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RESEARCH ARTICLE

Bayesian assessment of an existing bridge: A case study

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This article presents a case study involving the assessment of an existing bridge, starting with simple methods and ending with a probabilistic analysis, the latter emphasizing Bayesian methods. When assessing an existing bridge, it is common practice to collect information from the bridge in the form of samples. These samples are in general of small size, raising the question of how the corresponding statistical uncertainty can be taken into account on reliability estimates. The case study illustrates how Bayesian methods are especially suitable to deal with that source of uncertainty. Another strong point of the Bayesian methods is their ability to combine the information contained in the samples collected from the bridge with prior information, if any. This aspect will also be illustrated through the case study.

Keywords: Existing bridges; Probabilistic assessment; Bayesian updating; Statistical uncertainty; Predictive model.

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1. Introduction

Structural assessment of existing bridges has become an activity with increasing importance due to the ageing of transportation networks. The fact that a significant number of bridges need repair, together with the occurrence of serious accidents, has led, in recent years to extensive inspection campaigns of existing bridges. Most countries do not have, however, specific documentation to support structural assessment of existing structures. Therefore, when assessing an existing bridge, the codes developed for new structures are used, which is not satisfactory. Indeed, several bridges were classified as unsafe by applying design codes, while their reliability was found to be high enough, as demonstrated by means of probabilistic evaluation (Lauridsen et al. 2007). This shows that a structural assessment using design codes might lead to unnecessary waste of funds with repair and strengthening. Hence, it is not surprising that probabilistic methods have been gaining increasing acceptance, particularly in the assessment domain.

Among those methods, Bayesian approach must be mentioned. In fact, in structural assessment, it is very common to collect information from the bridge under evaluation in order to improve the assessment. As demonstrated in this work, Bayesian methods are useful in structural assessment, for they allow the updating of probabilistic models by adding the latest information, without ignoring the oldest. Moreover, samples collected from the bridge are often of limited size, due to both costs and induced damage. The corresponding statistical uncertainty affects the reliability estimate of the bridge and must therefore be included in the analysis. Bayesian approach has been considered the appropriate tool to deal with statistical uncertainty (Engelund and Rackwitz 1992).

Bayesian methods have been widely used by several researchers in structural engineering. Geyskens et al. (1998) used Bayesian methods to quantify uncertainties inherent to the modeling process, in particular statistical uncertainty related to the unobservable parameters in such models. Bayesian updating has also been used in the context of reliability assessment of existing structures. For example, Strauss et al. (2008) employed Bayesian updating to obtain more accurate reliability estimates combining past data and monitoring data. Enright and Frangopol (1999) used Bayesian techniques for combining information from both inspection data and engineering judgment in order to better predict strength loss and time-variant reliability of deteriorating reinforcing concrete structures.

In this article, a case study involving the assessment of an existing bridge is presented, in which Bayesian methods were also applied. The bridge has reached a high level of deterioration and its replacement was considered the only viable solution. It was decided to replace the bridge by a new one within one year, when this study was made (Summer 2010). However, it was necessary to evaluate if the bridge could be kept in service until its replacement or if its reliability for the period in question was not acceptable, in which case the bridge should be immediately closed. Although the bridge had failed the assessment based on traditional criteria, a subsequent probabilistic assessment demonstrated that its reliability was sufficient, and the bridge was kept in service during the time required for its replacement.

2. Basis of the Bayesian paradigm

The fundamental element of the Bayesian paradigm consists of assigning probabilities to all unknown quantities (Bernardo 2009). According to this paradigm, all uncertainties, regardless of their nature (random or epistemic), must be described by means of probability distributions. This extends substantially the scope of probability theory. In fact, a substantial part of engineering problems, in which the reliability problems are no exception, have epistemic nature (Kiureghian and Ditlevsen 2009). According to the Bayesian paradigm, it makes sense to assign probabilities not only to random and unpredictable quantities but also to unknown states. These states, usually known as states of nature (Benjamin and Cornell 1970), refer to fixed quantities, but unknown for some reason.

An example of fixed quantities, but normally unknown, are the parameters $\boldsymbol{\theta} = (\theta_1, \theta_2, \dots)$ of probabilistic models. Assigning a probability distribution to parameters $\boldsymbol{\theta}$ arises as a convenient way to describe the uncertainty in them. This uncertainty, usually known as statistical uncertainty (a especial case of epistemic uncertainty), arises from to finite size of samples used to estimate probabilistic parameters. As the sample size grows, the mean values of the parameters $\boldsymbol{\theta}$ approximate their true values and their variances decrease.

Modelling the parameters $\boldsymbol{\theta}$ as random variables has two main advantages. Firstly, it guarantees that the statistical uncertainty is properly included in any subsequent probabilistic calculations. Secondly, it allows the probabilistic models to be updated as new data becomes available.

To review the main Bayesian terminology, consider a bi-parametric model $f_X(x \mid \boldsymbol{\theta})$, where $\boldsymbol{\theta} = (\theta_1, \theta_2)$. Let $f(\theta_1, \theta_2)$ be the joint probability density function (PDF) of those parameters, which describes the current uncertainty about them. As soon as a sample $\varepsilon = \{x_1, \dots, x_n\}$ from X is observed, Bayes' Theorem makes it possible to update the distribution $f(\theta_1, \theta_2)$ into $f(\theta_1, \theta_2 \mid \varepsilon)$ (Ditlevsen and Madsen 1996):

$$f(\theta_1, \theta_2 \mid \varepsilon) = c \cdot L(\theta_1, \theta_2 \mid \varepsilon) \cdot f(\theta_1, \theta_2), \tag{1}$$

where c is a constant, called normalization constant, and $L(\theta_1, \theta_2 \mid \varepsilon)$ is the likelihood of the sample ε, given by $L(\theta_1, \theta_2 \mid \varepsilon) = \prod_{i=1}^{n} f_X(x_i \mid \theta_1, \theta_2)$.

The distribution $f(\theta_1, \theta_2)$ is usually known as *prior distribution* and $f(\theta_1, \theta_2 \mid \varepsilon)$ as *posterior distribution*. To apply the Bayes' theorem it is necessary to assign to $\boldsymbol{\theta}$ some prior distribution. When there is no relevant prior information, it is common to use a non-informative distribution, which is characterized by having little impact on the posterior distribution, when compared with the impact of the sample information (Bernardo 2009).

Probabilistic computations involving the variable X should be made using its marginal distribution, considering X as a component of the random vector (X, θ_1, θ_2). The marginal distribution of X, termed *predictive distribution* in the Bayesian terminology, is given by:

$$f_X(x) = \int_{\Theta_1} \int_{\Theta_2} f_X(x \mid \theta_1, \theta_2)\, f(\theta_1, \theta_2)\, d\theta_2 d\theta_1, \tag{2}$$

before the sample is available, and by:

$$f_X(x \mid \varepsilon) = \int_{\Theta_1} \int_{\Theta_2} f_X(x \mid \theta_1, \theta_2)\, f(\theta_1, \theta_2 \mid \varepsilon)\, d\theta_2 d\theta_1, \tag{3}$$

after the sample is available. The former is called *prior predictive distribution* and the latter *posterior predictive distribution*, or updated predictive model.

In many cases it is not possible to get the closed form of the above integrals. In these cases the Monte Carlo Method (MCM) can be useful for drawing samples of X

without knowing its analytical form, through which any necessary calculations can be made. In the present study, an algorithm to draw a sample from the predictive model was developed. This algorithm combines the Acceptance-Rejection and the composition methods (Rubinstein 1981), and can be regarded as a direct algorithm. Indirect methods, as those based on Markov chains, have been used in the context of Bayesian analysis (Shao and Ibrahim 2000), but were not applied in the present study.

To apply the MCM it is convenient to factorize the distribution $f(\theta_1, \theta_2)$ in the form: $f(\theta_1, \theta_2) = f(\theta_1 \mid \theta_2) f(\theta_2)$. If θ_1 and θ_2 are independent variables, $f(\theta_1, \theta_2) = f(\theta_1) f(\theta_2)$. The algorithm runs then as follows:

1. Obtain $\{(\theta_1^{(1)}, \theta_2^{(1)}), \ldots, (\theta_1^{(N)}, \theta_2^{(N)})\}$, repeating, as many times as needed, the cycle (Acceptance-Rejection method):
   - draw $\theta_2 \sim f(\theta_2)$;
   - draw $\theta_1 \sim f(\theta_1 \mid \theta_2)$, where θ_2 is the generated value in the previous step;
   - draw $\theta_1 \sim f(\theta_1 \mid \theta_2)$, where θ_2 is the generated value in the previous step;
   - evaluate $L(\theta_1, \theta_2 \mid \varepsilon) = \prod_{i=1}^{n} f(x_i \mid \theta_1, \theta_2)$, using the sample $\varepsilon = \{x_1, \ldots, x_n\}$;
   - draw $u \sim \mathrm{unif}(0, 1)$;
   - if $u \le L(\theta_1, \theta_2 \mid \varepsilon)$ the pair (θ_1, θ_2) is accepted as belonging to $f(\theta_1, \theta_2 \mid \varepsilon)$;
2. obtain $\{x^{(1)}, \ldots, x^{(N)}\}$ in the following way (Composition method):
   - draw $x^{(1)} \sim f_X(x \mid \theta_1^{(1)}, \theta_2^{(1)})$;
   - draw $x^{(2)} \sim f_X(x \mid \theta_1^{(2)}, \theta_2^{(2)})$;

     ...
   - draw $x^{(N)} \sim f_X(x \mid \theta_1^{(N)}, \theta_2^{(N)})$;

It is observed that to apply the Acceptance-Rejection Method there is no need to know the constant c in (1). This means that any constant that multiplies the likelihood function is irrelevant. Hence we can multiply the likelihood function by any constant, providing that $0 < L(\theta_1, \theta_2 \mid \varepsilon) < 1$. An immediate conclusion is that the acceptance rate in the above algorithm can be greatly improved determining previously $L_{\max} = \max\{L(\theta_1, \theta_2 \mid \varepsilon)\}$. Once $L_{\max}$ is known, k can be defined so that $kL_{\max} = 1 \Leftrightarrow k = 1/L_{\max}$. The condition $u \le L(\theta_1, \theta_2 \mid \varepsilon)$ will then be substituted by the condition $u \le kL(\theta_1, \theta_2 \mid \varepsilon)$, becoming the algorithm more effective.

3. Brief description of the studied bridge

The bridge under analysis, built in the seventies, is composed by a deck of four longitudinal beams, joined by a slab, which receives two side walkways and a roadway of two lanes. The deck is supported by two abutments and two piers founded in the bed of the River Lis (see Figure 1). All structural elements are in reinforced, non-prestressed concrete. The total length of the deck is 60 m, distributed in three spans: 18.6, 22.8 and 18.6 m.

The bridge was located near the mouth of the river Lis, in Portugal, and was in a very advanced state of degradation, partly due to the high aggressiveness of the environment (marine environment), as Figure 2 shows. Several zones with exposed reinforcement bars exist, particularly in the beam at the sea side (beam on the left), where bars have significant loss of cross-section area.

Several campaigns of inspection and testing demonstrated very advanced levels of chloride contamination. Electrochemical tests revealed the existence of active corrosion, even in elements without visible signs of deterioration, namely in piers and abutments.

Figure 1. View of the bridge under study.

Figure 2. Deck photographs showing its deterioration state.

Table 1. Core testing results.

Core	Location	Mass [g]	Height [mm]	Section [mm^2]	Ultimate load [kN]	Strength [MPa]
1	Abutments	1870	99	7698	487.7	63.5
2	Abutments	1873	99	7698	503.9	65.5
3	Abutments	1885	99	7698	528.4	68.5
4	Piers	1160	84	5542	336.0	60.5
5	Piers	1140	84	5542	207.0	37.5
6	Piers	1151	84	5542	190.0	34.5
7	Deck	1868	99	7698	344.8	45.0
8	Deck	1876	99	7698	314.3	41.0
9	Deck	1887	99	7698	342.0	44.5

Since the level of degradation was significant mainly in the deck, safety concerns related mostly to the carrying capacity of the superstructure. According to previous studies, the safety of the piers and abutments, including foundations, raised no concerns. Thus, this study dealt only with the reliability of the superstructure.

The procedure considered for the safety assessment of the bridge is depicted in Figure 3. In the first step, the structure is evaluated using a simple approach, based on direct use of relevant codes in a semi-probabilistic approach. Should the structure fail to comply with the codes, a more detailed analysis should be employed. Reliability tools can be used at this stage, but to limit costs, only information already available is used. If the structure is still considered unsafe, the results on this step are used to identify the critical random variables, for which additional data is required. Bayesian updating is then used to define new distribution for the key random variables and the reliability analysis is repeated. Only if the structure is deemed unsafe after all these steps, should mitigation measures be applied, including retrofitting or replacement.

Next section describes the preliminary safety analysis based on traditional, semi-probabilistic methods.

4. Semi-probabilistic analysis

4.1 *Materials*

In order to characterize the concrete of the bridge, nine cores were extracted: three from the deck, three from the piers and three from the abutments. Table 1 shows the results of core testing. As it can bee seen, there is considerably dispersion on the strength results.

Since the objective of this study was to evaluate the carrying capacity of the superstructure, it would make sense to use only the cores taken from the deck. On the other hand, the original design prescribed identical properties for the concrete of the three elements (abutments, piers and deck), so assuming all cores as belonging to the same population was legitimate. Therefore, it seemed reasonable to estimate the characteristic strength of the concrete using either all cores or only the deck ones, whichever would lead to the lowest strength estimate.

To estimate the characteristic value of concrete compressive strength, a probabilistic model was chosen. The normal and lognormal models are in general considered ade-

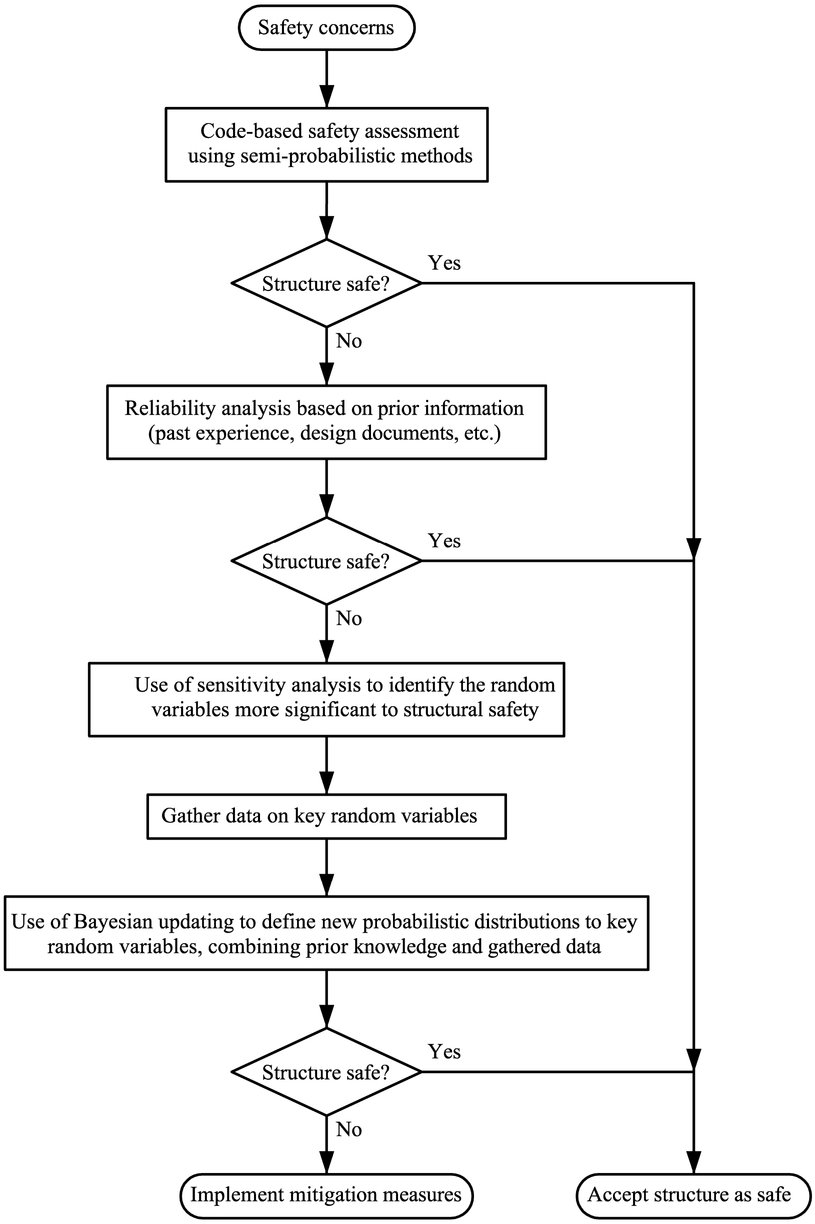


Figure 3. Procedure used in the assessment of the case study bridge.

quate to describe concrete strength (JCSS 2001, Wisniewski 2007). In the present case the lognormal model was used, following Wisniewski (2007) recommendations for high variability concrete strength. In addition, to take into account the statistical uncertainty originated by the small sample available, the Bayesian predictive model of a lognormal population was used, also described in detail later.

Using then the Bayesian predictive model proposed in Eurocode (EN1990 2002), Annex D, assuming no prior knowledge, the following estimates concerning concrete strength were obtained: f_{ck} = 29.7 MPa considering all cores and f_{ck} = 36.6 MPa considering only the cores produced from the deck. Choosing the small value, the design value of the

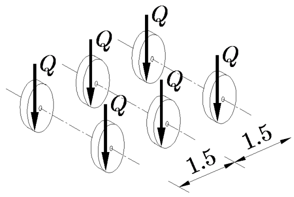


Figure 4. Portuguese standard vehicle for II-class bridges. $Q = 50$ kN.

concrete strength was estimated in $f_{cd} = 29.7/1.5 = 19.8$ MPa.

Regarding reinforcing steel, the original design plans prescribed A-40 grade, which corresponds to the characteristic 0.2% proof stress $f_{s0.2k} = 400$ MPa. Considering the partial safety factor of 1.15, the design yield strength of reinforcement is $f_{yd} = 348$ MPa.

4.2 *Loads*

Since the concerns regarding the bridge lied mainly on the deck safety to ultimate limit states, the loads of interest are the permanents loads and the traffic loads. Regarding permanent loads (self-weight of structural and non-structural elements), the following densities were considered: (1) reinforced concrete - 25 kN/m^3; (2) plain concrete (side walkways) - 24 kN/m^3; (3) roadway surface - 24 kN/m^3; (4) metallic guards - 77 kN/m^3.

Regarding traffic loads, the Portuguese code (RSA 1983) stipulates two classes of bridges, depending on the traffic intensity of the road served by the bridge. In the present case, considering that the bridge serves a secondary roadway, with essentially light vehicles, the bridge was ranked as class II, which corresponds to a lighter traffic. The same code prescribes two types of traffic loads, to be applied separately. The first models a heavy truck using a set of concentrated loads, while the second a set of vehicles using a distributed load. The first load, which in the present bridge resulted in higher internal forces, is composed by a tridem (3 axes), with a total weight of 300 kN (see Figure 4).

As mentioned above, the bridge deck is composed by four longitudinal beams, whose distance is approximately equal to the vehicle width. Thus when a wheel is aligned with a beam, the other is aligned with a second beam. It was assumed that, when a wheel is placed on a beam, the corresponding internal-forces are resisted only by that beam, i.e., no transference between beams was considered. This corresponds to consider the four beams loaded simultaneously, or, equivalently, considering the simultaneous presence of two vehicles, side-by-side, which is not unrealistic.

4.3 *Preliminary analysis*

A linear elastic model of the deck was developed for each beam, in accordance with their influence width (Figure 5) and subjected to the loads described previously. The outer beam governs the assessment, due to the weight of sidewalk. Based on the resulting sectional-forces, the safety regarding ultimate limit states (bending and shear) was evaluated, considering the criteria prescribed in Portuguese codes. Details of that analysis can be found in Jacinto (2011).

Longitudinal reinforcement at mid-spans and at supports are disposed in several layers. As a first approximation, a 50% reduction in area was assumed for the first layer at mid-spans, while others layers were assumed intact. As it is well known, resistance regarding bending and shear of concrete beams depends on good bonding between reinforcements

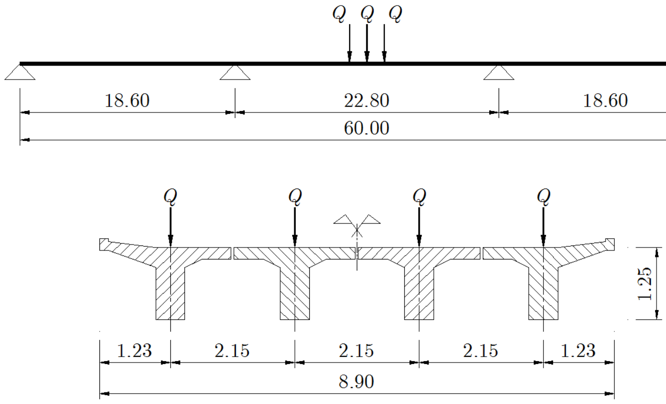


Figure 5. Structural model. Dimensions in meters.

and concrete. In sound concrete structures, bonding properties do not, in general, cause any concern, but in structures with corroded reinforcements and spalling concrete (caused by the expansion of corrosion products), bonding must be investigated. This problem was analyzed in Jacinto (2011), having been concluded that there was no reasons expect any reduction in bonding properties and the existence of damaged concrete did not significantly affected the carrying capacity of this particular bridge.

The main results of the preliminary analysis were:

(1) the bridge failed to meet the safety criteria. The critical limit state is bending at mid-span of the central span.
(2) Cross-sections at supports had a reasonable safety margin in bending.
(3) The bridge presents a reasonable safety margin in shear.

It was concluded then that the critical scenario consisted in the formation of a plastic hinge at mid-span of the central span, because this was the cross-section with greater safety deficit. However, considering the redundancy of the structure, a plastic hinge at mid-span does not determines the collapse of the structure. Since cross-sections at supports had a reasonable margin of safety, a plastic analysis of the superstructure could have been considered, as described in Jacinto (2011).

5. Probabilistic analysis

In this section the probability of the event $M_E > M_R$ (bending ultimate limit state) in the critical cross-section is determined, where M_E represents the applied moment and M_R the moment resistance. The applied moment is given by:

$$M_E = M_g + M_{\Delta g} + M_Q, \tag{4}$$

where M_g is the moment due to self-weight, $M_{\Delta g}$ is the moment due to additional permanent loads and M_Q is the moment due to live loads.

The applied moment M_E was evaluated through a linear elastic frame model. Although the problem involves ultimate limit states and significant cracking, yielding and moment redistribution are expected, this model represents a simple and conservative approach, compatible with current design practice. Indeed, the actual moments at the critical cross-section tend to be smaller than the elastic ones, as a result of the moment transference from that section to the supports. The error associated with M_E and the corresponding uncertainty will be taken into account by a specific random variable, θ_E.

The moment resistance M_R was calculated using the rectangular diagram method. To take into account the uncertainty originated by this method, the random variable θ_R was introduced. Thus the limit state function is as follows:

$$Z = \theta_R M_R - \theta_E M_E, \tag{5}$$

and the problem consists of determining the probability $p_f = P(Z < 0)$.

The failure probability p_f was evaluated for the period of one year, since it was the deadline for the replacement of the bridge. Hence, regarding the live load, the annual maxima distribution was of interest.

The following section describes all the random variables considered in the reliability analysis. The uncertainty associated with geometric quantities (cross-section dimensions and span lengths) can be neglected when compared with the uncertainty associated with loads and material properties. Hence, cross-section dimensions and lengths will be modeled as deterministic.

5.1 *Basic variables and transformation models*

5.1.1 Self-weight of the structural concrete

The density of the cores extracted from the bridge (Table 1) reveals little variability. In fact, the computed coefficient of variation (COV), for the nine cores, is smaller than 1%. However the self-weight γ_c must include reinforcements, which varies from location to location, causing the variability of the self-weight to increase. The provisions of the Danish guideline (Vejdirektoratet 2004) were adopted, which recommends for γ_c a normal distribution with mean equal to 25 kN/m^3 and COV equal to 5%.

Regarding the moment M_g (bending moment at mid-span due to self-weight of structural elements), the elastic linear model described earlier resulted in a bending moment of 608.2 kNm considering a density of 25 kN/m^3. Since the moment M_g is a linear function of γ_c and the basic variables concerning dimensions of structural elements are being considered as deterministic, it follows that the distribution of M_g is also normal, with a coefficient of variation (COV) of 5%. This COV corresponds the standard deviation of $0.05 \times 608.2 = 30.4$ kNm. Thus the probabilistic model that describes M_g is:

$$M_g \sim \mathrm{N}(608.2, 30.4) \quad [\mathrm{kNm}]. \tag{6}$$

5.1.2 Additional permanent load

Danish guideline (Vejdirektoratet 2004) recommends for the additional permanent load a normal model with a COV of 10% and mean equal to the nominal value. The bending moment at the critical cross-section due to this load was estimated in 108.4 kNm, so that the probabilistic model to be adopted is:

$$M_{\Delta g} \sim \mathrm{N}(108.4, 10.8) \quad [\mathrm{kNm}]. \tag{7}$$

5.1.3 Traffic loads

In this subsection the probability distribution of the variable Q ={Load introduced by each wheel of the standard vehicle that are crossing the bridge at a given time} will be discussed (see Figure 4). The variable representing the maximum of Q in n years will be denoted by Q_n. According to the Portuguese code (RSA 1983), which uses $n = 50$ years as the reference period, the 0.95-quantile of Q_{50}, denoted by Q_{50k}, is 50 kN.

It will be assumed that Q follows a normal distribution, in agreement with Vejdirektoratet (2004) and BRIME (2001). Therefore the maximum of Q in n years, Q_n, tends asymptotically to the Gumbel model (Ang and Tang 2007). Thus it will be considered that $Q_n \sim \text{Gumb}(u_n, \alpha_n)$, whose cumulative distribution is given by:

$$F_{Q_n}(x) = \exp\Big(- \exp\big(- \alpha_n(x - u_n)\big)\Big). \tag{8}$$

where α_n and u_n are the model parameters, related to the mean and standard deviation by:

$$\mu_{Q_n} = u_n + \frac{\gamma}{\alpha_n}; \qquad \sigma_{Q_n} = \frac{\pi}{\sqrt{6}\,\alpha_n}, \tag{9}$$

where $\gamma \cong 0.57722$ (Euler constant).

It can be demonstrated that if Q_n follows a Gumbel distribution, the same applies to the variable Q_1. Another important result is that the parameter α is invariant to the reference period n, that is, $\alpha_1 = \alpha_n = \alpha$. The parameter u_n is related to u_1 through the expression:

$$u_n = u_1 + (1/\alpha)\ln n. \tag{10}$$

The characteristic value (0.95-quantile) of the Gumbel distribution, which can be obtained inverting Equation (8), is given by:

$$Q_{nk} = \mu_{Q_n}(1 + 1.866\, V_{Q_n}), \tag{11}$$

where μ_{Q_n} and V_{Q_n} represent, respectively, the mean and COV of Q_n.

The probabilistic model of Q_{50} will be defined so that $Q_{50k} = 50$ kN, in accordance with the Portuguese code RSA (1983). Considering that there is an infinity of distributions with that characteristic value, it is necessary to specify one additional parameter. In *Commentary on CAN/CSA-S6-00* (CAN/CSA-S6-00 2000) there is a comment that recommends for traffic loads on roadway bridges a coefficient of variation V of 0.035 concerning the annual maxima distribution. The authors of the research project BRIME (2001), based on studies of real traffic have obtained coefficients of variation (for annual maxima) of the same order of magnitude. It may seem at first a very low COV but, given that in one year several thousand of vehicles cross a typical bridge (even bridges on roads with moderate traffic intensity), it is not surprising that the annual maxima has low variability. The variability of the maximum in 50 years is even lower.

In the present study it was adopted for the distribution of the maximum in 50 years $V_{Q_{50}} = 0.05$, which, in light of the above comments, can be considered as a conservative value. Thus, assuming that $Q_{50k} = 50$ kN, the above equations yielded the following model:

$$Q_1 \sim \text{Gumb}(38, 0.56) \quad [\text{kN}]. \tag{12}$$

According to the beam model described earlier, the bending moment at the critical cross-section due to the three wheels of the vehicle, each one introducing 50 kN, was estimated in 506.3 kNm. Thus, the transformation model for the bending moments due to traffic loads is:

$$M_{Q_1} = (506.3/50)\, Q_1 \quad [\text{kNm}]. \tag{13}$$

5.1.4 Concrete strength

Table 1 shows the core testing results of nine cores taken from the bridge, three from the abutments, three from the piers and three from the deck. As seen previously, considering all cores instead of only the cores taken from the deck, a lower characteristic strength is obtained, so that it was decided to use all cores. The mean, standard deviation and coefficient of variation of that sample are as follows:

$$\bar{f}_c = 51.2 \times 10^3 \text{ kN/m}^2; \quad s = 13.2 \times 10^3 \text{ kN/m}^2; \quad V = 0.26. \tag{14}$$

To describe the strength f_c of the concrete, both the normal and lognormal models have been recommended in the literature (JCSS 2001, Wisniewski 2007). In the present case, since the COV of this property is relatively high ($V = 0.26$), considering a normal model would result in non-negligible probability of negative values. This is physically impossible and would result in erroneous results. For this reason the lognormal model was considered more appropriate.

Considering the lognormal distribution to model the strength f_c and assuming that $\mu_{fc} = \bar{f}_c$ and $\sigma_{fc} = s$, the lognormal distribution parameters are $\mu_X = 10.81$ and $\sigma_X = 0.25$. The model for $Y = f_c$ is then:

$$f_{f_c}(y) = \text{LN}(y \mid 10.81, 0.25). \tag{15}$$

This model, however, does not take into account the statistical uncertainty, that is, the uncertainty originated by the fact that the parameters μ_X and σ_X were estimated from a finite sample. It is important to evaluate the impact of this uncertainty on the reliability estimation of the bridge.

The Bayesian approach has been widely accepted as the appropriate tool to deal with statistical uncertainty (Engelund and Rackwitz 1992). The predictive Bayesian model of the concrete strength (lognormal population) can be readily obtained from the affinity between the normal and lognormal models. Suppose that $f_c \sim \text{LN}$ and that the sample $f_c = (f_{c1}, \ldots, f_{cn})$ of concrete strengths is available. Thus the sample $(x_1, \ldots, x_n) = (\ln f_{c1}, \ldots, \ln f_{cn})$ belongs to a normal population. Let $\bar{x}$ be the mean of this sample and s_X its standard deviation. Assuming that there is no relevant prior information, the Bayesian predictive model for $X = \ln f_c$ is given by (Bernardo 2009):

$$f_X(x) = \text{St}\left(x \mid \bar{x}, s_X\sqrt{1 + \frac{1}{n}}, n - 1\right), \tag{16}$$

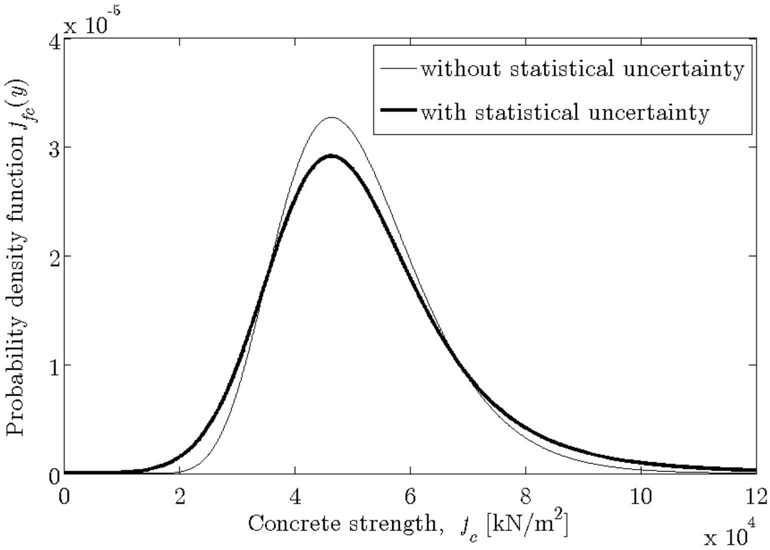


Figure 6. Comparison of the models for concrete strength: without statistical uncertainty (Equation (15)) and with statistical uncertainty (Equation (18)).

where $\mathrm{St}(x \mid a, b, \nu)$ is the three parameters t-Student distribution, a is the mean (which coincides with the sample mean $\bar{x}$), $b = s_X\sqrt{1 + 1/n}$, $\nu = n - 1$ is number of freedom degrees and n is the sample size.

The predictive Bayesian model for f_c is determined considering that $f_c = e^X$, as follows:

$$F_{f_c}(y) = P(f_c < y) = P(e^X < y) = P(X < \ln y) = F_X(\ln y).$$

Applying derivatives to this equation, it follows immediately that $f_{f_c}(y) = (1/y)f_X(\ln y)$, that is:

$$f_{f_c}(y) = (1/y)\ \mathrm{St}\left(\ln y \mid \bar{x}, s_X\sqrt{1 + \frac{1}{n}}, n - 1\right). \tag{17}$$

Considering now the sample of cores available, the following model was obtained:

$$f_{f_c}(y) = (1/y)\ \mathrm{St}\left(\ln y \mid 10.81, 0.276, 8\right). \tag{18}$$

This is the predictive Bayesian model of the strength of the concrete of the bridge, which includes appropriately the effect of the statistical uncertainty. Figure 6 compares this model with the model expressed in Equation (15). As it can be seen, statistical uncertainty causes the weight of the distribution tails to increase.

It was found that the models corresponding to Equations (15) and (18) resulted in similar estimates of the reliability of the bridge, that is, the statistical uncertainty induced by the limited sample of cores have a relatively small impact, so it could be neglected in the present case. Moreover, it shows that extracting more cores from the bridge would not have a significant impact of the safety assessment.

5.1.5 Reinforcing steel strength

As mentioned before, the original design plans of the bridge specified ribbed and cold worked grade A-40 reinforcement steel. The strength of this type of steel is characterized by the 0.2% proof stress, here denoted by $f_{0.2}$. The Portuguese code in use when the bridge was built prescribed for A-40 grade the characteristic value ($f_{0.2k}$) of 40 kgf/mm^2, or 400 MPa.

Concerning the probabilistic model of $f_{0.2}$ the recommendation of *Probabilistic Model Code* (PMC) was adopted (JCSS 2001). Regarding yield stress, which can be seen as equivalent to 0.2% proof stress, PMC recommends a standard deviation σ equal to 30 MPa, which can be separated into three independent sources of variability: (1) variability between different mills ($\sigma_1 = 19$ MPa), (2) variability from batch to batch within the same mill ($\sigma_2 = 22$ MPa), and (3) variability within a batch ($\sigma_3 = 8$ MPa). It was decided to consider in the present study all three sources of variability and hence $\sigma = 30$ MPa was adopted, although probably the steel of the deck came from the same mill.

With regard to the mean μ of the 0.2% proof stress, PMC recommends $\mu = f_{\mathrm{nom}} + 2\sigma$, where f_{nom} is the nominal 0.2% proof stress, 400 MPa for A-40 grade. Thus for this grade the mean is $\mu = 400 + 2 \times 30 = 460$ MPa. Therefore the probabilistic model adopted for reinforcing steel strength was:

$$f_{0.2} \sim \mathrm{N}(460, 30) \quad [\mathrm{MPa}]. \tag{19}$$

This model has resulted from a recommendation of a relatively recent code (JCSS 2001) when compared with the age of the bridge. It is appropriate to discuss if such a recommendation is applicable to the bridge under assessment. In this respect it is interesting to note that, regarding the safety factor for reinforcing steel, the old Portuguese code in use at the time of designing the bridge (about 40 years ago) indicated 1.15, which is still recommended by recent codes. This shows that the confidence about the steels produced in that period did not change since then, suggesting that the model expressed in Equation. (19) is adequate.

The COV of this model is $30/460 = 0.065$. Coefficients of variation of about 10% have been reported (Wisniewski 2007). However, these refer to populations involving various producers, and not a single site. It is believed, therefore, that the standard deviation of 30 MPa fits the steel used in the bridge.

5.1.6 Cross-section of the reinforcing steel

The variability of steel strength, as described by the model for $f_{0.2}$, already includes the variability of the cross-section area of the reinforcing bars. The reason is that the stresses $f_{0.2}$ are in general obtained dividing the forces measured in tensile tests by the nominal area, not by the real one. Therefore, there is no need to consider uncertainty in cross-section area of the reinforcing steel, except that arising from the lack of knowledge regarding the section loss due to corrosion.

The mid-span of the central span cross section presents two layers of reinforcement, the first with $6\phi25$ and the second with $5\phi25$ (Figure 7). Reinforcement loss was only considered in the first layer, while the second layer was considered intact. To model the state of section loss, the variable i_{res} was introduced as:

$$i_{\mathrm{res}} = \frac{A_{\mathrm{res}}}{A}, \tag{20}$$

where A_{res} is the residual section area and A represents the original section area. The variable i_{res}, here called *residual section index*, is then comprised between 0 and 1, where 0 corresponds to total loss and 1 corresponds to intact section.

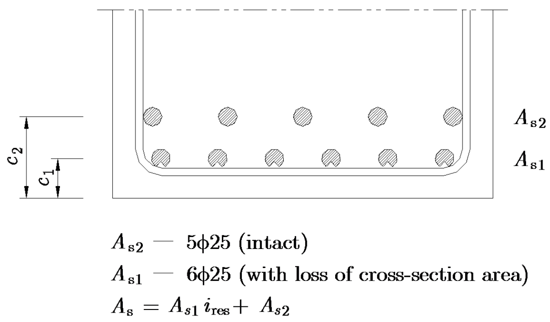


Figure 7. Reinforcements of the critical cross-section.

Since the residual section of reinforcement bars is unknown, the variable $X = i_{\text{res}}$ was modeled as a random variable, in accordance with the Bayesian interpretation of probability. Since this variables has well defined limits (0-1), it was modeled with a Beta distribution, whose PDF is given by:

$$f_X(x \mid \alpha, \beta) = c \cdot x^{\alpha-1}(1-x)^{\beta-1} \quad (0 < x < 1, \quad \alpha > 0, \quad \beta > 0), \tag{21}$$

where c is a normalization constant.

The reliability of the bridge had a significant sensitivity to the residual section index, so it was important to model this parameter as accurately as possible. It was decided to use the Bayesian paradigm, allowing the model to incorporate further information, taken from the bridge. Thus the parameters of the model, α and β, were modeled themselves as random variables. Initially little was known about these parameters, whereby it was decided to model them with a uniform distribution in the interval $[1, 8]$. This interval ensured all foreseeable shapes of the PDF were considered.

Figure 8 shows the predictive Bayesian histogram of the residual section index $X = i_{\text{res}}$ considering that α and β are independent and with uniform distribution within the interval $[1, 8]$. As shown in Figure 8, the mean of the distribution is 0.5, which is consistent with the preliminary analysis made earlier.

5.1.7 Reinforcement position

Since the position of the reinforcements is not accurately known, the distances c_1 and c_2 (Figure 7) were modelled as random variables, with uniform distributions. The following probabilistic models were adopted:

$$c_1 \sim \text{Unif}(0.04, 0.06) \quad [\text{m}], \tag{22}$$

$$c_2 \sim \text{Unif}(0.09, 0.13) \quad [\text{m}]. \tag{23}$$

These models were based on the following positioning tolerances: $c_1 = 0.05 \pm 0.01$ and $c_2 = 0.11 \pm 0.02$.

5.1.8 Uncertainty in the transformation models

In the limit state function $Z = \theta_R M_R - \theta_E M_E$, described earlier, two important transformation models are used: the resistance model and the structural model. The first relates basic variables (material properties and geometric quantities) with bending strength and the second relates actions and other basic variables with load effects. The uncertainty in these models is described through the variables θ_R and θ_E, which in accordance with

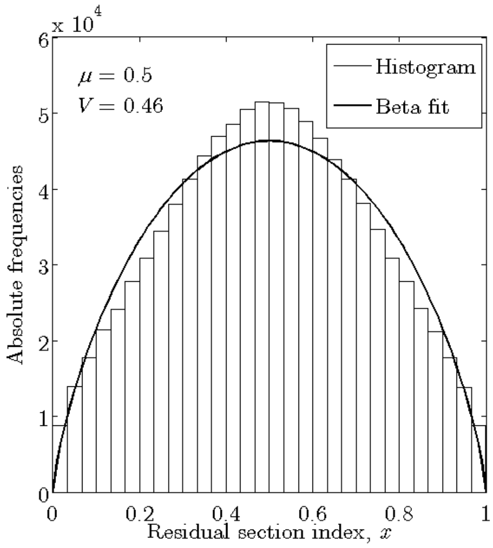


Figure 8. Bayesian predictive histogram of the residual section index, $X = i_{\mathrm{res}}$, considering that α and β are independent and uniformly distributed within the interval $[1, 8]$.

Probabilistic Model Code (JCSS 2001) were modelled with lognormal distributions.

Scarce information exists on the uncertainty regarding both the resistance and the load effect models for existing structures. In terms of resistance models, the bending resisting model is very reliable, and a low uncertainty can be assumed. Regarding the effects of loads, different authors have proposed the use of improved structural models, such as considering the non-linear behaviour of the structure (Strauss et al. 2009, Bergmeister et al. 2009), aiming at minimizing the uncertainty in the model. In the present case, a simple model, similar to those used in the design phase, was employed. For this reason, the uncertainty in the load effect model was taken similar to that recommended by Danish guideline (Vejdirektoratet 2004) concerning structural models with normal accuracy.

To define the mean and standard deviation of the variables θ_R and θ_E it is important to bear in mind their meaning. The mean constitutes a measure of the model accuracy and the standard deviation a measure of its precision. Accuracy of the transformation models defines their ability to predict values close to the actual values, and precision their ability to predict values with little scatter. Lack of accuracy and precision might be the result of the existence of other variables that affect the model response and that are not being considered in the model, or might be simply the result of lack of knowledge.

Regarding the structural model, it is useful to remember the three types of equations involved in the model: (1) equilibrium, (2) constitutive laws and (3) boundary conditions. For the first type, it can be stated with confidence that they are satisfied. Concerning the second type, the structural model developed has assumed linear elastic behaviour for the materials, which deviates from the reality, especially since high loads are considered (ultimate limit states). However, since the first cross-section to exhibit non-linear behaviour is the mid-span cross-section, the calculated bending moments M_E in that section tend to be higher than the real ones, due to transfer of bending moments from mid-span to supports. Thus, from this point of view, the structural model deviates from reality, but in the safe side. This would correspond to adopt θ_E with mean less than one. Concerning boundary conditions, a source of error, and hence a source of uncertainty, would be for example the occurrence of foundations movements, not taken into account in the structural model, which assumes rigid supports. This assumption was judged as

Table 2. Probabilistic models for each basic variable.

Variable	Symbol	Unit	Distribution	Parameters		Note
Bending moment due to self-weight	M_g	kNm	Normal	$\mu = 608.2$	$\sigma = 30.2$	
Bending moment due to add. dead load	$M_{\Delta g}$	kNm	Normal	$\mu = 108.4$	$\sigma = 10.8$	
Weight introduced by a vehicle wheel	Q_1	kN	Gumbel	$u = 38.0$	$\alpha = 0.56$	(1)
Concrete strength	f_c	kN/m^2	Lognormal	$a = 10.81$	$b = 0.25$	
Reinforcing steel strength	$f_{0.2}$	kN/m^2	Normal	μ =460E3	σ =30E3	
Residual section index	i_{res}	–	Beta	variable	variable	(2)
Bottom dist. of the 1$^{\text{st}}$ layer of reinf. steel	c_1	m	Uniforme	$a = 0.04$	$b = 0.06$	
Bottom dist. of the 2$^{\text{nd}}$ layer of reinf. steel	c_2	m	Uniforme	$a = 0.09$	$b = 0.13$	
Structural model uncertainty	θ_E	–	Lognormal	$\mu = 1.0$	$V = 0.05$	(3)
Resistance model uncertainty	θ_R	–	Lognormal	$\mu = 1.0$	$V = 0.05$	(3)

(1) Annual maxima.
(2) Parameters considered variables, according to the Bayesian paradigm.
(3) The mean and COV shown refers to the parameters of distribution itself and not the parameters of the underlying normal distribution.

satisfied with a reasonable degree of certainty, considering the type of foundations (pile foundations) and the age of the bridge (about forty years).

When using frame models, *Probabilistic Model Code* (JCSS 2001) recommends for θ_E a mean μ of 1.0 and a COV of 0.10. The authors believe that, for the present case, this COV is excessive, as it was defined for new structures rather than existing structures. The Danish guideline Vejdirektoratet (2004) recommends $V = 0.04$ for structural models with good accuracy, $V = 0.06$ for structural models with normal accuracy and $V = 0.09$ for structural models with poor accuracy. In the present study, the authors adopted $V = 0.05$, leading to the following model:

$$\theta_E \sim \mathrm{LN}(\mu_{\theta_E} = 1.0, V_{\theta_E} = 0.05). \tag{24}$$

With respect to the variable θ_R, the resistance moment M_R was computed using a rectangular stress distribution in the compressed zone and taking the following assumptions: (1) ultimate strain in concrete equal to 0.035; (2) elasto-plastic diagram for the steel, without limit strain; (3) Bernoulli assumption (plane sections remain plane) and (4) perfect bond between steel and concrete. It is well known that these assumptions lead to satisfactory results, having good agreement with laboratory tests.

The *Probabilistic Model Code* (JCSS 2001) recommends the model $\theta_R \sim \mathrm{LN}(\mu_{\theta_R} = 1.2, V_{\theta_R} = 0.15)$. These model parameters (mean and COV) do not seem suitable in the face of the above comments. In this study the authors adopted a model in agreement with the recommendations found in Melchers (1999):

$$\theta_R \sim \mathrm{LN}(\mu_{\theta_R} = 1.0, V_{\theta_R} = 0.05). \tag{25}$$

Table 2 summarizes the probabilistic models described in this section.

5.2 *Reliabilty analysis*

Once the probabilistic models have been defined, the probability $p_f = P(Z < 0)$ was computed. The failure probability was evaluated using both the Monte Carlo Method (MCM) and FORM. MCM yielded $\beta = 3.04$ and FORM $\beta = 2.96$. There is then a differ-

Table 3. FORM sensitivity coefficients.

Variable	Symbol	α	α^2
Bending moment due to self-weight	M_g	0.18	0.031
Bending moment due to add. dead load	$M_{\Delta g}$	0.06	0.004
Weight introduced by a vehicle wheel	Q_1	0.14	0.020
Concrete strength	f_c	-0.02	0.000
Reinforcing steel strength	$f_{0.2}$	-0.47	0.219
Residual section index	i_{res}	-0.71	0.502
Bottom dist. of the 1st layer of reinf. steel	c_1	0.00	0.000
Bottom dist. of the 2nd layer of reinf. steel	c_2	0.08	0.007
Structural model uncertainty	θ_E	0.33	0.108
Resistance model uncertainty	θ_R	-0.33	0.108
	Σ		1.00

ence of 2.6% between the methods, attributed to the fact that FORM is an approximate method. Since MCM can be considered an exact method, the estimate $\beta = 3.04$ was considered correct.

It is now necessary to compare this reliability index with the target reliability, β_T. The only European official recommendation concerning to target reliability levels is that contained in the standard EN1990 (2002), which recommends $\beta_T = 3.8$ for a 50 years reference period and reliability class RC2 (medium consequences). The period of 50 years must be regarded as a reference related to the life time of the structure, not strictly 50 years (Steenbergen and Vrouwenvelder 2010). Thus, in the present case, the target reliability index for one year was taked as $\beta_T = 3.8$. The bridge hence does not fulfill the reliability criterion stated in EN1990 (2002). It should be noted, however, that the above reliability would be considered acceptable in the USA (Casas and Wisniewski 2013).

A reliability analysis should be accompanied by a sensitivity analysis. Table 3 shows the FORM sensitivity coefficients α. The sensitivity coefficient constitutes a measure of the impact that each variable has on the estimated reliability index. Consequently, it is a measure of the potential improvement in reliability estimates that could be obtained if additional data is gathered.

Figure 9 plots the squares of the FORM sensitivity coefficients. As shown, the residual section index i_{res} is the variable with the largest sensitivity coefficient, followed by the strength of the reinforcing steel, $f_{0.2}$, and the variables θ_E and θ_R. This results showed that it was justifiable any attempt to collect more data concerning cross-section loss of the reinforcing bars. The very low sensitivity coefficient concerning the concrete strength, f_c, shows that there was no need of more tests concerning the concrete strength.

6. Bayesian updating of the residual section index

6.1 *Collection of information on residual areas of reinforcement*

The residual section index i_{res} (used to quantify the remaining cross-section area of the corroded reinforcing steel) was the variable with the greatest impact on the estimated reliability of the bridge, for which $\alpha^2 = 0.5$. This means that it was advisable to seek for more information about i_{res}, if possible, in order to reduce its uncertainty. With this

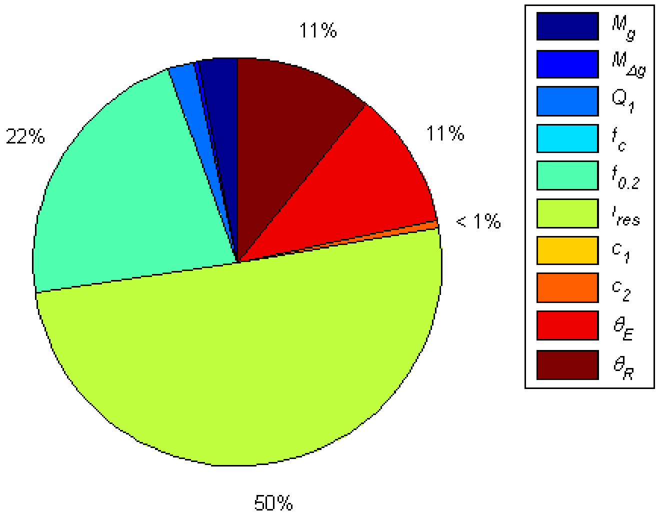


Figure 9. Squares of FORM sensitivity coefficients, α^2.

Figure 10. Condition state of the critical cross-section.

purpose in mind, a set of measurements of diameters of corroded reinforcing bars were carried out. Due to the difficulty in accessing the mid-span of the central span section and the increased risk resulting from additional damage in this cross-section, several locations, near the South abutment, were selected as representative of the condition of the critical cross-section (Figure 10).

Firstly, it was observed that, in that cross-section, the concrete cover had not yet detached, except for the inside edge. Thus, the works started by selecting an area of the beam near the South abutment with concrete not yet detached, and a small window was opened to reveal the actual condition of the reinforcing bars. Figure 11 shows the window being opened, which was afterwards sealed. It was possible to verify the very good condition of the bars, transverse and longitudinal, both being of the ribbed type. By using a vernier caliper, the bar diameters were measured, the stirrups being of 10 mm and the longitudinal of 25 mm, which is in accordance with the original design plans.

Figure 11. Window to inspection the condition of reinforcement bars.

Figure 12. Diameter measurement of an exposed bar.

This observation made it possible to conclude that in areas where there was no detached concrete, it is probable that the reinforcing bars had no significant loss of cross-section.

Subsequently, a bar located in an area with exposed reinforcement was measured Figure 12. A preliminary observation seemed to indicate that the bar had considerable loss of cross-section, but after cleaning it by a steel brush, it was found out that the diameter was still 25 mm, showing thus that it is possible to find exposed bars without cross-section loss. Next measurement focused on a bar extremely corroded, located in the edge of the beam, Figure 13, whose diameter was 17 mm. Lastly, an edge bar, in a condition very similar to the edge of the critical cross-section, was measured (see Figure 14.) The measured diameter was 18 mm.

Figure 13. Diameter measurement of an extremely corroded bar.

Figure 14. Diameter measurement of an edge corroded bar.

In brief, the diameters measured were: 25, 25, 17 and 18 mm. The residual section index i_{res} can be expressed in terms of bar diameters as:

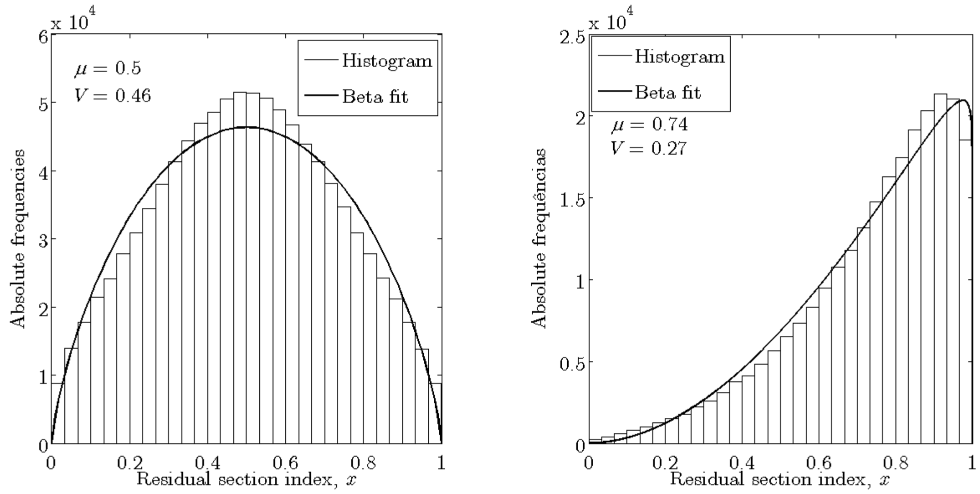
$$i_{\text{res}} = \frac{A_{\text{res}}}{A} = \frac{\pi\phi_{\text{res}}^2/4}{\pi\phi^2/4} = \left(\frac{\phi_{\text{res}}}{\phi}\right)^2, \tag{26}$$

which gave the following sample of the variable i_{res}:

$$\varepsilon = \{0.99, 0.99, 0.46, 0.52\}. \tag{27}$$

6.2 *Updating of the residual section index predictive model*

To generate via MCM a posterior predictive sample (or updated sample) of the residual section index, the algorithm described earlier was implemented in a MATLAB routine. Figure 15 shows the histogram of the generated sample, together with the prior his-



(a) Prior histogram (b) Posterior histogram

Figure 15. Predictive histograms of the residual section index, $X = i_{\text{res}}$.

togram. As shown, the observed sample ε caused a significant probabilistic mass to move to the right. There was thus a substantial change in the expectations regarding the steel reinforcement loss.

The updated probabilistic model of the residual section index i_{res} was used to update the estimated reliability of the bridge, for which $\beta = 3.9$ was obtained. This reliability index represents an increase of 28% when compared with the initial estimate. This example shows how significantly the reliability estimate can change when using information collected from a bridge in assessment.

The value $\beta = 3.9$ is greater than the target reliability index, showing that the risk of failure of the bridge could be considered acceptable in one year and the bridge was then kept in service during this time.

6.3 *Additional considerations about the Bayesian model*

The Bayesian paradigm provides a formal mechanism for changing probabilities, or changing beliefs, in the Bayesian sense. To better appreciate this point, Figure 16 shows the evolution of the predictive histogram of the i_{res}-variable as measurements were successively been made, namely, $x = \{0.99\}$, $x = \{0.99, 0.99\}$, $x = \{0.99, 0.99, 0.46\}$ and $x = \{0.99, 0.99, 0.46, 0.52\}$.

It is noted that after the first observation, $x = \{0.99\}$, the predictive histogram has suffered a major change, showing that the prior histogram was little informative, with small impact on the final histogram. It should be mentioned also that the final histogram is independent of the sequence of observations.

7. Conclusions

When assessing an existing bridge, in general due to safety concerns, one must start by employing simple methods of safety, as the method of partial safety factors. If the bridge fails the assessment, the decision to strengthen the bridge should be carefully considered. Experience has shown that bridges that do not meet traditional safety criteria, might

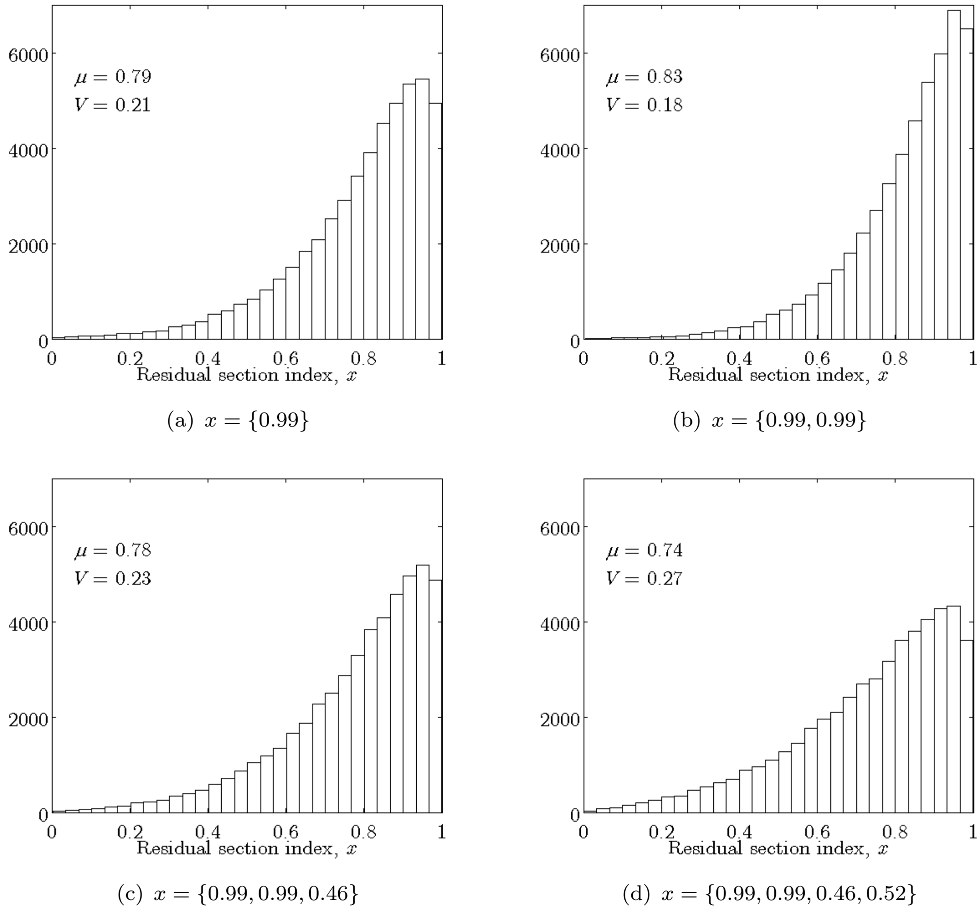


Figure 16. Evolution of the predictive histogram of the residual section index, $X = i_{\text{res}}$, as observations were being available.

have acceptable levels of reliability, as seen in the case study presented.

In most situations the decision of strengthening a bridge should not be taken without first carrying out a probabilistic assessment of the problem. If the probabilistic analysis leads to the conclusion that the reliability is acceptable, the funds saved with correcting measures could be employed more effectively in maintenance work.

The probabilistic assessment allows modelling consistently the different sources of uncertainty, which are specific to the problem at hand. Moreover, a probabilistic analysis makes it possible to perform a sensitivity analysis, showing which variables must be investigated by collecting new information from the bridge in order to reduce their uncertainty. Once the new information is collected, Bayesian methods can then be applied in order to update the probabilistic distributions of those variables.

Besides allowing the updating of probabilistic models, the case study highlighted another strong point of Bayesian methods: they assure that the uncertainty caused by the use of small sample size gathered from de structure is always taken into account.

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