DE GRUYTER
Journal of Applied Geodesy 2014; 8(4): 265–272

Research Article

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# Optimal GPS/accelerometer integration algorithm for monitoring the vertical structural dynamics

**Abstract:** The vertical structural dynamics is a crucial factor for structural health monitoring (SHM) of civil structures such as high-rise buildings, suspension bridges and towers. This paper presents an optimal GPS/accelerometer integration algorithm for an automated multi-sensor monitoring system. The closed loop feedback algorithm for integrating the vertical GPS and accelerometer measurements is proposed based on a 5 state extended KALMAN filter (EKF) and then the narrow moving window Fast Fourier Transform (FFT) analysis is applied to extract structural dynamics. A civil structural vibration is simulated and the analysed result shows the proposed algorithm can effectively integrate the online vertical measurements produced by GPS and accelerometer. Furthermore, the accelerometer bias and scale factor can also be estimated which is impossible with traditional integration algorithms. Further analysis shows the vibration frequencies detected in GPS or accelerometer are all included in the integrated vertical deflection time series and the accelerometer can effectively compensate the short-term GPS outages with high quality. Finally, the data set collected with a time synchronised and integrated GPS/accelerometer monitoring system installed on the Nottingham Wilford Bridge when excited by 15 people jumping together at its mid-span are utilised to verify the effectiveness of this proposed algorithm. Its implementations are satisfactory and the detected vibration frequencies are 1.720 Hz, 1.870 Hz, 2.104 Hz, 2.905 Hz and also 10.050 Hz, which is not found in GPS or accelerometer only measurements.

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**Keywords:** Global Ppositioning System (GPS), accelerometer, integration, vibration, extended Kalman filter, dynamic deflection monitoring

DOI 10.1515/jag-2014-0024
Received October 22, 2014; accepted October 28, 2014

## 1 Introduction

The multi-sensor integration system for structural dynamics monitoring of large civil structures is drawing more and more attention in recent years and the essential structural modal parameters (e.g. natural frequency, mode shape and modal damping) extracted from the field measurements are essential for understanding the structural health conditions [13, 14, 16]. To overcome the shortcoming of individual sensors, various integration approaches have been developed to fuse the data sets of the Global Positioning System (GPS) receivers and the accelerometers that are installed together. Since 1990s, researchers from the University of Nottingham have conducted extensive studies on bridge deformation monitoring and accelerometers are usually used to compensate the deficiencies of GPS positioning such as low sampling rate and the high level multipath signature. The integration concept was systematically developed and an acceleration aided adaptive filtering technique was also adopted to extract the bridge dynamics from highly contaminated deflection signal by multipath [10]. The natural frequencies were also accurately identified from GPS/accelerometer off-line measurements [11, 17]. Hide [8] investigated the use of GPS and navigation grade INS to monitor the Forth Road Bridge in Scotland, UK. It demonstrated that high precision position and orientation information can be extracted from the integrated system. The integrated system by using RTK GPS positioning and accelerometers was also used to monitor the dynamic responses of other large-scale structures and the systematic analysis of the measured data has demonstrated that the two sensors complement each other in monitoring the static, quasi-static and dynamic deflections of the structures [1]. In order to further explore the benefits of different systems, a more effective and reliable data fusion technique should be developed. Chan [2] presented a GPS/accelerometer data integration processing technique based on empirical mode decomposition (EMD) and an adaptive filter. The simulation tests demonstrated that the measurement accuracy of the deflection is significantly improved. For combining GPS/accelerometer data sampled at different data rates, a multi-rate Kalman filtering approach was proposed to improve the positioning accuracy [19]. A comparative analysis showed that frequency-based deflection extraction approach is most appropriate for extracting precise structure displacement [9]. However, there are many unknowns in bridge monitoring that need to be further investigated as pointed out by Meng [12].

The basic methods used for GPS/ accelerometer integration can be summarised as follows: (1) the collected measurements without strict time synchronisation are analysed separately to extract vibration parameters and validate each other; (2) the time synchronised measurements are fused by post-processing for extracting structural dynamics; (3) the fully automated on-line integration system and data processing algorithm are developed for structural dynamics and structural health monitoring. Due to the algorithm and implementation complexity, an ideal online GPS/accelerometer integration algorithm is still not available and a feasibility study work that was sponsored by the European Space Agency to the first author's team had been started in 2013 to address these issues [6].

As a part of this ESA work and sponsored by other sources, this paper focuses on developing an extended Kalman filter based integration algorithm for fusing the vertical deflection measurements of a suspension bridge with RTK GPS positioning and a triaxial accelerometer. The algorithm can calibrate the acceleration and velocity corrections online by a closed loop feedback without sensor calibration in advance. The Fast Fourier Transform (FFT) is also adopted for precisely extracting the vibration dynamics. The simulated test and field experiment on the Wilford suspension bridge demonstrate that the fusion algorithm is satisfactory.

## 2 Accelerometer error model

The error sources of a Micro Electromechanical System (MEMS) grade accelerometer include sensor noise, bias drift and scale factor errors. The acceleration measurements can be modelled with Eq. 1 [20]:

$$\tilde{a} = (1 + f)a + b + v_a \tag{1}$$

where $\tilde{a}$ is a raw acceleration, $a$ is the true acceleration provided by the sensor. $v_a$ is the sensor noise assumed to be zero mean Gauss white noise ($v_a \sim N(0, \sigma_a^2)$) that is caused by the electronic interference. For an MEMS sensor, it is well known that acceleration suffers from high frequency noise. $f$ is the sensor's scale factor usually described with a first order Gauss-Markov model [4]:

$$f = -\frac{1}{\tau_f} + w_f \tag{2}$$

where $\tau_f$ is correlation time, $w_f$ is Gaussian white noise. The quantity $b$ in Eq. (1) is the sensor bias which can also be modelled as a Gauss-Markov process:

$$b = -\frac{1}{\tau_b} b + w_b \tag{3}$$

where $\tau_b$ is correlation time, $w_b$ is Gaussian white noise.

## 3 Extended Kalman filter (EKF)

Considering a nonlinear discrete system, the state $\mathbf{x}_k$ can be described as Eq. (4) [18]:

$$\mathbf{x}_k = f_{k-1}(\mathbf{x}_{k-1}) + \mathbf{w}_{k-1} \tag{4}$$

and the noisy nonlinear combination of the system states can be measured by $\mathbf{y}_k$ and expressed as:

$$\mathbf{y} = h_k(\mathbf{x}_k) + \mathbf{v}_k \tag{5}$$

where $f_{k-1}(\bullet)$ is the state transition function from epoch $k-1^{\text{th}}$ to $k^{\text{th}}$, $\mathbf{w}_k$ is the process noise at epoch $k-1^{\text{th}}$ with a covariance matrix $\mathbf{Q}_{k-1}$, and $h_k(\bullet)$ is the transition function between the state vector $\mathbf{x}_k$ and the observation vector $\mathbf{y}_k$. Eq. (5) is the measurement model with the measurement noise $\mathbf{v}_k$ whose covariance matrix is $\mathbf{R}_k$. $\mathbf{w}_k$ and $\mathbf{v}_k$ are both white noise and uncorrelated. The solution of EKF is a recursive procedure which contains prediction step that is given by

$$\begin{cases} \hat{\mathbf{x}}_k^- = f_{k-1}\left(\hat{\mathbf{x}}_{k-1}^+\right) \\ \mathbf{P}_k^- = \mathbf{\Phi}_{l,k-1}\mathbf{P}_{k-1}^+\mathbf{\Phi}_{k,k-1}^T + \mathbf{Q}_{k-1} \end{cases} \tag{6}$$

where $\hat{\mathbf{x}}_{k-1}^+$ is the posteriori state vector at epoch $k-1$, $\hat{\mathbf{x}}_k^-$ is the priori state vector at epoch $k$, $\mathbf{\Phi}$ is the state transition matrix from epoch $k-1$ to $k$, and $\mathbf{P}_k^-$ is the priori covariance matrix of $\hat{\mathbf{x}}_k^-$.

The update step is provided as

$$\begin{cases} \mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k)^{-1} \\ \hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{y}_k - h_k(\hat{\mathbf{x}}_k^-)) \\ \mathbf{P}_k^+ = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^- \end{cases} \tag{7}$$

where $\mathbf{K}_k$ is the Kalman gain matrix, $\hat{\mathbf{x}}_k^+$ is the posteriori state vector at epoch $k$, and $\mathbf{P}_k^+$ is the posteriori covariance matrix of $\hat{\mathbf{x}}_k^+$.

The transition matrices of linear and observation matrixes is:

$$\mathbf{\Phi}_{k,k-1} \sim \left.\frac{\partial f_{k-1}}{\partial x}\right|_{x=\hat{\mathbf{x}}_{k-1}^+} \tag{8}$$

$$\mathbf{H}_k \sim \left.\frac{\partial h_k}{\partial x}\right|_{x=\hat{\mathbf{x}}_k^-} \tag{9}$$

# 4 GPS/accelerometer integration algorithm

## 4.1 Description of the integration system

For a single axial accelerometer, the measured deflection $p(t)$ and deflection velocity $v(t)$ varying with time can be described as a differential equation [5]:

$$\dot{p}(t) = v(t) \tag{10}$$

$$\dot{v}(t) = a(t) \tag{11}$$

where $a(t)$ is the measured acceleration. The deflection $p(t)$ can be acquired with real-time kinematic (RTK) GPS positioning or other sensors. The accelerometer measurements should be transformed to the vertical direction for integration with vertical GPS measurements.

The fusion model of GPS/accelerometer integration for the vertical dynamics of a civil structure can be considered as a single channel equation similar to the GPS/INS integration. The error states are $\delta p = \tilde{p} - p$, $\delta v = \tilde{v} - v$, $\nabla b$ and $\nabla f$. Considering the center discrepancy between the GPS antenna and the accelerometer, one more error state $\delta L$ is included and the dynamic equation for the integration system can be expressed as Eq. 12:

$$\dot{\mathbf{x}} = \underbrace{\begin{bmatrix} 0 & 1 & & & \\ & 0 & 1 & 1 & \\ & & -\frac{1}{\tau_b} & & \\ & & & -\frac{1}{\tau_f} & \\ & & & & 0 \end{bmatrix}}_{\mathbf{F}} \mathbf{x} + \underbrace{\begin{bmatrix} 0 \\ v_a \\ w_b \\ w_f \\ w_L \end{bmatrix}}_{\mathbf{w}} \tag{12}$$

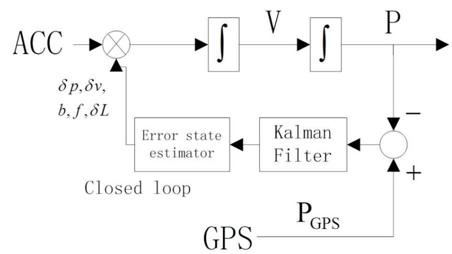


**Fig. 1.** Schematic for a Closed Loop Feedback EKF Filter.

and $\mathbf{x}$ is the error state vector given by:

$$\mathbf{x} = \begin{bmatrix} \delta p & \delta v & \nabla b & \nabla f & \delta L \end{bmatrix}^T \tag{13}$$

where $\delta L$ is modelled as a random walk process and $w_L$ as Gauss white noise. At time epoch $t$, the corresponding error observation equation can be described as Eq. 14:

$$\begin{aligned} y(t) &= r_{GPS}(t) - r_{Acc}(t) \\ &= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x} + v_p(t) \end{aligned} \tag{14}$$

where $r_{GPS}(t)$ is the observed vertical GPS deflection, $r_{Acc}(t)$ is the double integral deflection of the acceleration and is the Gaussian white noise of the measurements.

## 4.2 Closed loop feedback algorithm

The closed loop feedback algorithm is usually applied to GPS/INS integration system [7]. In this paper, the GPS/accelerometer integration algorithm is realised with a closed loop feedback algorithm based on an extended Kalman filter (Figure 1). The accelerometer sensor errors are modelled by biases and scale factor errors, which are estimated by the online Kalman filter and fed back to calibrate the raw accelerometer measurements according to Eq. (15) to limit the error growth.

$$a = \frac{\tilde{a} - b}{1 + f} \tag{15}$$

# 5 Simulation trial

The authors simulated a vertical deflection of a bridge with frequency of 0.8 Hz, 2 Hz and 5 Hz and the corresponding amplitudes are 0.005 m, 0.01 m and 0.008 m, respectively. Then the simulated deflection signal is formed as:

$$\begin{aligned} x(t) &= A_1 \cdot \sin(2\pi f_1) \\ &+ A_2 \cdot \sin\left(2\pi f_2 + \frac{\pi}{4}\right) \\ &+ A_3 \cdot \sin\left(2\pi f_3 + \frac{\pi}{3}\right) \end{aligned}$$

**Table 1.** Detailed process parameters used for the simulated accelerometer [CROSSBOW, 2002].

| Simulated parameters | Values | Units |
|---|---|---|
| Range | ±10 | g |
| Bias | 12 | mg |
| Scale factor error | 4000 | ppm |
| Velocity random Walk | 0.5 | $m/s/hr^{1/2}$ |
| Correlation time, $\tau_b$ | 200 | s |
| Correlation time, $\tau_f$ | 1000 | s |

where $A_1$ is 0.01 m and $f_1$ is 2 Hz, $A_2$ is 0.008 m and $f_2$ is 5 Hz, $A_3$ is 0.005 m and $f_3$ is 0.8 Hz. That is, the simulated acceleration is

$$\begin{aligned} a(t) &= -A_1(2\pi f_1)^2 \sin(2\pi f_1) \\ &\quad - A_2(2\pi f_2)^2 \sin\left(2\pi f_2 + \frac{\pi}{4}\right) \\ &\quad - A_3(2\pi f_3)^3 \sin\left(2\pi f_3 + \frac{\pi}{3}\right) \end{aligned}$$

An integrated GPS/accelerometer bridge vertical monitoring system is simulated with above settings. The GPS vertical measurements are simulated with the deflection signal added with white measurement noise ($v_p \tilde{N}(0, 0.01^2)$) and the sampling rate is 5 Hz, a total of 60 s data is simulated. The acceleration is assumed to be collected with an accelerometer sampled at the rate of 100 Hz, the high sampling rate can offer some benefits for extracting high-frequency vibration in bridge monitoring. The simulated accelerometer specifications in the measurements are the same as Crossbow IMU400CC [3] given by Table 1. These parameters are used to form the stochastic model described in Section 2.

It is apparent that only the vibrations of 0.8Hz and 2Hz (the Nyquist frequency of 5 Hz GPS data) could be extracted from the GPS measurements with the peak-picking approach (Figure 2). The vibration signatures between 2 Hz and 5 Hz can be easily extracted with FFT from the acceleration measurements whilst the 0.8 Hz vibration is hardly to be identified for its low power (Figure 3).

To assess the effectiveness of the proposed method, the deflection results derived from the integration algorithm and corresponding frequency domain analysis are shown in Figure 4, and the estimation errors compared to the true signal are plotted in Figure 5. The estimated amplitude of the deflection is consistent with the simulated signal and the RMS error of the estimated deflection is 4.1 mm. This is to say that the integration algorithm can provide better performance than GPS-only or accelerometer-only deflection monitoring strategy. Excellent velocity estimates can also be obtained even without

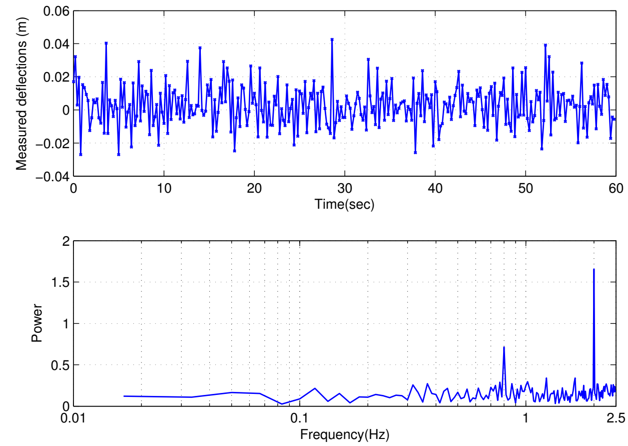


**Fig. 2.** The Simulated GPS Measurements and FFT Analysis.

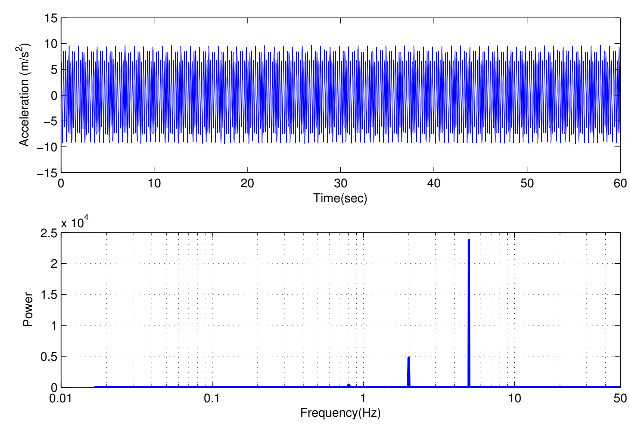


**Fig. 3.** The Simulated Accelerations and FFT Analysis.

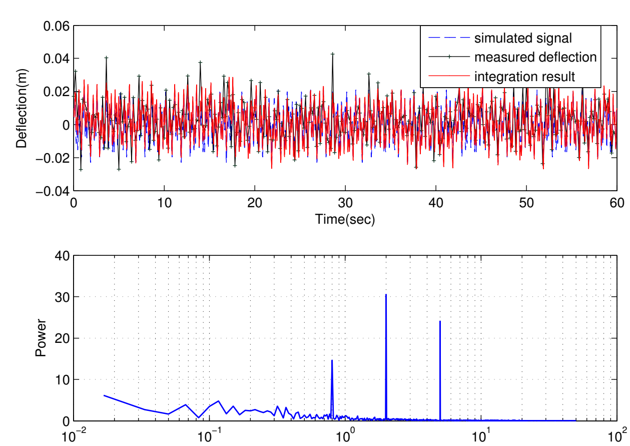


**Fig. 4.** The Estimated Deformations and Their Corresponding Vibration Frequencies.

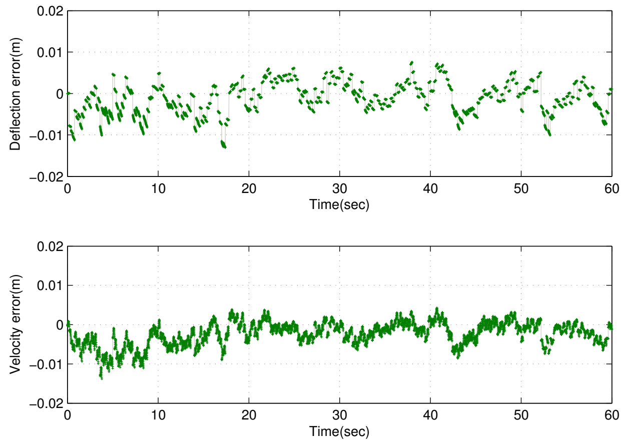


Fig. 5. The Estimated Deflection and Velocity Errors.

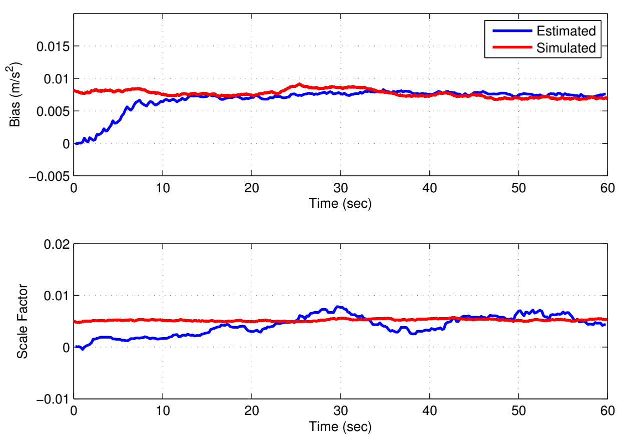


Fig. 6. The Estimated Accelerometer Sensor Errors (Bias and Scale Factor).

GPS velocity observations, which is also important for real-time monitoring and warning.

The integration approach has the benefit to identify all vibration frequency compared to GPS-only or accelerometer-only data. It is shown in Figure 4 that three simulated vibration frequencies can be accurately extracted from the estimated deflection series. The estimated accelerometer errors are shown in Figure 6, from this graph it is evident that the estimated sensor errors, including bias and scale factor errors, are compatible with the simulated errors and the estimated error states are fed back to correct the raw measurements.

The integrated system can also improve the system integrity for real-time monitoring. During the GPS blockages only the accelerometer measurements can be used for the vertical deflection of the bridge and the accumulated systematic errors will be corrected once the GPS is available again. A 3s (30s-33s) GPS blockage that is equivalent to 15

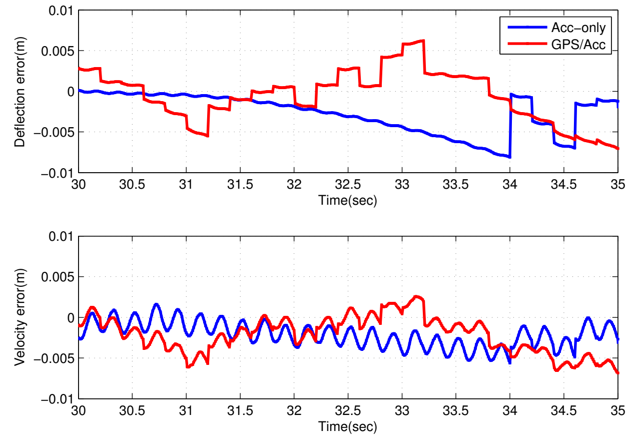


Fig. 7. The Performance of GPS/accelerometer Integration during GPS blockage.

epochs is simulated and the estimation errors during the period are given in Figure 7. It shows that the accelerometer can provide satisfactory result in a short GPS outage, and the performance recovers at 33th second immediately when GPS is available.

## 6 The Wilford suspension bridge trial

The Wilford suspension bridge is 69m long and 3.7m wide with a steel deck covered by a floor of wooden slats [10]. The bridge is possibly 100 years old and was extensively utilised as a test bed by staff of the University of Nottingham during their past research (Figure 9). The data set utilised in this paper was collected with GPS and a triaxial accelerometer from a trial carried out on 15th of May in 2003. In this paper, the vertical deflection measurements are used for testing the integration algorithm and extracting the vertical vibrations. The GPS sampling rate was set to 10 Hz, and the accelerometer recorded the data at the frequency of 80Hz. For more details about the trial see [11].

### 6.1 Multipath isolation based on a Chebyshev filter

The GPS coordinates of the rover receiver on the bridge in the WGS-84 coordinate system was transformed into the coordinates in the local reference datum in advance before integrated with acceleration. The time span of the GPS data set is from GPS second 387704.1 to GPS second

Fig. 8. Wilford bridge over the River Trent in Nottingham, UK.

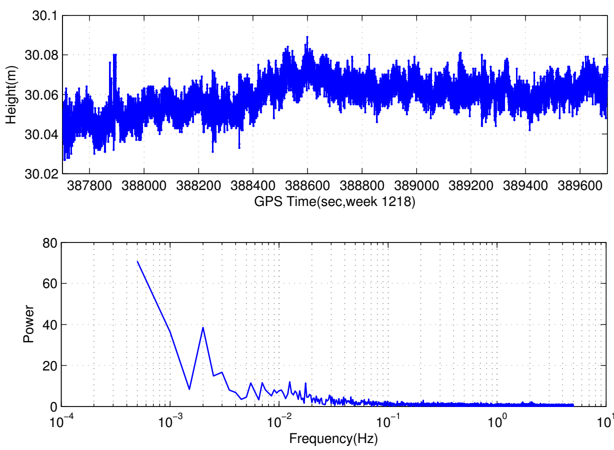


Fig. 9. GPS Vertical Deflections and Vibration.

389704.0 in the GPS week 1218, with a total of 20000 observations.

During the experiment, several forced excitation tests had been carried out. The resolved vertical deflection time series and its spectra are shown in Figure **??**. The whole data set shows a clear low frequency character, which is the effect of multipath. It is hardly to identify structural dynamics from the original time series without removing this multipath signature. It was found that the dominant signals of low frequency nature between 0.001 Hz to 0.2 Hz are multipath and flicker noise effects [15]. Their effects could be removed with a Chebyshev type 3-order digital filter designed to extract long-period component and then the dynamic deflections were extracted from the frequencies from 0.2 Hz above (Figure 10).

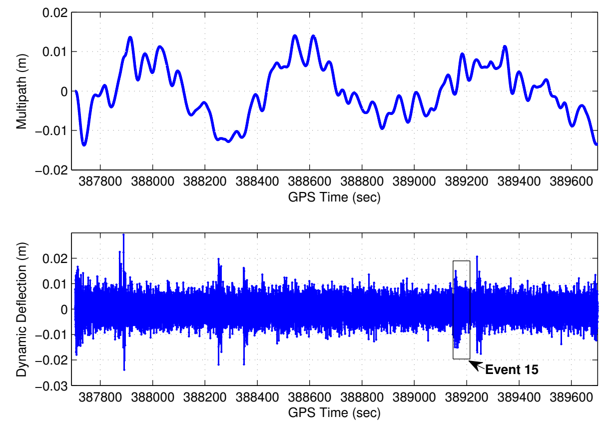


Fig. 10. Extracted Long-period Component and Dynamic Deflections.

## 6.2 Integrated algorithm for detecting the vertical dynamics

In this paper, the excitation test event 15 as shown in Figure 10 when 15 people jumped together at the mid-span of the Wilford Bridge was used for analysis. Considering the relatively low precision of GPS measurements in the vertical direction and the main excitations occur in the vertical direction, the vertical component of the data set was selected to test the proposed integration algorithm. The vertical deflections measured with GPS and the extracted frequencies are shown in Figure 11. The extracted vertical structural vibrations have two dominant frequencies of 1.717 Hz and 2.083 Hz. The un-calibrated accelerometer measurements and their corresponding frequencies are shown in Figure 12. It is obvious that raw acceleration contains a clear bias component, which will have an adverse impact on the state estimate without correction. From the spectra of accelerometer records, the detected frequencies are 1.702 Hz, 1.869 Hz, 2.103 Hz, and 2.904 Hz respectively.

The proposed integration algorithm is adopted for this real-life monitoring data fusion and the integrated vertical deflection time series is shown in Figure 13. Figure 14 shows the estimated velocity and it is evident that the starting epoch is affected by the bias and scale factor of the accelerometer. The estimated sensor calibration parameters are shown in Figure 15. By fitting the line, it is obtained that the accelerometer bias is 0.37 $m/s^2$ and the scale factor is 1100 ppm. The integration algorithm filters most GPS measurements noise and the extracted vibration frequencies from the integrated deflection time series are 1.703 Hz, 1.853Hz, 2.104 Hz, 2.905 Hz, and 10.050 Hz. A summary of the extracted frequencies from the GPS measurements, the accelerometer measurements and the in-

**Table 2.** Vibration Frequencies Detected by GPS and Accelerometer Measurements.

| Vibration Freq. | GPS (Hz) | ACC. (Hz) | GPS/ACC Integration (Hz) | comments |
|---|---|---|---|---|
| 1 | 0.001~0.020 | - | - | Multipath effects earsed by a bandpass filter |
| 2 | 1.717 | 1.702 | 1.720 | Detected by both GPS and ACC |
| 3 | 2.083 | 2.103 | 2.104 | Detected by both GPS and ACC |
| 4 | - | 1.869 | 1.870 | Detected by ACC. only |
| 5 | - | 2.904 | 2.905 | Detected by ACC only |
| 6 | - | - | 10.050 | Detected by GPS/ACC. |

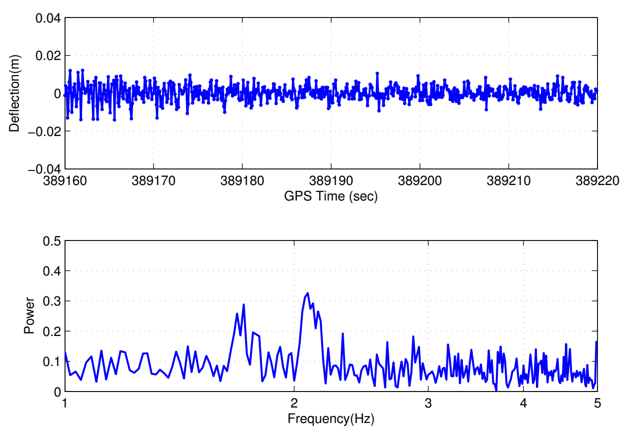


**Fig. 11.** Vertical Deflections Measured with GPS and the Vibration Frequencies.

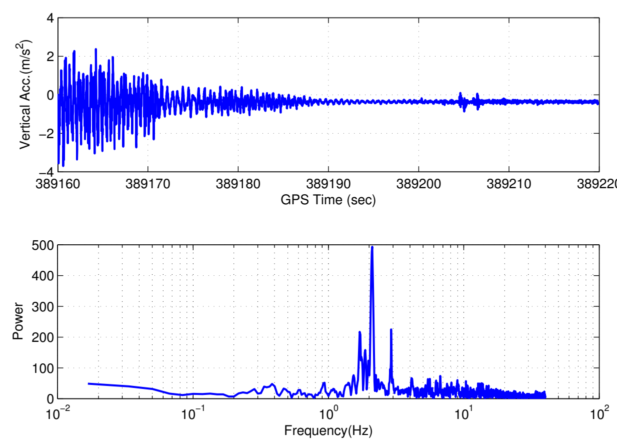


**Fig. 12.** Raw Accelerations and Identified Frequencies.

tegration process is listed in Table 2. The spectra of the integrated deflection time series match well with that of GPS measurements and acceleration time series except the vibration of 10.050 Hz that is not detected in GPS-only or accelerometer-only monitoring, which may be another high frequency structural dynamics. The integration approach can extract more vibration frequencies than separately extracted from GPS-only and accelerometer-only measurements.

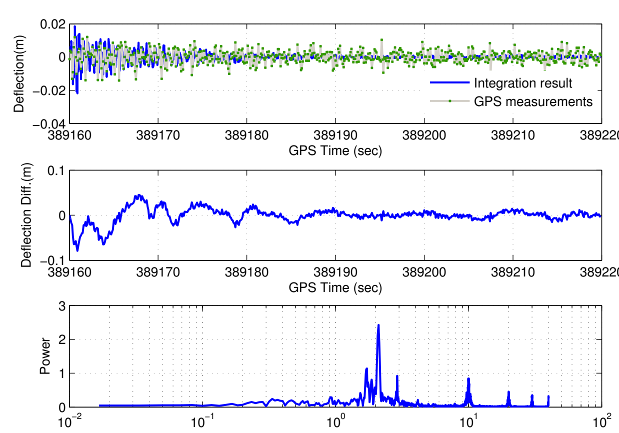


**Fig. 13.** The Performance of the Integration Algorithm and Vibration Frequencies Extraction.

## 7 Conclusions

This paper introduces an online integration algorithm to integrate GPS and MEMS grade accelerometer measurements and thereafter the peak-picking approach is used to extract vibration frequencies. It has verified that the integrated vertical deflection time series contains not only all the vibration frequencies detected by GPS or accelerome-

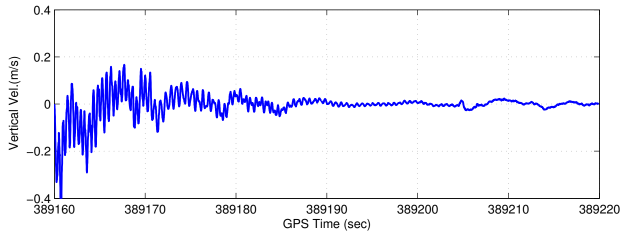


**Fig. 14.** Estimated Velocity Time Series.

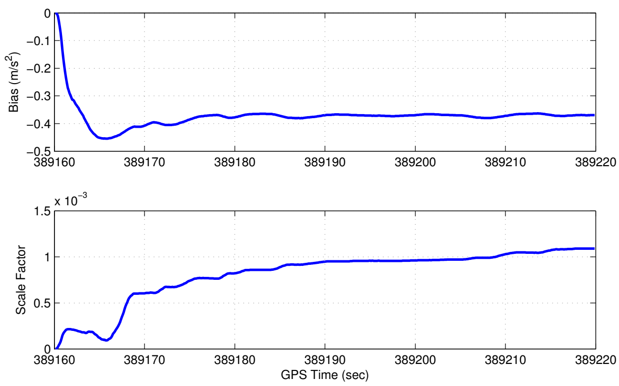


**Fig. 15.** Estimated Calibration Parameters: Bias and Scale Factor.

ter only measurements but also extra frequencies that cannot be obtained easily with observations from a sole sensor system. The algorithm suits for online application which can be applied to establish structural health monitoring system. Further research should focus on more reliable integration algorithm (e.g. robust integration algorithm) and more precise acceleration integral algorithm.

**Acknowledgement:** This study is supported by the Program for New Century Excellent Talents in University with a grant number as NCET-13-1019 and partially sponsored by the Fundamental Research Funds for the Central Universities with a grant number as 2013RC16.

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