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# THE STABILITY OF ARTICULATED TIPPING TRAILER UNITS 

by Simon G Pickering, BEng

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## CONTENTS

Page No
TABLE OF CONTENTS ..... (i)
ABSTRACT ..... (iii)
ACKNOWLEDGEMENTS ..... (v)
NOTATION ..... (vi)
CHAPTER 1 INTRODUCTION
1.1 Size of the problem ..... 1
1.2 Trailer construction ..... 2
1.3 Theoretical model ..... 4
CHAPTER 2 GENERAL BACKGROUND
2.1 General description of tipping trailer units and operation ..... 7
2.2 Loading modes ..... 10
2.3 Design features ..... 13
2.4 Roll over models ..... 17
2.5 General discussion and conclusions ..... 18
CHAPTER 3 BASIS OF THE THEORETICAL MODEL
3.1 Introduction ..... 30
3.2 Co-ordinate system ..... 32
3.3 Assumptions ..... 33
3.4 Theoretical basis ..... 37
3.5 Outline of the overall solution procedure ..... 55
CHAPTER 4 FINITE ELEMENT ANALYSES
4.1 Introduction ..... 64
4.2 Finite element meshes ..... 66
4.3 Design investigations ..... 72
4.4 Loading of finite element meshes ..... 73
4.5 Flexibility influence and stiffness matrices ..... 75
4.6 Discussion ..... 78

## CHAPTER 5 DESIGN INVESTIGATIONS

5.1 Introduction975.2 Reference chassis ..... 98
5.3 The effect of using changes on stability ..... 102
5.4 The effect of increasing the torsional stiffness of the cross members ..... 103
5.5 The effect of reducing the trailing arm stiffness ..... 104
5.6 Discussion ..... 105
CHAPTER 6 GENERAL DISCUSSION AND CONCLUSION
6.1 Technique ..... 152
6.2 Theoretical model ..... 152
6.3 Implications ..... 156
6.4 Requirements of further investigations ..... 157
REFERENCES ..... 159
APPENDIX A AIRBAG STIFFNESS
A1 Introduction ..... 162
A2 Theoretical prediction of airbag force/weight relationship ..... 162
A3 Experimental procedure ..... 163
A4 Results ..... 164
APPENDIX B TYRE STIFFNESS
B1 Theory ..... 172
B2 Experimental equipment and procedure ..... 174
B3 Results ..... 176
APPENDIX C COMPUTER PROGRAM AND FLOW CHART
Subroutine descriptions ..... 188
APPENDIX D PROGRAM USER GUIDE
D1 Program execution ..... 271
D2 Changing program variables ..... 173
D3 Changing flexibility, stiffness and influence matrices ..... 274
APPENDIX E FINITE ELEMENT ANALYSES AND FLEXIBILITY, INFLUENCE AND STIFFNESS MATRICES
E1 Introduction ..... 278
E2 Flexibility matrices ..... 278
E3 Influence matrices ..... 279
E4 Stiffness matrix ..... 280


#### Abstract

When an articulated tipper unit is being loaded or is tipping, it is unlikely to be standing on perfectly level ground. Also, the centre of gravity of the load is unlikely to be in the centre of the body. Hence the loads carried by the suspension and tyres on one side of the tipper will be greater than those on the other side. This uneven loading will cause the tyres and suspension on one side of the tipper unit to deform more than those on the other side. It will also cause the chassis to deform; the twisting about its longitudinal axis being the most significant mode of deformation. As a result of thase deformations caused by the uneven loading, the position of the centre of gravity will be shifted even further towards the more heavily loaded side. This will cause even more uneven loading and further deformations.


Under stable conditions a situation will exist at which the position of the centre of gravity, the deformations and the forces transmitted through the system are compatible. Instability, resulting in roll-over would occur if the overall centre of gravity of the load, body, chassis etc. were to fall outside the area bounded by the contact of the wheel with the ground, before a stable condition was reached.

Many factors influence the roll stability. To increase stability, an understanding of the influence of components of the lorry on the stability is required. In order to achieve this, a theoretical model of an articulated tipper was developed which will allow roll-over predictions to be made for a given lorry in likely attitudes. In this model dimensions and stiffness of the lorry components can be altered to assess their influence on roll stability.

The previous theoretical roll-over models were based on lumped mass systems, representing various parts of the lorry inter-connected by compliant elements. Certain flexibilities such as the tyres, suspension units, etc. could be obtained from the respective components manufacturers but the tractor and trailer chassis flexibilities are unknown. To overcome this problem the flexibilities were obtained from full scale static tilt tests. This is a very expensive undertaking, providing a limited means in which to assess those elements of trailer design which are important in improving stability, without further recourse to more tilt tests. It was decided that the finite element method should be used to model the tractor and trailer, in order to determine the important deformations. Once the finite element model is created it is relatively straight forward to make changes to the structure. Hence an assessment of component contribution to roll stability can be undertaken relatively inexpensively.

Whilst a vehicle operator should always endeavour to discharge the payload with the vehicle standing on level ground, practical situations arise where this is not possible. This may be due to the absence of level ground or poor judgement by the operator, which may result in the vehicle being tipped on a lateral ground slope. As a result of this, the maximum ground slope angle considered for the theoretical model is limited to eight degrees, as this position is at least twice the severity of ground slope on which a vehicle should normally be tipped.

For each trailer design, the magnitude of the load, position of the load, ram length and ground slope can be varied in any combination. Four payloads and up to nine payload positions are considered, varying the ground slope from 0 to 8 degrees and varying the ram length from 2 to 8 meters. Also, three further chassis configurations, based on the reference chassis were modelled to investigate the contribution of important component flexibilities on roll stability.

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## NOTATION

## Forces

$\mathrm{A}_{\mathrm{L}}, \mathrm{A}_{\mathrm{R}} \quad$ left and right airbag forces, see Fig. 3.5
$A_{R 1}$ and $A_{R 2}$ forces at the top and bottom of the right hand airbag of the 1st suspension, see Fig. 3.5
$A_{R 3}$ and $A_{R 4} \quad$ forces at the top and bottom of the right hand airbag of the 2nd suspension, see Fig. 3.5
$\mathrm{A}_{\mathrm{RS}}$ and $\mathrm{A}_{\mathrm{R} 6}$
forces at the top and bottom of the right hand airbag of the 3 rd suspension, see Fig. 3.5
$\underset{A_{R 4}, A_{R S}, A_{R 6}}{A_{R}}\left(A_{R 3}\right.$,
column oí forces applied to the right hand airbags
$\underset{\sim}{B}\left(B_{y}, B_{z}\right)$
body force vector
$\underset{\sim}{\mathbf{F}},{\underset{\sim}{\mathbf{F}}}^{\prime},{\underset{\sim}{\mathbf{F}}}^{*}$,
column matrices of all important chassis forces
$\underset{\left(G_{L 1}, G_{L 2}\right.}{ }, G_{L 3}$,
${\underset{\sim}{L}}_{L}\left(H_{L x}, H_{L y}, H_{L z}\right) \quad$ left hinge force vector
${\underset{\sim}{\mathrm{H}}}_{\mathrm{R}}\left(\mathrm{H}_{\mathrm{Rx}}, \mathrm{H}_{\mathrm{Ry}}\right) \quad$ right hinge force vector

L
$\underset{\sim}{\mathbf{P}}\left(\mathbf{P}_{\mathrm{x}}, \mathbf{P}_{\mathbf{y}}, \mathbf{P}_{\mathbf{z}}\right)$
payload force vector
$\underset{\sim}{\mathbf{R}}\left(\mathbf{R}_{\mathrm{x}}, \mathrm{R}_{\mathrm{y}}, \mathrm{R}_{\mathrm{z}}\right)$
ram force vector
$\mathrm{T}\left(\mathrm{T}_{\mathrm{L}}, \mathrm{T}_{\mathrm{R}}\right)$
$\underset{\sim}{W}\left(W_{x}, W_{y}, W_{v}\right) \quad$ wind force vector
$\underline{W}_{T}\left(W_{T y}, W_{T z}\right) \quad$ tractor self-weight vector
${\underset{\sim}{\mathrm{W}}}_{\mathrm{c}}\left(\mathrm{W}_{\mathrm{cy}}, \mathrm{W}_{\mathrm{cz}}\right) \quad$ chassis self-weight vector

Position Vectors (relative to origin of $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ coordinate system)

| $\underline{\text { b }}\left(\mathrm{b}_{\mathrm{x}}, \mathrm{b}_{\mathrm{y}}, \mathrm{b}_{\boldsymbol{z}}\right)$ | position of the centre of gravity of body |
| :---: | :---: |
| L ( $\mathrm{p}_{x}, \mathrm{p}_{y}, \mathrm{p}_{2}$ ) | position of the centre of gravity of payload |
| $\underline{\sim}_{1}\left(h_{l x}, h_{l y}, h_{10}\right)$ | position of the ieft hinge |
|  | position of the right hinge |
| $\underline{\sim}_{\text {r }}\left(\mathrm{r}_{\mathrm{bx}}, \mathrm{r}_{\mathrm{b}}, \mathrm{r}_{\mathrm{b}}\right)$ | position of the bottom of ram |
| $\underline{\sim}_{1}\left(r_{x}, r_{t y}, r_{v i}\right)$ | position of the top of ram |
| $\underset{\sim}{w}\left(w_{x}, w_{y}, w_{z}\right)$ | position of the centroid of wind pressure |
| $\xrightarrow{\text { ¢ }}$ | position of the top of the ram relative to the bottom of the ram |

## Displacements

$\underset{\sim}{u},{ }_{\sim}^{u}$
${\underset{\sim}{u}}^{u_{k}}\left(u_{b l}, v_{b l}, w_{b}\right)$
$\underline{u}_{\mathrm{hr}}\left(\mathrm{u}_{\mathrm{hr}}, \mathrm{v}_{\mathrm{hr}}, \mathrm{w}_{\mathrm{hr}}\right)$
$\underline{u}_{t}\left(u_{t r}, v_{t}, w_{r t}\right)$
$\mathrm{v}_{\mathbf{1 1}}$ and $\mathrm{v}_{\mathrm{az}}$
$\mathbf{v}_{\mathbf{3}}$ and $\mathrm{v}_{\mathrm{A}}$
$v_{2 s}$ and $v_{\mathbf{a} 6}$
column matrix of all important chassis displacements
left hinge displacement vector right hinge displacement vector
bottom of ram displacement vector
y components of displacement of the upper and lower, right hand airbag ends of 1st suspension, see Fig. 3.5
y components of displacement of the upper and lower, right hand ends of the 2nd suspension, see Fig. 3.5
y components of displacement of the upper and lower, right hand ends of the 3rd suspension, see Fig. 3.5
 the right hand airbags
vertical displacement of cantilever beam tip rotation of cantilever beam tip hinge bar rotation about X axis, see Fig. 3.1 slope of the ground, see Fig. 3.1 hinge bar rotation about Y axis, see Fig. 3.1

## Others

$\mathrm{a}, \mathrm{b}$ dimensions of the rigid frame attached to the tip of the cantilever, see Fig. 3.2

E

I
$\left[I^{(\alpha)}\right],\left[I^{(b)}\right],\left[I^{(c)}\right] \quad$ influence matrices relating the important chassis displacements to the important chassis forces for different airbag conditions
$\left[I^{(a)}\right],\left[I^{(b)}\right],\left[I^{(c)}\right] \quad$ influence matrices relating the tie bar forces to the important chassis forces for different airbag conditions
$\left[I^{(n)}\right],\left[I^{(b)}{ }_{G}\right],\left[I^{(c)}\right]\left[I^{(d)}\right]$ influences matrices relating the ground/tyre reaction forces to the important chassis forces (or displacement) for different airbag conditions
$\left[I^{(b)}\right]$
influence matrix relating the right hand airbag displacements to the important chassis forces
[K]

1
$l_{0}$
n

Q
$\mathbf{X}, \mathbf{Y}, \mathbf{Z}$
$\beta$
inclination of the body relative to the $x$ axis
$\boldsymbol{\gamma}$
$\delta$
angle between the vectors ${\underset{\sim}{r}}$, and p
vector product
scalar product
$+$
vector addition
vector subtraction

## CHAPTER ONE

## INTRODUCTION

### 1.1 Size of the problem

Articulated tipping units are used in many different industries for transporting many different types of payloads. In 1984 Keen et al [1] estimated that the number of vehicles having tipping facilities (articulated and rigid) was 120,000 with articulated tippers accounting for $15 \%$. Following the amendments to the Construction and Use Regulations, which came into effect on $1^{\text {st }}$ May 1983, the maximum gross vehicle weight of articulated tipper units was increased from 32 to 38 tonnes. Due to the increase in payload capacity, articulated tipping trailers have become more popular.

Information from official sources on the extent of tipper lorry rollovers during discharge of loads is limited to those where fatalities and or serious injuries result, as notification of non injury type incidents are not mandatory. A survey of hauliers by Keen et al [1], with a combined fleet of 600 tipper, found that the number of static rollovers accounted for $0.75 \%$ of the total fleet. Scaling up suggested an annual occurrence of 900 static rollovers for the UK!

Many factors influence the roll stability of articulated tippers, while they are discharging their payloads. To increase stability, an understanding of the influence of components of the lorry on stability is required. This work describes a theoretical model of an articulated tipper which will allow rollover prediction to be made for a
given lorry in likely attitudes. In this model dimensions and stiffnesses of the lorry components, payload magnitude and position body attitude and transverse ground slope can be altered to assess their influence on roll stability.

### 1.2 Trailer construction

There are two main categories into which tipper lorries can be grouped, rigid and articulated tipper units. The rigid tipper has a maximum gross vehicle weight of 30 tonnes, with the payload body and cab supported on one chassis, mounted on two steering wheels and four driving wheels. The articulated tipper has a maximum gross vehicle weight of 38 tonnes, with the payload body supported by the trailer chassis, mounted on six wheels and the driver's cab supported by the tractor chassis mounted on two steering wheels and two driving wheels. The tractor and trailer unit are detachable and are connected by a coupling known as the "fifth wheel".

This work is only concerned with 38 tonne articulated tipper units. A typical tractor and trailer unit is shown in Fig. 1.1.

### 1.2.1 Trailer design

The design of articulated tippers has been concerned more with the static strength of the vehicle, than its stability characteristics in either dynamic or tipping situations. The object of this work is to study the static tipping of articulated tipper units and develop a mathematical model which predicts the stability characteristics for a given lorry
configuration. The mathematical model incorporates all of the important tractor and trailer flexibilities which can be changed to enable an assessment of their contribution to stability. This will allow designs to be produced based on stability criteria as well as strength criteria.

### 1.2.2 Trailer structures

Lorry chassis structures may be comprised of all open section channel or two longitudinal I beams inter-connected by cross members to form a ladder type structure. Open section channel chassis are used for rigid vehicles to form a ladder type structure. Two longitudinal I beams inter-connected by cross members forming a ladder type structure are used for articulated tipper units.

Mild steel is widely used in the construction of the ladder type structures for articulated tippers. The cross members are constructed from closed sections (rectangular, circular etc.) as the section is efficient in resisting bending and torsional forces. The longitudinal members are constructed from I beams as the section is efficient in resisting bending forces due to vertical loads.

The payload is transported in the "body" which is usually constructed from an aluminium alloy. The body is supported at three points, during tipping, by two hinges at the rear of the chassis and by the lifting ram at the front of the chassis. The chassis has to predominantly resist twisting about the longitudinal axis and bending about the transverse axis during tipping.

The forces generated by the payload the self weight of the vehicle are transmitted through the suspension system, which are more commonly air suspension, and tyres to ground reactions.

Chapter 3 describes the method of modelling the air suspension in the theoretical model.

### 1.3 Theoretical model

No published material has been found relating to the stability of articulated tipping trailer units. Previous work has been undertaken on the dynamic stability of non tipping trailer negotiating corners. Theoretical models developed to predict vehicle stability in such conditions relies upon full scale experimental tests to provide flexibility data. This is not only an expensive undertaking, but makes further design investigations difficult to undertake without further experimental testing.

This work is only concerned with the static stability of articulated tipping trailer units.

The theoretical model developed to predict vehicle stability incorporates the following flexibilities.
(i) Tractor and trailer chassis structure
(ii) Tractor and trailer axles
(iii) Tractor leaf spring suspensions
(iv) Trailer air suspension and height control mechanism
(v) Tractor and trailer tyres

The important flexibilities (i), (ii) and (iii) are determined using the finite element method. General flexibility matrices are developed using the finite element results, to enable stability characteristics to be determined, for a given lorry configuration, for any payload magnitude and position and body attitude.

Chapter 4 describes the finite element model.

The flexibilities (iv) and (v) are determined experimentally as described in Appendices A and B and are included in the F.E. model.


Fig. 1.1 A typical articulated tipping trailer unit

## CHAPTER 2

## GENERAL BACKGROUND

### 2.1 General description of tipping trailer units and operation

A schematic of a typical tipping trailer unit is shown in Fig. 2.1, it consists of a tractor unit connected to a trailer unit by a fifth wheel coupling. The tractor unit provides the motive force to pull the trailer and in most cases; provides the hydraulic pressure to raise the trailer's body by means of a telescopic ram. A tractor unit has a flexible chassis with a pair of single steering wheels and a pair of dual driving wheels connected to the chassis by leaf springs, as shown in Fig. 2.2.

A trailer unit is comprised of a flexible chassis, suspension and tyres supporting a body used for transporting the payload, as shown in Fig. 2.3. The longitudinal members (usually I sections) with circular, rectangular, hexagonal or similar hollow cross-members. The cross-members are welded to the longitudinal members with the joints often stiffened by local reinforcing, as indicated in Fig. 2.4. Steels or aluminium alloys are used in the manufacture of the chassis.

There are two main types of suspension used on articulated tippers, ie. leaf springs and air suspension. Because the modern trend is towards the use of air-bag suspension systems, only this type of suspension is described. Typically there are three air-bags on each side of the trailer, connected to three pairs of wheels, as indicated in Fig. 2.5. Each air-bag is connected to a wheel by a "trailing arm",
which rotates about the pivot mounting and each wheel is attached to an axle tube, which in turn is attached to a pivot arm on the left and right side of the trailer, shown in Fig. 2.5. The distance between the middle axle and chassis frame, known as the 'ride height' is automatically maintained by the airbags; each set of three airbags, on the left and right hand sides of the chassis, are pressurised independently. Because there is a maximum pressure limit, during tipping, the ride height may not be maintained due to excessive loading on one side of the trailer.

The tyres now predominantly used in modern tipping trailer units are so called "super singles", which, as the name suggests, are one large tyre instead of two smaller tyres used side by side. Flexibility data exists so that these tyres can be modelled mathematically using three mutually perpendicular springs.

The telescopic ram used to raise and lower the body usually have four or five stages. These rams can be used to tip the body to between 40 and 50 degrees, relative to the horizontal. Since this type of ram has a relatively low resistance to transverse deformation, it can be regarded as a strut, with ball joints at the ram to body and ram to chassis connecting points, for modelling purposes.

The fifth wheel and king pin unit which connects the tractor unit to the trailer allows relative rotation between the tractor and trailer about the vertical and transverse axis. However, only limited relative rotation of the tractor and trailer can occur about the longitudinal axis. The relative longitudinal rotation, which is not intended for design purposes, is due to the inevitable relative vertical displacement which must exist for
practical purposes. This limited, relative vertical movement has a non-linear effect on the roll stability However, since this "play" can only contribute to the behaviour when roll-over is in progress it has not been taken into account in the modelling presented in this thesis.

The body, in which the payload is transported, is usually made from an aluminium alloy to minimise the weight of the trailer. It is attached to the chassis by two hinges at the rear and by the ram at the front. For the purpose of calculation, in this thesis it is assumed that the body is rigid and that during tipping operation, the body is only connected to the chassis via the hinges and the ram.

Standing on a firm, level site is the ideal situation in which to tip the body, in order to discharge the payload. However, tipping takes place under a wide range of conditions on sites such as building sites, landfill sites etc. and the vehicle operator may not always be able to find a firm and level place to tip. Hence, in practice some vehicles are tipped on uneven, inclined and soft ground and in these adverse conditions, the possibility of roll over exists. Although, with experience, an operator may be able to judge, by eye, whether a site is reasonably level. The operator cannot be sure that there isn't soft ground under one side of the trailer and that the payload will not discharge in a non-uniform manner, moving the payload centre of gravity from the longitudinal centre line of the body; a combination of these events may lead to vehicle roll over.

If the payload sticks in the body of a trailer at maximum tip angle, with the body fully tipped an operator may drive the vehicle, and then apply the brakes sharply, in an attempt to free the stuck payload. It has been suggested by Keen et al [1] that if a vehicle is driven over irregular terrain, under these conditions, a pendulum motion can occur and that this could lead to roll over; this problem is not specifically addressed in this thesis.

### 2.2 Loading modes

When stationary, the external forces experienced by a tipping trailer unit are due to the gravitational 'oad, wind loads and tyre reaction forces.

When standing on horizontal ground the gravitational forces only act normal to the ground and therefore there are no resultant transverse or longitudinal forces. However, when standing on a slope, components of the gravitational force many be experienced in the transverse and longitudinal directions.

The unsprung masses of a tipping trailer unit are defined as those masses which are not directly supported by the vehicles suspension. These masses on the trailer consist of the wheels, hubs and axles. For modelling purposes, the unsprung mass of each axle may be divided into two lumped masses located at the centre of each tyre. No detailed data, relating to the mass distribution, was obtainable for the tractor. Only the gross vehicle weight was obtained, and therefore the mass was taken to be entirely sprung for modelling purposes.

The trailer sprung masses, which are those supported by the trailer suspension, are the chassis mass and all of the accessories which are located on, or attached to, the chassis. The PAFEC finite element package [2] was used to obtain some of the information used in the model developed to assess the stability of a tipping trailer unit, part of the sprung mass is determined using the density and the volume of the material used for constructing the chassis. The remaining sprung masses were modelled by locating a lumped mass at the centre of gravity determined using the finite element package. The tractors sprung mass is comprised of the chassis, cab and engine, together with all the associated accessories.

The tractor's gross weight and the position of its centre of gravity were obtained from the information provided by the manufacturer. As no information was available relating to the distribution of the tractor mass, it was modelled using a single lumped mass, positioned at the centre of gravity of the tractor unit.

The present work is concerned with the stability of a vehicle tipping in a static position. Although there is no published information relating to this problem, work has been undertaken relating the stability of vehicles travelling round corners Sweatman et al [3], Kemp et al [4], Miller et al [5] and Isermann [6]. Where a vehicle travels around a corner of radius $\mathbf{r}$, speed $\mathbf{v}$ and mass m , a transverse force, acting in the radial direction, of magnitude $\mathrm{mv}^{2} / \mathrm{r}$ is induced. If this force should become large enough, the vehicle can roll over. This previous work has been concerned with predicting roll over under cornering conditions, by comparing results obtained from static tilt tests, where the concerning force $\operatorname{mv}^{3} \mathrm{r}$ was replaced by a
component of the gravitational force, ie. $\mathrm{mg} \cos \theta$, where $\theta$ is the transverse tilt angle and $g$ is the acceleration due to gravity.

In the present investigation the body and payload are assumed to be rigid. A single force, representing the body self weight is taken to act along its centre line, whereas a single force, representing the payload, may be taken to act at a suitable position, which can be defined by the userof the theoretical model. During the analysis, the position of the payload relative to the body remains unchanged, although the position of the body can change as the body/chassis contact points move due to deformation of the chassis, tyres, suspension etc.

Wind loading can produce significant forces, especially when the vehicle is at the maximum tip angle. A transverse wind force has been included in the theoretical model. However, the method for obtaining the magnitude and line of action of wind forces was outside the scope of the present work. Previous work by Hollis [7] undertaken on determining wind forces on crane structures, up to 20 m high, showed that the measured values could be as much as 100 to $150 \%$ greater than those predicted, shows that this problem is relevant to the tipping trailer situation.

Conventional leaf springs suspension exert forces on the chassis which are proportional to the suspension deformation. However, with air suspension the airbag exerts a combined force which is related to the sprung mass (including payload and body weight). Under normal conditions, the pressures on the left hand and right hand sets of airbags are increased or decreased independently to maintain a fixed "ride height" for the centre axle. However, once a maximum allowed pressure is reached
on one side, a valve closes and the set of three airbags compress under constant mass of gas conditions; the behaviour is taken to be adiabatic compression of the gas for modelling purposes, the airbag forces are treated as externally applied loads.

### 2.3 Design features

Although no research work, related to the stability of tipping trailer units, operating under tipping conditions, has been published, a number of experimental $[3,4,5,6$, $8,9,10]$ and theoretical $[3,4,5,6,10,11,12,13]$ investigations on stability under cornering conditions have been published.

A survey undertaken by Miller and Barter [5] found that operators were surprised at the lack of warning they had obtained before roll over occurred. This was put down to a lack of "feel" in the drivers cab for the trailer roll behaviour. Kemp, Chinn and Brock [4] indicated that design changes could improve roll stability performance. Although the greatest reduction in roll over accidents might be achieved by operators accepting that roll over is caused by travelling at too high a speed for a particular radius of turn, it is clear that designers have a significant role to play in minimising the possibility of roll over instability.

Some of the major details over which the design has control include:
(i) the tyre and suspension type and operating conditions.
(ii) dimensions and materials, which may be varied in order to increase the chassis stiffness, particularly the torsional stiffness about the longitudinal axis.
(iii) the body materials and construction, which can be altered to minimise weight and increase rigidity.

The tyre and suspension stiffness were found by Laird [10] to significantly effect roll stability. It was found that the tractor drive suspension and trailer suspension could significantly affect roll stability, while the tractor front suspension only has a moderate influence.

Encouraged by legislative procedures, air suspension systems have become more popular than conventional leaf springs. However, the design of air suspension systems is still relatively new and further significant improvements are likely to occur. Current suspension systems predominantly use a combination of traditional leaf springs and air suspension bags, as indicated in Fig. 2.5. Current air bag designs, only accommodate loading along their longitudinal axis. Therefore leaf springs have been used to resist the large multi-directional forces resulting from side scrub between tyres and road, bumping into curbs or verges etc. It has been estimated by Dickson-Simpson [14] that these forces can be more than three times the nominal axle load. The air suspension system, modelled in this thesis, see Fig. 2.5, consists of elliptic leaf springs, each hinged to the chassis at one end, and supported at the other end by an air bag. Each axle tube is then rigidly clamped to the middle of the left and right leaf springs. Simultaneous displacements of the left and right tyres are primarily accommodated by the air suspension bags. However, when
uneven loading on the tyres occurs (when cornering or tipping on uneven ground, etc.) the leaf springs and axle tube, which in effect form an anti-roll bar system, accommodate the uneven displacements.

Due to the trend towards the use of air suspension systems on commercial vehicles, height control valves have been developed to automatically govern the vehicles design ride height. This is accomplished by adjusting the pressure in the air bags in response to changes in vehicle loading which force the valve leverto move up or down as the chassis to axle spacing changes. A consistent design height is one advantage of air suspension over mechanical suspension. To maintain the design height while using a minimum amount of air, a time delay is incorporated into the design. The time delay feature prevents the valve from allowing air flow to or from the airbags during momentary changes in the chassis to axle spacing. In addition to the time delay, the valve may be biased to the centre position, so that it is slow to open but quick to close, thus eliminating the tendency for the valve to oscillate, when repeated small variations in load occur.

A typical control valve operation, described by Hillebrand [15], is shown in Fig. 2.6. The valve is designed with a minimal "dead band" shown in Fig. 2.6a. This is the neutral zone where the lever arm attached to the axle can move without allowing air to enter or leave the air bag. The "dead band" is kept to a minimum to maintain an accurate spacing between the axle and chassis and minimise air usage.

The valve is attached to the vehicle frame and the lever arm is connected to the middle axle only through a linkage. On tipping trailer units there are two height control valves for left and right air bags . When tipping it is assumed that the left and right airbags operate independently. As the valve is opened the air flow into the airbag is shown in Fig. 2.6b. As the valve closes the air flow into the air as is shown in Fig. 2.6c. This combination of delayed opening and rapid closing provides a biasing tendency towards the centre closed position.

An increase in the torsional rigidity of a typical trailer chassis about this longitudinal axis, was shown by Holmes [8] to improve its stability and handling. Also, by eliminating fifth wheel play, it was shown by Sweatinan and Mai [12] that the transverse acceleration could be increased by $4 \%$ before first wheel lift occurred.

The most important design parameters, effecting roll stability is obviously the height of the centre of gravity of the trailer. Sweatman and Mai [12] showed that for a typical trailer design, an increase in the height of the centre of gravity of $26 \%$ (1.95 m to 2.35 m ) caused the first wheel lift acceleration to reduce by $26 \%$. Also, they showed that increasing the trailer axle track width by $10 \%$ resulted in an increase in first wheel lift acceleration of $10 \%$.

In experiments conducted by MIRA, [16], a fifth wheel, which was constructed with a load cell, to measure vertical, longitudinal and transverse loads was used to determine fifth wheel dynamic loads during driving conditions. These loads were required in order to perform realistic long term fatigue tests under laboratory conditions. The typical and maximum forces measured are given in Table 2.1.

Since 1978, some trailer designs have been constructed using extruded sections welded to aluminium plates, to form a strong light weight box,Brock [17]. There are two basic types oí box design which are now commonly used. The first design is constructed from extruded " $U$ " shaped channels welded to a large rail forming the top rail of the body. Aluminum plate forms the skin of the body to contain the payload. The other design is constructed from extruded aluminium channels which form the side and floor supports, as shown in Fig. 2.7. These are welded to a large rail which forms the floor corners of the body. Aluminium plate is again used for the skin of the body. Finite element analyses [17] indicated that, for the latter design, the bottom pillar joints, shown in Fig. 2.7, were the most highly stressed joints.

The stiffness of the bottom rail and the way in which it is connected to the floor sections is intrinsic to construction strength. Fabrication procedures and the adoption of MIG welding procedures have enabled improved quality and enhance fatigue life performance.

### 2.4 Roll over models

A thorough literature survey and discussions with manufacturers of tipping trailer units has shown that all previous work on articulated roll stability has been directed towards non-tipping trailers, negotiating corners. The theoretical models, developed to predict the cornering roll stability, have all been based on unsprung and sprung masses, connected by compliant elements under the influence of gravitational and a model
transverse forces. In 1970)theoretical |was developed by Isermann [6] which
included the flexibilities of the tyres, the suspension and tractor unit chassis with a rigid trailer chassis. This was later refined by Kemp et al [4] and Sweatman et al [5] to incorporate a torsional stiffness associated with the trailer. One of the major draw backs with these models is that a theoretical estimation of the various flexibilities is required or, alternatively, a vehicle has to be experimentally tested under static rolling conditions so that measurements of various displacements can be obtained in order to give an indication of the trailers torsional stiffness etc. In 1984 Sweatman [12] developed a further model which reduced the number of variables to be measured experimentally to three, namely the percentage of wheel load transferred laterally in the trailer and the drive and steering axle groups of the tractor unit during a tilt test. Again the model is based on a set of constant masses interconnected by massless compliant elements.

The above models have been used to assess a number of design parameters in order to determine their effect on vehicle roll stability. Structural changes to an actual chassis would require expensive tilt tests to be undertaken, to enable accurate assessment of their effect on roll stability. Therefore, in order to make an initial assessment of possible design changes, these models have been found to be useful.

### 2.5 General discussions and conclusions

No information, which relates directly to the roll stability of articulated tipping trailer units in operation, has been published in the open literature. Although some of the results of work related to cornering roll stability is applicable in a general sense, to
static tipping roll stability, little or no information exists for the types of forces generated by a tipped vehicle and on the effect of vehicle design parameters on roll stability during this mode of operation.

Published theoretical models, relating to roll stability are all based on rigid masses interconnected by compliant elements. Application of these methods requires full scale experimental tilt testing, in order to determine important compliancies. In the present work, the finite element method is introduced to eliminate the need for static tilt tests. Also air bag suspension systems are modelled so that the forces generated by the tipped body, on the chassis, can be properly determined, in order to accurately model the roll stability of an articulated tipper unit. All of the important design parameters can be changed in order to assess the influence of them on the roll stability, without the need for static tilt tests. Also, although the required finite element analyses are simple small deformation, linear elastic analyses, the computer program written to perform the stability calculations is iterative and can cope with the large movements of the payload.


Fig. 2.1 A schematic diagram of a tipping trailer unit

| GVW | 19500 | $17000^{* *}$ |
| :--- | ---: | :---: |
| Front Axle | 6700 | $6700^{*}$ |
| Rear Axle | 13000 | 10500 |
| GCW/GTW | 44000 | 38000 |



A 1080 UNLADEN. 980 LADEN B 3110 UNLADEN


## Chassis Dimensions mm

| WB | Wheelbase | 3400 | 3700 |
| :--- | :--- | ---: | ---: |
| OAL | Overall Length | 5619 | 5919 |
| AF | Rear overhang | 934 | 934 |
| BBC\# | Bumper to back of air intake | 2220 | 2220 |
| CA@ | Back of air intake to axle centre | 2465 | 2765 |
| Turning | Circle Diameter(Kerb to Kerb) | 12100 | 13000 |
| FWP Fifth Wheel Position for 16.5m OAL | 350 | 650 |  |

F.W.P. For accurate fith wheel positioning, in terms of axle weights and dimensions, consult SSS program

Note 1) Height measurements calculated on 295/80R 22.5 tyres.
2) Do not use this drawing for 5 th. wheel mounting, refer to $F$ superstructures manual and coachbuilder drawing.
\# Day cab-320. @ Day cab +320

## Chassis Kerb Weights Kg (Tolerance $\mathbf{\pm 2 \%}$ )

| Front Axle | 4560 | 4570 |
| :--- | :--- | :--- |
| Rear Axle | 1930 | 1940 |
| Total Kerb Weight | 6490 | 6510 |

Note:Weights for vehicles to standard specification i.e. sleeper cab, R1400 gearbox, twin plate clutch, 550 litre fuel tank(full), 295/80R22.5 tyres, water and tools, but excluding spare wheel and carrier.

Fig. 2.2 A Typical tractor unit






| 妾 | - | N $\omega$ | $\omega$ | 0 | 0 | $\checkmark$ | $\infty$ | $\bullet$ | $\vec{O}$ | - | $\vec{\sim}$ | $\vec{\omega}$ | $\stackrel{\rightharpoonup}{0}$ | \| $\vec{\square}$ | - | $\stackrel{\rightharpoonup}{\infty} \mid \stackrel{\rightharpoonup}{0}$ | O | $\sim$ | N | W | N | 0 | N | ${ }^{\sim}$ | - | O | ${ }_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\left\|\begin{array}{l} 0 \\ 0 \\ 0_{3} \\ \text { on } \end{array}\right\|$ | \% | 硕 |  |  |  |  |  |  |  |
| 운 | $N$ | $\omega$ - | - - | $\rightarrow$ | $\checkmark$ | - | N | $N$ | N | $N$ | N | $\sim 9$ | a | - | $\rightarrow-$ | $\rightarrow$ | $\rightarrow-$ | N | N | $\rightarrow$ | N | N | N | 0 |  |  |  |
|  | $\left\|\begin{array}{l} \frac{3}{2} \\ \sum_{2} \\ \frac{D}{2} \\ \hline \end{array}\right\|$ |  | $970971 \times 1 \mathrm{M} 5 \times 571$ - 38 n 1 |  |  |  |  | $=$ | $=$ |  | $==\begin{gathered} \frac{n}{7} \\ \frac{7}{7} \\ \frac{n}{n} \\ \frac{m}{0} \\ \end{gathered}$ |  |  |  |  | $=$ $=$ ¢ $=$ |  |  |  | $\left.\begin{array}{\|l\|} \hline 0 \\ \frac{1}{0} \\ 2 \\ 0 \\ 0 \end{array} \right\rvert\,$ | $1=$ | $=$ <br> $=$ |  | $\frac{7}{2}$ |  |  |  |



Fig. 2.4 Connection of chassis cross-members to longitudinal members


Fig. 2.5 Airbag suspension system
(a)


Fig 2.6 Typical suspension height control valve operating modes


Table 2.1 - Dynamic fifth loads occuring once per km for a 32 tonne fully laden articulated lorry

| LOADING MODE | DYNAMIC LOADS |  |
| :--- | :---: | :---: |
|  | Average | Maximum |
| LONGITUDINAL <br> LOADING (+ve load, <br> trailer pushing tractor) | +20 kN to -40 kN | +120 kN |
| VERTICAL LOADING <br> (+ve load downards) | $\pm 7 \mathrm{kN}$ | $\pm 24 \mathrm{kN}$ |
| LATERAL LOADING <br> (+ve loading applied <br> from Kerbside) | $\pm 20 \mathrm{kN}$ | +33 kN |

## CHAPTER 3

## BASIS OF THE THEORETICAL MODEL

### 3.1 Introduction

When an articulated tipper unit is being loaded or is tipping, it is unlikely to be standing on perfectly level ground. Also, the centre of gravity of the load is unlikely to be in the centre of the body. Hence the loads carried by the suspension and tyres on one side of the tipper will be greater than those on the other sicic. This uneven loading will cause the tyres and suspension on one side of the tipper unit to deform more than those on the other side. It will also cause the chassis to deform; the twisting about its longitudinal axis is the most significant mode of deformation. As a result of these deformations, caused by the uneven loading, the position of the centre of gravity moves even further towards the more heavily loaded side. This causes even more uneven loading and further deformations.

Under stable conditions, a situation will exist at which the position of the centre of gravity, the deformations and the forces transmitted through the system are compatible. Instability, resulting in roll-over would occur if the overall centre of gravity of the load, body, chassis etc. were to fall outside the area bounded by the contact of the wheels with the ground.

Although the stresses within the chassis, suspension, tyres etc. may be well within the linear elastic limits for the materials used, the position of the centre of gravity of the
load can be significantly altered by the elastic deformations. The movement of the centre of gravity is largest when tipping is in progress. In order to analyse this type of non-linear, elastic system, an iterative procedure must be adopted. The basic procedure is illustrated in Section 3.4.1 using a simple example.

In some articulated tipper units, a further non-linearity arises from the use of heightcontrolled, pneumatically pressurised, airbag suspension units. In such systems, the distance between the axle of the centre pair of wheels and the chassis, is maintained constant on both, by increasing or decreasing the pressure in the airbags on each side independently. Under normal driving conditions this reduces the significant effect that the suspension deformations have on the overall movement of the centre of gravity. However, other deformations, such as the tyre compression, chassis twisting etc. still cause significant movements of the centre of gravity.

Two further non-linear situations can result from this type of airbag suspension. Firstly, if the pressure to one side should become zero (gauge), no further control is exerted, ie. the airbag is effectively removed from that side. Secondly, above a maximum allowable pressure, a cut-off valve operates and the compression of the fixed volume of air trapped in the system at this stage controls the deformation of the suspension. Under these conditions, an effectively polytropic compression of the gas occurs.

Other non-linear effects, such as clearance in the hinges connecting the body to the chassis and separation of the fifth wheel plate and the king pin have not been
included. However, unless wear is very great and/or the tipper unit is in an unstable condition, these effects are likely to be small compared to those described above.

The theoretical model developed for predicting the behaviour of a tipping trailer unit, under tipping conditions, requires only one set of finite element analyses to calculate the flexibility matrix for a particular chassis and tractor. A computer program was developed based on the theoretical model. It requires the use of the chassis finite element results, tyre stiffnesses, air-bag characteristics, etc. It can be used to assess the stability of the particular tipping trailer unit, for any combination of ground slope, payload and tipping angle.

### 3.2 Co-ordinate system

The theoretical model requires a system to define the initial undeformed and subsequent deformed positions of the tipping trailer unit. A trailer datum, coordinate axis, ground slope and ram length are required.
(i) The trailer datum and coordinate axis are used to describe the position of the elements in the finite element model in an undeformed and deformed position. The coordinate system is also used to describe the position of the ram to body contact when tipped and the body and payload centre of gravity in an undeformed or deformed position. The datum position is at the centre of the chassis/body hinges. The +X direction is defined from the datum along a line parallel to the ground in the forward direction. The +Z direction is defined from the datum along a line passing through the right body chassis hinge in
a undeformed state when viewed from the rear of the trailer. The +Y direction is upwards (perpendicular to the XZ plane), they form a right handed system of axes, as shown in Fig. 3.1.
(ii) The ground slope is used to describe the trailer unit's position (XZ plane) relative to the horizontal, and is a rotation about the X axis only. The ground slope, $\theta_{\mathrm{p}}$, is shown in Fig. 3.1. Only a positive ground slope (clockwise rotation about the X axis) is considered.
(iii) The ram length is used to describe the tipped position of the body, and varies between 2 and 8 meters.

### 3.3 Assumptions

The main assumptions made in this work are as follows:

### 3.3.1 The body

The body discussed in Section 2.1 is assumed to be rigid and to be connected to the chassis by a hinge bar at the back and a ram at the front. The connection at the hinge bar is assumed to have no clearance in it. It is further assumed that there is negligible friction in this hinge and that the force acting along the hinge bar, due to its contact with the chassis, ie. in the transverse direction of the tipper unit, occurs at the left hinge. The connection between the ram and the body is assumed to be a pin-joint with negligible friction.

### 3.3.2 The chassis (Trailer \& Tractor)

The mainrails and cross-members of the trailer and tractor chassis described in Section 2.1 are assumed to stretch, compress, bend and twist according to beam theory. That is, local deformations at the connections between cross-members and mainrails, and at load application points are assumed to be negligible. The components welded to the underside of the chassis, containing the pivot bush (hinge), which connects the trailing arm to the chassis, is also represented by a cantilever beam.

The trailer chassis is connected to the body by a hinge bar at the back and a ram at the front. It is assumed that there is negligible friction in the hinge and that the transverse force acting on the chassis, at the hinge bar, occurs on the upper side of the chassis. The connection between the ram and the chassis is assumed to be a pinjoint with negligible friction.

The fifth wheel and king pin unit, described in Section 2.1, connects the chassis to the fifth wheel plate on the tractor unit. It is assumed to allow relative rotation of the fifth wheel plate and the chassis plate, about an axis perpendicular to these plates, through the king pin, and about a transverse axis in the $\mathbf{Z}$ direction through the king pin only. Other displacements of the tractor and chassis, at the point of contact between the king pin and the fifth wheel plate, are assumed to be the same for the tractor and chassis. That is, it is assumed that there is no clearance in the connection between the king pin and fifth wheel and that there is no separation between the fifth
wheel plate and chassis plate. It is also assumed that friction effects in the region of contact between the tractor and chassis is negligible.

The trailing arms on the suspension, which connect the axle to the chassis, described in Section 2.1, are assumed to be connected by hinges. That is, free (frictionless) relative rotation of the chassis and trailing arms is allowed about an axis through the point of contact which runs transverse relative to the chassis. Other deflections of the chassis and trailing arms at the points of contact between them, are assumed to be the same, ie. it is assumed that there is no clearance in the hinges.

Each airbag is assumed to apply a force to the chassis at the point on the chassis which is on the centre-line of the airbag. It is further assumed that the force acts along the axis of the airbag, ie. the resistance to motion, of one end of the airbag relative to the other, perpendicular to its axis, is assumed to be negligible.

### 3.3.3 The suspension

The suspension systems, described in Section 2.1, are comprised of trailing arms and torsion beams and are assumed to stretch, compress, bend and twist according to beam theory. That is, local deformations at the connections between these components and the mainrails and at load application points are assumed to be negligible. Also, it is assumed that the trailing arm seats rigidly attach the trailing arms to the torsion beams. The shock absorbers, which are intended to damp out vibrations when the trailer is in motion have little effect when the trailer is stationary. The shock absorbers are therefore assumed to be inactive in the present analysis.

Each airbag is assumed to apply a force to a trailing arm at the point on the trailing arm which is on the axis of the airbag. It is further assumed that the force acts along the axis of the airbag, ie. the resistance to transverse motion of one end of an airbag relative to the other, perpendicular to its axis, is assumed to be negligible.

The forces exerted by the airbags, when operating under pressure control, are assumed to be proportional to the pressure. This has been verified experimentally; details are given in Appendix A. Once the maximum pressure is achieved in the airbags on one side, and the cut-off valve operates, the force, $F$, in the three bags on that side and the sum of the lengths of the three airbags, $\Sigma \mathrm{l}$, are assumed to be governed by a polytrophic gas law, ie. $\mathrm{F}(\Sigma 1)^{\mathrm{n}}=\mathrm{a}$ constant. This has been verified experimentally and the value of n has been determined; details are given in Appendix A.

Each of the trailer wheel/tyre units, which is connected to the end of a torsion bar, is assumed to be represented by three springs. Each of these three springs has a stiffness which represents the relationship between the force and displacements of the axis of the wheel, in the radial, rolling and transverse directions, assuming no slip between the tyre and the ground.

The vertical spring stiffness was obtained experimentally; details are given in Appendix B. The fore/aft and transverse spring stiffnesses were also determined experimentally, however the manufacturers data was used, as the experimental procedure over estimated them, as explained in Appendix B.

Each of the tractor wheel/tyre units is assumed to be represented by three mutually perpendicular springs. The stiffnesses were obtained from manufacturers information.

### 3.3.4 Ground conditions

All contact areas between wheels and the ground (tractor and trailer) are assumed to lie on a plane with all lower wheels at the same elevation and all upper wheels at the same elevation. The ground is assumed to be rigid, ie. the axles only move due to compression of the tyres, no compression of soft or loose ground is included because this would be difficult to define.

### 3.4 Theoretical basis

### 3.4.1 Simple illustration of principle

For the tipping trailer unit, the body containing the payload is assumed to be rigid and the rest of the system, comprising the tractor unit, the trailer chassis, suspension and tyres, is assumed to undergo linear elastic deformations. A tipping trailer unit is similar to a number of other engineering systems in the sense that small, linear elastic deformations of some parts of the system can cause large displacements of the positions of the loads (eg. the centre of gravity of the payload, the top of the ram and the line of action of the wind load).

The simple cantilever beam, of length 1 , made of a material with a Young's modulus of $E$ and with a cross-section having a second moment of area about the horizontal axis of I, loaded by a constant, relatively small load, $L$, carried on a rigid frame, as shown in Fig. 3.2a, illustrates the type of problem for which this is the case. In the undeformed position, the end of the beam is subjected to a force $L$ and a moment of magnitude La, as shown in Fig. 3.2b. However, these cause displacement $\Delta^{(1)}$ and rotation $\phi^{(1)}$ of the tip of the cantilever, where

$$
\begin{equation*}
\Delta^{(1)}=\frac{L l^{3}}{3 E I}+\frac{L a l^{2}}{2 E I} \tag{3.1a}
\end{equation*}
$$

and

$$
\begin{equation*}
\phi^{(1)}=\frac{L l^{2}}{2 E I}+\frac{L a l}{E I} \tag{3.1b}
\end{equation*}
$$

resulting in a movement of the rigid frame, and hence a change in the load position, as shown in Fig. 3.2c. In this deformed position, the load on the beam is still L , but the moment becomes $L(a+b \theta)$, assuming that $\theta$ is small enough for $\sin \phi \simeq \phi$ and $\cos$ $\phi \simeq 1$.

Hence, using the updated load and moment, the displacement and rotation of the tip of the beam, $\Delta^{(2)}$ and $\phi^{(2)}$, can be obtained, ie.,

$$
\begin{equation*}
\Delta^{(2)}=\frac{L l^{3}}{3 E I}+\frac{L\left(a+b \phi^{(1)}\right) l^{2}}{2 E I} \tag{3.2a}
\end{equation*}
$$

and

$$
\begin{equation*}
\phi^{(2)}=\frac{L l^{2}}{2 E I}+\frac{L\left(a+b \phi^{(1)}\right) l}{E I} \tag{3.2b}
\end{equation*}
$$

Although, in this example a further iteration is not necessary.
If the process is repeated i times, the displacement, $\Delta^{(i)}$, and rotation, $\phi^{(\mathrm{i})}$, of the tip of the cantilever are given by

$$
\begin{equation*}
\Delta^{(i)}=\frac{L l^{3}}{3 E I}+\frac{L\left(a+b \phi^{(i-1)}\right) l^{2}}{2 E I} \tag{3.3a}
\end{equation*}
$$

and

$$
\begin{equation*}
\phi^{(i)}=\frac{L l^{2}}{2 E I}+\frac{L\left(a+b \phi^{(i-1)}\right) l}{E I} \tag{3.3b}
\end{equation*}
$$

By continuing the iterative procedure until the differences between $\Delta^{(i)}$ and $\Delta^{(i-1)}$ and between $\phi^{(i)}$ and $\phi^{(i-1)}$ are negligible, the actual solution is obtained. At this stage, the load and moment transmitted to the beam, through the displaced rigid frame, are compatible with the displacement and rotation of the tip of the beam calculated using these forces and moments.

### 3.4.2 Principle applied to tipping trailer

The whole of the following description and calculation applies to one initial position of the body, ie. one fixed ram length. It has to be repeated for other ram lengths to analyse the complete tipping operation of a particular truck, unloading in a particular position. The body (assumed to be rigid) of the tipping trailer corresponds to the rigid frame in Fig. 3.2a. It is loaded by the payload $\underset{\sim}{\mathbf{P}}$, and wind load $\underset{\sim}{\mathbb{W}}$. As shown in Fig. 3.3, the equilibrium of the body is maintained by forces ${\underset{\sim}{\mathrm{H}}}_{\mathrm{L}}$ at the left hand hinge, ${\underset{\sim}{H}}_{\mathrm{R}}$ at the right hand hinge and $\underset{\sim}{\mathrm{R}}$ at the bottom of the ram (which is considered as part of the body). Just like the tip of the cantilever in Fig. 3.2a, the positions of these reactions change under load, because ${\underset{\sim}{H}}_{L},{\underset{\sim}{R}}_{\mathrm{R}}$ and $\underset{\sim}{\mathrm{R}}$ are supported
by the flexible chassis. Instead of Equ. 3.2, a stiffness matrix for the flexible parts of the vehicle (chassis, tyres and suspension etc.) has to be used to obtain the displacements of the connecting 'points' between the body and chassis (see Fig. 3.4). A similar iterative procedure to that used in the simple cantilever example allows the true displacements to be determined.

There are two other important differences between the simple illustration and the tipping trailer. The distributed weight of the chassis, wheels and suspension of the trailer, ${\underset{\sim}{W}}_{c}$, and the weight of the tractor, ${\underset{\sim}{W}}_{T}$, are significant loads on the chassis, which have to be included in the calculation of the displacements. Also, the airbags exert forces which have to be determined.

The three airbags on each side are connected so that they exert six identical forces, $A_{L}$, on the left and six other identical forces, $A_{R}$, on the right, as shown in Fig. 3.5. However, there are four possible operating conditions which exist for the airbag suspension system. Under normal conditions, the airbag pressure is continuously changed to maintain 'height control' (called ' H '), ie., the distance between each side of the central wheel axle and the chassis is maintained at a constant value. If the pressure in one of the sets of airbags reaches a maximum value, a cut-off valve operates and polytropic compression of the fixed mass of gas (called ' ${ }^{\prime}$ ') controls the airbag force and hence the deformations of the suspension on this side, while the other side can still be height controlled. Also, if the gauge pressure becomes zero for one of the sets of airbags, no further control is exerted on that set of airbags (called ' $F$ '). Hence the airbags will be effectively removed form one side while the
other side may be either height controlled, or controlled by the polytropic compression of a fixed mass of gas. In summary, for rolling to the right, the four possible airbag conditions are:
(a) the left and right side airbag pressures are both adjusted to give height control on both sides (ie., LH/RH),
(b) the left airbag pressure is adjusted to give height control on this side while the right side is controlled by the polytropic compression of a fixed mass of gas (ie., LH/RC),
(c) the left airbag is inoperative (gauge pressure of zero) while the right airbag pressure is adjusted to give height control on this side (ie., LF/RH) and
(d) the left airbag is removed while the right side is controlled by the polytropic compression of a fixed mass of gas (ie., LF/RC).

### 3.4.3 Basis of the tipping trailer analysis

The forces on the rigid body, and hence on the elastic parts of the system, (in a deformed position) are given in the free body diagrams shown in Fig. 3.3. The origin of the orthogonal, cartesian, co-ordinate system $(\mathbf{X}, \mathbf{Y}, \mathbf{Z})$ is in the middle of the axis of the hinges which allow the body to be tipped. $\underset{\sim}{\mathrm{H}}$ and $\underset{\sim}{\mathrm{H}}$ are the reactions at the centres of the left and right hinge, $\underset{\sim}{\mathbf{B}}$ is weight of the body, $\underset{\sim}{\mathrm{P}}$ the magnitude of the payload and $\underset{\sim}{W}$ is the resultant of the wind forces acting on the body. $\underset{\sim}{\mathrm{R}}$ is the force in the ram which lifts the front end of the body during tipping. In the analysis these forces are split into their cartesian components.

The components of rotation about the X and Y directions of the hinge-bar, which connects the body to the trailer chassis, are defined by $\theta$ and $\psi$, as indicated in Fig. 3.1 and Fig. 3.4.

The points on the chassis at which the deformations control the displaced position of the body and payload are the connection points with the hinge bar (the left, L, and right, R, rear positions on the chassis, see Fig. 3.3) and the connection point with the bottom of the ram. Although analytical solutions, such as those applicable to the deflection and rotation of the cantilever (ie. Equation 3.1), cannot be obtained to relate the deformations of the important points on the chassis to the loads applied to the chassis, the finite element method can be used to obtain the required relationships for each of the four possible suspension conditions.

Unlike the force, L , in the simple illustration in Fig. 3.2, the forces $\underset{\sim}{\underset{\sim}{H}}, \underset{\sim}{\underset{R}{\underset{R}{R}}}$ and $\underset{\sim}{\mathrm{R}}$ vary during the iteration. For each of the four suspension conditions (a, b, c, d in Section 3.4.2) the finite element program was run for one load at a time, making this load unity. The required displacements, due to each unit load, were stored and later multiplied by the $i$ th values of the loads to calculate the $(i+1)$ th values of deformation.

The 14 unit loads were:
a) the cartesian components of the body contact forces, $\mathrm{H}_{\mathrm{Lx}}, \mathrm{H}_{\mathrm{Ly}}, \mathrm{H}_{\mathrm{L} z}, \mathrm{H}_{\mathrm{Rx}}, \mathrm{H}_{\mathrm{Ry}}$, $R_{x}, R_{y}$, and $R_{z}$. (NB. $H_{R z}=0$ because there must be axial clearance between the body and chassis at the hinge bar);
b) the distributed weight of the tractor, $\mathrm{W}_{\mathrm{Ty}}$ and $\mathrm{W}_{\mathrm{Tz}}$;
c) the distributed weight of the chassis, suspension and wheels, $W_{C y}$ and $W_{C_{z}}$;
d) the airbag forces, $A_{L}$ and $A_{R}$, each acting in six positions, as shown in Fig. 3.5.

The reactions to each of these loads were taken where the ten wheels touch the ground (see Fig. 3.3).

The 15 important cartesian components of displacement were:
a) at the connections to the body, $u_{h l}, v_{h l}, w_{h l}, u_{h r}, v_{h r}, w_{h r}, u_{r b}, v_{r b}$ and $w_{r t}$; (suffices refer to hinges at left and right and the ram bottor. end);
b) at the upper ends of the airbags $v_{a 1}, v_{\mathbf{2} 3}$ and $v_{\mathbf{a}}$;
c) at the lower ends of the airbags, $v_{22}, v_{24}, v_{26}$;

Suffices 1 and 2 refer to the 1 st airbag, 3 and 4 to the 2 nd airbag and 5 and 6 to the 3rd airbag, see Fig. 3.5.

### 3.4.3a Suspension on both sides operating under height control (LH/RH)

In the finite element analysis, in order to model the situation when both sides are height controlled, tie bar elements were placed between $E$ and $B$ and between $C$ and D (see Fig. 3.5) to keep the distances EB and CD constant. In the truck, the tie bars do not exist, there are only height sensing devices, which do not transmit any forces between EB or CD. Therefore the forces exerted by the airbags must be adjusted during each iteration to make the tie bar forces zero. Hence, an influence matrix,
$\left[I^{2}\right]$, relating the important displacements to the forces can be obtained, such that

$$
\begin{equation*}
\underset{\sim}{u}=\left[I^{(a)}\right] \underset{\sim}{F} \tag{3.4a}
\end{equation*}
$$

where $\left[I^{a}\right]$ is the flexibility matrix obtained from the finite element work,

$$
\begin{equation*}
{\underset{\sim}{u}}^{t}=\left[u_{r b}, v_{r b}, w_{r b}, u_{h l}, v_{h b}, w_{h b}, u_{h r}, v_{h r}, w_{h r}, v_{a l}, v_{a 2}, v_{a 3}, v_{a 4}, v_{a s}, v_{a 6}\right] \tag{3.4b}
\end{equation*}
$$

and

$$
\begin{equation*}
{\underset{\sim}{F}}^{t}=\left[R_{x x}, R_{y}, R_{z}, H_{L x}, H_{L y}, H_{L z}, H_{R x}, H_{R y}, W_{T y}, W_{T z}, W_{c y}, W_{c z}, A_{L}, A_{R}\right] \tag{3.4c}
\end{equation*}
$$

[N.B. The superscript, $t$, used in equations $4 b$ and $4 c$, and in subsequent equations, refers to the transpose of the matrix].

Equations 3.4 are the equivalent, for the tractor and trailer system, of Equations 3.1 for the simple cantilever problem used to illustrate the approach.

As well as determining the important displacements, the components of the forces in the tie bars can be obtained from the finite element results for each unit load applied to the finite element mesh; hence, it is possible to determine an influence matrix, $\left[\mathrm{I}_{\mathrm{T}}{ }^{(\mathrm{a}}\right]$, which relates the tie bar forces, $\underset{\sim}{\mathrm{T}}$, to the force vector, $\underset{\sim}{\mathrm{F}}$, ie.

$$
\begin{equation*}
\underset{\sim}{T}=\left[I_{T}^{(a)}\right] \underset{\sim}{F} \tag{3.5a}
\end{equation*}
$$

where

$$
\begin{equation*}
{\underset{\sim}{T}}^{t}=\left[T_{L}, T_{R}\right] \tag{3.5b}
\end{equation*}
$$

The relationships between $A_{L}$ and $A_{R}$ and the other forces can be obtained from equation 3.5 a by imposing the condition $\underset{\sim}{T}=\underline{0}$.

From the finite element analyses, the relationship between the tyre forces transmitted to ground, $\underset{\sim}{G}$, and the other forces, $\underset{\sim}{\mathrm{F}}$, can be obtained, ie.

$$
\begin{equation*}
\underset{\sim}{G}=\left[I_{G}^{(a)}\right] \underset{\sim}{F} \tag{3.6}
\end{equation*}
$$

The initial components of the force vector, $\underset{\sim}{\mathrm{F}}$, at the connection points between the body and chassis, related to the undeformed position of the system can be obtained by considering the equilibrium of the body and payload in the undeformed position. In the undeformed position, $\theta$ and $\psi$, indicated in Fig. 3.4, are both zero. The force and moment equilibrium equations for the body and payload, in vector form, are as follows:

$$
\begin{equation*}
\underset{\sim}{\boldsymbol{H}_{L}}+{\underset{\sim}{H}}_{R}+\underset{\sim}{R}+\underset{B}{B}+\underline{P}+\underset{W}{W}=\underline{0} \tag{3.7a}
\end{equation*}
$$

and

$$
\begin{equation*}
\underset{\sim}{\underset{L}{H}} \wedge h_{l}+\underset{\sim}{\underset{R}{H}} \wedge h_{r}+\underset{\sim}{R} \wedge r_{t}+\underset{\sim}{B} \wedge \underset{\sim}{b}+\underset{\sim}{P} \wedge \underset{\sim}{p}+\underset{\sim}{W} \wedge \underset{\sim}{w}=\underset{\sim}{0} \tag{3.7b}
\end{equation*}
$$

When expanded, equations 3.7 produce six linear equations with eight unknown forces, shown in Fig. 3.6, ie., $\mathrm{R}_{\mathrm{x}}, \mathrm{R}_{\mathrm{y}}, \mathrm{R}_{\mathrm{z}}, \mathrm{H}_{\mathrm{Lx}}, \mathrm{H}_{\mathrm{L} y}, \mathrm{H}_{\mathrm{L} z}, \mathrm{H}_{\mathrm{Rx}}$ and $\mathrm{H}_{\mathrm{Ry}}$. By assuming that the ram is pin-jointed at both ends, the ram load components $\left(R_{x}, R_{y}\right.$ and $\left.R_{z}\right)$ can be related to the resultant ram force $\underset{\sim}{\mathrm{R}}$, which must act along the axis of the ram. In the initial undeformed position, the co-ordinates of the top and bottom of the ram
are known from the dimensions of the vehicle. Therefore, the vector, ${\underset{\sim}{r}}_{b}$, which defines the position of the top of the ram relative to the bottom of the ram, which in turn defines the line of action of the ram force, $\underset{\sim}{R}$, can be obtained, ie.,

$$
\begin{equation*}
{\underset{\sim}{t b}}_{r_{t}}^{r_{t}}-\underset{\sim}{r_{b}} \tag{3.8}
\end{equation*}
$$

Hence equations 3.4 can be solved to obtain the components of the chassis forces, $\underset{\sim}{\mathrm{F}}$, at the connection points between the chassis and body, related to the undeformed position of the body and payload. Using these forces together with the tractor and chassis self weight forces, equations 3.5 a can be used, with $\underset{\sim}{T}=\underset{\sim}{0}$, to obtain the suspension forces, $A_{L}$ and $A_{R}$. Hence, the complete force vector, $\underset{\sim}{F^{(1)}}$, related to the undeformed position can be determined. Substituting $\underset{\sim}{F}{ }^{(1)}$ into equations 3.4 and 3.6 allows a first estimates for the important chassis displacements, ${\underset{\sim}{u}}^{(1)}$, and the left, rear tyre force, $\mathrm{G}_{\mathrm{L} 3}{ }^{(1)}$, to be determined.

Having determined ${\underset{\sim}{u}}^{(1)}$ values, the updated loads, ${\underset{\sim}{F}}^{(2)}$, can be obtained using the equilibrium equations 3.7 with the updated position vectors ${\underset{\sim}{h}}^{(1)},{\underset{\sim}{h}}^{(1)}{\underset{\sim}{r}}^{(1)},{\underset{\sim}{b}}^{(1)}, \mathbf{p}^{(1)}$ and $\underset{\sim}{\mathbf{w}}{ }^{(1)}$. The facts that the body is assumed to be rigid, the ram length is constant during the iteration process and the small changes in distance between the centre of the hinge bar and the bottom of the ram during the deformation process are insignificant (further explanation is given later) allows the updated, position vectors to be related to the initial position vectors.

The vector defining the position of the top of the ram, $\underline{r}_{t}\left(\right.$ or ${\underset{\sim}{r}}^{(1)})$, is at right angles to the vector defining the position of the left hinge, ${\underset{1}{h}}_{1}\left(\operatorname{or}_{\boldsymbol{h}_{1}}{ }^{(1)}\right)$; displacement of the
body does not affect this, because the body is assumed to be rigid. Therefore, the scalar product of the vectors $\underline{r}_{1}\left(\operatorname{or}{\underset{\sim}{r}}_{1}^{(1)}\right)$ and $\underline{\boldsymbol{h}}_{1}\left(\operatorname{or} \underline{\underline{h}}_{1}{ }^{(1)}\right)$ is zero, ie.,

$$
\begin{equation*}
{\underset{\sim}{t}}^{(1)} \cdot \underline{h}_{l}^{(1)}=0 \tag{3.9}
\end{equation*}
$$

Also, if the distance between the centre line of the hinge bar and the bottom of the ram, $r_{b}$, is constant (finite element analysis showed the changes in this distance to be negligible) and the ram length remains constant, then in the displaced position, the angle between the vectors ${\underset{\sim}{r}}^{\text {t }}\left(\operatorname{or}{\underset{\sim}{r}}^{(1)}\right)$ and ${\underset{\sim}{r}}_{b}\left(\operatorname{or}{\underset{\sim}{r}}^{(1)}\right)$, defined as $\gamma$, would remain constant. Hence,

$$
\begin{equation*}
{\underset{\sim}{r}}_{t}^{(1)} \cdot{\underset{\sim}{r}}_{b}^{(1)}=r_{t} r_{b} \cos \gamma \tag{3.10}
\end{equation*}
$$

and

$$
\begin{align*}
& \left(r_{t x}^{(1)}-r_{b x}^{(1)}\right)^{2}+\left(r_{t y}^{(1)}-r_{b y}^{(1)}\right)^{2}+\left(r_{t z}^{(1)}-r_{b z}^{(1)}\right)^{2}  \tag{3.11}\\
& \quad=\left(r_{t x}-r_{b x}\right)^{2}+\left(r_{t y}-r_{b y}\right)^{2}+\left(r_{t z}-t_{b z}\right)^{2}
\end{align*}
$$

\left. From equations 3.9, 3.10 and 3.11, ${\underset{\sim}{r}}^{(1)}{\underset{\sim}{r}}^{\left(r_{x}\right)},{\underset{\sim}{r}}^{(1)}{ }^{(1)},{\underset{\sim}{r}}^{(1)}\right)$ can be determined. Having obtained ${\underset{\sim}{r}}^{(1)}$ the other unknown position vectors, ie. $\underline{p}^{(1)},{\underset{\sim}{b}}^{(1)}$ and ${\underset{\sim}{w}}^{(1)}$ are all obtained in a similar manner. For example, the triangle defined by the centre of the hinge bar, the top of the ram and the centre of gravity of the payload remains fixed in shape, because the body/payload is assumed to be rigid. If the angle between the vectors ${\underset{\sim}{r}}^{1}$ and $\mathbf{p}$ is taken to be $\delta$ (say), this angle can be obtained on the basis of the undeformed position of the chassis, ie.,

$$
\begin{equation*}
\underset{\sim}{r} \cdot \underset{\sim}{p}=r_{t} p \cos \delta \tag{3.12}
\end{equation*}
$$

In the displaced position, $\delta$ remains constant, therefore

$$
\begin{equation*}
{\underset{\sim}{t}}_{(1)}^{\sim} \cdot{\underset{\sim}{p}}^{(1)}=r_{t} p \cos \delta=r_{t} \cdot \underset{\sim}{p} \tag{3.13}
\end{equation*}
$$

Also, and

$$
\begin{gather*}
{\underset{\sim}{p}}^{(1)} \cdot{\underset{\sim}{l}}_{\underset{\sim}{(1)}}=\underset{\sim}{p} \cdot h_{l}  \tag{3.14}\\
\left(p_{x}^{(1)}\right)^{2}+\left(p_{y}^{(1)}\right)^{2}+\left({\underset{p}{z}}_{(1)}\right)^{2}=p^{2} \tag{3.15}
\end{gather*}
$$

The solution of equations 13,14 and 15 allow $p^{(1)}\left(p_{x}, p_{y}, p_{z}\right)$ to be determined. By replacing $\underline{p}^{(1)}$ by ${\underset{\sim}{b}}^{(1)}$ or ${\underset{\sim}{w}}^{(1)}$, then ${\underset{\sim}{b}}^{(1)}$ and ${\underset{\sim}{w}}^{(1)}$ can also be obtained.

With the updated position vectors, ie., ${\underset{\sim}{\mathbf{h}}}_{1}^{(1)},{\underset{\sim}{r}}^{(1)},{\underset{\sim}{r}}^{(1)},{\underset{\sim}{b}}^{(1)},{\underset{p}{ }}^{(1)}$ and ${\underset{\sim}{\mathbf{w}}}^{(1)}$, equations 3.7 can be solved to obtain the updated components of the forces between the body and chassis. These forces, together with the tractor and chassis self-weight forces can be used in equation 3.5 , with $\underset{\sim}{T}=\underset{\sim}{0}$, to obtain updated suspension forces. Hence, an updated total force vector, $\underset{\sim}{\underset{\sim}{F}}{ }^{(2)}$, can be obtained. An improved set of displacements, ${\underset{\sim}{u}}^{(2)}$, and rear, left-hand tyre force, $G_{L 3}{ }^{(2)}$, can be obtained using equations 3.4 and 3.6, respectively. These updated displacements can then be used to obtain the updated position of the top of the ram $\boldsymbol{r}_{1}^{(2)}$, as indicated by equations 3.9, 3.10 and 3.11 . Then, the updated positions of the other important points can also be obtained and used to obtain a further update of the chassis load vector. This process is repeated until either the changes in displacement are negligible or an unstable solution, indicating a rollover condition, is obtained. The onset of instability is taken to be when the rear, left-hand tyre reaction force $\mathrm{G}_{\mathrm{L} 3}$ becomes zero.

A similar approach can be used to assess stability with the other three suspension conditions. The way in which these analyses differ from the one described above are outlined below.

### 3.4.3b Left suspension height controlled and right suspension governed by polytropic compression of a fixed mass of gas (LH/RC)

In this case, a tie bar is placed between C and D (see Fig. 3.5), as in the previous case. However, the tie bar is removed from the right hand side because the distance between E and B can decrease, governed by the polytropic compression of a fixed mass of gas in the three, connected, right hand airbags. Since the force in the right hand suspension, $A_{R}$, is related to the deformation of the system, an iterative procedure is required in this case. Again, unit forces $R_{x}=1 \ldots H_{R y}=1, \ldots W_{c z}$ $=1, \mathrm{~A}_{\mathrm{L}}=1$ and $\mathrm{A}_{\mathrm{R}}=1$, are applied in turn to the finite element mesh. For these units forces the left hand tie bar force, $\mathrm{T}_{\mathrm{L}}$, and the important chassis displacements, at points where the body and ram are attached, ie., $\underset{\sim}{\mathrm{u}}$ are determined. As before, $\underset{\sim}{\mathrm{u}}$ can be related to $\underset{\sim}{F}$, ie.,

$$
\begin{equation*}
\underset{\sim}{u}=\left[I^{(b)}\right] \underset{\sim}{F} \tag{3.16a}
\end{equation*}
$$

where a subset of matrix equation 16 a , related to the right hand side air-bag displacement, is

$$
\begin{equation*}
\underset{\sim}{v_{a}}=\left[I_{a}^{(b)}\right] \underset{\sim}{F} \tag{3.16b}
\end{equation*}
$$

Also, as before, the tie-bar force, $\mathrm{T}_{\mathrm{L}}$, can also be related to $\underset{\sim}{\mathrm{F}}$, ie.

$$
\begin{equation*}
T_{L}=\left[I_{T}^{(b)}\right] \underset{\sim}{F} \tag{3.16c}
\end{equation*}
$$

and the tyre forces, $\underset{\sim}{G}$, can be related to the $\underset{\sim}{F}$, ie.

$$
\begin{equation*}
\underset{\sim}{G}=\left[I_{G}^{(b)}\right] \cdot \underset{\sim}{F} \tag{3.16d}
\end{equation*}
$$

In addition to the above, the polytropic compression of the fixed mass of gas dictates that

$$
\begin{equation*}
A_{R}\left(3 l_{o}+\sum_{i=1}^{6} v_{a i}\right)^{n}=\text { constant }(=Q) \tag{3.17}
\end{equation*}
$$

where $l_{o}$ is the length of gas contained in the constant cross-sectional area cylinders, at the limiting pressure, ie., when the pressure supply is cut off, and $\Sigma \mathrm{v}_{\mathrm{ai}}$ is the sum of the changes of length of the three right hand airbags. The constant is obtained by substituting the instantaneous values of $A_{R}$ and $v_{a i}$ when $A_{R}$ (which is proportional to the pressure) reaches the limiting value. The values of $n$ and $Q$ (for a given mass of gas) have been determined by performing experiments on airbags.

In order to obtain the solution to equations 3.16 and 3.17 , an iterative procedure is adopted. A first estimate, $A_{R}{ }^{(1)}$ of the force in the right hand airbags is substituted into equation 3.16 c with $\mathrm{T}_{\mathrm{L}}=0$, to obtain a first estimate, $\mathrm{A}_{\mathrm{L}}{ }^{(1)}$, of the force in the left hand airbags. These first estimates for the airbag forces are then substituted into equation 3.16 b to obtain a first estimate, ${\underset{\sim}{a}}^{(1)}$, for the displacements of the ends of the right hand airbags. The ${\underset{\sim}{a}}^{(1)}$ values are then substituted into equation 3.17 to obtain an updated estimate, $\mathrm{A}_{\mathrm{R}}{ }^{(2)}$, for the force in the right hand airbags. This
iterative procedure is continued until negligible change in the forces and displacements occur between one iteration and the next. At this stage, the $\underset{\sim}{\mathrm{F}}$ matrix is substituted into equation 3.16a to obtain the updated displacements, ${\underset{\sim}{u}}^{(2)}$, which are used to obtain the new displaced position of the body and payload. As for the case described in Section 3.4a, the procedure is repeated, iteratively, until the changes in displacement and forces, including the right-hand airbag values are negligible.

### 3.4.2c Left suspension airbags inoperative and right suspension height controlled (LF/RH)

When the gauge pressure falls to zero in the left hand suspension, the airbags become inoperative and the forces $A_{L}$ no longer act. For this condition, it is therefore only necessary to place a tie bar between E and B (Fig. 3.5) and to leave the left suspension free. By obtaining the displacements, $\underset{\sim}{\mathbf{u}}$, and right-handed tie bar force, $T_{R}$, from the finite element results for each unit load case $R_{x}=1, \ldots W_{c z}=1$ and $A_{R}=1$, the influence matrices $\left[I^{(c)}\right],\left[I_{T}^{(c)}\right]$ and $\left[I_{G}^{(c)}\right]$ can be determined, such that

$$
\begin{gather*}
\underset{\sim}{u}=\left[I^{(c)}\right] \underset{\sim}{F}  \tag{3.18a}\\
\boldsymbol{T}_{R}=\left[I_{T}^{(c)}\right] \underset{\sim}{\boldsymbol{F}} \tag{3.18b}
\end{gather*}
$$

and

$$
\begin{equation*}
\underset{\sim}{G}=\left[I_{G}^{(c)}\right] \underset{\sim}{F} \tag{3.18c}
\end{equation*}
$$

Note, in this case, the force vector, ${\underset{\sim}{F}}^{\prime}$, does not contain left-hand airbag force, ie. $A_{L}=0$, and the influence matrices $\left[I^{(c)}\right]$ and $\left[I_{T}{ }^{(\mathrm{c})}\right]$ are smaller than the corresponding $\left[I^{(a)}\right],\left[I^{(b)}\right],\left[I_{T}^{(a)}\right]$ or $\left[I_{T}{ }^{(b)}\right]$ matrices.

In operation, with height control between $E$ and $B, T_{R}=0$ would be applicable. Therefore, for a given set of forces, at the connections between the body and chassis, and using tractor and chassis components of self-weight, the right hand airbag force, $A_{R}$, can be obtained by setting $T_{R}=0$ in equation 3.18b. This value of $A_{R}$ is then substituted into equation 3.18a, along with the other forces to obtain the displacement, $\underset{\sim}{u}$, which define the updated displaced position of the body and payload. An iterative procedure is again adopted until the changes in displacements and forces are negligible.

### 3.4.2d Left suspension airbags inoperative and right suspension governed by polytropic compression of a fixed mass of gas (LF/RC)

In this case, $A_{L}=0$ and the left hand tie bar is removed, because the left hand suspension system is inoperative, ie., no tie bar is present between $C$ and $D$, as well as $A_{L}=0$. Because the maximum pressure, and hence the maximum allowed value of $A_{R}$, has been reached, no tie bar is included between $E$ and $B$ either (see Fig. 3.5). Therefore, the method of applying unit loads in order to determine the influence matrices cannot be applied in this situation, because the finite element mesh would act as a mechanism.

By applying unit displacements to each of the important positions on the chassis in $\operatorname{turn}\left(\right.$ ie., $u_{\mathrm{r}}=1$, or $\mathrm{v}_{\mathrm{r}}=1$ or $\mathrm{w}_{\mathrm{r}}=1$ or $\mathrm{u}_{\mathrm{hl}}=1$ or $\mathrm{v}_{\mathrm{hl}}=1$ or $\mathrm{w}_{\mathrm{hl}}=1$ or $\mathrm{u}_{\mathrm{hr}}=$ 1 or $\mathrm{v}_{\mathrm{hr}}=1$ ) and to the six right-hand airbag connection points in turn (ie., $\mathbf{v}_{\mathbf{a} 1}=1$ or $\mathrm{v}_{\mathrm{a} 2}=1 \ldots$ or $\mathrm{v}_{26}=1$ ), while restraining each of the other points, the reaction forces at the restrained positions can be obtainerd. These reaction forces form components in a stiffness matrix, such that,

$$
[K] \underset{\sim}{u}=\left\{\begin{array}{l}
F_{\sim}^{*}  \tag{3.19a}\\
\tilde{A_{R}}
\end{array}\right\}
$$

where $\underset{\sim}{F}$ is the same as $\underset{\sim}{F}$, but with the terms $A_{L}$ and $A_{R}$ omitted and

$$
\begin{equation*}
A_{R}^{t}=\left(A_{R 1}, A_{R 2}, A_{R 3}, A_{R 4}, A_{R S}, A_{R \sigma}\right) \tag{3.19b}
\end{equation*}
$$

which are the forces corresponding to the right-hand airbag displacement, $\underline{\sim}_{\mathbf{a}}$. In reality, all of the $A_{R i}$ values ( $i=1$ to 6 ) must be the same, because the airbags are interconnected.

Matrix equation 3.19a can be rewritten as

By eliminating $\underset{\sim}{u}$ ", which is the same as $\underset{\sim}{u}$ but with the ${\underset{\sim}{v}}^{\mathbf{a}}$ terms omitted, from matrix equation 3.20 and rearranging, an expression for ${\underset{\sim}{a}}^{2}$ can be obtained, ie.,

$$
\begin{equation*}
\underset{\sim}{v_{a}}=\left[\left[K_{22}\right]-\left[K_{21}\right]\left[K_{11}\right]^{-1}\left[K_{12}\right]\right]^{-1}\left\{A_{\underset{R}{ }}-\left[K_{21}\right]\left[K_{11}\right]^{-1} \underset{\sim}{F}{ }^{\prime \prime}\right\} \tag{3.21}
\end{equation*}
$$

In addition to the above, the right hand side airbag forces, $A_{R}$, are related to the airbag displacement, $\underset{\sim}{\underset{a}{2}}$, via the gas equation which governs the polytropic compression of the fixed mass of gas, ie., equation 3.17 is applicable.

The solution, in this case, also requires an iterative approach. A first estimate, $\mathbf{A}_{\mathrm{R}}{ }^{(1)}$, of the forces in the right hand airbags (each component in ${\underset{\sim}{R}}^{\mathbf{A}}$ is taken to be the same) is substituted into equation 3.21 to obtain the first estimate for the right hand side airbag displacements, $\mathrm{v}_{\mathrm{a}}{ }^{(1)}$. These airbag displacements can then be used in equation 3.17 to obtain an updated airbag force, $\underset{\sim}{\underset{\sim}{A}}{ }^{(2)}$. This iteration, between equations 3.21 and 3.17, is continued until negligible changes in the airbag forces and displacements occur between one iteration and the next. Having obtained the airbag forces and displacement, equation 3.20 can be rearranged to allow the other chassis displacements, $\underset{\sim}{\text { u }}$, to be determined, ie.,

$$
\begin{equation*}
\underset{\sim}{u}{ }_{\sim}^{\prime \prime}=\left[K_{11}\right]^{-1}\left\{\underset{\sim}{F}{\underset{\sim}{\prime \prime}}^{\prime \prime}-\left[K_{12}\right]{\underset{\sim}{v}}_{a}\right\} \tag{3.22a}
\end{equation*}
$$

These updated chassis displacements, $\underset{\sim}{u}$ ", can now be used to obtain the updated, displaced position of the body and payload. Also, the forces at the contact points between the tyres and the ground can be related to $\underline{u}$, ie.,

$$
\begin{equation*}
\underset{\sim}{G}=\left[I_{\underline{G}}^{(d)}\right] \underset{\sim}{u} \tag{3.22b}
\end{equation*}
$$

An iterative procedure is again adopted until the changes in displacement and forces are negligible.

### 3.5. Outline of the overall solution procedure

The theoretical model, which enables the positions of the important points on the body, body/chassis contact point reactions, suspension forces, chassis displacements and tyre forces to be determined, requires a method to govern the sequence of operations. The theoretical model involves the solution of complicated equations, repetitive iterations and the multiplication of many matrices. This type of problem lends itself to solution by digital computer. Hence a program was written for an Akhter 286 PC using Turbo Basic; with this language, modifications are relatively easy and separate sub-routines can be written.

For each ground slope, ram length, wind loading and payload, the iterative calculation procedure used to establish the airbag operating conditions, deformed position, tyre force, etc., is outlined in the flow diagram shown in Fig. 3.7. A separate loop is applicable for each of the four possible airbag conditions. Within each of the four loops, there are some common features. The important position vectors (ie. ${\underset{\sim}{h}}_{1},{\underset{\sim}{r}}_{r},{\underset{\sim}{r}}^{r}, \underset{\sim}{b}, \underset{\sim}{p}$ and $\underset{\sim}{w}$ ) are determined using equations 3.9 to 3.15 and the direction of the ram is obtained using equation 3.8. With these position and direction vectors, equations 3.7 a and 3.7 b are solved to determine the forces on the body (ie. $\underset{\sim}{\mathrm{R}},{\underset{\sim}{\mathrm{H}}}_{\mathrm{L}}$ and ${\underset{\sim}{\mathrm{H}}}_{\mathrm{R}}$ ). Hence, the equal and opposite forces applied to the chassis by the hinge bar, connecting the chassis to the body, and the bottom of the ram are obtained. Using the appropriate airbag conditions, the airbag forces can be determined and hence the total force vector, $\underset{\sim}{F}$, applied to the chassis can be obtained. This force vector, $\underset{\sim}{F}$, is then used with the influence matrices (for the
appropriate airbag conditions) to obtain the updated displacements. The procedure is continued until the changes in the predicted displacements are negligible. The tyre forces are then calculated.

The program's initial execution of each iteration is governed by the undeformed position of the tractor and trailer and body. For the first iteration, the transverse ground slope is taken to be zero and the body is considered in the untipped position. For this ram length, after each iteration of the ground slope is increased in $0.1^{\circ}$ interval until a ground slope of $10^{\circ}$ is reached or until instability occurs, ie. the rear left wheel force becomes zero. The ground slope is then zeroed and the ram length is increased by 0.5 m . Further iterations are again undertaken increasing the ground slope up to $10^{\circ}$ or until instability occurs. The program's execution is terminated when the ram length reaches 8 m and instability occurs for the given ground slope.

Appendix C, "Computer Program and Flow Chart", describes the program subroutines and how they operate in a flow diagram and the program coding.

Appendix D, "Program User Guide" gives the necessary information to enable the user to run the program, use the results and change positional, loading and flexibility/stiffness matrices variables.

$B C$ - underformed hinge bar
DE - deformed hinge bar

Fig. 3.1 Coordinate system and angular movement of hinge bar


Fig. 3.2 Simple, cantilever example, to illustrate the interaction between small, elastic deformation and load position


Fig. 3.3 Free body diagrams of
(i) the body and payload in a displaced position,
(ii) the ram in a displaced position, and
(ii) the tractor unit, trailer chassis, suspension and tyres combination in an undeformed position.


Fig. 3.5 Schematic diagram representing the rear part of the chassis, the suspension and tyres
The forces $A_{L}$ and $A_{R}$ represent the forces applied by the left and right airbags. EB
and CD are the height control positions.

Fig. 3.6 Force and moment equilibrium equations expanded into matrix form


and payload.

## CHAPTER FOUR

## FINITE ELEMENT ANALYSES

### 4.1 Introduction

The previous theoretical roll-over models were based on lumped mass systems, representing various parts of the lorry inter-connected by compliant elements. Certain flexibilities such as the tyres, suspension units etc. could be obtained from the respective component manufacturers but the tractor and trailer chassis flexibilities are unknown. To overcome this problem the flexibilities of these components were obtained from full scale tests. This is a very expensive undertaking, providing a limited means in which to assess those elements of trailer design which are important in improving stability, without further recourse to more tilt tests.

It was decided that the finite element method should be used to model the tractor and trailer, in order to determine the important deformations. Once the finite element model is created it is relatively straight forward to make changes to the structure. Hence an assessment of component contribution to roll stability can be undertaken relatively inexpensively.

### 4.1.1 Flexible elements

Finite element analysis allows a structure to be simulated by dividing it into a number of elements which model its flexibility. This enables a relationship to be established between the structures applied loads and its displacement.

The tractor's components which are considered to be flexible are the chassis, suspension and tyres. The driver's cab is not considered as it is connected to the chassis by very flexible springs.

The trailer's components which are considered to be flexible are the chassis, suspension, tyres and axial tubes. The trailer's body is considered to be rigid as its flexibility is thought to have a second order effect on roll over stability. The fifth wheel model enables rotation between the tractor and trailer about the vertical ( $\mathbf{Y}$ axis) and transverse ( Z axis) axes only.

The finite element line diagram shown in Figs. 4.1 and 4.2 represent the Tractor and Trailer structure respectively, with node numbers denoted by a prefix " $N$ " and the remaining numbers referring to element numbers. The element numbers can be used in conjunction with Table 4.1 to obtain the element property number. This can then be used in conjunction with Table 4.2 to obtain the element's mechanical properties (ie. second moment of area, stiffness, cross sectional area etc.) depending on the type of element.

### 4.1.2 Suspension operating modes

The suspension operating modes discussed in Section 3.4.2 affect the loading and boundary/loading conditions for the finite element model. For the three suspension operating modes ( $\mathrm{a}, \mathrm{b}, \mathrm{c}$ ) where at least one side is height controlled, the displacement and forces are derived from unit load flexibility and influence matrices.

As the air suspensions are represented by only forces, the finite element model of the suspension forms a mechanism. To overcome the problem of loading a mechanism, tie bars are inserted in the finite element model to maintain the required ride height. A tie bar is only inserted in the left or right suspension if it is height controlled.

For the remaining suspension operating mode (d) where neither side is height controlled, a tie bar cannot be inserted in the finite element model to overcome the problem of loading a mechanism. It is therefore not possible for the displacement and frrces of such a structure to be obtained from unit load, flexibility matrices. Instead a unit displacement stiffness matrix was generated to determine the chassis displacements and suspension forces. A module within the PAFEC finite element package allows prescribed displacements to be imposed on the structure. The nodes of interest, where forces or displacements are required, are restrained in the appropriate directions.

### 4.2 Finite element meshes

The finite element mesh is constructed from six different elements and modules. These are used to model component flexibility, fifth wheel and suspension hinges and tyre/ground contacts.

### 4.2.1 Elements and modules

(i) Simple Beam Element: A straight simple beam element with a node at each end of the shear centre. The element is connected to the remainder of the structure by these nodes or by offset nodes. The element formulation includes bending in two directions, axial force and twisting. The main application of this element is for frame structures and stiffened plates.
(ii) Eight Noded Facet Shell Element: A flat thin shell element which can carry bending and membrane loads. The main applications are for shell problems in which either or both in-plane and out-of-plane effects are important.
(iii) Spring Element: A spring element which connects any pair of nodes with translatory stiffness in the $\mathbf{x}, \mathrm{y}, \mathrm{z}$ directions and with rotational stiffness about the $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ axes. The element is used whenever a discrete flexibility or stiffness needs to be introduced.
(iv) Mass Element: The mass element is used to model concentrated masses with a stiffness greater than those of the surrounding structure.
(v) Repeated Freedoms: The repeated freedom module is used to join two nodes which are to have one or more identical freedoms. The module can be used to provide relative movement between coincident nodes in order to model a hinge or slide connection.
(vi) Restraints: The restraints module is used to restrain the freedom of a node. The node can have all or just one of its six freedoms restrained.
(vii) Gravity: The gravity module is used to simulate the self weight of a structure by applying an acceleration of $9.81 \mathrm{~m} / \mathrm{s}^{2}$, or whatever the user defines.

### 4.2.2 Tractor chassis

The tractor unit described in Section 2.1, models the flexibilities of the chassis, suspension, tyres and fifth wheel. The finite element line diagram of the tractor unit, shown in Fig. 4.1 represents these details. The element numbers are given and the corresponding property values (depending on which type of element is being considered) can be obtained from Table 4.1 and 4.2.

The tractor chassis is divided into a series of beam elements to represent the ladder structure, with dimensions being obtained from detail drawings and the measurement of an actual vehicle. The chassis beams are channel sections welded together forming a ladder type structure, with the intersection of cross members and longitudinal members being treated as rigid joints. Beam elements 175 to 200 describe the chassis.

The leaf spring suspensions are modelled by translation springs, with their stiffnesses being obtained from manufacturers information. The front and rear suspension
springs are connected between the chassis beams and the axle beam and are described by elements 213,214 and 215,216 respectively. In order to model the leaf springs the ends of each spring are made to have the same $U_{x}$ and $U_{z}$ displacement, with the $\mathrm{U}_{\mathrm{y}}$ displacement being independent of each other, hence achieving a flexible connection in the y direction only.

The tractor has front and rear axles connected to the chassis by leaf springs and roll bars with single steering wheels attached to the front axle and dual driving wheels attached to the rear axle. The front and rear rn! bars-are described by elements 227 to 238 , the front and rear axles are described by elements 219 to 226 and the tyres are described by elements 210 to 121 . The tyre model is described in Section 4.2.5.

The tractor's mass is modelled by applying unit loads to the centre of gravity in the $\mathbf{Y}$ and Z directions. From gross vehicle axle weights the position of the centre of gravity could be determined in the fore/aft direction but the vertical position was obtained from an estimation made by the manufacturer.

### 4.2.3 Fifth wheel connection

The fifth wheel coupling described in Section 2.1 is modelled using the repeated freedoms module between nodes on the tractor unit and trailer unit. This allows rotation between the tractor and trailer unit about the Y and Z axes. The play between components which allows limited rotation about the X axis between the tractor and trailer is not modelled. Three nodes $(292,293,294)$ shown in Fig. 4.2
on the tractor and three corresponding nodes $(43,40,47)$ shown in Fig. 4.2 on the trailer were constrained to have certain displacements the same. The nodes 292, 43 and 294,47 were constrained to have the same $\mathrm{U}_{\mathrm{y}}$ displacements, while the nodes 293, 40 were constrained to have the same $U_{x}$ and $U_{z}$ displacements.

### 4.2.4 Trailer chassis

The trailer unit described in Section 2.1 models the flexibilities of the chassis suspension, tyres and king pin. The finite element line diagram shown in Fig. 4.2 represents these details. The element numbers are given and the corresponding property values (depending on which type of element is being considered) can be obtained from Table 4.1 and 4.2.

The chassis is divided into a series of beam elements with appropriate second moments of area and plate elements to represent the ladder structure, with dimensions being obtained from detail drawings. As the chassis beams vary in cross-section along their length, the average value of the second moment of area at the ends of each beam are used. The intersection of cross members and longitudinal members have been treated as rigid joints. The fifth wheel plate is represented by plate elements interconnected with beam elements. Plate elements 1 to 20 and beam elements 21 to 127 describe the chassis, see Fig. 4.2.

Each suspension unit consists of two airbags. The middle suspension system is described by nodes 180-211, 199-220, an axle tube described by beam elements 179 , $252,253,173$ two suspension supports described by beam slements 133,148 , two trailing arms described by beam elements $134,135,149,150$ and two tyres described by spring elements $137,163,152,166$ as shown in Fig. 4.2. The trailing arms on either side pivot about nodes 226 and 229 as the airbag height changes. In order to model the pivot the suspension support beam 133 and trailing arm beam 134 and 135 connection is made to have the same $U_{x}, U_{y}, U_{z}$ displacements rotation. The $z$ rotation is made independent of each other, hence achieving a pivot connection. This arrangement is repeated for the opposite side.

The tyres on the tractor and trailer are modelled using a vertical spring and two horizontal springs connected to the vertical spring at a distance of a tyre radius below the tyre centre. The base of the vertical spring is restrained in the vertical direction, with the fore/aft and lateral freedoms being restrained by the two horizontal springs. The other ends of these springs are completely restrained as shown in Fig. 4.3. The tyres on the trailer are described by the elements $132,137,142,147,152$ and 157 as shown in Fig. 4.2. The tyres on the tractor are described by 201, 203, 205, 207, 209, and 211 as shown in Fig. 4.1.

### 4.3 Design investigations

Individual lorry components can be altered in the finite element model allowing an assessment of their contribution to roll stability to be made. Following discussions with the trailer manufacturer, three areas of chassis design were changed individually to assess their effect on roll stability.
(i) A 3 mm plate was attached between the left and right longitudinal chassis I beams, being located along their neutral axes.. The plate is attached from the hinge bar, nodes 186 and 205, to the third torsion box cross member, nodes 174 and 193 as shown in Fig. 4.1. It is thought that this would stiffen the rear of the chassis when subjected to fore/aft loading.
(ii) The three large torsion box cross members were doubled in size, from 100 x $200 \times 5 \mathrm{~mm}$ to $200 \times 200 \times 5 \mathrm{~mm}$. These elements are attached between nodes 182 to 201, 178 to 197 and 174 and 193 as shown in Fig. 4.2. It is thought that this would increase the torsional stiffness of the chassis as a whole.
(iii) The six trailing arm leaf springs were reduced in size to double their bending flexibility. The leaf springs are connected between nodes 214 to 212,211 to 209, 208 to 206,223 to 221,220 to 218 and 217 to 215 , as shown in Fig. 4.3.

The loads applied to the vehicle, described in Section 2.2, are used to obtain the displacements from the finite element analyses. The iterative procedure adopted in Chapter 3 to analyse this type of non-linear, elastic system, necessitates the need for a general procedure for determining the displacement. There are two solutions required to determine the F.E. displacement, depending upon the suspension operating conditions.

For the three suspension operating conditions a, b or c, described in Section 3.4.2, in which at least one suspension side is height controlled, unit loads (one newton) are individually applied to the F.E. model, at nodes where forces are applied on the real structure. The forces are always applied to the F.E. model in the positive $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ directions (except for the suspension forces), representing the body/chassis contact forces at nodes $186,205,314$, the tractor self weight at node 313 , the chassis self weight using the gravity module, the left suspension forces at nodes $176,208,180$, $211,184,214$ and the right suspension forces at nodes 195, 217, 199, 220, 203, 233. The body/chassis contact forces are modelled by applying individual unit forces to the right hinge in the $\mathbf{X}, \mathrm{Y}, \mathrm{Z}$ direction, to the right hinge in the $\mathbf{X}, \mathrm{Y}$ directions and to the ram/chassis contact point in the $X, Y, Z$ direction. The tractor unit self weight in the vertical and lateral directions is modelled by applying individual unit forces in the $\mathrm{Y}, \mathrm{Z}$ direction. The unsprung and sprung mass of the actual trailer and the finite element model are different as only the masses of the flexible components are taken into account. Therefore, the trailers actual sprung and unsprung mass is modelled by
locating an additional mass at the trailers centre of gravity and at the ends of each axle and applying an individual unit acceleration in the $Y$ and $Z$ direction using the gravity module of the finite element package. The suspension forces are modelled by applying three pairs of forces to each side, with each pairs of forces representing the expansion forces of each airbag. The six forces on one side are applied simultaneously and in the direction of the actual forces. The forces are applied simultaneously as the suspension forces on each side are all equal.

For the remaining suspension operating conditions, (d) in which neither side is height controlled, unit displacements (one mm) are individually applied to the F.E. model at nodes where forces are applied to the structure. The displacements are always applied to the F.E. model in the positive $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ directions, (except for the suspension displacements), to the body/chassis contact points at nodes $186,205,314$, the tractor self weight at node 313 , the chassis sprung weight at node 315 , the chassis unsprung weight at nodes $316,317,318$ and the right suspension forces at nodes 195, 217, 199, 220, 203, 223.

The body/chassis contact point forces are modelled by applying individual unit displacements to the left hinge in the $X, Y, Z$ directions to the right hinge in the $X$, $\mathbf{Y}$ direction and to the ram/chassis contact point in the $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ directions. The tractor self weight is modelled by applying individual unit displacements to the centre of gravity in the $\mathrm{Y}, \mathrm{Z}$ directions. The chassis unsprung weight is modelled by applying individual unit displacements to the middle of each axle in the $\mathrm{Y}, \mathrm{Z}$ directions. The right suspension forces are modelled by applying individual unit
displacements in the $Y$ direction representing the expansion forces of each airbag. Although the displacements are applied individually it is still assumed that the right suspension forces are all equal.

To obtain the tyre forces in the Y direction the restraint in the Y direction at the tyre/ground contact node is released and unit displacements are individually applied in the y direction to the nodes 231 to 233 .

### 4.5 Flexibility, influence and stiffness matrices

For the three suspension operating conditions, (a, b, c described in Section 3.4.2) in which at least one suspension side is height controlled, the unit load finite element results enable flexibility and influence matrices to be generated. From these matrices, chassis displacements, suspension displacements, suspension forces and tyre forces can be determined for any given loading condition. For each flexibility or influence matrix described there are three types which are directly related to the suspension operating condition. The matrices will be discussed in general with the specific matrices given at the ends of this section for the original chassis design.

For the remaining suspension operating condition, (described in Section 3.4.2), in which neither suspension is height controlled, unit displacement finite element results enable a stiffness matrix to be generated. From this single matrix chassis displacement, suspension displacement, suspension forces and tyre forces can be determined for any given loading condition. The matrix will be discussed in general with the specific matrix given at the end of this section for the original chassis design.

### 4.5.1 Chassis displacement flexibility matrix

To determine the chassis displacements at the body/chassis contact points for any given loading condition, a unit load flexibility matrix was generated using the results from the finite element model. At the nodes of interest, the displacement at these points, resulting from the individually applied unit loads to the finite element model, form the coefficients of the unit load flexibility matrix. These results are used to form the unit load flexibility matrix which gives a relationship between the body/chassis contact forces, the tractor self weight, the chassis self weight, the suspension forces and the chassis displacements at the body/chassis contact point. The unit load flexibility matrices relating to the suspension operating conditions $a, b$, and $c$ are given in Tables 4.3, 4.4 and 4.5 respectively.

### 4.5.2 Suspension displacement flexibility matrix

To calculate the compression of the left airbags, the displacement of the two ends of each airbag are required. To determine these displacements for any given loading condition a unit load flexibility matrix was generated using the results from the finite element model in the same manner as the unit load chassis, displacement flexibility matrix. These results are used to form the unit load flexibility matrix, which gives a relationship between the body/chassis contact forces, the tractor self weight, the chassis self weight, the suspension forces and the suspension displacements at the ends of each airbag. The suspension displacements form the coefficients of the unit load suspension displacement flexibility matrix. The unit load flexibility matrices
relating to the suspension operating conditions $a, b, c$ are given in Tables 4.6, 4.7 and 4.8 respectively.

### 4.5.3 Suspension force influence matrix

To determine the left and right suspension forces for any given loading condition, a unit load influence matrix was generated using the results from the finite element model. The tie bar forces resulting from the individually applied unit loads to the finite element model form the coefficients of the unit load influence matrix. From these results the unit load influence matrix can be generated which gives a relationship between the body/chassis contact forces, unit tractor self weight, the chassis self weight and the suspension forces. The unit load influence matrices relating to the suspension operating conditions $\mathrm{a}, \mathrm{b}$ and c are given in Tables 4.9, 4.10 and 4.11 respectively.

### 4.5.4 Tyre force influence matrix

To determine the vertical tyre forces in the trailer wheels for any given loading condition, a unit load influence matrix was generated using the results from the finite element model. The forces in the tyre springs resulting from the individually applied unit loads to the finite element model form the coefficients of the unit load influence matrix. These results are used to form the unit load influence matrix which gives a relationship between the body/chassis contact forces the tractor self weight, the chassis self weight, the suspension forces and the vertical tyre forces. The unit load
influence matrices relating to the suspension operating conditions $\mathrm{a}, \mathrm{b}$ and c are given in Tables 4.12, 4.13 and 4.14 respectively.

### 4.5.5 Stiffness matrix

For the remaining suspension operating condition, d , described in Section 3.4.2, in which neither suspension is height controlled, unit displacements are applied to the finite element model instead of unit forces, with the nodes of interest being restrained in the desired directions. The reaction forces resulting from the individually applied unit displacements to the finite element model form the coefficient of the unit displacement stiffness matrix. The finite element results enable a unit displacement stiffness matrix, shown in Table 4.15, to be generated from which the chassis displacements, the suspension displacements, the suspension forces and the tyre forces can be determined for any given loading condition. The method of calculating the unknown displacement and forces is fully described in Section 3.4.3.

### 4.6 Discussion

Two different methods have been devised for determining chassis displacements, suspension displacements, suspension forces and tyre forces using the finite element method. Depending upon the suspension operating condition, unit forces are used to generate the flexibility and influence matrices where at least one suspension is height controlled, with unit displacements being used to generate the stiffness matrix where neither suspension is height controlled.

The validity of each method for determining displacements and forces is confirmed by cross checking the forces and displacements obtained at the point at which the analysis changes the suspension operating condition. The four different methods can all be cross checked in a similar manner to confirm their validity, as the analysis changes from one suspension operating condition to another.

The continuity of the graphs shown in Figs. 5.1 to 5.4, show that good agreement has been obtained between the forces and displacements determined by either suspension operating condition at the point of cross over.


Fig. 4.1 Trailer finite element line diagram


Table 4.1a - Element Property Numbers

| Element <br> Numbers | Property Numbers | Element Numbers | Property Numbers |
| :---: | :---: | :---: | :---: |
| 1-20 | 28 | 112 | 13 |
| 21-30 | 7 | 113 | 14 |
| 31-40 | 2 | 114 | 15 |
| 41-42 | 3 | 115 | 16 |
| 43-52 | 6 | 116 | 17 |
| 53-60 | 1 | 117 | 18 |
| 61-62 | 3 | 118 | 19 |
| 63-68 | 4 | 119 | 20 |
| 69-80 | 5 | 120 | 21 |
| 81-82 | 3 | 121 | 10 |
| 83-88 | 4 | 123 | 9 |
| 89-97 | 1 | 124 | 8 |
| 98 | 11 | 125 | 9 |
| 99 | 12 | 126 | 8 |
| 100 | 13 | 127 | 9 |
| 101 | 14 | 128 | 22 |
| 102 | 15 | 129 | 24 |
| 103 | 16 | 130 | 31 |
| 104 | 17 | 132 | 26 |
| 105 | 18 | 133 | 22 |
| 106 | 19 | 134 | 24 |
| 107 | 20 | 135 | 31 |
| 108 | 21 | 137 | 26 |
| 109 | 1 | 138 | 22 |
| 110 | 11 | 139 | 24 |
| 111 | 12 | 140 | 31 |

Table 4.1b - Element Property Numbers

| Element <br> Numbers | Property <br> Numbers | Element <br> Numbers | Property <br> Numbers |
| :---: | :---: | :---: | :---: |
| 142 | 26 | 210 | 34 |
| 143 | 22 | 211 | 35 |
| 144 | 24 | 212 | 34 |
| 145 | 31 | 213, 214 | 33 |
| 147 | 26 | 215, 216 | 32 |
| 148 | 22 | 217, 218 | 36 |
| 149 | 24 | 219-226 | 37 |
| 150 | 31 | 227-232 | 38 |
| 152 | 26 | 233, 234 | 18 |
| 153 | 22 | 235, 236 | 40 |
| 154 | 24 | 237-242 | 41 |
| 155 | 31 | 243, 244 | 18 |
| 157 | 26 | 245 | 42 |
| 162-167 | 29 | 246 | 17 |
| 169-174 | 23 | 247 | 17 |
| 175-200 | 36 | 248, 249 | 8 |
| 210 | 35 | 250-255 | 23 |
| 202 | 34 | 256 | 43 |
| 203 | 35 |  |  |
| 204 | 34 |  |  |
| 205 | 35 |  |  |
| 206 | 34 |  |  |
| 207 | 35 |  |  |
| 208 | 34 |  |  |
| 209 | 35 |  |  |

Table 4.2a - Property Number Values

| Beam Property Number | $\mathrm{I}_{\mathrm{yy}}(\mathrm{mm})^{4} \times 10^{6}$ | $\mathrm{I}_{z z}(\mathrm{~mm})^{4} \times 10^{6}$ | $\mathrm{K}(\mathrm{mm})^{4} \times 10^{3}$ | Area (mm) ${ }^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 3.663 | 13.667 | 92.5 | 3250 |
| 2 | 1.464 | 9.193 | 93.333 | 2800 |
| 3 | 2.46 | 0.393 | 60.0 | 1800 |
| 4 | 0.571 | 5.343 | 77.666 | 6570 |
| 5 | 0.189 | 6.351 | 755.208 | 3625 |
| 6 | 0.813 | 2.686 | 10.208 | 1225 |
| 7 | 0.388 | 7.801 | 43.52 | 2040 |
| 8 | 15.224 | 5.124 | 11834 | 2900 |
| 9 | 0.362 | 1.298 | 8125 | 1000 |
| 10 | 2.569 | 2.569 | 5138 | 1717 |
| 11 | 3.664 | 25.025 | 0.0942 | 3458 |
| 12 | 3.665 | 69.249 | 0.0988 | 4008 |
| 13 | 3.665 | 105.286 | 0.1019 | 4375 |
| 14 | 3.664 | 125.099 | 0.0952 | 4162 |
| 15 | 3.664 | 170.164 | 0.0965 | 4406 |
| 16 | 3.664 | 186.195 | 0.09695 | 4488 |
| 17 | 3.664 | 174.025 | 0.0966 | 4430 |
| 18 | 3.664 | 162.496 | 0.0964 | 4372 |
| 19 | 3.664 | 151.253 | 0.0961 | 4314 |
| 20 | 3.664 | 140.986 | 0.0957 | 4258 |
| 21 | 3.664 | 140.986 | 0.0957 | 4258 |
| 22 | 8.919 | 17.756 | 0.0241 | 2875 |
| 23 | 80.55 | 80.55 | 16109 | 49700 |
| 24 | 2.084 | 0.777 | 1926 | 3908 |
| 31 | 1.042 | 0.389 | 963 | 1954 |
| 36 | 1.585 | 27.551 | 30.744 | 2598 |
| 37 | 100000 | 10000 | 100 | 1000 |
| 38 | 0.134 | 0.134 | 267.803 | 1297.2 |
| 39 | 0.397 | 0.397 | 794.716 | 2234.6 |
| 40 | 0 | 0 | 0 | 900 |
| 42 | 1.113 | 1.113 | 2226.26 | 1995 |

Table 4.2b - Property Number Values

| Spring Property <br> Number | Spring Stiffness | Spring Stiffness | Spring Stiffness |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{K}_{\mathrm{x}}(\mathrm{N} / \mathrm{mm})$ | $\mathrm{K}_{\mathrm{y}}(\mathrm{N} / \mathrm{mm})$ | $\mathrm{K}_{\mathrm{z}}(\mathrm{N} / \mathrm{mm})$ |
| 26 | - | 1070 | - |
| 29 | 749 | - | 412 |
| 32 | - | 872 | - |
| 33 | - | 403 | - |
| 34 | 755 | - | 350 |
| 35 | - | 981 | - |
| Mass rroperty | Mass (kg) | - |  |
| 41 | 17.666 |  |  |
| 43 | 1220 |  |  |
| Plate Property <br> Number | Plate Thickness <br> (mm) |  |  |
| 28 | 8 |  |  |


| Table 4.3-Displacement flexibility matrix, original chassis, case 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{H}_{\mathrm{Lx}}$ | $\mathrm{H}_{\mathrm{Rx}}$ | $\mathrm{H}_{\text {Ly }}$ | $\mathrm{H}_{\mathrm{Ry}}$ | $\mathrm{H}_{\mathrm{L}}$ | $\mathrm{R}_{\mathrm{x}}$ | $\mathrm{R}_{\mathrm{y}}$ | $\mathrm{R}_{2}$ | $\mathrm{W}_{\text {Ty }}$ | $\mathrm{W}_{\mathrm{Tz}}$ | $\mathrm{W}_{\mathrm{xy}}$ | $\mathrm{W}_{\mathrm{cz}}$ | $\mathrm{A}_{\mathrm{R}}$ | $\mathrm{A}_{\mathrm{L}}$ |
| N186-X | 0.1562 | 0.0919 | 0.0839 | 0.0505 | -0.1082 | 0.1163 | -0.0135 | 0.0408 | -0.0122 | 0.0286 | 0.028 | -0.0043 | 0.0926 | 0.0693 |
| N186-Y | 0.0839 | 0.0505 | 1.3528 | 0.7072 | 0.1345 | 0.0442 | -0.799 | 0.0088 | -0.0777 | 0.0015 | 0.2506 | 0.0145 | 2.2495 | 1.7450 |
| N186-Z | -0.1082 | 0.1082 | 0.1345 | -0.1345 | 1.2801 | 0 | 0 | $\begin{aligned} & -0.083 \\ & 6 \\ & \hline \end{aligned}$ | 0 | -0.0630 | 0 | 0.3613 | 0.1051 | -0.1050 |
| N250-X | 0.0919 | 0.1562 | 0.0505 | 0.0839 | 0.1802 | 0.1163 | -0.0135 | $0.0408$ | -0.0122 | -0.0286 | 0 | 0.0043 | 0.0693 | 0.0296 |
| N250-Y | 0.0505 | 0.0839 | 0.7072 | 1.3528 | -0.1345 | 0.0442 | -0.799 | $0.0088$ | -0.0777 | -0.0012 | 0.2056 | -0.0145 | 1.7450 | 2.2495 |
| N250-Z | -0.1082 | 0.1082 | 0.1345 | -0.1345 | 1.2785 | 0 | 0 | -0.836 | 0. | -0.0630 | 0 | 0.3613 | 1.051 | -0.1050 |
| N314-X | 0.1163 | 0.1163 | 0.0442 | 0.0442 | 0 | 0.1185 | -0.0126 | 0.0150 | -0.0112 | 0 | 0.0043 | 0 | 0.0667 | 0.0667 |
| N314-Y | -0.0135 | 0.0135 | 0.0800 | -0.0800 | 0 | -0.0126 | 0.2826 | 0.004 | 0.2639 | 0 | 0.0579 | 0 | 0.0316 | 0.0316 |
| N314-Z | 0.0408 | -0.0408 | 0.0088 | -0.0088 | -0.0836 | 0 | 0 | 0.3203 | 0 | 0.4085 | 0 | 0.1219 | 0.069 | -0.0070 |

Table 4.4 - Displacement flexibiliyt matrix, original chassis, case 2

|  | $\mathrm{H}_{\text {Lx }}$ | $\mathrm{H}_{\mathrm{Rx}}$ | $\mathrm{H}_{\mathrm{L} y}$ | $\mathrm{H}_{\mathrm{Ry}}$ | $\mathrm{H}_{\mathrm{L} 2}$ | $\mathrm{R}_{\mathrm{x}}$ | $\mathrm{R}_{\mathrm{y}}$ | $\mathrm{R}_{2}$ | $\mathrm{W}_{\text {Ty }}$ | $\mathrm{W}_{\text {Tz }}$ | $\mathrm{W}_{\mathrm{xy}}$ | $\mathrm{W}_{\mathrm{cz}}$ | $\mathrm{A}_{\text {R }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N186-X | 0.1567 | 0.0922 | 0.1011 | 0.0511 | -0.1046 | 0.1166 | -0.0133 | 0.0411 | -0.0122 | 0.0287 | 0.056 | -0.0040 | 0.0954 |
| N186-Y | 0.1011 | 0.0615 | 1.965 | 0.8703 | 0.2617 | 0.0572 | -0.0736 | 0.0167 | -0.0763 | 0.0043 | 0.3481 | 0.0263 | 1.6727 |
| N186-Z | -0.1047 | 0.1105 | 0.2617 | -0.1006 | 1.3065 | 0.0027 | 0.0013 | -0.0820 | 0.0003 | -0.0624 | 0.0203 | 0.3638 | 0.0877 |
| N250-X | 0.0922 | 0.1564 | 0.0615 | 0.0868 | 0.1105 | 0.1165 | -0.0134 | -0.0406 | -0.0122 | -0.0285 | 0.0046 | 0.0045 | 0.1092 |
| N250-Y | 0.0551 | 0.0868 | 0.8703 | 1.3963 | -0.1006 | 0.0477 | -0.0782 | -0.0067 | -0.0773 | -0.0003 | 0.2766 | -0.0113 | 2.4966 |
| N250-Z | -0.1047 | 0.1105 | 0.2617 | -0.1006 | 1.3049 | 0.0027 | 0.0013 | -0.0820 | 0.0003 | -0.0624 | 0.0203 | 0.3638 | 0.0877 |
| N314-X | 0.1167 | 0.1165 | 0.0572 | 0.0477 | 0.0027 | 0.1187 | -0.0124 | 0.0002 | -0.0112 | 0.0007 | 0.0064 | 0.0003 | 0.0863 |
| N314-Y | -0.0133 | -0.0134 | -0.0736 | -0.0783 | 0.0013 | -0.0124 | 0.2827 | 0.0001 . | -0.2640 | 0 | 0.0589 | 0.0001 | 0.0413 |
| N314-Z | 0.0411 | -0.0407 | 0.0167 | -0.0067 | -0.0820 | 0.002 | 0.0001 | 0.6212 | 0 | 0.4086 | 0.0013 | 0.1220 | 0.050 |



|  | $\mathrm{H}_{\mathrm{L} \times}$ | $\mathrm{H}_{\mathrm{Rx}}$ | $\mathrm{H}_{\mathrm{L},}$ | $\mathrm{H}_{\mathrm{Ry}}$ | $\mathrm{H}_{\mathrm{L}}$ | $\mathrm{R}_{\mathrm{x}}$ | R ${ }_{\text {y }}$ | $\mathrm{R}_{\mathrm{z}}$ | $\mathrm{W}_{\text {Ty }}$ | $\mathrm{W}_{\text {Tz }}$ | $\mathrm{W}_{\mathrm{xy}}$ | $\mathrm{W}_{\mathrm{cz}}$ | $\mathrm{A}_{\mathrm{R}}$ | $\mathrm{A}_{\mathrm{L}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N186-X | 0.1564 | 0.0922 | 0.0868 | 0.0615 | -0.1105 | 0.1165 | -0.0134 | 0.0407 | -0.0122 | 0.0285 | 0.0045 | -0.0045 | 0.1092 | 0.1035 |
| N186-Y | 0.0868 | 0.0551 | 1.3963 | 0.8703 | 0.1007 | 0.0477 | -0.0782 | 0.0067 | -0.0773 | 0.0003 | 0.2766 | 0.0113 | 2.4966 | 2.2539 |
| N186-Z | -0.1105 | 0.1047 | 0.1007 | 0.2617 | 1.3065 | -0.0027 | -0.0013 | -0.0820 | -0.0003 | -0.0624 | -0.0203 | 0.3638 | -0.0876 | -0.5017 |
| N250-X | 0.0922 | 0.1567 | 0.0551 | 0.1012 | 0.1046 | 0.1166 | -0.0133 | -0.0410 | -0.0122 | -0.0286 | 0.0055 | 0.0040 | 0.0954 | 0.1464 |
| N250-Y | 0.0615 | 0.1012 | 0.8703 | 1.9650 | -0.2616 | 0.0572 | -0.0736 | -0.0167 | -0.0763 | -0.0043 | 0.2481 | -0.0263 | 2,6727 | 4.1598 |
| N250-Z | -0.1105 | 0.1047 | 0.1007 | -0.2617 | 1.3049 | -0.0027 | -0.0013 | -0.0820 | -0.003 | -0.0624 | -0.0202 | 0.3638 | -0.0876 | -0.5018 |
| N314-X | 0.1165 | 0.1167 | 0.0477 | 0.0572 | -0.0027 | 0.1187 | -0.0124 | -0.0002 | -0.0001 | -0.0001 | 0.0064 | -0.0002 | 0.0863 | 0.1071 |
| N314-Y | -0.0133 | -0.0133 | -0.0783 | -0.0736 | -0.0013 | -0.0214 | 0.2827 | -0.0001 | -0.2640 | 0 | 0.0589 | -0.0001 | 0.0413 | 0.0515 |
| N314-Z | 0.0407 | -0.0411 | 0.0067 | -0.0167 | -0.0820 | -0.0002 | -0.0001 | 0.06212 | 0 | 0.4086 | -0.0013 | 0.1220 | -0.0050 | -0.0315 |

Table 4.6-Suspension displacement flexibility matrix, original chassis, case 1

| Unit forces | $\mathrm{H}_{\mathrm{L} \times}$ | $\mathrm{H}_{\mathrm{R} x}$ | $\mathrm{H}_{\mathrm{L}}$ | $\mathrm{H}_{\mathrm{Ry}}$ | $\mathrm{H}_{\mathrm{Lz}}$ | $\mathrm{R}_{\mathrm{x}}$ | Ry | $\mathrm{R}_{2}$ | $\mathrm{W}_{\text {Ty }}$ | $\mathrm{W}_{\text {T2 }}$ | $\mathrm{W}_{\mathrm{xy}}$ | $\mathrm{W}_{\mathrm{cz}}$ | $\mathrm{A}_{\mathrm{R}}$ | $\mathrm{A}_{\mathrm{L}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N195-Y | 0.0148 | -0.0023 | 0.3534 | 0.5399 | -0.1094 | 0.0012 | 0.0511 | -0.0065 | 0.00386 | -0.0075 | 0.2202 | -0.0176 | 1.1810 | 1.3474 |
| N217-Y | 0.0103 | -0.0041 | -0.2113 | -0.5139 | -0.0614 | -0.0010 | -0.0528 | -0.0042 | -0.0430 | -0.0064 | 0.0013 | -0.0127 | -1.7704 | -2.1217 |
| N199-Y | 0.0287 | 0.0279 | 0.4941 | 0.8413 | -0.01186 | 0.0238 | -0.0047 | -0.0072 | -0.0109 | -0.0043 | 0.2329 | -0.0165 | 1.4093 | 1.6999 |
| N220-Y | 0.0304 | 0.0248 | 0.5108 | 0.8104 | -0.1158 | 0.00236 | -0.0047 | -0.0069 | -0.0109 | -0.0049 | 0.2322 | -0.0168 | 1.3457 | 1.4240 |
| N203-Y | 0.0447 | 0.0679 | 0.6508 | 1.2129 | -0.1301 | 0.0388 | -0.0606 | -0.0084 | -0.0603 | -0.0019 | 0.2549 | -0.0150 | 1.6568 | 2.1038 |
| N223-Y | -0.0219 | -0.0199 | -0.5461 | -0.3118 | -0.0760 | -0.0165 | 0.0119 | -0.0040 | 0.0145 | -0.0018 | -0.0148 | -0.01117 | -2.1747 | -2.3856 |

Table 4.7 - Suspension displacement flexibility matrix, original chassis, case 2

|  | $\mathrm{H}_{\mathrm{L} \times}$ | $\mathrm{H}_{\mathrm{Rx}}$ | $\mathrm{H}_{\text {Ly }}$ | $\mathrm{H}_{\mathrm{Ry}}$ | $\mathrm{H}_{\mathrm{L}}$ | $\mathrm{R}_{\text {x }}$ | Ry | $\mathrm{R}_{\mathbf{z}}$ | $\mathrm{W}_{\text {Ty }}$ | $\mathrm{W}_{\text {Tz }}$ | $\mathrm{W}_{\mathrm{xy}}$ | $\mathrm{W}_{\mathrm{cz}}$ | $\mathrm{A}_{\mathrm{R}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N195-Y | 0.01878 | 0.0002 | 0.4946 | 0.5575 | -0.0801 | 0.0131 | 0.0526 | -0.0047 | 0.0389 | -0.0068 | 0.2426 | -0.0149 | 1.5613 |
| N217-Y | 0.0054 | -0.0072 | -0.3871 | -0.2008 | -0.0979 | -0.0047 | -0.0546 | -0.0064 | -0.0438 | -0.0073 | -0.0266 | -0.0161 | -2.3880 |
| N199-Y | 0.0328 | 0.0305 | 0.6413 | 0.8805 | -0.0881 | 0.0269 | -0.0031 | -0.0053 | -0.0106 | -0.0035 | 0.2563 | -0.0137 | 1.9229 |
| N220-Y | 0.0301 | 0.0246 | 0.5002 | 0.8076 | -0.1180 | 0.0234 | -0.0048 | -0.0071 | -0.0110 | -0.0030 | 0.2306 | -0.0170 | 1.4081 |
| N203-Y | 0.0492 | 0.0708 | 0.8093 | 1.2552 | -0.0971 | 0.0422 | -0.0587 | -0.0063 | -0.0599 | -0.0011 | 0.2712 | -0.0120 | 2.3441 |
| N223-Y | -0.0302 | -0.0252 | -0.8407 | -0.3903 | -0.1372 | -0.0227 | -0.0088 | -0.0078 | 0.0139 | -0.0033 | -0.0617 | -0.0174 | -2.8321 |

Table 4.8 - Suspension displacement flexibility matrix, original chassis, case 4

| Unit forces <br> Unit force displacements | $\mathrm{H}_{\text {Lx }}$ | $\mathrm{H}_{\mathrm{Rx}}$ | $\mathrm{H}_{\text {Ly }}$ | $\mathrm{H}_{\mathrm{Ry}}$ | $\mathrm{H}_{\mathrm{L} \boldsymbol{L}}$ | R ${ }_{\text {, }}$ | R ${ }_{\text {y }}$ | $\mathrm{R}_{2}$ | $\mathrm{W}_{\text {Ty }}$ | $\mathrm{W}_{\text {Tz }}$ | $\mathrm{W}_{\mathrm{xy}}$ | $\mathrm{W}_{\mathrm{cz}}$ | $\mathrm{A}_{\mathrm{R}}$ | $\mathrm{A}_{\mathrm{L}}{ }^{-}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N195-Y | 0.0215 | 0.0082 | 0.04532 | 0.9145 | -0.1873 | 0.0180 | 0.0550 | -0.0114 | 0.0394 | -0.0094 | 0.2800 | -0.0249 | 1.7487 | 2.5165 |
| N217-Y | 0.0849 | -0.0062 | -0.02330 | -0.2314 | -0.0453 | -0.0027 | -0.0536 | -0.0032 | -0.0432 | -0.0060 | -0.0110 | -0.0112 | -1.8879 | -2.3635 |
| N199-Y | 0.0372 | 0.0413 | 0.6218 | 1.3204 | -0.2182 | 0.0340 | 0.0003 | -0.0103 | -0.0099 | -0.0068 | 0.3092 | -0.0258 | 2.1354 | 3.1951 |
| N220-Y | 0.0248 | 0.0160 | 0.4270 | 0.4961 | -0.0505 | 0.0170 | -0.0080 | -0.0029 | -0.0117 | -0.0034 | 0.1822 | -0.0108 | 0.8695 | 0.4434 |
| N203-Y | 0.0551 | 0.0842 | 0.8047 | 1.7908 | -0.2501 | 0.0511 | -0.0543 | -0.0158 | -0.5090 | -0.0049 | 0.3380 | -0.0262 | 1.5236 | 3.9072 |
| N223-Y | -0.0234 | -0.0233 | -0.5692 | -0.3987 | -0.0579 | -0.0183 | 0.0110 | -0.0029 | 0.0143 | -0.0014 | -0.0287 | -0.0100 | -2.3064 | -2.6568 |

Table 4.9-Suspension force influence matrix, original chassis, case 1

92
Table 4.12 - Tyre force influence matrix, original chassis, case 1

| Unit forces <br> Unit force displacements | $\mathrm{H}_{\text {Lx }}$ | $\mathrm{H}_{\mathrm{R} \times}$ | $\mathrm{H}_{\text {Ly }}$ | $\mathrm{H}_{\mathrm{Ry}}$ | $\mathrm{H}_{\mathrm{L} 2}$ | $\mathrm{R}_{\text {x }}$ | R ${ }_{\text {r }}$ | $\mathrm{R}_{\mathbf{z}}$ | $\mathrm{W}_{\text {Ty }}$ | $\mathrm{W}_{\text {Tt }}$ | $\mathrm{W}_{\mathrm{xy}}$ | $\mathrm{W}_{\mathrm{cz}}$ | $\mathrm{A}_{\mathrm{R}}$ | $\mathrm{A}_{\mathrm{L}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N231-Y | 0.0105 | -0.0102 | -0.0413 | 0.0412 | -0.088 | 0.0001 | 0 | -0.0059 | 0.0018 | -0.0091 | -0.0826 | -0.0181 | 0.911 | 0.669 |
| N232-Y | -0.0192 | -0.0274 | -0.854 | -0.434 | -0.0163 | -0.0214 | -0.0106 | -0.0098 | -0.0022 | -0.0071 | -0.2446 | -0.0238 | -1.74 | -1.39 |
| N233-Y | -0.0014 | 0.0017 | -0.168 | 0.168 | -0.109 | 0.001 | 0 | -0.0058 | 0 | -0.0027 | -0.0826 | -0.0167 | 0.810 | 0.770 |
| N234-Y | -0.0102 | 0.0105 | 0.0412 | -0.0413 | 0.088 | 0.0001 | 0 | 0.0059 | 0 | 0.0091 | -0.0826 | 0.0181 | 0.669 | 0.911 |
| N235-Y | -0.0274 | -0.0192 | -0.434 | -0.854 | 0.163 | -0.0214 | -0.0106 | 0.0098 | -0.0022 | 0.0071 | -0.2446 | 0.0238 | -1.39 | -1.74 |
| N236-Y | 0.0017 | -0.0014 | 0.168 | -0.168 | 0.109 | 0.0001 | 0 | 0.0058 | 0 | 0.0027 | -0.0826 | 00167 | 0.770 | 0.810 |

Table 4.13 - Tyre force influence matrix, original chassis, case 2

| Unit force <br> Unit forces <br> displacements | $\mathrm{H}_{\mathrm{Lx}}$ | $\mathrm{H}_{\mathrm{Rx}}$ | $\mathrm{H}_{\mathrm{Ly}}$ | $\mathrm{H}_{\mathrm{Ry}}$ | $\mathrm{H}_{\mathrm{L} z}$ | $\mathrm{R}_{\mathrm{x}}$ | $\mathrm{R}_{\mathrm{y}}$ | $\mathrm{R}_{\mathrm{z}}$ | $\mathrm{W}_{\mathrm{Ty}}$ | $\mathrm{W}_{\mathrm{Tz}}$ | $\mathrm{W}_{\mathrm{xy}}$ | $\mathrm{W}_{\mathrm{cz}}$ | $\mathrm{A}_{\mathrm{R}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{N} 231-\mathrm{Y}$ | 0.085 | -0.0115 | -0.112 | 0.0224 | -0.103 | -0.0014 | -0.0007 | -0.0068 | -0.0001 | -0.0095 | -0.0938 | -0.0194 | -0.5620 |
| $\mathrm{~N} 232-\mathrm{Y}$ | -0.0134 | -0.0237 | -0.647 | -0.379 | -0.120 | -0.017 | -0.0084 | -0.0071 | -0.017 | -0.0060 | -0.2116 | -0.0198 | -1.080 |
| $\mathrm{~N} 233-\mathrm{Y}$ | -0.0056 | -0.0010 | -0.318 | -0.129 | -0.140 | -0.003 | -0.0015 | -0.0678 | -0.0003 | -0.0034 | -0.1062 | -0.0195 | 0.544 |
| $\mathrm{~N} 234-\mathrm{Y}$ | -0.0082 | -0.0118 | 0.112 | -0.025 | 0.103 | 0.0016 | 0.0008 | 0.0068 | 0.0002 | 0.0095 | -0.0713 | 0.0194 | 1.02 |
| $\mathrm{~N} 235-\mathrm{Y}-$ | -0.0331 | -0.0228 | -0.636 | -0.908 | -0.121 | -0.0256 | -0.0127 | 0.0072 | -0.0026 | 0.0061 | -0.2768 | 0.0199 | -2.05 |
| N236-Y | 0.0059 | 0.0013 | 0.318 | -0.129 | 0.140 | 0.0033 | 0.0016 | 0.0077 | 0.0003 | 0.0034 | -0.059 | 0.0195 | 1.04 |

Table 4.14 - Tyre force influence matrix, original chassis, case 4

|  | $\mathrm{H}_{\mathrm{L} \times}$ | $\mathrm{H}_{\mathrm{Rx}}$ | $\mathrm{H}_{\mathrm{L},}$ | $\mathrm{H}_{\mathrm{R} \boldsymbol{y}}$ | $\mathrm{H}_{L 2}$ | $\mathrm{R}_{\mathrm{x}}$ | $\mathrm{R}_{\mathrm{y}}$ | $\mathrm{R}_{2}$ | $\mathrm{W}_{\text {Ty }}$ | $\mathrm{W}_{\text {T } 2}$ | $\mathrm{W}_{\mathrm{xy}}$ | $\mathrm{W}_{\mathrm{cz}}$ | $\mathrm{A}_{\mathrm{R}}$ | $\mathrm{A}_{\mathrm{L}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N231-Y | 0.0117 | -0.0082 | -0.0225 | 0.112 | -0.103 | 0.0016 | 0.0008 | -0.0068 | 0.0002 | -0.0095 | -0.7125 | -0.01944 | 1.0 | 0.890 |
| N232-Y | -0.0228 | -0.0331 | -0.908 | -0.636 | -0.121 | -0.0256 | -0.0127 | -0.0072 | -0.0026 | -0.0061 | -0.2768 | -0.199 | -2.05 | -2.02 |
| N233-Y | 0.0013 | 0.0059 | -0.129 | 0.318 | -0.140 | 0.0044 | 0.0016 | -0.0078 | 0.0003 | -0.0034 | -0.0588 | -0.0195 | 1.04 | 1.24 |
| N234-Y | -0.0115 | 0.0085 | 0.0224 | -0.112 | 0.103 | -0.0014 | -0.0007 | -0.0068 | -0.0001 | 0.0095 | -0.938 | 0.0194 | 0.562 | 0.690 |
| N235-Y- | -0.0237 | -0.0134 | -0.379 | -0.637 | 0.120 | -0.0170 | -0.0008 | 0.0071 | -0.0018 | 0.0060 | -0.2116 | 0.0198 | -1.08 | -1.10 |
| N236-Y | -0.0010 | -0.0056 | 0.129 | -0.318 | 0.140 | -0.0030 | -0.0015 | -0.0078 | -0.0003 | 0.0034 | -0.1062 | 0.0195 | 0.544 | 0.344 |


|  | $\mathrm{H}_{\text {L }}$ | $\mathrm{H}_{4}$, | $\mathrm{H}_{4}$ | $\mathrm{H}_{2}$ | $\mathrm{H}_{7}$ | $\mathrm{R}_{2}$ | R | $R_{1}$ | $w_{\text {w }}$ | $w_{\text {w }}$ | $w_{\text {mp }}$ | $w_{\text {ant }}$ | $\mathrm{w}_{\text {- }}$ | $\mathrm{w}_{\text {¢ }}$ | $w_{\sim}$ | ${ }^{*}$ | $w_{\pi}$ | $w_{\mathrm{n}}$ | An | $\mathrm{A}_{3}$ | A | A | As, | As |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N186-X | 49187 | -2497 | 2785 | 10570 | . 2948 | -57919 | -1715 | -839 | 4599 | -6917 | 960 | 2205 | 1156 | 1161 | 929 | 1743 | 232 | . 52 | -5098 | -617 | 4363 | -739 | 224 | .594 |
| N186-Y | -2497 | 1654 | -721 | 63 | -213 | 2575 | 519 | 37 | -865 | 454 | -151 | -280 | -721 | -10 | -1489 | 493 | 41 | 18 | 739 | 97 | -77 | 461 | -113 | 952 |
| N186-z | 2785 | -721 | 2745 | -2550 | 836 | -228 | -6 | 7 | . 329 | -339 | . 66 | 48 | -71 | -69 | -39 | -1992 | 2 | -0.1 | 327 | 43 | 44 | 45 | -49 | 25 |
| N200-X | 10569 | 63 | -2550 | 5330 | -16054 | -61830 | -1891 | 914 | .768 | 8539 | -113 | -3120 | 81 | -1602 | 155 | -20317 | 275 | 59 | 3176 | 74 | 1436 | -52 | 13563 | -98 |
| N20S-Y | -2948 | -213 | 836 | -16051 | 258220 | 18491 | 951 | -273 | 1271 | -3219 | -83 | 1506 | -71 | 617 | -32 | 494 | -116 | -38 | -28689 | 52 | 139440 | 45 | -37004 | 20 |
| N314X | -57919 | 2575 | -288 | -61829 | 18491 | 124720 | 3673 | -78 | -3472 | -1554 | . 787 | 869 | -1194 | 423 | -1058 | 565 | . 508 | -6 | 2147 | 506 | . 5941 | 764 | -15556 | 667 |
| N314Y | $-1785$ | 516 | -6 | -1891 | 943 | 3674 | 9520 | -9 | -1736 | -198 | . 234 | 172 | -54 | 56 | 3 | -20 | . 5626 | - 5 | -1674 | 152 | 2309 | 37 | -1367 | -2 |
| N3142 | -841 | 38 | 7 | 918 | -279 | -71 | 1 | 2535 | -74 | . 514 | -15 | 49 | -19 | 66 | -16 | 36 | 0. | -1005 | 67 | 10 | 10 | 12 | 256 | 10 |
| N3is-y | 4598 | -865 | -329 | . 768 | 1271 | -3471 | -1073 | . 72 | 1179 | -3193 | -2624 | 2286 | -1892 | 958 | -990 | 367 | 193 | -39 | -11540 | 1670 | 4994 | 1210 | -2218 | 634 |
| N315-2 | -6917 | 454 | -339 | 8539 | -3219. | -155 | -167 | -513 | -3193 | 13055 | . 632 | -8458 | -529 | -2947 | -320 | . 771 | 29 | -33 | 6644 | 406 | . 3773 | 339 | 3670 | 205 |
| N316-Y | 960 | -151 | -66 | $-113$ | -83 | -787 | -236 | -15 | -2624 | -632 | 8260 | 440 | -376 | 174 | -194 | 69 | 30 | -13 | 1655 | .3935 | -1007 | 240 | 328 | 124 |
| N316-Z | 2205 | -280 | 48 | -3121 | 1506 | 869 | 143 | 48 | 2286 | -8455 | 442 | 9424 | 363 | -146 | 184 | -83 | -22 | 28 | -5414 | -285 | 3672 | . 232 | -2246 | -117 |
| N317-Y | 1155 | -721 | -71 | 82 | -71 | -1194 | -53 | -19 | -1891 | . 529 | . 376 | 362 | 8378 | 88 | -261 | 134 | 14 | 4 | 1115 | 242 | 741 | -009 | 252 | 167 |
| N317-2 | 1161 | -10 | 469 | -1603 | 617 | 422 | 54 | 66 | 957 | -2497 | 174 | $-146$ | 92 | 5698 | 106 | -1645 | -9 | 5 | -2066 | -112 | 1240 | -60 | -919 | -68 |
| N318-Y | 929 | -1489 | -39 | 155 | -32 | -1057 | 3 | -16 | -090 | . 320 | -194 | 184 | 261 | 106 | 8545 | 48 | 6 | -1 | 565 | 125 | -113 | 167 | 126 | -119 |
| N318-2 | 1743 | 492 | -1992 | -2318 | 494 | 565 | -20 | 36 | 367 | -771 | 69 | -3 | 134 | -1645 | 51 | 5312 | -1 | 1 | 397 | 4 | -1472 | -3 | . 353 | -33 |
| N313-Y | 232 | -41 | 2 | 275 | -116 | . 508 | -5626 | 1 | 194 | 28 | 30 | -22 | 14 | -9 | 6 | -1.2 | 5910 | 0.2 | 172 | -19 | -186 | $\rightarrow$ | 149 | 4 |
| N313-2 | -52 | 19 | -0.1 | 59 | -38 | -6 | -3 | -1005 | -39 | -33 | 13 | 28 | 4 | 5 | -1 | 1 | -0.2 | 1547 | 128 | 8 | -124 | 3 | 63 | 1 |
| N195-Y | -5088 | 739 | 327 | 3175 | $-28689$ | 2145 | -1690 | 66 | -11540 | 6644 | 1656 | -5412 | 1114 | -2062 | 565 | 397 | 171 | 128 | 53949 | -1051 | -80065 | . 712 | 65800 | -362 |
| N217-Y | -617 | 97 | 43 | 74 | 52 | 506 | 152 | 9 | 1670 | 406 | -3934 | -283 | 242 | -112 | 125 | 4 | -19 | 8 | -1051 | 2515 | 641 | -155 | -208 | -80 |
| N199-Y | 4363 | -778 | 414 | 1437 | 139440 | -5940 | 2332 | 11 | 4994 | -3773 | -1008 | 3670 | . 741 | 1239 | +12 | -1470 | . 184 | . 123 | -80065 | 641 | 183970 | 474 | -25804 | 264 |
| N220-Y | .738 | 461 | 45 | -52 | 45 | 764 | 34 | 12 | 1210 | 339 | 240 | -232 | 4010 | . 56 | 167 | -85 | -9 | 3 | . 712 | -159 | 474 | 2365 | -161 | - 107 |
| N203-Y | 2245 | -113 | - 59 | 13560 | -37804 | -15556 | -1403 | 250 | -2218 | 3670 | 328 | -2246 | 252 | . 919 | 126 | -353 | 148 | 64 | 65800 | -208 | -25804 | -161 | 556980 | -31 |
| N223-Y | . 594 | 952 | 25 | . 98 | 20 | 677 | -2 | 10 | 634 | 205 | 124 | -117 | 167 | -68 | +118 | . 31 | 4 | 1 | -362 | -800 | 264 | -107 | -1 | 2634 |
| N231-Y | 93 | -13 | -6 | -0.05 | -62 | -90 | 4 | -1 | -153 | -50 | . 1740 | 32 | . 33 | 13 | -17 | 6 | 1 | -2 | 761 | 40 | -10 | 21 | 153 | 11 |
| N232-Y | 77 | -52 | $+$ | 11 | 19 | -85 | -6 | -2 | . 143 | -29 | . 27 | 14 | -1722 | -10 | -19 | 12 | 1 | -0.2 | 372 | 17 | 114 | 428 | $-40$ | 12 |
| N233-Y | 63 | -109 | 1 | 16 | -82 | . 78 | 1 | -1 | . 71 | -22 | -14 | 14 | -19 | 12 | -1710 | -11 | 0.4 | -0.1 | 7 | 9 | 311 | 12 | 203 | 421 |
| N234Y | -77 | 10 | 5 | -2 | 63 | 71 | -0.2 | 1 | 141 | 40 | - 303 | -25 | 27 | . 10 | 14 | -s | -0.5 | 2 | .756 | 479 | 405 | -17 | -152 | 9 |
| N23S. Y | -65 | 4 | 4 | -10 | -20 | 72 | 5 | 1 | 123 | 23 | 23 | -20 | 317 | 11 | 16 | -11 | $-1$ | 0.2 | -361 | -15 | -122 | 471 | 43 | -10 |
| N236-Y | -54 | 93 | $-1$ | -14 | 81 | 67 | -0.8 | 1 | 61 | 19 | 12 | -12 | 17 | -10 | -327 | 12 | 0.3 | 0.1 | 3 | -8 | -315 | -11 | -201 | 464 |

Table 4.15-Unit displacement stiffness matrix, original chassis, case d

## CHAPTER FIVE

## DESIGN INVESTIGATIONS

### 5.1 Introduction

Whilst a vehicle operator should always endeavour to discharge the payload with the vehicle standing on level ground, practical situations arise where this is not possible. This may be due to the absence of level ground or poor judgement by the operator, which may result in the vehicle being tipped on a lateral ground slope. As a result of this, the maximum ground slope angle considered for the theoretical model is limited to ten degrees, as this position is at least twice the severity of ground slope on which a vehicle should normally be tipped.

For each trailer design the magnitude of the load, position of the load, ram length and ground slope can be varied in any combination. Four payload and up to nine payload positions are considered, varying the ground slope from 0 to $\mathbf{8}$ degrees and varying the ram length from 2 to 8 meters. Also, three further chassis configurations, described in Section 4.3, based on the reference chassis were modelled to investigate the contribution of important component flexibilities to stability.

From the theoretical results, three types of graphs are obtained giving details of the stability of the vehicle. The first set of graphs give details of how the rear left trailer tyre force varies with ground slope and ram length for a given payload magnitude and position. The ground slope varies between 0 to 10 degrees in steps of 1 degree for
each graph. The graphs for the four chassis configurations are shown on one page for each payload magnitude and position, to enable a direct comparison of their stability characteristics and are shown in Fig. 5.1 to Fig. 5.4. The graph also shows what condition the suspension is operating in. The data points are only marked if the as discursed in section 3.4.2. suspension is operating using condition, $b, c$ or d. The data points are labelled only where the suspension operating condition changes (eg. $a$ to $b$ or $b$ to $c$ etc). The data points following a labelled data point are the same (ie. no change in the suspension operating condition).

The second set of graphs give details of how the rear left trailer tyre force varies with payload magnitude and horizontal and vertical payload position for a 4 degree ground slope and a 4 meter ram length and are shown in Fig. 5.5 to Fig. 5.10.

The third set of graphs give details of a stability envelope for each chassis configuration for a given payload magnitude and position. The graphs give details of what ram length and ground slope give rise to a zero rear left trailer tyre force for a given payload magnitude and position and are shown in Fig. 5.11 to Fig. 5.13.

### 5.2 Reference chassis

The reference chassis is the tipper chassis in production at the moment, and is described fully in Section 2.1. The stability criterion is quantified by the rear left trailer tyre force for a given payload magnitude and position. For the present analysis the vehicle is considered to be in a stable position as long as the rear left tyre force is greater than zero.

For stability comparisons between different chassis configurations, where the tyre force is greater than zero, the larger the tyre force the more stable the vehicle is for the given situation. Where the tyre force has reached zero the larger the ram length the more stable the vehicle is for the given situation.

Nine payload positions are considered for the present analyses to demonstrate how the initial payload position within the body effects the vehicles stability. As the body is assumed to be rigid the payload remains the same position relative to the body throughout the analysis. The positions are described with the vehicle standing on horizontal ground with the body untipped. The positions lie in the X-Y plane (ie. $z=0$ ) and are spaced symmetrically about the centroid of the body. The payload of centre of gravity positions and their co-ordinates are shown in Fig. 5.14.

### 5.2.1 General tipping behaviour

The suspension's operating mode, discussed in Section 3.4.2, is dependent upon the payload magnitude and position within the body, ground slope and ram length for a given chassis. From the stability graphs in Fig. 5.1 to 5.4 which show how the rear left tyre force varies with ram length and ground slope for a given payload magnitude and position, described in the previous section, it can also be seen how the suspension operating condition changes.

For smallest payload of 2500 kg the suspension operates using predominantly mode a with mode b being used in the more extreme body attitudes greater than $\theta_{\mathrm{p}}=5^{\circ}$ and
$\mathrm{RL}=6 \mathrm{~m}$ when considering payload position nine, as shown in Fig. 5.1i. The position of the payload in the fore/aft direction increases the occurrence of suspension mode $b$. However for each of the fore/aft positions considered, the vertical position does not significantly change the suspension mode.

As the payload increases to 10000 kg the suspension still operates using mode a with mode $b$ being used more frequently in body attitudes greater than the $\boldsymbol{\theta}_{\mathrm{p}}=3^{\circ}$ and RL $=5 \mathrm{~m}$ when considering payload position nine, as shown in Fig. 5.2i. The position of the payload in the fore/aft direction increases the occurrence of suspension $b$, mode $b$ with the vertical position beginning to influence the occurrence of mode $b$.

For the 17500 kg payload the suspension operates under all four operating modes. Where the payload is positioned at the rear of the body the suspension operates using mode a with mode c being used in body attitudes greater than $\theta_{p}=2^{\circ}$ and $\mathrm{RL}=7$ m , when considering payload position seven as shown in Fig. 5.3g. With the payload positioned mid-way along the body the suspension operates under mode a with modes $\mathrm{b}, \mathrm{c}, \mathrm{d}$ occurring in body attitudes greater than $\theta_{\mathrm{p}}=2^{\circ}$ and $\mathrm{RL}=7 \mathrm{~m}$, when considering payload position eight as shown in Fig. 5.3h. When the payload is positioned in the most forward position the suspension operates under mode $a$ and $b$ with modes $\mathbf{c}$ and $\mathbf{d}$ only occurring in extreme body attitudes. Suspension mode $b$ and d occurs in body attitudes greater than $\theta_{\mathrm{p}}=2^{\circ}$ and $\mathrm{RL}=6 \mathrm{~m}$ when considering payload position nine as shown in Fig. 5.3i.

With the maximum payload of 25000 kg the suspension operates under modes a and c, with the latter being most prevalent. It should be noted that the most forward payload positions three, six and nine are not considered for this payload as they are impractical positions for this payload. The suspension operates using mode a with mode c being used in body attitudes greater than $\theta_{\mathrm{p}}=1^{\circ}$ and RI $=4 \mathrm{~m}$, when considering payload position seven, as shown in Fig. 5.4e. For each of the fore/aft positions, considered, the vertical position does not significantly influence the suspension mode.

The payload magnitudes and the position of the payload centre of gravity's effect on the rear left tyre force can be conveniently demonstrated from the second set of stability graphs, by comparing the rear left tyre force for all nine payload positions with the vehicle standing on a ground slope of $4^{\circ}$ at a 4 m ram length. From Figs. 5.5 to Fig. 5.10 it can be seen that the tyre force is not proportional to load, for loads above 10000 kg . The graphs in Fig. 5.5 to Fig. 5.7 show how the rear left tyre force varies with the fore/aft position and payload magnitude of three vertical payload this heights. For loads of above 10000 kg tyre force is not proportional to the change n in the fore/aft payload position and is made worse with increasing the vertical height. The graphs in Fig. 5.8 to Fig. 5.10 show how the tyre force varies with vertical height and payload magnitude for three fore/aft payload positions. For loads above 17500 kg the tyre force is not proportional to the change in vertical height and is made worse with increasing the fore/aft position.

### 5.2.2 Rollover condition

When the trailer's rear left tyre force approaches or becomes zero it is in an unstable position and rollover is imminent. From the first set of stability graphs in Fig. 5.1 to Fig. 5.4 the stability characteristics of a trailer can be determined for a range of payload magnitudes and positions. To obtain a more general view of the rollover characteristics the ram length and ground slope data at which the graphs cross the X -axis (ie. zero tyre force) can be plotted against each other to produce the graphs in Fig. 5.11 to Fig. 5.13. From these results a stability envelope is created for a payload magnitude and position, which show that the vehicle is in a stable position below the graph line and in an unstable position above the graph line. As some payload position and magnitude combinations do not cause the vehicle to become unstable ie. rollover, not all payload positions are shown. Each payload magnitude and its unstable payload positions are shown on one graph. It should be noted that the 2500 kg payload does not have a stability envelope graph due to the tyre force remaining positive for all of the payload positions considered.

### 5.3 The effect of design changes on stability

To quantify the increase or decrease in stability resulting from design changes, the rear left tyre force, for a given body attitude, ground slope, payload and chassis configuration, is compared to the reference chassis.

### 5.3.1 The effect of introducing a sheardeck on stability

The sheardeck, described in Section 4.3 made a reasonable improvement to the stability characteristics.

For the 2500 kg payload the chassis change hardly effected the stability characteristic. Considering payload position 9, shown in Fig. 5.1i, $\theta_{p}=8^{\circ}$ and $R L=8 \mathrm{~m}$, the rear left tyre force is increased from 3047 N to 3509 N , a $15 \%$ increase.

For the 10000 kg payload the chassis change has a more noticeable effect on the stability characteristics coisidering payload position 9 , shown in Fig. 5.2i, $\boldsymbol{\theta}_{\mathrm{p}}=4^{\circ}$ and $R L=8 \mathrm{~m}$, the rear left tyre force is increased from 2795 N to 4005 N, a $43 \%$ increase.

For the 17500 kg payload the chassis change makes a noticeable effect on the stability characteristics. Considering payload position 9, shown in Fig. 5.3i, $\theta_{p}=2^{\circ}$ and $\mathrm{RL}=8 \mathrm{~m}$, the rear left tyre force is increased from 6188 N to 8531 N , a $38 \%$ increase.

For the 25000 kg payload the chassis change has a noticeable effect on the stability characteristics. Considering payload position 5, shown in Fig. 5.4d, $\theta_{p}=1^{\circ}$ and $\mathrm{RL}=8 \mathrm{~m}$, the rear left tyre force increases from 21511 N to 24714 N , a $15 \%$ increase.

### 5.4 The effect of increasing the torsional stiffness of the cross members

The torsion chassis, described in Section 4.3, made a significant improvement to the stability characteristics. The improvement to stability is quantified in the same manner as described in Section 5.3.

For the 2500 kg payload the chassis change has made a reasonable effect upon the stability characteristics. Considering payload position 9 , shown in Fig. 5.1i, $\theta_{p}=8^{\circ}$ and $\mathrm{RL}=8$, the rear left tyre force is increased from 3047 N to 4203 N , a $38 \%$ increase.

For the 10000 kg payload the chassis change has a large effect on the stability characteristics. Considering payload position 9, shown in Fig. 5.2i, $\theta_{\mathrm{p}}=4^{\circ}$ and $\mathrm{RL}=8 \mathrm{~m}$, the rear left tyre force is increased from 2795 N to 6714 N , a $140 \%$ increase.

For the 17500 kg payload the chassis change has a large effect on the stability characteristic. Considering payload position 9, shown in Fig. 5.3i, $\boldsymbol{\theta}_{\mathrm{p}}=2^{\circ}$ and $\mathrm{RL}=8 \mathrm{~m}$, the rear left tyre force is increased from 6188 N to 14703 N , a $138 \%$ increase.

For the 25000 kg payload the chassis change has a noticeable effect on the stability characteristic. Considering payload position 5 , shown in Fig. 5.4d, the $\theta_{p}=1^{\circ}$ and $\mathrm{RL}=8 \mathrm{~m}$, the rear left tyre force increases from 21511 N to 32017 N a $49 \%$ increase.

### 5.5 The effect of reducing the trailing arm stiffness

The trailing arm chassis, described in Section 4.3, made little improvement to the stability characteristics. The improvement to stability is quantified in the same manner as described in Section 5.3.

For the 2500 kg payload the chassis change hardly effected the stability characteristics. Considering payload position 9, shown in Fig. 5.1i, $\theta_{\mathrm{p}}=8^{\circ}$ and $\mathrm{RL}=8 \mathrm{~m}$, the rear left tyre force is increased from 3047 N to 3436 N, a $13 \%$ increase.

For the 10000 kg payload the chassis change had a slightly adverse effect on the stability characteristic. Considering payload position 9, shown in Fig. 5.2i, $\theta_{\mathrm{p}}=4^{\circ}$ and $\mathrm{RL}=8 \mathrm{~m}$, the rear left tyre force is reduced from 2795 N to 2500 N , an $11 \%$ decrease.

For the 17500 kg payload the chassis change had a slightly adverse effect on the stability characteristic. Considering payload position 8, shown in Fig. 5.3i, $\theta_{\mathrm{p}}=2^{\circ}$ and $\mathrm{RL}=8 \mathrm{~m}$, the rear left tyre force is reduced from 11122 to 10549 N , a $5 \%$ decrease.

For the 25000 kg payload the chassis change had a noticeable adverse effect on the stability characteristic. Considering payload position 5, shown in Fig. 5.4d, $\theta_{p}=1^{\circ}$ and $\mathrm{RL}=8 \mathrm{~m}$ the rear left tyre force decreases from 21511 N to 11479 N , a $47 \%$ decrease.

### 5.6 Discussion

From the design investigations it can be seen, for a given chassis configuration, how the rear left tyre force varies with payload magnitude and position, ram length and ground slope. From the results it was determined that the rear left tyre force does not vary linearly with these variables and that the non-linear model developed is necessary to determine the stability of a tipping trailer.

The results enable a stability envelope to be generated which gives detail of what ground slope and ram length give rise to a stable/unstable vehicle for a given payload magnitude and position. Future work may be able to utilise these results tc develop a system which would not allow the vehicle operator to tip in unstable conditions.

The design investigation demonstrates how the stability characteristics can be assessed for a given chassis configuration and how subsequent chassis modifications can be assessed in improving vehicle stability. The typical stability envelope for all four chassis configurations, is shown in Fig. 5.15 for a 25000 kg payload, in payload position 8. From the results the chassis modifications showed that the torsional stiffness of the cross-members make an important contribution to the vehicle's stability. The introduction of a sheardeck to the chassis made a small but significant increase in the vehicle stability, with the trailing stiffness being found to make little contribution to the vehicle's stability, and in some case an adverse effect resulted.

The program could also be used to investigate the effect of other design parameters on stability
(i) tyre and suspension flexibilities
(ii) suspension operating conditions
(iii) different chassis design (dimensions and materials)
(iv) the importance of matching a tractor and trailer flexibilities (eg. a stiff tractor suspension combined with a stiff trailer suspension etc.)
(v) wind loading

With some modifications to the program the effect of other design parameters on stability could also be investigated
(vi) a flowing payload during tipping
(vii) trailer body flexibility (currently assumed rigid).

Unit load or unit displacement stress matrices can be generated from the finite element analyses undertaken, these results would indicate which parts of the chassis are under or over stressed, during tipping highlighting those areas that would benefit from design changes.

|  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $1$ | $1$ |  | $11$ |  |  |  |
|  |  | - |  |  | 0.15 | \% |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  | $1$ |  |  |  |  |  |  |
|  |  |  |  |  | 8 | \% |  | \% |


(N) 2030n 3x.L igat rext




Ram Length (M)
level ground
$2^{\circ}$ ground slope
$4^{0}$ ground slope
$6^{\circ}$ ground slope
$8^{\circ}$ ground slope

















26000


26000 Smerdeck Chereh

level ground
ground slope
ground slope
$6^{\circ}$ ground slope
$8^{\circ}$ ground slope



















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level ground
$2^{\circ}$ ground slope
$4^{\circ}$ ground slope
$6^{\circ}$ ground slope
$8^{\circ}$ ground slope

| $\prime \prime \prime \prime \prime \prime \prime \prime$ |  |
| :--- | :--- |
| 0 | $\prime \prime$ |












Fig 5.5 Rear left Tyre force versus payload magnitude in fore/aft payload
positions 1,2 and 3 .

Fig 5.6 Rear left Tyre force versus payload magnitude in fore/aft payload


Fig 5.7 Rear left Tyre force versus payload magnitude in fore/aft payload



Fig 5.9 Rear left lyre force versus payload magnitude in vertical payload
positions 2,5 and 8 .


Fig 5.10 Rear left Tyre force versus payload magnitude in vertical payload

17500 Kg payload

$$
\text { Fig } 5.11 \text { Stability envelopes for a } 10000 \mathrm{Kg} \text { payload in payload positions }
$$

$3,6,8$, and 9. (based on maximm ram length/zero rear left tyre forces)
 - payload position 3 _ payload position 6 _- payload position $8 \_$payload position 9

Fig 5.12 Stability envelopes for a 17500 Kg payload in payload positions 2,3,5 6,8 and 9. (based on maximum ram length/zero rear left tyre forces)

Fig 5.13 Stability envelopes for a 25000 Kg payload in payload positions $\mathbf{1 , 2 , 4}$
5,7 and 8.(based on maximum ram length/zero rear left tyre forces)


$\begin{aligned} & \text { paylaod position } 2 \\ & 25000 \mathrm{Kg} \text { payload }\end{aligned}$
for the four different chassis configurations.(based on maximum ram

## CHAPTER SIX

## GENERAL DISCUSSION AND CONCLUSION

### 6.1 Technique

Theoretical models for predicting the stability behaviour of non-tipping trailers negotiating corners have been established by a number of people. However, no specific work has been found, in published material, relating to tipping trailer stability. The fundamenta! ideas associated with the dynamic non-tipping trailer models can be used in the static tipping trailer situation. The main criticism of the past models are that they require full scale experimental tests to determine important flexibilities (eg. chassis torsional stiffness and that the contribution of important flexibilities towards lorry stability cannot be assessed very easily. Also, air suspensions have not been considered At the start of this project in January 1989, it was decided that the model developed for assessing the tipping stability of a trailer, should not require experimental tests to determine important flexibilities, and that design investigations should be relatively easy and cheap to establish the effect of component flexibilities on stability.

### 6.2 Theoretical model

It was decided that the finite element method, described in Section 1, would be used to model the important flexibilities of the lorry. This enabled the important flexibilities of the lorry to be determined, without the requirement of full scale tilt
tests. It also enables design investigations to be undertaken relatively easily and quickly. However, not all flexibilities can be determined theoretically. Experimental investigations, described in Appendix A and B had to be undertaken in order to determine the flexibilities of the airbags and tyres.

### 6.2.1 Finite element model

Although large movements of the body and payload occur during tipping, these result from small linear elastic deformations of the body to chassis connection points, tyres and suspensions. The finite element model therefore uses beam plate and spring elements which are capable of dealing with small linear elastic deformations. Only the basic beam elements were used to model the chassis structure. However, brick elements could be used instead of the beam elements, which may result in a more accurate model but would significantly complicate the model, and subsequently increasing the cost of running the finite element model design investigations.

The tyres were modelled accurately using linear springs, with the flexibilities obtained from manufacturers information. It was thought at first that they might be incorrect. The vertical flexibility was confirmed to be correct through experimental work described in Appendix B, due to difficulties in the experiment the transverse and fore/aft flexibilities were not confirmed.

The relatively recent developments in suspension systems has seen the proliferation of air bag suspensions, mainly due to the airbag providing a flexible support
irrespective of the loading carried, as described in Appendix B. These supports are comprised of a rubber airbag inflated by air, with a mechanical high control device. The ability for the air suspension to increase or decrease the pressure in the airbag, depending on the load carried, results in a non linear system, described in Section 3.4.2. Experimental investigations into the load/deflection characteristics of an airbag, described in Appendix B, enabled a relationship to be determined of the form

$$
F L^{n}=\text { Const }
$$

Whereas leaf springs could be modelled using a simple linear spring, the operation of airbags and the non linear characteristics prevented this, and a novel approach had to be developed. The novel method of applying non linear loadings to a linear elastic structure, modelled by the finite element method, was successfully achieved and is described in Chapter 3.

Although the accuracy of the finite element model may be improved by the use of beam element with local flexibilities or by the use of brick elements, the method of solution is still good. The method of solution can still be used to highlight areas in the tipping trailer units that influence the stability characteristics, even if the flexibility matrices could be modelled more accurately. Also, the computer program written to execute the many matrix multiplications and iteration procedures has been shown to produce converged solutions with continuity as the suspension changes between the different operating conditions $a, b, c$, and d, as described in Section 4.6.

### 6.2.2 Solution procedure

The idea of using the results from finite element model to predict the flexibility of the tractor and trailer unit chassis and the realisation that the finite element package on its own could not be readily used to analyse the non linear loading and combination of payload magnitudes and positions, led to the development of general matrices for the calculation of forces and displacements. This required large unit load flexibility and influence matrices or unit displacement matrices described in Section 4.5 to be generated from the finite element results. This optimised the number of finite element runs required so that the stability characteristics of a lorry could be determined for any combiration of payload magnitude and position and ground slope without further finite element runs.

Extensive processing of the finite element results, due to the non linearity of the problem is required by the PC program to determine the required trailer tyre forces. The continuity of the stability graphs, discussed in Chapter 5, shows that the flexibility matrices are accurate, relative to each other.

### 6.2.3 Configuration of flexibility data

The general force and displacement matrices derived from the finite element results are incorporated into a PC based program. The data for each matrix is inputted by hand. This is a laborious task and would benefit from a more sophisticated data handling system, between the finite element package and the PC program. Any
obvious mis-readings were later corrected, by editing the stored data files. Errors can be identified using a number of methods. Firstly there is a degree of symmetry within each matrix and this can be used to cross-check data values. Secondly the program can be run with a symmetrical payload load (ie. $\mathrm{Z}=0$ ) and a zero ground slope, resulting in symmetrical vertical and horizontal displacement of the left and right hinges and zero transverse hinge displacement. The vertical tyre forces should be the same for corresponding left and right tyres and the transverse and fore/aft tyre forces should be zero. Thirdly the continuity of forces and displacements can be cross-checked before and after the suspension operating condition changes.

### 6.3 Implications

The design changes have been investigated in order to assess the influence of the respective components on roll stability, and to demonstrate how design changes can be analysed.

The design changes investigated in Section 5.3, highlight that there is scope for improving the roll stability of articulated tipping vehicles.

The effect of the changes on roll stability compared to the reference chassis can be seen in Fig. 5.15. The graph gives details for a payload magnitude of 2500 kg and payload position of $x=4465, y=1533,20$, stability envelope for each chassis configuration. The vehicle is in a stable position below the line and in an unstable position above the line, for each respective chassis configuration. The reference
chassis enables a maximum ram length of 5.4 m to be reached, for $\theta \mathrm{p}=3^{\circ}$, before the onset of instability. The trailing arm chassis enables a ram length of 5.0 m . The sheardeck chassis enabled a ram length of 5.7 m , and the torsion chassis which made the greatest improvement, enabling a ram length of 7.4 m to be reached, for the same body attitude before the onset of instability was reached.

Also, that the traditional chassis configuration comprising of two I-beams connected by cross members, may not be the optimum design and further design investigations could lead to alternative chassis designs.

### 6.4 Requirements for further investigations

The current model allows dimensions and flexibilities of lorry components to be changed in order to assess their influence on roll stability. The solution procedure of the theoretical model is correct, however the flexibilities derived from the current finite element model have not been verified. In order to have complete confidence in the flexibilities coefficients, the results from a full scale test should be compared against the F.E. results. While the current flexibility matrices may contain inaccuracies, they can still be used to assess design changes.

There are a number of areas in which the model accuracy could be improved. Firstly the tapered I beams have been modelled as uniform I beams, using average beam properties based on the two ends of the beam due to limitations of the finite element program.

Secondly inter-connections between cross members and the main I beams have been treated as rigid joints. Incorporating local flexibilities would enable simple beam plate elements to model the chassis more accurately. on shown by Beermann [18] and Megson $[19]$

Otherwise brick elements could be used instead of the plate and beam elements to model the chassis enabling tapered sections and local flexibilities to be modelled. The accuracy achieved through the use of brick elements in the finite element model has to be considered against the increased complication producing the F.E. model, the increase in computing time to run the finite element program and the increase in difficulty in making design changes.

Currently, fifth wheel separation has not been taken into account and that the rotation of the tractor and trailer at the fifth wheel are the same. It has been shown in the dynamic roll-over situation by Sweatman [3] that fifth wheel separation reduces roll stability. Introducing fifth wheel separation capability would further increase the model accuracy.

A useful inclusion in the computer program would be the facility to calculate stresses within the beam elements. Unit load or unit displacement stress matrices could be generated in the same manner as the flexibility matrices. This would enable the design engineers to identify those areas that are over or understressed, and that would benefit from design investigations.

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## APPENDIX A

## AIRBAG STIFFNESS


#### Abstract

The aim of this experimental work was to determine the force/deflection characteristics of an airbag for given initial air pressures while operating at the normal ride height of 381 mm .

The actual loading and support conditions were found to be difficult to apply with existing equipment. Therefore, simpler, reasonably representative loading and support conditions were imposed. Although these loading and support conditions were incorrect, it was considered that the results obtained could be used as an approximate check on the manufacturers data.


## A1 Introduction

An airbag is a rubber/fabric bellow which contains a column of compressed air. The rubber below itself does not significantly contribute to load support, this is mainly due to the column of compressed air. The air spring can only support axial loads with the existence of a suitable mechanism to eliminate sideways movement. When considering axial loads for an airbag, an initial ride height and pressure must be selected before loading can take place.

## A2 Theoretical prediction of airbag force/height relationship

A typical curve of force versus height is shown in Fig. A1(a). It was assumed that the force displacement graph can be represented by an equation of the form

$$
\begin{equation*}
F L^{n}=C \tag{A1}
\end{equation*}
$$

The constant n and C are determined from the intercept and slopes of log-log graphs of F and L and are equivalent to those of an adiabatic compression of a gas.

## A3 Experimental procedure

Because the airbags have a normal operating height of 381 mm , the first set of tests were set up using this height, and varying the initial pressure only. Five different initial pressures of $25,40,70$ and 80 psi were used to obtain the results shown by Fig. A1(a)-A1(e). The airbag was placed in an Instron universal testing machine and allowed to expand to a height of 381 mm . Following pressurisation to 25, 40, 5570 or 80 psi the air supply was closed. It was then unloaded to a 500 mm height and then compressed to a 250 mm height or up to a 100 psi pressure, whichever came first, and then unloaded back to 381 mm height.

A second set of tests were setup using a constant airbag height and varying the air supply pressure only. Four different airbag heights of 300, 325, 375, 400 and 425 mm were used to obtain the results in Fig. A2. The airbag was placed in an Instron universal testing machine and allowed to expand to one of the predetermined height previously mentioned. It was then pressurised to 90 psi in steps of 10 psi and the corresponding axial force was recorded.

## Results

From the experimental work three important results were obtained which enabled the airbag to be accurately modelled. Firstly it was found that when the airbag was compressed with a constant mass of air, equation A1 could predict the force/deflection curve accurately, using suitable values for the constants. The
theoretical and experimental force/deflection curves are shown in Fig. A1(a)-A1(e). The constants $\gamma$ and C, for equation A1, are shown in Table A1, for the five different initial pressures considered.

Secondly, it was found that the airbag axial force was not influenced by its height and was directly related to air pressure only, where the mass of air within the airbag is not constant. The experimental results showing the airbag force/pressure relationship for four different fixed heights are shown in Fig. A2.

Thirdly, it can be seen from Fig. A2 that the maximum airbag force of 30 kN can be used to indicate when the airbag air supply reaches its maximum pressure of 80 psi . Thus preventing the airbags from maintaining height control and allowing the airbags to compress according to equation A1.

From these results the airbags on one side have the same axial force as they are always at the same pressure due to being interconnected. When the suspensions are operating below their maximum working pressure of 80 psi (ie. the mass of air within the airbags is not constant), the airbag force is proportional to airbag pressure. When the suspensions are operating above their maximum working pressure (ie. the mass of air within the airbags is constant), the airbag force is determined from equation A1.

The value of $n=1.57$ is constant for all airbag operating condition as they change from height control to compression of a fixed mass of gas always occurs when the air pressure reaches 80 psi . It is assumed that the force/deflection characteristics of three
airbags side by side, as in the case of the actual trailer air bags, can be represented by a modified form of equation A1. Where $F$ is the airbag force, 1 is the combined length of the three displaced airbags and C the constant in equation A1. C is unknown as no experimental data exists for the compression of three airbags. Instead, C is calculated using the previous height controlled airbag forces and lengths substituted into equation A1.





## APPENDIX B

## TYRE STIFFNESS

The aim of this experimental work is to determine the flexibility of a stationary, loaded "super single" lorry wheel in the direction of motion, lateral direction and vertical direction.

## B1 Theory

## B1.1 In-plane horizontal forces

When considering the deformation of the wheel in the direction of motion during roll over, shown in Fig. B1, the axle is prevented from rotating by the action of the brake. Hence, a movement $\mathbf{M}_{\mathrm{i}}$, applied at the axle results in a pair of reaction forces, $\mathrm{F}_{\mathrm{i}}$, at the axle and the tyre/ground contact. When determining the stiffness $\mathrm{K}_{\mathrm{x}}=\mathrm{F}_{\mathrm{i}} / \delta_{\mathrm{i}}$ of a tyre when not connected to a trailer, it is more convenient to restrain the top and bottom of the tyre to determine $K_{x}$, shown in Fig. B2. The stiffness of the tyre when connected to the trailer will be the same as the stiffness $\mathrm{K}_{\mathrm{x}}$ determined experimentally.

The gradient obtained from the graph of $\boldsymbol{\delta}_{i}$ versus $F_{i}$ shown schematically in Fig. B3, is used to determine the flexibility $\mathbf{h}_{\mathbf{i}}$.

$$
\text { Tyre deflection }=\text { hi } * \text { Fi. }
$$

Therefore, the in-plane horizontal stiffness of the tyre $K_{x}$ is given by

$$
\begin{equation*}
k_{x}=\frac{1}{h_{i}} \tag{Bi}
\end{equation*}
$$

## B1.2 Out-of-plane horizontal forces

When considering the deformation of the wheel when loaded in the lateral, horizontal direction to motion during roll over, shown in Fig. B4, the axle is prevented from rotating by the axle to which it is connected. Hence, a horizontal force, $\mathrm{F}_{0}$ applied at the axle results in a reaction $F_{0}$, at the tyre/ground contact and a moment $M_{0}$ when determining the stiffness, $\mathrm{K}_{\mathrm{z}}=\mathrm{F}_{0} / \delta_{0}$, of a tyre when not connected to a trailer, it is more convenient to restrain the top and bottom of the tyre to determine $\mathrm{K}_{2}$, shown in Fig. B5. The actual stiffness of the tyre, when connected to the trailer, will be the same as the stiffness $\mathrm{K}_{\mathbf{z}}$, determined experimentally.

The gradient obtained from the graph of $\delta_{o}$ versus $F_{o}$, shown in Fig. B6, is used to determine the flexibility, $\mathrm{h}_{\mathrm{o}}$

$$
\text { Tyre deflection }=h_{0} * F_{0}
$$

Therefore, the out-of-plane horizontal stiffness of the tyre $K_{\mathbf{z}}$ is given by

$$
\begin{equation*}
L_{z}=1 / h_{0} \tag{B2}
\end{equation*}
$$

## B1.3 Vertical forces

The vertical load, $\mathrm{F}_{\mathrm{v}}$ is applied to the wheel through its axle, as shown in Fig. B7. However, it is experimentally more convenient to apply a diameter load to the whole wheel, as shown in Fig. B8. The gradient of a graph of $\delta_{v}$ versus $F_{v}$, shown in Fig. B9, is the same for axle loading as for diametral loading, due to the measuring of displacement at the axle during diametral loading. The gradient, $h_{v}$ of the graph of $\delta_{v}$ is the flexibility of the wheel in the vertical direction.

The stiffness of the tyre, $\mathrm{K}_{\mathrm{y}}$ in the vertical direction is given by

$$
\begin{equation*}
L_{y}=1 / h_{v} \tag{BX}
\end{equation*}
$$

## B2 Experimental equipment and procedures

The tyre and rig used to apply a vertical compressive load are shown in Fig. B10. In the position shown an in-plane horizontal force can be applied to the hub of the tyre. In order to apply an out-of-plane horizontal force the tyre is rotated $90^{\circ}$ about the vertical axis. To avoid the possibility of slip (due to limiting friction) at the contact between the tyre and the flow or load cell, two Araldite moulds of the vertically loaded tyre tread, under a load of 40 kN , were made. The tyre was placed in the grooves in the Araldite mould during testing. The Araldite moulds at the top and bottom of the tyre were held in the $\mathbf{X}$ and $\mathbf{Z}$ directions to prevent movement under loading. The effect of the depth of penetration of the Araldite moulds into the tyre was investigated by carrying out full tread penetration and half tread penetration (ie. 6 mm and 12 mm penetrations).

Equations B1, B2, B3, can be used together with experimental measurements to calculate the tyre flexibilities $K_{x}, K_{y}$, and $K_{z}$. In particular the gradients of the graphs of $\delta_{\mathrm{i}}$ versus $\mathrm{F}_{\mathrm{i}}$, $\delta_{\mathrm{o}}$ versus $\mathrm{F}_{\mathrm{o}}$ and $\delta_{\mathrm{v}}$ versus $\mathrm{F}_{\mathrm{v}}$ are required.

## B2.1 In-plane horizontal tyre loading (to obtain $\mathbf{K}_{x}$ )

The tyre was placed in a framework securely bolted to the floor and loaded vertically by a hydraulic ram to 40 kN , which is the approximate load on each tyre under operating conditions. The horizontal load was applied using dead weights, with a wire attached to the hub of the tyre and a pulley attached to the framework to convert the vertical load into the required horizontal load. The horizontal displacement was measured using two dial gauges, at a distance of 330 mm above and below the tyre centre, with the average of these displacements taken for the horizontal displacement of the tyre.

## B2.2 Out-of-plane horizontal tyre loading (to obtain $\mathbf{K}_{\mathbf{z}}$ )

The out-of-plane horizontal tyre stiffness was obtained in the same manner as the in-plane horizontal tyre stiffness, except that the tyre was placed in the framework when rotated $90^{\circ}$ about the vertical. The tyre was loaded in the vertical and horizontal direction in the same manner as for the in-plane tyre test. The horizontal displacement was also measured and used in the same manner as previously described in relation to the in-plane loading.

## B2.3 Vertical tyre loading (to obtain $K_{y}$ )

The tyre was placed in the framework and loaded in increments of 5 kN from 0 to 45 kN . The vertical displacement was measured using two dial guages, at a distance of 330 mm either side of the tyre centre, with the average of these displacements taken for the vertical displacement of the tyre hub.

## B3 Results

The experimental results, for a 6 mm tyre tread depth in the Araldite mould, are shown in Figs. B11, B12, and B13, with only the final result for the 12 mm tread given. The gradients for these graphs were obtained using a linear regression analysis. The gradients and corresponding tyre stiffnesses for the in-plane, out-ofplane and vertical loadings are given in Tables B1 , B 2, and B 3, respectively. The tyre stiffnesses are compared with the manufacturers' stiffnesses in Table B. 4.

The different tread depths used in the Araldite mould did not make any significant difference to the stiffnesses obtained. The $K_{\mathrm{x}}$ and $\mathrm{K}_{\mathbf{z}}$ stiffnesses obtained are greater than the manufacturers' supplied stiffness. The $\mathrm{K}_{\mathrm{y}}$ stiffness agreed well with the manufacturers' stiffness.

The present results were found to be within $67 \%, 8 \%$ and $32 \%$ of the manufacturers' results for the $k_{x}, k_{y}$ and $k_{z}$ stiffnesses, respectively. The level of agreement, bearing in mind that the loading and support conditions were not correct, was considered to be good enough to allow the manufacturers' data to be confidently used.


Fig. B1 Free body diagram for in-plane horizo,ical loading of the wheel


Fig. B2 Free body diagram for applying an inplane horizontal force ${ }_{2}{ }^{2 \mathrm{~F}} \mathrm{i}$, to the wheel


Fig. B3 Schematic diagram of $\delta_{i}$ versus $F_{i-}$


Fig. B4 Free body diagram for out of plane loading of the wheel


Fig. B5 Free body diagram for applying an out of plane horizontal force, $2 F_{0}$, to the wheel


Fig. B6 Schematic diagram of $\mathcal{S}_{0} \underbrace{}_{0-}$


Fig. B7 Free body diagram for vertical wheel loading


Fig. B8 Free body diagram for applying a vertical Fprce, $F_{v}$, to the wheel.




Fig. B10 Loading rig used to apply a vertical force $F_{v}$ to the tyre


Fig. B11 Variation of $\delta_{\mathrm{FI}}$ with $\mathrm{f}_{\mathrm{l}}$, for loading and unloading, with a 6 mm tread depth in the Araldite mould.


Fig. B12 Variation of $\delta_{\text {ro }}$ with $F_{0}$, for loading and unloading, with a 6 mm tread depth in the Araldire mould.


Fig. B13 Variation of with $F_{\mathbf{v}}$, for loading and unloading, with a 12 mm araldite mould.
Table B1: In-plane, Horizontal Loading Fesults

| Depth of mould tread <br> $(\mathrm{mm})$ | $\mathrm{h}_{\mathrm{i} 1}(\mathrm{~mm} / \mathrm{N})$ | $\mathrm{h}_{\mathrm{i} 2}(\mathrm{~mm} / \mathrm{N})$ | $\mathrm{h}_{\mathrm{i}}(\mathrm{mm} / \mathrm{N})$ | $\mathrm{k}_{\mathrm{x}}(\mathrm{N} / \mathrm{mm})$ |
| :---: | :---: | :---: | :---: | :---: |
| 6 | $4.068 \times 10^{-4}$ | $4.363 \times 10^{-4}$ | $4.216 \times 10^{-4}$ | 1186 |
| 12 | $4.200 \times 10^{-4}$ | $4.575 \times 10^{-4}$ | $4.388 \times 10^{-4}$ | 1140 |

Table B2: Out-of-plane, Horizontal Loading Results

| Depth of mould tread <br> $(\mathrm{mm})$ | $\mathrm{h}_{01}(\mathrm{~mm} / \mathrm{N})$ | $\mathrm{h}_{02}(\mathrm{~mm} / \mathrm{N})$ | $\mathrm{h}_{\mathrm{o}}(\mathrm{mm} / \mathrm{N})$ | $\mathrm{k}_{\mathrm{z}}(\mathrm{N} / \mathrm{mm})$ |
| :---: | :---: | :---: | :---: | :---: |
| 6 | $8.968 \times 10^{-4}$ | $9.633 \times 10^{-4}$ | $9.301 \times 10^{-4}$ | 538 |
| 12 | $8.843 \times 10^{-4}$ | $9.599 \times 10^{-4}$ | $9.221 \times 10^{-4}$ | 542 |

Table B3: Vertical Loading Results

| Depth of mould tread, $(\mathrm{mm})$ | $\mathrm{h}_{\mathrm{v}}(\mathrm{mm} / \mathrm{N})$ | $\mathrm{k}_{\mathrm{y}}(\mathrm{N} / \mathrm{mm})$ |
| :---: | :---: | :---: |
| 12 | $8.666 \times 10^{-4}$ | 1154 |

Tableß4:Comparison of Manufacturers Tyre Stiffnesses with Experimental Results

| Depth of mould tread (mm) | $\mathrm{k}_{\mathrm{x}}(\mathrm{N} / \mathrm{mm})$ | $\mathrm{k}_{\mathrm{y}}(\mathrm{N} / \mathrm{mm})$ | $\mathrm{k}_{\mathrm{z}}(\mathrm{N} / \mathrm{mm})$ |
| :---: | :---: | :---: | :---: |
| 6 | 1186 | - | 538 |
| 12 | 1140 | 1154 | 542 |
| Manufacturers' value | 712 | 1070 | 412 |

## APPENDIX C

## COMPUTER PROGRAM AND FLOW CHART

The program was written on an Akhter 286 PC using Turbo Basic. This language was chosen as a novice programmer would be able to modify the program relatively easily, while the language offered a more sophisticated program construction compared to ordinary Basic. The main advantage is that completely separate subroutines can be developed independently, enabling a large program to be split into manageable sections. The program subroutines are described in conjunction with the program flow chart, shown in Fig. C 1 (a) to Fig . $\mathrm{C} 1(\mathrm{c})$ and the corresponding Turbo Basic program(s) shown in Fig. C2 to C28.

## C1 Start

The program is controlled by the main program TIPPER.BAS, shown in Fig. C2 which contains the subroutine described next. The subroutines are linked together through the main program, allowing variables to be passed between the subroutines. The subroutines are executed in a sequence described by the routine CONTROL.BAS, shown in Fig. C3.

## C2 Input routines

(i) The first input routine contains the undeformed co-ordinates of the hinges, ram/chassis contact, ram/body contact and body centre of gravity, and the
beam mass. These variables are fixed for each trailer under consideration with the coding described by the routine INPUTR.BAS. shown in Fig. C4.
(ii) The second input routine contains the undeformed co-ordinates of the payload centre of gravity, the paylnad mass, the initial ground slope and the ram length, all of which can be determined by the operator. The program is currently set to automatically consider four payload masses in the nine payload positions. The ram length is varied from 2 to 8 in steps of 1 with the ground slope varying from $0^{\circ}$ to $10^{\circ}$ in steps of $0.1^{\circ}$, for each ram length considered. The coding describing the subroutine is given in VARIAB.BAS, shown in Fig. C5.

## C3 Initialise variables routine

The variables used during the program operation are set to zero at the beginning of each new payload magnitude, or payload position, or ground slope, or ram length considered. The coding describing this routine is given in ZERO.BAS, shown in Fig. C6.

## C4 Vector positional routine

The co-ordinates of the top of the ram, payload centre of gravity and body mass centre of gravity are determined using the vector analysis described in Section 3.4.1. For the first iteration for each new body or chassis position considered, their co-
ordinates are determined for a rigid chassis, with subsequent iterations taking into account chassis deformations. The coding describing this routine is given in VECTOR1.BAS and VECTOR.BAS, shown in Fig. C7 and Fig. C8.

## C5 Ram direction routine

The ram direction is described by three cosine resolving functions which independently relate the direction of the ram to the $\mathrm{X}, \mathrm{Y}$, or Z axis. These functions are used in the body equilibrium analysis described.in Section 3.4.2 The coding describing this routine is given in RAM.BAS, shown in Fig. C9.

## C6 Body equilibrium matrix routine

The body equilibrium matrix, $\operatorname{EQM}(6,6)$ is formed using the co-ordinates of the hinges and the ram/body contact, and the ram resolving functions described in Section 3.4.2. The coding describing this routine is given in EQUILB.BAS, shown in Fig. C10.

## C7 Body equilibrium matrix inversion routine

The body equilibrium matrix EQM(6,6), described in Section 3.4.2, is inverted to form EQMI(6,6), in order to calculate the body reactions. The coding describing this routine is given in INVERT1.BAS, shown in Fig. C11.

The co-ordinates of the payload and body mass centres of gravity, the payload mass and the body mass, are used to form the force and moment coefficients of the loading matrix LM(6), described in Section 3.4.2. The coding describing this routine is given in BODYF.BAS, shown in Fig. C12.

## C9 Body reactions routine

The inverted body equilibrium matrix EQM1 $(6,6)$ is multiplied by the loading matrix LM(6) to give the body/chassis contact point reaction forces described in Section 3.4.2. The coding describing this routine is given in BODYF.BAS, shown in Fig. C12.

## C10 Suspension routine

This routine chooses the appropriate suspension forces and suspension operating condition, which is compatible with the chassis forces based on one of the four suspension operating conditions described in Section 3.4.3. There are four different matrices corresponding to the four suspension operating conditions, which enable the suspension forces to be calculated. The coding describing this routine is given in SUSP.BAS, CASEa.BAS, CASEb.BAS, CASEc.BAS and CASEd.BAS, show in Fig. C13 to Fig. C17.

There are four different matrices for calculating chassis deformations, which correspond to the four suspension operating conditions described in Section 3.4.3. Depending upon the suspension condition, the appropriate matrix is chosen to calculate the chassis deformations, described in Section 3.4.3. The coding describing this routine is given in FLEXI.BAS, FLEXI1.BAS, FLEXI2.BAS, FLEXI3.BAS, FLEXI4.BAS and DISP.BAS, shown in Fig. C18 to Fig. C23.

## C12 Tyre force routine

There are four different matrices for calculating tyre forces, which correspond to the four suspension operating conditions described in Section 3.4.3. Depending upon the suspension condition, the appropriate matrix is chosen to calculate the tyre forces, described in Section 3.4.3. The coding describing this routine is given in TYRE.BAS, TYRE1.BAS, TYRE2.BAS, TYRE3.BAS and TYRE4.BAS, shown in Fig. C24 to Fig. C27.

## C13 Converge routine

At the end of each iteration except the first, a check is made to see if the procedure has converged or has become unstable. The displacement of the top of the ram is compared with the previous displacement and if the change in this is within the specified tolerance of 1 mm , the solution is deemed to have converged, else a further
iteration is undertaken. After convergence the tyre forces are examined and as long as the forces are positive (indicating a downwards force) the trailer is in a stable position. The ram length, tyre force, payioad position and payload magnitude are then written to a file. If the trailer is in a stable position, this process is repeated, increasing the ground slope after every converged iteration, so long as the trailer remains stable, up to a maximum ground slope of $10^{\circ}$.

If the tyre force becomes negative (indicating an unstable trailer position) or when the ground slope is equal to $10^{\circ}$, the ram length is increased by 1 m and the ground slope is re-set to zero. The program is then run again for this new body position. When the ram length is 8 m and the ground slope is $10^{\circ}$, the program will have considered the current payload mass and position in 147 body attitudes and ground slopes. At this point, the program can choose a new payload mass and position which is dependent upon the operators supplied data. The program is initiated again until all payload mass and positions have been considered. The program is then terminated. The coding describing this routine is given in CONVERGE.BAS, shown in Fig. C28.

## Fig. C1(a) Program flow chart



Initialise coordinates of
(i) Hinges
(ii) Ram/Chassis contact
(iii) Ram/Body contact
(iv) Body centre of gravity


Input user variables
(i) Nine Payload centre of gravity
(ii) Four Payload masses
(iii) Ground slope $=0$
(iv) Ram Length $=2 \mathrm{~m}$

Input routine (i)

Input routine (ii)

Initialise variables
Zero- Chassis displacements Suspension forces
Tyre forces
Body reactions
A
Determine coordinates of
(i) Top of ram
(ii) Payload centre of gravity
(iii )Body centre of gravity
using vector analysis
$r_{t} \cdot h_{1}=\left|r_{t}\right| *\left|h_{1}\right| * \cos$
$r_{t} \cdot r_{b}=\left|r_{t}\right| *\left|r_{b}\right|^{*} \cos$
$r_{t}{ }^{2}=r_{t x}{ }^{2}+r_{t y}{ }^{2}+r_{t z}{ }^{2}$

## Determine aram direction cosines using coordintes of top and bottom of ram.

Form equilibrium Matrix, $\operatorname{EQM}(6,6)$ using
(i) Left hinge coordinates
(ii) Right hinge coordinates
(iii) Ram base coordinates
(iv) Ram top coordinates
(v) Ram direction cosines

1



- S.G.PICKERING

```
******
    '
    ' This is the main program described in Appendix Cl which links
the
    ' subroutines together.
```

******
cls
DIM DYNAMIC lhc\#(2,35), rhc\#(2,35), rb\#(2,35),rt\#(2,35),wc\#(2,35)
DIM DYNAMIC bcg\#(2,35),pcg\#(2,35),BF\#(5,35),BR\#(7,35)
DIM DYNAMIC CHL\#(15),flex\#(8,15), uchassis\# (8, 35)
DIM DYNAMIC SUSPR\#(35),SUSPL\#(35), brconst" ${ }^{\prime \prime}(35)$
DIM DYNAMIC eqm\# 5,5 ), eqmi\# $(5,5)$
DIM DYNAMIC k11\#(17,17) , k11i\#(17,17), k12\#(17,5), k21\#(5,17)
, k22\#(5,5)
DIM DYNAMIC prescribe\#(29,23) , k21k!11\#(5,17)
k21k11ik12\# (5,5)
DIM DYNAMIC k22k21k11ik12\#(5,5), k22k21k11ik12i\#(5,5)
DIM DYNAMIC a\#(5) , fab\#(5) , fch\#(5) , fch1\#(5,17) ,
FCH2\#(23), uab\# (5,35)
DIM DYNAMIC TYREF\# 5,35 ), uch\# $(23,35)$, FLCHECK (35)

POSI\% (4, 8), PCGX\# ( 8 ) , PCGY\# ( 8 ), $\operatorname{PCGZ\# (8),\operatorname {PM}\% (4),\operatorname {PMS}(4,8)~}$

CALL VARIABS

```
for y%=1 to 4
    for }x%=1\mathrm{ to }
    if POSI%(Y%,x%)=1 then
        PM# = PM% (Y%)*9.81
        pcg#(0,0)=PCGX# (x%)
        pcg#(1,0)=PCGY# (x%)
        pcg#(2,0)=PCGZ#(x%)
        BS=STRS(x%)
        FILENS=PMS(Y%,X%)
        AS="PAYLOAD POSITION="+BS
```

ITER=0
rl\#=2000
focount $=0$
FLAG3=0

```
    PRINT FILENS,AS
```

OPEN "A", \#1,FILENS
WRITE\#1,AS
CLOSE\#1

DO UNTIL rl\# > 8000
print "ram length="; rl\#;pm\#;pcg\#(0,0);pcg\#(1,0);pcg\#(2,0)
OPEN "A", \#1,FILEN\$
WRITE\#1, rl\#
CLOSE\#1

```
    concount=0
    FLAGC=0
    thetap#=0
    thetactr=0
    tyre$= "go"
    DO UNTIL tyreS="stop" or thetactr>20.1
print "thetap=";thetactr
        CALL CONTROL
        CALL ZERO
        thetactr=thetactr+1/2
        thetap#=thetap#+3.1415927/(180*2) 'increment by 1/2 degree
in rads.
    ITER=1
    n=0
    l=0
    m=0
    pcg# (0,0) = PCGX# (x% )
    pcg#(1,0)= PCGY# (x%)
    pcg# (2,0)=PCGZ# (x%)
    LOOP
    rl#=rl#+1000
    focount=focount+1
LOOP
end if
    AS=" "
    B$=" "
    C$=" "
    FILEN$=""
    DS=""
    ES=""
next x%
next y%
stop
```

```
    SINCLUDE "VARIABS.BAS"
```

    SINCLUDE "VARIABS.BAS"
    $INCLUDE "ZERO.BAS"
    $INCLUDE "ZERO.BAS"
    $INCLUDE "CONTROL.BAS"
    $INCLUDE "CONTROL.BAS"
    $INCLUDE "INPUTR.BAS"
    $INCLUDE "INPUTR.BAS"
    $INCLUDE "VECTOR1.BAS"
    $INCLUDE "VECTOR1.BAS"
    $INCLUDE "VECTOR.BAS"
    $INCLUDE "VECTOR.BAS"
    $INCLUDE "RAM.BAS"
    $INCLUDE "RAM.BAS"
    $INCLUDE "EQUILIB.BAS
    $INCLUDE "EQUILIB.BAS
    $INCLUDE "BODYF.BAS"
    ```
    $INCLUDE "BODYF.BAS"
```

\$SEGMENT
\$INCLUDE "SUSP.BAS"
\$INCLUDE "CASE1.bAS"
SINCLUDE "CASE2.BAS"
SSEGMENT
sINCLUDE "CASE3.BAS"
SSEGMENT
\$INCLUDE "CASE4.BAS"
SINCLUDE "INVERT1.BAS"
SINCLUDE "FLEXI.BAS"
SINCLUDE "FLEXI1.BAS"
SINCLUDE "FLEXI2.BAS"
SINCLUDE "FLEXI3.BAS"
SINCLUDE "FLEXI4.BAS"
SSEGMENT
SINCLUDE "DISP.BAS"
SINCLUDE "TYRE.BAS"
$\hat{\beta}$ include "TYRE1.BAS"
SINCLUDE "TYRE2.BAS"
\$INCLUDE "TYRE3.BAS"
\$INCLUDE "TYRE4.BAS"
SINCLIJE "CONVERGE.BAS"
SINCLUDE "OUTP.BAS"
end

SUB CONTROL

' This subroutine controls the execution of the subroutines listed below.


SHARED CONVERGED\%, $n$, ITER
IF $n=0$ AND ITER $=0$ THEN
CALL INPUTR
END IF

LABEL. LOOPC:
CALL VECTOR1

CALL RAM
CALL EQUILIB
CALL INVERT1
CALL BODYF
CALL SUSP

CALL FLEXI

CALL DISP
CALL TYRE
CALL CONVERGE
IF CONVERGED? $=0$ THEN
$\mathrm{n}=\mathrm{n}+1$
GOTO LABEL. LOOPC
END IF

END SUB

```
SUB INPUTR
SHARED rt#(),rb#(),wc#(),bcg#()
SHARED pcg#(),lhc#(),rhc#(),1,m,n
SHARED rl#,thetap#,BM#,PM#,WX#,WY#,WZ#
```


******
- This subroutine contains the undeformed coordinates of
' the trailer/body hinges, ram/chassis contact, ram/body
contact,
' body $c$. of $g$. and the body mass
,
1 lhc\#(O) - LEFT HINGE X CO-ORD.
lhc\#(1) - LEFT HINGE Y CO-ORD.
lhc\#(2) - LEFT HINGE Z CO-ORD.
*
-
- THE BODY ATTITUDE IS SPECIFIED BY DEFINING THE
THE GROUND SLOPE ANGLE, thetap\#, AND THE LENGTH
OF THE RAM , rl\#.
- THE APPLIED LOADS TO THE BODY AND THEIR RESPECTIVE
POSITIONS ARE GIVEN .
****
$1=m=n=0$
$r t \#(0,0)=8977.2: r t \#(1,0)=2029.5: r t \#(2,0)=1 \mathrm{E}-06$
$\operatorname{rb\# }(0,0)=8977.2: \operatorname{rb\# }(1,0)=0.0: \operatorname{rb\# }(2,0)=1 \mathrm{E}-06$
wC\# $(0,0)=4488.6:$ wC\# $(1,0)=1014.75:$ wC\# $(2,0)=-1247.0$
$\operatorname{bcg\# }(0,0)=4488.6: \operatorname{bcg} \#(1,0)=1014.75: \operatorname{bcg\# }(2,0)=1 \mathrm{E}-06$
lhc\# $(0,0)=1 \mathrm{E}-06: \operatorname{lhc\# }(1,0)=1 \mathrm{E}-06: \operatorname{lhc\# }(2,0)=-700$
$\operatorname{rhc} \#(0,0)=1 \mathrm{E}-06: \operatorname{rhc}(1,0)=1 \mathrm{E}-06: \operatorname{rhc}(2,0)=700$
$\mathrm{BM} \#=1250 * 9.81$
end sub

SUB VARIABS
SHARED POSI\% (), PCGX£(), PCGY£(), PCGZ£(), PM\$()
 *************
' This routine contains user input variables, which are the undeformed
1 coordinates of the payload centre of gravity, payload mass and the
initial ground slope and ram length.
 *************
'Line 12
'PAYLOAD CENTRE OF GRAVITY X COORDINATES STORED IN PCGX£(8) IN mm
1
----

PCGX\# ( 0 ) $=2992$
$\operatorname{PCGX\# }(1)=4488$
PCGX\# (2) $=5984$
PCGX\# ( 3 ) $=2992$
PCGX\# (4) $=4488$
PCGX\# (5) $=5984$
PCGX\# (6) $=2992$
PCGX\# (7) $\mathbf{~} 4488$
PCGX\# ( 8 ) $=5984$
'Line 28
' PAYLOAD CENTRE OF GRAVITY Y COORDINATES STORED IN PCGY£(8) IN mm
1.

PCGY\# ( 0 ) $=676$
PCGY\# (1) $=676$
PCGY\# (2) $=676$
PCGY\# (3) $=1014$
PCGY\# (4) $=1014$
PCGY\# (5) $=1014$
PCGY\# (6) $=1350$
PCGY\#(7)=1350
PCGY\# (8) $=1350$

[^0]PCGZ\#(0) $=1 \mathrm{E}-06$
PCGZ\# (1) $=1 \mathrm{E}-06$ PCGZ\#(2) $=1 \mathrm{E}-06$ PCGZ\# (3) $=1 E-06$ PCGZ\# (4) $=1 E-06$ PCGZ\# (5) $=1 \mathrm{E}-06$ PCGZ\# (6) $=1 \mathrm{E}-06$ PCGZ\# (7) $=1 \mathrm{E}-06$ PCGZ\# (8) $=1 \mathrm{E}-06$
'Line 60
'PAYLOAD MASSES STORED IN PME(3) IN kg
$\operatorname{PM\# }(0)=2500$
PM\# (1) $=10000$
PM\# ( 2 ) $=17500$
PM\# (3) $=25000$
'Line 72
'OUTPUT FILENAMES FOR 2500 kG PAYLOAD ( 9 FILES)
1---

```
PMS(0,0)="A:\P2500-1.ROL"
PMS(0,1)="A:\P2500-2.ROL"
PMS(0, 2)= "A:\P2500-3.ROL"
PMS(0,3)= "A:\P2500-4.ROL"
PMS(0,4)="A:\P2500-5.ROL"
PMS(0,5)="A:\P2500-6.ROL"
PMS(0,6)="A:\P2500-7.ROL"
PMS(0,7)= "A:\P2500-8.ROL"
PMS(0,8)="A:\P2500-9.ROL"
```

'Line 90
'OUTPUT FILENAMES FOR 10000 kG PAYLOAD ( 9 FILES)

----
$\operatorname{PMS}(1,0)=" A: \ P 10000-1 . R O L "$ $\operatorname{PMS}(1,1)=" A: \ P 10000-2$. ROL" $^{\prime \prime}$ $\operatorname{PMS}(1,2)=" A: \backslash P 10000-3$. ROL" $^{\prime \prime}$ $\operatorname{PMS}(1,3)=" A: \ P 10000-4$. ROL" $\operatorname{PMS}(1,4)=" A: \ P 10000-5 . R O L "$ PMS (1,5)="A: $\backslash$ P10000-6.ROL" $\operatorname{PMS}(1,6)=" A: \backslash P 10000-7 . R O L "$ PMS $(1,7)=" A: \backslash P 10000-8 . R O L "$ PMS (1, 8) = "A: \P10000-9.ROL"
'Line 106
'OUTPUT FILENAMES FOR 17500 kG PAYLOAD (9 FILES)
'----
----
$\operatorname{PMS}(2,0)=$ "A: $\backslash$ P17500-1.ROL" $\operatorname{PMS}(2,1)=" A: \backslash P 17500-2 . R O L "$ $\operatorname{PMS}(2,2)=" A: \ P 17500-3$. ROL" $\operatorname{PMS}(2,3)=" A: \ P 17500-4$. ROL" $\operatorname{PMS}(2,4)=" A: \backslash P 17500-5$. ROL $^{\prime \prime}$ PMS $(2,5)=$ "A: $\backslash P 17500-6$. ROL $^{\prime}$ $\operatorname{PMS}(2,6)=" A: \backslash P 17500-7 . R O L "$ $\operatorname{PMS}(2,7)=$ "A: $\backslash$ P17500-8.ROL" $\operatorname{PMS}(2,8)=" A: \backslash P 17500-9 . R O L "$
$\operatorname{PMS}(3,0)=" A: \ P 25000-1 . R O L "$ $\operatorname{PMS}(3,1)=$ "A: $\backslash P 25000-2$. ROL" $^{\prime \prime}$ $\operatorname{PMS}(3,2)=" A: \backslash \operatorname{P25000-3.ROL"}$ $\operatorname{PMS}(3,3)=$ "A: $\backslash P 25000-4$. ROL" $^{\prime \prime}$ $\operatorname{PMS}(3,4)=" A: \ P 25000-5 . R O L "$ PMS (3,5) = "A: $\backslash P 25000-6 . R O L "$ $\operatorname{PMS}(3,6)=" A: \ P 25000-7 . R O L "$ $\operatorname{PMS}(3,7)=$ "A: $\backslash P 25000-8$. ROL" $^{\prime}$ $\operatorname{PMS}(3,8)=$ "A: $\backslash$ P25000-9.ROL"

END

Fig. C6 Zero.bas

SUB ZERO

```
*******
' This subroutine sets the program's variables to zero at the ' beginning of each new payload magnitude or payload mass or ground
slope or ram length considered.
```



```
*******
```

SHARED lhc\#(), rhc\#(), rb\#(), rt\#(), wc\#()
SHARED bcg\#(), pcg\#(), BF\# (), BR\# ()
SHARED CHL\#(), flex\#(), uchassis\# ()
SHARED SUSPR\#(), SUSPL\#(), brconst\#()
SHARED a\#() , fab\#() ,fch\#(), fch1\#(), FCH2\#(), uab\#()
SHARED TYREF\#(), uch\#(), FLCHECK ()
FOR $x=0$ TO 2
FOR $\mathrm{Y}=1$ TO 20
$\operatorname{lhc\# }(x, y)=0$
rhc\# $(x, y)=0$
$\operatorname{rb} \#(x, y)=0$
$r$ t\# $(x, y)=0$
wc\# $(x, y)=0$
bcg\# $(x, y)=0$
pcg\# $(x, y)=0$
NEXT Y
NEXT $x$

## ERASE BF\#,BR\#

ERASE CHL\#,flex\#, uchassis\#
ERASE SUSPR\#, SUSPL\#, brconst\#
ERASE fCh\# , FCH2\#, uab\#
ERASE TYREF\#, uch\#, FLCHECK
DIM DYNAMIC BF\# (5,35), BR\#(7,35)
DIM DYNAMIC CHL\#(15),flex\#(8,15), uchassis\#(8,35)
DIM DYNAMIC SUSPR\#(35),SUSPL\#(35),brconst\#(35)
DIM DYNAMIC fch\#(5), FCH2\#(23), uab\# $(5,35)$
DIM DYNAMIC TYREF\#(5,35), uch\#(23,35), FLCHECK(35)
END SUB

## SUB VECTOR1

1 VECTOR ANALYSIS BY S.G.PICKERING 28/1/91

****大**
'This subroutine calls the routine VECTOR.BAS for the payload c. of g . .
'the body $c$. of $g$. and ram/body contact to determine their positional
' coordinates using vector analysis
 ********

SHARED rt\# () , rb\# (), wc\# () , bcg\# ()
SHARED pcg\# (), lhc\#(), rhc\# (), rl\#, l, m, n
CALL VECTOR(rt\# (), rb\# ( ), lhc\# ( ) , 0, 1, m, n)
$m=m+1$
CALL VECTOR(bcg\# (), rt\# ( ), 1hc\# ( ), 1, 1, m, n)
CALL VECTOR(pCg\#(), rt\# (), lhc\# (), 1, 1, m, n)
$1=1+1$
if $n=0$ then
$n=n+1$

- t*************************
' hinge and ram base co-ords are modified by subtracting gravity load disp-
' lacements as these have been taken into account in the chassis FE mesh.
1**************************
$\operatorname{lhc\# }(0, n)=\operatorname{lhc\# }(0, n-1)+0.2714-0.778$
$1 \mathrm{hc}(1, n)=1 \mathrm{hc}(1, n-1)+10.176-4.946$
lhc\# $(2, n)=1 h c \#(2, n-1)$
$\operatorname{rhc\# }(0, n)=\operatorname{rhc}(0, n-1)+0.2714-0.778$
rhc\# $(1, n)=\operatorname{rhc}(1, n-1)+10.176-4.946$
rhc\# ( $2, n$ ) $=\operatorname{rhc} \#(2, n-1)$
$\operatorname{rb\# }(0, n)=r b \#(0, n-1)+0.2542-0.706$
$\operatorname{rb\# }(1, n)=r b \#(1, n-1)+1.667+16.809$
rb\# $(2, n)=r b \#(2, n-1)$
end if
end sub

231 SUB VECTOR(a\#(2),b\#(2), c\#(2),flag, 1, m, n)


'This routine contains the coding for multiplying vectors to determine
'the positional coordinates of the important points on the body
 ********

```
240 SHARED rl#
250 'N.B a#(,)-uses l subscript
260 ' b#(,)-uses m subscript
270 ' c#(,)-uses n subscript
280
290
300
310
320
330
340
350
360
370
380 if ABS(b#(0,m)) <1e-06 then
390 b#(0,m)=1e-06
400 end if
410 if ABS(b#(1,m)) < le-06 then
420 b#(1,m)=1e-06
430 end if
440 if ABS(b#(2,m)) < le-06 then
450 b# (2,m)=1e-06
460 end if
470 if ABS(C#(O,n)) < 1e-06 then
480 c#(0,n)=1e-06
500 end if
510 if ABS(C#(1,n)) < 1e-06 then
520 c#(1,n)=1e-06
530 end if
540 if ABS(c#(2,n)) < 1e-06 then
550 c#(2,n)=1e-06
560 end if
570
580 if flag=0 then
590 'CALC a# VECTOR LENGTH
600 a#=SQR(a#(0,0)`2+a#(1,0)`2+a#(2,0)-2)
610 'CALC b# vector length
620 b#=SQR(b#(0,m)`2+b#(1,m)`2+b#(2,m)`2)
630 'c# vector length
640 c#=700.0
650 ab#=rl#
6 6 0
ac#=SQR((a#(0,0)-c#(0,0))^2+(a#(1,0)-c#(1,0))^2+(a#(2,0)-c#(2,
0))`2)
670 end if
```

680

690 if $f$ lag=1 then
$700{ }^{\prime}$ CALC a\# VECTOR LENGTH
$710 \mathrm{a}=\mathrm{SQR}\left(\mathrm{a} \mathrm{\#}(0,0)^{\sim} 2+a \#(1,0) \sim 2+a \#(2,0) \sim 2\right)$
$720{ }^{\text {' CALC }} \mathrm{b} \#$ vector length
$730 \mathrm{~b} \#=\operatorname{SQR}\left(b \#(0,0)^{\sim} 2+b \#(1,0) \sim 2+b \#(2,0){ }^{-2}\right)$
740 'c\# vector length
750 c \# $=700.0$
760 'calc ab\# vector lngth
7 7
$a b \#=\operatorname{SQR}\left((a \#(0,0)-b \#(0,0)){ }^{-2+(a \#(1,0)-b \#(1,0))^{\sim} 2+(a \#(2,0)-b \#(2, ~}\right.$ 0)) ${ }^{-2)}$

7 8 0
$a c \#=\operatorname{SQR}\left((a \#(0,0)-c \#(0,0))^{\sim} 2+(a \#(1,0)-c \#(1,0))^{\sim} 2+(a \#(2,0)-c \#(2\right.$,
0)) ~2)

790 end if

810 'calc cos(gamma) ie angle between vectors a\# 6 b\#
820 g\# = ( $a \#^{\prime-2+b \#-2-a b \#-2) /(2 * a \# * b \#) ~}$
830 'calc cos(delta) le angle between vectors a\# \& c\#

850 'calc ak\#=dj\#+ej\#(aj\#) ie ak\# in terms of aj\#
860 'where $d j \#=$
888
0
$d j \#=(a \# * c \# * d \# * b \#(0, m)-a \# * b \# * g \# * c \#(0, n)) /(c \#(2, n) * b \#(0, m)-L \#(2$, m) ${ }^{\star} \mathrm{c} \#(0, n)$ )

900 ' and
9110
ej\# $=(b \#(1, m) * c \#(0, n)-c \#(1, n) * b \#(0, m)) /(c \#(2, n) * b \#(0, m)-b \#(2, m)$
*C\# (0, n) )
930 'calc ai\#=fi\#+gi\# (aj\#) ie ai\# in terms of aj\#
940 'where fi
950 fi\# = (a\#kb\#tg\#-dj\#*b\#(2,m))/b\#(0,m)
970 'and $g i=$
980 gi\# $=(-e j \# * b \#(2, m)-b \#(1, m)) / b \#(0, m)$
' sub ai\# \& aj\# into
1010 'ai\#"2+aj\#^2+ak\#"2=a\#"2 to give
10020
'ak\#"2(gi\#"2+ej\#^2+1)+ak\#(2*fi\#*gi\#+2*dj\#*ej\#)+fi\#^2+dj\#-2-a\#^2=0

## 1030

1040 ae\# $=\mathrm{g} 1 \#^{\wedge}{ }^{-2+e j \#}{ }^{-} 2+1$
1050 be\# $=2$ *fi\#*gi\#+2*dj\#*ej\#
1060 ce\#=fi\#^2+dj\#"2-a\#~2
1080 ' CALC ROOTS
1090 disc\#=be\#*be\#-4.0*ae\#*ce\#
1100 if ABS(disc\#/be\#) < 1E-04 THEN
1120 disc\# $=0.0$
1130 'print "suspected error in quadratic 2?2???7?7?7??7?"
1140 end if
1150 if ae\# $=0$ then
1160 xj0\#=-ce\#/be\#
$1170 \quad$ check $=1$
1180 elseif disc\# $>=0$ then
1190 xj1\#=(-be\#+SQR(disc\#))/(2*ae\#)
1200 xj2\#=(-be\#-SQR(disc\#))/(2*ae\#)
1210 check=2
1220 else
1230 print "STOP error in vector quadratic "
1250 end

1290 a\# (1, (1+1))=xj0\#
1300 a\#( $2,(1+1))=d j \#+e j \# * x j 0 \#$
1310 a\# $(0,(1+1))=f i \#+g i \# * x j 0 \#$
1320 elseif check $=2$ then
1330 'calc original cross product of a \& b for $k$ term only
1340 acrossb1 $=a \#(0,0)$ *b\# ( 1,0$)$-a\# ( 1,0 ) *b\# ( 0,0 )
1350
1360
xj1\#
1370
1380 xil\#=ri\#+gi\#*xj1\#
1380 xk1\#=dj\#+ej\#*xj1\#
1390 acrossb2=xil\#*b\# (1,m)-xj1\#*b\# (0,m)
1410 'calc current cross product of a \& b for $k$ term only using
xj2\#
1420 xi2\#=fi\#+gi\#*xj2\#
1430 xk2\#=dj\#+ej\#*xj2\#
1440 acrossb3 $=$ xi2\#*b\# ( $1, \mathrm{~m}$ )-xj2\#*と\# $(0, \mathrm{~m})$
1455 if acrossb $2=0$ and acrossb3=0 then
1456 check $1=0$
1457 goto 1480
1458 end if
1460 check1=acrossb1/acrossb2 'using xj1\# co-ord
1470 check2macrossb1/acrossb3 'using xj2\# co-ord
1480 if check $>=0$ then
1490 a\# $(0,1+1)=x i 1 \#$
1500 a\# $(1,1+1)=x j 1 \#$
1510 a\#(2,(1+1))=xk1\#
1520 elseif check2 $>=0$ then
1530 a\# $(0,1+1)=x i 2 \#$
1540 a\# $(1,1+1)=x j 2 \#$
1550 a\#(2,(1+1))=xk2\#
1560 else
1570 print
"error in choosing $y$ value on body?3??3???" stop
1590 end
1600 end if
1610 end if
1930 end sub

SUB RAM
SHARED rt\# (), rb\# (), n
SHARED rl\#, nx\#, ny\#, nz\#
 *******
'This routine determines the ram resolving cosines functions nx\#, ny\#, nz\#
'to enable $X, Y, Z$ components to be calculated.

****** 大
'bottom of ram co-ords is stored in rb\# $(2,20)$
'top of ram co-ords stored in rt\# 2,20 )
'ram direction co-sines stored in nx, ny, nz

```
nx#=(rt#(0,n)-rb#(0,n))/(rl#)
ny#=(rt#(1,n)-rb#(1,n))/(rl#)
nz#=(rt#(2,n)-rb#(2,n))/(rl#)
end sub
```


## SUB EQUILIB

shared eqm\#( ), thetap\#, nx\#, ny\#, nz\#, wX\#, WY\#, WZ\#, BM\#, PM\#, alpha\# shared rt\#(), rb\# (), wc\#(), bcg\#(), pcg\#(), lhc\#(), rhc\#(), BF\#(), n

```
|*************************************************************
****
', EQUILIB.MAT - VALUES ARE ASSIGNED TO THE EQUILIBRIUM MATRIX
    *
* eqm() AND THE LOADING MATRIX BF() .
****
'alpha=1 ASSUMES LEFT HINGE TAKES ALL LATERAL LOADING
alpha#=1
for r=0 to 5
for c=0 to 5
eqm# (r,c)=0.0
next c,r
eqm#(0,0)=1.0
eqm#(0,1)=1.0
eqm#(0,2)=0.0
eqm#(0,3)=0.0
eqm#(0,4)=0.0
eqm#(0,5)=nx#
eqm#(1,0)=-1hc#(2,n)
eqm#(1, 1)=-rhc# (2,n)
eqm#(1,2)=0.0
eqm#(1,3)=0.0
eqm#(1,4)=alpha#*lhc#(0,n)+(1-alpha#)*rhc#(0,n)
eqm#(1,5)=-nx#*rt#(2,n)+nz#*rt#(0,n)
eqm#(2,0)=0.0
eqm#(2,1)=0.0
eqm#(2, 2)=-1hc#(2,n)
eqm# (2,3)=-rhc# (2,n)
eqm#(2,4)=alpha#*lhc#(1,n)+(1-alpha#)*rhc#(1,n)
eqm#(2,5)=-ny#*rt#(2,n)+nz#*rt#(1,n)
eqm# (3,0)=0.0
eqm#(3,1)=0.0
eqm#(3,2)=1.0
eqm#(3,3)=1.0
eqm#(3,4)=0.0
eqm#(3,5)=ny#
eqm#(4,0)=0.0
eqm#(4,1)=0.0
eqm# (4,2)=0.0
eqm#(4,3)=0.0
eqm#(4,4)=1.0
eqm#(4,5)=nz#
eqm# (5,0)=-1hc#(1,n)
eqm# (5,1)=-rhc#(1,n)
eqm# (5,2)=lhc#(0,n)
eqm# (5,3)=rhc# (0,n)
egm#(5,4)=0.0
eqm#(5,5)=-nx#*rt#(1,n)+ny#*rt#(0,n)
```

Fig. C11 Invert1.bas

```
SUB INVERT1
shared eqm#(), eqmi#()
```


******
' INVERT - THE EQUILIBRIUM MATRIX IS STORED IM EQM() • TO SOLVE
*
' FOR BODY REACTIONSN THE EQUILIBRIUM HAS TO BE
INVERTED.*
THIS ROUTINE INVERTS EQM() AND STORES THE INVERTED
MATRIX IN EQMI().
******
for $r=0$ to 5
for $C=0$ to 5
eqmi\# $(r, c)=0$
test\#( $r, c$ ) =eqm\# ( $r, c$ )
next $\mathrm{c}, \mathrm{r}$
for $x=0$ to 5
eqmi\# ( $x, x$ ) $=1$
next $x$
for $x=0$ to 5
for $y=x$ to 5
d\#=eqm\# $(x, y):$ if $d \#=0$ or $d \#=1$ goto LABEL. ONE
for $k=x$ to 5
eqm\# $k, y)=$ eqm\# $(k, y) / d \#$
next $k$
for $k=0$ to 5
eqmi\# $(k, y)=$ eqmi\# $(k, y) / d \#$
next $k$
next $y$
if $\mathrm{x}=5$ goto LABEL. TWO
for $y=x+1$ to 5
if eqm\# $(x, y)=0$ goto LABEL.THREE
for $k=x$ to 5
eqm\# $(k, y)=e q m \#(k, y)-e q m \#(k, x)$
next $k$
for $k=0$ to 5
eqmi\# $(k, y)=$ eqmi\# $(k, y)$-eqmi\# $(k, x)$
next $k$
LABEL.THREE:
next y
next x

LABEL. TWO:

```
for }x=0\mathrm{ to 5
if eqm#(x,x)=1 then next }
if x<>6 then print "*****NOT INV*******equilib matrix": end
    for }x=5\mathrm{ to 1 step -1
    for Y=x-1 to 0 step -1
        d#=eqm#(x,Y)
            for k=x to 5
                eqm#(k,y)=eqm#(k,y)-eqm#(k,x)*d#
            next k
                for k=0 to 5
                eqmi#(k,Y)=eqmi#(k,Y)-eqmi##(k,x)*d#
            next k
        next y
    next x
end sub
```

SUB BODYF
 eqm\# ( ), thetap\#, psi\#, Ja\#, nx\#, nY\#, nz\#, WX\#, WY\#, WZ\#, BM\#, PM\#, alpha\# s $\quad$ h a remed eqmi\# (), rt\#(), rb\#(),wc\#(),bcg\# (),pcg\# (), lhc\#(),rhc\#(), BR\#(), n shared brconst\#(),BF\#()
 ********

- VALUES ARE ASSIGNED TO THE BODY LOADING MATRIX BF\# (5)
 *********

BF\# $(0, n)=-W X \#$
BF1a\# = -WZ\#*wC\# (0, n) +WX\#*WC\# (2, n)-BM\#*SIN(thetap\#)*bcg\# (0, n)
BFIb\# = - PM\#*SIN (thetap\#)*pcg\# $(0, n)$
BF\# (1, n) $=$ BF1a\#+BF1b\#
BF2a\# $=-\operatorname{BM} \# *(\operatorname{SIN}($ thetap\# $) * b c g \#(1, n)+C O S($ thetap\# $) * b c g \#(2, n))$
BF2b\# $=-\mathrm{PM} \# *(\operatorname{SIN}($ thetap\# ) *pcg\# (1, $n)+\operatorname{COS}($ thetap\# $) * \operatorname{pcg} \#(2, n))$
BF2C\# $=-$ WZ\#*WC\# $(1, n)+W Y \# *$ wc\# $(2, n)$
BF\# $(2, n)=B F 2 a \#+B F 2 b \#+B F 2 C \#$
BF\# (3, n) $=$ COS (thetap\#) * ( PM + BM\#) -WY\#
BF\# (4, n) $=-$ SIN( thetap\#)* (PM\#+BM\#) -WZ\#
BF\# $(5, n)=\operatorname{COS}($ thetap\# $) *(B M \# * b c g \#(0, n)+P M \# * p c g \#(0, n))-W Y \# * w C \#(0$, n)+WX\#*wC\#(1,n)

```
l*************************************************************
******
' OBTAIN BODY REACTIONS BY MULTIPLYING INVERTED EQUILIBRIUM
*
' MATRIX , eqmi#(), BY APPLIED BODY LOADS MATRIX BF#().
    *
******
```

```
for x=0 to 5
BR# (x,n)=0
    for }y=0\mathrm{ to }
        BR#(x,n)=BR#(x,n)+eqmi#(x,y)*BF#(y,n)
    next y
next x
```

l**** change sign of body reactions so they can be applied
'**** directly to chassis
for $x=0$ to 5
BR\# $(x, n)=-\operatorname{BR\# }(x, n)$
next $x$
brconst\# (n) $=$ BR\# (5, $n$ )
BR\# (5,n)=brconst\# (n)*nx\#
BR\# ( $6, n$ ) $=$ brconst\# $(n){ }^{\text {* }}$ nY\#
BR\#(7,n)=brconst\#(n)*nz\#

```
SUB SUSP
SHARED SUSPL#(),SUSPR#(),n,FLCHECK(),thetactr
```


## *******

- This routine determines the appropriate suspension operating - condition and calls the appropriate suspension force matrix
 *******

```
PMAX=30000
FLCHECK(n)=0
```

IF thetactr > 0 THEN
IF FLCHECK $(\mathrm{n}-1)=3$ THEN
CALL CASE3
FLCHECK (n)=3

## ELSE

```
IF FLCHECK \((n-1)=4\) THEN
        CALL CASE4
        FLCHECK (n) \(=4\)
    IF SUSPL\# (n) < O THEN
        CALL CASE3
        FLCHECK ( n ) \(=3\)
    END IF
    END IF
END IF
END IF
```

IF FLCHECK ( $\mathbf{n}$ ) = 0 THEN
CALL CASE1
FLCHECK ( n ) = 1
IF SUSPL\# $(n)<0$ THEN '(1)
IF SUSPR\# (n) > PMAX THEN '(2)
CALL CASE4
FLCHECK ( $n$ ) $=4$
1f SUSPL\# ( n ) < 0 then ' (3)
SUSPL\# $(n)=0$
SUSPR\# (n) $=0$
CALL CASE2
FLCHECK ( $\mathbf{n}$ ) $=\mathbf{2}$
if SUSPR\#(n) > PMAX THEN '(4)
SUSPL\# ( $n$ ) $=0$
SUSPR\# $(n)=0$
CALL CASE3
$\operatorname{FLCHECK}(\mathrm{n})=3$
end if '(4)
end if '(3)
ELSE '(2)
SUSPL\# ( n ) $=0$
SUSPR\#(n)=0
CALL CASE2
FLCHECK $(n)=2$
END IF
(2)

ELSE '(1)

```
IF SUSPR#(n) >= PMAX THEN
    CALL CASE4
    FLCHECK(n)=4
END IF
```

END IF
END IF
END SUB

SUB CASE1
SHARED BR\#(),n,thetap\#, CHL\#(),SUSPR\#(),SUSPL\#()
SHARED uchassis\#()
dim suspm1\#(1,13)

**********
' This routine contains the suspension force matrix for suspension
' operating condition condition a.
' Suspension flexibility matrix is formed (suspm\#()).The two equations *
' are solved to give right and left suspension forces, using
' the condition of zero relative displacement between the middle axle *
' and chassis. This is determined by placing two tie bars between the *
' middle axle and chassis and deriving a flexibility matrix
' which relates tie bar forces to body reactions, chassis and tractor
' self weight and suspension forces. Setting the tie bar forces to zero

- simulates the desired condition enabling suspension forces to be obtained.
 **********
suspm1\# 0,0$)=-0.28627 e-01$
suspm1\# $(0,1)=-0.18198 e-01$
suspm1\# $(0,2)=-1.0168$
suspm1\# ( 0,3 ) $=-0.27087$
suspm1\# $(0,4)=-0.21127$
suspm1\# $(0,5)=-0.21508 e-01$
suspm1\# $(0,6)=-0.10598 e-01$
suspm1\# 0,7 ) $=-0.13069 e-01$
suspm1\# $(0,8)=-0.22033 e-02$
suspm1\# $(0,9)=-0.52137 e-02$
suspm1\# $(0,10)=-0.16194$
suspm1\# $(0,11)=-0.01963$
suspm1\# $(0,12)=-3.1736$
suspm1\# $(0,13)=-1.5410$
suspm1\# ( 1,0 ) $=-0.18186 e-01$
suspm1\# (1,1)=-0.28627e-01
suspm1\# $(1,2)=-0.2708$
suspm1\# $(1,3)=-1.0168$
suspm1\# (1, 4) =0.21124
suspm1 \# (1,5) $=-0.21502 e-01$
suspm1\# (1, 6) $=-0.10605 e-01$
suspm1\# (1, 7) $=0.13083 \mathrm{e}-01$
suspm1\# $(1,8)=-0.22123 e-02$
suspm1\# ( 1,9 ) $=0.52242 e-02$
suspm1\# $(1,10)=-0.16192$
suspm1\# (1,11) $=0.01963$
suspm1\# $(1,12)=-1.5413$
suspm1\# $(1,13)=-3.1733$


# PAGE <br> NUMBERING <br> AS <br> ORIGINAL 

```
l****** body reactions,tractor weight, chassis weight and
suspension
l****** forces (not known yet) are stored in chl(14).
CHL# (0)=BR# (0,n)
CHL# (1)=BR# (1,n)
CHL#(2)=BR# (2,n)
CHL# (3)=BR# (3,n)
CHL# (4)=BR# (4,n)
CHL# (5)=BR# (5,n)
CHL# (6)=BR#(6,n)
CHL# (7)=BR#(7,n)
CHL# ( 8) = -63666.9*cos(thetap#)
CHL# (9)=63666.9*sin(thetap#)
CHL#(10)=-47088*cos(thetap#)
CHL#(11)=47088*sin(thetap#)
' CHL#(12)-unknown yet (SUSPL)
1 CHL#(13)-unknown yet (SUSPR)
A#=0
B#=0
C#=0
D#=0
'PRINT A#,B#,C#,D#
for }x=0\mathrm{ to 11
    A#=A#-suspm1#(1,x)*CHL#(x)/suspm1#(1, 13)
next x
    B#=-suspm1#(1,12)/suspm1#(1,13)
for }x=0\mathrm{ to }1
    C#=C#-suspm1#(0, x)*CHL#(x)/suspm1#(0,12)
next x
    D#=-suspm1#(0,13)/suspm1#(0,12)
SUSPR#(n)=(A#+B#*C#)/(1-B#*D#)
SUSPL#(n)=C#+D#*SUSPR#(n)
CHL#(12)=SUSPL#(n)
CHL#(13)=SUSPR#(n)
END SUB
```

SUB CASE
SHARED BR\# (), n, thetap\#, CHL\# (), SUSPR\#(),SUSPL\# ()
shared uchassis\#()
DIM suspm2\#(12)
 **********
' This routine contains the suspension force matrix for suspension
' operating condition b.
' Suspension flexibility matrix is formed (suspm\#()). The equation *
' is solved to give the right hand suspension force only, using
' the condition of zero relative displacement between the middle axle *
' and chassis. This is determined by placing a tie bar between the *
' middle axle and chassis and deriving a flexibility matrix
' which relates tie bar force to body reactions, chassis and tractor
' self weight and suspension forces. Setting the tie bar force to zero

- simulates the desired condition enabling suspension force to be determined
' for the RIGHT SUSPENSION ONLY. LEFT SUSPENSION FORCE =0
 **********
suspm2\# ( 0 ) $=-0.04669$
suspm2\# (1) $=-0.046757$
suspm2\# (2) $=-1.2827$
suspm2\# (3) $=-1.2864$
suspm2\# (4) $=0.0010223$
suspm2\# (5) $=-0.042903$
suspm2\# (6) $=-0.021143$
suspm2\# (7) $=-0.8287 e-04$
suspm2\# (8) $=-0.0043983$
suspm2\# (9) $=0.42915 e-04$
suspm2\# (10) $=-0.3215$
suspm2\#(11)=9.65717e-05
suspm2\# (12)=-4.7075
'****** body reactions, tractor weight, chassis weight and suspension
l****** forces (not known yet) are stored in chl(14).
for $x=0$ to 15
CHE \# ( $x$ ) $=0$
next $x$
CML\# ( 0 ) $=\operatorname{BRZ}(0, n)$
CHL\# (1) $=\operatorname{BRZ}(1, n)$
CHL\# (2) $=$ BR\# $(2, n)$
CHL\# (3) $=\operatorname{BRZ}(3, n)$
CHL\# (4) $=\operatorname{BR\# }(4, n)$
CHL\# (5) $=\operatorname{BR\# }(5, \mathrm{n})$
CHL\# (6) $=$ BR\# $(6, n)$
CHL\# (7) $=\operatorname{BR\# }$ (7,n)
CHE\# ( 8 ) $=-63666.9 * \cos ($ thetap\# $)$

```
CHL# (9)=63666.9*sin(thetap#)
CHL# (10) =-47088* cos( thetap#)
CHL#(11) =47088*sin(thetap#)
' CHL#(12)-unknown yet (SUSPR)
a# =0
for x=0 to 11
    a#=a#+CHL#(x)*suspm2#(x)
next x
SUSPR#(n)=a#/-suspm2#(12)
CHL#(12)=SUSPR#(n)
a#=0
for }x=0\mathrm{ to }1
    a#=a#+CHL#(x)*suspm2#(x)
next x
end sub
```

```
SUB CASE3
SHARED k11#() , k111#() , k12#() , k21#() , k22#()
SHARED prescribe#() , k21k11i##() , k21k111k12#()
SHARED k22k21k111k12#() , k22k21k11ik12i#()
SHARED fab#() , fch#(), fch1#() , FCH2#()
SHARED BR#(), CHL#() , n ,thetap#,SUSPR#(), uab#()
SHARED FLAGC, FLAG3
SHARED const#
|*************************************************************
**夫夫**太
' this routine contains the suspension force matrix for
suspension
/ operating condition d
"*************************************************************
*******
```



- FORCES IN FCH2 ARE DETERMINED FROM CHASSIS FORCES
STORED IN CHL\#(15) AND UNSPRUNG AXLE FORCES.

FCH2\#(0)=BR\#(0,n)
FCH2\# (1) $=\operatorname{BR\# }(2, n)$
FCH2\# (2) $=\operatorname{BR} \#(4, n)$
FCH2\# (3) $=\operatorname{BR} \#(1, n)$
FCH2\# (4) $=$ BR \# ( $3, n$ )
FCH2\# (5) $=$ BR\# $(5, n)$
$\operatorname{FCH} 2 \#(6)=\operatorname{BR\# }(6, n)$
FCH2\# (7) $=\operatorname{BR} \#(7, n)$
FCH2\# ( 8 ) $=-2350 * 9.81 * \cos ($ thetap\# )
FCH2\# ( 9 ) $=2350 * 9.81 * \sin ($ thetap\# )
FCH2\# ( 10 ) $=-800 * 9.81 * \cos ($ thetap\#)
FCH2\# (11) $=800 * 9.81 * \sin ($ thetap\# )
FCH2\# (12) $=-800 * 9.81 * \cos ($ thetap\# )
FCH2\# (13) $=800 * 9.81 * \sin ($ thetap\# )
FCH2\# (14) $=-800 * 9.81 * \cos ($ thetap\# )
FCH2\# ( 15 ) $=800 * 9.81 * \sin ($ thetap\# )
FCH2\# (16) $=-6490 * 9.81 * \cos ($ thetap\#)
FCH2\# (17) $=6490 * 9.81 * \sin ($ thetap\# )
Tractor
IF FLAG3 $=1$ THEN
GOTO LABEL.MISS
END IF
FLAG3=1
a\# $(0)=1$
a\# $(1)=-1$
a\# (2) $=1$
a\# (3) $=-1$
a\# $(4)=1$
a\# $(5)=-1$
prescribe\＃$(0,0)=49187$
prescribe\＃$(0,1)=-2946.7$
prescribe\＃$(0,2)=2785.0$

```
prescribe#(0,3)=10570.0
prescribe#(0,4)=-2948.0
prescribe#(0,5)=-57919.0
prescribe#(0,6)=-1715.2
prescribe#(0,7)=-839.1
prescribe#(0,8)=4599.1
prescribe# (0,9)=-6916.7
prescribe#(0,10)=960.4
prescribe#(0,11)=2204.4
prescribe#(0,12)=1155.7
prescribe# (0,13)=1160.9
prescribe#(0,14)=929.1
prescribe#(0,15)=1743.3
prescribe#(0,16)=231.6
prescribe#(0,17)=-52.2
prescribe#(0,18)=-5088.2
prescribe#(0,19)=-617.0
prescribe#(0,20)=4362.9
prescribe#}(0,21)=-738.
prescribe# (0, 22)=2244.7
prescribe#(0,23)=-593.7
```

prescribe\# $(1,0)=-2946.7$
prescribe\# $(1,1)=1654.0$
prescribe\# $(1,2)=-721.0$
prescribe\# $(1,3)=62.8$
prescribe\# $(1,4)=-212.9$
prescribe\# (1,5) $=2574.7$
prescribe\# (1,6) $=519.3$
prescribe\# ( 1,7 ) $=37.0$
prescribe\# (1, 8) $=-865.1$
prescribe\# $(1,9)=454.1$
prescribe\# $(1,10)=-151.2$
prescribe\# $(1,11)=-279.9$
prescribe\# $(1,12)=-720.5$
prescribe\# $(1,13)=-9.6$
prescribe\# $(1,14)=-1489.3$
prescribe\# $(1,15)=492.8$
prescribe\# $(1,16)=-41.0$
prescribe\# $(1,17)=18.5$
prescribe\# $(1,18)=738.8$
prescribe\# $(1,19)=97.3$
prescribe\# $(1,20)=-778.5$
prescribe\# $(1,21)=460.7$
prescribe\# $(1,22)=-113.3$
prescribe\# $(1,23)=952.4$
prescribe\# $(2,0)=2785.0$
prescribe\# $(2,1)=-721.0$
prescribe\# $(2,2)=2744.9$
prescribe\# $(2,3)=-2550.0$
prescribe\# $(2,4)=836.0$
prescribe\# $(2,5)=-221.0$
prescribe\# $(2,6)=-6.3$
prescribe\# $(2,7)=7.5$
prescribe\# $(2,8)=-329.2$
prescribe\# $(2,9)=-339.4$
prescribe\# $(2,10)=-66.5$

```
prescribe#(2,11)=47.6
prescribe#(2,12)=-70.9
prescribe# (2,13)=-469.4
prescribe# (2,14)=-38.8
prescribe# (2,15)=-1991.9
prescribe#(2,16)=2.3
prescribe#(2,17)=-0.1
prescribe#(2,18)=327.4
prescribe#(2,19)=42.7
prescribe#(2,20)=414.0
prescribe# (2,21)=45.4
prescribe#(2,22)=-459.1
prescribe#(2,23)=24.8
prescribe#(3,0)=10569
prescribe#(3,1)=62.9
prescribe#(3,2)=-2550.0
prescribe#(3,3)=53220.0
prescribe# (3,4)=-16054.0
presc-ive#(3,5)=-61830
prescribe#(3,6)=-1891.1
prescribe#(3,7)=914.
prescribe#(3,8)=-768
prescribe#(3, 3)=8538.5
prescribe#(3,10)=-113.4
prescribe#(3,11)=-3120.4
prescribe#( }3,12)=81.
prescribe#(3,13)=-1602.8
prescribe#(3,14)=155.2
prescribe#(3,15)=-2317.4
prescribe#(3,16)=275.3
prescribe#(3,17)=59.0
prescribe#(3,18)=3176.1
prescribe#(3,19)=73.7
prescribe#(3,20)=1436.1
prescribe# (3,21)=-51.5
prescribe# (3,22)=13563.0
prescribe#(3,23)=-98.5
prescribe#(4,0)=-2948.1
prescribe#(4,1)=-212.9
prescribe#(4,2)=836.0
prescribe#(4,3)=-16051
prescribe#(4,4)=0.25822e06
prescribe# (4,5)=18491
prescribe#(4,6)=950.8
prescribe#(4,7)=-273.2
prescribe#(4,8)=1271.0
prescribe#(4,9)=-3129.1
prescribe#(4,10)=-82.8
prescribe#(4,11)=1505.7
prescribe#(4,12)=-70.8
prescribe# (4,13)=617.0
prescribe#(4,14)=-31.7
prescribe#(4,15)=493.8
prescribe# (4,16)=-115.8
prescribe#(4,17)=-38.0
prescribe#(4,18)=-28689.0
```

```
prescribe#(4,19)=51.8
prescribe# (4, 20)=0.13944e06
prescribe#(4, 21)=45.1
prescribe# (4, 22)=-0.37073e06
prescribe#(4, 23)=20.2
prescribe#(5,0)=-57917.0
prescribe#(5,1)=2575.2
prescribe## (5,2)=-228.0
prescribe# (5,3)=-61829
prescribe#(5,4)=18491
prescribe#(5,5)=0.12472e06
prescribe# (5,6)=3672.5
prescribe#(5,7)=-78.0
prescribe#(5,8)=-3472.2
prescribe#(5,9)=-1554.2
prescribe#(5,10)=-786.9
prescribe#(5,11)=868.8
prescribe#(5,12)=-1194.0
prescribe#(5,13)=422.5
prescribe# (5,14)=-1057.5
prescribe#(5,15)=565.1
prescribe#}(5,16)=-508.
prescribe#(5,17)=-6.0
prescribe#(5,18)=2146.5
prescribe#(5,19)=506.2
prescribe# (5,20)=-5941.3
prescribe#(5, 21)=764.0
prescribe#(5, 22)=-15556.0
prescribe#(5,23)=676.6
prescribe#(6,0)=-1705.3
prescribe#(6,1)=515.9
prescribe##(6,2)=-6.3
prescribe#(6,3)=-1891.4
prescribe## (6,4)=942.9
prescribe# (6,5)=3674.3
prescribe# (6,6)=9520.0
prescribe#(6,7)=-8.7
prescribe#(6,8)=-1736.2
prescribe# (6,9)=-198.5
prescribe#(6,10)=-234.1
prescribe#(6,11)=171.7
prescribe# (6, 12)=-54.2
prescribe#(6, 13)=55.8
prescribe#(6,14)=3.1
prescribe# (6, 15)=-20.2
prescribe#(6,16)=-5626.1
prescribe# (6,17)=-4.8
prescribe#(6,18)=-1674.5
prescribe#(6,19)=151.5
prescribe#(6, 20)=2309.3
prescribe#(6, 21)=36.6
prescribe#(6, 22)=-1367.1
prescribe# (6, 23)=-2.0
prescribe#(7,0)=-841.5
prescribe#(7,1)=37.7
```

prescribe\#(7,2)=7.5
prescribe\# (7,3)=917.9
prescribe\# (7,4) $=-278.9$
prescribe\# (7,5) $=-71.1$
prescribe\# (7,6)=1.2
prescribe\# (7,7)=2535.0
prescribe\# $(7,8)=-74.1$
prescribe\# $(7,9)=-514.3$
prescribe\# $(7,10)=-14.8$
prescribe\# $(7,11)=49.0$
prescribe\# $(7,12)=-18.9$
prescribe\# $(7,13)=66.2$
prescribe\# $(7,14)=-16.0$
prescribe\# (7,15) $=35.9$
prescribe\# $(7,16)=0.6$
prescribe\# $(7,17)=-1004.8$
prescribe\# $(7,18)=67.1$
prescribe\# $(7,19)=9.5$
prescribe\# $(7,20)=10.0$
prescribe\# (7,21)=12.1
prescribe\# $(7,22)=255.7$
prescribe\#(7,23)=10.2
prescribe\# $(8,0)=4598.4$
prescribe\# $(8,1)=-865.0$
prescribe\# $(8,2)=-329.2$
prescribe\# $(8,3)=-768.0$
prescribe\# $(8,4)=1271.1$
prescribe\# $(8,5)=-3471.0$
prescribe\# $(8,6)=-1702.9$
prescribe\# $(8,7)=-72.4$
prescribe\# $(8,8)=11796.0$
prescribe\# $(8,9)=-3193.1$
prescribe\# $(8,10)=-2623.5$
prescribe\# $(8,11)=2286.4$
prescribe\# $(8,12)=-1891.7$
prescribe\# $(8,13)=957.6$
prescribe\# $(8,14)=-990.3$
prescribe\# $(8,15)=367.5$
prescribe\# $(8,16)=194.7$
prescribe\# $(8,17)=-39.2$
prescribe\# $(8,18)=-11540$
prescribe\# $(8,19)=1669.6$
prescribe\# $(8,20)=4994.3$
prescribe\# $(8,21)=1210.0$
prescribe\# $(8,22)=-2218.1$
prescribe\# ( 8,23 ) $=633.5$
prescribe\# (9,0) $=-6916.9$
prescribe\# $(9,1)=454.2$
prescribe\# (9,2) $=-339.4$
prescribe\# (9,3)=8538.6
prescribe\# $(9,4)=-3129.1$
prescribe\# $(9,5)=-1554.6$
prescribe\# $(9,6)=-167.4$
prescribe\# $(9,7)=-512.8$
prescribe\# $(9,8)=-3192.7$
prescribe\# (9,9)=13055

```
prescribe#(9,10)=-631.7
prescribe#(9,11)=-8454.7
prescribe#(9,12)=-529.5
prescribe#(9,13)=-2947.4
prescribe#(9,14)=-319.7
prescribe#(9,15)=-770.8
prescribe# (9,16)=29.2
prescribe# (9,17)=-33.1
prescribe# (9,18)=6643.7
prescribe#(9,19)=406.3
prescribe#(9,20)=-3772.9
prescribe#(9,21)=338.6
prescribe#(9,22)=3670.0
prescribe# (9,23)=204.5
prescribe#(10,0)=960.2
prescribe#(10,1)=-151.2
prescribe#(10,2)=-66.5
prescribe#(10,3)=-113.4
prescribe# (10,4)=-82.7
prescribe#(10,5)=-786.7
prescribe#(10,6)=-236.3
prescribe# (10,7) =-14.6
prescribe#(10,8)=-2623.5
prescribe#(10,9)=-631.6
prescribe#(10,10)=8260.1
prescribe#(10,11)=439.8
prescribe#(10,12)=-375.9
prescribe#(10,13)=174.1
prescribe#(10,14)=-194.3
prescribe#(10,15)=68.8
prescribe#(10,16)=30.1
prescribe#(10,17)=-12.8
prescribe#(10,18)=1655.2
prescribe# (10,19)=-3934.8
prescribe#(10,20)=-1006.9
prescribe#(10,21)=240.5
prescribe#(10, 22)=327.9
prescribe#(10,23)=124.3
prescribe#(11,0)=2204.7
prescribe#(11,1)=-280.0
prescribe#(11, 2)=47.6
prescribe#(11,3)=-3120.5
prescribe#(11,4)=1505.5
prescribe#(11,5)=868.6
prescribe#(11,6)=142.9
prescribe#(11,7)=47.7
prescribe#(11,8)=2286.3
prescribe# (11,9)=-8454.7
prescribe#(11,10)=441.7
prescribe#(11,11)=9424.0
prescribe#(11,12)=362.5
prescribe#(11,13)=-145.9
prescribe#(11,14)=183.6
prescribe#(11,15)=-83.1
prescribe#(11,16)=-22.9
prescribe#(11,17)=28.3
```

```
prescribe#(11,18)=-5413.6
prescribe#(11,19)=-285.0
prescribe#(11, 20)=3671.9
prescribe#(11,21)=-231.9
prescribe#(11,22)=-2245.5
prescribe#(11,23)=-117.4
prescribe#(12,0)=1155.4
prescribe#(12,1)=-720.5
prescribe#(12,2)=-70.9
prescribe#(12,3)=81.5
prescribe# (12,4)=-70.8
prescribe#(12,5)=-1193.6
prescribe#(12,6)=-53.3
prescribe#(12,7)=-18.6
prescribe#(12,8)=-1891.3
prescribe#(12,9)=-529.4
prescribe#(12,10)=-376.0
prescribe#(12,11)=362.4
prescribe#(12,12)=8378.1
prescribe#(12,13)=87.5
prescribe#(12,14)=-260.9
prescribe#(12,15)=133.6
prescribe#(12,16)=14.4
prescribe#(12,17)=-4.5
prescribe# (12,18)=1114.6
prescribe# (12,19)=241.8
prescribe# (12, 20) =-741.3
prescribe#(12,21)=-4009.4
prescribe#(12,22)=251.8
prescribe#(12,23)=166.8
prescribe#(13,0)=1161.2
prescribe#(13,1)=-9.5
prescribe# (13,2)=-469.3
prescribe#(13,3)=-1602.9
prescribe#(13,4)=616.9
prescribe#(13,5)=421.7
prescribe#(13,6)=53.8
prescribe#(13,7)=65.9
prescribe#(13,8)=957.2
prescribe#(13,9)=-2947.2
prescribe#(13,10)=174.3
prescribe# (13,11)=-146.0
prescribe#(13,12)=91.7
prescribe#(13,13)=5968.0
prescribe#(13,14)=105.8
prescribe#(13,15)=-1645.0
prescribe#(13,16)=-8.9
prescribe#(13,17)=4.6
prescribe#(13,18)=-2065.7
prescribe# (13,19)=-112.1
prescribe#(13,20)=1239.7
prescribe#(13,21)=-60.0
prescribe#(13,22)=-918.7
prescribe#(13,23)=-67.6
```

prescribe\# (14,0)=928.9
prescribe\# $(14,1)=-1489.3$
prescribe\# $(14,2)=-38.8$
prescribe\# $(14,3)=155.3$
prescribe\# $(14,4)=-31.6$
prescribe\# $(14,5)=-1057.3$
prescribe\# (14,6)=3.2
prescribe\# $(14,7)=-15.8$
prescribe\# $(14,8)=-990.3$
prescribe\# $(14,9)=-319.7$
prescribe\# $(14,10)=-194.3$
prescribe\# $(14,11)=183.6$
prescribe\# $(14,12)=-260.8$
prescribe\# $(14,13)=105.8$
prescribe\# $(14,14)=8545.0$
prescribe\# $(14,15)=48.4$
prescribe\# $(14,16)=6.1$
prescribe\# $(14,17)=-1.1$
prescribe\# $(14,18)=565.4$
prescribe\# $(14,19)=125.0$
prescribe\# $(14,20)=-412.5$
prescribe\# (14,21)=166.8
prescribe\# $(14,22)=125.5$
prescribe\# (14, 23) $=-4118.9$
prescribe\# $(15,0)=1743.3$
prescribe\# $(15,1)=492.5$
prescribe\# $(15,2)=-1991.9$
prescribe\# $(15,3)=-2317.6$
prescribe\# $(15,4)=493.7$
prescribe\# $(15,5)=564.7$
prescribe\# $(15,6)=-20.1$
prescribe\# $(15,7)=35.6$
prescribe\# $(15,8)=367.5$
prescribe\# $(15,9)=-770.7$
prescribe\# $(15,10)=68.8$
prescribe\# $(15,11)=-83.1$
prescribe\# $(15,12)=133.7$
prescribe\# $(15,13)=-1645.0$
prescribe\# $(15,14)=51.2$
prescribe\# $(15,15)=5312.0$
prescribe\# $(15,16)=-1.2$
prescribe\# $(15,17)=1.0$
prescribe\# $(15,18)=397.2$
prescribe\# $(15,19)=-44.2$
prescribe\# $(15,20)=-1471.5$
prescribe\# $(15,21)=-85.4$
prescribe\# $(15,22)=-352.7$
prescribe\# (15, 23) $=-33.4$
prescribe\# $(16,0)=231.7$
prescribe\# $(16,1)=-40.9$
prescribe\# $(16,2)=2.3$
prescribe\# $(16,3)=275.2$
prescribe\# $(16,4)=-115.5$
prescribe\# $(16,5)=-508.3$
prescribe\# $(16,6)=-5625.6$
prescribe\# $(16,7)=0.7$
prescribe\# $(16,8)=194.1$

```
prescribe#(16,9)=27.9
prescribe#(16,10)=29.9
prescribe#(16,11)=-22.0
prescribe#(16,12)=14.3
prescribe#(16,13)=-8.8
prescribe#(16,14)=6.1
prescribe#(16, 15)=-1.2
prescribe#(16, 16)=5910.0
prescribe# (16,17)=0.2
prescribe#(16,18)=172.1
prescribe#(16,19)=-19.3
prescribe#(16, 20)=-186.4
prescribe#(16, 21)=-9.2
prescribe#(16, 22)=148.7
prescribe#(16, 23)=-3.9
```

prescribe\# (17,0) $=-52.2$
prescribe\# (17,1)=18.6
prescribe\# $(17,2)=-0.1$
prescribe\# $(17,3)=58.9$
prescribe\# $(17,4)=-38.0$
prescribe\# (17,5) $=-6.0$
prescribe\# $(17,6)=-3.4$
prescribe\# (17, 7) $=-1004.6$
prescribe\# $(17,8)=-39.0$
prescribe\# $(17,9)=-32.8$
prescribe\# $(17,10)=-12.9$
prescribe\# $(17,11)=28.3$
prescribe\# $(17,12)=-4.5$
prescribe\# $(17,13)=4.6$
prescribe\# $(17,14)=-1.1$
prescribe\# $(17,15)=1.0$
prescribe\# $(17,16)=-0.2$
prescribe\# $(17,17)=1547.0$
prescribe\# $(17,18)=128.4$
prescribe\# $(17,19)=8.3$
prescribe\# $(17,20)=-124.2$
prescribe\# $(17,21)=2.9$
prescribe\# $(17,22)=63.4$
prescribe\# $(17,23)=0.7$
prescribe\# $(18,0)=-5088.2$
prescribe\# $(18,1)=738.8$
prescribe\# $(18,2)=327.4$
prescribe\# $(18,3)=3175.1$
prescribe\# $(18,4)=-28689$
prescribe\# $(18,5)=2145.0$
prescribe\# $(18,6)=-1689.7$
prescribe\# (18,7)=65.5
prescribe\# $(18,8)=-11540$
prescribe\# $(18,9)=6644.3$
prescribe\# $(18,10)=1656.1$
prescribe\# $(18,11)=-5412.1$
prescribe\# $(18,12)=1114.1$
prescribe\# $(18,13)=-2062.2$
prescribe\# $(18,14)=565.4$
prescribe\# $(18,15)=397.2$
prescribe\# $(18,16)=171.0$
prescribe\# (18, 17) $=127.9$
prescribe\# $(18,18)=53949$
prescribe\# $(18,19)=-1050.8$
prescribe\# $(18,20)=-80065$
prescribe\# $(18,21)=-712.4$
prescribe\# $(18,22)=65800$
prescribe\# $(18,23)=-361.7$
prescribe\# $(19,0)=-616.9$
prescribe\#(19,1)=97.2
prescribe\# $(19,2)=42.7$
prescribe\#(19,3)=73.7
prescribe\# (19,4)=51.8
prescribe\# $(19,5)=506.0$
prescribe\# $(19,6)=152.4$
prescribe\# $(19,7)=9.4$
prescribe\# $(19,8)=1669.6$
prescribe\# $(19,9)=406.3$
prescribe\# $(19,10)=-3934.2$
prescribe\# $(19,11)=-283.4$
prescribe\# $(19,12)=241.8$
prescribe\# $(19,13)=-112.1$
prescribe\# $(19,14)=125.0$
prescribe\# $(19,15)=-44.3$
prescribe\# $(19,16)=-19.4$
prescribe\# $(19,17)=8.2$
prescribe\# $(19,18)=-1050.8$
prescribe\# $(19,19)=2515.0$
prescribe\# $(19,20)=640.6$
prescribe\# $(19,21)=-154.7$
prescribe\# $(19,22)=-208.0$
prescribe\# (19,23) $=-80.0$
prescribe\# $(20,0)=4362.8$
prescribe\# $(20,1)=-778.4$
prescribe\# $(20,2)=414.0$
prescribe\# (20, 3 ) $=1437.2$
prescribe\# $(20,4)=0.13944 \mathrm{e} 06$
prescribe\# $(20,5)=-5939.7$
prescribe\# $(20,6)=2332.1$
prescribe\# $(20,7)=11.2$
prescribe\# $(20,8)=4994.2$
prescribe\# $(20,9)=-3773.4$ prescribe\# $(20,10)=-1007.6$
prescribe\# $(20,11)=3669.8$
prescribe\# $(20,12)=-740.9$
prescribe\# $(20,13)=1239.3$
prescribe\# $(20,14)=-412.0$
prescribe\# $(20,15)=-1469.8$
prescribe\# $(20,16)=-184.5$
prescribe\# $(20,17)=-123.3$
prescribe\# $(20,18)=-80065$
prescribe\# $(20,19)=640.6$
prescribe\# $(20,20)=0.18397 \mathrm{e} 06$
prescribe\# $(20,21)=473.6$
prescribe\# $(20,22)=-0.2488 e 06$
prescribe\# $(20,23)=263.9$
prescribe\# $(21,0)=-738.5$
prescribe\# $(21,1)=460.7$
prescribe\# $(21,2)=45.4$
prescribe\# $(21,3)=-51.5$
prescribe\# $(21,4)=45.1$
prescribe\# $(21,5)=763.9$
prescribe\# $(21,6)=34.1$
prescribe\# $(21,7)=11.9$
prescribe\# $(21,8)=1210.0$
prescribe\# $(21,9)=338.6$
prescribe\# $(21,10)=240.5$
prescribe\# $(21,11)=-231.8$
prescribe\# $(21,12)=-4009.8$
prescribe\# $(21,13)=-56.4$
prescribe\# $(21,14)=166.8$
prescribe\# $(21,15)=-85.5$
prescribe\# $(21,16)=-9.2$
prescribe\# $(21,17)=2.9$
prescribe\# $(21,18)=-712.4$
prescribe\# $(21,19)=-154.7$
prescribe\# $(21,20)=473.6$
prescribe\# $(21,21)=2564.6$
prescribe\# $(21,22)=-160.8$
prescribe\# $(21,23)=-106.7$
prescribe\# $(22,0)=2244.8$
prescribe\# $(22,1)=-113.4$
prescribe\# $(22,2)=-459.1$
prescribe\# $(22,3)=13560$
prescribe\# $(22,4)=-0.37074 \mathrm{e} 06$
prescribe\# $(22,5)=-15556$
prescribe\# $(22,6)=-1403.1$
prescribe\# $(22,7)=249.8$
prescribe\# $(22,8)=-2217.9$
prescribe\# $(22,9)=3670.1$
prescribe\# $(22,10)=328.0$
prescribe\# $(22,11)=-2245.7$
prescribe\# $(22,12)=251.8$ prescribe\# $(22,13)=-918.9$ prescribe\# $(22,14)=125.6$ prescribe\# $(22,15)=-352.9$ prescribe\# $(22,16)=147.7$ prescribe\# $(22,17)=63.7$ prescribe\# $(22,18)=65800$
prescribe\# $(22,19)=-208.0$
prescribe\# $(22,20)=-0.2488 \mathrm{e} 06$
prescribe\# $(22,21)=-160.8$
prescribe\# $(22,22)=0.55698 \mathrm{e} 06$
prescribe\# $(22,23)=-80.7$
prescribe\# $(23,0)=-593.6$
prescribe\# $(23,1)=952.4$
prescribe\# $(23,2)=24.8$
prescribe\# $(23,3)=-98.5$
prescribe\# $(23,4)=20.2$
prescribe\# $(23,5)=676.5$
prescribe\# $(23,6)=-2.0$
prescribe\# $(23,7)=10.1$
prescribe\# $(23,8)=633.5$
prescribe\# $(23,9)=204.5$
prescribe\# $(23,10)=124.3$
prescribe\# $(23,11)=-117.4$
prescribe\# $(23,12)=166.9$
prescribe\# $(23,13)=-67.6$
prescribe\# $(23,14)=-4118.2$
prescribe\# $(23,15)=-31.4$
prescribe\# $(23,16)=-3.9$
prescribe\# $(23,17)=0.7$
prescribe\# $(23,18)=-361.7$
prescribe\# $(23,19)=-80.0$
prescribe\# $(23,20)=263.9$
prescribe\# $(23,21)=-106.7$
prescribe\# $(23,22)=-80.7$
prescribe\# $(23,23)=2633.9$
prescribe\# (24,0)=92.9
prescribe\# $(24,1)=-12.7$
prescribe\# $(24,2)=-6.1$
prescribe\# $(24,3)=-0.04$
prescribe\# $(24,4)=-62.2$
prescribe\# $(24,5)=-89.7$
prescribe\# $(24,6)=-3.9$
prescribe\# $(24,7)=-1.4$
prescribe\# $(24,8)=-155.1$
prescribe\# $(24,9)=-50.5$
prescribe\# $(24,10)=-1739.6$
prescribe\# $(24,11)=31.9$
prescribe\# $(24,12)=-33.3$
prescribe\# $(24,13)=12.9$
prescribe\# $(24,14)=-17.1$
prescribe\# $(24,15)=5.7$
prescribe\# $(24,16)=1.0$
prescribe\# $(24,17)=-1.9$
prescribe\# $(24,18)=760.8$
prescribe\# $(24,19)=439.6$
prescribe\# $(24,20)=-410.2$
prescribe\# $(24,21)=21.3$
prescribe\# $(24,22)=153.3$
prescribe\# $(24,23)=10.9$
prescribe\# $(25,0)=77.0$
prescribe\# $(25,1)=-52.0$
prescribe\# $(25,2)=-4.3$
prescribe\# (25,3)=11.0
prescribe\# (25,4)=18.9
prescribe\# $(25,5)=-85.0$
prescribe\# $(25,6)=-5.9$
prescribe\# $(25,7)=-1.1$
prescribe\# $(25,8)=-143.4$
prescribe\# $(25,9)=-28.9$
prescribe\# $(25,10)=-26.9$
prescribe\# $(25,11)=23.7$
prescribe\# $(25,12)=-1721.8$
prescribe\# $(25,13)=-9.6$
prescribe\# $(25,14)=-18.9$
prescribe\# $(25,15)=12.2$
prescribe\# $(25,16)=1.2$
prescribe\# $(25,17)=-0.2$
prescribe\# $(25,18)=372.4$
prescribe\# $(25,19)=17.3$
prescribe\# $(25,20)=113.7$
prescribe\# $(25,21)=428.2$
prescribe\# $(25,22)=-40.3$
prescribe\# $(25,23)=12.1$
prescribe\# $(26,0)=63.4$
prescribe\# $(26,1)=-109.0$
prescribe\# $(26,2)=0.6$
prescribe\# $(26,3)=16.0$
prescribe\# $(26,4)=-81.6$
prescribe\# $(26,5)=-77.8$
prescribe\# $(26,6)=0.8$
prescribe\# $(26,7)=-1.3$
prescribe\# $(26,8)=-71.3$
prescripet $(26,9)=-22.3$
prescribe\# $(26,10)=-14.4$
prescribe\# $(26,11)=14.3$
prescribe\# $(26,12)=-19.3$
prescribe\# (26.13) $=11$. 6
prescribe\# $(26,14)=-1710.1$
prescribe\# $(26,15)=-11.3$
prescribe\# $(26,16)=0.4$
prescribe\# $(26,17)=-0.1$
prescribe\# $(26,18)=3.4$
prescribe\# $(26,19)=9.3$
prescribe\# $(26,20)=311.0$
prescribe\# $(26,21)=12.4$
prescribe\# $(26,22)=202.5$
prescribe\# $(26,23)=420.5$
prescribe\# $(27,0)=-77.5$
prescribe\# $(27,1)=10.2$
prescribe\# $(27,2)=5.0$
prescribe\# $(27,3)=-2.0$
prescribe\# $(27,4)=62.7$
prescribe\# $(27,5)=77.5$
prescribe\# $(27,6)=-0.2$
prescribe\# $(27,7)=1.2$
prescribe\# $(27,8)=140.7$
prescribe\# $(27,9)=40.2$
prescribe\# $(27,10)=-303.3$
prescribe\# $(27,11)=-24.7$
prescribe\# $(27,12)=27.2$
prescribe\# $(27,13)=-10.0$
prescribe\# $(27,14)=13.9$
prescribe\# $(27,15)=-4.6$
prescribe\# $(27,16)=-0.5$
prescribe\# $(27,17)=1.7$
prescribe\# $(27,18)=-756.4$
prescribe\# $(27,19)=-479.3$
prescribe\# $(27,20)=405.2$
prescribe\# $(27,21)=-17.4$
prescribe\# $(27,22)=-152.4$

```
prescribe#(27,23)=-8.9
prescribe#(28,0)=-64.8
prescribe#(28,1)=44.4
prescribe#(28,2)=3.6
prescribe#(28,3)=-10.2
prescribe# (28,4)=-19.7
prescribe#(28,5)=72.4
prescribe#(28,6)=5.4
prescribe#(28,7)=1.3
prescribe# (28,8)=123.3
prescribe# (28,9)=23.3
prescribe#(28,10)=22.9
prescribe# (28,11)=-19.8
prescribe#(28,12)=-317.1
prescribe# (28,13)=10.6
prescribe#(28,14)=16.2
prescribe#(28,15)=-10.8
prescribe# (28,16)=-1.1
prescribe# (28,17)=0.2
prescribe#(28,18)=-360.6
prescribe# (28,19)=-14.7
prescribe#(28,20)=-121.5
prescribe# (28,21)=-470.6
prescribe#(28,22)=43.0
prescribe#(28,23)=-10.3
prescribe#(29,0)=-53.5
prescribe#(29,1)=93.2
prescribe#(29,2)=-1.0
prescribe#(29,3)=-14.3
prescribe#(29,4)=81.2
prescribe# (29,5)=66.6
prescribe# (29,6)=-0.8
prescribe# (29,7)=1.1
prescribe#(29,8)=60.9
prescribe#(29,9)=18.9
prescribe#(29,10)=12.4
prescribe#(29,11)=-12.4
prescribe#(29,12)=16.5
prescribe#(29,13)=-10.5
prescribe#(29,14)=-327.0
prescribe#(29,15)=11.8
prescribe# (29,16)=-0.3
prescribe# (29,17)=0.1
prescribe# (29,18)=2.6
prescribe#(29,19)=-7.9
prescribe#(29, 20)=-315.4
prescribe# (29,21)=-10.6
prescribe#(29, 22)=-201.2
prescribe#(29,23)=-464.1
```

[^1]```
|**********************************
' determine k11#(17,17) inverted and
- store in k111#(17,17)
```



```
'
for x=0 to 17
    for }\textrm{y}=0\mathrm{ to 17
        k1l#(x,y)=prescribe#(x,y)
    next y
next x
    ' call invert2
for }x=0\mathrm{ to }1
        for }\textrm{y}=0\mathrm{ to 17
        k11i##(x,y)=0
    next y
next x
for }x=0\mathrm{ to 17
    kl1i#(x,x)=1
next x
for }x=0\mathrm{ to 17
    for }\textrm{y}=\textrm{x}\mathrm{ to 17
```

```
        d#=k11#(x,y) : if d#=0 or d#=1 goto LABEL.ONE2
```

        d#=k11#(x,y) : if d#=0 or d#=1 goto LABEL.ONE2
            for k=x to 17
            for k=x to 17
                k11#(k,y)=k11#(k,y)/d#
                k11#(k,y)=k11#(k,y)/d#
            next k
            next k
            for k=0 to 17
            for k=0 to 17
                k11i#(k,y)=k11i#(k,y)/d#
                k11i#(k,y)=k11i#(k,y)/d#
        next k
        next k
    LABEL.ONE2:
LABEL.ONE2:
next y
if x=17 goto LABEL.TWO2
for y=x+1 to 17
if k11\#(x,y)=0 goto LABEL.THREE2
for k=x to 17
k11\#(k,y)=k11\#(k,y)-k11\#(k,x)
next k
for k=0 to 17
k11i\#(k,y)=k11i\#(k,y)-k11i\#(k,x)
next k

```
LABEL. THREE2:
    next \(y\)
next \(x\)

LABEL . TWO2:
for \(x=0\) to 17
if \(k 11 \#(x, x)=1\) then next \(x\)
if \(x<>17+1\) then print "*****NOT INV*******invert2": end
```

for }x=17\mathrm{ to }1\mathrm{ step -1

```
    for \(y=x-1\) to 0 step -1
        \(\mathrm{d} \#=\mathbf{k} 11\) \# ( \(\mathbf{x}, \mathrm{Y}\) )
            for \(k=x\) to 17
                \(k 11 \#(k, Y)=k 11 \#(k, Y)-k 11 \#(k, x) \star d \#\)
            next \(k\)
            for \(k=0\) to 17
                \(k 11 i \#(k, Y)=k 111 \#(k, Y)-k 111 \#(k, x) \star d \#\)
            next \(k\)
        next \(y\)
next \(x\)

' determine k21\#(5,17) and multiply
' by k111\#(17,17) and
' store in k21k111\#(5,17)

for \(x=18\) to 23
for \(Y=0\) to 17
k21\# \(((x-18), Y)=\) prescribe\# \((x, y)\)
next \(y\)
next \(x\)
```

for }x=0\mathrm{ to }
for }y=0\mathrm{ to }1
k21k111\#=0.0
for z=0 to 17
k21k111\#=k21k11i\#+k21\#(x,z)*k111\#(z,Y)
next z
k21k11i\#\#(x,Y)=k21k111\#\#
next }
next x
'**********************************
'determine k12(17,5) and multiply
'k21k11i\#(5,17) by k12\#(5,17) and
'store in k21k11ik12\#(5,5).
|**********************************
'
for }x=0\mathrm{ to }1

```
for \(\mathrm{y}=18\) to 23
k12\#( \(x,(y-18))=\) prescribe\# ( \(x, y\) )
next \(y\)
next \(x\)
```

for x=0 to 5
for }\textrm{y}=0\mathrm{ to 5
k21k11ik12\#=0.0
for z=0 to 17

```
            k21k11ik12\#=k21k11ik12\#+k21k11i\# (x, z)*k12\# (z,y)
            next \(z\)
        k21k11ik12\# (x,y)=k21k11ik12\#
    next \(y\)
next \(x\)

'determine \(k 22(5,5)\) and subtract
'k21k11ik12\#(5,5) from it and
'store in k22k21k11ik12\# (5,5).

```

for }x=18\mathrm{ to }2
for }Y=18\mathrm{ to }2
k22\#((x-18),(y-18))=prescribe\# (x,y)
next }
next x
for }x=0\mathrm{ to }
for Y=0 to 5
k22k21k11ik12\#(x,y)=k22\#(x,y)-k21k111k12\#(x,y)
next Y
next x

```
' CALL INVERT3
for \(x=0\) to 5
    for \(y=0\) to 5
        k22k21k11ik12i\# ( \(x, y\) ) \(=0\)
    next \(y\)
next \(x\)
for \(x=0\) to 5
    k22k21k11ik12i\# ( \(x, x\) ) \(=1\)
next \(x\)
for \(x=0\) to 5
    for \(\mathrm{Y}=\mathrm{x}\) to 5
```

d\#=k22k21k11ik12\#(x,y) : if d\#=0 or d\#=1 goto LABEL.ONE3
for k=x to 5
k22k21k11ik12\#(k,y)=k22k21k11ik12\#(k,y)/d\#
next k

```
```

    for k=0 to 5
        k22k21k11ik12i#(k,Y)=k22k21k11ik12i#(k,Y)/d#
    next k
LABEL.ONE3:
next Y
if }x=5\mathrm{ goto LABEL.TWO3
for }Y=x+1 to
if k22k21k11ik12\#(x,Y)=0 goto LABEL.THREE3
for k=x to 5
k22k21k111k12\#(k,y)=k22k21k111k12\#(k,Y)-k22k21k111k12\#(k,x)
next k
for k=0 to 5
k22k21k11ik12i\#(k,Y)=k22k21k11ik12i\#\#(k,y)-k22k21k111k12i\#\#(k,x)
next k
LABEL.THREE3:
next Y
next x
LABEL.TWO3:
for }x=0\mathrm{ to }
1f k22k21k111k12\#(x,x)=1 then next x
if x<>5+1 then print "*****NOT INV*******invert3": end
for }x=5\mathrm{ to 1 step -1
for y=x-1 to 0 step -1
d\#=k22k21k111k12\#(x,y)
for k=x to 5
k22k21k11ik12\#(k,Y)=k22k21k111k12\#(k,Y)-k22k21k111k12\#(k,x)*d\#
next k
for k=0 to 5
k22k21k11ik12i\#\#(k,Y)=k22k21k111k12i\#(k,Y)-k22k21k111k121\#(k,x)*d\#
next k
next Y
next x
l**********************************
'determine k22k21k11ik12i\#(5,5) multiplied
'by airbag force unity matrix a\#(5) and
'store in fab\#(5).
1*********************************

```
```

for }x=0\mathrm{ to }
fab\#(x)=0
for }\textrm{y}=0\mathrm{ to }
fab\#(x)=fab\#(x)+k22k21k11ik12i\#(x,y)*a\#(y)
next y
PRINT "Fab(";x;")=";fab\#(x)
next x
'PRINT
1*************************************
'determine k22k21k11ik12i\#(5,5) multiplied
'by k21k11i\#(5,5) and
'store in fch\#(5,5)
|**********************************
for x=0 to 5
for y=0 to 17
fch3\#=0.0
for z=0 to 5
fch3\#=fch3\#+k22k21k11ik12i\#(x,z)*k21k11i\#(z,y)
next z
fch1\#(x,y)=fch3\#
next Y
next x
LABEL.MISS:
l************************************
'determine fch1\#(5,17) multiplied
'by chassis force unity matrix FCH2\#(17) and
'store in fch\#(17).
l***********************************
for x=0 to 5
fch\#(x)=0
for }\textrm{y}=0\mathrm{ to }1
fch\#(x)=fch\#(x)+fch1\#(x,y)*FCH2\#(y)
next y
next x
IF FLAGC = O THEN
constl\#=uab\#(0,n-1)-uab\#(1,n-1)+uab\#(2,n-1)-uab\#(3,n-1)+uab\#(4
,n-1)-uab\#(5,n-1)
const\#=30000*(3*381+Const1\#)`1.57
END IF
FLAGC=1
print "constant=",const\#,"**********************Case3"
count1\#=20000
count2\#=200000
step1\# = ( count2\#-count1\#)/10
eqn\#=1
do until eqn\# > -0.000001 and eqn\# < 0.000001
f\#=count1\#

```
```

uab\#(0,n)=fab\#(0)*f\#-fch\#(0)
uab\#(1,n)=fab\#(1)*f\#-fch\#(1)
uab\#(2,n)=fab\#(2)*f\#-fch\#(2)
uab\#(3,n)=fab\#(3)*f\#-fch\#(3)
uab\#(4,n)=fab\#(4)*f\#-fch\#(4)
uab\#(5,n)=fab\#(5)*f\#-fch\#(5)
eqn1\#=(3*381+uab\# (0,n)-uab\#(1,n)+uab\#(2,n)-uab\# (3,n)+uab\#(4,n)
-uab\#(5,n))
eqn\#=(f\#-(1/1.57))*eqn1\#-(const\#)"(1/1. 57)
if eqn\# <-0.000001 then
count1\#=countl\#+stepl\#
end if
if eqn\# >0.000001 then
count2\# =count1\#
count1\# =countl\#-stepl\#
step1\# = (count2\#-count1\#)/10
end if
loop
f1\#=const\#/((3*381+uab\#(0,n)-uab\#(1,n)+uab\#(2,n)-uab\#(3,n)+uab
\#(4,n)-uab\#(5,n))^1.57)
SUSPR\#(n)=f\#
'print
'print uab\#(0,n),"case3", fch\#(0), f\#, fab\#(0)
'print uab\#(1,n)
'print uab\#(2,n)
'print uab\#(3,n)
'print uab\#(4,n)
'print uab\#(5,n)
'input azt
'stop
END SUB

```

SUB CASE4
S H A \(\quad\) H \(\quad\) R BR\# (), n, thetap\#, CHL\# ( ), SUSPR\# (), SUSPL\# ( ) , uab\# () , uchassis\# ()

DIM suspm4\#(13), flex4a\#(5,13)
SHARED FLAGC
SHARED const\#
 **********
'This routine contains the suspension force matrix and chassis flexibility
' matrix for suspension operating condition \(c\).
' Suspension flexibility matrix is formed (suspm\#()). The equation *
' is solved to give the right hand suspension force only, using *
' the condition of zero relative displacement between the middle axle *
' and chassis. This is determined by placing a tie bar between the *
' middle axle and chassis and deriving a flexibility matrix
1 which relates tie bar force to body reactions,chassis and tractor
- self weight and suspension forces. Setting the tie bar force to zero
' simulates the desired condition enabling suspension force to be determined
- for the left SUSPENSION
 * * * * * * * * * *
```

IF FLAGC = 0 THEN
const1\#=uab\#(0,n-1)-uab\#(1,n-1)+uab\#(2,n-1)-uab\#(3,n-1)+uab\#(4
,n-1)-uab\#(5,n-1)
const\#=30000*(3*381+Constl\#)^1.57
END IF
FLAGC=1
'PRINT "CONST=";const\#
f1\#=const\#/((3*381+uab\#(0,n-1)-uab\#(1,n-1)+uab\#(2,n-1)-uab\#(3,
n-1)+uab\#(4,n-1)-uab\#(5,n-1))^1.57)
f\#=f1\#

```
suspm4\#(0) \(=-0.046744\)
suspm4\# (1) \(=-0.04670\)
suspm4\# (2) \(=-1.2866\)
suspm4\# (3) \(=-1.2832\)
suspm4\# (4) \(=-0.0011292\)
suspm4\#(5) \(=-0.042928\)
suspm4\# (6) \(=-0.021167\)
suspm4\# (7) \(=-0.5722 e-04\)
suspm4\# ( 8 ) \(=-0.0044022\)
suspm4\# (9) \(=-0.17166 e-04\)
suspm4\# (10) \(=-0.3231\)
suspm4\# (11) \(=-1.0596 e-04\)
suspm4\# (13) \(=-4.7075\)
suspm4\# (12)=-4.6992
'****** body reactions,tractor weight, chassis weight and suspension '****** forces (not known yet) are stored in chl(14).
```

for x=0 to 15
CHL\#(x)=0
next x

```
f2\# = 1
do until f2\# < 1 and \(f 2 \#>-1\)
CHL\# ( 0 ) \(=\operatorname{BR} \#(0, \mathrm{n})\)
CHL\# (1) \(=\) BR\# \((1, n)\)
CHL\# (2) \(=\operatorname{BR} \#(2, n)\)
CHL\# ( 3 ) = BR\# ( \(3, n\) )
CHL\# (4)=BR\# (4, n)
CHL\# (5) \(=\) BR\# \((5, n)\)
CHL\# ( 6 ) \(=\) BR\# ( \(6, \mathrm{n}\) )
CHL\# (7) \(=\) BR\# \((7, n)\)
CHL\# ( 8 ) \(=-63666.9 * \cos\) ( thetap\#)
CHL\# ( 9 ) \(=63666.9 * \sin (\) thetap\# )
CHL\# (10) \(=-46597.5 * \cos (\) thetap\#)
CHL\# ( 11 ) \(=46597.5 *\) sin(thetap\#)
CHL\# (12) \(=\mathrm{f}\) \#
- CHL\#(13)-unknown yet (SUSPL)
a\# = 0
for \(x=0\) to 12
    a\#=a\#+CHL\# (x)*suspm4\# (x)
next \(x\)
SUSPL\#(n)=a\#/-suspm4\#(13)
CHL\# ( 13 ) =SUSPL\# ( \(n\) )
flex4a\# \((0,0)=0.02146\)
flex4a\# \((0,1)=0.00823\)
flex4a\# \((0,2)=0.45320\)
flex4a\# \((0,3)=0.91450\)
flex4a\# \((0,4)=-0.1873\)
flex4a\# \((0,5)=0.01804\)
flex4a\# \((0,6)=0.05501\)
flex4a\# \((0,7)=-0.01135\)
flex4a\# \((0,8)=0.03941\)
flex4a\# \((0,9)=-0.00941\)
flex4a\# \((0,10)=0.27982\)
flex4a\# \((0,11)=-0.02487\)
flex4a\# \((0,13)=1.7487\)
flex4a\# \((0,12)=2.5165\)
flex4a\#( 1,0 ) \(=0.084921\)
flex4a\# \((1,1)=-0.00626\)
flex4a\# \((1,2)=-0.2320\)
flex4a\# (1,3) \(=-0.23140\)
flex4a\# \((1,4)=-0.04533\)
flex4a\# \((1,5)=-0.00266\)
flex4a\# \((1,6)=-0.05354\)
flex4a\#(1,7)=-0.00316
flex4a\# (1, B) \(=-0.04315\)
flex4a\# (1,9) \(=-0.00597\)
flex4a\# \((1,10)=-0.01099\)
flex4a\# (1,11) \(=-0.01120\)
flex4a\# (1,13) \(=-1.8879\)
flex4a\#(1,12) \(=-2.3635\)
flex4a\#(2,0) \(=0.03724\)
flex4a\# (2,1)=0.04131
flex4a\# \((2,2)=0.62180\)
flex4a\# \((2,3)=1.32040\)
flex4a\# \((2,4)=-0.2182\)
flex4a\# \((2,5)=0.03396\)
flex4a\# \((2,6)=0.00032\)
flex4a\# ( 2,7 ) \(=-0.01333\)
flex4a\# \((2,8)=-0.00987\)
flex4a\# \((2,9)=-0.00675\)
flex4a\# \((2,10)=0.30916\)
flex4a\# \((2,11)=-0.02576\)
flex4a\# \((2,13)=2.13540\)
flex4a\# \((2,12)=3.19510\)
flex4a\# \((3,0)=0.02481\)
flex4a\#(3,1) \(=0.01596\)
flex4a\# (3,2) \(=0.4270\)
flex4a\# (3,3) \(=0.4961\)
flex4a\# \((3,4)=-0.05050\)
flex4a\# \((3,5)=0.01699\)
flex4a\# \((3,6)=-0.008\)
flex4a\# \((3,7)=-0.00289\)
flex4a\# \((3,8)=-0.01161\)
flex4a\# (3,9) \(=-0.00337\)
flex4a\# \((3,10)=0.1822\)
flex4a\# \((3,11)=-0.01077\)
flex4a甘( 3,13 ) \(=0.8695\)
flex4a\# \((3,12)=0.4434\)
flex4a\# \((4,0)=0.05506\)
flex4a\# \((4,1)=0.08421\)
flex4a\# (4, 2) \(=0.8047\)
flex4a\# (4,3)=1.7908
flex4a\# \((4,4)=-0.2501\)
flex4a\# \((4,5)=0.05111\)
flex4a\# \((4,6)=-0.05432\)
flex4a\# \((4,7)=-0.0158\)
flex4a\# (4, 8) \(=-0.05903\)
flex4a\# \((4,9)=-0.004908\)
flex4a\# (4,10) \(=0.33796\)
flex4a\# \((4,11)=-0.02616\)
flex4a\# \((4,13)=2.5326\)
flex4a\# \((4,12)=3.9072\)
flex4a\# \((5,0)=-0.02343\)
flex4a\# (5,1) \(=-0.02233\)
flex4a\# \((5,2)=-0.5692\)
flex4a\# (5,3) \(=-0.3987\)
flex4a\# \((5,4)=-0.0579\)
flex4a\# \((5,5)=-0.01829\)
flex4a\# (5, 6) \(=0.01101\)
flex4a\# \((5,7)=-0.00289\)
flex4a\# \((5,8)=-0.014302\)
flex4a\# (5,9) \(=-0.01369\)
flex4a\#(5,10) \(=-0.02866\)
flex4a\# \((5,11)=-0.01002\)
flex4a\# \((5,13)=-2.3064\)
flex4a\# \((5,12)=-2.6568\)
```

for $x=0$ to 5
uab\# $(x, n)=0$
for $y=0$ to 13
uab\# ( $\mathrm{x}, \mathrm{n}$ ) $=$ uab\# $(\mathrm{x}, \mathrm{n})+\mathrm{flex4a}$ ( $\mathrm{x}, \mathrm{y})$ *CHL\# $(\mathrm{y}) / 1000.0$
next $y$
'print uab\#(x,n)
next $x$

```
f\#=const\#/( (3*381+uab\#(0,n)-uab\#(1,n)+uab\#(2,n)-uab\#(3,n)+uab\# (4, n)-
\(\operatorname{SUSPR\# }(\mathrm{n})=\mathrm{f} \#\)
f2\#=f\#-f1\# f1\#=f\# loop
end sub
```

SUB FLEXI
SHARED FLCHECK(),uchassis\#(),n

```

\section*{} ***\&***
' This routine calls the appropriate chassis flexibility matrix
- for the current suspension operating condition
 *******
```

IF FLCHECK(n)=1 THEN

```

\section*{CALL FLEXII}
'PRINT "CALLING CASE1" ELSEIF FLCHECK \((n)=2\) THEN
CALL FLEXI2
'PRINT "CALLING CASE2"
ELSEIF FLCHECK(n)=3 THEN
'PRINT "CALLING CASE3"
CALL FLEXI3
ELSEIF FLCHECK( n ) \(=4\) THEN
CALL FLEXI4
'PRINT "CALLING CASE4"
END IF
end sub

SUB FLEXI1
SHARED BR\＃（），n，thetap\＃，CHL\＃（），SUSPR\＃（），SUSPL\＃（）
SHARED uchassis\＃（），uab\＃（）
DIM flex1\＃（8，13），flexla\＃（5，13）
 ＊＊＊＊＊＊＊＊＊＊
＇This routine contains the chassis flexibility matrix for determining
＇displacements for suspension operating condition a．

＊＊＊大 大＊＊＊大＊＊

CHL\＃（ 0 ）\(=\) BR\＃（ \(0, n\) ）
CHL\＃（1）＝BR\＃（1，n）
CHL\＃（2）\(=\) BR\＃（ \(2, n\) ）
CHL\＃（3）\(=\) BR\＃（ \(3, \mathrm{n}\) ）
CHL\＃（4）\(=\operatorname{BR} \#(4, n)\)
CHL\＃（5）\(=\) BR\＃\((5, n)\)
CHL\＃（ 6 ）\(=\) BR\＃\((6, n)\)
CHL\＃（ 7 ）\(=\) BR\＃（ \(7, \mathrm{n}\) ）
CHL\＃（ 8 ）\(=-63666.9 * \cos\)（ thetap\＃）
CHL\＃（9）\(=63666.9 * \sin (\) thetap\＃）
CHL\＃（ 10 ）\(=-47088 * \cos (\) thetap\＃）
CHL\＃（ 11 ）\(=47088 * \sin (\) thetap\＃）
CHL\＃（12）\(=\) SUSPL\＃（ \(n\) ）
CHL\＃（13）\(=\operatorname{SUSPR}\)（ \(n\) ）
flex1\＃（0，0）＝0．15622
flexl\＃\((0,1)=0.091892\)
flexl\＃\((0,2)=0.0839\)
flex1\＃（0，3）\(=0.0505\)
flex1\＃（0，4）\(=-0.1082\)
flex1\＃\((0,5)=0.11628\)
flex1\＃\((0,6)=-0.013469\)
flex1\＃\((0,7)=0.040835\)
flex1\＃\((0,8)=-0.012217\)
flex1\＃\((0,9)=0.028565\)
flexl\＃\((0,10)=0.002803\)
flex1\＃\((0,11)=-0.004309\)
flex1\＃\((0,12)=0.0926\)
flex1\＃\((0,13)=0.0693\)
flex1\＃（1，0）＝0．083911
flex1\＃（1，1）\(=0.050547\)
flexl\＃（1，2）\(=1.3528\)
flexl\＃（1，3）\(=0.7072\)
flex1\＃\((1,4)=0.1345\)
flex1\＃\((1,5)=0.044217\)
flex1\＃\((1,6)=-0.079945\)
flex1\＃（1，7）\(=0.008794\)
flex1\＃（1，8）\(=-0.077668\)
flex1\＃（1，9）\(=0.001147\)
flex1\＃\((1,10)=0.25059\)
flex1\＃\((1,11)=0.01449\)
flex1\＃\((1,12)=2.2495\)
flex1\＃（1，13）＝1．7450
flex1\# \((2,0)=-0.10824\)
flex1\#(2,1)=0.10823
flex1\# \((2,2)=0.1345\)
flexl\# \((2,3)=-0.1345\)
flex1\# \((2,4)=1.2801\)
flexl\# \((2,5)=0\)
flex1\# \((2,6)=0\)
flex1\#(2,7) \(=-0.083615\)
flexl\# \((2,8)=0\)
flexl\# \((2,9)=-0.063022\)
flexl\# \((2,10)=0\)
flex1\# \((2,11)=0.361328\)
flex1\# \((2,12)=0.1051\)
flexl\# \((2,13)=-0.1051\)
flex1\#(3,0) \(=0.091889\)
flexl\#(3,1) \(=0.15622\)
flex1\#(3, 2) \(=0.0505\)
flex1\#(3, 3 ) \(=0.0839\)
flex1\# \((3,4)=0.1082\)
flex1\# \((3,5)=0.11628\)
flex1\# \((3,6)=-0.013469\)
flex1\# \((3,7)=-0.040825\)
flex 1 H(3, 8\()=-0.012217\)
flex1\# \((3,9)=-0.028555\)
flex1\# \((3,10)=0\)
flexl\#(3,11) \(=0.004309\)
flex1\# \((3,12)=0.0693\)
flex1\# \((3,13)=0.0926\)
flexl\# (4,0) \(=0.05054\)
flex1\# \((4,1)=0.083924\)
flexl\#(4,2) \(=0.7072\)
flex1\# (4, 3) \(=1.3528\)
flex1\# (4,4) \(=-0.1345\)
flex1\# (4,5) \(=0.044217\)
flex1\# \((4,6)=-0.079947\)
flex1\# \((4,7)=-0.008803\)
flex1\# \((4,8)=-0.077671\)
flex1\# \((4,9)=-0.001155\)
flex1\# \((4,10)=0.25059\)
flexl\# \((4,11)=-0.01448\)
flex1\#(4,12) \(=1.7450\)
flex1\# (4,13) \(=2.2495\)
flex1\# \((5,0)=-0.10824\)
flex1\# (5,1) \(=0.10823\)
flex1\# \((5,2)=0.1345\)
flexl\# (5,3) \(=-0.1345\)
flex1\# (5,4) \(=1.2785\)
flexl\# (5,5) \(=0\)
flex1\# \((5,6)=0\)
flex1\# \((5,7)=-0.083615\)
flex1\# \((5,8)=0\)
flexl\# \((5,9)=-0.063022\)
flex1\# \((5,10)=0\)
flexl\# \((5,11)=0.3613498\)
flex1\# \((5,12)=0.1051\)
flex1\# \((5,13)=-0.1050\)
flex1\# \((6,0)=0.11628\)
flex1\# \((6,1)=0.11628\)
flex1\# \((6,2)=0.0442\)
flex1\# \((6,3)=0.0442\)
flexl\# \((6,4)=0\)
flexl\# (6,5) \(=0.11845\)
flex1\# \((6,6)=-0.012583\)
flex1\# \((6,7)=0.014970\)
flex1\# (6, 8) \(=-0.01119\)
flexl\# \((6,9)=0\)
flex1\# \((6,10)=0.0042985\)
flex1\# \((6,11)=0\)
flex1\# \((6,12)=0.0667\)
flex1\# \((6,13)=0.0667\)
flex1\# (7,0) \(=-0.01347\)
flex1\# \((7,1)=-0.01347\)
flexl\# \((7,2)=-0.0800\)
\(f 1 \operatorname{ex} 1 \#(7,3)=-0.0800\)
flexl\# (7,4)=0
flex1\# (7.5) \(=-0.012583\)
flex1\# (7,6)=0.28262
flex \(\#(7,7)=0.000408\)
flex1\# \((7,8)=0.26394\)
flexl\# (7,9) =0
flex1\# (7,10) =0.05793
flex1\# (7, 11) \(=0\)
flex1\# \((7,12)=0.0316\)
flex1\# \((7,13)=0.0316\)
flex1\# (8,0) \(=0.040836\)
flex1\# \((8,1)=-0.040836\)
flex1\# \((8,2)=0.0088\)
flex1\# \((8,3)=-0.0088\)
flex1\# \((8,4)=-0.0836\)
flex1\# \((8,5)=0\)
flex1\# \((8,6)=0\)
flex1\# ( 8,7 ) \(=0.32029\)
flex1\# (8, 8) \(=0\)
flex1\# ( 8,9\()=0.40854\)
flex1\# \((8,10)=0\)
flex1\# (8, 11) \(=0.121884\)
flex1\# (8, 12) \(=0.0069\)
flex1\# \((8,13)=-0.0070\)
for \(x=0\) to 8
uchassis\# \((x, n)=0\)
for \(Y=0\) to 13
uchassis\# ( \(x, n\) ) \(=\) uchassis\# \((x, n)+f l e x 1 \#(x, y) * C H L \#(Y) / 1000.0\)
next \(y\)
next \(x\)
flexla\# \((0,0)=0.01475\)
flexla\# \((0,1)=-0.00232\)
flex1a\# \((0,2)=0.35340\)
flexla\# \((0,3)=0.53990\)
flexla\# \((0,4)=-0.10940\)
flexla\# \((0,5)=0.01012\)
flex1a\# \((0,6)=0.05110\)
flexla\# \((0,7)=-0.00654\)
flexla\# \((0,8)=0.03859\)
flexla\# \((0,9)=-0.00749\)
flex1a\# \((0,10)=0.22016\)
flex1a\# \((0,11)=-0.01764\)
flexla\# \((0,12)=1.1810\)
flexla\# \((0,13)=1.3474\)
flexla\# \((1,0)=0.01032\)
flexla\# \((1,1)=-0.00408\)
flex1a\# (1, 2) \(=-0.21130\)
flexla\# (1, 3) \(=-0.15390\)
flexla\# \((1,4)=-0.06140\)
flexla\# \((1,5)=-0.00102\)
flexla\# \((1,6)=-0.05277\)
flex1a\# \((1,7)=-0.00415\)
flexla\# \((1,8)=-0.04298\)
flexla\# \((1,9)=-0.00636\)
flexla\# \((1,10)=1.3456 e-03\)
flexla\# \((1,11)=-0.01269\)
flexla; \((1,12)=-1.7704\)
flexla\# \((1,13)=-2.1217\)
flex1a\# \((2,0)=0.02867\)
flex1a\# \((2,1)=0.02783\)
flexla\# \((2,2)=0.49410\)
flexla\# \((2,3)=0.84130\)
flexla\# \((2,4)=-0.11860\)
flexla\# \((2,5)=0.02383\)
flex1a抹 \((2,6)=-0.00468\)
flex1a\# \((2,7)=-0.00717\)
flexla\# \((2,8)=-0.01091\)
flexla\# \((2,9)=-0.00429\)
flexla\# \((2,10)=0.23287\)
flexla\# \((2,11)=-0.01651\)
flexla\# \((2,12)=1.4093\)
flex1a\# \((2,13)=1.6999\)
flex1a\# \((3,0)=0.03043\)
flexla\# \((3,1)=0.02480\)
flexla\# \((3,2)=0.51080\)
flexla\# \((3,3)=0.81040\)
flex1a\# \((3,4)=-0.11580\)
flex1a\# \((3,5)=0.02364\)
flexla\# \((3,6)=-0.00472\)
flex1a\# \((3,7)=-0.00694\)
flexla\# (3, 8) \(=-0.01093\)
flex1a\# (3,9) \(=-0.00499\)
flex1a\# (3,10) \(=0.23224\)
flexla\# \((3,11)=-0.01684\)
flexla\# \((3,12)=1.3457\)
```

flexla\#(3,13)=1.4240
flex1a\#(4,0)=0.04472
flex1a\#(4,1)=0.06794
flexla\#(4,2)=0.6508
flex1a\#(4,3)=1.2129
flex1a\#(4,4)=-0.13010
flexla\#(4,5)=0.03888
flexla\#(4,6)=-0.06035
flexla\#(4,7)=-0.00837
flex1a\#(4,8)=-0.06029
flexla\#(4,9)=-0.00194
flex1a\#(4,10)=0.24591
flexla\#(4,11)=-0.01501
flexla\#(4,12)=1.6568
flex1a\#(4,13)=2.1038
flex1a\#(5,0)=-0.02188
flexla\#(5,1)=-0.01988
flexla\#(5,2)=-0.54610
flex1a\#(5,3)=-0.31180
flexla\#(5,4)=-0.07600
flexla\#(5,5)=-0.01645
flexla\#(5,6)=0.01192
flexla\#(5,7)=-0.00401
flexla\#(5,8)=0.01449
flexla\#(5,9)=-0.00182
flexla\#(5,10)=-0.01482
flexla\#(5,11)=-0.01171
flex1a\#(5,12)=-2.1747
flex1a\#(5,13)=-2.3856
for x=0 to 5
uab\#(x,n)=0
for }y=0\mathrm{ to }1
uab\#(x,n)=uab\#(x,n)+flex1a\#(x,y)*CHL\#(y)/1000.0
next y
next x
end sub

```

SUB FLEXI2
SHARED BR\#(), n, thetap\#, CHL\# (), SUSPR\#(), SUSPL\#(), uab\#()
shared uchassis\#()
DIM flex2\#(8,12), flex2a\#(5,12)
 **********
' This routine contains the chassis flexibility matrix for determining
' displacements for suspension operating condition b.

**********
CHL\# (0) \(=\) BR\# \((0, n)\)
CHL\# ( 1 ) \(=\operatorname{BRZ}(1, \mathrm{n})\)
CHL\# ( 2 ) \(=\) BR\# \((2, n)\)
CHL\# ( 3 ) \(=\operatorname{BRZ}(3, n\) )
CHL\# (4)=BR\# \((4, n)\)
CHL\# (5) \(=\operatorname{BRZ}(5, n)\)
CHL\# ( 6 ) \(=\) BR \# \((6, n)\)
CHL\# (7) \(=\) BR\# ( \(7, \mathrm{n}\) )
CHL\# ( 8 ) \(=-63666.9 * \cos\) ( thetap\#)
CHL\# (9) \(=63666.9 * \sin (\) thetap\# )
CHL\# (10) \(=-47088 * \cos (\) thetap\# )
CHL\# (11) \(=47088 * \sin (\) thetap\# \()\)
CHL\# (12)=SUSPR\#(n)
flex2\# \((0,0)=0.15671\)
flex2\# \((0,1)=0.092201\)
flex2\# \((0,2)=0.1011\)
flex2\# (0,3) \(=0.0551\)
flex2\# \((0,4)=-0.1046\)
flex2\# \((0,5)=0.11664\)
flex2\# \((0,6)=-0.01329\)
flex2\# \((0,7)=0.041056\)
flex2\# \((0,8)=-0.01218\)
flex2\# \((0,9)=0.028653\)
flex2\# \((0,10)=0.00555\)
flex2\# \((0,11)=-0.00398\)
flex2\# \((0,12)=0.0954\)
flex2\#(1,0) \(=0.10114\)
flex2\# (1,1) \(=0.061504\)
flex2\# (1, 2) \(=1.9650\)
flex2\# (1, 3) \(=0.8703\)
flex2\# ( 1,4 ) \(=0.2617\)
flex2\# \((1,5)=0.057164\)
flex \(2 \#(1,6)=-0.073557\)
flex \(2 \#(1,7)=0.01666\)
flex2\# \((1,8)=-0.076338\)
flex2\# (1,9) \(=0.004285\)
flex2\# \((1,10)=0.34809\)
flex2\# \((1,11)=0.02631\)
flex2\# (1, 12) \(=2.6727\)
flex2\# \((2,0)=-0.10466\)
flex2\# \((2,1)=0.11050\)
flex2\# \((2,2)=0.2617\)
flex2\# \((2,3)=-0.1006\)
flex2\# \((2,4)=1.3065\)
flex2\# \((2,5)=0.002684\)
flex2\# \((2,6)=0.001328\)
flex2\# \((2,7)=-0.081978\)
flex2\# \((2,8)=0.000278\)
flex2\# \((2,9)=-0.062368\)
flex2\# \((2,10)=0.02025\)
flex2\# \((2,11)=0.3638\)
flex2\# \((2,12)=0.0877\)
flex2\# (3,0) \(=0.092198\)
flex2\# \((3,1)=0.15642\)
flex2\# \((3,2)=0.0615\)
flex2\# (3, 3) \(=0.0868\)
flex2\# (3,4) \(=0.1105\)
flex2\# \((3,5)=0.11651\)
flex2\# \((3,6)=-0.013355\)
flex2\# \((3,7)=-0.040683\)
flex2\# (3, 8) \(=-0.012193\)
flex2\# \((3,9)=-0.028498\)
flex2\# \((3,10)=0.00455\)
flex2\# \((3,11)=0.00452\)
flex2\# \((3,12)=0.1092\)
flex2\# \((4,0)=0.055131\)
flex2\# (4,1) \(=0.086843\)
flex2\# (4, 2) \(=0.8703\)
flex2\# \((4,3)=1.3963\)
flex2\# (4,4) \(=-0.1006\)
flex2\# \((4,5)=0.047666\)
flex2\# \((4,6)=-0.078245\)
flex2\# (4, 7) \(=-0.006707\)
flex2\# \((4,8)=-0.077317\)
flex2\# \((4,9)=-0.000319\)
flex2\# \((4,10)=0.27656\)
flex2\# \((4,11)=-0.011333\)
flex2\# \((4,12)=2.4966\)
flex2\# \((5,0)=-0.10466\)
flex2\#(5,1) \(=0.11050\)
flex2\#(5,2) \(=0.2617\)
flex2\#(5,3) \(=-0.1006\)
flex2\# (5, 4) =1.3049
flex2\# \((5,5)=0.002684\)
flex2\# \((5,6)=0.001328\)
flex2\# (5, 7) \(=-0.081978\)
flex2\# \((5,8)=0.000278\)
flex2\# \((5,9)=-0.062368\)
flex2\# \((5,10)=0.02026\)
flex2\# \((5,11)=0.363796\)
flex2\# (5,12) \(=0.0877\)
flex2\# (6,0) \(=0.11665\)
flex2\# \((6,1)=0.11651\)
flex2\# \((6,2)=0.0572\)
flex2\# (6, 3) \(=0.0477\)
flex2\# \((6,4)=0.0027\)
flex2\# (6,5) \(=0.11872\)
flex2\# (6, 6) \(=-0.012448\)
```

flex2\#(6,7)=0.000168
flex2\#(6,8)=-0.011162
flex2\#(6,9)=0.000068
flex2\#(6,10)=0.00636
flex2\#(6,11)=2.51086e-05
flex2\#(6,12)=0.0863
flex2\#(7,0)=-0.01329
flex2\#(7,1)=-0.013356
flex2\#(7, 2)=-0.0736
flex2\#(7,3)=-0.0783
flex2\#(7,4)=0.0013
flex2\#(7,5)=-0.012448
flex2\#(7,6)=0.28269
flex2\#(7,7)=0.000083
flex2\#(7,8)=0.26395
flex2\#(7,9)=0.000033
flex2\#(7,10)=0.05894
flex2\#(7,11)=1.22324e-04
flex2\#(7,12)=0.0413
flex2\#(8,0)=0.041058
flex2\#(8,1)=-0.040695
flex2\#(8,2)=0.0167
flex2\#(8,3)=-0.0067
flex2\#(8,4)=-0.0820
flex2\#(8,5)=0.000167
flex2\#(8,6) =0.000081
flex2\#(8,7)=0.62117
flex2\#(8,8)=0.000015
flex2\#( 8,9)=0.40858
flex2\#( 8,10)=0.001253
flex2\#(8,11)=0.122037
flex2\#(8,12)=0.0050
for x=0 to 8
uchassis\#(x,n)=0
next x
for }x=0\mathrm{ to }
for y=0 to 12
uchassis\#(x,n)=uchassis\#(x,n)+flex2\#(x,y)*CHL\#(y)/1000.0
next y
next x

```
- flexibility matrix for right suspension disps.
flex2a\# \((0,0)=0.018728\)
flex2a\# \((0,1)=0.000204\)
flex2a\# ( 0,2 ) \(=0.4946\)
flex2a\# (0,3) \(=0.5775\)
flex2a\# \((0,4)=-0.0801\)
flex2a\# (0,5) \(=0.013101\)
flex2a\# \((0,6)=0.052578\)
flex2a\# \((0,7)=-0.004723\)
flex2a\# \((0,8)=0.0389\)
flex2a\# \((0,9)=-0.006763\)
flex2a\# (0,10) \(=0.2426\)
flex2a\# \((0,11)=-0.01492\)
flex \(2 a\) \# \((0,12)=1.5613\)
flex2a\#(1, 0) \(=0.00537\)
flex2a\# \((1,1)=-0.007224\)
flex2a\# \((1,2)=-0.3871\)
flex2a\#(1,3) \(=-0.2008\)
flex2a\# (1,4) \(=-0.0979\)
flex2a\# (1,5) \(=-0.004738\)
flex2a\# (1, 6) \(=-0.054578\)
flex2a\# (1,7) \(=-0.00641\)
flex2a\# (1, 8) \(=-0.043866\)
flex2a\#(1,9) \(=-0.007265\)
flex2a\# (1, 10) \(=-0.02664\)
flex2a\# (1,11) \(=-0.01608\)
flex2a\# (1, 12) \(=-2.3880\)
flex2a\# \((2,0)=0.032808\)
flex2a\# \((2,1)=0.030458\)
flex2a\# \((2,2)=0.6413\)
flex2a\# ( 2,3 ) \(=0.8805\)
flex2a\# \((2,4)=-0.0881\)
flex2a\# ( 2,5 ) \(=0.026942\)
flex2a\# \((2,6)=-0.003141\)
flex2a\# ( 2,7 ) \(=-0.005281\)
flex2a\# \((2,8)=-0.01059\)
flex2a\# ( 2,9 ) \(=-0.003536\)
flex2a\# \((2,10)=0.2563\)
flex2a\# \((2,11)=-0.01367\)
flex2a\# \((2,12)=1.9229\)
flex2a\# (3, 0) \(=0.030137\)
flex2a\# \((3,1)=0.024613\)
flex2a\#(3,2) \(=0.5002\)
flex2a\# (3, 3) \(=0.8076\)
flex2a\# \((3,4)=-0.1180\)
flex2a\# \((3,5)=0.023416\)
\(f l e x 2 a\) \# \((3,6)=-0.00483\)
flex2a\#(3,7) \(=-0.007074\)
flex2a\# \((3,8)=-0.01095\)
flex2a\# \((3,9)=-0.005042\)
flex2a\# \((3,10)=0.2306\)
flex2a\# \((3,11)=-0.01704\)
flex2a\# \((3,12)=1.4081\)
flex2a\# (4, 0) \(=0.049183\)
flex2a\# \((4,1)=0.070781\)
flex2a\# \((4,2)=0.8093\)
flex2a\# (4, 3) \(=1.2552\)
flex2a\# \((4,4)=-0.0971\)
flex2a\# (4,5) \(=0.042237\)
flex2a\# (4,6) \(=-0.058694\)
flex2a\# (4,7) \(=-0.006332\)
flex2a\# (4,8) \(=-0.059944\)
flex2a\# (4,9) \(=-0.001126\)
flex2a\# \((4,10)=0.2712\)
flex2a\# \((4,11)=-0.01195\)
```

flex2a\#(4,12)=2.3441
flex2a\#(5,0)=-0.030173
flex2a\#(5,1)=-0.025156
flex2a\#(5,2)=-0.8407
flex2a\#(5,3)=-0.3903
flex2a\#(5,4)=-0.1372
flex2a\#(5,5)=-0.022681
flex2a\#(5,6)=0.008841
flex2a\#(5,7)=-0.007793
flex2a\#(5,8)=0.01385
flex2a\#(5,9)=-0.003325
flex2a\#(5,10)=-0.06174
flex2a\#(5,11)=-0.01739
flex2a\#(5,12)=-2.8321
for x=0 to 5
uab\#(x,n)=0
for y=0 to 12
uab\#(x,n)=uab\#(x,n)+flex2a\#(x,y)*CHL\#(y)/1000.0
next y
next x
end sub

```

SUB FLEXI3
```

H
A
R
E
D
FCH2\#(), SUSPR\#(),k11i\#(),k12\#(), n, uchassis\#(), uab\#(),uch\# () DIM k12uabfch2\#(17),k12uab\#(17)
'******************************************************************) *******
' This routine calculates the chassis displacements for suspension

- operating condition d

```

``` *******
```

 **********
' Suspension STIFFNESS matrix is formed PRESCRIBE\#().The FIRST FIVE
' ROWS ARE MULTIPLIED BY FCH2\# () , WHICH IS WHERE ALL THE APPLIED LOADS ARE
' STORED , TO GIVE LEFT AND RIGHT HINGE Displacements
 **********

```
FCH2#(18)=SUSPR#(n)
FCH2#(19)=SUSPR#(n)
FCH2#(20)=SUSPR#(n)
FCH2#(21)=SUSPR#(n)
FCH2#(22)=SUSPR#(n)
FCH2#(23)=SUSPR#(n)
```

for $x=0$ to 17
k12uab\# $=0.0$
for $\mathrm{y}=0$ to 5
k12uab\# =k12uab\#+k12\# ( $x, y$ )*uab\# ( $y, n$ )
next $y$
k12uab\# (x)=k12uab\#
next $x$
for $x=0$ to 17
k12uabfch2\# ( $x$ ) = FCH2\# ( $x$ )-k12uab\# ( $x$ )
next $x$
for $x=0$ to 17
uch\# $=0.0$
for $y=0$ to 17
uch\# =uch\#+k111\# ( $x, y$ )*k12uabfch2\# ( $y$ )
next $y$
uch\# ( $x, n$ ) $=$ uch\#
next $x$
uchassis\# ( $0, \mathrm{n}$ ) $=\operatorname{uch} \#(0, n)$
uchassis\# ( $1, n$ ) $=$ uch\# $(1, n)$
uchassis\# $(2, n)=u c h \#(2, n)$
uchassis\# $(3, n)=u c h \#(3, n)$
uchassis\# $(4, n)=u c h \#(4, n)$
'assuming uchassis\#(5,n)=uchassis\#(2,n)
uchassis\# ( $5, n$ ) $=$ uch\# $(2, n)$
uchassis\# $(6, n)=\operatorname{uch} \#(5, n)$
uchassis\# $(7, n)=\operatorname{uch} \#(6, n)$
uchassis\# $(8, n)=\operatorname{uch} \#(7, n)$
uch\# (18, $n$ ) =uab\# ( $0, n$ )
uch\# (19, n) =uab\# (1, n)
uch\# ( $20, n$ ) =uab\# ( $2, n$ )
uch\# $(21, n)=\operatorname{uab} \#(3, n)$
uch\# $(22, n)=\operatorname{uab} \#(4, n)$
uch\# $(23, n)=$ uab\# $(5, n)$
end sub

```
SUB FLEXI4
SHARED BR#(),n,thetap#,CHL#(),SUSPR#(),SUSPL#(),uchassis#()
DIM flex4#(8,13)
l**************************************************************
**********
'This routine contains the chassis flexibility matrix for
determining
/ displacements for suspension operating condition c
l***************************************************************
**********
```

```
CHL#(0)=BR#(0,n)
CHL#(1)=BR#(1,n)
CHL#(2)=BR# (2,n)
CHL#(3)=BR# (3,n)
CHL#(4)=BR# (4,n)
CHL#(5)=BR#(5,n)
CHL# (6)=BR#(6,n)
CHL#(7)=BR#(7,:.)
CHL# ( 8) = -63666.9*cos(thetap#)
CHL# (9) =63666.9*sin(thetap#)
CHL# (10) =-47088*cos(thetap#)
CHL#(11)=47088*sin(thetap#)
CHL#(12)=SUSPL#(n)
CHL#(13)=SUSPR#(n)
```

flex4\#(0,0) $=0.15642$
flex4\# $(0,1)=0.092201$
flex4\# $(0,2)=0.0868$
flex4\# $(0,3)=0.0615$
flex4\# $(0,4)=-0.1105$
flex4\# $(0,5)=0.11651$
flex4\# $(0,6)=-0.013355$
flex4\# (0,7) $=0.040693$
flex4\# $(0,8)=-0.012194$
flex4\# $(0,9)=0.028507$
flex4\# $(0,10)=0.004545$
flex4\# $(0,11)=-0.0045217$
flex4\# $(0,12)=0.1092$
flex4\# $(0,13)=0.1035$
flex4\#(1, 0) $=0.086829$
flex4\# (1, 1) $=0.055139$
flex4\#(1, 2)=1.3963
flex4\# (1, 3) $=0.8703$
flex4\# ( 1,4 ) $=0.1007$
flex4\# $(1,5)=0.047666$
flex4\# ( 1,6 ) $=-0.078242$
flex4\# (1, 7) $=0.006697$
flex4\# $(1,8)=-0.077314$
flex4\# ( 1,9 ) $=0.000309$
flex4\# ( 1,10 ) $=0.27656$
flex4\# (1,11)=0.011344
flex4\# (1,12) $=2.4966$
flex4\# $(1,13)=2.2539$
flex4\#(2,0) $=-0.11051$
flex4\# (2, 1) =0. 10465
flex4\# $(2,2)=0.1007$
flex4\# $(2,3)=-0.2617$
flex4\# $(2,4)=1.3065$
flex4\# (2,5) $=-0.002694$
flex4\# (2, 6) $=-0.001324$
flex4\# (2,7) $=-0.081975$
flex4\# $(2,8)=-0.000274$
flex4\# $(2,9)=-0.062363$
flex4\# $(2,10)=-0.02025$
flex4\# $(2,11)=0.363796$
flex4\# $(2,12)=-0.0876$
flex4\# (2, 13) $=-0.5017$
flex4\# (3, 0) $=0.092198$
flex4\# $(3,1)=0.15671$
flex4\# (3, 2) $=0.0551$
flex4\# $(3,3)=0.1012$
flex4\# $(3,4)=0.1046$
flex4\# $(3,5)=0.11664$
flex4\# $(3,6)=-0.013289$
flex4\# (3.7) $=-0.041046$
flex4\# $(3,8)=-0.012179$
flex4\# (3, 9) = -0.028642
flex4\# $(3,10)=0.005475$
flex4\# (3, 11) $=0.0039788$
flex4\# $(3,12)=0.0954$
flex4\# $(3,13)=0.1464$
flex4\# (4,0) $=0.061492$
flex4\# $(4,1)=0.10116$
flex4\# $(4,2)=0.8703$
flex4\# $(4,3)=1.9650$
flex4\# $(4,4)=-0.2616$
flex4\# (4,5)=0.057163
flex4\# $(4,6)=-0.073561$
flex4\# $(4,7)=-0.016678$
flex4\# $(4,8)=-0.076342$
flex4\# (4,9) $=-0.0043$
flex4\# $(4,10)=0.34807$
flex4\# $(4,11)=-0.026295$
flex4\# $(4,12)=2.6727$
flex4\# $(4,13)=4.1598$
flex4\# $(5,0)=-0.11051$
flex4\# $(5,1)=0.10465$
flex4\# (5, 2) $=0.1007$
flex4\# $(5,3)=-0.2617$
flex4\# $(5,4)=1.3049$
flex4\# $(5,5)=-0.002694$
flex4\# $(5,6)=-0.001324$
$f \operatorname{lex} 4 \#(5,7)=-0.081975$
flex4\# (5, 8) $=-0.000274$
flex4\# $(5,9)=-0.062363$
flex4\# (5,10) $=-0.02024$
flex4\# $(5,11)=0.363796$
$f l e x 4 \#(5,12)=-0.0876$
flex4\# (5.13) $=-0.5018$

```
flex4#(6,0)=0.11651
flex4#(6,1)=0.11665
flex4#(6,2)=0.0477
flex4#(6,3)=0.0572
flex4#(6,4)=-0.0027
flex4#(6,5)=0.11872
flex4#(6,6)=-0.012448
flex4#(6,7)=-0.000165
flex4#(6,8)=-0.011162
flex4#(6,9)=-0.000065
flex4#(6,10)=0.0063609
flex4#(6,11)=-0.0002489
flex4#(6,12)=0.0863
flex4#(6,13)=0.1071
flex4#(7,0)=-0.013355
flex4#(7,1)=-0.013291
flex4#(7,2)=-0.0783
flex4#(7,3)=-0.0736
flex4#(7,4)=-0.0013
flex4#(7,5)=-0.012448
flex4#(7,6)=0.28268
flex4#(7,7)=-0.000082
flex4#(7,8)=0.26395
flex4#(7,9)=-0.000033
flex4#(7,10)=0.058947
flex4#(7,11)=-0.0001245
flex4#(7,12)=0.0413
flex4#(7,13)=0.0515
flex4#(8,0)=0.040696
flex4#(8,1)=-0.041058
flex4#(8,2)=0.0067
flex4#(8,3)=-0.0167
flex4#(8,4)=-0.0820
flex4#(8,5)=-0.000166
flex4#(8,6)=-0.000083
flex4#(8,7)=0.62117
flex4#(8,8)=-0.000019
flex4#(8,9)=0.40858
flex4# ( 8,10)=-0.001255
flex4#(8,11)=0.12204
flex4#(8,12)=-0.0050
flex4#(8,13)=-0.0315
for \(x=0\) to 8
uchassis#(x,n)=0.0
    for y=0 to 13
uchassis#(x,n)=uchassis#(x,n)+flex4#(x,y)*CHL#(y)/1000.0
    next y
next x
end sub
```

```
SUB DISP
shared lhc#(),rhc#(),rb#(),uchassis#(),n
|*************************************************************
*****************
!
|
' Hinge and ram co-ordinates are updated using chassis
displacements
' taking into consideration the origin movement. Firstly the
origin
' movement is determined from averaging the displacements of the
' two hinges in each of the co-ordinate directions.
' Then the displacements at the hinges and ram base are
modified to
' give zero origin movement by subtracting the origin
displacements
' previously determined from their actual displacements.
,
,
l*******4+****************************************************
******************
```

- determine origin movement.
xorigin=(uchassis\# ( $0, n$ ) +uchassis\# ( $3, n$ ))/2
yorigin=(uchassis\#(1,n)+uchassis\#(4,n))/2
zorigin=(uchassis\# ( $2, n$ ) +uchassis\# (5,n))/2
' print xorigin, yorigin,zorigin
' determine modified chassis displacements
lhc\# $(0, n+1)=\operatorname{lhc\# }(0,0)+$ uchassis\# ( $0, n$ )-xorigin


rhc\# $\left.(0, n+1)=r h c \#(0,0)+u c^{2}\right)$
rhc\# $(1, n+1)=$ rhc\# ( 1,0 ) +uchassis\# ( $4, n$ )-yorigin
rhc\# $(2, n+1)=r h c \#(2,0)+u c^{2}$ assis\# (5,n)-zorigin
rb\# $(0, n+1)=r b \#(0,0)+$ uchassis\# $(6, n)-x o r i g i n$
$\operatorname{rb\# }(1, n+1)=\operatorname{rb} \#(1,0)$ +uchassis\# $(7, n)$-yorigin
rb\# ( $2, n+1$ ) $=\operatorname{rb} \#(2,0)$ +uchassis\# ( $8, n)$-zorigin
end sub


## Fig．C24 Tyre．bas

```
SUB TYRE
SHARED FLCHECK(),TYREF#(),n,tyre$
```

 ＊＊＊＊＊＊＊
＇This routine calls the appropriate tyre force matrix for the －current suspension operating condition
 ＊夫夫＊大 大＊

IF $\operatorname{FLCHECK}(\mathrm{n})=1$ THEN
CALL TYRE1
－PRINT＂CALling tyrel＂
ELSEIF FLCHECK（ $n$ ）$=2$ THEN CALL TYRE2
－PRINT＂CALLING TYRE2＂ ELSEIF FLCHECK（n）＝3 THEN
＇PRINT＂CALLING TYRE3＂ CALL TYRE3
ELSEIF FLCHECK（ n ）$=4$ THEN CALL TYRE4
－PRINT＂CALLING TYRE4＂
END IF
FOR $X=0$ TO 5
IF TYREF\＃$(X, n)<0$ THEN tyres＝＂stop＂
END IF
NEXT X
end sub

## SUB TYRE1

SHARED BR\# (), n, thetap\#, CHL\#(), SUSPR\#(), SUSPL\# (), TYREF\# ()
DIM tyrel\# $(5,13)$

******* * * *
' This routine contains the chassis tyre force matrix for suspension
' operating condition a

**********
tyre1\# $(0,0)=0.0105$
tyre1\# $(0,1)=-0.0102$
tyre1\# $(0,2)=-0.0413$
tyrel\# $(0,3)=0.0412$
tyre1\# $(0,4)=-0.088$
tyre1\# $(0,5)=0.000138$
tyrel\# $(0,6)=0.0000209$
tyre1\# $(0,7)=-0.00593$
tyrel\#(0,8) $=0.00184$
tyre1\# $(0,9)=-0.00912$
tyre1\# $(0,10)=-0.0826$
tyre1\# $(0,11)=-0.0181$
tyre1\# $(0,12)=0.911$
tyrel\# $(0,13)=0.669$
tyre1\# (1,0) $=-0.0192$
tyre1\#(1,1)=-0.0274
tyrel\# (1, 2) $=-0.854$
tyre1\#(1, 3) $=-0.434$
tyre1\#(1,4) $=-0.163$
tyrel\# (1,5) $=-0.0214$
tyre1\# (1, 6) $=-0.0106$
tyre1\# $(1,7)=-0.00978$
tyre1\# (1, 8) $=-0.00219$
tyrel\# (1,9) $=-0.00709$
tyre1\# ( 1,10 ) $=-0.2446$
tyrel\# $(1,11)=-0.0238$
tyrel\# $(1,12)=-1.74$
tyrel\# $(1,13)=-1.39$
tyre1\# $(2,0)=-0.00137$
tyrel\# $(2,1)=0.00166$
tyrel\# $(2,2)=-0.168$
tyrel\# ( 2,3 ) $=0.168$
tyre1\# $(2,4)=-0.109$
tyrel\# $(2,5)=0.000139$
tyrel\# $(2,6)=0.0000153$
tyrel\# ( 2,7 ) $=-0.00583$
tyre1\# $(2,8)=0.0000131$
tyrel\# $(2,9)=-0.00266$
tyrel\# $(2,10)=-0.0826$
tyre1\# $(2,11)=-0.0167$
tyrel\#(2,12)=0.810
tyrel\# $(2,13)=0.770$
tyrel\#(3,0) $=-0.0102$
tyre1\# $(3,1)=0.0105$
tyre1\#(3,2)=0.0412
tyrel\# $(3,3)=-0.0413$
tyrel\# $(3,4)=0.088$
tyre1\# $(3,5)=0.000138$
tyre1\# $(3,6)=0.0000138$
tyrel\# $(3,7)=0.00592$
tyre1\# $(3,8)=0.0000122$
tyre1\# (3, 9) $=0.00912$
tyre1\# $(3,10)=-0.0826$
tyrel\# $(3,11)=0.0181$
tyrel\# $(3,12)=0.669$
tyrel\#(3,13)=0.911
tyre1\# $(4,0)=-0.0274$
tyrel \# (4,1) $=-0.0192$
tyrel\# (4, 2) $=-0.434$
tyre1\#(4, 3) $=-0.854$
tyre1\# $(4,4)=0.163$
tyre1\# $(4,5)=-0.0214$
tyre1\# $(4,6)=-0.0106$
tyrel\# (4, 7) $=0.0098$
tyrel\# $(4,8)=-0.00219$
tyrel\# $(4,9)=0.0071$
tyrel \# (4,10) $=-0.2446$
tyre1\# $(4,11)=0.0238$
tyre1\#(4,12) $=-1.39$
tyre1\# $(4,13)=-1.74$
tyrel\#(5,0) $=0.00165$
tyre1\#(5,1) $=-0.00137$
tyrel\# (5,2) $=0.168$
tyre1\# (5,3) $=-0.168$
tyrel\#(5,4) $=0.109$
tyre1\# (5,5) $=0.000141$
tyre1\# $(5,6)=0.0000147$
tyrel\# $(5,7)=0.00582$
tyre1\# $(5,8)=0.0000128$
tyrel\# $(5,9)=0.00266$
tyre1\# $(5,10)=-0.0826$
tyre1\# $(5,11)=0.0167$
tyrel\# $(5,12)=0.770$
tyrel\# $(5,13)=0.810$
CHL\# ( 0 ) $=$ BR \# ( $0, n$ )
CHL\# ( 1 ) $=\operatorname{BR} \#(1, n)$
CHL\# (2) $=$ BR\# $(2, n)$
CHL\# (3) $=\operatorname{BR} \#(3, n)$
CHL\# (4) $=$ BR \# ( $4, n$ )
CHL\# (5) $=$ BR\# $(5, n)$
CHL\# ( 6 ) $=$ BR \# $(6, n)$
CHL\# (7) $=$ BR\# $(7, n)$
CHL\# ( 8 ) $=-63666.9 * \cos$ ( thetap\#)
CHL\# ( 9 ) $=63666.9 * \sin ($ thetap\# )
CHL\# ( 10 ) $=-47088 * \cos$ ( thetap\#)
CHL\# (11)=47088*sin(thetap\#)
CHL\# (12)=SUSPL\# (n)
CHL\# (13)=SUSPR\#(n)

```
SUB TYRE2
SHARED BR#(),n,thetap#,CHL#(),SUSPR#(),SUSPL#(),TYREF#()
DIM tyre2#(5,12)
|*************************************************************
**********
' This routine contains the chassis tyre force matrix for the
suspension
- operating condition b
```



```
************
```

tyre2\# $(0,0)=0.0085$
tyre2\# $(0,1)=-0.0115$
tyre2\# $(0,2)=-0.112$
tyre2\# $(0,3)=0.0224$
tyre2\# $(0,4)=-0.103$
tyre2\# $(0,5)=-0.00136$
tyre2\# $(0,6)=-0.00071$
tyre2\# $(0,7)=-0.00683$
tyre2\# $(0,8)=-0.00013$
tyre2\# $(0,9)=-0.00949$
tyre2\# $(0,10)=-0.09378$
tyre2\# $(0,11)=-0.01944$
tyre2\# $(0,12)=0.5620$
tyre2\# (1, 0) $=-0.0134$
tyre2\# $(1,1)=-0.0237$
tyre2\# (1,2) $=-0.647$
tyre2\# (1, 3) $=-0.379$
tyre2\# (1, 4) $=-0.120$
tyre2\# $(1,5)=-0.0170$
tyre2\# $(1,6)=-0.00843$
tyre2\# (1,7) $=-0.00712$
tyre2\# (1,8) $=-0.00174$
tyre2\# (1,9) $=-0.00603$
tyre2\# $(1,10)=-0.2116$
tyre2\# $(1,11)=-0.01979$
tyre2\# $(1,12)=-1.0800$
tyre2\# $(2,0)=-0.00558$
tyre2\# $(2,1)=-0.00102$
tyre2\# (2,2) $=-0.318$
tyre2\# ( 2,3 ) $=0.129$
tyre2\# $(2,4)=-0.140$
tyre2\# $(2,5)=-0.00302$
tyre2\# $(2,6)=-0.00154$
tyre2\# (2,7) $=-0.00775$
tyre2\# $(2,8)=-0.00031$
tyre2\# $(2,9)=-0.00343$
tyre2\# $(2,10)=-0.10623$
tyre2\# $(2,11)=-0.01953$
tyre2\# (2,12)=0.544
tyre2\# $(3,0)=-0.00822$
tyre2\#(3,1) $=0.0118$
tyre2\# $(3,2)=0.112$
tyre2\# $(3,3)=-0.0225$
tyre2\# (3,4)=0.103

```
tyre2\# (3,5) \(=0.00163\)
tyre2\# \((3,6)=0.00075\)
tyre2\# \((3,7)=0.00683\)
tyre2\# \((3,8)=0.00016\)
tyre2\# \((3,9)=0.00949\)
tyre2\# \((3,10)=-0.07125\)
tyre2\# \((3,11)=0.01944\)
tyre2\# \((3,12)=1.02\)
tyre2\# \((4,0)=-0.0331\)
tyre2\# (4,1) \(=-0.0228\)
tyre2\# (4, 2) \(=-0.636\)
tyre2\# (4, 3) \(=-0.908\)
tyre2\# (4, 4) \(=0.121\)
tyre2\# \((4,5)=-0.0256\)
tyre2\# (4, 6) \(=-0.0127\)
tyre2\# (4, 7) \(=0.0072\)
tyre2\# (4,8) \(=-0.0026\)
tyre2\# (4,9)=0.00607
tyre2\# \((4,10)=-0.27684\)
tyre2\# \((4,11)=0.01987\)
tyre2\# \((4,12)=-2.05\)
tyre2\# \((5,0)=0.00586\)
tyre2\#(5,1) \(=0.0013\)
tyre2\# \((5,2)=0.318\)
tyre2\# (5,3) \(=-0.129\)
tyre2\# (5, 4) \(=0.140\)
tyre2\# (5,5) \(=0.0033\)
tyre2\# \((5,6)=0.00157\)
tyre2\# \((5,7)=0.00774\)
tyre2\# \((5,8)=0.00034\)
tyre2\# \((5,9)=0.00342\)
tyre2\# \((5,10)=-0.0588\)
tyre2\# (5,11) \(=0.01953\)
tyre2\# (5,12) \(=1.04\)
CHL\# ( 0 ) \(=\) BR\# \((0, n\) )
CHL\# (1) \(=\) BR\# \((1, n)\)
CHL\# (2) \(=\) BR\# \((2, n)\)
CHL\# (3) \(=\) BR\# \((3, n)\)
CHL\# (4)=BR\# (4, n)
CHL\# (5) \(=\operatorname{BR} \#(5, n)\)
CHL\# ( 6 ) \(=\operatorname{BR} \#(6, n)\)
CHL\# (7) \(=\) BR\# \((7, n)\)
CHL\# ( 8 ) \(=-63666.9 * \cos\) ( thetap\#)
CHL\# (9) \(=63666.9 * \sin (\) thetap\# )
CHL\# ( 10 ) \(=-47088 * \cos (\) thetap\# )
CHL\# (11) \(=47088 * \sin (\) thetap\# )
CHL\# (12) \(=\) SUSPR\# ( \(n\) )
```

for $x=0$ to 5
TYREF\# $(x, n)=0$
for $\mathrm{Y}=0$ to 12
TYREF\# $(x, n)=$ TYREF\# $(x, n)+$ tyre2\# $(x, y) * C H L \#(y)$
next $y$
next $x$

```
SUB TYRE4
SHARED BR#(), n, thetap#,CHL##(),SUSPR#(),SUSPL#(),TYREF#()
DIM tyre4#(5,13)
```


夫れ ** * * * * *
' This routine contains the chassis tyre force matrix for the
suspension
' operating condition $C$


*     *         *             *                 *                     *                         *                             *                                 *                                     *                                         * 

tyre4\#(0,0) $=0.0117$
tyre4\# $(0,1)=-0.00822$
tyre4\# $(0,2)=-0.0225$
tyre4\# $(0,3)=0.112$
tyre4\# $(0,4)=-0.103$
tyre4\# ( 0,5 ) $=0.00163$
tyre4\# (0,6) $=0.000757$
tyre4\# ( 0,7 ) $=-0.00683$
tyre4\# ( 0,8 ) $=0.000172$
tyre4\# (0,9) $=-0.00949$
tyre4\# $(0,10)=-0.07125$
tyre4\# $(0,11)=-0.01944$
tyre4\# $(0,12)=1.02$
tyre4\# $(0,13)=0.890$
tyre4\# ( 1,0 ) $=-0.0228$
tyre4\# $(1,1)=-0.0331$
tyre4\# $(1,2)=-0.908$
tyre4\# (1, 3) $=-0.636$
tyre4\#(1,4) $=-0.121$
tyre4\# (1,5) $=-0.0256$
tyre4\# (1,6) $=-0.0127$
tyre4\# (1,7) $=-0.00718$
tyre4\# $(1,8)=-0.00263$
tyre4\# ( 1,9 ) $=-0.00605$
tyre4\# $(1,10)=-0.2768$
tyre4\# $(1,11)=-0.01987$
tyre4\# $(1,12)=-2.05$
tyre4\# $(1,13)=-2.02$
tyre4\# (2,0)=0.0013
tyre4\# ( 2,1 ) $=0.00586$
tyre4\# $(2,2)=-0.129$
tyre4\# $(2,3)=0.318$
tyre4\# $(2,4)=-0.140$
tyre4\# $(2,5)=0.0033$
tyre4\# $(2,6)=0.00157$
tyre4\# $(2,7)=-0.00775$
tyre4\# $(2,8)=0.000337$
tyre4\# $(2,9)=-0.00343$
tyre4\# $(2,10)=-0.0588$
tyre4\# $(2,11)=-0.01953$
tyre4\# $(2,12)=1.04$
tyre4\# $(2,13)=1.24$
tyre4\# $(3,0)=-0.0115$
tyre4\#(3,1)=0.0085
tyre4\#(3,2)=0.0224
tyre4\# $(3,3)=-0.112$
tyre4\# (3, 4) $=0.103$
tyre4\# $(3,5)=-0.00136$
tyre4\# $(3,6)=-0.000723$
tyre4\# $(3,7)=0.00683$
tyre4\# $(3,8)=-0.000141$
tyre4\# $(3,9)=0.00949$
tyre4\# $(3,10)=-0.09378$
tyre4\# $(3,11)=0.01944$
tyre4\# $(3,12)=0.562$
tyre4\# $(3,13)=0.690$
tyre4\# (4,0) $=-0.0237$
tyre4\# $(4,1)=-0.0134$
tyre4\# (4,2) $=-0.379$
tyre4\# (4,3) $=-0.647$
tyre4\# (4, 4) $=0.3$ ?. 0
tyre4\# $(4,5)=-0.0170$
tyre4\# $(4,6)=-0.000843$
tyre4\# $(4,7)=0.00713$
tyre4\# $(4,8)=-0.00175$
tyre4\# $(4,9)=0.00604$
tyre4\# $(4,10)=-0.21160$
tyre4\# $(4,11)=0.01977$
tyre4\# $(4,12)=-1.08$
tyre4\# $(4,13)=-1.10$
tyre4\# $(5,0)=-0.00102$
tyre4\# (5, 1) $=-0.00557$
tyre4\# (5, 2) $=0.129$
tyre4\# $(5,3)=-0.318$
tyre4\# $(5,4)=0.140$
tyre4\# (5,5) $=-0.00302$
tyre4\# $(5,6)=-0.00154$
tyre4\# $(5,7)=0.00775$
tyre4\# $(5,8)=-0.000311$
tyre4\# $(5,9)=0.00343$
tyre4\# $(5,10)=-0.10622$
tyre4\# $(5,11)=0.01953$
tyre4\# $(5,12)=0.544$
tyre4\# $(5,13)=0.344$
CHL\# ( 0 ) $=$ BR\# $(0, n)$
CHL\# ( 1 ) $=\operatorname{BRZ}(1, n)$
CHL\# (2) $=$ BR\# $(2, n)$
CHL\# ( 3 ) $=$ BR\# $(3, n)$
CHL\# (4)=BR\# $(4, n)$
CHL\# (5)=BR\# $(5, n)$
CHL\# (6) $=\operatorname{BR\# }(6, \mathrm{n})$
CHL\# (7) $=$ BR\# $(7, n)$
CHL\# ( 8 ) $=-63666.9 * \cos$ (thetap\#)
CHL\# ( 9 ) $=63666.9 * \sin ($ thetap\# )
CHL\# ( 10 ) $=-47088 * \cos ($ thetap\# $)$
CHL\# (11) $=47088 * \sin ($ thetap\# )
CHL\# ( 12 ) =SUSPL\# ( $n$ )
CHL\# (13) $=\operatorname{SUSPR}$ ( $n$ )
for $x=0$ to 5 TYREF\# $(x, n)=0$
for $\mathrm{y}=0$ to 13
TYREF\# $(x, n)=$ TYREF\# $(x, n)+$ tyre1\# $(x, y) * C H L \#(y)$
next $y$
next $x$
'for $x=0$ to 5
'PRINT "TYREF1="; TYREF\#(x,n)
'next $x$
end sub

## SUB CONVERGE

s h a r el rt\# (), n, SUSPL\# ( ) , SUSPR\# (), CONVERGED\%, rl\#, thetap\#, TYREF\# () , focount
SHARED tyrel\#(), concount, const\#, FLCHECK(), FILENS, thetactr CONVERGED\% = 0
 ************
' This routine determines if the procedure has converged by comparing , the current and previous displacements of the top of the ram. $A$
1 displacement of $<1 \mathrm{~mm}$ is deemed to be convergence.


if $n>1$ then
if $r t \#(0, n)-r t \#(0, n-1)<1$ and $r t \#(0, n)-r t \#(0, n-1)>-1$ then
if $r t \#(1, n)-r t \#(1, n-1)<1$ and $r t \#(1, n)-r t \#(1, n-1)>-1$ then
if rt\#(2,n)-rt\#(2,n-1) < 1 and $r t \#(2, n)-r t \#(2, n-1)>-1$ then

- convergence has occured, iteration is terminated 'print "stop , convergence has occured, iteration is terminated"
if thetactr=0 or thetactr=1 or thetactr=2 or thetactr=3 or thetactr=4 or thetactr=5 or thetactr=6 or thetactr=7 or thetactr=8 or thetactr=9 or thetactr=10 then OPEN "A", \#1 ,FILENS

```
WRITE#1,TYREF#(2,n)
WRITE#1, FLCHECK(n)
```

close \#1
end if
concount=concount+1
CONVERGED\% $=1$
'CALL OUTP
end if
end if
end if
end if
END SUB

## APPENDIX D

## PROGRAM USER GUIDE

The program user guide is aimed at giving the information necessary to enable the user to run the program, use the results, change the nine payload centre of gravity coordinates and the four payload masses and input new flexibility, stiffness and influence matrices when a new finite element model is to be considered.

## D1 Pruglam Execution

The program can be executed in two different ways. Firstly, when the user is using the Turbo Basic package, the program can be executed by the run command in the menu. The program is complied to memory only and then run, after which the program will have to be re-complied if further runs are required. The advantage of compiling to memory is that it is quick to swap between running and modifying a program compared to an executable program, which is then run as a stand alone program without the aid of the Turbo Basic package. Although the program can be run immediately, the executable program cannot be modified in any way. This guide will only concern itself with the former method of program execution as this is the most useful.

The disc in Fig. D1 contains the Turbo Basic program subroutines and is found attached to the inside of the rear cover. To run the program, the user must have a copy of the Turbo Basic package. To invoke the Turbo Basic package, the user must
insert this disc into an IBM compatible PC and type TB when in drive A. This command will invoke the Turbo Basic menu which the user will use to edit the subroutines and to run the program. To run the program and interpret the results, the user must follow instructions (i) to (v) below, having invoked the Turbo Basic package.
(i) Place the cursor over the FILE menu (using the arrow keys) and press return to reveal its submenu.
(ii) Place the cursor over the LOAD option and press return twice to give the list of files on drive A :
(iii) Place the cursor over the filename TIPPER.BAS and press return, which loads this file into the screen editor and returns the cursor to the main menu.
(iv) Place the cursor of the RUN menu and press return, which will compile TIPPER.BAS to memory and then run it. The compile time is approximately 1 minute and the run time for 9 payload positions and four payload masses is approximately 24 hours.
(v) The program's rules are written to C: drive (as writing the data to the hard disk instead of the floppy disc reduces the running time). The results for each payload position and mass are stored in separate files, with 36 files required for the current program. For each ram length considered the rear left tyre force is recorded for ground slopes of $0^{\circ}$ to $10^{\circ}$ in steps of $1^{\circ}$. The trailer type, payload mass and position and ram length are recorded with each set of tyre fore results. The results can be used in conjunction with a graphs package to obtain the stability graphs shown in Figs. 5.1(a) to 5.4(f).

The program variables which the user can change for a given trailer design, are the 9 payload centre of gravity co-ordinates, 4 payload masses and the 36 output filenames and their destination (ie. A:, B: or C: drive). These variables are all stored in the Turbo Basic program VARIABS.BAS.

The payload centre of gravity, $X, Y$ and $Z$ co-ordinates are stored in the variable matrices PCGX\#(8), PCGY\#(8) and PCGZ\#(8), respectively. The 9 X-co-ordinates are located under line 12 and can have any value in the range 0 to 8976 mm . The 9 Y-co-ordinates are located under line 28 and can have any value in the range 0 to 2028 mm . The 9 Z-co-ordinates are located under line 45 and can have any value in the -1014 to 1014 mm .

The payload masses are stored in the variable matrix PM\#(3). The 4 masses are located under line 60 and can have any value in the range 0 to $\mathbf{2 5 , 0 0 0} \mathbf{K g}$.

The output filenames are stored in the variable matrix PM $\$(3,8)$ and can have up to a seven letter prefix and a three letter suffix. The output filenames for the first, second, third and fourth payload masses are located under line 72, 90, 106 and 122, respectively.

To change any of the above variables, the user must repeat steps (i) to (iii) described in Section D1 loading VARIABS.BAS, instead of TIPPER.BAS. To modify this file
the cursor must select the EDIT menu, allowing modifications to be made. When the desired modifications have been made, the user must press the ESC key to return to the main menu, where the modified file is saved. To save the file, the user must place the cursor over the FILE menu and press return, which will reveal the save command.

## D3 Changing flexibility, stiffness and influence matrices

To determine chassis displacements, suspension displacements, suspension forces and trailer tyre forces 13 unit load flexibility, stiffness and influence matrices are required. The matrices are generated using the lorry finite element models for suspension conditions one to four, described in Section 3.4.3.

## D3.1 Flexibility matrices

The unit load flexibility matrices allow the hinges, ram/chassis contact and airbag displacements to be determined, for any loading arrangement for suspension operating conditions one, two and four. The hinge and ram/chassis contact displacement flexibility matrix variable names and their subroutine locations are shown in Table D1. The airbag displacement flexibility matrix variable names and their subroutine locations are shown in Table D2.

## D3.2 Influence matrices

The unit load influence matrices allow the suspension and tyre forces to be determined for any loading arrangement for suspension operating conditions one, two and four. The suspension for influence matrix variable names and their subroutine locations are shown in Table D3. The tyre force influence matrix variable names and their subroutine locations are shown in Table D4.

## D3.3 Stiffness matrix

The stiffness matrix allows hinge ram/chassis contact and airbay displacements and the suspension and tyre forces to be determined for any loading arrangement, for suspension operating condition $d$. The stiffness matrix variable name is PRESCRIBE\# $(29,23)$ and its subroutine name is CASE3.BAS.
Table D1

| CHASSIS DISPLACEMENT FLEXIBILITY MATRICES LOCATION |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| SUSPENSION <br> CONDITION | ONE | TWO | THREE | FOUR |
| SUBROUTINE <br> NAME | FLEX1.BAS | FLEX2.BAS | CASE3.BAS | FLEX4.BAS |
| VARIABLE <br> NAME | FLEXI\# (8, 13) | FLEX2\# (8, 13) | PRESCRIBE\# (29, 33) C | FLEX4\# (8, 13) |

Table D2

| AIRBAG DISPLACEMENT FLEXIBILITY MATRICES LOCATION |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SUSPENSION <br> CONDITION | ONE | TWO | THREE |  |  |  |
| SUBROUTINE NAME | FLEX1.BAS | FLEX2.BAS | $\cdot$ CSE3.BAS | FOUR |  |  |
| VARIABLE NAME | FLEX1A\# (5,13) | FLEX2A\# (5,13) | PRESCRIBE\# (29, 33) | FLEX4A\# (5, 13) |  |  |

Table D3

| SUSPENSION FORGE INFLUENCE MATRICES LOCATION |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| SUSPENSION <br> CONDITION | ONE | TWO | THREE | FOUR |
| SUBROUTINE NAME | CASE1.BAS | CASE2.BAS | CASE3.BAS | CA54.BAS |
| VARIABLE NAME | SUSPM\# $(1,13)$ | SUSPM2\# $(1,13)$ | PRESCRIBE\# $(29,33)$ | SUSPM4\# (1,13) |

Table D4

| TYRE FORCE INFLUENCE MATRICES LOCATION |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SUSPENSION <br> CONDITION | ONE | TWO | THREE | FOUR |  |
| SUBROUTINE NAME | TYRE1.BAS | TYRE2.BAS | CASE3.BAS | TYRE4.BAS |  |
| VARIABLE NAME | TYRE1\# (5, 13) | TYRE2\# (5,13) | PRESCRIBE\# $(29,33)$ | TYRE4\# (5, 13) |  |

## APPENDIX E

## FINITE ELEMENT ANALYSES AND FLEXIBILITY, INFLUENCE AND STIFFNESS MATRICES

## E1 Introduction

To determine chassis displacements, suspension forces and tyre forces, a total of thirteen flexibility, influence and stiffness matrices are required for a given articulated tipper design. The flexibility and influence inatrices are used when the suspension is operating under conditions $\mathbf{a} \mathbf{b}$ or $\mathbf{c}$. The stiffness matrix is used when the suspension is only operating under condition d. This appendix gives details, shown in Tables 4.3 to 4.15, of these matrices for the original chassis, and describes how the different types of matrix are obtained. Also, a typical PAFEC data file is shown in Fig. E1.

## E2 Flexibility matrices

The flexibility matrices, shown in Tables 4.3 to 4.8, enable the displacement of the body/chassis contact points and the airbags to be determined for any loading condition. For each matrix the top row gives details, for each column, of the force and its respective direction in which it is applied to the structure and its corresponding node in the PAFEC model. For Tables 4.3 to 4.5 the left column gives details of the chassis displacement to which the row coefficients correspond. For Tables 4.6 to 4.8 the left column gives details of the airbag displacement to which the row coefficients correspond.

The matrix coefficients are determined a column at a time. A unit force is applied in the direction of the force which corresponds to that column. The LOADS module in the PAFEC data program, shown in Fig. E1, is used to apply the unit force to the finite element mesh. The displacement at the nodes corresponding to the body/chassis contact points and the end of the airbags, resulting from the unit force form the column matrix coefficients.

## E2 Influence matrices

The influence matrices, shown in Tables 4.9 to 4.14, enable the suspension and tyre forces to be determined for any loading condition. For each matrix the top row gives details, for each column, of the force and its respective direction in which it is applied to the structure and its corresponding node in the PAFEC model. For Tables 4.9 to 4.11 the left column gives details of the suspension forces to which the row coefficients correspond. For Tables 4.12 to 4.14 the left column gives details of tyre forces to which the row coefficients correspond.

The matrix coefficients are determined a column at a time. A unit force is applied in the direction of the force which corresponds to that column. The LOADS module in the PAFEC data program, shown in Fig. E1, is used to apply the unit force to the finite element mesh. The forces in the tie bars resulting from the unit force form the column matrix coefficients for the suspension force influence matrices shown in Tables 4.9 to 4.11. The forces in the springs representing the trailer tyre resulting from the unit force form the column matrix coefficients for the tyre force influence matrices shown in Tables 4.12 to 4.14.

The stiffness matrix, shown in Table 4.15, enables the chassis displacements, suspension forces and tyre forces, to be determined for any loading condition. The matrix top row gives details, for each column, of the force and its respective direction in which it is applied to the structure and its corresponding node in the PAFEC model. The left column gives details of the chassis displacements (N186-X to N314-X), chassis and tractor sprung and unsprung weight (N315-Y to N313Z), suspension forces (N195-Y to N223-Y) and tyre forces (N231-Y to N236-Y) to which the row coefficients correspond.

The matrix coefficients are determined a column at a time. A unit displacement is applied in the direction of the force which corresponds to that column. The PRESCRIBE.DISPLACEMENTS module in the PAFEC data program, shown in Table E1, is used to apply the unit displacement to the finite element meshes. The reaction forces resulting from the unit displacement form the column matrix coefficients for the stiffness matrix shown in Table 4.15.

## STJNLD(TWOTIE.SUSPLJ)

RUNPAFEC(RUN=SUSPL)
TITLE CASE1 TWO TIERODS CONTROL
OPLIB. TWOTIE
FULL. SAVE. OUTPUT
REACTIONS
CONTROL.END
NODES
AXIS $=1$
NODE. NUMB X $\quad \mathbf{Y} \quad$ Z

C 1-79 : PLATE NODES

23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
C NODE 40
40
41
42
43
44

| 1 | 9535.5 | -22.0 | -700.0 |
| :--- | :--- | :--- | :--- |
| 2 | 9535.5 | -22.0 | -537.5 |
| 3 | 9535.5 | -22.0 | -375.0 |
| 4 | 9535.5 | -22.0 | -253.3 |
| 5 | 9535.5 | -22.0 | -131.5 |
| 6 | 9535.5 | -22.0 | 0.0 |
| 7 | 9535.5 | -22.0 | 131.5 |
| 8 | 9535.5 | -22.0 | 253.3 |
| 9 | 9535.5 | -22.0 | 375.0 |
| 10 | 9535.5 | -22.0 | 537.5 |
| 11 | 9535.5 | -22.0 | 700.0 |
| 12 | 9406.0 | -22.0 | -700.0 |
| 13 | 9406.0 | -22.0 | -375.0 |
| 14 | 9406.0 | -22.0 | -131.5 |
| 15 | 9406.0 | -22.0 | 131.5 |
| 16 | 9406.0 | -22.0 | 375.0 |
| 17 | 9406.0 | -22.0 | 700.0 |
| 18 | 9276.5 | -22.0 | -700.0 |
| 19 | 9276.5 | -22.0 | -537.5 |
| 20 | 9276.5 | -22.0 | -375.0 |
| 21 | 9276.5 | -22.0 | -253.0 |
| 22 | 9276.5 | -22.0 | -131.5 |

Z
-700.0
-537.5
-375. 0
-131.5
0.0
253.3
375.0
537.5
700.0
-700. 0
-375.0
131.5
375.0
700.0
-537.5
-253.0
$9276.5-22.0$ 0.0
$9276.5-22.0 \quad 131.5$
$9276.5-22.0 \quad 253.0$
9276 - $22.0 \quad 375.0$
$9276.5-22.0 \quad 537.5$
$9276.5-22.0 \quad 700.0$
9183.1 -22.0 -700.0
9183.1 -22.0 -375.0
9183.1 -22.0 -131.5
9183.1 -22.0 131.5
9183.1 -22.0 375.0
9183.1 -22.0 700.0
$9089.7-22.0 \quad-700.0$
$9089.7-22.0 \quad-537.5$
9089.7 -22.0 -375.0
$9089.7-22.0 \quad-253.3$
$9089.7-22.0 \quad-131.5$
IS KING PIN
9089.7 -22.0 0.0
$9089.7-22.0 \quad 131.5$
$9089.7-22.0 \quad 253.3$
9089.7 -22.0 375.0
9089.7 -22.0 537.5

| 45 | 9089.7 | -22.0 | 700.0 |
| :---: | :---: | :---: | :---: |
| 46 | 8977.2 | -22.0 | -700.0 |
| 47 | 8977.2 | -22.0 | -375.0 |
| C NODES | 48 \& 49 LOC | CATE RAM | PIN |
| 48 | 8977.2 | -22.0 | -131.5 |
| 49 | 8977.2 | -22.0 | 131.5 |
| 50 | 8977.2 | -22.0 | 375.0 |
| 51 | 8977.2 | -22.0 | 700.0 |
| 52 | 8864.7 | -22.0 | -700.0 |
| 53 | 8864.7 | -22.0 | -537.5 |
| 54 | 8864.7 | -22.0 | -375.0 |
| 55 | 8864.7 | -22.0 | -253.3 |
| 56 | 8864.7 | -22.0 | -131.5 |
| 57 | 8864.7 | -22.0 | 0.0 |
| 58 | 8864.7 | -22.0 | 131.5 |
| 59 | 8864.7 | -22.0 | 253.3 |
| 60 | 8864.7 | -22.0 | 375.0 |
| 61 | 8864.7 | -22.0 | 537.5 |
| 62 | 8864.7 | -22.0 | 700.0 |
| 63 | 8741.3 | -22.0 | -700.0 |
| 64 | 8741.3 | -22.0 | -375.0 |
| 65 | 8741.3 | -22.0 | -131.5 |
| 66 | 8741.3 | -22.0 | 131.5 |
| 67 | 8711.3 | -22.0 | 375.0 |
| 68 | 8741.3 | -22.0 | 700.0 |
| 69 | 8618.0 | -22.0 | -700.0 |
| 70 | 8618.0 | -22.0 | -537.5 |
| 71 | 8618.0 | -22.0 | -375.0 |
| 72 | 8618.0 | -22.0 | -253.0 |
| 73 | 8618.0 | -22.0 | -131.5 |
| 74 | 8618.0 | -22.0 | 0.0 |
| 75 | 8618.0 | -22.0 | 131.5 |
| 76 | 8618.0 | -22.0 | 253.3 |
| 77 | 8618.0 | -22.0 | 375.0 |
| 78 | 8618.0 | -22.0 | 537.5 |
| 79 | 8618.0 | -22.0 | 700.0 |
| C FITH | WHEEL BEAM | NODES | 80167 |
| 80 | 9535.5 | 67.5 | -700 |
| 81 | 9535.5 | 67.5 | -537.5 |
| 82 | 9535.5 | 67.5 | -375.0 |
| 83 | 9535.5 | 67.5 | -253.3 |
| 84 | 9535.5 | 67.5 | -131.5 |
| 85 | 9535.5 | 67.5 | 0.0 |
| 86 | 9535.5 | 67.5 | 131.5 |
| 87 | 9535.5 | 67.5 | 253.3 |
| 88 | 9535.5 | 67.5 | 375.0 |
| 89 | 9535.5 | 67.5 | 537.5 |
| 90 | 9535.5 | 67.5 | 700.0 |
| 91 | 9276.5 | 57.0 | -700.0 |
| 92 | 9276.5 | 57.0 | -537.5 |
| 93 | 9276.5 | 57.0 | -375.0 |
| 94 | 9276.5 | 57.0 | -253.3 |
| 95 | 9276.5 | 57.0 | -131.5 |
| 96 | 9276.5 | 57.0 | 0.0 |
| 97 | 9276.5 | 57.0 | 131.5 |
| 98 | 9276.5 | 57.0 | 253.3 |
| 99 | 9276.5 | 57.0 | 375.0 |
| 100 | 9276.5 | 57.0 | 537.5 |

101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116
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127

| 9276.5 | 57.0 |  |
| :--- | :--- | :--- |
| 8864.7 | 15.90 | -131.5 |
| 8864.7 | 15.90 | 0.00 |
| 8864.7 | 15.90 | 131.5 |
| 8618.0 | 83.0 | -700.0 |
| 8618.0 | 83.0 | -537.5 |
| 8618.0 | 83.0 | -375.0 |
| 8618.0 | 83.0 | -253.3 |
| 8618.0 | 83.0 | -131.5 |
| 8618.0 | 83.0 | 0.0 |
| 8618.0 | 83.0 | 131.5 |
| 8618.0 | 83.0 | 253.3 |
| 8618.0 | 83.0 | 375.0 |
| 8618.0 | 83.0 | 537.5 |
| 8618.0 | 83.0 | 700.0 |
| 9535.5 | 57.0 | -700.0 |
| 9406.0 | 57.0 | -700.0 |
| 9276.5 | 57.0 | -700.0 |
| 9183.1 | 57.0 | -700.0 |
| 9089.7 | 57.0 | -700.0 |
| 8977.2 | 57.0 | -700.0 |
| 8864.7 | 57.0 | -700.0 |
| 8741.3 | 57.0 | -700.0 |
| 8618.0 | 57.0 | -700.0 |
| 9535.5 | 15.9 | -375 |
| 9406.0 | 15.5 | -375 |
| 9276.5 | 15.9 | -375 |
| 9276.5 | 45.5 | -375 |
| 9183.1 | 45.5 | -375 |
| 9089.7 | 45.5 | -375 |
| 8977.2 | 45.5 | -375 |
| 8864.7 | 45.5 | -375 |
| 8741.3 | 45.5 | -375 |
| 8618.0 | 45.5 | -375 |
| 9276.5 | 54.5 | -131.5 |
| 9183.1 | 54.5 | -131.5 |
| 9089.7 | 54.5 | -131.5 |
| 8977.2 | 54.5 | -131.5 |
| 8864.7 | 54.5 | -131.5 |
| 8741.3 | 54.5 | -131.5 |
| 8618.0 | 54.5 | -131.5 |
| 9276.5 | 54.5 | 131.5 |
| 9183.1 | 54.5 | 131.5 |
| 9089.7 | 54.5 | 131.5 |
| 8977.2 | 54.5 | 131.5 |
| 8864.7 | 54.5 | 131.5 |
| 8741.3 | 54.5 | 131.5 |
| 8618.0 | 54.5 | 131.5 |
| 9535.5 | 15.9 | 375.0 |
| 9406.0 | 15.9 | 375.0 |
| 9276.5 | 15.9 | 375.0 |
| 9276.5 | 45.5 | 375.0 |
| 9183.1 | 45.5 | 375.0 |
| 9089.7 | 45.5 | 375.0 |
| 8977.2 | 45.5 | 375.0 |
| 8864.7 | 45.5 | 375.0 |
| 8741.3 | 45.5 | 375.0 |
| 8618.0 | 45.5 | 375.0 |
|  |  |  |

159

| 9535.5 | 57.0 | 700.0 |
| :--- | :--- | :--- |
| 9406.0 | 57.0 | 700.0 |
| 9276.5 | 57.0 | 700.0 |
| 9183.1 | 57.0 | 700.0 |
| 9089.7 | 57.0 | 700.0 |
| 8977.2 | 57.0 | 700.0 |
| 8864.7 | 57.0 | 700.0 |
| 8741.3 | 57.0 | 700.0 |
| 8618.0 | 57.0 | 700.0 |
| 8530.0 | 57.0 | -700.0 |
| 7210.0 | 15.5 | -700.0 |
| 6905.0 | -53.0 | -700.0 |
| 6680.0 | -140.0 | -700.0 |
| 6680.0 | 2.0 | -700.0 |
| 5332.5 | -170.5 | -700.0 |
| 3985.0 | -201.0 | -700.0 |
| 3985.0 | -292.0 | -700.0 |
| 3093.0 | -191.0 | -700.0 |
| 3093.0 | -297.0 | -700.0 |
| 2670.0 | -186.5 | -700.0 |
| 2670.0 | -263.0 | -700.0 |
| 1778.0 | -176.5 | -700.0 |
| 1778.0 | -268.0 | -700.0 |
| 1355.0 | -172.0 | -700.0 |
| 1355.0 | -234.0 | -700.0 |
| 463.0 | -162.0 | -700.0 |
| 463.0 | -240.0 | -700.0 |
| 6205 |  |  |
| 0.0 | 0.0 | -700.0 |
| 8530.0 | 57.0 | 700.0 |
| 7210.0 | 15.5 | 700.0 |
| 6905.0 | -53.0 | 700.0 |
| 6680.0 | -140.0 | 700.0 |
| 6680.0 | 2.0 | 700.0 |
| 5332.5 | -170.5 | 700.0 |
| 3985.0 | -201.0 | 700.0 |
| 3985.0 | -292.0 | 700.0 |
| 3093.0 | -191.0 | 700.0 |
| 3093.0 | -297.0 | 700.0 |
| 2670.0 | -186.5 | 700.0 |
| 2670.0 | -263.0 | 700.0 |
| 1778.0 | -176.5 | 700.0 |
| 1778.0 | -268.0 | 700.0 |
| 1355.0 | -172.0 | 700.0 |
| 1355.0 | -234.0 | 700.0 |
| 463.0 | -162.5 | 700.0 |
| 463.0 | -240.0 | 700.0 |
| 0.00 | 0.0 | 700.0 |
| 4032.75 | -652.0 | -700.0 |
| 3438.0 | -652.0 | -700.0 |
| 3093.0 | -652.0 | -700.0 |
| 2717.75 | -623.0 | -700.0 |
| 2123.0 | -623.0 | -700.0 |
| 1778.0 | -623.0 | -700.0 |
| 1402.75 | -594.0 | -700.0 |
| 808.0 | -594.0 | -700.0 |
| 463.0 | -594.0 | -700.0 |
| 4032.75 | -652.0 | 700.0 |
| 7 |  |  |

$216 \quad 3438.0 \quad-652.0 \quad 700.0$
217 3093.0 $-652.0 \quad 700.0$
$218 \quad 2717.75$-623.0 700.0
$219 \quad 2123.0-623.0 \quad 700.0$
220 1778.0 $-623.0 \quad 700.0$
221 1402.75-594.0 700.0
222 808.0 $\quad$ 594.0 0 . 0
223 463.0 -594.0 700.0
C OX FREEDOM FOR SUSPENSION NODES
225 4032.75-653.0 -700.0
226 2717.75-624.0 -700.0
227 1402.75-595.0 -700.0
$228 \quad 4032.75-653.0 \quad 700.0$
$229 \quad 2717.75-624.0 \quad 700.0$
$230 \quad 1402.75-595.05 \quad 700.0$
231 3438.0 -1252.0 -925.0
$232 \quad 2123.0-1223.0$-925.0
233 808.0 -1194.0 -925.0
$234 \quad 3438.0$-1252.0 925.0
$235 \quad 2123.0-1223.0 \quad 925.0$
236
C 237
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255
$808.0 \quad-1194.0 \quad 925.0$
$9089.7-1272.0 \quad 0.0$

C TRACTOR NODES
256
257
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261
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267
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269
270
271
272
7389.7
7889.7
-376.7 381.5
-381.3 381.5
$-385 \quad 381.5$
$\begin{array}{lll}8289.7 & -385 & 38 \\ 8989.7 & -391.4 & 381.5\end{array}$
$9089.7 \quad-392.4 \quad 381.5$
$9189.7 \quad-393.3 \quad 381.5$
$10280.7 \quad-403.4 \quad 381.5$
10959.7 -409.6 381.5
11389.7 -413.6 381.5
$12089.7-420 \quad 431.5$
12789.7 -426.4 481.5
13289.7 -431.1 481.5
$7390.3 \quad-376.7 \quad-381.5$
$7889.7 \quad-381.3 \quad-381.5$
$8289.7 \quad-385 \quad-381.5$
8989.7 -391.4 -381.5
$9089.7 \quad-392.4 \quad-381.5$

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C 297
C 298
299
300
301
302
9189.7
10280.7
10959.7
11389.7
12089.1 - 120 - 431.5
12789.7 -426.4 -481.5
13289.7 -431.1 -481.5
$8289.7-703.51086$
$8289.7 \quad-703.5 \quad 753.5$
12089.7 -703.5 981
$8289.7 \quad-703.5 \quad-753.5$
8289.7 -703.5 -1086
12089.7 -703.5 -981
$8289.7-1240.51086$
$8289.7 \quad-1240.5 \quad 753.5$
12089.7 -1240.5 981
8289.7 -1240.5 -753.5
$8289.7-1240.5-1086$
$\begin{array}{ccc}12089.7 & -1240.5 & -981 \\ 9089.7 & 9.5 & 381.5\end{array}$
$9089.7 \quad 9.5 \quad 0$
$\begin{array}{rcc}9089.7 & 9.5 & -381.5 \\ 9089.7 & -392.4 & 0 \\ 10959.7 & -340.5 & 0\end{array}$
10959.7
000
000

C ROLL BARS

| 303 | 12789.7 | -425.4 | 481.5 |
| :--- | :--- | :--- | :--- |
| 304 | 12789.7 | -425.4 | -481.5 |
| 305 | 12091.6 | -703.5 | 431.5 |
| 306 | 12091.6 | -703.5 | -431.5 |
| 307 | 7889.7 | -383.0 | 381.5 |
| 308 | 7889.7 | -383.0 | -381.5 |
| 309 | 8289.7 | -705.0 | 381.5 |
| 310 | 8289.7 | -705.0 | -381.5 |

C MID-SUSPENSION CHASSIS NODES
311 2123-182.6-700
$312 \quad 2123-182.6700$
313 10959.7-340.50
314 8977.2-22.0 0
C SPRUNG CHASSIS C.OF.G
315 3985,-292,0
C FRONT AXLE MID POINT
3163438 -588.5 0
C MIDDLE AXLE MID POINT
$3172123-559.50$
C REAR AXLE MID POINT
318808 -530.5 0
ELEMENTS

| NUMB | GROUP. NUMB | ELEM. TYPE | PROPS |  | TOPOLOGY |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 144210 | 28 | 1820 | 3 | 1912 | 13 |
| 2 | 144210 | 28 | 20223 | 5 | 2113 | 14 |
| 3 | 144210 | 28 | 22245 |  | 2314 | 15 |
| 4 | 144210 | 28 | 24267 | 9 | 2515 | 16 |



| 63 | 1 | 34200 | 4 | 20 | 30128 | 129 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 64 | 1 | 34200 | 4 | 30 | 37129 | 130 |
| 65 | 1 | 34200 | 4 | 37 | 47130 | 131 |
| 66 | 1 | 34200 | 4 | 47 | 54131 | 132 |
| 67 | 1 | 34200 | 4 | 54 | 64132 | 133 |
| 68 | 1 | 34200 | 4 | 64 | 71133 | 134 |
| 69 | 1 | 34200 | 5 | 22 | 31135 | 136 |
| 70 | 1 | 34200 | 5 | 31 | 39136 | 137 |
| 71 | 1 | 34200 | 5 | 39 | 48137 | 138 |
| 72 | 1 | 34200 | 5 | 48 | 56138 | 139 |
| 73 | 1 | 34200 | 5 | 56 | 65139 | 140 |
| 74 | 1 | 34200 | 5 | 65 | 73140 | 141 |
| 75 | 1 | 34200 | 5 | 24 | 32142 | 143 |
| 76 | 1 | 34200 | 5 | 32 | 41143 | 144 |
| 77 | 1 | 34200 | 5 | 41 | 49144 | 145 |
| 78 | 1 | 34200 | 5 | 49 | 58145 | 146 |
| 79 | 1 | 34200 | 5 | 58 | 66146 | 147 |
| 80 | 1 | 34200 | 5 | 66 | 75147 | 148 |
| 81 | 1 | 34200 | 3 | 9 | 16149 | 150 |
| 82 | 1 | 34200 | 3 | 16 | 26150 | 151 |
| 83 | 1 | 34200 | 4 | 26 | 33152 | 153 |
| 84 | 1 | 34200 | 4 | 33 | 43153 | 154 |
| 85 | 1 | 34200 | 4 | 43 | 50154 | 155 |
| 86 | 1 | 34200 | 4 | 50 | 60155 | 156 |
| 87 | 1 | 34200 | 4 | 60 | 67156 | 157 |
| 88 | 1 | 34200 | 4 | 67 | 77157 | 158 |
| 89 | 1 | 34200 | 1 | 11 | 17159 | 160 |
| 90 | 1 | 34200 | 1 | 17 | 28160 | 161 |
| 91 | 1 | 34200 | 1 | 28 | 34161 | 162 |
| 92 | 1 | 34200 | 1 | 34 | 45162 | 163 |
| 93 | 1 | 34200 | 1 | 45 | 51163 | 164 |
| 94 | 1 | 34200 | 1 | 51 | 62164 | 165 |
| 95 | 1 | 34200 | 1 | 62 | 68165 | 166 |
| 96 | 1 | 34200 | 1 | 68 | 79166 | 167 |
| 97 | 1 | 34200 | 1 | 69 | 168124 | 4168 |
| 98 | 1 | 34200 | 11 | 168 | 169 |  |
| 99 | 1 | 34200 | 12 | 169 | 170 |  |
| 100 | 1 | 34200 | 13 | 170 | 171 |  |
| 101 | 1 | 34200 | 14 | 171 | 173 |  |
| 102 | 1 | 34200 | 15 | 173 | 174 |  |
| 103 | 1 | 34200 | 16 | 174 | 176 |  |
| 104 | 1 | 34200 | 17 | 176 | 178 |  |
| 105 | 1 | 34200 | 18 | 178 | 311 |  |
| 106 | 1 | 34200 | 19 | 180 | 182 |  |
| 107 | 1 | 34200 | 20 | 182 | 184 |  |
| 108 | 1 | 34200 | 21 | 184 | 186 |  |
| 109 | 1 | 34200 | 1 | 79 | 187167 | 187 |
| 110 | 1 | 34200 | 11 | 187 | 188 |  |
| 111 | 1 | 34200 | 12 | 188 | 189 |  |
| 112 | 1 | 34200 | 13 | 189 | 190 |  |
| 113 | 1 | 34200 | 14 | 190 | 192 |  |
| 114 | 1 | 34200 | 15 | 192 | 193 |  |
| 115 | 1 | 34200 | 16 | 193 | 195 |  |
| 116 | 1 | 34200 | 17 | 195 | 197 |  |
| 117 | 1 | 34200 | 18 | 197 | 312 |  |
| 118 | 1 | 34200 | 19 | 199 | 201 |  |
| 119 | 1 | 34200 | 20 | 201 | 203 |  |
| 120 | 1 | 34200 | 21 | 203 | 205 |  |



| 1763 | 34200 | 36 | 257258 |
| :---: | :---: | :---: | :---: |
| 1773 | 34200 | 36 | 258259 |
| 1783 | 34200 | 36 | 259260 |
| 1793 | 34200 | 36 | 260261 |
| 1803 | 34200 | 36 | 261262 |
| 1813 | 34200 | 36 | 262263 |
| 1823 | 34200 | 36 | 263264 |
| 1833 | 34200 | 36 | 264265 |
| 1843 | 34200 | 36 | 265266 |
| 1853 | 34200 | 36 | 266267 |
| 1863 | 34200 | 36 | 267279 |
| 1873 | 34200 | 36 | 256268 |
| 1883 | 34200 | 36 | 268269 |
| 1893 | 34200 | 36 | 269270 |
| 1903 | 34200 | 36 | 270271 |
| 1913 | 34200 | 36 | 271272 |
| 1923 | 34200 | 36 | 272273 |
| 1933 | 34200 | 36 | 273274 |
| 1943 | 34200 | 36 | 274275 |
| 1953 | 34200 | 36 | 275276 |
| 1963 | 34200 | 36 | 276277 |
| 1973 | 34200 | 36 | 277278 |
| 1983 | 34200 | 36 | 278279 |
| 1993 | 34200 | 36 | 261273 |
| 2003 | 34200 | 36 | 264276 |
| C TRACTOR SPRINGS |  |  |  |
| C FRONT TYRES |  |  |  |
| 2013 | 301003 | $\begin{array}{ll}35 & 282288\end{array}$ |  |
| 2023 | 30100 | 342880 |  |
| 2033 | 301003 | $\begin{array}{llll}35 & 285 & 291\end{array}$ |  |
| 2043 | 301003 | 342910 |  |
| C REAR TYRES |  |  |  |
| 2053 | 301003 | $\begin{array}{lll}35 & 280 & 286\end{array}$ |  |
| 2063 | 30100 | $34 \quad 2860$ |  |
| 2073 | 301003 | $\begin{array}{llll}35 & 281 & 287\end{array}$ |  |
| 2083 | 301003 | 342870 |  |
| 2093 | 301003 | 35283289 |  |
| 2103 | 301003 | 342890 |  |
| 2113 | 301003 | 35284290 |  |
| 2123 | 301003 | 342900 |  |
| C FRONT | SUSPENS | SIONS |  |
| 2133 | 301003 | 33265301 |  |
| 2143 | 301003 | 33277302 |  |
| C REAR | SUSPENS | SIONS |  |
| 2153 | 30100 | 32258299 |  |
| 2163 | 30100 | 32270300 |  |
| C FITH | WHEEL B | BEAMS |  |
| 2173 | 34200 | 36 | 292293 |
| 2183 | 34200 | 36 | 293294 |
| C TRACTOR AXLES 29329 |  |  |  |
| C REAR |  |  |  |
| 2193 | 34200 | 37 | 280281 |
| 2203 | 34200 | 37 | 281299 |
| 2213 | 34200 | 37 | 299300 |
| 2223 | 34200 | 37 | 300283 |
| 2233 | 34200 | 37 | 283284 |
| C FRONT |  |  |  |
| 2243 | 34200 | 37 | 282301 |



```
285 291 13
280 286 13
281 287 13
283 289 13
284 290 13
C TRACTOR SUSPENSION
265 301 13
277 302 13
299 258 13
270 300 13
C FIFTH WHEEL
260 292 123456
272 294 123456
C FIFTH WHEEL/PLATE
29243 2
293 40 13
294 37 2
C ROLL BAR NODES
305 301 123
306 302 123
307 257 123
308 269 123
309 299 123
310 300 123
303 266 123
304 278 123
RESTRAINTS
NODE.NUMB DIRECTION
C CHASSIS TYRES
231 2456
232 2456
233 2456
234 2456
235 2456
236 2456
C TRACTOR TYRES
288 2456
291 2456
286 2456
2872456
2892456
290 2456
BEAMS
SECTION.NUMB MATE.NUMB IYY IZZ AXIS.NUMB
TORSIONAL.CONSTANT AREA
\begin{tabular}{|c|c|c|c|c|c|}
\hline & \[
\begin{gathered}
1 \\
3250
\end{gathered}
\] & 3.663E06 & 13.677E06 & 1 & 92.5E03 \\
\hline 2 & \[
2800
\] & 1.464 EO 6 & 9.193 E 06 & 1 & \(93.333 \mathrm{E03}\) \\
\hline 3 & \[
\begin{gathered}
1 \\
1800
\end{gathered}
\] & 2.46E06 & \(0.393 E 06\) & 1 & 60.0E03 \\
\hline 4 & \[
\begin{gathered}
1 \\
6570
\end{gathered}
\] & 0.571 E06 & 5.343E06 & 1 & 77.666E03 \\
\hline 5 & \[
\begin{gathered}
1 \\
3625
\end{gathered}
\] & 0.189 E 06 & \(6.351 E 06\) & 1 & 755.208E03 \\
\hline 6 & \[
\begin{gathered}
1 \\
1225
\end{gathered}
\] & 0.813 E 06 & 2.686E06 & 1 & 10.208E03 \\
\hline 7 & 1 & 0.388 E 06 & 7.801E06 & 1 & \(43.520 E 03\) \\
\hline
\end{tabular}
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[^0]:    'Line 45
    'PAYLOAD CENTRE OF GRAVITY Z COORDINATES STORED IN PCGZ£(8) IN mm
    -
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[^1]:     ' FORCES IN FCH2 ARE DETERMINED FROM CHASSIS FORCES ' STORED IN CHL\# (15) AND UNSPRUNG AXLE FORCES.
    1

