Average Transmit Power of Adaptive ZF Very Large Multi-User and Multi-Antenna Systems

Dian-Wu Yue and Yichuang Sun

Abstract

In this paper, we investigate adaptive zero-forcing (ZF) uplink transmission for very large multi-user multi-antenna (MUMA) systems in Rayleigh fading environments. We assume that the number of antennas at the base station (BS) (denoted as M) is not less than the number of users (denoted as K) with each having single antenna, and power control can be done at the transmitter(s) as channel condition changes. Under constraints of individual rates and maximum transmit powers, we adopt the optimal transmit strategy of minimizing the total average transmit power (ATP). We derive and give individual ATP expressions for each link with short- and long-term rate constraints, respectively. Numerical results show that the individual ATP for each link with short term rate constraint is quite close to its long term counterpart when $M-K$ is large, and its corresponding outage probability can be designed to be nearly zero at the same time. Finally, we present two simple adaptive transmission schemes with constant transmit power satisfying short- and long-term rate constraints, respectively. Both of them are easy to implement, and asymptotically optimal when $M-K$ grows without bound.

This work was presented in part at the 9th European Wireless Conference, Guildford, UK, April 16-18, 2013.

This work was supported in part by the Scheme of Research Exchanges with China and India, the Royal Academy of Engineering, the Specialized Research Fund for the Doctoral Program of Higher Education under Grant 20132125110006, the National Natural Science Foundation of China under Grant 61371091 and 61301228, and the Fundamental Research Funds for the Central Universities under Grant 3132013334.

Dian-Wu Yue is with the College of Information Science and Technology, Dalian Maritime University, Dalian, Liaoning 116026, China (e-mail: dianwuyue@yahoo.com). Yichuang Sun is with the School of Engineering and Technology, University of Hertfordshire, College Lane, Hatfield, Herts AL10 9AB, UK, (Email: y.sun@herts.ac.uk).

Index Terms

Virtual MIMO, average transmit power, multi-user, multi-antenna, zero-forcing, very large MIMO.

I. INTRODUCTION

Wireless transmission using multiple antennas has attracted much interest in the past couple of decades due to its capability to exploit the tremendous capacity inherent in MIMO channels. Various aspects of wireless MIMO systems have been studied intensively, especially the important capacity aspect [1]. Whilst single-user systems have been well investigated, multi-user systems including classical multiple access (MA) and broadcast (BC) systems nowadays have become the focus of theoretical analysis and practical design of MIMO communications [2]. Theoretically, the maximum-likelihood multiuser detector and "dirty paper coding" can be used to obtain optimal performance for the MA and BC systems, respectively. However, they induce a significant complexity burden on the system implementation, especially for a large MUMA system. Therefore, linear effective processing schemes, in particular zero-forcing (ZF) receiving or precoding, are of particular interest as low-complexity alternatives [3]- [5].

Recently, there exist a lot of interests in MUMA with a very large antenna array at the BS, which means a array comprising a few hundreds of antennas simultaneously serving tens of users [6]- [11]. These very large MIMO systems can offer higher data rates, increased link reliability, and potential power savings since the transmitted RF energy can be more sharply focused in space while many random impairments can be averaged out. The analysis and design of very large MIMO systems is at the moment a fairly new research topic [7]. In [8], ZF precoding performance is studied for measured very-large MIMO downlink channels. It is shown that there exist clearly benefits with an excessive number of BS antennas [8]. In [9]

, [10] and [11], with ZF receivers authors give uplink capacity analysis of single-cell, single-cell distributed, and multi-cell very large MIMO systems, respectively, derive bounds on the achievable sum rate in both small and large-scale fading environments, and provide asymptotic performance results when the number of antennas grows without bound. It should be pointed out that all of ZF system schemes investigated in these three papers do not involve the adaptive transmission. Adaptive transmission techniques that can utilize the resources efficiently have always been of great interest in the field of wireless communications, especially for the current virtual MIMO systems [12]. Therefore, in this regard we will consider adaptive ZF processing with power control.

On the other hand, with explosive growth of high-data applications, more and more energy is consumed in wireless networks to guarantee QoS. Therefore, energy efficiency (EE) communications have been paid increasing attention under the back ground of limited energy resource and environment-friendly transmission behaviors [13], [14]. As for the information-theoretic aspect, most literature about EE mainly focused on point-to-point scenarios and the impact of practical issues on EE is not fully exploited. Thus, research on EE needs to be extended to multi-user and/or multi-cell cases as well as considering the practical issues such as transmission associated circuit energy consumption, which is of great significance to practical system design. As for the advanced techniques that will be used in future wireless systems, such as OFDMA, MIMO and relay, existing research has proved that larger EE can be achieved through EE design. However, most work is still in the initial stage, and more effort is needed to investigate potential topics such as those listed in [13]. This motivates our study of adaptive ZF very large MUMA systems from the EE aspect.

Moreover, in [15]- [17], authors addressed beamforming transmission schemes in a MIMO broadcast or multiple access channel, and investigated such an optimization problem that minimizes the ATP under a set of given rate or signal-to noise ratio constraints. In their discussion, however, the number of used antennas was assumed to be not large while the number of users could be allowed to be large enough for obtaining the multi-user gain. Similar to [15]- [17], we will also pursue such an optimal strategy: minimizing the total ATP under given rate constraints. But obviously different from these papers, we will specially analyze the power benefits obtained from the large antenna array at the BS.

In this paper, we will consider two system optimization problems involving two different kinds of rate constraints respectively: short term and long term. The short term rate constraints are particularly relevant to delay-sensitive services such as speech, real-time video and network game [16]. On the other hand, the long term rate constraints are closely relevant to delay-insensitive services such as email and file transfer for data networks.

The rest of the paper is organized as follows. In Section II, we describe the system model and present the optimization problems for short-and long-term rate constraints, respectively. In Section III, we derive ATP expressions for short and long term rate constraints, respectively. Further, we provide some numerical results to make ATP comparisons. In Section IV, we provide two simple adaptive ZF transmission schemes based on the constant transmit power for short-and long-term rate constraints, respectively. Finally, in Section V we conclude the paper.

II. System Model And Optimization Formulation

A. *system model*

We first consider a virtual MIMO system, concretely, a MUMA-MA system. The system includes one BS (or an access point) equipped with an array of M antennas, and the BS serves $K \leq M$ users, each user having one antenna. These users transmit their data in the same time-frequency resource. The $M \times 1$ received vector at the BS is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}. \tag{1}$$

where $\mathbf{H}$ represents the $M \times K$ channel matrix between the BS and the K users; $\mathbf{x}$ is the $K \times 1$ vector of symbols simultaneously transmitted by the the K users; and $\mathbf{n}$ is a vector of additive white and zero mean complex Gaussian noise. Without loss of generality, we take the noise variance to be one. The channel $\mathbf{H} = [h_{mk}]$ models independent fast fading, geometric attenuation, and log-normal shadow fading [9]. Thus the channel coefficient between the m-th antenna of the BS and the k-th user can be written as

$$h_{mk} = g_{mk}\sqrt{z_k} \tag{2}$$

where g_{mk} is the small-scale fading coefficient from the k-th user to the m-th antenna of the BS. It is further assumed that g_{mk} is independent and identically distributed (i.i.d) and follows complex Gaussian distribution of zero mean and unit variance. $\sqrt{z_k}$ models the geometric attenuation and shadow fading which is assumed to be independent over m and be constant over many coherence time intervals and known a priori. As a large-scale fading coefficient, the value of z_k changes very slowly over time. Now let $\mathbf{G} = [g_{mk}]$ and $\mathbf{Z} = \text{diag}(z_1, z_2, \ldots, z_K)$. Then $\mathbf{H} = \mathbf{G}\mathbf{Z}^{1/2}$. Moreover, $\mathbf{x}$ can be rewritten as

$$\mathbf{x} = \mathbf{P}^{1/2}\mathbf{s} \tag{3}$$

where $\mathbf{P} = \text{diag}(p_1, p_2, \ldots, p_K)$ with p_k being the transmitted power of the k-th user; and $\mathbf{s} = (s_1, s_2, \ldots, s_K)^T$ with s_k being the corresponding transmitted signal with unit power, and ^{T} representing the transpose. We assume that the BS has perfect CSI, i.e., it knows $\mathbf{H}$, and it employs the linear ZF detection. Thus, after using the ZF detector the received signal vector is given by

$$\mathbf{r} = \mathbf{H}^{\dagger}\mathbf{y} \tag{4}$$

where $\mathbf{H}^{\dagger} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$, ^{H} represents the complex conjugate transpose and † denotes the pseudo-inverse operation. From (1) the k-th elements of $\mathbf{r}$ can be written as

$$r_k = \sqrt{p_k}s_k + \mathbf{H}_k^{\dagger}\mathbf{n}. \tag{5}$$

where $\mathbf{H}_k^{\dagger}$ is the k-th row of $\mathbf{H}^{\dagger}$. Then the instantaneous signal to noise ratio (SNR) is given by [18]

$$\text{SNR}_k = p_k\lambda_k \tag{6}$$

where

$$\lambda_k = \frac{1}{[\mathbf{W}^{-1}]_{kk}} \tag{7}$$

with $\mathbf{W} = \mathbf{H}^H\mathbf{H}$. The parameter λ_k can be interpreted as the effective channel gain to the k-th user [4]. For adaptive transmission, under the time-varying channel conditions, all the K users will dynamically change their transmit power. In practice, depending on $\mathbf{H}$, the dynamical power allocation $\{p_k, k = 1, 2, \ldots, K\}$ should be done at the BS rather than at the users, and then the BS forwards the value of p_k to the k-th user. For $\{p_k : k = 1, 2, \ldots, K\}$, our optimal power allocation policy is adaptive in time and involves the following statistical power measure

$$\mathcal{P} = \mathbb{E}\{\sum_{k=1}^{K} p_k\} \tag{8}$$

where $\mathbb{E}\{\cdot\}$ stands for the expectation operator.

It should be noticed that the mentioned-above system model can be extended to the distributed MIMO one studied in [10], and the corresponding theoretical analysis is not difficult. However, in this regard, we are going to focus on the relationship among the total ATP, the numbers of BS antennas and users.

B. Optimization formulation for short term rate constraints

We will investigate how to optimally design such a wireless MUMA system that is subject to the QoS requirements. As already mentioned before, we aim at minimizing the total ATP while each data stream achieves its individual satisfying rate. For the rate constraints, we will consider two cases of short and long terms, respectively. Moreover, we add the maximum transmit power constraints for practical applications since too high transmit power in communication systems poses a problem in designing power amplifiers or draining batteries at the transmitters [19]. For the case with short term rate constraints, the corresponding optimal problem can be formulated as

$$\begin{cases} \underset{\{p_k: 1\leq k\leq K\}}{\text{Minimize}} & \mathcal{P}; \\ \text{Subject to} & R_k = \log_2(1+\text{SNR}_k) \geq \overline{R}(k); \\ & p_k \leq \overline{p}_{\max}(k), \;\; 1 \leq k \leq K. \end{cases} \tag{9}$$

Define

$$\overline{\text{SNR}}(k) = 2^{\overline{R}(k)} - 1 \tag{10}$$

and

$$\overline{\lambda}_{\text{out}}(k) = \frac{\overline{\text{SNR}}(k)}{\overline{p}_{\max}(k)}. \tag{11}$$

Then based on (6), the optimal problem (9) can be translated into K individual optimization problems, and each corresponds to one user:

$$\begin{cases} \underset{\{p_k\}}{\text{Minimize}} & \mathbb{E}\{p_k\}; \\ \text{Subject to} & \lambda_k p_k \geq \overline{\text{SNR}}(k), \\ & \lambda_k \geq \overline{\lambda}_{\text{out}}(k). \end{cases} \tag{12}$$

From (12) we can get such a conclusion that the adaptive power allocation policy should adopt the form of similar channel inversion, precisely,

$$p_k = \frac{\overline{\text{SNR}}(k)}{\lambda_k}, \;\; k = 1, 2, \ldots, K. \tag{13}$$

Moreover, when $\lambda_k < \overline{\lambda}_{\text{out}}(k)$, it implies that $p_k > \overline{p}_{\max}(k)$. At this time when the channel with the k-th user is possibly in deep fading, in order to save transmit power, the system should let the k-th user have a transmit outage temporarily.

It should be pointed out that the above optimization formulation with the short term rate or SNR constraints results in such a system design scheme which can also provide the MUMA system with stable and reliable detection performance all the time. This is specially convenient to further concatenate a constant rate channel code such as a powerful LDPC or Turbo code [12].

C. Optimization formulation for long term rate constraints

For the situation with long term rate constraints, we will also introduce a transmit outage for the k-th user when $\lambda_k < \overline{\lambda}_{\text{out}}(k)$. Denote the transmit probability without outage by

$$P_0(k) = \int_{\overline{\lambda}_{\text{out}}(k)}^{\infty} f_k(\lambda_k) d\lambda_k \tag{14}$$

where $f_k(\lambda_k)$ is the p.d.f. of the random variable λ_k. Then in this situation the optimal problem is formulated as

$$\begin{cases} \underset{\{p_k: 1 \le k \le K\}}{\text{Minimize}} & \mathcal{P}; \\ \text{Subject to} & p_k = 0 \;\; \text{when} \;\; \lambda_k < \overline{\lambda}_{\text{out}}(k); \\ & \frac{\mathbb{E}\{R_k\}}{P_0(k)} \ge \overline{R}(k), \;\; 1 \le k \le K. \end{cases} \tag{15}$$

This optimization problem can be also translated into K individual optimization problems, and each corresponds to one user:

$$\begin{cases} \underset{\{p_k\}}{\text{Minimize}} & \mathbb{E}\{p_k\}; \\ \text{Subject to} & p_k = 0 \;\; \text{when} \;\; \lambda_k < \overline{\lambda}_{\text{out}}(k) \\ & \frac{\mathbb{E}\{R_k\}}{P_0(k)} \le \overline{R}(k). \end{cases} \tag{16}$$

Applying Lagrange multiplier method to each of the above individual optimization problems, we get the following family of unconstrained optimization problems parameterized by multipliers $\omega_k > 0, \quad 1 \le k \le K$:

$$\underset{p_k}{\text{Min}} \int_{\overline{\lambda}_{\text{out}}(k)}^{\infty} [p_k - \omega_k(R_k - \overline{R}(k))] f_k(\lambda_k) d\lambda_k \tag{17}$$

Applying $R_k = \log_2(1+\text{SNR}_k)$ and (6), we can easily obtain an optimum solution as follows:

$$p_k = \begin{cases} \omega_k - \lambda_k^{-1} & \text{for} \lambda_k > \lambda_0(k); \\ 0 & \text{for} \lambda_k \le \lambda_0(k). \end{cases} \tag{18}$$

where $\lambda_0(k) = \max\{\omega_k^{-1}, \overline{\lambda}_{\text{out}}(k)\}$. We denote ω_k^{-1} by $\overline{\lambda}_{\text{out1}}(k)$. $\overline{\lambda}_{\text{out1}}(k)$ can be found by solving

$$\int_{\overline{\lambda}_{\text{out}}(k)}^{\infty} R_k f_k(\lambda_k) d\lambda_k = P_0(k)\overline{R}(k). \tag{19}$$

Now we define with the knowledge of calculus

$$\Upsilon(k) = \frac{\int_{\overline{\lambda}_{\text{out}}(k)}^{\infty} \log_2(1+\lambda_k) f_k(\lambda_k) d\lambda_k}{P_0(k)} = \log_2 \widehat{\lambda}_{\text{out}}(k). \tag{20}$$

Then with the help of (18) and (19), we obtain a simple expression of parameter $\overline{\lambda}_{\text{out1}}(k)$

$$\overline{\lambda}_{\text{out1}}(k) = 2^{\Upsilon(k) - \overline{R}(k)} = \frac{\widehat{\lambda}_{\text{out}}(k)}{1 + \overline{\text{SNR}}(k)}. \tag{21}$$

The above optimum solution will provide convenience for us to produce numerical results and make comparison with the short term counterpart.

III. Minimum Average transmit Power and Outage Probability

A. Adaptive scheme under short term rate constraints

In order to derive expressions for minimum average total transmit power and individual outage probability, we need to have the p.d.f. of the channel gains $\{\lambda_k, \ i = 1, 2, \ldots, K\}$. For this reason, we first introduce the following useful lemma given in [18].

Lemma 1: Throughout the paper, we let $\Psi = M - K$. Then

$$f_k(\lambda_k) = \frac{\lambda_k^{\Psi} e^{-\lambda_k/z_k}}{z_k^{\Psi+1}\Psi!}. \tag{22}$$

Obviously, when the k-th channel gain $\lambda_k < \overline{\lambda}_{\text{out}}(k)$ $(1 \leq k \leq K)$, there exists a transmit outage for the k-th data stream. The corresponding individual outage probability for the k-th data stream is given by

$$P_{\text{out}}^{(k)} = \int_0^{\overline{\lambda}_{\text{out}}(k)} f_k(\lambda_k) d\lambda_k. \tag{23}$$

With the help of Lemma 1, we can derive $P_{\text{out}}^{(k)}$ and have the following proposition, which involves the incomplete gamma function.

Proposition 1:

$$P_{\text{out}}^{(k)} = 1 - P_0(k) = \frac{\gamma(\Psi + 1, \overline{\lambda}_{\text{out}}(k)/z_k)}{\Psi!} \tag{24}$$

where $\gamma(q, x)$ is just the incomplete gamma function (See Page 454 of [20]).

The derivation of the ATP $\mathcal{P}$ in (8) involves the complementary incomplete gamma function. With the help of Lemma 1, we can easily derive the ATP and obtain the following result.

Proposition 2: Let $\rho_s^{(k)}(\overline{\text{SNR}}(k))$ denote the average needed transmit power for k-th data stream achieving the receive SNR, $\overline{\text{SNR}}(k)$. Then

$$\rho_s^{(k)}(\overline{\text{SNR}}(k)) = \frac{\overline{\text{SNR}}(k)\Gamma(\Psi, \overline{\lambda}_{\text{out}}(k)/z_k)}{z_k \Psi!} \tag{25}$$

In particular, when $\overline{\lambda}_{\text{out}}(k) = 0$, the above equation can be simplified to

$$\rho_s^{(k)}(\overline{\text{SNR}}(k)) = \frac{\overline{\text{SNR}}(k)}{z_k \Psi}. \tag{26}$$

In (25), $\Gamma(q, x)$ is just the complementary incomplete gamma function(See Pages 454 and 456 of [20]).

Example 1: Let $K = 1$. Denote $\vartheta = \overline{\lambda}_{\text{out}}(1)/z_1$ and $\overline{\text{SNR}}(1)/z_1 = \overline{\text{SNR}}$ for short. Then

$$\mathcal{P}_s = \begin{cases} \dfrac{\overline{\text{SNR}}}{(M-1)+\frac{\vartheta^{M-1}}{(M-2)!\sum_{j=0}^{M-2}\frac{\vartheta^j}{j!}}}, & \text{if } M \geq 2; \\ \overline{\text{SNR}}[\Gamma(0,\vartheta)e^{\vartheta}], & \text{if } M = 1. \end{cases} \tag{27}$$

B. Adaptive scheme under long term rate constraints

For the case with long term rate constraints, the derivations of the average transmit power and individual outage probability are similar to the case with short term rate constraints. In particular, the corresponding expressions of individual outage probability are the same as in the short term rate case except that the symbol $\overline{\lambda}_{\text{out}}(k)$ in those equations is replaced by $\lambda_0(k)$. However, in order to make a good comparison between average needed transmit powers for the two adaptive schemes of long and short terms, and also in order to provide convenience for the system design, we revisit the derivation of optimum solution in Subsection II.C, and hope that the following constraint can be met $\lambda_0(k) = \overline{\lambda}_{\text{out}}(k)$. It is found that this can hold if we can have $\overline{\lambda}_{\text{out1}}(k) \leq \overline{\lambda}_{\text{out}}(k)$. From (21) this means that we must meet such a *constraint condition*:

$$\overline{\text{SNR}}(k) \geq \frac{\widehat{\lambda}_{\text{out}}(k)}{\overline{\lambda}_{\text{out}}(k)} - 1. \tag{28}$$

Here $\widehat{\lambda}_{\text{out}}(k) = 2^{\Upsilon(k)}$, and $\Upsilon(k)$ is defined in (20). Again using Lemma 1, $\Upsilon(k)$ can be expressed as

$$\Upsilon(k) = \frac{\log_2 e \sum_{j=0}^{\Psi} \Gamma(j, \overline{\lambda}_{\text{out}}(k)/z_k)/j!}{\Gamma(\Psi+1, \overline{\lambda}_{\text{out}}(k)/z_k)/\Psi!} + \log_2 \overline{\lambda}_{\text{out}}(k). \tag{29}$$

The above derivation is not difficult, but involves a process employing the following expression involving a special function [21]:

$$\begin{aligned} \jmath_q(x) &= \int_1^{\infty} \ln(t) t^{q-1} e^{-xt} dt \\ &= \frac{(q-1)!}{x^q} \sum_{j=0}^{q-1} \frac{\Gamma(j,x)}{j!}. \end{aligned} \tag{30}$$

As long as the constraint condition (28) is satisfied, the expressions of individual outage probability for the long term scheme will be completely the same as in the short term case.

Now we consider to derive the average minimum transmit power with the constraint condition (28). After employing again Lemma 1, we can finally obtain the following result.

Proposition 3: Let $\rho_l^{(k)}(\overline{\mathrm{SNR}}(k))$ denote the average needed transmit power for the k-th user achieving the received SNR, $\overline{\mathrm{SNR}}(k)$. Then

$$\rho_l^{(k)}(\overline{\mathrm{SNR}}(k)) = \begin{cases} \frac{P_0}{\overline{\lambda}_{\mathrm{out1}}(k)} - \frac{\Gamma(\Psi,\overline{\lambda}_{\mathrm{out}}(k)/z_k)}{z_k \Psi!}, & \text{if (28) holds;} \\ \frac{P_1}{\overline{\lambda}_{\mathrm{out1}}(k)} - \frac{\Gamma(\Psi,\overline{\lambda}_{\mathrm{out1}}(k)/z_k)}{z_k \Psi!}, & \text{otherwise.} \end{cases} \tag{31}$$

where $P_1 = \Gamma(\Psi+1, \overline{\lambda}_{\mathrm{out1}}(k)/z_k)/\Psi!$, $\overline{\lambda}_{\mathrm{out1}}(k) = 2^{\Upsilon(k)-\overline{R}(k)}$ and $\Upsilon(k)$ can be computed using (29). Moreover, it easily follows that $P_0 = \Gamma(\Psi+1, \overline{\lambda}_{\mathrm{out}}(k)/z_k)/\Psi!$.

C. Numerical Results and Comparison

By numerical simulation we mainly observe the link performance behaviors for the MUMA system. For the k-th link, we let $\frac{\overline{\mathrm{SNR}}(k)}{z_k} = \mathrm{SNR}$ and $\frac{\overline{\lambda}_{\mathrm{out}}(k)}{z_k} = \lambda_{\mathrm{out}}$ for simplicity.

We first simulate the individual outage probability (OP) for each user using Proposition 1. In order to provide convenient for making OP comparison between SUSA and MUMA systems, we firstly evaluate the OP $P_{\mathrm{out}} = 10^{-\upsilon}$ for SUSA systems by setting exponent υ. We call υ as a SUSA outage exponent (OE). For example, we set $\upsilon = 1$ and $\upsilon = 1.2$, then $P_{\mathrm{out}} = 10^{-1}$ and $P_{\mathrm{out}} = 6.3 \cdot 10^{-2}$ for the SUSA system, respectively. Based on Proposition 1 we can compute the corresponding OPs $P_{\mathrm{out}} = 1.8 \cdot 10^{-4}$ and $P_{\mathrm{out}} = 4.4 \cdot 10^{-5}$ for the MUMA system with $\Psi = 2$, respectively. And for the MUMA system with $\Psi = 6$, the corresponding OPs become $P_{\mathrm{out}} = 2.6 \cdot 10^{-11}$ and $P_{\mathrm{out}} = 9.3 \cdot 10^{-13}$, respectively. Table I provides computed results for the MUMA system with $\Psi = 2$ and $\Psi = 6$ when υ is set from 0.4 to 1.8. Table I shows that the MUMA system with $\Psi = 6$ almost has not system outage when $\upsilon \geq 0.6$. This implies also that the OP will be better if $\Psi > 6$. In addition, it can be found from the table that the corresponding λ_{out} is always appropriate for constraining the corresponding maximum transmit power. For this reason, in the following we will always set $\lambda_{\mathrm{out}} = 1.1 \cdot 10^{-1}$ whose corresponding OE is $\upsilon = 1$ if needed. it should be pointed out that the OP with $\Psi = 2$ is still relatively high as seen in Table I.

We now observe the ATP behavior of the MUMA system that can be evaluated by Proposition 2. For the fixed OE $\upsilon = 1$ or letting $\lambda_{\text{out}} = 1.1 \cdot 10^{-1}$, Fig.1 plots the individual ATP under different numbers of $\Psi = 1, 2, 6$, and includes the corresponding ATP with $\lambda_{\text{out}} = 0$ for comparison. As expected, the individual ATP decreases as Ψ increases, and the ATP with $\lambda_{\text{out}} > 0$ is closer to the one with $\lambda_{\text{out}} = 0$ as Ψ increases. It can be found that when $\Psi = 6$ they are almost the same for various values of constraint SNR.

We now focus on the adaptive transmit scheme under the long term rate constraints. First of all, we should consider the constraint condition of SNR in optimization design which is given in (28). Still fix $\lambda_{\text{out}} = 1.1 \cdot 10^{-1}$. The first figure in Fig.2 plots the SNR under constraint condition for the MUMA system with various numbers of Ψ. From this figure, the constraint SNR varies with Ψ, and becomes gradually higher with increasing Ψ. When $\Psi = 8$, SNR is equal to 19dB. For that, we fix the constraint SNR, SNR $= 19$dB and in the second figure of Fig.2 plot the individual ATP based on Proposition 3 with the constraint condition (28). It is obviously found from this figure that the individual ATP gradually becomes small as Ψ increases.

In order to make ATP comparison between the adaptive ZF scheme with short term rate constraints and the one with long term rate constraints, The first figure in Fig.3 plots two individual ATP curves based on Propositions 2 and 3 under the condition of fixing SNR $=$ 19dB and $\lambda_{\text{out}} = 1.1 \cdot 10^{-1}$. It is obviously found from this figure that the ATP curve with short term rate constraints is closer to the one with long term rate constraints as Ψ increases. Especially, when $\Psi = 20$, they are nearly the same. For comparison, in the asymptotical case with $\lambda_{\text{out}} = 0$, the first figure also plots the individual ATP curve for the adaptive ZF scheme with short term rate constraints. As expected, it can be seen that the curve with $\lambda_{\text{out}} = 0$ is almost the same as the corresponding curve with $\lambda_{\text{out}} = 1.1 \cdot 10^{-1}$. On the other hand, the two adaptive schemes possess possibly different outage probabilities. For this reason, the second figure in Fig.3 plots the outage probability parameter (OPP) under the long term rate constraint $\lambda_{\text{out}1}$. When $\Psi > 8$, the difference of $\lambda_{\text{out}1} - \lambda_{\text{out}}$ gradually becomes large as Ψ increases. Even although it still follows from Table I that $P_{\text{out}} \approx 0$ when $\Psi \geq 6$. Therefore, when the number of antennas is large enough, the two adaptive schemes will have the same power savings and near-zero outage probability.

Recently, [9] and [10] provided analysis results about the average achievable rate for very large MIMO systems with ZF detections. It should be noticed that in the discussion of [9] and [10], the transmit power is fixed without adaption. For this reason, we make comparison between our adaptive scheme and the traditional scheme with fixed transmit power. By using the ATP values given in Fig.3 under short term rate constraints, Table II provides computed OP results for the traditional scheme. It can be found from Table II that the OP with the traditional scheme keeps quite high and slowly increases as Ψ grows large while the OP with our adaptive scheme rapidly tends to zero. This implies that the traditional scheme with large antenna array can not be used in practice. Moreover, by using the ATP values given in Fig.3 under long term rate constraints, Fig.4 plots the average achievable rate for the traditional scheme. It can be seen from Fig.4 that the average achievable rate slowly increases as Ψ grows large, and is obviously smaller than the one with our adaptive scheme.

IV. Adaptive Transmission Schemes with Constant Transmit Power

In this section, we will consider a very simple adaptive transmission scheme with one or two transmit power levels. This kind of transmission systems facilitates the use of the most power-efficient and cheap power amplifiers/analog components [25].

A. Adaptive scheme with short term rate constraints

In the short term case, for the k-th user we now consider a very simple adaptive transmission scheme with two transmit power levels, in which the k-th user only uses a couple of power levels: $\overline{p}_{\text{con}}(k)$ and $\overline{p}_{\text{max}}(k)$ when it transmits. In such a case, the feedback design will become quite simple: the BS needs at most 2-bit feedback power control information to the k-th user. If the maximum transmit power $\overline{p}_{\text{max}}(k)$ is fixed, then the individual optimization problem will be simplified to how to choose another power $\overline{p}_{\text{con}}(k)$ to minimize the individual ATP. Still let $\overline{\lambda}_{\text{out}}(k) = \frac{\overline{\text{SNR}}(k)}{\overline{p}_{\text{max}}(k)}$ and define $\overline{\lambda}_{\text{con}}(k) = \frac{\overline{\text{SNR}}(k)}{\overline{p}_{\text{con}}(k)}$. Then the individual ATP is given by

$$\begin{aligned}
&\mathbb{E}p_k = \overline{p}_{\text{con}}(k) \int_{\overline{\lambda}_{\text{con}}(k)}^{\infty} f_k(\lambda_k) d\lambda_k \\
&+\overline{p}_{\text{max}}(k) \int_{\overline{\lambda}_{\text{out}}(k)}^{\overline{\lambda}_{\text{con}}(k)} f_k(\lambda_k) d\lambda_k \\
&= \frac{\overline{\text{SNR}}(k)}{\overline{\lambda}_{\text{con}}(k)} (1 - F_k(\overline{\lambda}_{\text{con}}(k))) \\
&+ \frac{\overline{\text{SNR}}(k)}{\overline{\lambda}_{\text{out}}(k)} (F_k(\overline{\lambda}_{\text{con}}(k)) - F_k(\overline{\lambda}_{\text{out}}(k))).
\end{aligned} \tag{32}$$

where $F_k(\cdot)$ is the distribution function of λ_k. The optimization solution of $\overline{p}_{\text{con}}(k)$ satisfies

$$\frac{\partial \mathbb{E}p_k}{\partial \overline{\lambda}_{\text{con}}(k)} = 0. \tag{33}$$

By applying Lemma 1, (33) can be written as

$$\sum_{i=0}^{M-K} \frac{\overline{\lambda}_{\text{con}}(k)^i}{z_k^i i!} = \left(\frac{1}{\overline{\lambda}_{\text{out}}(k)} - \frac{1}{\overline{\lambda}_{\text{con}}(k)}\right) \frac{\overline{\lambda}_{\text{con}}(k)^{\Psi+2}}{z_k^{\Psi+1} \Psi!}. \tag{34}$$

This belongs to the problem of finding a root in a polynomial equation. When $M = K$, we can easily get the solution as follows:

$$\overline{\lambda}_{\text{con}}(k) = \frac{\overline{\lambda}_{\text{out}}(k)}{2} + \sqrt{z_k \overline{\lambda}_{\text{out}}(k) + \overline{\lambda}_{\text{out}}(k)^2/4}. \tag{35}$$

On the other hand, if $\Psi \to +\infty$, then we must have that $\overline{\lambda}_{\text{con}}(k) \to +\infty$ or $\overline{p}_{\text{con}}(k) \to 0$. This means again that we can use only low enough power to communicate for a very large MIMO system.

Dependence on the concrete cases of λ_k, the BS may allocate its feedback 2 bits as follows:

a) When $\lambda_k \in (\overline{\lambda}_{\text{con}}(k), +\infty)$, the BS feedback 1 bit "1";

b) When $\lambda_k \in (\overline{\lambda}_{\text{out}}(k), \overline{\lambda}_{\text{con}}(k))$, the BS feedback 2 bits "01";

c) When $\lambda_k \in (0, \overline{\lambda}_{\text{out}}(k))$, the BS feedback 2 bits "00" .

For the mentioned-above feedback scheme, the average bit amount of feedback is given by

$$\begin{aligned}\bar{B} &= 1 \cdot (1 - F(\overline{\lambda}_{\text{con}}(k))) \\ &+2 \cdot (F(\overline{\lambda}_{\text{con}}(k)) - F(\overline{\lambda}_{\text{out}}(k))) + 2 \cdot F(\overline{\lambda}_{\text{out}}(k)) \\ &= 1 + F(\overline{\lambda}_{\text{con}}(k)).\end{aligned} \quad (36)$$

Obviously, $\bar{B} \approx 1$ if $F(\overline{\lambda}_{\text{con}}(k)) \approx 0$. This can be satisfied when $\Psi \to +\infty$.

Finally, we simulate the simple adaptive scheme with a couple of transmit power levels in the following two figures. We set $\lambda_{\text{out}} = 1.1 \cdot 10^{-1}$ and SNR $= 10$ dB for various Ψ. Fig.5 plots the individual ATP with the simple adaptive scheme and the corresponding ATP expressed in Proposition 2 for comparison. As can be seen in Fig.5, the ATP with constant transmit power becomes lower and lower as Ψ increases constantly, and is always worse than the counterpart without constant transmit power. But we still find from this figure that as Ψ increases the ATP gap decreases from about 4dB to 2dB. It can be estimated that the ATP with the simple scheme will be asymptotically zero as the ATP with the optimal scheme when $\Psi \to +\infty$.

In addition to the ATP, the first figure of Fig.6 plots the probability distribution of λ_{out} which corresponds to the constant transmit power. As Ψ increases, the probability becomes smaller and smaller. But it can be found by simulation that there exists a "floor" phenomena when Ψ is very large. The second figure of Fig.6 plots the needed feedback bits. It can be easily seen from this figure that for a very large MUMA system the amount of the needed feedback bits is almost 1 bit, which implies a good agreement with the theoretical analysis result.

B. Adaptive scheme with long term rate constraints

Under long term rate constraints, we now consider the simplest transmit scheme in which each user only uses one transmit power level (denoted as $\overline{p}_{\text{con}}$) to communicate. In this case, the BS does not need to feedback the power control information to each user when the large scale fading parameter z is unchanged. For the k-th user, let $\overline{p}_{\max}(k) = \infty$. Then $\overline{\lambda}_{\text{out}}(k) = 0$. Further, its average achievable rate with constant transmit power $\overline{p}_{\text{con}}(k)$ can be given by

$$\begin{aligned}\mathbb{E}R_k &= \int_0^\infty \log_2(1+\overline{p}_{\text{con}}(k)\lambda_k) f_k(\lambda_k) d\lambda_k \\ &= \frac{\phi^{\Psi+1}\log_2 e}{e^{-\phi}\Psi!}\sum_{n=0}^{\Psi}(-1)^{\Psi-n}\binom{\Psi}{n}\jmath_{n+1}(\phi) \qquad (37)\end{aligned}$$

where $\jmath_q(x)$ is the special function defined in (30) and $\phi = \frac{1}{z_k\overline{p}_{\text{con}}(k)}$. Given the constrained rate $\overline{R}(k) = \log_2(1+\overline{\text{SNR}}(k))$,with (37) we can only determine precisely $\overline{p}_{\text{con}}(k)$ by numerical computation. In the following, by using Jensen's inequality we give a simple lower bound for $\overline{R}(k)$, which is very convenient for us to estimate the needed power $\overline{p}_{\text{con}}(k)$.

$$\begin{aligned}\overline{R}(k) &= \mathbb{E}\{\log_2(1+\overline{p}_{\text{con}}(k)\lambda_k)\} \\ &\geq \log_2\{1+\frac{\overline{p}_{\text{con}}(k)}{\mathbb{E}\{\lambda_k^{-1}\}}\} \\ &= \log_2(1+\overline{p}_{\text{con}}(k)z_k\Psi). \qquad (38)\end{aligned}$$

The last equality is derived by using the property $\mathbb{E}\{\lambda_k^{-1}\} = \frac{1}{z_k\Psi}$, which is in fact the special case of (26). Therefore, if we make use of

$$\overline{p}_{\text{con}}(k) = \frac{\overline{\text{SNR}}(k)}{z_k\Psi}, \qquad (39)$$

we can satisfy the design requirement. It is interesting to find that the short term average transmit power for the k-th user from Proposition 2 is just equal to

$$\rho_s^{(k)}(\overline{\text{SNR}}(k)) = \overline{p}_{\text{con}}(k) = \frac{\overline{\text{SNR}}(k)}{z_k \Psi}. \tag{40}$$

Under the constraint of SNR $= 10$ dB, Fig.7 plots the achievable average rate with constant transmit power and plots the lower bound for comparison. From Fig.7 it can be seen that as Ψ increases, the achievable average rate gets slowly close to the lower bound.

It should be noticed that the simple adaptive scheme may be asymptotically optimal since it obtains the same optimal solution as the short term optimal scheme since in Section III the optimal ATP with the short term scheme has been shown to be asymptotically the same as the one with long term scheme in term of the ATP when Ψ is large enough.

V. Conclusions

Adaptive transmission techniques in wireless communications can utilize the system resources efficiently and provide satisfying QoS. In this paper, for MUMA systems we have investigated adaptive ZF transmission technique with power control under the rate and power constraints. Mainly based on the statistics of effective channel gains, we have derived and presented expressions for the minimum ATP. Furthermore, we have developed two simple adaptive transmission schemes only using one or two transmit power levels. It is shown by our analysis that for a very large MUMA system, the system stable performance and user QoS (including outage probability) can always be met only by dynamically changing the number of antennas at the BS.

References

[1] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity Limits of MIMO Channels," *IEEE Journal on Selected Areas in Communications,* Vol. 21, No. 5, pp. 684-702, June 2003.

[2] D. Gesbert, M. Kountouris, R. W. Heath Jr., C.-B. Chae, and T. Sälzer, "Shifting the MIMO paradigm," *IEEE Sig. Proc. Mag.,* vol. 24, no. 5, pp. 36-46, Sep. 2007.

[3] C. B. Peel, B. M. Hochwald and A. L. Swindlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication-part I: channel inversion and regularization," *IEEE Trans. on Communicatios,* vol. 53 no. 1, pp. 195-202, Jan.2005.

[4] T. Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamming," *IEEE Journal on Selected Areas in Communications,* vol. 24, no. 3, pp. 528-541, Mar.2006.

[5] A. Wiesel, Y. C. Eldar and S. Shamai, "Zero-forcing precoding and generalized inverses," *IEEE Trans. on Signal Processing,* vol. 56, no. 9, pp. 4409-4418, Sep. 2008.

[6] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of BS antennas," *IEEE Trans. Wireless Commun.,* vol. 9, no. 11, pp. 3590-3600, Nov. 2010.

[7] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, "Scaling up MIMO: Opportunities and challenges with very large arrays," *IEEE Signal Process. Mag.,* vol. 30, no. 1, pp. 40-60, Jan. 2013.

[8] X. Gao, O. Edfors, F. Rusek, and F. Tufvesson, "Linear Pre-Coding Performance in Measured Very-Large MIMO Channels," *in Proc. IEEE Vehicular Technology Conf. (VTC),* San Francisco, CA, US, Sept. 2011, pp. 1-5.

[9] H. Q. Ngo, E. G. Larsson, and T. L. Marzetta, "Energy and Spectral Efficiency of Very Large Multiuser MIMO Systems," *IEEE Trans. Commun.,* vol. 61, no. 4, pp. 1436-1449, Apr. 2013.

[10] M. Matthaiou, C. Zhong, M. R. Mckay, and T. Ratnarajah, "Sum rate analysis of ZF receivers in distributed MIMO systems," *IEEE Journal on Selected Areas in Communications,* vol. 31, no. 2, pp. 180-191, Feb. 2013.

[11] H. Q. Ngo, M. Matthaiou, T. Q. Duong, and E. G. Larsson, "Uplink Performance Analysis of Multicell MU-SIMO Systems with ZF receivers," *IEEE Trans. Veh. Technol.,* vol. 62, no. 9, pp.4471-4483, Nov. 2013.

[12] G. Caire and K. R. Kumar, "Information Theoretic foundations of adaptive code and modulation," *Proceedings of the IEEE,* vol. 95, no. 12, pp.2274-2298, Dec.2007.

[13] G. Y. Li, Z. Xu, C. Xiong, C. Yang, S. Zhang, Y. Chen, and S. Xu, "Energy-efficient wireless communications: tutorial, survey, and open issues, " *IEEE Wireless Communications,* vol. 18, no. 6 pp. 28-35, Dec. 2011.

[14] J. Joung, C. K. Ho, and S. Sun, "Spectral efficiency and energy efficiency of OFDM systems: Impact of power amplifiers and countermeasures," *IEEE Journal on Selected Areas in Communications,* vol. 32, no. 2, pp. 208-220, Feb.2014.

[15] M. Schbert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Transactions on Vehicular Technology,* vol. 53, no. 1, pp.18-28, Jan.2004.

[16] P. Wang and P. Li, "On Maximum eigenmode beamforming and multi-user gain," *IEEE Transactions on Information Theory,* vol. 57, no. 7, pp.4170-4186, Jul. 2011.

[17] D.-W. Yue and J. Yuan, “On the power of MIMO broadcast systems under SNR constraints with limited feedback,” *IEEE Transactions on Vehicular Technology,* vol. 60, no. 1, pp.349-353, Jan. 2011.

[18] M. Matthaiou, N. D. Chatzidiamantis, G. K. Karagiannidis, and J. A. Nossek, “ZF detectors over correlated K Fading MIMO channels,” *IEEE Trans. on Communicatios,* vol. 59, no. 6, pp. 1591-1603, Jun.2011.

[19] M.Dohler, R. W. Heath,Jr., A. Lozano, C. B. Papadias, R. A. Valenzuela, “Is the PHY layer dead?” *IEEE Communications Magazine,* vol.49, no.4, pp.159-165, April 2011.

[20] L. C. Andrews, *Speical Functions of Mathematics for Engineering,* Oxford: Oxford University Press, 1998.

[21] M. -S. Alouini and A. J. Goldsmith, “Capacity of Rayleigh fading channels under different adaptive transmission and diversity-combining techniques,“ *IEEE Trans. on Vehicular Technology,* vol.48, no.4, pp. 1165-1181, July 1999.

[22] H. Lütkepohl, *Handbook of Matrices,* NewYork: John Wiley & Sons, 1996.

[23] H. Hassibi and B. M. Hochwald, “How much training is needed in muliple-antenna wirelss links?” *IEEE Trans. on Information Theory,* Vol.49, No. 4, pp. 951-963, Apr. 2003.

[24] C. Wang, E. K. S. Au, R. D. Murch, W. H. Mow, R. S. Cheng, and V. Lau, “On the performance of the MIMO zero-forcing receiver in the presence of channel estimation error,” *IEEE Trans. on Wireless Communicatios,* vol. 6, no. 3, pp. 805-810, Mar. 2007.

[25] S. K. Mohammed and E. G. Larsson, “Single-user beamforming in large-scale MISO systems with per-antenna constant-envelope constraints: the doughnut channel,” *IEEE Trans. on Wireless Communicatios,* vol.11, no. 11, pp. 3992-4005, Nov. 2012.

TABLE I

OP COMPARISON BETWEEN SUSA AND MUMA SYSTEMS

SUSA OE	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
$\overline{\lambda}_{\text{out}}(k)$	$5.1 \cdot 10^{-1}$	$2.9 \cdot 10^{-1}$	$1.7 \cdot 10^{-1}$	$1.1 \cdot 10^{-1}$	$6.5 \cdot 10^{-2}$	$4.1 \cdot 10^{-2}$	$2.5 \cdot 10^{-2}$	$1.6 \cdot 10^{-2}$
SUSA OP	$4.0 \cdot 10^{-1}$	$2.5 \cdot 10^{-1}$	$1.6 \cdot 10^{-1}$	$1.0 \cdot 10^{-1}$	$6.3 \cdot 10^{-2}$	$4.0 \cdot 10^{-2}$	$2.5 \cdot 10^{-2}$	$1.6 \cdot 10^{-2}$
$\Psi = 2$ OP	$1.5 \cdot 10^{-2}$	$3.3 \cdot 10^{-3}$	$7.5 \cdot 10^{-4}$	$1.8 \cdot 10^{-4}$	$4.4 \cdot 10^{-5}$	$1.1 \cdot 10^{-5}$	$2.7 \cdot 10^{-6}$	$6.7 \cdot 10^{-7}$
$\Psi = 6$ OP	$1.1 \cdot 10^{-6}$	$2.6 \cdot 10^{-8}$	$7.7 \cdot 10^{-10}$	$2.6 \cdot 10^{-11}$	$9.3 \cdot 10^{-13}$	$3.5 \cdot 10^{-14}$	$1.3 \cdot 10^{-15}$	$1.1 \cdot 10^{-16}$

TABLE II

OP COMPARISON BETWEEN THE ADAPTIVE SCHEME AND THE TRADITIONAL SCHEME

Ψ	2	4	6	8	10	12	14	16
Adaptive Scheme	$5.2 \cdot 10^{-3}$	$1.8 \cdot 10^{-4}$	$4.7 \cdot 10^{-6}$	$9.9 \cdot 10^{-8}$	$1.7 \cdot 10^{-9}$	$2.6 \cdot 10^{-11}$	$3.4 \cdot 10^{-13}$	$4.1 \cdot 10^{-15}$
Traditional Scheme	$6.0 \cdot 10^{-1}$	$7.6 \cdot 10^{-1}$	$8.5 \cdot 10^{-1}$	$9.0 \cdot 10^{-1}$	$9.3 \cdot 10^{-1}$	$9.5 \cdot 10^{-1}$	$9.7 \cdot 10^{-1}$	$9.8 \cdot 10^{-1}$
ρ_s	15.97	12.98	11.22	9.97	9.00	8.21	7.54	6.96

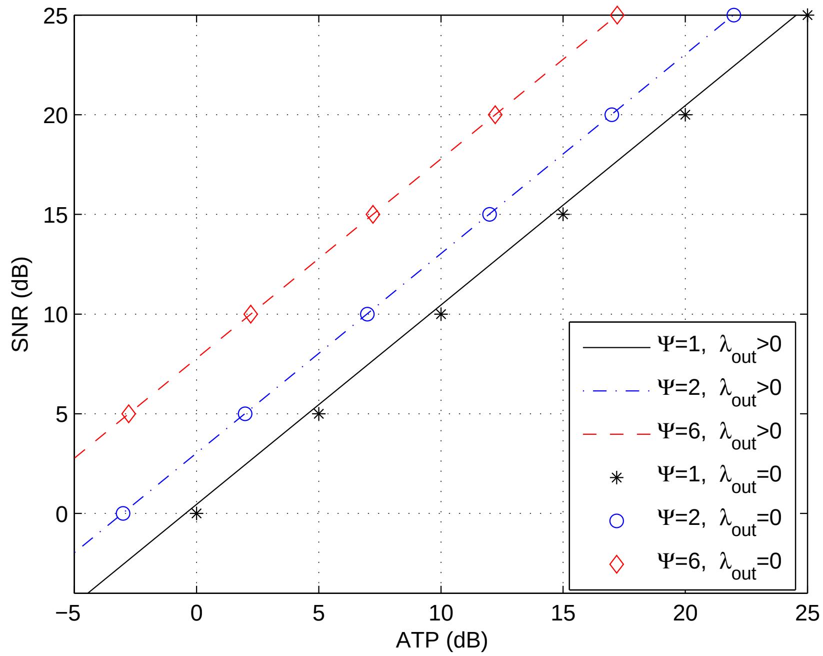

Fig. 1. Average transmit power for different short term constraint SNR

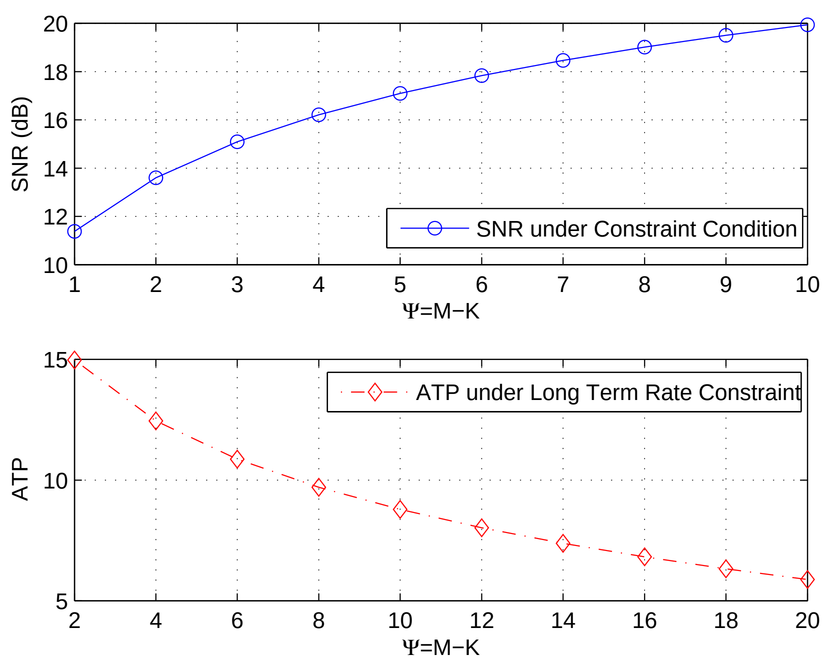

Fig. 2. SNR under constraint condition and ATP under fixed long term constraint SNR for different Ψ

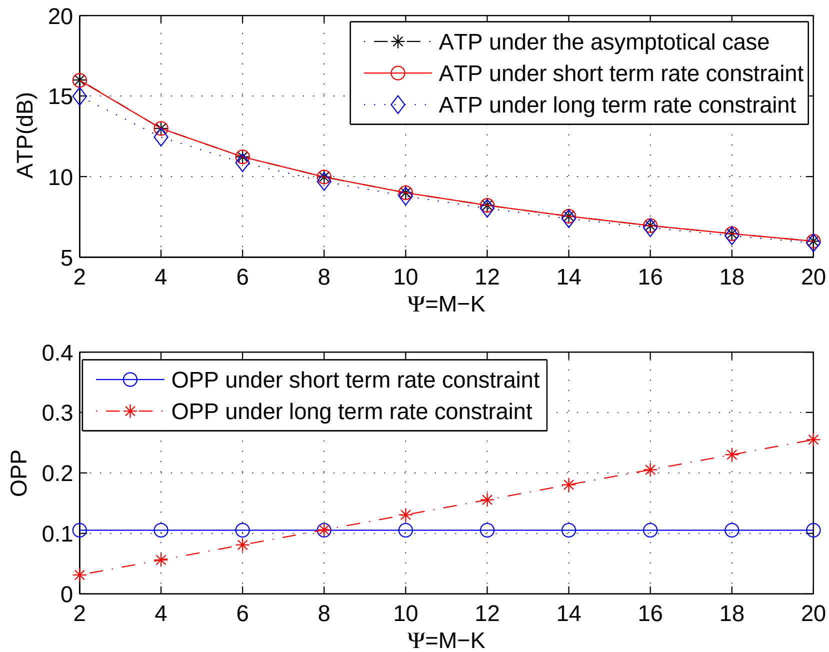

Fig. 3. ATP comparison between the two short and long term schemes for different Ψ

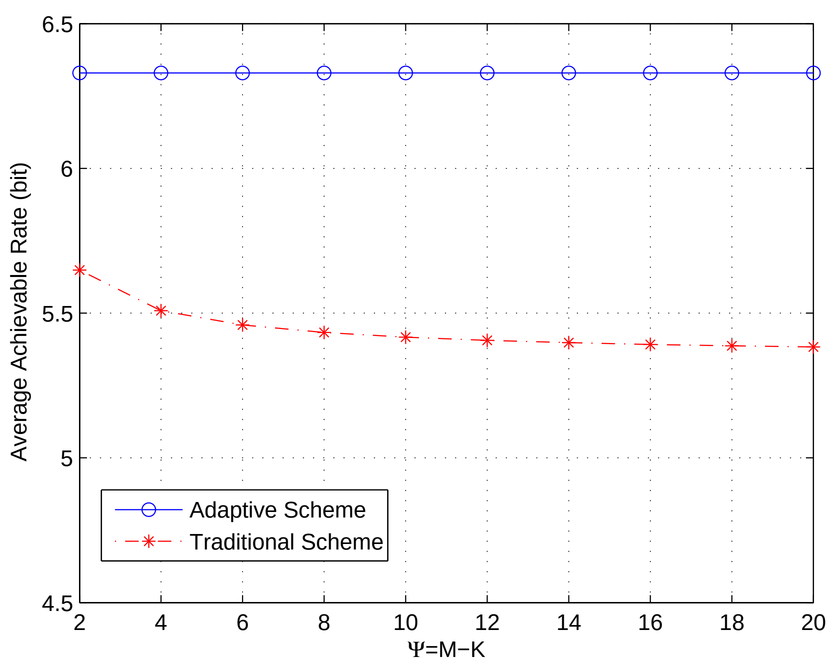

Fig. 4. Long term rate comparison between the adaptive scheme and the traditional scheme

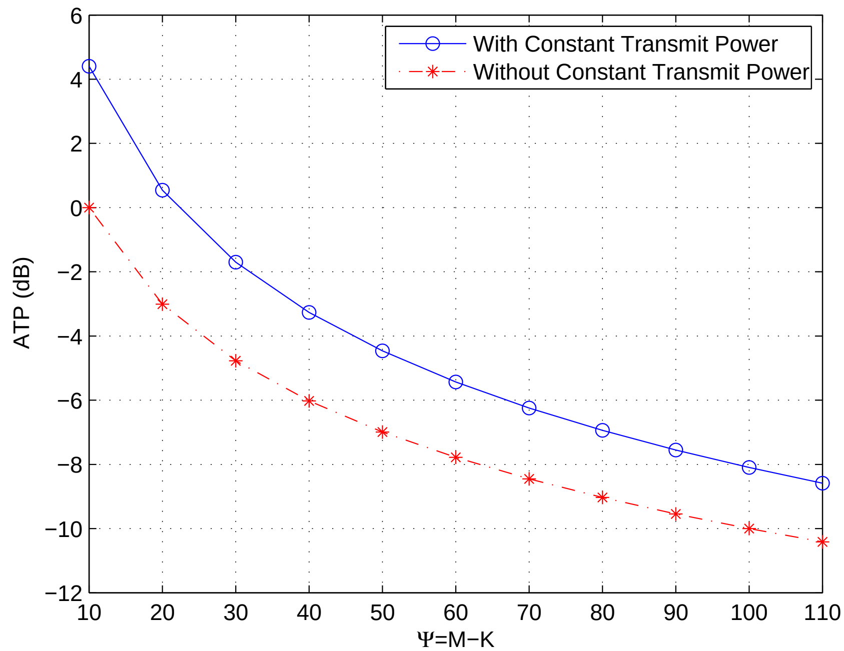

Fig. 5. Average transmit power with short term rate constraint and constant transmit power for different Ψ

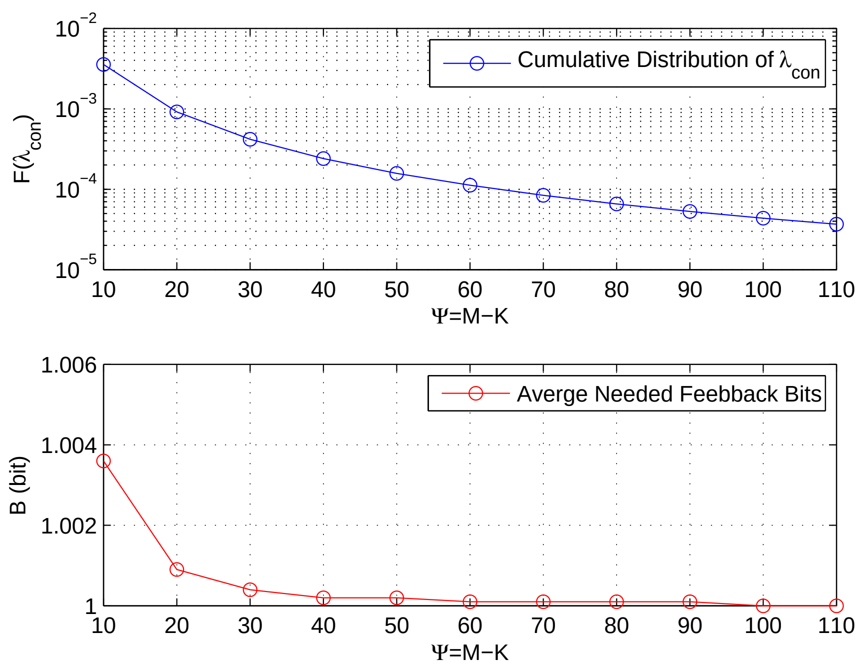

Fig. 6. Probability distribution of $\overline{\lambda}_{\text{con}}$ and average needed feedback bits $\overline{B}$ for different Ψ

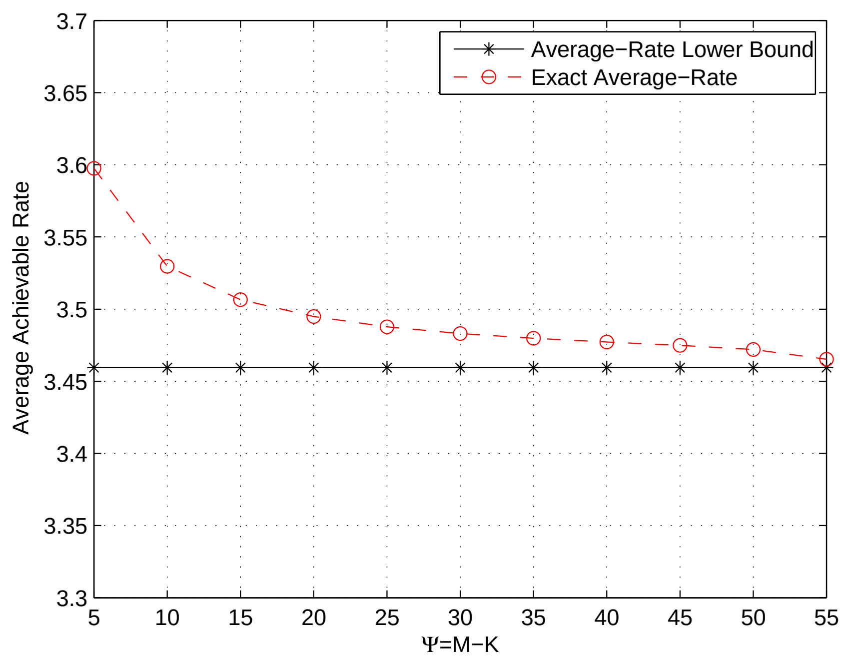

Fig. 7. Average long term rate and its lower bound with constant transmit power for different Ψ