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Literature Research in Relevant Fields to Understand Pressure Relief Valve Leak Tightness in a Static Closed State

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Abstract

Currently, no review of literature exists which attempts to understand the leakage phenomenon of metal-to-metal seal contact Pressure Relief Valves (PRV) for static closed positions as they reach the set pressure point. This paper attempts to do just that by drawing on inspiration from other research areas such as: metal-to-metal contact and gasket seals. The key topics of interest surrounding the leakage of fluid through a gap are: fluid flow assumptions; surface characteristics and its deformation; and experimental techniques used to quantify leakage. The fluid flow assumptions relating to the gap height such as transmissivity and diffusivity are found to be directly linked to the surface roughness and the surfaces deformations. Traditionally the summing method has been used to represent two rough surfaces at a micro scale from which the Tsukizoe and Hisakado theory has been applied for deformation of the micro contact in a plastic manner. The path the fluid also takes through the gap is investigated with recent work using computational methods to determine that path. Current experimental leakage quantification techniques are also discussed. Finally, the future development of PRV static leakage is examined.

Peer-review under responsibility of the organizing committee of ICPVT-14

Keywords: Contact; Surface characteristics; Metal-to-metal seal; leakage; surface deformation; Safety valve.

1. Introduction

There are many commercially available valves which perform different functions. Essentially valves can function as isolators, diverters, flow reversal prevention and reduce pressures within a service system. The service characteristics range from fluid type, fluid characteristics, pressure, temperature, chemical resistance and finally

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operational and maintenance requirements [1]. Appropriate valve selection is dependent on complete knowledge of the required function and the service characteristics.

Nomenclature

h	parallel gap height	q_d	Fick's law (volumetric flow rate) per unit width
R	gas constant	Δp	change in fluid pressure
T	temperature	∇c	mass fraction of species
P_0	inlet Pressure	$\boldsymbol{D}$	diffusivity tensor
P_1	outlet pressure	$\mathcal{D}$	molecular diffusion coefficient
Kn_2	Knudsen number at outlet	μ	viscosity
$\boldsymbol{K}$	transmissivity tensor	ρ	density
q_v	Poiseulle Law (volumetric flow rate) per unit width	σ	accommodation coefficient

As the operating pressure within a valve reaches the set pressure, the sensitivity off the valve opening prior to reaching an equilibrium (set pressure = operating pressure) increases. To be able to seal the valve up to 90% of the set pressure and higher requires research into the leak tightness of the valve.

When choosing a valve it has been recommended to factor in the leakage since leak tightness has a direct effect on the operational and maintenance requirements [2]. Depending on the service, especially if the fluid is hazardous to humans or the environment, the leak tightness is of the highest interest. Regardless, for any service characteristic and function the leak tightness will have a direct effect on the overall pressure of the system, therefore the leak tightness is equitable to a direct cost in operations and maintenance.

This paper attempts to try and identify the current understanding and technological knowledge of leak tightness of metal-to-metal contact valves in particular Pressure Relief Valves (PRV). Within a PRV the seat and disc would be in contact and would constrain the fluid (as shown in Fig. 1). Research in this field in direct relation to valves is scarce. However, inspiration can be drawn from relevant fields such as: metal-to-metal contact and gasket seals. When these contacting surfaces come in contact in parallel to each other a finite gap or path is present which is dictated by the surface finish such as; profile, roughness, waviness, flatness etc. Subsequently if there is a driving internal pressure the fluid can pass through the path and exit the valve. It has been stated that the minimum leakage rate which can be obtained for one time seal applications is 'less that 10-8 atmospheric cc/sec of helium' [3].

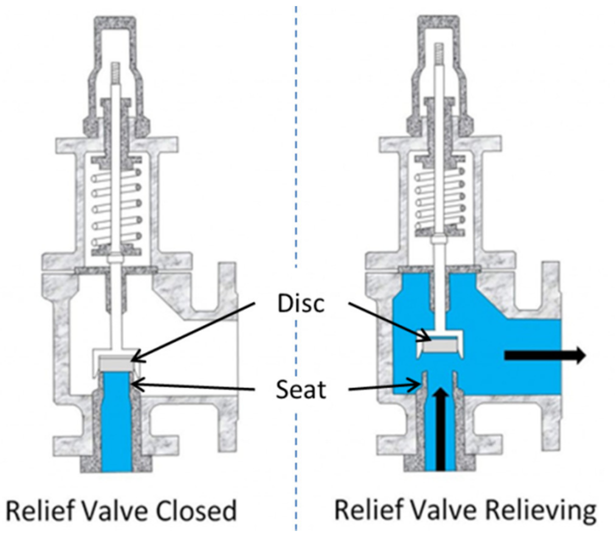

Fig. 1. PRV closed (static) and open (relieving). Blue region represents the fluid [4]

Previous work suggests that the leakage rate is either viscous laminar flow, molecular flow or, in the intermediate or transitional regime, or a mix of both. Assuming the non-contact area through the gap to be a finite length and the fluid to be isothermal and viscous compressible/incompressible, Poiseuille flow equations are utilized either for a circular cross section [5] or parallel plates. More recent work shows the development of fluid flow due to diffusion flux based on high to low concentration regions through the contacting surfaces using Fick's Law for incompressible isothermal fluids. The lineic flow rate characteristics which determines the gap height for both Posieuille flow and diffusion are generally put into two tensors; transmissivity (*K*) and diffusivity (*D*) [5, 6, 9-12].

The true area of actual contact between two parallel faces is only a small fraction of the nominal area due to the aperture. To describe the path the fluid would take, the chaotic surface must be characterised. Methods to characterise the surface have been created by many authors such as the MOTIF procedure, self-affine fractal analysis and simplistic geometry. Recently this has been adopted into leak tightness projects to determine its effect on the gap height [9-12] & [14-18].

The contact area is dependent upon the aperture of both surfaces in contact and the deformation magnitude is dependent upon the normal load applied and the effective surface hardness of the softer of the two materials. Depending upon the loading the surface aperture can become plastic in areas while the whole structure remains predominantly elastic. Taking this into consideration, the flow path will change depending on the load. Attempts have been made to analytically and computationally describe the surface roughness, elastic (Herts Theory [18]), elastic-perfectly plastic [21] and perfectly plastic rigid (Tsukizoe and Hisakado [12] deformation and understand its effect on the gap height or aperture field. [9, 12, 14, 18-20].

To verify the leakage rate experimentally British, ASME and API, standards can be used. Recent experimental work by Haruyama *et al*. [21], and Geoffroy and Prat [22] has shown promising methods of detecting and quantifying leakage and its link to the transmisimivty and diffusivity tensors K and D.

More specific research into PRV leakage which detracts from surface finish has been conducted by Ritchie [23] which examines the effect of misalignment of the valve and its effect on pressure drop.

Computationally to model a whole seat and disc of a PRV with the surface roughness and waviness would be possible, but intensive and other techniques such as multi-scale modelling [29, 30] could be considered instead.

Here introduce the paper, and put a nomenclature if necessary, in a box with the same font size as the rest of the paper. The paragraphs continue from here and are only separated by headings, subheadings, images and formulae. The section headings are arranged by numbers, bold and 10 pt. Here follows further instructions for authors.

2. Leakage: fluid flow assumptions

Depending on the service characteristics the fluid type and characteristics such as pressure, temperature and contaminants will vary. Generally high performance valves will be capable of withstanding compressible and incompressible Newtonian fluids which range from -196 ^{0}C to 540 ^{0}C and pressurised up to 440 bar [6, 8].

Assuming the valve has not opened and the seat and disc is in contact, only the surface deformations at a micro scale form an aperture field through which the fluid can escape out with the PRV.

In previous studies [5, 6, 9-12] on gaskets, valves and metal surface contacts, the fluid flux has been assumed to be laminar and either; viscous isothermal compressible/incompressible and equated using Poiseulle Law q_v (volumetric flow rate) per unit width in Eq. (1), or; diffusive, equated using Fick's law q_d (volumetric flow rate) per unit width in Eq. (2), as shown below:

$$q_v = -\frac{\boldsymbol{K}}{\mu}\Delta p \tag{1}$$

$$q_d = -\rho\mathcal{D}\boldsymbol{D}\nabla c \tag{2}$$

where μ is the viscosity, p is the fluid pressure, ρ is the density, $\mathcal{D}$ is the molecular diffusion coefficient and c is the mass fraction of the species. $\boldsymbol{K}$ and $\boldsymbol{D}$ are respectively the transmissivity and diffusivity tensors. These allow the lineic flow rate characteristics to be described at the scale of the surface. Based on a parallel gap with a height of h these terms can be described as:

$$\boldsymbol{K} = \frac{h^3}{12} \tag{3}$$

$$\boldsymbol{D} = h \tag{4}$$

Majority of authors have adopted a parallel gap [9-12] rather than a circular cross section [5]. This generalisation is more appropriate since the diameter of flow path does not need to be known, rather the separation between the contact surfaces is adequate. Also this is only applicable to asperities with local small slopes ($h \ll 1$ or $< 10^0$) [11]. The connection between the transmissivity and diffusivity has been theoretically analysed by Geoffroy and Prat [12] and they conclude that the dependence of the fluid transition in either a transmissivity or diffusivity form is defined by both the gap size and applied load. The caveat with this theory is that a uniform gap height, surface form, waviness and roughness are present with a flat surface deforming it.

Depending on the rarefaction of the fluid, it could also be in the transition or slip flow regime [5, 6]. This is likely since the surface roughness is at a micro-scale and therefore the gap height is also likely to be similar. This can be verified by calculating the Knudsen number and has been accounted for by Gorash *et al.* [13] in an analytical model which is an extension of the Poiseuille's Law for fluid flux through a parallel gap written as:

$$q = \frac{h^3}{24\mu RT}\left[P_0^2 - P_1^2 + 12\frac{2-\sigma}{\sigma}Kn_2 P_1\left(P_0 - P_1\right)\right]\left|\frac{P_0^2 - P_1^2}{P_1}\right| \tag{5}$$

where σ is the accommodation coefficient, R is the gas constant, T is the temperature and Kn_2 is the Knudsen number at the outlet. The second more subtle assumption here is that the micro fluid flow is based on the outlet not the inlet.

3. Representation of surface roughness at micro-scale and its effect on fluid flow path

Micro-scale geometry of a surface profile is chaotic in nature making it important to be able to represent the area in an effective form since this has a direct effect on the gap height, h. There are methods available which make it possible to represent the surface roughness of a model at a micro-scale level. These can be in the form of generic surfaces such as a sinusoidal waves [12] or vibrational Eigen modes [10] or wedges [9]. There are more analytical methods based on the surface available such as the sum surface [14], MOTIF procedure [15] and fractal analysis [11, 16, 17] which all have benefits and limitations. The benefits and limitations are discussed below and its effect on the gap size.

The sum surface allows the direct analysis of two contacting surfaces. This technique is used by many authors and based upon a theory created by Tsukizoe and Hisakado [18], which states, it can be assumed that, 'the contact between two rough surfaces can be regarded as the equivalent to the contact between an imaginary rough surface having an appropriate effective topography and a perfectly flat surface' and 'the contacting asperities deform in an ideal plastic manner so that, providing no interference from neighbouring asperities occurs and that work hardening does not take place' as that shown in Fig. 2. Essentially the flat surface is assumed rigid perfectly-plastic surface. These are accepted and applied assumptions in references [9, 12, 14]. The main limitation with the sum surface technique is that the actual surface has to be measured physically. A further limitation is that representation of the surface is generally in a 2D format and an 'effective' gap size, h, has to be calculated and is generally based on an average values as demonstrated by [9].

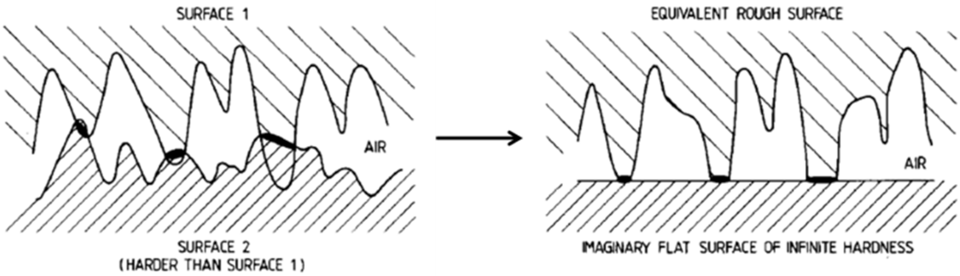

Fig. 2. Sum surface technique- generation of an equivalent rough surface. Black areas indicate micro-contacts. [9]

More recently, the surface roughness of both lapped and sand blasted surfaces has been represented as self-affine fractal surfaces combined with the sum surface technique contacting a rigid-perfectly plastic flat surface [11]. By using fractal analysis the self-affine fractal surface is based on two parameters and is created using a power law. Using this technique the surface roughness can be modelled in 3D, representing the whole aperture field. 2D representation can be created using this technique but an 'effective gap size', h, would have to be employed when calculating leakage rate. Vallet *et al.* show good agreement when comparing the fractal surface representation to the 'real' surfaces for the lapped surfaces. This shows that for lapped surfaces self-affine fractal surfaces can be used to generically replicate surfaces and represent the aperture field. The main limitation of this method is that the mathematics is intensive and requires the use of computational programs such as MATLAB. By modelling the whole aperture field all possible fluid flow paths can be represented and more accurate gap sizes through the valley and peaks can be calculated.

4. Micro material deformation of rough surfaces and its effect on fluid flow path

When two surfaces each with their own unique roughness's come into contact, the actual contact area is much less than the nominal area. The magnitude of the contact area is dependent on the load applied. So, the accurate deformation of the surface roughness's is of great importance since the voids between the surface roughness's is the gap size (2D) or aperture field (3D) through which the fluid will flow. Also a small change at a micro scale will reverberate in a larger change over a macro scale.

Tsukizoe and Hisakado theory essentially assumes the flat surface is rigid-perfectly plastic and the summed surface is perfectly-plastic and as said these are accepted and applied assumptions in references [9, 12, 14].

Using slip line field theory, Mitchell and Rowe [9] have incorporated the effects of the perfectly-plastic isotropic deformation structural response of two-dimensional wedges to represent the surface roughness in contact with a rigid-perfectly-plastic flat surface which is based on the theory discussed above. The slip line theories main limitation is that it is used to model plastic deformation in plain strain only for a solid represented as a rigid-perfectly-plastic flat surface [19]. It is shown that for all contact pressures there is a specific deformed wedge angle found and crucially there is a point at which a maximum leakage rate for specific wedge angle over the seat length. The limitations of the findings are concurrent with the fluid flow assumptions about the gap size and the simplified representation of the surface roughness.

Assuming a simplistic, but effective sinusoidal shape geometry to represent the surface roughness in contact with a flat surface [12], it is shown that: as the load on the gasket is increased, the incompressible fluid flows from a radial direction, to a circumferential spiral fluid flow through the valleys as shown in Fig. 3. It is also shown that there is a very small region over which the transition from circumferential to radial (or vice-versa) occurs and the diffusive and viscous flows are mixed. It is concluded that: the radial leakage (which is related to the transmissivity tensor) is of most critical since it is very sensitive to the gap size. However, this theory can only be valid for surfaces which depict a predominantly sinusoidal shape contacting a flat face.

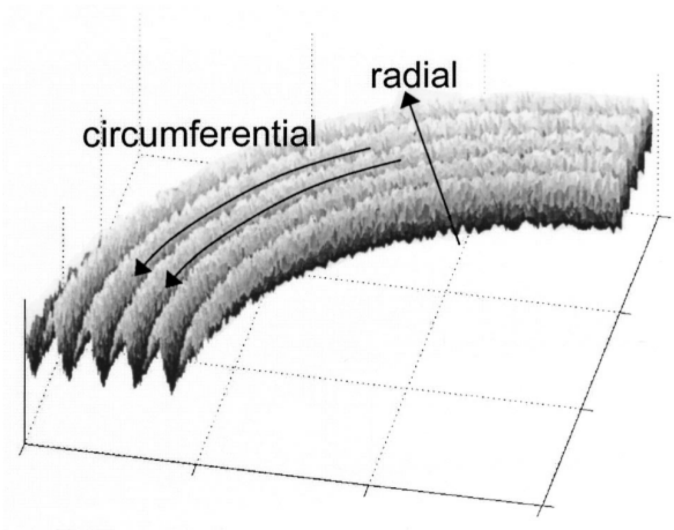

Fig. 3. Illustration of the crest and valley sinusoidal representation and the fluid flow path in either a radial or circumferential manner. [12]

Another analytical technique which accounts for elastic deformation is the Hertz theory [18]. The Hertz elastic deformation theory is only applicable for surfaces with purely spherical aperture contacting shapes. It is employed by Man *et al.* [20] for two 3D random rough surfaces in contact. As the two rough surfaces come into contact and deform due to the loading applied, the leakage path through the aperture field is dictated by the largest gap from one end of the specimen to the other and is calculated using a ‘recognition algorithm’ (Fig. 4). This fluid flow path recognition is a simplistic, but effective theory to allow one to understand if the micro-contact under specific loads will either leak or not for a material.

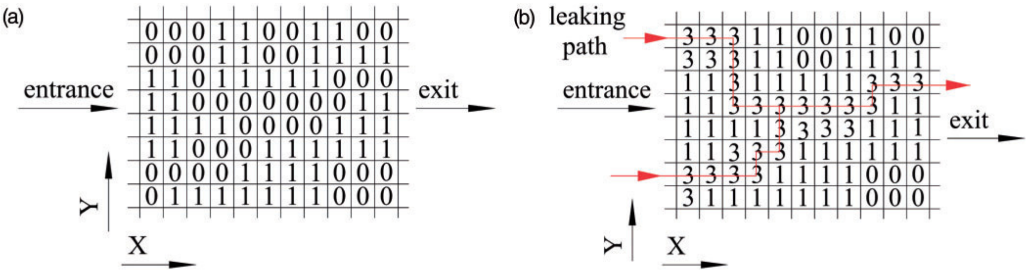

Fig. 4. Recognition algorithm diagram of leak path identification where 0 is no gap and 3 is the fluid flow path. [20]

5. Macro scale deformation and its effect in fluid flow path

Using Finite Element Analysis (FEA) (ANSYS), Gorash et al. [13] have shown that at a macro-scale the contact length of the seat and disc of a PRV is reduced and is further exasperated due to cyclic opening and closing of the valve. Gorash et al. assumed the contact force to be normal to the face of the seat and disc modelled as 2D. Using elements PLANE 183 (Seat and disc model), COMBIN14 (spring), CONTA172 (seat contact) and TARGE169 (disc contact) and assuming that for a gas and liquid the fluid flow pressure acting on the seat and disc is parabolic and linear respectively, they have shown that the once the spring preload and the internal set pressure is applied, the contact edges of the disc become plastics for a monotonic material response. This plastic response of the seat is exasperated towards the middle of the disc when cyclic material response is considered. This essentially means the fluid flow path is increased; effective contact area and length would reduce meaning a higher leakage rate.

6. Experimental leakage rate

The common way to calculate the leakage rate of a PRV is based on British standards, API standards or ASME standards. This method requires the PRV to be set to 90% of its set pressure using a gas. The outlet of the PRV is closed off with a pipe attached to expel the increase in pressure (i.e. leakage) in some water in the form of bubbles. The leakage has to be less than a specified amount for it to be used in-service.

Based on the average surface roughness, there have been attempts to relate this to the leakage rate through a gasket. Haruyama et al. [21] created an experiment which quantified the leakage rate of helium through a bolted flange with a new gasket placed between. They concluded that the leakage rate is highest for rough surfaces of Ra=3.5 μm when a low load force is applied. When a maximum of 400 MPa of force is applied on the flange then leakage rate is the least and is similar for all Ra's being 1.5, 2.5 and 3.5 μm. However, the material properties of the gasket and the flange are not known and so links between the material deformation and leakage is difficult to comment on.

Another method used by Marie and Lasseux [22] allows quantification of leakage flow of solvents at a micro or nano-scale through a rough metal contact for both viscous and diffusive fluid flow separately for contact pressures up to 700MPa. Using the leakage results Equations (1-4), they have managed to find the diffusive and viscous properties from which the effective gap size has been estimated.

7. PRV specific issues relating to leakage

Now that an understanding of the work currently completed on micro deformation and its effect on leakage has been analysed it is important to consider factors which are unique to a PRV valve which could also cause it to leak.

Depending on the design of the valve, one issue which has been highlighted is that the guide pin can be displaced easily which causes the seat to rotate or displace. This rotation/displacement of the seat causes the valve to leak and subsequently the set pressure decreases. Ritchie [23] examined this issue and created an analytical model to understand the reduction of seat pressure due to the misalignment angle of the seat. Assuming the valve leaks only when the set pressure has been reached, it was shown that for a disc with radius of 8.47 mm and 155 N of applied force on the seat, the set pressure (100 PSI) decreases by 10% for a misalignment of 1.225 degrees for the seat.

8. Summary and discussion

As it has been shown, previous analytical work accounts for the gap height either in the form of h or h^3 for a parallel gap. This is mainly been accounted for only laminar viscous incompressible/ compressible or diffusive flow through the aperture field, with more recent work in accounting for the rarefaction of the flow. However, the parallel gap assumption is limited to small asperities $h \ll 1$ and there has to be an 'effective' height used for a 2D surface model which is geometrically represented as simple surfaces. While for a 3D surface models computer programs such as MATLAB or equivalent can be used to model lapped surfaces with high accuracy as self-affine fractal surfaces, but are extensive.

In reality Tsukizoe and Hisakado and the slip-line theory is not completely valid for multiple asperity contacts demonstrated by Uppal and Probert's [24] experiment. They showed that for a multiple asperity surface in contact with a harder flat surface, under a high load the shallower valleys do rise (0.1 μm – 0.3 μm) while deep valleys do not, while Tsukizoe and Hisakado theory considers no movement in the valleys. So, there is an element of elastic behaviour and plastic strain hardening occurring which would have to be considered.

Instead of analytical techniques to describe the deformation of two rough surfaces, FEA programs such as ANSYS could be utilized to describe elastic perfectly plastic interface deformation as shown by Megalingam and Mayuram [25]. The FEA would require some form of verification of deformation accuracy, but if achieved, this may allow a more accurate representation of the fluid flow path.

A theory created by Prat and Geoffrey ties to link the diffusive and circumferential fluid flow to either radial or circumferential, but has not been verified and is only applicable to predominant sinusoidal shaped surfaces in contact with a flat surface.

Gorash et al, have shown that there is a deformation at a macro scale due to the spring force and the pressure of the fluid, which has to be considered to begin. Their work has shown the contact is not uniform between a seat and disc of a PRV rather; it begins and ends within the seat length. So there is a need to understand how the macro and micro contact areas link to give the actual contact area.

The disc rotation on the seat causing leakage maybe more of a design problem rather than a research based problem. The reason for this is that there are clearances prevalent throughout the seat and disc allowing this motion to occur. Also it is understood that the spring force under compression may not be perpendicular to the compression axis of the spring.

9. Future development

Instead of analytical techniques to describe the deformation of two rough surfaces, FEA programs such as ANSYS could be utilized to describe elastic perfectly plastic deformation of the interface as shown by Megalingam and Mayuram using actual 3D or 2D scanned surfaces. There is other literature particularly by Thompson [26-28], which gives further guidance on multi-scale modelling and optical measurements of the surface aperture which is then transferred into ANSYS. The FEA would require some form of verification of the surface deformation accuracy, but if achieved, this will allow a more accurate representation of the fluid flow path.

This could be taken further by modelling the surface as a representative surface using self-affine fractal surface or MOTIF procedure vs. actual surface in FEA and see the difference in the contact area.

Further investigations would be required to understand the effect of grain boundaries on the deformation of surface roughness at a micro scale.

Using the FEA deformed surface results of the deformed surface for both the actual surface and representative, CFD could be employed to better understand the leakage rate through a small micro scaled sample to find the effective or actual gap size through the aperture field by recognising the fluid flow path.

The leap has to be made to the more informative results which occur at a macro-scale. Gorash *et al.* macro scale work has to be interlinked with the micro scale leakage rates. To do this multi-scale modelling would have to be employed to work out the effective gap height. Both Thompson [29], and Jackson and Streator [30] have achieved this using FEA and analytical techniques in the form of effective contact area. From this macro-micro linked model the effective gap height can be found. This would then require a final verification against an actual PRV (with no guide pin movement issues which could cause leakage) with set-up similarities used by Marie and Lasseux [22]. Using Marie and Lasseux analysis technique the effective gap height for transmissivity can be found from the experiment and compared against the computational results.

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