Absorbing state phase transition with competing quantum and classical fluctuations

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Stochastic processes with absorbing states feature examples of non-equilibrium universal phenomena. While the classical regime has been thoroughly investigated in the past, relatively little is known about the behavior of these non-equilibrium systems in the presence of quantum fluctuations. Here we theoretically address such a scenario in an open quantum spin model which in its classical limit undergoes a directed percolation phase transition. By mapping the problem to a non-equilibrium field theory, we show that the introduction of quantum fluctuations stemming from coherent, rather than statistical, spin-flips alters the nature of the transition such that it becomes first-order. In the intermediate regime, where classical and quantum dynamics compete on equal terms, we highlight the presence of a bicritical point with universal features different from the directed percolation class in low dimension. We finally propose how this physics could be explored within gases of interacting atoms excited to Rydberg states.

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Introduction. — Non-equilibrium phenomena can be found in many different contexts, ranging from chemical reactions to disease-spreading. Analogously to the equilibrium case, non-equilibrium ensembles can show the emergence of universal behavior, signaling the irrelevance of the microscopic details of the dynamics for macroscopic observables. This occurs when such out-of-equilibrium systems start to act collectively [1–4]. A distinction can be made depending on the presence or absence of detailed balance [5–8], between systems which evolve towards a stationary equilibrium state [9] (e.g., quenched systems coupled to thermal baths [10]) or that preserve their non-equilibrium character even in the long-time limit, representing flux equilibrium states.

Directed percolation (DP) [11] constitutes an instance of a classical phase transition to an *absorbing state*, i.e., a state which can be reached, but not left by the dynamics, and represents a simple instance of a broader class of intrinsically non-equilibrium phase transitions [11–14]. Despite its robustness, its experimental observation has so far been elusive [15], with a single exception [16, 17]. However, it was recently suggested to realize and explore DP dynamics in cold gases of atoms excited to high-lying Rydberg states [18]. In this work, we harness the opportunities that result from the fact that Rydberg gases are actually open *quantum* systems to go beyond the realm of classical physics (see also [19]), and establish a generalised absorbing state phase transition in the presence of quantum fluctuations. *Driven-dissipative* systems constitute indeed an ideal platform for the investigation of the interplay between classical and quantum effects, and have been recently addressed in a broad range of experiments. The spectrum includes light-driven semiconductor heterostructures [20], arrays of driven microcavities [21, 22], cold atoms in optical lattices [23], cavities [24, 25] and microtraps [26–28]. Several among these instances employ excitation of the atoms to high-lying Rydberg orbitals [29–31] in order to achieve strong interatomic interactions and to study cooperative effects [32–36].

In these systems, the driving/dissipation not only introduces coherence loss, but also explicitly violates the equilibrium conditions at the microscopic level [7, 37]. It is thus a challenge to identify to what extent the non-equilibrium and quantum nature of the dynamics impact on the macroscopic phase diagram and phase transition properties. Oftentimes, upon coarse graining such systems lose their quantum character and equilibrium conditions are effectively restored [38–43]. But there are instances where non-equilibrium [44, 45] and quantum [46, 47] aspects persist even at asymptotically large wavelength. The transition we highlight here does not fall into the DP universality class, and its origin can be unambiguously traced back to the presence of coherent dynamics. More precisely, the latter introduces a first-order non-equilibrium phase transition without counterpart in the purely classical DP problem. This discontinuous phase transition terminates in a bicritical point which even asymptotically at large distances and in dimensions $d < 2$, does not feature the symmetries underlying DP, or any equilibrium problem.

Model. — We reproduce a quantum variant of the *contact process* (for an introduction see [11]). Basically, it consists of a lattice of "active" and "inactive" sites, where the former can spontaneously decay to inactive, whereas activation can only occur in the proximity of already active sites. Thus, the fully-inactive state is absorbing. Specifically, we consider a lattice of quantum two-level systems with spacing r. On every site k we define the basis $|a_k\rangle$ (active) and $|i_k\rangle$ (inactive), the density of active sites $n_k = |a_k\rangle\langle a_k|$ and the ladder operators $\sigma_k^+ = |a_k\rangle\langle i_k|$ and $\sigma_k^- = |i_k\rangle\langle a_k|$. Under the action of Markovian noise sources, the state ρ of the sys-

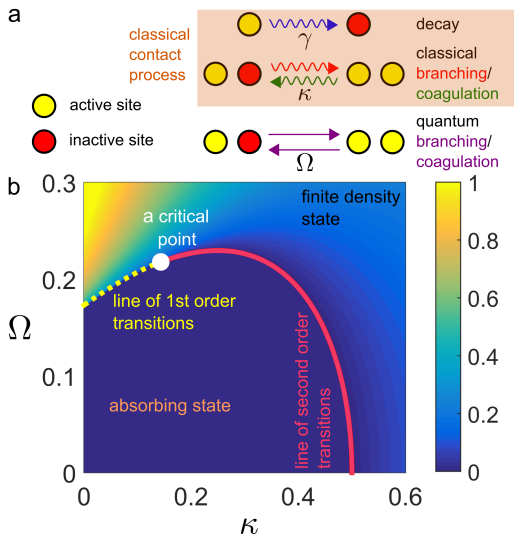

Figure 1. (Color online) (a) Fundamental processes. We consider a lattice whose sites admit two states: active (red) and inactive (green). Active sites decay to inactive at a rate γ. Proliferation of active sites is possible through classical (rate κ) and quantum (strength Ω) branching. (b) One-dimensional ($z = 2$) phase diagram constructed from the effective action (5) in saddle-point approximation (color code corresponds to density of active sites). All parameters are measured in units of γ. In the classical limit ($\Omega = 0$) the system exhibits a continuous (2nd order) directed percolation phase transition between an absorbing state and a finite-density one. This transition extends into the quantum regime (thick red line) up to the critical point α. In the quantum limit ($\kappa = 0$) a first-order transition is found which also extends into the classical regime (dashed yellow line) up to point α. In the neighborhood of this line, a narrow region of coexistence of two attractive stationary solutions is present, which is not resolved here. The high values of the density reached in the active phase stem from neglecting higher orders in n in the action, which would otherwise enforce $n \leq 1/2$.

tem evolves according to the Lindblad equation [48, 49] $\dot{\rho} = -i\,[H,\rho] + \sum_{a,k} \mathbf{D}[L_{a,k}]\rho$ [see sketch in Fig. 1], where

$$H = \Omega \sum_k C_k\, \sigma_k^x \quad \text{with} \quad C_k = \sum_{j\,\mathrm{nn}\,k} n_j \tag{1}$$

is the quantum Hamiltonian, $\sigma_k^x = \sigma_k^+ + \sigma_k^-$, and "nn k" denotes nearest neighbors (nn) of site k; $\mathbf{D}[X]\rho = X\rho X^\dagger - (X^\dagger X\rho + \rho X^\dagger X)/2$ is the dissipator and $L_{a,k}$ are the so-called jump operators, with indices a (process type), and k (lattice site). These jump operators are chosen to define a modified contact process [11], which is known to feature a DP transition, and include *decay* $L_{\mathrm{d},k} = \sqrt{\gamma}\sigma_k^-$ ($|a_k\rangle \to |i_k\rangle$) and — for every neighbor j of k — *branching* $L_{\mathrm{b},j,k} = \sqrt{\kappa}\, n_j\, \sigma_k^+$ (an active site can activate a neighboring one $|a_j i_k\rangle \to |a_j a_k\rangle$) and *coagulation* $L_{\mathrm{c},j,k} = \sqrt{\kappa}\, n_j\, \sigma_k^-$ (the inverse process $|a_j a_k\rangle \to |a_j i_k\rangle$). The "constraint" operator C_k in H represents the simplest choice reproducing the requirement of an active site nearby to flip a spin; this makes H the "minimal quantum equivalent" of the noisy branching/coagulation above. Similar constrained Hamiltonians have been studied in the past with a focus on quantum glassy behavior [19] and many-body localization [50, 51].

Equations of motion and density path integral. — We infer here the properties of the phase diagram by exploiting an effective path integral description for the density variable n_k alone. We start by deriving the Heisenberg-Langevin equations of motion (EOM) [52] for the single-site operators n_k, σ_k^x and $\sigma_k^y = -i\sigma_k^+ + i\sigma_k^-$. For convenience we introduce the coordination number z (number of nearest neighbors per lattice site) and the shorthand $P_k^{x/y} = \sigma_k^{x/y} \sum_{j\,\mathrm{nn}\,k} \sigma_j^x$. In the following, we also measure all times and energies in units of γ, i.e., we set $\gamma = 1$:

$$\dot{n}_k = -n_k + \left[\Omega\sigma_k^y - \kappa(2n_k - 1)\right] C_k + \hat{\xi}_k^n, \tag{2}$$

$$\dot{\sigma}_k^x = \Omega P_k^y - \tfrac{z\kappa+1}{2}\sigma_k^x - \kappa\sigma_k^x C_k + \hat{\xi}_k^x, \tag{3}$$

$$\dot{\sigma}_k^y = \Omega P_k^x - \tfrac{z\kappa+1}{2}\sigma_k^y - \left[\Omega(4n_k - 2) + \kappa\sigma_k^y\right] C_k + \hat{\xi}_k^y. \tag{4}$$

The quantum noise terms $\hat{\xi}_k^\alpha$ consider the fluctuations of the bath and depend on the structure of the jump operators. They show vanishing averages but non-trivial, Markovian correlations, which for the present setup are (in rescaled units) $\langle\hat{\xi}_k^x\hat{\xi}_{k'}^x\rangle = \langle\hat{\xi}_k^y\hat{\xi}_{k'}^y\rangle = \delta_{k,k'}$, $\langle\hat{\xi}_k^n\hat{\xi}_{k'}^n\rangle = \delta_{k,k'}n_k$, $\langle\hat{\xi}_k^x\hat{\xi}_{k'}^y\rangle = -i\delta_{k,k'}$, $\langle\hat{\xi}_k^n\hat{\xi}_{k'}^x\rangle = -\delta_{k,k'}\sigma_k^+$ and $\langle\hat{\xi}_k^n\hat{\xi}_{k'}^y\rangle = i\delta_{k,k'}\sigma_k^+$ up to leading order in the density [53].

In the following, we work in the continuum limit $(k,t) \to (\vec{x},t) \equiv X$ and derive an effective path integral for the density field n_X via a Martin-Siggia-Rose (MSR) construction [3, 56–58], presented in the supplemental material [53]. Crucially, the $\sigma^{x,y}$-fields are gapped, and thus can be integrated out perturbatively. The resulting long-wavelength field theory depends on the density variable n alone, and is obtained by additionally performing a derivative expansion of the action. It reads

$$\begin{aligned} S_n = &\int_X \tilde{n}_X\Big[(\partial_t - D\nabla^2 + \Delta)n_X + u_3 n_X^2 + u_4 n_X^3\Big] \\ &- \int_X \left[\tfrac{1}{2}\tilde{n}_X^2 n_X + \mu_4\tilde{n}_X^2 n_X^2\right] \equiv S_n^{(1)} + S_n^{(2)}, \end{aligned} \tag{5}$$

where $D = r^2\kappa$ represents a diffusion constant and $\Delta = 1 - z\kappa - \frac{8z^2\Omega^2}{(z\kappa+1)^3}$, $u_3 = 2z\left(\kappa - \frac{2z\Omega^2}{z\kappa+1}\right)$, $u_4 = \frac{8z^2\Omega^2}{z\kappa+1}$ and $\mu_4 = \frac{2z^2\Omega^2}{(z\kappa+1)^2} + \frac{128z^4\Omega^4}{(z\kappa+1)^6}$ are the microscopic coupling constants. The *response field* $\tilde{n}$ encodes the linear response properties of n under small perturbations.

We emphasize two key properties of the action (5): First, the absence of a density-independent Markovian noise level $\sim T\tilde{n}_X^2$ (necessarily present in classical systems in thermal equilibrium). This is characteristic of DP dynamics, which feature the absence of density fluctuations in the absorbing state $n_X = 0$ and consequently

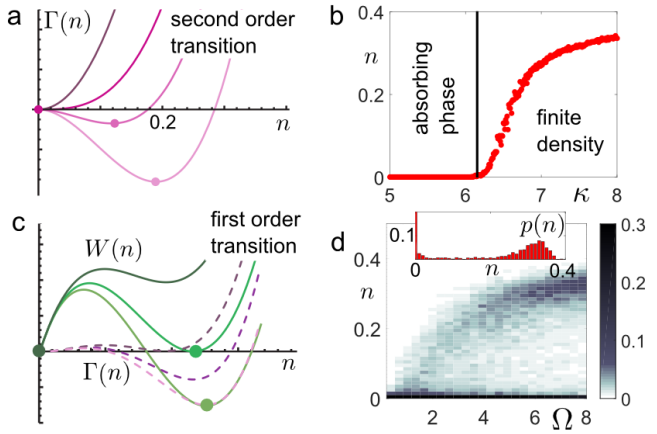

Figure 2. Effective potential and phase transitions. (a) Behavior of the effective potential $\Gamma(n)$ (arbitrary units) across the second order phase transition. Dots mark the minima of $\Gamma(n)$. The transition occurs when Δ in Eq. (6) changes sign. (b) Stationary state density in the classical limit ($\Omega = 0$) as a function of κ (chain of 200 sites, average over 10^3 realizations per point), obtained via Monte Carlo simulations starting from a completely active configuration and stopped at time $\gamma t = 10^4$. The data show the characteristic behavior of a continuous phase transition around $\kappa_c \approx 6.2$. (c) Effective potential $\Gamma(n)$ (dashed lines) and corresponding "optimal-path" potential $W(n)$ (solid lines), see Eq. (7), across the first-order transition. At the transition point, $W(n_1 = 0) = W(n_2) = 0$. (d) Steady-state histogram of the density in the quantum limit $\kappa = 0$ (12 spins) obtained via a quantum-jump Monte Carlo (QJMC) method, indicating a first-order transition ($\Omega_c \approx 2$) as Ω increases. Two stable stationary solutions, one with zero and one with finite density, emerge. The inset displays a section of the histogram taken at $\Omega = 8$.

a multiplicative kernel $\propto n_X$. An additive noise introduced by the dissipative terms $L_d = \sqrt{\gamma}\sigma^-$ only occurs in the eliminated spin variables $\sigma^{x,y}$. Second, the presence of a non-zero coherent coupling $\Omega \neq 0$ – i.e. the intrinsic quantum effect – leads to the appearance of non-zero couplings u_4 and μ_4 as well as a negative contribution to u_3. This additional "quantum" scale Ω breaks a fundamental symmetry of the DP class (specified below) and strongly modifies the phase diagram compared to the purely-dissipative model [see Fig. 1].

Effective potential and mean-field phase diagram. — The discussion of the various phases and transitions of the system is considerably simplified by realizing that the deterministic contribution to the action $S_n^{(1)}$ can be written as $\int_X \tilde{n}_X \left[\partial_t n_X - D\nabla^2 n_X + \frac{\delta\Gamma(n_X)}{\delta n_X}\right]$, where

$$\Gamma(n) = \frac{\Delta}{2}n^2 + \frac{u_3}{3}n^3 + \frac{u_4}{4}n^4 \qquad (6)$$

is a local effective potential. In the absence of fluctuations Γ characterizes the mean-field phases, which are determined by the properties around its minima.

The corresponding phase diagram is shown in Fig. 1(b). The active phase is identified by $\Delta < 0$, $u_4 \geq 0$ and $u_3 > 0$, which leads to a single minimum of the effective potential at finite density. On the other hand, when both Δ and u_4 are positive, there is a local minimum of Γ at $n = 0$. For negative and sufficiently strong cubic coupling $u_3 < -2\sqrt{u_4\Delta}$, there exists a second local minimum at finite density $n > 0$. In this regime, the mean field evolution features two attractive fixed points and the thermodynamic phase is determined within the optimal path approximation in phase space [59].

Three different types of phase transitions from the active to the inactive state can be thus identified, their nature depending on the specific choice of parameters and the dimensionality. When the gap Δ vanishes with both $u_3, u_4 > 0$ the system undergoes a second order phase transition [see Fig. 2(a)], corresponding to a diverging correlation length $\xi = 1/\sqrt{|\Delta|} \to \infty$. Numerical evidence for this transition is presented in panel (b) of Fig. 2, which displays the stationary density of active sites obtained for $\Omega = 0$ in a chain of 200 sites. For $\Delta > 0$ and $u_3 \leq -2\sqrt{u_4\Delta}$, the transition from the active to the inactive phase takes place instead at finite correlation length $\xi = 1/\sqrt{|\Delta|} < \infty$. The form of the effective potential $\Gamma(n)$ suggests a first-order transition line in this regime featuring the coexistence of the zero and finite-density solutions. This case, however, requires additional care due to the specific form of the noise, as detailed further below.

The α point in Fig. 1(b) located at $\Delta = u_3 = 0$ represents a bicritical point at which both the line ($\Delta > 0, u_3 = -2\sqrt{\Delta u_4}$) and the line of continuous transitions ($\Delta = 0, u_3 > 0$) terminate. At this point, the quartic potential term u_4 provides the leading non-linearity.

Fluctuations at the continuous transition. — The competition between quantum and classical dynamics strongly affects the nature of the active-to-inactive transition. In the absence of the coherent coupling, $u_4, \mu_4 = 0$, the action (5) is equivalent to the so-called Reggeon field theory for classical DP [60]. It features — upon rescaling the fields — the characteristic *rapidity inversion* symmetry, which leaves the system invariant under the transformation $n \leftrightarrow -\tilde{n}$ and $t \to -t$ [3, 7, 59]. For $u_4 > 0$, this symmetry is broken by the microscopic action. The implications depend on the dimension d: For $d > 2$, u_4 is RG irrelevant and can be discarded in the infrared-dominated dynamics close to the continuous transition. Consequently, in $d > 2$, rapidity-inversion is restored and the line of continuous transitions displays universal scaling behavior corresponding to classical DP.

At the α point [white dot in Fig. 1(b)], $u_3 = 0$ and the leading-order coupling becomes u_4. For $d > 2$, the continuous transition at this point is governed by mean-field scaling behavior, since u_4 is RG-irrelevant and cannot introduce infrared divergent corrections to the vanishing couplings u_3, Δ. On the other hand, for $d < 2$, u_4 becomes RG relevant and generates a non-trivial RG flow of Δ and u_3 on the entire second order transition line.

This leads to a violation of rapidity-inversion which persists at long wavelength, and thus drives the system away from the DP critical point to a different non-equilibrium universality class, without specific symmetries. In $d < 2$, therefore, only the isolated point $\kappa = 1/z$, $\Omega = 0$ lies in the DP class, while the presence of quantum fluctuations imprints a new universal scaling behavior on the entire line, including the α point. In $d = 2$, the scaling of the fluctuation corrections to u_4 determines whether this coupling becomes relevant, making the scenario equivalent to $d < 2$, or irrelevant, which has to be determined by an RG analysis.

Non-equilibrium discontinuous transition. — For ($\Delta > 0, u_3 < -2\sqrt{\Delta u_4}$) the effective potential Γ displays two distinct minima, $n_1 = 0$ and $n_2 = \frac{|u_3|}{2u_4} + (\frac{u_3^2}{4u_4^2} - \frac{\Delta}{u_4})^{1/2}$, suggesting a first-order phase transition. The actual transition line lies where the finite-density minimum becomes statistically preferred. In equilibrium, this would be the point at which the minima of Γ are at the same height. However, the present non-equilibrium noise shows more pronounced fluctuations at larger densities and thus favors n_1 over n_2. To estimate the steady state distribution function $P(n)$, we apply the optimal path approximation to the action [3, 59]; this involves treating the coefficient $\Xi(n) = \frac{1}{2}n + \mu_4 n^2$ of $\tilde{n}^2$ as a kind of mean-field, density-dependent temperature. It yields [53]

$$P(n) = \tfrac{1}{Z} e^{-V\,W(n)}, \text{ with } W(n) = \int_0^n dm \tfrac{\partial\Gamma/\partial m}{\Xi(m)}, \quad (7)$$

with volume V and normalization Z. Both potentials $W(n)$ and $\Gamma(n)$ vanish in n_1 and share the finite-density minimum n_2. In the thermodynamic limit $V \to \infty$, $P(n) \to \delta(n - n_l)$, where $l = 1, 2$ depending on which one is the global minimum of W, accounting for the physical constraint $n \geq 0$. The transition occurs when $W(n_2) = 0$, which identifies the non-equilibrium first-order line [dashed line in Fig. 1(b)]. Due to the non-equilibrium nature of the fluctuations, this does not coincide with the naive prediction $\Gamma(n_2) = 0$, as shown in Fig. 2(c). In Fig. 2(d) we report the full-counting statistics of the density n obtained via QJMC techniques [61] for a chain of 12 spins. Despite the presence of strong finite-size effects, a bimodal structure is still highlighted for large values of Ω. This implies that trajectories bunch together around two possible values, the absorbing one and a finite-density one, and is a signature of the aforementioned coexistence.

Realization with Rydberg atoms. — Atoms excited to Rydberg states are employed in current experiments to study many-body effects [26, 32, 34, 62–70]. Recently, several theoretical studies addressed the semiclassical limit of these systems [71, 72] connecting their dynamics to that of constrained classical ones [72, 73]. Reasoning along the same lines of Ref. [18], we discuss below an implementation which should permit the exploration of the physics discussed above.

The internal structure of Rydberg atoms can be approximated as a ground state $|\mathrm{GS}\rangle \equiv |i\rangle$ (inactive site) and an excited one $|\mathrm{Ryd}\rangle \equiv |a\rangle$ (active site). Rydberg gases feature strong van-der-Waals interactions in state $|a\rangle$ [29–31], which rapidly decay as r^{-6} with the inter-particle distance r. For the sake of simplicity, we approximate them here as nearest-neighbor terms V_{nn} in a one-dimensional configuration.

Quantum branching/coagulation is realized via coherent driving by a laser field of Rabi frequency Ω and detuning Δ_L with respect to the atomic transition frequency; fixing $\Delta_L = -V_{\mathrm{nn}}$ enables an "anti-blockade" [71, 74, 75] mechanism which favors the excitation of a Rydberg atom next to an already excited one, e.g. $|iai\rangle \to |iaa\rangle$. Differently from the idealized model above, the constraint requires here a *single* excitation nearby, and processes such as $|aia\rangle \to |aaa\rangle$ are highly suppressed. The Hamiltonian is therefore approximately given by $H_{\mathrm{ryd}} = \Omega \sum_k C'_k \sigma^x_k$ where $C'_k = n_{k-1} + n_{k+1} - 2n_{k-1}n_{k+1}$.

To generate the incoherent branching/coagulation the atoms are coupled (with coupling g) to a second equally-detuned light field with strong phase noise (dephasing rate $\lambda \gg g$) [76]; for a correlation length shorter than the interatomic distance, the bath is modeled as independent bosonic modes b_k, $b_k^\dagger$ acting on each lattice site. The effective equation of motion for the atoms is obtained by performing second order perturbation theory in the small parameter g/λ [18, 77, 78]. The resulting master equation for the reduced atomic density matrix ρ is

$$\dot{\rho} = \frac{4g^2}{\lambda} \sum_k \left(\langle b_k^\dagger b_k \rangle \mathbf{D}[C'_k \sigma_k^+] + \langle b_k^\dagger b_k + 1 \rangle \mathbf{D}[C'_k \sigma_k^-] \right) \rho.$$

For sufficiently high ($\langle b_k^\dagger b_k \rangle \gg 1$) and homogeneous ($\langle b_k^\dagger b_k \rangle \approx \langle b_m^\dagger b_m \rangle$) intensity, one can identify $\kappa = (4g^2 \langle b_k^\dagger b_k \rangle)/\lambda$, leading to the branching/coagulation jump operators: $L^{\mathrm{ryd}}_{\mathrm{b},k} = \sqrt{\kappa}\, C'_k\, \sigma_k^+$ and $L^{\mathrm{ryd}}_{\mathrm{c},k} = \sqrt{\kappa}\, C'_k\, \sigma_k^-$. The final process is radiative decay from the Rydberg state to the ground state, with jump operator $L^{\mathrm{ryd}}_{\mathrm{d},k} = \sqrt{\gamma}\, \sigma_k^-$ [31].

Although the microscopic formulation of the dynamics is slightly different from the previously-discussed model – in particular, atoms with more than one excited neighbor are brought off-resonance – the resulting phase structure is similar, as the EOMs only differ from Eqs. (2-4) by RG-irrelevant higher-order density terms.

Outlook. — We have investigated the effects of quantum dynamical processes on a prototypical absorbing-state phase transition. We highlighted the emergence of a richer structure in the phase diagram, which includes both a discontinuous and a continuous non-equilibrium transition. In low dimension $d < 2$ the presence of a quantum coherent process leads to a breaking of the only fundamental symmetry of DP in a way that persists at long wavelengths, and thus leads to a phase transition

of a different nature. In equilibrium, the interplay between classical (thermal) and quantum fluctuations typically leads to a dimensional crossover [2, 79]. The present work shows that out of equilibrium the picture is not as straightforward and opens the path for further investigations in this field, including the quantitative characterization of the new universality class.

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