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## INVESTMENT UNCERTAINTY UNDER STRINGENT UK DECARBONISATION TARGETS

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### Abstract

The legally binding UK greenhouse gas emissions targets were in part derived using deterministic variants of UK MARKAL. UCL, under UKERC, has developed a new two-stage stochastic variant, which provides additional near-term insights for policy makers under future uncertainties. Significant uncertainties remain as to the level of effort required by the UK to avoid dangerous warming. In this paper, the use of cumulative CO<sub>2</sub> targets, equivalent to 80% and 90% reductions by 2050 allow comparison between current UK policy and the modelled results. Deterministic scenarios result in steep near-term decarbonisation in part due to a social discount rate, proportional to the cumulative target. Under uncertain future cumulative CO<sub>2</sub> emission targets, the cost of the hedging strategy is related asymmetrically to the weighting of future scenarios. When the cumulative CO<sub>2</sub> targets are equally weighted, the near-term investment strategy lies close to that of the deterministic 90% CO<sub>2</sub> target. This indicates that steep near-term decarbonisation is important given exponentially rising cumulative welfare costs with increasingly stringent cumulative emission targets.

### 1 Introduction

Meeting legally binding targets for the reduction of CO<sub>2</sub> emissions is now the key priority for policy makers. The Climate Change Act [1] enshrined in law an 80% reduction in UK greenhouse gas emissions from 1990 levels by 2050. Various UK Government policies have been implemented to begin the transition towards a low-carbon future [2]. Energy system modelling using UK MARKAL has been an important part of the process, giving policy makers an understanding of and insights into the trade-offs between the different technologies and abatement options between sectors.

The use of the MARKAL framework has attracted criticism. For example, bottom-up energy-economic models, such as UK MARKAL, require a large number of assumptions, many of which are external to UK policy makers. Examples include future fossil fuel prices, technological availability and energy service demands. Previous studies using UK MARKAL conduct exhaustive sensitivity analysis in order to address these uncertainties. The deterministic approach in previous versions of UK MARKAL result in multiple decarbonisation trajectories. As such, it is difficult to resolve these multiple trajectories into a single and decisive near-term action. Understanding of future uncertainty is increasingly seen as important in order to hedge against the risk of a high-cost future, while minimising the cost of this hedge. Given the long lead-times and lifetimes of generation plant, near-term decision-making is crucial to avoid technology lock-in to a non-optimal energy system.

Under UKERC, the energy systems modelling team at UCL have developed a stochastic variant to UK MARKAL. By considering uncertainty, Stochastic MARKAL provides a more plausible representation of socially optimal investment decisions under severe carbon constraints in an uncertain future. Key insights from this model are the generation of a single hedging strategy, identification of robust options under a range of uncertainties, the resulting feasibility of meeting long-term targets and metrics such as the expected value of perfect information (EVPI).

This paper aims to further the understanding of near term investment strategies under CO<sub>2</sub> decarbonisation pathways in the UK to provide insights to policy makers. This approach combines both deterministic and stochastic variants of UK MARKAL with a variety of carbon emission trajectories, cumulative and percentage reduction targets.

## **2 Methodology**

This section describes UK MARKAL and the new stochastic variant. A discussion of current UK policy follows explaining the difference between stock and flow perceptions of CO<sub>2</sub> emissions.

### **2.1 UK MARKAL**

UK MARKAL is a well-regarded tool used for analysis of the UK energy system and has provided the basis for several previous studies [3,4,5]. UK MARKAL is the UK specific version of the MARKAL paradigm of environment-energy-economic-engineering models. Operating in standard mode, UK MARKAL seeks to minimise the total discounted cost of the energy system, encompassing both supply and demand technologies and commodities. The MARKAL Elastic Demand variant optimises a partial equilibrium, maximising consumer and producer surplus, analogous to social welfare.

UK MARKAL incorporates the resources, transformations, conversions, losses, and demands found in the UK energy system. Through time, exogenously imposed constraints, such as a CO<sub>2</sub> emission cap or commodity tax, influence a move from one energy landscape to another. The least cost trajectory followed involves investment in new energy transformation and conversion processes from a database of technologies. These technologies represent existing and future uses of new or existing resources, also subject to availability constraints and costs, in order to meet forecast energy service demands. The exogenously specified energy service demands are defined with demand elasticities, enabling a response to changes in price. The objective function, the maximisation of consumer and producer surplus, seeks to balance the size of the energy system with the energy required to meet projections of energy service demands. Policies are modelled through constraints on resources and technologies, or through taxes and subsidies on emissions or energy carriers.

Key outputs from UK MARKAL include metrics such as welfare cost, the system cost or ‘marginal price’ of commodities as well as the changing structure of the resulting energy system over time. To view the key assumptions and structure of the UK MARKAL model see [6].

## 2.2 Stochastic MARKAL

Stochastic MARKAL enables the quantification of future uncertainty to inform near-term investment decisions. The two-stage stochastic variant computes a hedging strategy by minimising the sum of the expected costs of the recourse strategies that correspond to multiple predefined future states of the world. Each state of the world is weighted according to the likelihood of that state of the world occurring. In other words, the hedging strategy minimises exposure to the variation in cost that could occur given the state of the worlds defined. See [7] for a description of the objective function.

Key assumptions made when defining a stochastic model run include the period in which uncertainty is resolved – the *resolution period*, the choice of future ‘states of the world’ and the weightings assigned to these states of the world. The choice of resolution period affects the length of the hedging strategy and recourse strategies, which in turn affect the freedom available to the model to generate useful insights. Interest in a particular period will also dictate the resolution period. Understanding the choice of future states of the world is important when analysing the model output. The states of the world reflect a predefined perspective of future possibilities and the hedging strategy is computed solely from these future states of the world. Lastly, the value of the weighting assigned to each state of the world directly affects its contribution to the hedging strategy

In addition to the above assumptions, the two-stage stochastic variant of MARKAL is limited to representation of a maximum of nine future states of the world. It is possible to explore combinations of variables, although this is constrained by the above limit. Secondly, the two-stage stochastic structure limits the representation of uncertainty to one period. A current multi-core desktop PC solves stochastic problems in less than 10 minutes.

The stochastic objective function used in this paper is that of expected cost criterion, which calculates a weighted average cost. This assumes that we are confident in the probability weightings assigned to each state of the world and that the investor is risk neutral. An alternative is to use an expected utility criterion, where the variance in expected cost is minimised according to a *risk aversion factor* specified by the modeller [7]. However, this requires the use of a non-linear solver and has not yet been fully tested at UCL.

As perfect foresight models that assume optimal behaviour and competitive markets, UK MARKAL and Stochastic MARKAL represent ‘best-cases’ for the achievability and a lower bound for the costs of long-term energy policies.

## 2.3 Definition of scenarios - the rationale for cumulative emission targets

In this initial study, the discount rate is set at a social level of 3.5% and fossil fuel prices are derived from UK Government projections [8]. Existing UK policy is included although the EU-ETS is excluded.

Table 1 lists the major assumptions in the latest version of UK MARKAL

Key parameter	Description									
Conversion factors	GDP deflators: (2000 = 100), 2005 = 116.9, 2008 = 123.9 (Source: <a href="http://www.berr.gov.uk/files/file41491.pdf">www.berr.gov.uk/files/file41491.pdf</a> ) Exchange rates: \$/£ = 1.8, €/£ = 1.4 (Source: <a href="http://www.hmrc.gov.uk/exrate/usa.htm">www.hmrc.gov.uk/exrate/usa.htm</a> ) Physical: 1 MTOE = 11.6 TWhr = 48.9 PJ									
Discount and hurdle rates	Global discount rate of 3.5% Hurdle rates are implemented on conservation in residential and commercial sectors (8.75%); and transport technologies (7.0% for public transport, 7.0% for hydrogen private transport, 5.25% for battery and methanol private transport)									
Fossil Fuel Price 2000-2050 (2000£/GJ)	Oil	4.12	9.35	6.41	6.87	7.33	7.79	8.25	8.25	
	Gas	1.93	4.47	4.47	4.85	5.16	5.47	5.70	5.70	
	Coal	0.91	2.97	2.23	1.62	1.62	1.62	1.62	1.62	1.62
Biomass Imports	Import constraint (increasing geometrically to 1260PJ by 2050)									
Energy service demand elasticities	25% maximum reduction. Own price elasticity range from 0.25 to 0.61 dependent on specific ESD									
Policy variables	As of 2008 Energy Bill. Note, no EU-ETS price in reference case									
Technologies	As in [6] with additions including biomass CCS, infrastructure costs by scale and distance, additional district heat/CHP options, up-rated CCS costs and efficiencies, restricted capacity (30%) of residential heat pumps and night storage									

In providing recommendations to the UK Government, the Committee on Climate Change (2008) used a climate model, MAGICC [11], to explore the likely global emissions scenarios required to limit the risk of an increase over pre-industrial temperatures of 2°C. From this analysis, they established that a likely global reduction in greenhouse gases of approximately 34-46% below 1990 levels will be required by 2050.

The analysis extrapolated two greenhouse-gas reduction strategies for the UK, modelled in this study. Note that UK MARKAL models the CO<sub>2</sub> emissions arising from energy use and does not consider the other greenhouse gases. This analysis therefore excludes greenhouse gas emissions other than those from CO<sub>2</sub>. The two scenarios are equivalent to the Interim and Intended budgets both with an 80% reduction by 2050. The Interim budget requires a 29% reduction in CO<sub>2</sub> on 1990 levels by 2050 and the Intended budget, a 40% reduction in CO<sub>2</sub> from 1990 levels. The 29% reduction is a recommended minimum to achieve by 2020, although still stretching. For example, the reference case in UK MARKAL, incorporating no new climate policy, shows a 16% reduction in CO<sub>2</sub> levels by 2020 before a rapid increase post 2020. A reduction beyond this requires significant restructuring of the energy system.

Recent developments in climate science continue to stress the importance of large reductions in CO<sub>2</sub> emissions. It is therefore unlikely that the UK CO<sub>2</sub> emissions reduction target can be less than 80%, especially given the legal weight of the target. However, it may be necessary to increase the ambition of the greenhouse-gas emission target to 90% or more by 2050. The uncertainties that contribute to whether a further reduction will be necessary are as follows:

- i. Considerable uncertainty surrounding the quantity of abatement possible in non-CO<sub>2</sub> gases beyond 2020, especially in the agricultural sector

- ii. Considerable uncertainties in the measurement of non-CO<sub>2</sub> gases – future CO<sub>2</sub> emissions may have to fall further if undercounting has resulted in greater greenhouse gas concentrations than expected
- iii. Global near-term emissions are more than expected, e.g. if a global deal is delayed and business as usual continues
- iv. Sectors not yet covered by policy e.g. international shipping and aviation grow above forecast levels

The current recommendations focus on the trajectory required to meet the future target. However, the modelling that supported these choices of trajectory did not take the uncertainty in carbon target into account in an integrated manner. A decarbonisation trajectory should be:

- i. Feasible – sensible build rates, realistic demand reduction and internally consistent
- ii. Least-cost – equilibrium between energy system investment and demand reduction
- iii. Resilient to future uncertainties

The use of stochastic MARKAL allows all three points to be met. Exogenous bounds on technology investment and the quantity of demand reduction constrain the model to realistic operation. The second two points are in-built to the stochastic MARKAL formulation.

Another problem is to choose how to formulate the problem in modelling parlance. There is a related tension between current UK policymaking, which treats the problem of carbon emissions as a flow problem, and recent studies [12,13,14] that argue for treating CO<sub>2</sub> emissions as a stock problem. In MARKAL, cumulative emissions targets allow the model more freedom, specifically temporal, to achieve a potentially more optimal least-cost solution than under annual CO<sub>2</sub> emission constraints. Depending on the discount rate, this optimal solution either results in a delay or brings forward CO<sub>2</sub> abatement in comparison to an annually constrained solution. Figure 1 shows the difference between cumulative and annually constrained scenario, in which early action significantly undercuts the annually constrained scenario to avoid the deep and expensive cuts required in the final periods. Note that while cumulative emissions are the same for both scenarios the cumulative welfare costs are ~£10 B higher in the annually constrained scenario (~8% higher than the least-cost solution). Both [15] and [16] obtained similar results.

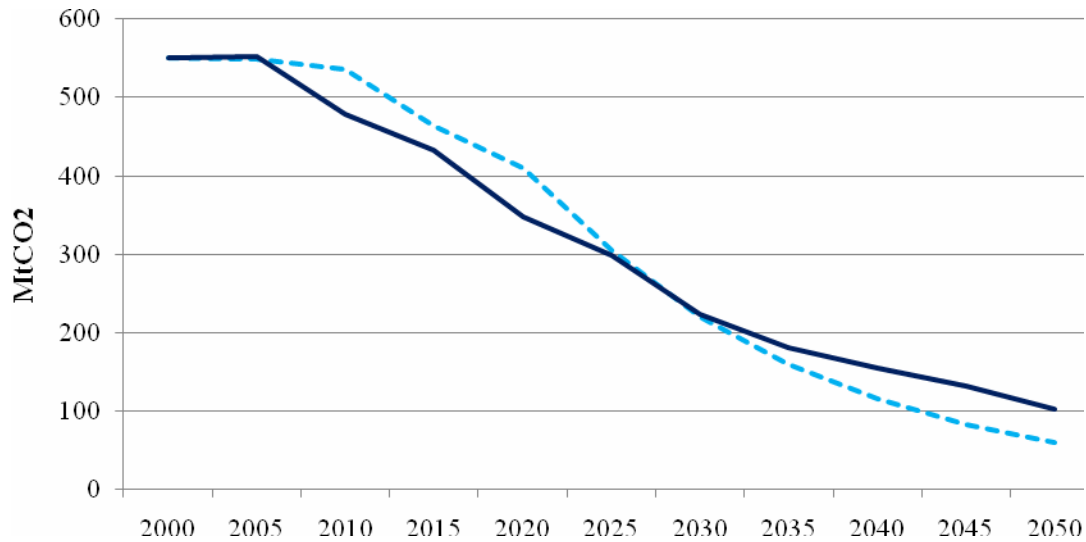


Figure 1 shows an annually constrained CO<sub>2</sub> trajectory and a cumulative equivalent under a social (3.5%) discount rate.

There are two main issues with a cumulative emission constraint. Firstly, the cumulative constraint considers only the years within the model horizon e.g. 2000-2050, while a stock treatment of CO<sub>2</sub> requires a longer view of the emissions. As a result, under cumulative constraints, the model tends to avoid deep cuts in later years, which may not be realistic moving beyond 2050 due to the problems of technology lock-in excluding further decarbonisation. Secondly, a cumulative emissions constraint results in significant early action from 2010 to 2025, relaxing the decarbonisation beyond 2030. The depths of the cuts in the 2010 period are unlikely to be realistic given the present UK trajectory. A solution to the first issue is to use an energy system model that runs beyond 2050, and UCL are currently performing analysis using a global model called TIAM-UCL that runs to 2100. However, the uncertainties across all parts of the model (technology cost, availability, fossil fuel price, global availability of carbon permits etc.) are so large, that it may be more effective to combine some of these into an uncertainty in carbon trajectories to 2050. This is the approach used by this paper – to investigate optimal near-term strategies given the uncertainties in future decarbonisation pathways. Future work will place this in a global context, with UK and global decarbonisation strategies compared and contrasted.

Despite the above issues, it seems sensible to use cumulative targets for modelling given the relevance to the stock perspective of current climate research. It is useful, however, to assess the feasibility of the decarbonisation trajectory for each scenario. Assuming an equal percentage reduction in CO<sub>2</sub> emissions from 2009 to 2020 and 2020 to 2050, the following table indicates the approximate differences in cumulative CO<sub>2</sub> emissions between the different trajectories under the Interim and Intended budgets suggested by the Committee on Climate Change.

Table 2 shows approximate estimates of cumulative CO<sub>2</sub> emissions of four decarbonisation trajectories derived using the UK MARKAL model

Cumulative CO <sub>2</sub> emissions 2000 to 2050 (BtCO <sub>2</sub> )	29% CO <sub>2</sub> reduction by 2020 (Interim)	40% CO <sub>2</sub> reduction by 2020 (Intended)
80% CO <sub>2</sub> reduction	~19.0	~17.9
90% CO <sub>2</sub> reduction	~17.2	~16.3

### 3 Results

#### 3.1 Comparison between cumulative (stock) and annual (flow) treatment of UK CO<sub>2</sub> emissions in UK MARKAL

Figure 2 shows the relationship between cumulative emission target scenarios from 2000 to 2050 and welfare cost in the UK MARKAL model. The blue line shows the cumulative welfare costs from a series of increasingly severe cumulative constraints. The black cross indicates the cumulative welfare cost of the current 29/80 target. It lies just above the blue line, which represents the least-cost frontier below which no scenario can sit, unless some model constraint were relaxed. Note that this frontier is only relevant for cumulative emissions from 2000 to 2050. It is evident that there is an exponential relationship between cumulative CO<sub>2</sub> emission target and the cumulative welfare cost of that reduction. Note that the model fails to solve with a CO<sub>2</sub> constraint below 12 BtCO<sub>2</sub> due to the paucity of abatement options available.

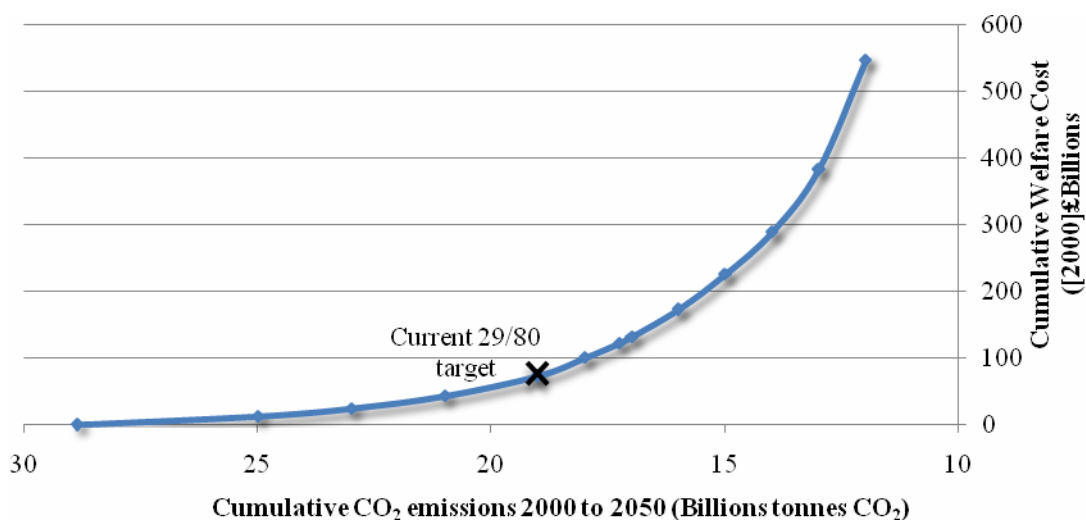


Figure 2 shows the relationship between cumulative welfare cost and cumulative emissions from 2000 to 2050

As shown in Figure 3, there is little difference in cumulative welfare cost between the cumulative constrained scenarios (represented by the dashed line) and the annual targets suggested by the Committee on Climate Change (black crosses with bold text). This suggests that the cumulative welfare cost is less sensitive to changes in decarbonisation trajectory than it is to the cumulative emissions reduction. However, the shape of the decarbonisation trajectory does have a limited impact, as shown by the deviation from the dotted line of each

of the crosses, although feasible scenarios (i.e. those that meet the criteria suggested in section 2.3) may lie in a restricted range above this line.

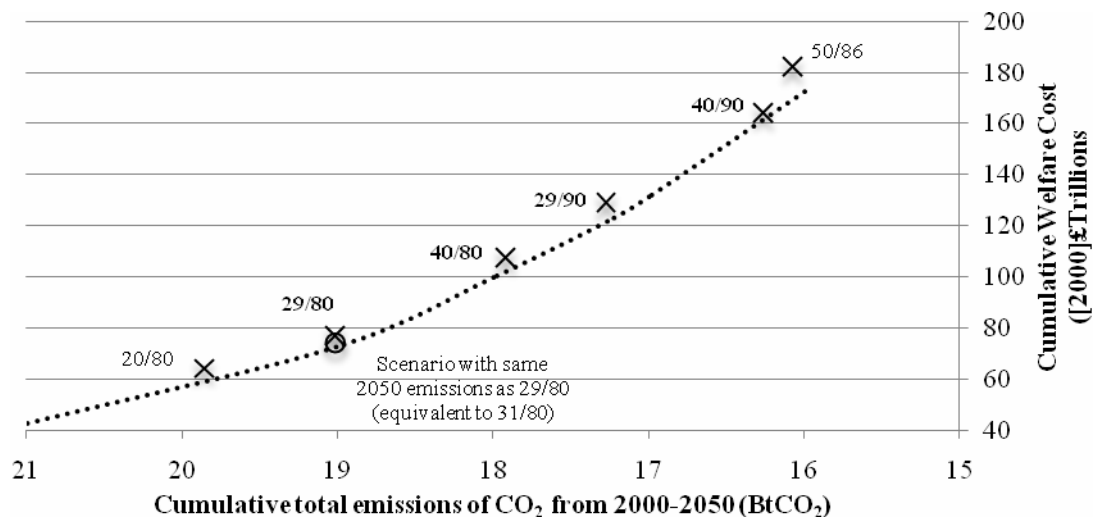


Figure 3 shows the relationship between cumulative CO<sub>2</sub> emissions from 2000 to 2050 for the theoretical optimum cumulative constraint scenarios and the proposed carbon targets (in bold).

Cumulative emissions scenarios of 17.5 BtCO<sub>2</sub> incur an approximate doubling in welfare cost in comparison to a 20 BtCO<sub>2</sub> scenario. A scenario below 16 BtCO<sub>2</sub> is roughly treble the cost of a 20 BtCO<sub>2</sub> scenario. Note that it is not possible using Figure 3 to estimate the cost of moving from one scenario to another (e.g. 29/80 to 29/90) in the case that new information is received. It is highly likely that the welfare cost would be higher than the increase shown, due to the delay in action. Even though the model follows the same CO<sub>2</sub> trajectory, subtle changes in the generation mix may result in extra costs if a policy shift is then required. This highlights the inflexibility of previous deterministic scenarios and suggests that the choice of a near-term trajectory that resolves the conflict between future decisions would be useful.

The magnitude of these welfare costs indicate that moving from one scenario to another is non-trivial. The remainder of this paper establishes stochastic scenarios that explore the optimum near-term investment strategy given the uncertainty in the future CO<sub>2</sub> targets.

### 3.2 Exploration of uncertainty surrounding emissions reductions and insights for near-term policy using the stochastic variant of UK MARKAL

In the scenarios presented below, cumulative CO<sub>2</sub> targets are derived from deterministic annually constrained emissions scenarios. This is to aid comparison with the 2980 and 2990 scenarios suggested by the Committee on Climate Change.

#### 3.2.1 Hedging strategies

Figure 4 shows the CO<sub>2</sub> emissions from 2 deterministic cumulative emission scenarios equivalent to 29/80 and 29/90 scenarios. The blue lines represent the stochastic scenario, while green lines the deterministic scenarios. Prior to 2020, the single blue line represents the stochastic hedging strategy, while there are two least-cost trajectories for the deterministic



scenarios. The model takes significant early action to address the more stringent 2990 equivalent target. Note that despite the equal probability weighting assigned to each of the future scenarios, the hedging strategy lies significantly lower than the average, only slightly above the deterministic 2990 trajectory. This indicates the dominance of the extra cost of increasing the stringency of the cumulative emissions target.

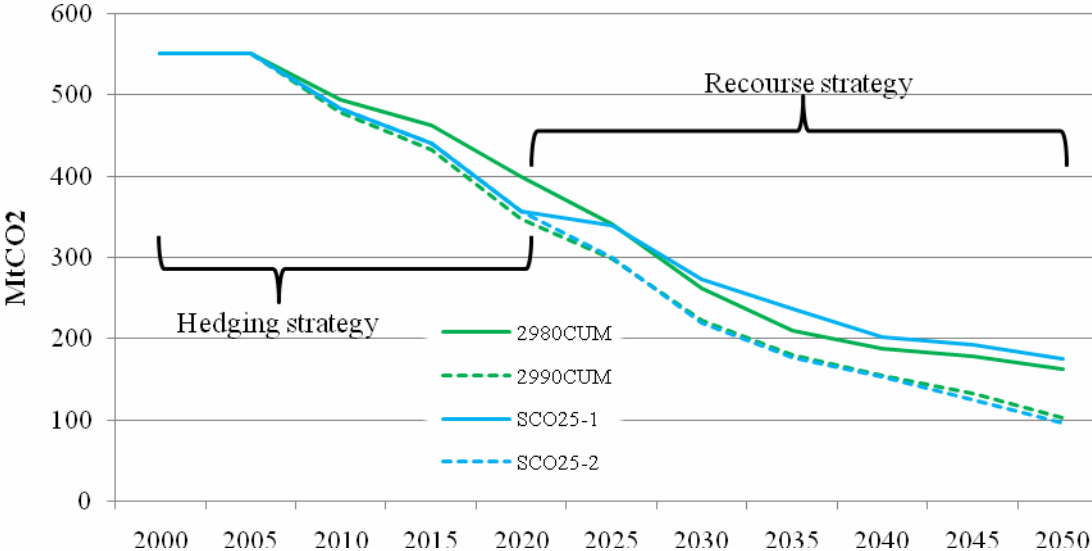


Figure 4 shows a stochastic scenario and deterministic scenario with cumulative emissions equivalent to 29/80 and 29/90

The stochastic scenario also demonstrates the trade off between periods resulting from the hedging strategy. For example, if in 2020, uncertainty over emissions reveals that a move to the more stringent scenario is not necessary, the model leads to an increase in CO<sub>2</sub> emissions in the recourse strategy over the 2980CUM deterministic scenario due. In contrast, if the move to the more stringent scenario is necessary, the hedge means that only minor extra effort is necessary.

By varying the probability weightings assigned to the future states of the world, it is possible to view the response of the model under different levels of uncertainty. For example, when the modeller assigns a 90% weighting in favour of the 2980 scenario, the model makes a smaller hedge towards the 2990 scenario. If this assessment is incorrect and the 2990 scenario is instead required, the energy system must then decarbonise more steeply in the recourse strategy. Figure 5 shows the inter-temporal trade-offs made across the full range of weightings.

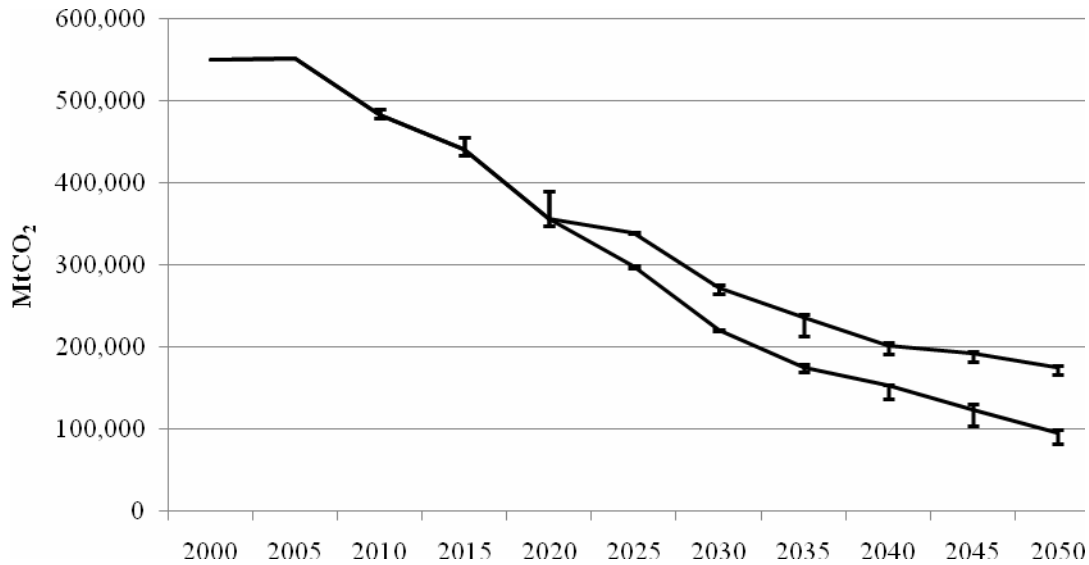


Figure 5 shows the results over a range of weightings applied to the future states of the world. Error bars show the range of movement in each period.

### 3.2.2 The cost of adjusting between scenarios (the cost of guessing incorrectly)

Using the Expected Value of Perfect Information (EVPI) where:

$$EVPI = \text{Expected Cost}_{\text{HEDGE}} - \sum \text{PROB}_{\text{SOW}} \times \text{Expected Cost}_{\text{SOW}}$$

We are able to measure the cost handicap introduced by the uncertainty (equation adapted from [7]). An alternative reading is that the EVPI is the value to the investor now, of knowing which deterministic trajectory to follow [17].

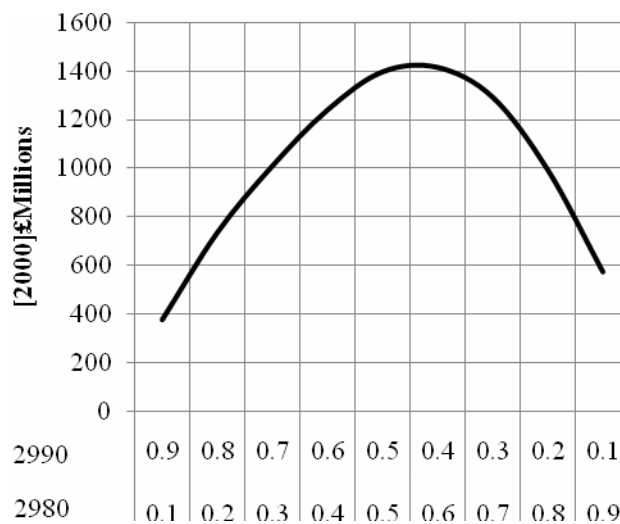


Figure 6 shows the response in EVPI to weighting

The EVPI of the stochastic scenarios varies between ~£0.5B and ~1.4B. The scenario with the larger uncertainties, i.e. those that attribute roughly equal weightings to either of the future scenarios, experience a larger EVPI than those in which the prior weighting is more certain. This follows, seemingly logically from the stochastic formulation. Note that the chart is asymmetric, indicating that the value of information regarding future costs is rather less if hedging towards the 2990 scenario, because the expected cost of ‘guessing wrong’ is less than in the opposite scenario.

#### **4 Conclusions**

Considering carbon dioxide emissions as a stock problem, the results show that the optimal decarbonisation pathway differ significantly from those proposed by the Committee on Climate Change. Under a social discount rate of 3.5%, early action results in severe CO<sub>2</sub> reduction to 2020 with a less severe emission cut from 2030 to 2050. However, this analysis ignores the potential issue of residual emissions in 2050. Future work, including modelling over longer timescales, will be required to assess this properly.

The two-stage stochastic variant of UK MARKAL allows a single near-term investment strategy to respond to multiple future uncertainties. This paper developed a simple stochastic scenario, assessing two potential cumulative emissions pathways equivalent to the interim pathways through 2020 to either an 80% or 90% reduction by 2050. The model showed that, under equal weighting of the outcomes, an optimum near-term investment strategy (i.e. one that minimises expected cost of the scenario and assumes a risk neutral investor) lies very close to the severe 2990 equivalent decarbonisation pathway. This is equivalent to a 40% reduction in CO<sub>2</sub> on 1990 levels by 2020. The cost of this hedging strategy (EVPI) is around £1.3B.

An analysis of future weightings shows that a range of decarbonisation pathways are optimal that depend on the confidence in one or the other pathway. These range from a 33% to 41% reduction in 2020 with a corresponding 86% to 70% reduction in 2050. The cost of the hedging strategies varies from £0.5B and £1.4B and follows a normal distribution with a minor left skew. The most expensive hedging strategy corresponds to the scenario with near-equal weighting 60% towards the 2080 equivalent scenario. Generally, hedging strategies become less costly as one moves away from equal weighting scenarios.

Finally, the development of the stochastic variant of UK MARKAL allows the model to address future uncertainties in a systematic manner. This paper has shown that under a social discount rate, significant near term decarbonisation is necessary given uncertainty in future cumulative emission targets.

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