Singular value decomposition-based robust cubature Kalman filtering for an integrated GPS/SINS navigation system

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Abstract: A new nonlinear robust filter is proposed in this paper to deal with the outliers of an integrated GPS/SINS navigation system. The influence of different design parameters for H_∞ cubature Kalman filter is analysed. It is found that when the design parameter is smaller, the robustness of the filter is stronger. However, the design parameter is easily out of step in the Riccati equation and the filter is easy to diverge. In this respect, a singular value decomposition algorithm is employed to replace Cholesky decomposition in the robust cubature Kalman filter. On the wider conditions for the design parameter, the new filter is more robust. The testing results demonstrate that the proposed filter algorithm is more reliable and effective in dealing with the outliers in the data sets produced by the integrated GPS/SINS system.

K E Y W O R D S

1. Singular Value Decomposition. 2. Cubature Kalman Filter. 3. Integrated Navigation System. 4. Nonlinear Filter

1. INTRODUCTION. The integration of strap-down inertial navigation system (SINS) and the Global Positioning System (GPS) has been widely utilised for real-time navigation, mobile mapping, Location-based Services, transport and many other applications. The Kalman filtering (KF) is the most common technique for data fusion of GPS and SINS (Grejner-Brzezinska et al., 1998)**.** However, the operations of the KF rely on the proper definition of dynamic and stochastic models and the standard KF can only be used to deal with linear systems (Yi and Grejner-Brzezinska, 2006). Furthermore, due to the nonlinear characteristics of the low-cost SINS error model and the uncertainty of the stochastic model, the KF estimation is not optimal and may produce an unreliable result, sometimes or even leads to filtering divergence (Geng and Wang, 2008).

Over the past few decades, nonlinear KF algorithms have been intensively investigated to deal with the nonlinear issues of low-cost SINS (Gustasson, 2010; Wendel et al., 2006). Recently proposed cubature Kalman filtering (CKF) , is a Gaussian approximation to the Bayesian filtering. It

has more accurate filtering performance than traditional method and less computational cost (Arasaratnam and Haykin, 2009). The CKF was introduced to deal with the data fusion of the integrated GPS/INS system (Sun and Tang, 2012). However, the standard CKF may still face difficulty in provision of stable results and cannot deal with the outliers in the measurements effectively. The robust cubature Kalman filtering (RCKF) based on an H∞ filter was proposed for integrated GPS/INS navigation applications (Liu et al., 2010).

The RCKF algorithm makes use of the H∞ robust filter to overcome the interference of outliers. For the H∞ robust filter, the given restrict parameter γ is used to show the bound level and assess the robustness of the H∞ filter to the uncertain interference (Simon, 2006). The parameter γ can be chosen appropriately according to the detailed performance index and there is a balance between system average accuracy and its robustness performance (Einicke and White, 1999). Certainly, γ must be larger than a positive number to output a normal filtering result. It is found that the smaller the value of γ the stronger the robustness of the filter is (Liu et al., 2010). However, a disproportionately small value can easily lead to a non-positive definite state covariance and cause filter divergence. Based on the error variance constraints or minimum variance principle, the modified robust filters were proposed (Hung and Yang, 2003; Shaked and Souza, 1995). A priori robust filter for linear uncertain systems was presented by (Yahali and Shake, 1996) which guarantees a certain bound on the error covariance of the estimate. Furthermore, the nonlinear robust Kalman filtering problem with norm-bound parameter uncertainties was also studied by Xiong et al (Xiong et al., 2012). However, how to improve the performance of a robust filter under a small given parameter γ was rarely investigated.

In this paper, the authors compare the performance of a robust cubature Kalman filter for the integrated GPS/SINS navigation applications under different given parameters γ. In order to maintain a high level of numerical stability, a singular value decomposition (SVD) algorithm is introduced to replace Cholesky decomposition in the RCKF. Land vehicle tests have been carried out to compare the performance of this algorithm with other cubature Kalman filter algorithms. The results show that the SVD based robust cubature Kalman filter (SVD-RCKF) algorithm can significantly improve the filtering stability and has better robustness to the impact of outliers.

The structure of this paper is as follows. Section 2 includes the nonlinear description of the H∞ filter and the calculation steps of RCKF based on SVD is presented in Section 3. Section 4 lists the formulas of the system and observation equations of the GPS/SINS system. Two test results and data analysis are given in Section 5. The final part of the paper is the preliminary conclusions attained through this study.

2. H∞ FILTER AND ITS NONLINEAR DESCRIPTION.

2.1 *The Principle of an H∞ filter.*   An H∞ filter is a typical implementation of the robust filtering theory (Simon, 2006). It defines a cost function as follows:

$$J = \frac{\sum_{k=1}^{N} \left\| \boldsymbol{x}_k - \hat{\boldsymbol{x}}_k \right\|^2}{\left\| \boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0 \right\|_{\boldsymbol{P}_0^{-1}}^2 + \sum_{k=1}^{N} \left(\left\| \boldsymbol{w}_k \right\|_{\boldsymbol{Q}_k^{-1}}^2 + \left\| \boldsymbol{v}_k \right\|_{\boldsymbol{R}_k^{-1}}^2 \right)} \tag{1}$$

where N is the total number of filtering time limit and $k = 1, 2, \cdots N$. $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are the unrelated system noise and measurement noise, and $\boldsymbol{Q}_k$ and $\boldsymbol{R}_k$ are their covariance matrices respectively. $\boldsymbol{x}_0$ is the initial value of the system state vector $\boldsymbol{x}_k$ while its covariance is $\boldsymbol{P}_0$,   whilst $\hat{\boldsymbol{x}}_0$ and $\hat{\boldsymbol{x}}_k$ are the

estimated values of x_0 and x_k. In this equation, $\left\|x_0 - \hat{x}_0\right\|_{P_0^{-1}}^2 = (x_0 - \hat{x}_0)^T P_0^{-1}(x_0 - \hat{x}_0)$. Other similar terms can be obtained recursively.

The goal of an optimal H∞ filter is to find an estimate x_k that minimizes J, under the condition $\hat{x}_k = \arg\min\left\|J\right\|_\infty$. Normally, it is difficult to get the analytical solution of an optimal H∞ filter problem. Therefore, we need find a suboptimal iterative algorithm. We can set a threshold value γ, which meets $\left\|J\right\|_\infty = \sup J \le \gamma^2$ ($\left\|J\right\|_\infty$ refers to the infinity-norm of J and $\sup$ is the supremum of a set). The threshold value γ is equivalent to the follow Riccati inequality (Chen and Yuan, 2009):

$$P_k^{-1} + H_k^T H_k - \gamma^{-2} L_k^T L_k > 0 \tag{2}$$

where, L_k is a user-defined matrix (assumed to be full rank). If we want to directly estimate x_k (as in the Kalman filtering), then we set $L_k = I$. H_k is the matrix of measurement function after linearization, and P_k is the covariance matrix of x_k.

For an H∞ filter, the estimation error in the most unfavourable conditions is controlled by the threshold value γ that is called the designed restrict parameter. When the designed restrict parameter γ is smaller, the robustness of a filter is stronger. When γ approaches infinity, the H∞ filter is approaching to the standard Kalman filter.

2.2 *The nonlinear description of an H∞ filter.* In order to apply the H∞ filter in the nonlinear models, the recursive Riccati equation for a linear model is transformed to implement nonlinear filter. Due to $P_{k/k-1} H_k^T = P_{xz,k}$, $H_k P_{k/k-1} H_k^T = P_{zz,k} - R_k$ (Yan et al., 2008), the formula for computing the state vector covariance matrix of the nonlinear H∞ filter can be modified as follows:

$$P_k = P_{k/k-1} - \begin{bmatrix} P_{xz,k} & P_{k/k-1} \end{bmatrix} \begin{bmatrix} P_{zz,k} - R_k + I & P_{xz,k}^T \\ P_{xz,k} & P_{k/k-1} - \gamma^2 I \end{bmatrix}^{-1} \begin{bmatrix} P_{xz,k}^T \\ P_{k/k-1}^T \end{bmatrix} \tag{3}$$

where, innovation covariance matrices $P_{zz,k}$ and cross-covariance matrix $P_{xz,k}$ can be gained by the nonlinear filtering methods such as cubature Kalman filter and unscented Kalman filter.

For the nonlinear models, we can calculate the mean and covariance of the state vectors by cubature point transformation instead of Taylor expansion as discussed by Arasaratnam (2009) and then we can obtain the nonlinear H∞ robust filter based on cubature Kalman filter.

3. ROBUST CUBATURE KALMAN FILTER BASED ON SVD. Based on a cubature Kalman filter frame, we developed a new filter algorithm by introducing an H∞ robust filter. It is called as a robust cubature Kalman filter (RCKF). However, after many times of iteration in RCKF, $P_{k/k-1}$ and P_k are very easy to lose their positive definiteness and this will consequently lead to the instability of the numerical value. Meanwhile a much smaller restrict parameter γ may lead to non-positive definiteness of $P_{k/k-1}$ and P_k. Therefore singular value decomposition (Gao et al., 2010) is applied in the calculation of the covariance matrix for RCKF instead of Cholesky decomposition in this paper.

Considering the follow discrete nonlinear system

$$x_k = f(x_{k-1}) + w_{k-1} \tag{4}$$

$$z_k = h(x_k) + v_k \tag{5}$$

where x_k and z_k are the state and measurements of the dynamic system; $f(\cdot)$ and $h(\cdot)$ are known nonlinear functions; w_{k-1} and v_k are the independent process and measurement Gaussian noise

sequences with zero means and co-variances $\boldsymbol{Q}_{k-1}$ and $\boldsymbol{R}_k$, respectively. Then the procedure of the robust cubature Kalman filter based on singular value decomposition (SVD-RCKF) can be expressed as follows:

1) Calculate the basic cubature sampling points $\boldsymbol{\xi}_i$ and weights ω_i based on the cubature rule.

$$\xi_i = \sqrt{\frac{m}{2}}[1]_i, \omega_i = \frac{1}{m}, i = 1, 2, \cdots m, m = 2n_x \tag{6}$$

where m is the number of cubature points, and n_x is dimension of state vector. [1] represents a

set that is similar to the following set of points: $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right\}$. Here, the generator is

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

2) Time update:

Factorize $\boldsymbol{P}_{k-1}$ based on SVD, we can get

$$\boldsymbol{P}_{k-1} = \boldsymbol{U} \begin{bmatrix} \boldsymbol{S} & 0 \\ 0 & 0 \end{bmatrix} \boldsymbol{V}^T$$

where $\boldsymbol{S} = diag\{s_1, s_2, \cdots s_n\}$ is a diagonal matrix. Normally, the co-variance matrix $\boldsymbol{P}_{k-1}$ is a symmetric one, so its eigenvalues are $s_i^2 (i = 1, 2, \cdots n)$ and $\boldsymbol{U} = \boldsymbol{V}$. Then the factorization formula can be described as follows:

$$\boldsymbol{P}_{k-1} = \boldsymbol{U}_{k-1} \boldsymbol{S}_{k-1} \boldsymbol{V}_{k-1}^T \tag{7}$$

Evaluate the cubature points $\boldsymbol{X}_{i,k-1}$

$$\boldsymbol{X}_{i,k-1} = \boldsymbol{U}_{i,k-1} \sqrt{s_i} \xi_i + \hat{\boldsymbol{x}}_{k-1} \tag{8}$$

Evaluate the propagated cubature points $\boldsymbol{X}_{i,k}^*$

$$\boldsymbol{X}_{i,k}^* = f(\boldsymbol{X}_{i,k-1}) \tag{9}$$

Estimate the predicted state $\bar{\boldsymbol{x}}_k$ and error covariance $\boldsymbol{P}_{k/k-1}$

$$\begin{cases} \bar{\boldsymbol{x}}_k = \sum_{i=1}^m \omega_i \boldsymbol{X}_{i,k}^* \\ \boldsymbol{P}_{k/k-1} = \sum_{i=1}^m \omega_i \boldsymbol{X}_{i,k}^* \boldsymbol{X}_{i,k}^{*T} - \bar{\boldsymbol{x}}_k \bar{\boldsymbol{x}}_k^T + \boldsymbol{Q}_{k-1} \end{cases} \tag{10}$$

3）Measurement update

SVD factorize

$$\boldsymbol{P}_{k/k-1} = \boldsymbol{U}_{k/k-1} \boldsymbol{S}_{k/k-1} \boldsymbol{V}_{k/k-1}^T \tag{11}$$

Calculate the cubature points

$$\boldsymbol{X}_{i,k/k-1} = \boldsymbol{U}_{i,k/k-1} \sqrt{s_i} \xi_i + \bar{\boldsymbol{x}}_k \tag{12}$$

Calculate the propagated cubature points of measurement vector $\boldsymbol{Z}_{i,k}$

$$\boldsymbol{Z}_{i,k} = h(\boldsymbol{X}_{i,k/k-1}) \tag{13}$$

Calculate the predicted measurement vector $\bar{z}_k$, innovation covariance matrices $\boldsymbol{P}_{zz,k}$ and cross-covariance matrix $\boldsymbol{P}_{xz,k}$

$$\begin{cases} \bar{z}_k = \sum_{i=1}^{m} \omega_i Z_{i,k} \\ P_{zz,k} = \sum_{i=1}^{m} \omega_i Z_{i,k} Z_{i,k}^{~T} - \bar{z}_k \bar{z}_k^{T} + R_k \\ P_{xz,k} = \sum_{i=1}^{m} \omega_i X_{i,k} Z_{i,k}^{~T} - \bar{x}_k \bar{z}_k^{T} \end{cases} \qquad (14)$$

Calculate the gain matrix K_k, updated state $\hat{x}_k$ and the corresponding covariance P_k

$$K_k = P_{xz,k} / P_{zz,k} \qquad (15)$$

$$\hat{x}_k = \bar{x}_k + K_k (z_k - \bar{z}_k) \qquad (16)$$

$$P_k = P_{k/k-1} - K_k P_{zz,k} K_k^{T} \qquad (17)$$

Considering the state covariance update formula of the H∞ robust filter:

$$P_k = P_{k/k-1} - \begin{bmatrix} P_{xz,k} & P_{k/k-1} \end{bmatrix} \begin{bmatrix} P_{zz,k} - R_k + I & P_{xz,k}^{T} \\ P_{xz,k} & P_{k/k-1} - \gamma^2 I \end{bmatrix}^{-1} \begin{bmatrix} P_{xz,k}^{T} \\ P_{k/k-1}^{T} \end{bmatrix} \qquad (18)$$

Formulas (4)-(16) and (18) constitute the calculation procedure of the robust cubature Kalman filter based on SVD (SVD-RCKF).

4. THE DYNAMIC AND OBSERVATION EQUATIONS OF A GPS/SINS SYSTEM. The loosely coupled GPS/SINS system is adopted. The state vectors are composed of the position and velocity error in earth-centered and earth-fixed frame (ECEF frame, e frame), attitude error between computer e frame and platform e' frame, gyros and accelerometers drift error in body frame(b frame), which can be expressed as (Petovello, 2003):

$$x_k = \begin{bmatrix} \Delta R^e & \Delta V^e & \varphi^e & \nabla^b & \varepsilon^b \end{bmatrix}^{T}.$$

The nonlinear differential error model for low-cost SINS is as follows:

$$\begin{cases} \Delta \dot{R}^e = \Delta V^e \\ \Delta \dot{V}^e = (I_{3\times3} - C_{e'}^e) f^{e'} + C_b^{e'} \nabla^b - 2\Omega_{ie}^e \Delta V^e \\ \dot{\varphi}^e = (I_{3\times3} - C_e^{e'}) \omega_{ie}^e - C_b^{e'} \varepsilon^b \\ \dot{\nabla}^b = 0 \\ \dot{\varepsilon}^b = 0 \end{cases} \qquad (19)$$

where ΔR^e and ΔV^e are the position and velocity error in the e frame, $\dot{\varphi}^e$ is the attitude error between computer e frame and platform e' frame, $I_{3\times3}$ is the 3×3 unit matrix, $C_{e'}^e$ is the rotation matrix between computer e frame and platform e' frame; $C_b^{e'}$ is the rotation matrix between body frame and platform e' frame; Ω_{ie}^e is the skew symmetric matrix of earth rotation rate ω_{ie}^e; ε^b and ∇^b are gyros and accelerometers drift error in the body frame.

The dynamical model of a loosely coupled GPS/INS system can be expressed as follows

$$\dot{x}_k = f(x_{k-1}) + w_k$$

Generally, the measurement model can be expressed as

$$z_k = \begin{bmatrix} \hat{R}_{INS}^e - \hat{R}_{GPS}^e \\ \hat{V}_{INS}^e - \hat{V}_{GPS}^e \end{bmatrix} = H x_k + v_k = \begin{bmatrix} I_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{bmatrix} x_k + v_k \qquad (20)$$

where $\hat{R}_{INS}^e$ and $\hat{V}_{INS}^e$ are computed position and velocity vectors by SINS in e frame, $\hat{R}_{GPS}^e$ and $\hat{V}_{INS}^e$ are the ones output by GPS, and v_k is the noise vector.

5. TEST CASES AND DATA ANALYSIS.   To demonstrate the performance of the SVD-RCKF algorithm, data was collected under real world conditions with a probe vehicle. The tests were performed in Xuzhou (China) and Nottingham (UK) respectively.

5.1 *Case study 1.* The first dataset was collected in China University of Mining and Technology (CUMT), Xuzhou, China in January 2011. The test had employed two GPS receivers and one low-cost IMU (SPAN-CPT). One of the GPS receivers was set on the rooftop as the reference station, and another one was placed on the top of the testing vehicle together with the IMU. The data was logged for post processing. The SPAN-CPT IMU consists of three-axis open-loop fiber optic gyroscope and three-axis MEMS accelerometers. The technical data is shown in Table 1. The specified parameters were used in setting up the Q estimation in filtering process. Figure 1 shows the ground track of the test vehicle. The update rate of INS is 100Hz and the one of GPS is 1Hz. The high accuracy double difference carrier-phase GPS position results are used as reference value.

Table 1. IMU technical specifications

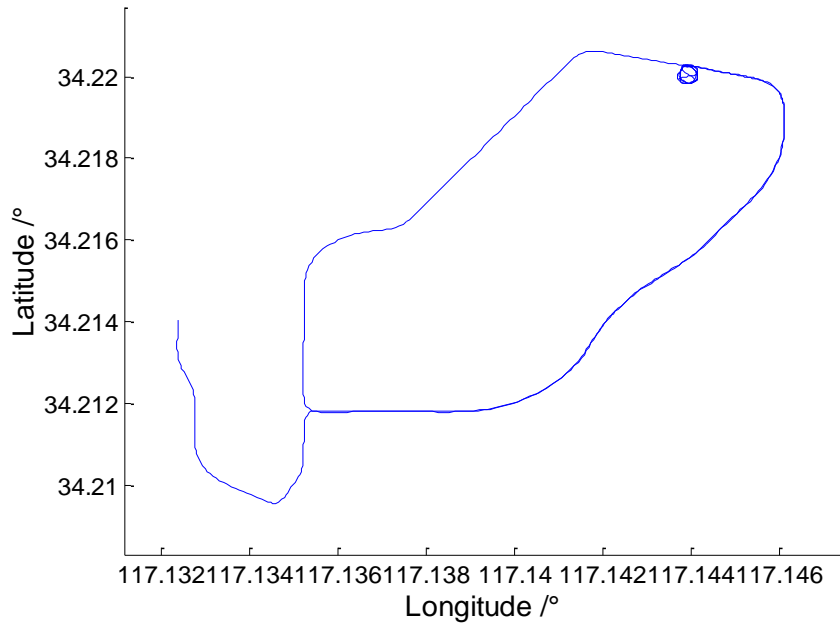|	SPAN-CPT	SPAN-LCI
Gyro Rate Bias	20 deg/hr	<1.0 deg/hr
Gyro Scale Factor	1500ppm	500ppm
Angle RW	0.067 deg/rt-hr	<0.15 deg/rt-hr
Acc. Bias	50mg	<1.0 mg
Acc. Scale Factor	4000ppm	<1000ppm
Velocity RW	55ug/rt-Hz	——

Figure 1. The vehicle trajectory of Case 1

The test results plotted in Figure 2 are based on the cubature Kalman filter. It is worth noting here that the results contain some outlier values due to the vehicle driving over the speed bumps. These indicate that the robustness of CKF needed to be improved.

Figure 3 shows the positioning error when using the robust cubature Kalman filter and the parameter γ is set as 2. Comparing Figures 2 and 3, it is apparent how effective the robust filter is. The error amplitude in Figure 3 is reduced by improving the robustness of the filter. Figure 4 is the position error result for the SVD based robust cubature Kalman filter (SVD-RCKF) with γ set as 2. Figures 3 and 4 show the almost same results when the parameter γ is set as 2. The reason is that

the Cholesky decomposition and SVD method can get a similar effect when the RCKF is relatively stable. Table 2 shows the statistic information for the different filter algorithms.

The performance of robust cubature Kalman filter with difference values of the parameter γ was further analysed. Table 3 presents the statistic information. For the RCKF, the larger value the parameter γ is, the less the robustness of the filter. Compared with the result of RCKF on the value of 2500, the result on the value of 1.5 just improved the performance by 6.5%, 7.1% and 3.7% in three directions. This indicates that RCKF can get the jarless robustness performance if the different design parameters are within limits.
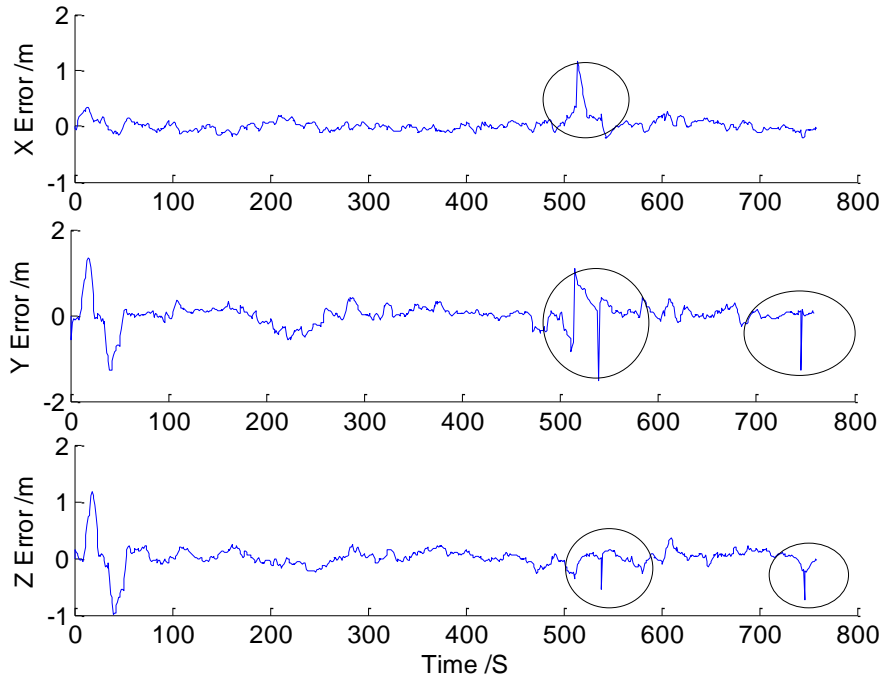

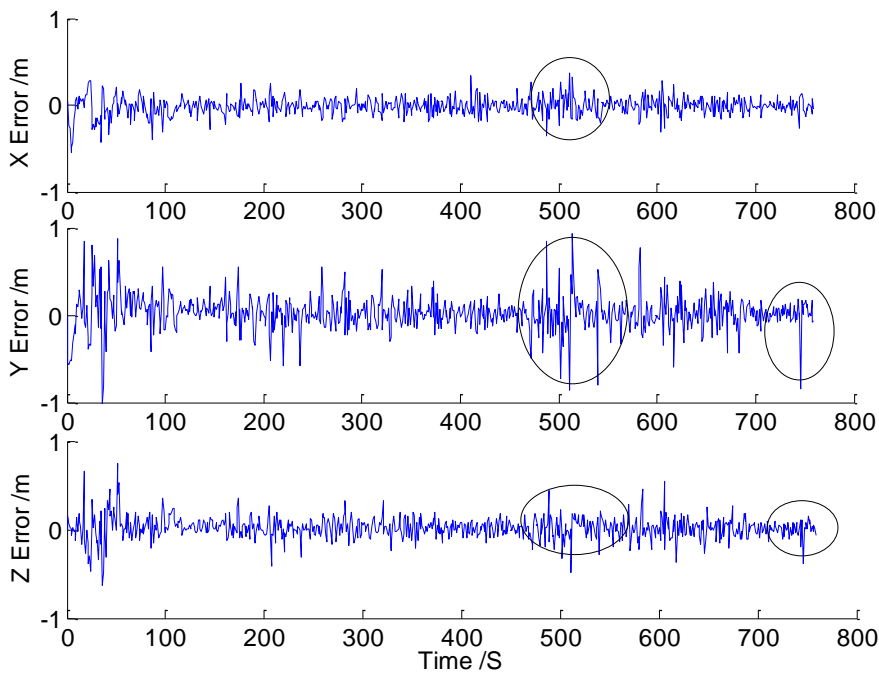Figure 2. The position error of CKF in Case 1

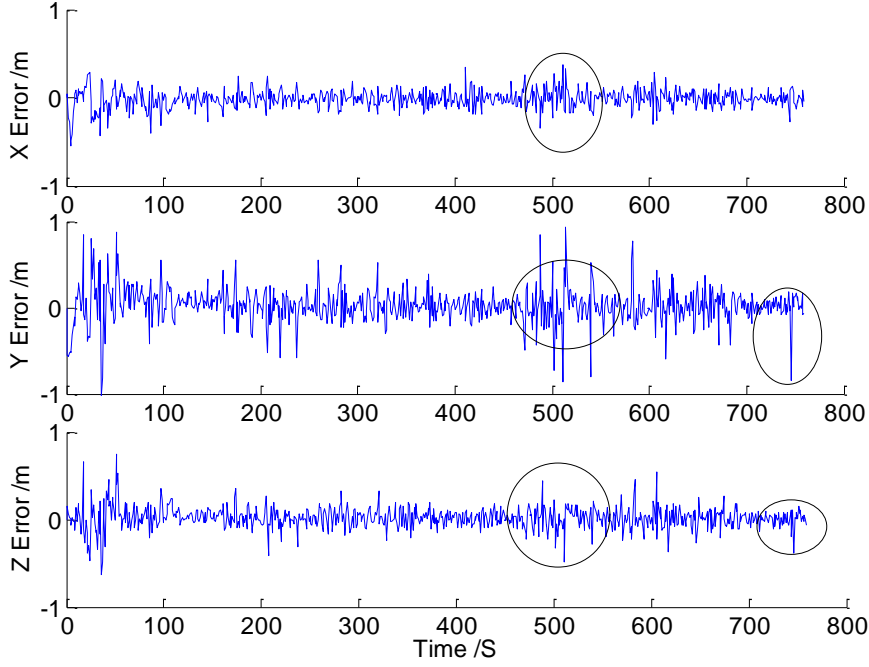Figure 3. The position error of RCKF ($\gamma = 2$) in Case 1

Figure 4. The position error of SVD-RCKF ($\gamma = 2$) in Case 1

Table 2. Position errors of different filters in Case 1($\gamma = 2$)

	RMS of Position Error (m)		
	X	Y	Z
CKF	0.129	0.229	0.116
RCKF	0.091	0.176	0.108
SVD- RCKF	0.091	0.176	0.108

Table 3. The position errors of different strict parameter in Case 1

Restrict parameter γ		0.5	0.7	1.0	1.4	1.5	2.0	2.5	25	250	2500
SVD- RCKF (Position Error in m)	X	0.850	0.554	0.04	0.088	0.087	0.091	0.092	0.093	0.093	0.093
	Y	1.013	0.394	0.069	0.167	0.168	0.176	0.178	0.182	0.182	0.182
	Z	0.851	0.379	0.053	0.105	0.105	0.108	0.109	0.109	0.109	0.109
RCKF (Position Error in m)	X	---	---	---	---	0.087	0.091	0.092	0.093	0.093	0.093
	Y	---	---	---	---	0.168	0.176	0.178	0.181	0.181	0.181
	Z	---	---	---	---	0.105	0.108	0.108	0.109	0.109	0.109

From the fundamental of the H$_\infty$ robust filter, we know that the smaller the restrict parameter γ the stronger the robustness of the filter. To get more robust performance, the case studies with smaller γ values were compared. The result is shown in Table 3. We can find that the RCKF cannot work well sometimes. The reason is that after many times of iteration in RCKF with a much smaller restrict parameter γ, $\boldsymbol{P}_{k/k-1}$ and $\boldsymbol{P}_k$ lose their positive definiteness and this consequently leads to the instability of the numerical value. In order to further improve the numerical stability of RCKF, the SVD-RCKF method is proposed. Figure 5 gives the position errors when using the new filter algorithm with γ set as 1. Compared with the result of RCKF with γ set as 2, the result of SVD-RCKF on the value of 1 improved the performance by 56.0%, 60.8% and 50.9% in three directions.

As expected, the results are better than the other algorithms previously mentioned. However, it was found that the relationship between γ and robustness of a filter does not exist anymore if γ is much smaller. That is because there is no solution of Riccati inequality if the γ is too smaller. For instance, the estimation error increases when γ is set smaller than 0.7 shown in Table 3.
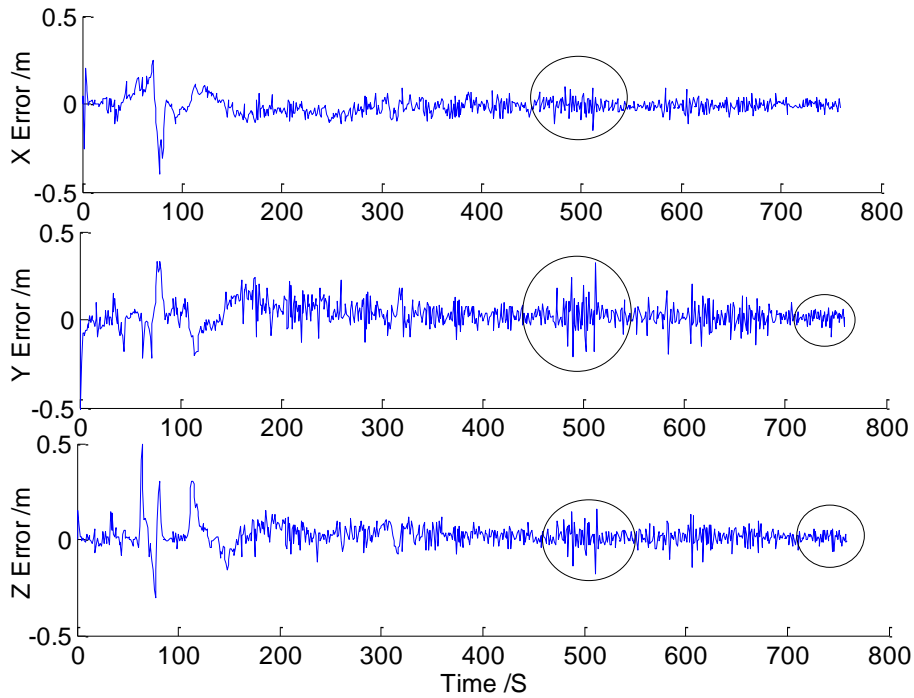

Figure 5.   The position error of SVD-RCKF ($\gamma = 1$) in Case 1

5.2 *Case study 2.* The second test was carried out in Nottingham, UK in November 2013. The test setup was similar with the first one. A GNSS antenna, a GNSS receiver and a SPAN-LCI IMU were mounted in a van and data was logged from the receiver's serial ports to a laptop for storage and processing. The vector between the IMU centre and GPS antenna was accurately surveyed using a total station and is considered known to within 1cm. A base station was set up on the roof of the NGB building to provide DGPS and RTK corrections. The update rate of INS is 200Hz and the one of GPS is 1Hz. The average baseline length was less than 3 km for the test. Figure 6 is the test trajectory and Figure 7 is a photograph taken for the van. The high accuracy real-time output results of a SPAN-CIMU system are used as the reference value and the double difference code GPS position and velocity results are used as the input measurements.

The position error of CKF is shown in Figure 8. It is indicated that there are many epochs of outlier data and the amplitude is a little big. The robustness of CKF should be enhanced to tackle the problem of gross errors or an inaccurate system.

Figure 9 shows the position error when using the SVD based robust cubature Kalman filter (SVD-RCKF) and the parameter γ is set as 3. As we can see from Figure 8 and Figure 9, the error amplitude in Figure 9 is reduced through improving the robustness of the filter. As the same as Case 1, the position error of RCKF is almost the same as SVD-RCKF, so the corresponding plot of RCKF is omitted in Case 2 for keeping the description concisely. Table 4 shows the statistic information of plots.
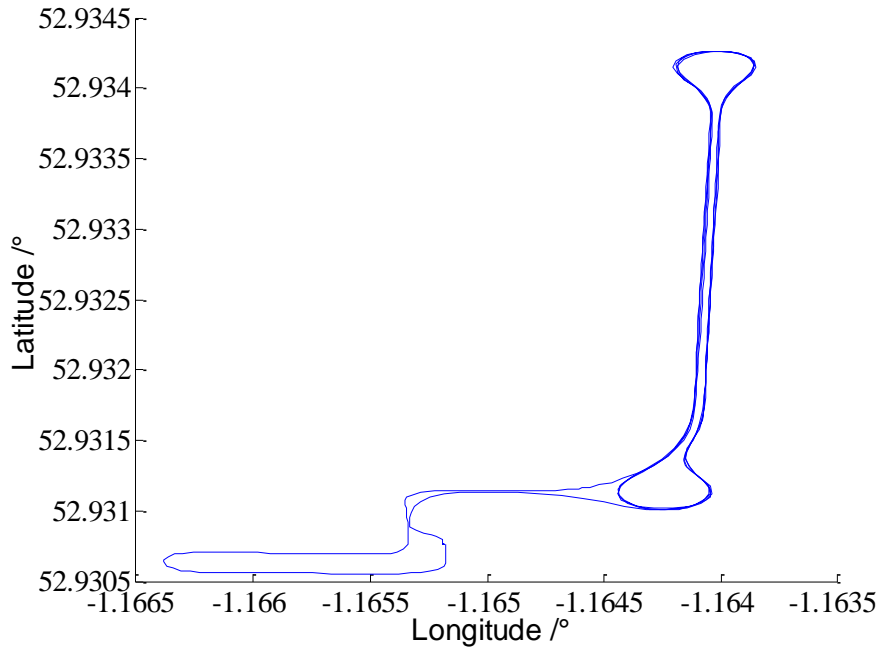
Figure 6. The vehicle trajectory of Case 2

Figure 7. The testing van in Case 2

As in Case 1, the performance of robust cubature Kalman filter with different values of the parameter γ was also compared. The result is very similar with Case 1. Table 5 shows the corresponding statistic result. From Table 5, we can know that the larger value the parameter γ is, the filter can get the worse robustness. Compared with the result of RCKF on the value of 5000, the result on the value of 3 just improved the performance by 7.0%, 0.0% and 6.2% in three directions. Similar with Case 1, this result demonstrates that RCKF can get the jarless robustness performance if the different design parameter is within limits.

Table 4 Position errors of different filters in Case 2 ($\gamma = 3$)

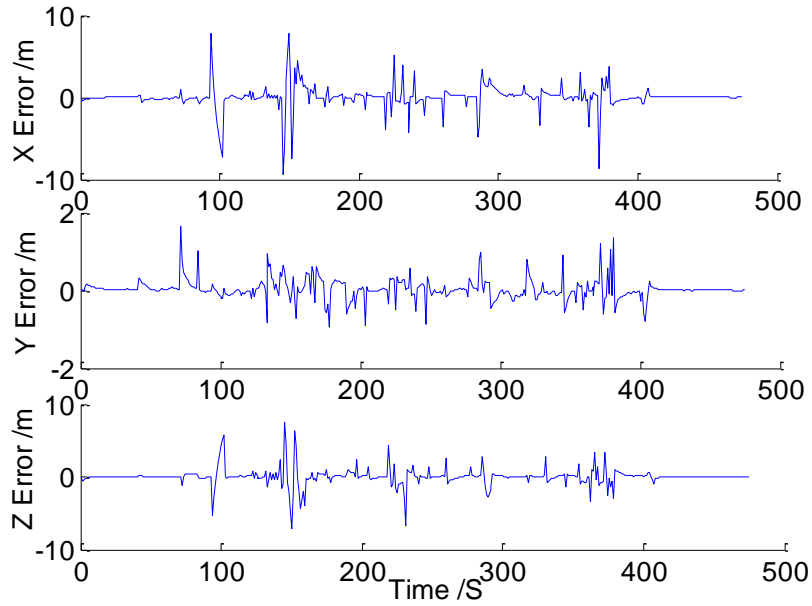|	RMS of Position Error (m)		
	X	Y	Z
CKF	1.430	0.264	1.185
RCKF	0.066	0.016	0.061
SVD- RCKF	0.066	0.016	0.061

Figure 8. The position error of CKF in Case 2
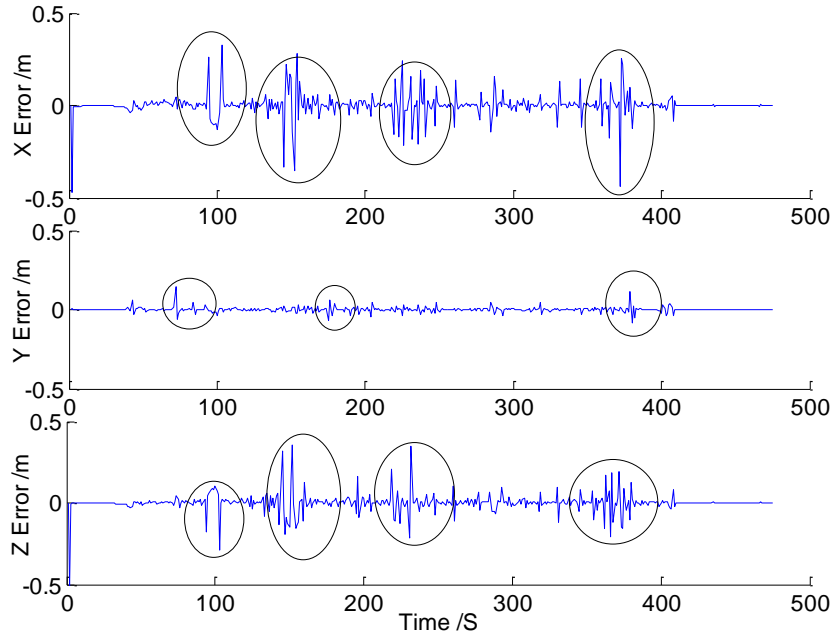

Figure 9. The position error of SVD-RCKF (γ = 3) in Case 2

To achieve more robust performance, the case studies with smaller γ were compared. The result of case 2 is shown in Table 5. We can find that the RCKF cannot work when the parameter γ is smaller than 2. The reason is that after many times of iteration in RCKF with a much smaller restrict parameter γ, $\boldsymbol{P}_{k/k-1}$ and $\boldsymbol{P}_k$ lose their positive definiteness and this consequently leads to the instability of the numerical value. As the same as Case 1, the SVD-RCKF is introduced to further improve the numerical stability of RCKF. Figure 10 gives the position errors when using the new filter algorithm with γ set as 1.44. Compared with the result of RCKF with γ set as 3, the result of SVD-RCKF on the value of 1.44 improved much the performance by 30.3%, 31.3% and 26.2% in three directions. As expected, the results are better than the other algorithms previously mentioned. However, the same problem as Case 1 is that the robust filter will diverge when the parameter γ is

set too small. That is because there is no solution of Riccati inequality if the γ is too smaller. In this case study, the estimation error shown in Table 5 increases when γ is set smaller than 1.44.

Different from Case 1, the outlier maybe still exist in the position error series. The reason could be that the test van turned the corners sharply in fairly high speed.
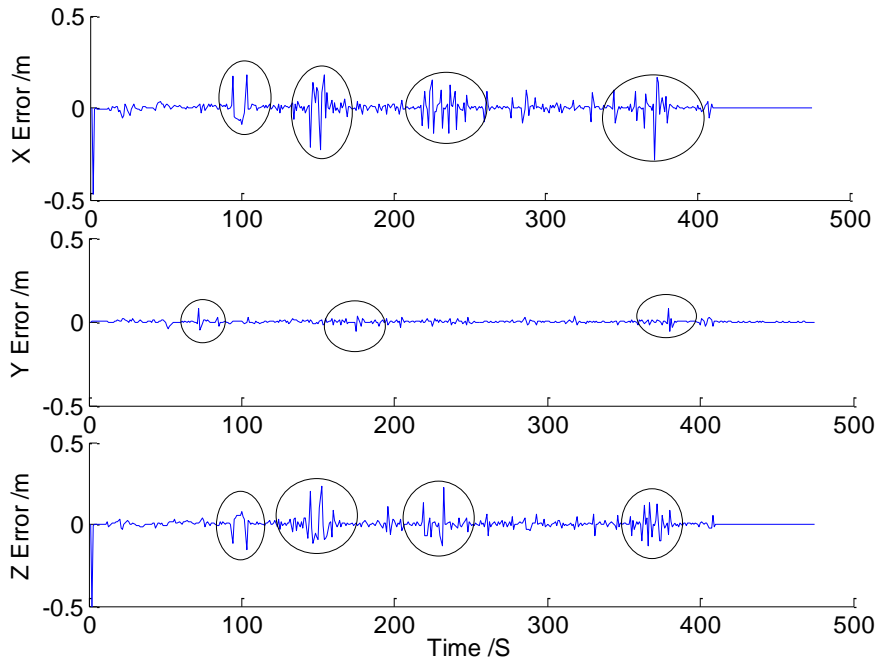

Figure 10. The position error of SVD-RCKF($\gamma =1.414$) in Case 2

Combining the analysis of Case 1 and Case 2, we know that the nonlinear robust filtering could gain a stable performance if the restrict parameter is assigned properly. If a more robust performance was expected, the restrict parameter should be assigned adventurously. However, due to the existence criteria of the Riccati inequality, the restrict parameter cannot be assigned unboundedly. From the analysis of two cases, the restrict parameter should equal or bigger than 1, and the optimal value should vary in different cases. Therefore, how to assign the parameter properly is the future work for nonlinear robust filter.

Table 5. The position errors of different strict parameter in case 2

Restrict parameter γ		0.8	0.866	1	1.414	1.732	2	3	4	5	50	500	5000
SVD- RCKF (Position Error /m)	X	1.576	0.375	0.112	0.046	0.055	0.059	0.066	0.068	0.069	0.071	0.071	0.071
	Y	1.632	0.362	0.118	0.011	0.016	0.018	0.016	0.016	0.016	0.016	0.016	0.016
	Z	2.630	0.333	0.277	0.045	0.052	0.056	0.061	0.063	0.063	0.065	0.065	0.065
RCKF (Position Error /m)	X	---	---	---	---	---	---	0.066	0.068	0.069	0.071	0.071	0.071
	Y	---	---	---	---	---	---	0.016	0.016	0.016	0.017	0.017	0.017
	Z	---	---	---	---	---	---	0.061	0.063	0.063	0.065	0.065	0.065

6. CONCLUSIONS.   The robust cubature Kalman filter based on the H$_\infty$ filter is very effective for detecting outlier data in GPS/SINS integration system. It has been found that the smaller restrict parameter γ can improve the overall performance of RCKF. However, it is also apparent that RCKF is easy to diverge if the parameter is too smaller. The robust cubature Kalman filter based on SVD is proposed to maintain stronger robustness on the wider conditions for the design parameters. Two case studies indicate that the new filtering method is effective. It is also anticipated that more work

needs to be carried out about how to set the optimal parameter and quantify the stability of SVD-RCKF.

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