway Corresponding Author: Andrew Renfree, Institute of Sport & Exercise Science, University of Worcester, Henwick Grove, Worcester, United Kingdom, WR2 6AJ Tel: +44 (0)1905 855376 Fax: +44 (0)1905 855132 Email: a.renfree@worc.ac.uk Preferred Running Head: Modelling collective behavior in 10,000 m running Abstract Word Count: 246 Text-Only Word Count: 3576 Number of Figures: 5

47 Abstract

48 <mark>Purpose</mark>

- 49 Drafting in cycling influences collective behaviour of pelotons. Whilst evidence for collective
- 50 behaviour in competitive running events exists, it is not clear if this results from energetic
- 51 savings conferred by drafting. This study modelled the effects of drafting on behavior in elite
- 52 **10,000 m runners.**

53 <mark>Methods</mark>

- 54 Using performance data from a men's elite 10,000 m track running event, computer simulations
- 55 were constructed using Netlogo 5.1 to test the effects of three different drafting quantities on
- 56 collective behaviour: no drafting, drafting to 3m behind with up to ~8% energy savings (a
- 57 realistic running draft); and drafting up to 3m behind with up to 38% energy savings (a realistic
- 58 cycling draft). Three measures of collective behaviour were analysed in each condition; mean
- 59 speed, mean group stretch (distance between first and last placed runner), and Runner
- 60 Convergence Ratio (RCR) which represents the degree of drafting benefit obtained by the
- 61 follower in a pair of coupled runners.

62 **Results**

- 63 Mean speeds were 6.32 ± 0.28 m.s⁻¹, 5.57 ± 0.18 m.s⁻¹, and 5.51 ± 0.13 m.s⁻¹ in the cycling draft,
- ⁶⁴ runner draft, and no draft conditions respectively (all *P*<0.001). RCR was lower in the cycling

draft condition, but did not differ between the other two. Mean stretch did not differ between

66 conditions.

67 **Conclusions**

- Collective behaviours observed in running events cannot be fully explained through energetic
 savings conferred by realistic drafting benefits. They may therefore result from other, possibly
 psychological, processes. The benefits or otherwise of engaging in such behavior are, as yet,
- 71 unclear.
- 72
- 73 <mark>Keywords</mark>
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- 75 Pacing, Endurance, Running, Modelling
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81 Introduction

82

Research has explored the mechanisms through which 'pacing', which reflects the strategy for 83 expending effort during athletic contests¹, is regulated. Whilst much of this work has focussed on 84 internal regulatory processes, including the role of the momentary Rating of Perceived Exertion 85 $(RPE)^2$, the Hazard Score³, and emotion^{4,5}, two recent reviews^{6,7} have suggested that regulation 86 87 is achieved through a continual process of decision-making. A key feature of these decision-88 making processes is that choices are made based on interpretation of data of either internal or external origin, which are 'perceived' to require a particular decision to be made and a course of 89 action taken at that moment in time. Indeed, Smits et al.⁷ have identified that in order to explain 90 91 athletic pacing decisions it may well be necessary to adopt an ecological approach that enhances understanding of how perception and action are coupled in determining behavior. Given that 92 athletes often compete in direct proximity to one another without separation due to individual 93 lane allocations, it is interesting that relatively few authors have explored the nature of 94 95 interactions between competitors in endurance athletic events, and their influence on pacing behaviors. 96

97

98 Different pacing strategies have been shown in elite female marathon runners resulting in

athletes achieving different absolute performance levels⁸, whereby slower athletes adopted

similar starting speeds to the faster athletes who finished in the leading positions. These overly

ambitious starting speeds resulted in progressive deceleration throughout the race, and overall
 race pacing profiles characterized by a 'positive split', whereby the second half of the race was

run more slowly than the first. Although similar findings are evident in elite male athletes at the

104 World Cross Country Championships^{9,10}, it is not clear why runners of differing performance

ability tend to adopt similar starting speeds. It may be evidence for a human tendency towards

106 collective behavior influencing pacing decisions, as in complex decision-making environments

the easiest decision is simply to do the same as everybody $else^{11}$, which may explain behaviors in

108 other human environments including pedestrian interactions¹² and market trading¹³. Although the

109 precise mechanisms underlying these behaviors are not fully understood, such complex

biological systems may well result from individual agents following simple rules governing the nature of their interactions with others¹⁴.

112

Among pelotons evidence indicates collective behavior self-organizes from cyclists' local 113 interactions. Pelotons are groups of cyclists coupled by the energy-savings of drafting¹⁵, and may 114 include as many as 200 individuals. Trenchard¹⁵ found that pelotons exhibit protocooperative 115 behavior, which emerges as a function of cyclists' capacity to share the most costly front 116 positions where aerodynamic drag is highest. As speeds vary, three main collective conditions 117 emerge: when speeds are low relative to the cyclists' maximal sustainable outputs (MSO), 118 individuals naturally cooperate by sharing the metabolically more costly front positions. In this 119 condition pelotons are compact and roughly circular in shape. As speeds increase eventually the 120 protocooperative threshold is reached whereby weaker cyclists are unable to share the costly 121 front-positions, and must maintain drafting positions to sustain the speed of the leading riders. In 122 this condition pelotons are single-file formation, and highly stretched. At yet higher speeds, 123 when weaker cyclists are unable to keep up with stronger cyclists even by drafting, a second 124 threshold is reached as cyclists decouple and form smaller sub-groups. Both protocooperative 125 126 and decoupling thresholds depend on the differentials between MSOs of the weaker and stronger

- riders, and the drafting quantity (which may be zero). Therefore, higher drafting quantities
- 128 permit greater MSO differential before either threshold is reached. A key prediction of
- 129 protocooperative behavior is that groups tend to sort so that the MSO variation range among the
- 130 group of cyclists approximately corresponds to the energy savings of drafting.
- 131

132 In running events the energetic savings from drafting are smaller due to the lower speeds

- achieved¹⁶⁻¹⁸, and the nature of any resulting protocooperative behavior is therefore largely
- unknown. Whilst Hanley¹⁹ has demonstrated that competitors in the World Half Marathon
- championships often form groups, and those athletes who run in groups throughout tend to
- display a greater 'endspurt' in the final stages, the reasons for this are unclear. It is plausible that
- this could result from the athletes achieving speeds whereby there *is* some energetic benefit from $\frac{20}{10}$
- drafting behavior²⁰, or because of a reduced cognitive load due to a reduction in the need to make continuous pacing decisions²¹, or some combination of the two.
- 140
- 141 Our aim therefore was to model collective behavior of a group of elite distance runners during
- 142 competition in order to determine the degree to which collective behavior may be influenced by
- energetic savings incurred through drafting. We hypothesized that models would suggest drafting
- benefits will influence collective behavior during a 10,000 m running race.
- 145

146 Method

- 147 A quasi-experimental design was used to address the aim of the study which had received prior
- 148 ethical approval from the University of Worcester. Final results and official split times
- (individual 100m segments) of all starters (n=32) in the Men's 10,000 m event at the 2013 IAAF
- 150 World Championships were accessed via the championship website
- 151 (http://media.aws.iaaf.org/competitiondocuments/pdf/4873/AT-10K-M-f--1--.RS7.pdf?v=-
- 152 <u>1733122098</u>) along with seasons best (SB) performances for all competitors. This event was
- selected because of the relatively homogenous performance characteristics of the competitors,
- and the high frequency of timing data available.
- 155
- 156 To analyze collective running dynamics, we adapted the modified 15 peloton simulation originally
- developed by Trenchard et al.²² This model incorporates maximal sustainable output (MSO)
- thresholds whereby cyclists decelerate when MSOs are exceeded relative to a pacesetter; and
- build upon Ratemero's peloton model²³ and flocking dynamics whereby group mean x and y
- 160 coordinate positions generate cohesion and separation parameters²³. Simulations were performed
- using Netlogo 5.1, a multi-agent computer modelling platform²⁴. The adapted runner model
- 162 involved simple modifications to the peloton threshold equations²², as follows:
- 163
- 164

- 166
- Where "RCR" is the "runner convergence ratio", describing two coupled runners whereby the
- 169 leader sets the pace and the follower may obtain a drafting benefit. If there is drafting quantity,
- 170 RCR reduces accordingly, and if there is no drafting quantity, RCR is simply a ratio of the
- 171 pacesetter's speed to the follower's MSO;
- 172

 $RCR = \underline{S_{front}}^{d}_{*}MSO_{follow} \quad (1)$

"S_{front}" is the front runner's speed, "MSO_{follow}" is the follower's MSO in terms of speed (m/s) 173 (for the purposes of this study we utilised the athletes SB times as representing MSO); and "d" is 174 the drafting coefficient obtained from: 175 176 $d = 0.62 - 0.0104d_w + 0.0104d_w + 0.0452d_w^2$ (2)177 178 Where d_w is distance between rear wheel of front rider, and front wheel of drafting rider in 179 180 meters. 181 Equation (2) was developed by Olds²⁵ using Kyle's published data¹⁶, which indicated energy 182 savings of up to approximately 38% in cyclists, depending on wheel spacing. Whilst this 183 equation does not reflect realistic drafting advantage for runners, we used it here as one of three 184 drafting quantities to test the effects of drafting on collective running dynamics. If wheel spacing 185 is 3m or greater, d is assumed to be 1 (no drafting benefit)²⁶. 186 187 For runners, since the speeds are considerably slower than in cycling, the drafting benefit (1-d)188 is smaller. Kyle¹⁶ found a 4% reduction in VO₂ at 6 m.s⁻¹ when drafting at 1m; Pugh¹⁷ found a 6.5% reduction at 4.5 m.s⁻¹ with a wind velocity of 6 m.s⁻¹ when drafting at 1m. Similarly, 189 190 Davies¹⁹ found 4% reduction at 6 m.s⁻¹ and 2% at 5 m.s⁻¹. 191 192 Further, applying empirical drafting quantities again reported in cycling by McCole et al.²⁶ we 193 derived the following regression equation: 194 195 $d' = -0.036 * s_{\text{front}} + 1.14$ (3)196 197 Where "S_{front"} is the speed in m.s⁻¹, d' is approximately 0.92 (8% reduction in metabolic 198 requirement), which is consistent with both the high end of the range of empirical findings noted, 199 and the actual mean speed of the runners in the Moscow $10,000 \text{ m} (5.98 \text{ m.s}^{-1})$. 200 201 Equation (3) is similar to (2) except d' is constant (0.92), whilst d varies according to distance up to $3m^*$. The empirical data¹⁶⁻¹⁸ does not clearly indicate whether drafting abruptly drops to zero 202 203 at 1m for runners, or whether it tails off up to 3m, as the evidence indicates for cyclists. Here we 204 err on the side of greater drafting benefit for runners to obtain clearer evidence of any effect that 205 drafting might have on collective running behavior. We infer negligible drafting benefit at 206 angles, but allow a 15 degree "comet's tail"²⁶ drafting effect to runners' sides, and zero at greater 207 angles. 208 209 Further, to obtain the drafting quantities for runners whereby drafting benefit decreases with 210 distance between runners, we applied the equation: 211 212 $d = 0.92 - 2.667 \times 10^{-3} dw + 3.667 \times 10^{-3} dw^{3}$ (4) 213 214 Thus if RCR > 1 for two runners, the follower cannot sustain the speed set by the leader and 215 must decelerate to a speed less than or equal to the speed equivalent to that runner's MSO, as 216 shown in the following equations, as adapted from Trenchard et al.²²: 217 218

First obtain the front runner's speed in excess of RCR = 1: 219 220 $Speed_e = (MSO * RCR) d$ 221 (5) 222 223 Where "Speed_e" is then the speed set by the leading runner in excess of the following runner's 224 possible speed at MSO. 225 226 Then obtain the speed for the following runner at MSO: 227 228 $Speed_{td} = MSO / d$ 229 (6)230 Where "Speed_{td}" is a runner's speed at his MSO, given the possible increase in speed facilitated 231 by the drafting benefit (if any). To obtain a runner's required speed reduction in order to resume 232 running at MSO, find the difference between $Speed_e$ and $Speed_{td}$: 233 234 $Speed_r = Speed_e - Speed_{td}$ 235 (7)236 If a runner incurs additional metabolic disruption as a result of the speed exceeding the metabolic 237 cost of running at their MSO, fatigue would be expected to induce decelerations to a speed below 238 his MSO, and not to a speed equivalent to MSO. To model this, we applied an additional random 239 deceleration factor: 240 241 Speed'_r = Speed_e - Speed_{td} + Δ s 242 (8)243 Where "Speed'r" is the final speed due to deceleration, where Δ is the noted small positive 244 random individual deceleration quantity. A relatively small random acceleration was generated 245 by adding a random quantity to the cohesion parameter noted earlier. 246 247 With these model adaptations, to test the effect of drafting on runners' collective dynamics, we 248 conducted 30 simulation trials for each of three experimental drafting quantities: 249 250 No drafting benefit ("no draft condition"). 251 1. 2. Drafting benefit up to 3 m behind other runners within a 15 degree cone centred around 252 the current heading of the runner ahead, using equation (3) ("runner draft condition"). 253 Drafting benefit up to 3 m behind other runners within a 15 degree cone, centred around 254 3. the current heading of the runner ahead using equation (2) ("cyclist draft condition"). 255 256 Simulation duration was 27:21.6 (1642 s) the fastest finishing time in the race. Accumulated 257 times for runners who were first at each 100 m were used as pacesetter splits for each 100 m 258 interval, converted to speeds (m.s⁻¹), as shown in Figure 1. 259 260 ****Insert Figure 1 near here**** 261 262 263

Thus there were 100 pacesetter speeds during each of the simulation races, with these speeds taken as stable during each intervening 100 m. Across the 90 simulated trials, runners constantly adjusted their speeds, distances and positions relative to pacesetter speeds and varying draft conditions, according to equations (5-8).

268

269 **insert Figure 2 near here**

270

271 Unknown was the effect of drafting quantities on runners' RCRs, speeds, and stretch. The RCR indicates whether there is any available energetic resources that would allow for accelerations 272 (i.e. if RCR<1, runners have metabolic "room" to accelerate). Stretch is the distance (m) between 273 the front runner and the last runner; in the simulation stretch equals the maximum x-coordinate 274 minus the minimum x-coordinate in which an agent appears, scaled to meters, a value that 275 changes constantly. To analyze the data, we used Excel 97-2003 and NCSS 2007 for descriptive 276 277 statistics and ANOVA. Statistical significance was accepted at P<0.01 due to the comparatively large sample of data from 30 simulation trials for each variable where each simulation second 278 279 (1642 s per simulation) represents a data point, yielding 49,260 data points for each of nine variables (RCR, stretch, speed; multiplied by: no draft, runner draft, and cyclist draft). Effect 280

- size was calculated using Cohen's d^{27} as an additional statistical metric. We apply Cohen's
- classified effect sizes *small* (d = 0.2), *medium* (d = 0.5), and *large* ($d \ge 0.8$).²⁷
- 283

284 **Results**

285

The mean speed maintained in the cyclist draft condition $(6.32 \pm 0.28 \text{ m.s}^{-1}, 99\% \text{ CI} = 6.317, -$ 6.323) was higher than in the no draft (*P* < 0.001, *d* = 2.907) and runner draft (*P* = < 0.001, *d* = 2.686) conditions. Speed also differed between the no draft (5.51 ± 0.13 m.s⁻¹, 99% CI = 5.506,

289 5.509) and runner draft conditions (5.57 \pm 0.18 m.s⁻¹, 99% CI = 5.568, 5.572) (P<0.001, d =

0.3553) (Figure 3), where there is low to medium effect, but effect overall is very low relative to
 the effects of the draft condition on speed.

292 293

294 **Insert Figure 3 near here**

295 296

The RCR was lower in the cyclist draft condition $(0.88 \pm 0.06, 99\% \text{ CI} = 0.8822, 0.8835)$ than in the no draft (*P* < 0.001, *d* = 2.0989) and runner draft (*P* < 0.001, *d* = 2.0512) conditions, and large effect. There were no differences, and small effect, found between the no draft (1.00 ± 0.04, 99% CI = 1.0011, 1.0021) and runner draft conditions (1.00 ± 0.04, 99% CI = 0.9984, 0.9993) (*P* = 0.1098; *d* = 0.0668) (Figure 4).

- 302
- 303 **Insert Figure 4 near here**

304 305

There were no differences in mean stretch between any of the drafting conditions (Figure 5), whereby in the cyclist draft condition it was 158.71 ± 113.28 m (99% CI =157.39, 160.02), in the no draft condition it was 125.42 ± 68.81 m (99% CI =124.62, 126.22), and the runner draft

309 condition it was 146.99 ± 85.89 m (99% CI = 145.99, 147.99)

310

311 **Insert Figure 5 near here**

312 Discussion

313

Our study sought to model the impact of three different drafting conditions on athlete

performance and collective behavior in 90 computer simulated running races using the original

data from the Men's 10,000 m event at the 2013 IAAF World Championships. We hypothesized

that the potential energetic benefits resulting from effective drafting would be more apparent in

the cycling draft condition compared with the running draft condition and that both would be

- 319 better than the no draft condition.
- 320

The mean speed and RCR results (Figures 3 & 4) demonstrated a similar pattern in that runners were able to maintain greater mean speed and lower RCR in the cyclist draft condition compared

were able to maintain greater mean speed and lower RCR in the cyclist draft condition compare with both other conditions. There was no difference in RCR between the runner and no draft

with both other conditions. There was no difference in KCK between the runner drafting

324 conditions. Although the difference in speed between the no drafting and runner drafting

325 conditions achieved our threshold for accepting statistical significance, it should be noted that the

effect size was much smaller than between the other conditions. Our results suggest that the

327 previously documented energetic benefits achievable through drafting in cycling studies are

unlikely to be realised in running events. In the more realistic simulated running condition, there

329 were very small performance benefits realised in terms of mean speed, and RCR did not differ

from the no draft condition. No differences were found in mean stretch between any of the draft

conditions (Figure 5) indicating that the overall spread of the field of athletes (from first to last

position) was not influenced by either the speed of the race or the RCR.

333

Since our results indicate no significant effect of drafting on collective dynamics, there is no 334 evidence that drafting has any bearing on the finding that acceleration capacity near the end of a 335 race is greater in athletes who have run as part of a group throughout¹⁹. This is somewhat 336 inconsistent with two-runner models whereby running behind can be an optimal strategy due in 337 part to the drafting benefit²⁷⁻²⁸. These two-runner models ²⁷⁻²⁸ however, involve faster speeds and 338 correspondingly higher drafting benefit, and do not necessarily extend to larger numbers of 339 runners where cumulative drafting benefit may be attained from more than one runner directly in 340 front. This therefore suggests that this acceleration capacity at the end of a race results from 341 lower levels of cognitive fatigue resulting from a reduced requirement to make continual 342 decisions relating to muscular work rate^{20, 21}, at least in larger groups and at slightly lower 343 speeds. It also suggest that the influence of the behavior of other competitors may be greater than 344 the influence of afferent feedback on metabolic status in determining the work rate selected, at 345 least in the early stages of a race. Towards the end, increasing metabolic disruption will cause 346 347 slower runners to further reduce their speed, thereby resulting in incomplete realisation of performance potential⁸. 348

349

Furthermore, protocooperative behavior theory suggests that groups will tend to sort such that
 the MSO range among group members is approximately equivalent to the percentage energy
 savings from drafting¹⁵. In this study, the MSO range among the runners was 6.73% (max MSO
 - min MSO/ max MSO), which is within the expected percent energy savings from drafting. This

might suggest, speculatively, that the group has "pre-sorted" through earlier competitions, and

thus narrowed to an MSO range equivalent to the energy saved by drafting. This suggestion is

consistent with the work of Hanley¹⁹ who demonstrated that in elite runners group sorting tends
 to occur among competitors within a narrow range of similar ability.

358

359 One limitation of our study is that we did not analyse positional change, which is a feature of protocooperative behavior that generally occurs at comparatively low outputs¹⁵. Since drafting 360 attenuates metabolic cost, we would expect high frequency positional change where there is high 361 drafting quantity. Even without drafting, when speeds fall sufficiently relative to mean runners' 362 MSO, we would expect some positional change as runners compete for desired tactical positions. 363 Conversely, at high relative speeds, we would expect runners to reduce the number and 364 frequency of positional changes within the group. Future studies may involve more specific 365 analysis of durations for which certain positions are maintained. Again, analysis of sub-group 366 formations were not undertaken here, and future studies may involve analysis of the mean MSO 367 of sub-groups that form during the race. It should also be acknowledged that runners may have 368 deliberately adopted specific intermediate positions due to perceived tactical benefits. However, 369

detailed analysis of the effects of tactical positioning on finishing position is beyond the scope of
 this study.

372

The finding that the effects of (realistic) drafting on collective behavior is negligible would be

expected to be especially relevant amongst groups of competitors of a lower performance level

375 (or who compete in longer events) than were studied in this analysis. This suggests that where

there is virtually no drafting advantage, runners tend to sort into groups of even narrower ranges

of ability (i.e. runners sort into groups whose members possess nearly identical MSO). A
 potential limitation of this study is that we used athlete's season's best performances as

potential limitation of this study is that we used athlete's season's best performances as
 individuals MSOs. We acknowledge that these may not be truly representative of absolute

performance capacity because of to the relative infrequency at which track events of this distance

are contested, and the tactical nature of many of these races. Nevertheless, we consider using

seasons best to be more appropriate than all time personal record for this purpose due to the

potentially long periods of time between this race and the setting of the personal record.

384

Our results show there are differences between simulations comparing no drafting with an unrealistic cycling drafting quantity, but there are smaller benefits realised at a more realistic

running drafting quantity. If there were a greater benefit from drafting, the competitive MSO

range might be greater, and so the results are not inconsistent with protocooperative theory. Also,

since realistic drafting does not influence collective dynamics, collective dynamics would appear

to be determined by mechanisms other than potential or perceived energetic savings.

391

Conclusion: Simulations indicate that the comparatively low drafting benefit obtained by
 runners does not have substantial effects on collective behavior. We would expect to see

substantial differences in collective behavior only if the drafting benefit is considerably higher,

395 likely somewhere between the realistic drafting quantity (up to $\sim 8\%$ for runners) and the drafting

396 quantity that cyclists experience. This finding indicates that group pacing behaviors in runners

are not dominated by drafting, and that other (probably psychological) factors determine

398 observed pacing behaviors. The results of our study are not inconsistent with protocooperative

behavior theory which contends that group sorting tends to converge on the range of maximal

400 abilities that is approximately equivalent to the energy saved by drafting. One implication of this

401		no drafting, groups will eventually sort so that groups contain		
402	runners of nearly identical potential performance capacity.			
403		key finding that collective behaviors in runners, at least from		
404		explained through the energetic savings obtained by drafting has		
405		al applications. It would suggest that athlete decision-making is		
406	5 1	layed by other competitors and may well result in the selection of		
407		. Interventions designed to improve the quality of athlete decision-		
408	making may result in better u	tilisation of existing physiological resources and greater realisation		
409	of potential performance capa	acity.		
410				
411	Future research may involve	video and/or more fine-grained speed data for positional and stretch		
412	dynamics, which may provide	e further insights into runners' collective dynamics, pacing		
413		poperative behavior theory. This study involved analysis of		
414	performance data from a sing	the elite championship 10,000 m race whereby reward is associated		
415		time achieved. It is not clear if similar results would be found in an		
416		a less homogenous sample of athletes, or in events of different		
417		as to whether deliberate engagement in collective race behaviors		
418		savings from drafting, reduce cognitive load, or both, is likely to be		
419		terms of maximising performance potential than would be selection		
420		gy that is typically considered optimal in events of this duration.		
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583	Captions for Figures
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586	Figure 1: Individual competitors' speeds with moving average of winner.
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589	Figure 2: Point in time from typical simulation trial showing individual maximal sustainable
590	speeds converted from each runner's season best 10,000 m times; group stretch is distance (m)
591	from first to last runner.
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594	Figure 3: Mean speeds in simulated races in three different drafting conditions (* <i>P</i> <0.01).
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597	Figure 4: Runner Convergence Ratio in simulated races in three different drafting conditions
598	(* <i>P</i> <0.01).
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601	Figure 5: Mean stretch at each 100 m point in three different drafting conditions
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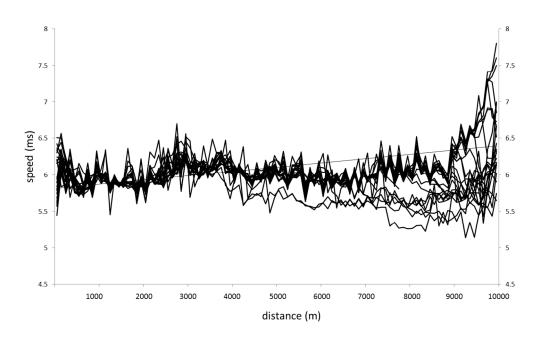


Figure 1: Individual competitors' speeds with moving average of winner. 355x215mm (300 x 300 DPI)



Figure 2: Point in time from typical simulation trial showing individual maximal sustainable speeds converted from each runner's season best 10,000 m times; group stretch is distance (m) from first to last runner. 262x109mm (96 x 96 DPI)

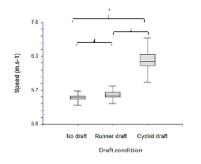


Figure 3: Mean speeds in simulated races in three different drafting conditions (*P<0.01). 359x152mm (96 x 96 DPI)



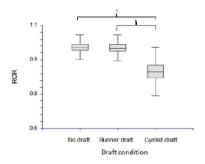


Figure 4: Runner Convergence Ratio in simulated races in three different drafting conditions (*P<0.01). 361x203mm (96 x 96 DPI)

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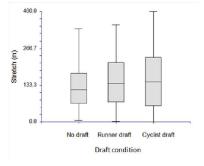


Figure 5: Mean stretch at each 100 m point in three different drafting conditions 359x152mm (96 x 96 DPI)

