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Measuring the inertial properties of a tennis racket

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Abstract

Simple and bifilar pendulum were used to measure the mass moments of inertia of three tennis rackets. The pendulum setups were filmed using an off-the-shelf camcorder, with a stopwatch in view to provide timing data. The measurement accuracy was assessed using calibration rods of known mass moment of inertia. The simple pendulum method was found to be most accurate (<1.0 % difference to theoretical value) when a square profile rod was used as a pivot. The bifilar pendulum was found to be very accurate (0.0% difference to theoretical value) but sensitive to non-parallel support wires. A Babolat Racket Diagnostic Centre (RDC) was assessed using four calibration rods, of known mass moment of inertia. Measurement agreement was greater than 99.0% for mass moments of inertias within the range of 220 – 380 kg·cm^2.

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Keywords: Tennis Racket; Moment of Inertia; Bifilar; Pendulum; Experiment.

1. Introduction

The performance of a tennis racket is, in part, determined by its mass and mass moments of inertia (MMOI). Brody et. al (2002) describe how an ideal racket exists for every player, but that the best specification is ultimately a compromise. For example, when considering a racket's mass and MMOIs, a heavier, high MMOI racket will be more powerful (Cross (2006), Cross and Bower (2006)), but requires more effort to swing. Lighter, low MMOI rackets are more maneuverable, but less stable and forgiving with mishit shots (Brody (2002).

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1.1. The mass moments of inertia of a tennis racket

Figure 1 illustrates the three MMOIs of a tennis racket – transverse, lateral and polar. Transverse MMOI is measured about the lateral, in-plane axis of the racket. It is used for racket selection and customization, as this rotation is synonymous with a tennis swing. However, a tennis racket has six degrees of freedom – three axes of translation and rotation. Therefore, modifying one MMOI will, in all likelihood, affect the other MMOIs.

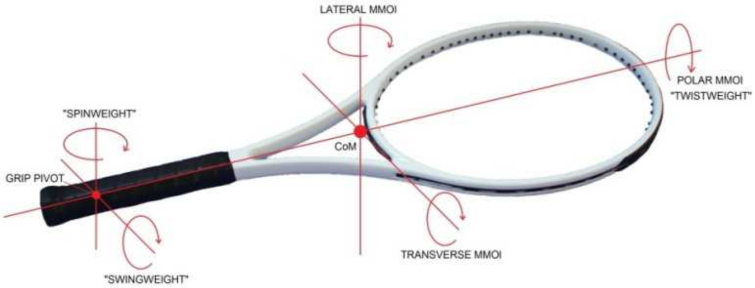

Fig. 1. The mass moments of inertia of a tennis racket.

The MMOIs are the measures of resistance to rotation about the principal axes through the center of mass (COM). However, the 'tennis standard' of MMOI is taken at an origin in the racket handle, four inches from the racket butt. MMOIs can be translated using the 'parallel axis theorem':

$$I_2 = I_1 + ML^2 \quad (1)$$

Where I_1 is the measured MMOI (kg·cm^2), I_2 is the translated MMOI (kg·cm^2), M is the mass of the racket (kg) and L is the distance between the two locations (cm). The translated transverse, lateral and polar MMOIs are colloquially known as 'swingweight', 'spinweight' and 'twistweight', respectively.

Swingweight customization is nowadays common for players seeking to maximize performance. Brody (1985) commented that, prior to his investigation; the importance of MMOI was unknown in tennis, despite a relatively simple measurement concept. Since then, several authors have expanded on his work (Brody (2002), Goodwill (2002), Fauteux-Brandt (2013), Brody (2005)). Choppin et. al. (2013) found a relationship between the transverse MMOI, swing style ('flat' or 'wristy') and the position of the 'ideal point' – the impact location of maximum outbound ball velocity. However, he advised caution before modifying the transverse MMOI, as it would likely change the lateral and polar MMOIs also.

This paper investigates the measurement techniques to measure the three MMOI's of a tennis racket.

1.2. Measuring the mass moments of inertia

MMOIs are measured by setting the racket up as a simple (Brody (2002), Goodwill (2002), Brody (2005)). or bifilar pendulum (Fauteux-Brandt (2013)) and measuring the oscillation period (figure 2). Goodwill measured transverse MMOI at the racket butt using a simple pendulum and the equation:

$$I_{BUTT} = \frac{T^2 g M L_1}{4\pi^2} \quad (2)$$

where T is the period of oscillation (s), g is gravity (cm·s^2), M is the mass of the racket (kg) and L_1 is distance from the pivot to the racket's COM (cm). He used a thin, light cylindrical rod attached at the racket butt to act as a pivot.

Fauteux-Brault (2013) measured the MMOI of a cricket bat using a bifilar set up and the equation:

$$I_{bifilar} = \frac{T^2 g M b^2}{4\pi^2 L_2} \quad (3)$$

where b is the horizontal distance from one support wire to the racket's COM (cm) and L_2 is the length of the support wires (cm) (see figure 2).

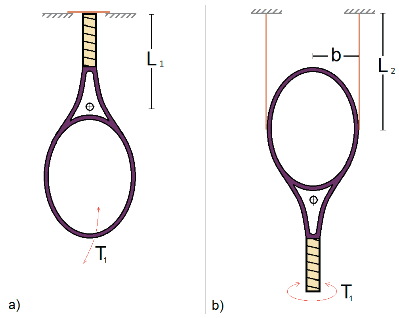

Fig. 2. Measuring MMOIs with a) simple pendulum (swing perpendicular to page) and b) bifilar pendulum (rotation about long axis).

Commercial devices, (e.g. the Babolat Racket Diagnostic Centre (RDC) (Babolat, France)), use a horizontal clamp arrangement to measure swingweight. Two springs, attached at a pivot four inches from the racket butt, create simple harmonic motion, from which the oscillation period can be measured. These machines use electronic timing to calculate swingweight.

1.3. Measurement error

Brody (1985, 2002) and Goodwill (2002) timed between 10 and 80 oscillations by eye. Timing error decreases with the number of oscillation measured, but Goodwill conceded that 50 oscillations was a practical limit. Jardin (2007) investigated the optimization of MMOI measurement using a bifilar pendulum jig from the aerospace industry. He explored several sources of error, concluding that:

- The objects COM should be centered to the support wires. An off-center COM would result in 'precessional' motions to the rotation required. Assuming competent set up, this effect is negligible.
- Using a high wire length to wire separation ratio reduces any error associated with non-parallel wires.
- The mass and strain dynamics of the support wires, and any effect from anchoring the wires, will have negligible effect for 'long' supports wires.

Measurement error in devices, such as the Babolat RDC, can increase over time, due to drift in the electronic components. Calibration rods can be used to periodically check the machine, but these must be sourced separately.

2. Method

Calibration rods of known MMOIs were measured with a Babolat RDC, simple and bifilar pendulum. The swing-, spin- and twistweights of three tennis rackets (ITF Development racket, Gamma Big Bubba racket and Tecnifibre TFlight 66 racket) were also measured with the pendulum and RDC.

Each pendulum test comprised five trials of 50 oscillations. The trials were filmed using a Panasonic HDC-SD9 handycam, filming at 25 frames per second. A Samsung Galaxy S4 smartphone was used to provide timing data to 1/100th second, with the screen kept in-shot for the 50 oscillations. The oscillation periods for each trial were then calculated via playback of each video. For the simple pendulum, the MMOIs at the pivot were calculated using (2) with the MMOI at the COM and swingweights and spinweights (rackets only) calculated using (1). For the bifilar pendulum, the MMOIs were calculated using (3).

2.1. Babolat RDC method

Four calibration bars were made using thinned walled aluminium tubing. The specifications of the rods were calculated using the equation:

$$I = \frac{1}{12}m[3(r_1^2 + r_2^2) + l^2] \quad (4)$$

where *m* is the mass of the rod (kg), r_1 is the outer radius (cm), r_2 is the inner radius (cm) and *l* is length (cm) (specifications can be found in Table 1). The four rods and three rackets were measured in a Babolat RDC, with mean values taken from 10 trials. Rotating the racket by 90° permitted spinweight measurements.

2.2. Simple pendulum method

A stainless steel rod was used to create a calibration object with an MMOI of 162.59 kg·cm^2 and swingweight 369.21 kg·cm^2, calculated using the equation:

$$I_{rod} = \frac{1}{12}ml^2 \quad (5)$$

where *m* is the mass of the rod (0.57600 kg) and *l* is length (29.00 cm). Mass was measured using a Mettler-Toledo PR503DR scale. Lengths were measured using a 100 cm steel rule.

Three pivots – two cylindrical steel rods of diameter 0.20 cm, and lengths 15.00 cm and 10.00 cm and a square profile rod with sides 0.40 cm and length 10.00 cm - were trialed using Goodwill's method (2002). Each pivot was attached to the calibration object in turn, using approximately 5 gr. of glue applied with a glue gun. The square rod was orientated to rest on one corner. Once secured, the pivot was rested on two lengths of Bosch aluminium profile, with a separation of approximately 8.00 cm. The square pivot was then attached to the butt of each racket in turn. The pivot was positioned in two orientations to measure both transverse and lateral swings.

2.3. Bifilar pendulum method

A stainless steel rod was used to create a calibration object with an MMOI of 7.97 kg·cm^2, calculated using the equation:

$$I_{rod} = \frac{1}{12}m(3r^2 + l^2) \quad (6)$$

where *m* is the mass of the rod (0.11350 kg), *r* is the radius (0.40 cm) and *l* is length (29.00 cm). The radius was measured using a set of Mitutoyo 500-191U veneer calipers.

The rod was attached to two 60.00 cm length of 'Mylon' wire (0.03 cm diameter and mass 0.008 kg) with a spacing of 27.20 cm – an inset of 0.90 cm at either end of the rod. The wires were anchored by wrapping excess length around the threads of two M10 bolts. The bolts were attached to Bosch aluminium profile with the same spacing of 27.20 cm. Two further pendulum arrangements were measured, with the anchor positions first moved apart by 1.0 cm (0.5 cm each side) and then moved closer together by 1.0 cm. The three rackets were then set up, using the widest string holes of the racket frame to tie off the support wires. The M10 bolts were repositioned to the racket widths for each test. The racket was twisted about its long axis to induce oscillations.

3. Results

Table 1 shows the specifications of the four calibrations bars, theoretical swingweights and the measured swingweights from the Babolat RDC.

Table 1. Calibration rod specifications, theoretical swingweight and mean Babolat RDC swingweight measurements

	Bar 1	Bar 2	Bar 3	Bar 4
Mass (kg), m	0.48348	0.49961	0.54226	0.58578
Outer radius, r_2 (cm)	1.91	1.91	1.91	1.91
Inner radius, r_1 (cm)	1.58	1.58	1.58	1.58
Length, l (cm)	49.00	50.70	55.00	59.50
COM to grip pivot, (cm)	14.34	15.19	17.34	19.59
Theoretical MMOI, (kg·cm^2)	97.5	107.8	137.5	173.7
Theoretical swingweight, (kg·cm^2)	**196.9**	**223.1**	**300.6**	**398.5**
Mean Babolat RDC swingweight, (kg·cm^2)	**201.1**	**224.2**	**299.8**	**404.9**

Figure 3 shows the Babolat RDC measured swingweights plotted against the difference between RDC and theoretical values, for the four calibration bars. A 2nd order polynomial fit is plotted with two linear plots to represent ±1% error.

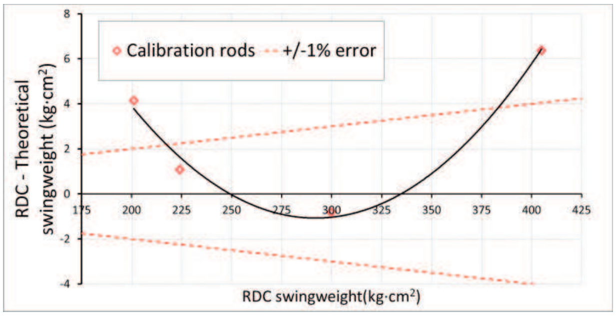

Fig. 3. Babolat RDC error values for four calibration rods with ±1% error plotted as dashed lines.

Table 2 shows the swingweight and spinweight for the three rackets measured with the Babolat RDC.

Table 2. Babolat RDC swingweight and spinweight measurements for three rackets

	ITF	Big bubba	Tecnifibre
RDC swingweight (kg•cm2)	333	445	198

	ITF	Big bubba	Tecnifibre
RDC spinweight (kg•cm2)	348	463	208

Table 3 shows the simple pendulum swingweight measurements for the calibration object using three pivots. The measured values are compared to the theoretical value for the calibration object.

Table 3. Simple pendulum swingweight measurements for the calibration object using three different pivots with standard deviations and comparison to theoretical values.

Pivot type	Long, cylindrical	Short, cylindrical	Square
Mean swingweight (kg·cm2)	362.51	362.59	366.44
Standard deviation, σ	0.10	0.23	0.21
Mean - theoretical swingweight (kg·cm2)	-6.71	-6.62	-2.77
% difference	**-1.8%**	**-1.8%**	**-0.8%**

Table 4 shows the simple pendulum swingweights and spinweight measurements for the three rackets. The values are compared to the measurements from the Babolat RDC shown in table 2.

Table 4. Simple pendulum swingweight and spinweight measurements for three rackets using the square pivot with standard deviations and comparison to Babolat RDC values.

	ITF	Big bubba	Tecnifibre
Mean swingweight (kg•cm2)	331.64	437.62	196.16
Standard deviation	0.32	0.56	0.26
Mean - RDC (kg•cm2)	-1.36	-7.38	-1.84
% difference	**-0.4%**	**-1.7%**	**-0.9%**

	ITF	Big bubba	Tecnifibre
Mean spinweight (kg•cm2)	346.98	455.61	206.57
Standard deviation	0.64	0.17	0.20
Mean - RDC (kg•cm2)	-1.02	-7.39	-1.43
% difference	**-0.3%**	**-1.6%**	**-0.7%**

Table 5 shows the bifilar pendulum MMOI measurements for the calibration object and three rackets. The values for the calibration object are compared to the theoretical values.

Table 5. Mean bifilar MMOI measurements for the calibration rod and rackets with standard deviation and comparison to theoretical values.

	Calibration object			Rackets		
	Parallel strings	Anchors 1.00 cm out	Anchors 1.00 cm in	ITF	Big Bubba	Tecnifibre
Mean MMOI (kg·cm^2)	7.97	7.65	8.22	14.33	17.49	10.34
Standard deviation, σ	0.02	0.02	0.03	0.03	0.08	0.04
Measured - theory	0.00	-0.32	0.25	-	-	-
% difference	**0.0%**	**-3.8%**	**3.8%**	-	-	-

4. Discussion

The 2nd order polynomial fit in figure 3 predicts the device is capable of measuring MMOIs to within 1% of the theoretical value over an approximate range of 220 – 380 kg·cm^2. Two of the rackets measured were outside of this range.

Filming the simple and bifilar pendulum proved an easy method to maintain high repeatability across multiple measurements, proven by the low standard deviations for all measurements. The method also permitted repeat measurement of individual trials, helping to reduce human error in timing the pendulum swings.

The three pivots trialed with the calibration object and simple pendulum show that the error is highest when using a cylindrical rod. This may be because the cylindrical rod creates a rolling pivot point, raising the axis of rotation about the knife edge.

Comparing the racket measurements for the simple pendulum and Babolat RDC showed the greatest agreement for the ITF Development racket. The lesser agreement for the other rackets is probably due to the Babolat RDC being less accurate for measurements outside of the range stated above.

The bifilar pendulum proved very accurate for the calibration object, although the results show a marked increase in error when the support wires were not parallel. The measurements of polar MMOI for the rackets are assumed to be of the same accuracy to the calibration object, as there was no theoretical value to compare against.

5. Conclusion

A simple and bifilar pendulum can be used in conjunction with a video camera to measure the MMOIs of a calibration object to within 1% of the theoretical value. The simple pendulum method used by Goodwill (2002) shows greatest agreement when a square pivot is used. The bifilar pendulum provides a novel measure of the polar moment of inertia of a tennis racket, but care must be taken to ensure the support wires are parallel. The Babolat RDC is an effective method to measure the MMOIs of a tennis racket, but accuracy is subject to drift in the electronic components – especially for MMOIs values at the extremes of the measurement range.

References

Brody, H., 1985 The moment of inertia of a tennis racket. The Physics Teacher, Apr.1985, pp. 213-216

Brody, H., Cross, R., Lindsey, C., 2002. The Physics and Technology of Tennis, Racquet Tech Publishing, Solana Beach, CA

Brody, H., 2005 A new twist of the twistweight of a tennis raquet. Racket Sports Industry magazine, Feb. 2005

Choppin, S., 2013. An investigation into the power point in tennis. Sports Engineering, 16(3), pp.173–180.

Cross, R., 2006. Raw Racket Power, 2006. Racket Sports Industry magazine, Feb. 2006

Cross R, Bower R., 2006. Effects of swing-weight on swing speed and racket power. Journal of sports sciences, 2006; 24:1 pp. 23-30.

Fauteux-Brault, S., 2012. Methods for modelling and validating stress distribution in cricket bats using finite element. Research project manuscript, Centre for Sports Engineering Research, Sheffield Hallam University.

Goodwill, S,. 2002. The dynamics of tennis ball impacts on tennis rackets. PhD thesis, Department of Mechanical Engineering, University of Sheffield

Jardin, M., 2007. Optimized measurements of UAV mass moment of inertia with a bifilar pendulum. Proceedings of Anerican Institute of Aeronautics and Astronautics, Guidance, Navigation and Control Conference and Exhibit, 2007, Hilton Head, South Caolina.