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R-Fuzzy Sets and Grey System Theory

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Abstract—This paper investigates the use of grey theory to enhance the concept of an R-fuzzy set, with regards to the precision of the encapsulating set of returned significance values. The use of lower and upper approximations from rough set theory, allow for an R-fuzzy approach to encapsulate uncertain fuzzy membership values; both collectively generic and individually specific. The authors have previously created a significance measure, which when combined with an R-fuzzy set provides one with a refined approach for expressing complex uncertainty. This pairing of an R-fuzzy set and the significance measure, replicates in part, the high detail of uncertainty representation from a type-2 fuzzy approach, with the relative ease and objectiveness of a type-1 fuzzy approach. As a result, this new research method allows for a practical means for domains where ideally a generalised type-2 fuzzy set is more favourable, but ultimately unfeasible due to the subjectiveness of type-2 fuzzy membership values. This paper focuses on providing a more effective means for the creation of the set which encapsulates the returned degrees of significance. Using grey techniques, rather than the arbitrary configuration of the original work, the result is a high precision set for encapsulation, with the minimal configuration of parameter values. A worked example is used to demonstrate the effectiveness of using grey theory in conjunction with R-fuzzy sets and the significance measure.

I. INTRODUCTION

The research conducted by Yang and Hinde in [1], was the first work that proposed the concept of an R-fuzzy set. It should be understood that the membership value of an R-fuzzy set, is itself a set, more specifically, it is a rough set. If using the voting method for example, the lower bound will contain all memberships values that have been agreed with by all in the consensus. Whereas, the upper bound will contain any membership value that has an affinity, but not necessarily absolute inclusion to the descriptor that the R-fuzzy set is being modelled for. There are several existing paradigms and concepts related to uncertainty, all of which have their own inherent difficulties in extracting clear and crisp information. The foundational understanding of sets from a classical sense is with regards to absolute inclusion, or absolute exclusion from the set. However, absoluteness can not always be guaranteed, more realistically, the inclusion of vagueness is often prevalent, this is heightened when considering perception based domains. For precise reasoning a crisp understanding is needed, but this becomes problematic when considering natural language. As

is evident in our daily communications, we are often obliged to use words which are themselves associated with inherent vagueness and ambiguity. Therefore, to mimic and understand human based reasoning, an entirely crisp, classical use of logic can not solely be relied upon. The concept of fuzzy provides the foundation that R-fuzzy is built upon.

One current enhancement to R-fuzzy sets is that of the significance measure, which was originally proposed by Khuman et al. in [2]. This provided the functionality needed for inspecting the importance of any membership value contained, within the membership set of an R-fuzzy set. By inferring from the coefficient value returned by the degree of significance for any given encapsulated membership value, one is able to understand its significance, relative to all other collected fuzzy membership values for any given R-fuzzy set. The extended work by Khuman et al. in [3], demonstrated that the significance and R-fuzzy paring allows for a connecting bridge such that, the higher order of detail one can expect from a generalised type-2 fuzzy approach, could be replicated to a high degree of success. Such is the effectiveness of this new research, one is able to use it for domains where a generalised type-2 fuzzy approach would be more ideal, but owing to its subjectiveness and computational complexities, not used.

Due to the versatility of grey theory, there are many areas of application and emerging domains such as natural language processing [4]. Grey theory places a particular interest on problem areas associated with poor information, small samples and high abstraction, a common trait of uncertain systems [5]. Grey provides a means to garner an informed and accurate conclusion based on what little, uncertain information is available. This is generally achieved through the processes of generating, excavating and extracting meaningful content.

The novelty of this paper is with regards to the addition of grey system theory, specifically, the use of the *typical* grey whitenisation weight function. This is used for the configuration of the membership function which encapsulates the returned degrees of significance for any given R-fuzzy set. By using the whitenisation weight function to plot the returned degree of significance, one is able to provide a more robust, versatile heuristic, as compared to the arbitrary selection of points given in the original paper [2]. Not only are the significance degrees correctly intersected, the minimal number

of parameter values are always used, instead of the various points that the original worked adopted.

Section II will describe the preliminaries for approximations, R-fuzzy sets and the significance measure. Also introduced is the grey whitenisation weight function. Section III presents the observations, using a worked example to demonstrate the benefit of a grey heuristic based approach, as compared to original arbitrary selection. Section IV provides the conclusion and summary of the paper.

II. PRELIMINARIES

We begin with *approximations*, the bounding that encompasses an R-fuzzy set.

A. Approximation Preliminaries

Definition 1 (*Approximations [6]*): Assume that $\Lambda = (\mathbb{U}, A)$ is an information system and that $B \subseteq A$ and $X \subseteq \mathbb{U}$. Set X can be approximated based on the information contained in B , via the use of a lower and upper approximation set.

The lower approximation should be understood to contain all observed objects that wholeheartedly belong to the set X with regards to the information contained in B . It is the union of all equivalence classes in $[x]_B$ which are absolutely contained within set X , and is given by:

$$\underline{B}X = \{x \mid [x]_B \subseteq X\} \quad (1)$$

$$\underline{B}(x) = \bigcup_{x \in \mathbb{U}} \{B(x) : B(x) \subseteq X\}$$

The upper approximation should be understood to contain all observed objects that have a *possible* overlap to the set X with regards to the information contained in B . It is the union of all equivalence classes that have a non-empty intersection with set X , and is given by:

$$\overline{B}X = \{x \mid [x]_B \cap X \neq \emptyset\} \quad (2)$$

$$\overline{B}(x) = \bigcup_{x \in \mathbb{U}} \{B(x) : B(x) \cap X \neq \emptyset\}$$

B. R-Fuzzy Set Preliminaries

We now present the concept of R-fuzzy sets, which makes use of the approximations as given in [Definition 1](#).

Definition 2 (*R-fuzzy sets [1]*): Let the pair $apr = (J_x, B)$ be an approximation space on a set of values $J_x = \{v_1, v_2, \dots, v_n\} \subseteq [0, 1]$, and let J_x/B denote the set of all equivalence classes of B . Let $(\underline{M}_A(x), \overline{M}_A(x))$ be a rough set in apr . The membership set of an R-fuzzy set A is a rough set $(\underline{M}_A(x), \overline{M}_A(x))$, where $x \in \mathbb{U}$, given by:

$$A = \{\langle x, (\underline{M}_A(x), \overline{M}_A(x)) \rangle \mid \forall x \in \mathbb{U}, \underline{M}_A(x) \subseteq \overline{M}_A(x) \subseteq J_x\} \quad (3)$$

$$A = \sum_{x \in \mathbb{U}} (\underline{M}_A(x), \overline{M}_A(x)) / x$$

Where \sum denotes the union of all admissible x elements over the universe of discourse. For each $x_i \in \mathbb{U}$, there will be an associated membership description $d(x_i)$ which describes the relationship of the element x_i with regards to the set $A \subseteq \mathbb{U}$. Assume C is a set of available evaluation criteria. For each pair $((x_i), c_j)$ where $x_i \in \mathbb{U}$ and $c_j \in C$, a subset $M_{c_j}(x_i) \subseteq J_x$ is created, given by:

$$M_{c_j}(x_i) = \{v \mid v \in J_x, v \xrightarrow{(d(x_i), c_j)} \text{YES}\} \quad (4)$$

The lower approximation for the rough set $M(x_i)$ is given by:

$$\underline{M}(x_i) = \bigcap_j M_{c_j}(x_i) \quad (5)$$

The upper approximation for the rough set $M(x_i)$ is given by:

$$\overline{M}(x_i) = \bigcup_j M_{c_j}(x_i) \quad (6)$$

Therefore the rough set approximating the membership $d(x_i)$ for x_i is given as:

$$M(x_i) = \left(\bigcap_j M_{c_j}(x_i), \bigcup_j M_{c_j}(x_i) \right) \quad (7)$$

C. Significance Measure

This section will present the significance measure originally proposed by Khuman et al. in [\[2\]](#).

Definition 3 (*Degree of significance*): Assume that an R-fuzzy set has already been established using the same notation given in [Definition 2](#). In doing so, one will already know of the available membership values contained in J_x , and the preferences given by all in the criteria set C . The total number of all generated subsets for a given R-fuzzy set is denoted by $|N|$. The number of subsets that contain the specific membership value one is inspecting is given by S_v . Each value $v \in J_x$ is evaluated by $c_j \in C$, the frequency of which is the number of times v occurred over $|N|$, this results in the significance measure given by:

$$\gamma_{\overline{A}}\{v\} = \frac{S_v}{|N|} \quad (8)$$

If the returned degree of significance for any given membership value is $\gamma_{\overline{A}}\{v\} = 1$, then it can be understood for the value being inspected, that it has been agreed upon by all in the criteria set C . Therefore it will categorically belong to the lower approximation:

$$\underline{M}_A = \{\gamma_{\overline{A}}\{v\} = 1 \mid v \in J_x \subseteq [0, 1]\} \quad (9)$$

Eq. (3) presented the notation of an R-fuzzy set, it also indicated that the lower approximation is a subset of the upper approximation $\underline{M}_A(x) \subseteq \overline{M}_A(x)$. Therefore, for any membership value to be given a $\gamma_{\overline{A}}\{v\} = 1$, one will know that it will undoubtedly also be included in the upper approximation. This is also the case for any returned degree of significance which is greater than 0:

$$\overline{M}_A = \{\gamma_{\overline{A}}\{v\} > 0 \mid v \in J_x \subseteq [0, 1]\} \quad (10)$$

D. Grey Theory

The use of whitenisation weight functions from grey theory provides for a heuristic based approach that the original R-fuzzy and significance framework lacked. Much like the original work, the whitenisation functions themselves are based on the returned degrees of significance for each R-fuzzy set that has been modelled. However, unlike the original arbitrary values decided for the function points, the grey approach provides a more efficient and streamlined perspective. By using an iterative process of optimisation and a combination of traditional triangular and trapezoidal membership functions, the encapsulated degrees of significance are precisely modelled using less parameter overhead.

There are several types of whitenisation functions, but this paper only considers the *typical* whitenisation function with fixed starting points. These starting points will be indicative of the starting and end points of the encapsulated candidate membership values contained within its R-fuzzy set. Rather than intersecting each apex height for each triggered membership value for a given set, the grey whitenisation function uses the minimum parameters required to contain all membership values with their correct corresponding intersections. The incorporation of a threshold value e to regulate the error rate of each whitenisation configuration is used to secure preciseness.

Definition 4 (*Typical weight function of whitenisation [7]*): Assume a continuous function with fixed end points, which are increasing on the left $L(x)$ and decreasing on the right $R(x)$, this is described as a typical weight function of whitenisation.

$$f(x) = \begin{cases} L(x) = \frac{x - x_1}{x_2 - x_1}, & \text{if } x \in [x_1, x_2] \\ 1, & \text{if } x \in [x_2, x_3] \\ R(x) = \frac{x_4 - x}{x_4 - x_3}, & \text{if } x \in (x_3, x_4] \end{cases} \quad (11)$$

Definition 4 describes the typical trapezoidal whitenisation function with fixed weights. For this to be transformed into a triangular whitenisation weight function, one simply replaces the interval at the apex $[x_2, x_3]$, with a single value.

III. OBSERVATIONS

For sake of continuity we consider the same example presented in [1] and [3]. We will employ the use of the whitenisation weight functions to demonstrate its effectiveness in retaining the secondary grade of detail garnered from the R-fuzzy and significance approach.

Example 1: $F = \{f_1, f_2, \dots, f_{10}\}$ is a set which is populated by 10 flights, where for each flight its noise level in decibels (dB) has been recorded. These 10 flights have all been recorded from a single specific airport, the values of which are given by $N = \{10, 20, 30, 50, 40, 70, 20, 80, 30, 60\}$. It can be easily inferred that for each N_i there is a corresponding F_i value. For example, f_1 has an associated noise value of 10(dB), f_2 has an associated noise value of 20(dB), and so on and so forth. For an R-fuzzy set to be generated one has to be made aware of the criteria set C , which in this case is populated by 6 individuals $C = \{p_1, p_2, \dots, p_6\}$, all of whom gave their own

perceptions based on the noise values for each of the flights contained in set F . **Table 1** shows these collected perceptions.

TABLE I. CORRESPONDING PERCEPTION VALUES AGAINST NOISE LEVELS

#	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
dB	10	20	30	50	40	70	20	80	30	60
p_1	NN	NN	NN	AC	AC	AN	NN	VN	NN	AN
p_2	NN	NN	AC	AC	AC	AN	NN	VN	AC	AN
p_3	NN	AC	AC	AN	AC	VN	AC	VN	AC	AN
p_4	NN	NN	NN	AC	AC	AN	NN	VN	NN	AN
p_5	NN	AC	AC	AC	AC	AN	AC	VN	AC	AN
p_6	NN	NN	AC	AC	AC	VN	NN	VN	AC	AN

The contained terms can be understood as meaning:

$$\begin{aligned} NN &\rightarrow \text{Not Noisy} & AC &\rightarrow \text{Acceptable} \\ AN &\rightarrow \text{A Bit Noisy} & VN &\rightarrow \text{Very Noisy} \end{aligned}$$

The fuzzy membership set J_x is created using a simple linear function:

$$\mu(f_i) = \frac{l_i - l_{min}}{l_{max} - l_{min}} \quad (12)$$

The resulting fuzzy membership set is given as follows:

$$J_x = \{0.00, 0.14, 0.29, 0.57, 0.43, 0.86, 0.14, 1.00, 0.29, 0.71\}$$

It is not expected that to know the precise decibel level of a flight, nor do individuals need to know when they converse between one another. As it was shown in [1], an R-fuzzy framework allows one to answer the question of, how can one express an objective type-1 fuzzy membership function if the precise decibel readings at not known? Assume that there was an additional flight f_{11} but with no known exact decibel reading. However, f_{11} has been given the description of being AC , what fuzzy membership value would one apply to this abstract concept? As it can be seen from **Table 1**, a single observation can have a variety of interpretations, no one single value could encapsulate the multitude of perceptions. Hence why a fuzzy set in this regard is no good, as a fuzzy approach would look to associate it to a single crisp value.

If we know that f_{11} is AC , we can set the descriptor to $d(f_{11}) = AC$, whereby any instance of AC occurring, its associated membership values are recorded. Using equation Eq. (4), the generated subsets for $M_{p_j}(f_{11})$ are given as follows:

$$\begin{aligned} M_{p_1}(f_{11}) &= \{0.57, 0.43\} \\ M_{p_2}(f_{11}) &= \{0.29, 0.57, 0.43\} \\ M_{p_3}(f_{11}) &= \{0.14, 0.29, 0.43\} \\ M_{p_4}(f_{11}) &= \{0.57, 0.43\} \\ M_{p_5}(f_{11}) &= \{0.14, 0.29, 0.57, 0.43\} \\ M_{p_6}(f_{11}) &= \{0.29, 0.57, 0.43\} \end{aligned}$$

Once collected, Eq. (5) and Eq. (6) are used to create the lower and upper approximations. The final generated R-fuzzy set is given by Eq. (7), therefore we are presented with:

$$M(f_{11}) = (\{0.43\}, \{0.14, 0.29, 0.43, 0.57\})$$

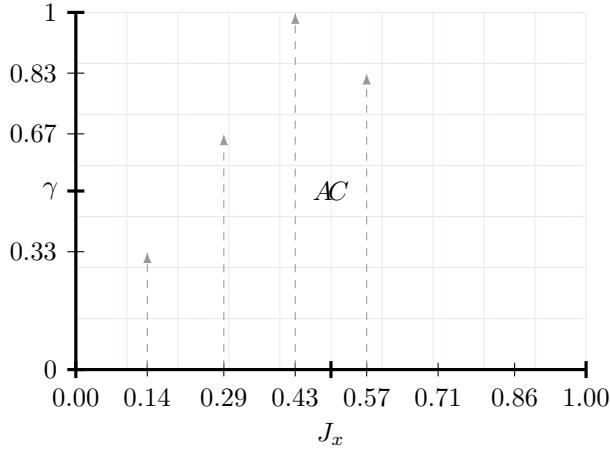


Fig. 1. A discrete representation for AC , given by the significance degree coefficient values

By using Eq. (8), one is able to calculate the degree of significance for each and every encapsulated fuzzy membership value, from J_x that has been triggered. The greater the returned coefficient values, the stronger criteria set C agreed to its sentiment.

one is able to obtain the following significance coefficient values for each of the membership values contained in J_x . The greater the value the greater its significance in relation to its descriptor, and the more individuals that agreed with its sentiment:

$$\begin{aligned} \gamma_{AC}\{0.00\} &= \frac{0}{6} = 0.00 & \gamma_{AC}\{0.14\} &= \frac{2}{6} = \frac{1}{3} = 0.33 \\ \gamma_{AC}\{0.29\} &= \frac{4}{6} = \frac{2}{3} = 0.67 & \gamma_{AC}\{0.43\} &= \frac{6}{6} = 1.00 \\ \gamma_{AC}\{0.57\} &= \frac{5}{6} = 0.83 & \gamma_{AC}\{0.71\} &= \frac{0}{6} = 0.00 \\ \gamma_{AC}\{0.86\} &= \frac{0}{6} = 0.00 & \gamma_{AC}\{1.00\} &= \frac{0}{6} = 0.00 \end{aligned}$$

The discrete visualisation for AC using the returned degrees of significance is presented in Fig. 1.

It is at this point the novelty of this paper becomes apparent, the original work by Khuman et al. in [2], [3] made use of arbitrary values to provide the convex hull of the encapsulated significance values. We will now make use of the whitenisation weight function given in Definition 4 to provide for a continuous representation. We will begin by structuring a triangular based whitenisation function. Using the values given in the membership set J_x , we have our initial left anchor point at 0.00. It would not be viable to have the set start at 0.43, as this would indicate its returned degree of significance is 0. As we already know the fuzzy membership value of 0.43 returned a significance degree of 1, we can have that act as the apex index. Once the membership value reaches 0.71, the significance degree returns a value of 0, so therefore this becomes the right most anchor point. For this initial state, the parameter values are given as $[0, 0.43, 0.71]$. Using a triangular membership characteristic function, similar to the one presented in Definition 4, we can cross check the degree of membership to the new membership set is the same

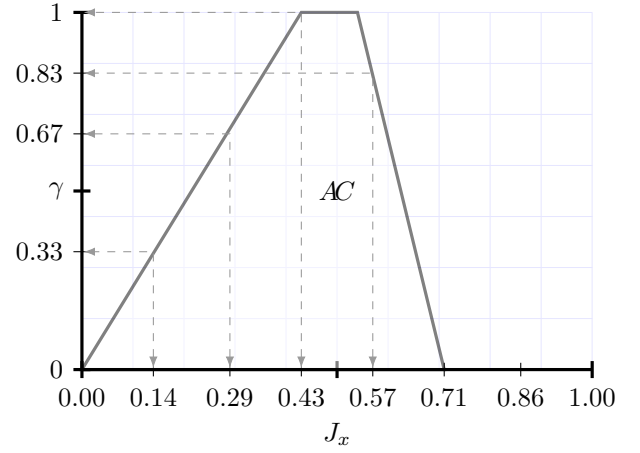


Fig. 2. A continuous representation for AC , given by the significance degree coefficient values with a trapezoidal whitenisation weight function of $[0, 0.43, 0.54, 0.71]$

as the returned degree of significance, the results are presented in Table 2.

The first column contains the membership values which belong to the membership set J_x . The second column presents the degrees of significance γ for each of the corresponding membership values from the membership set J_x , for the generated R-fuzzy set. The third column presents the returned degree of inclusion based on the initial state of the parameter values chosen for the whitenisation weight function. The membership values in J_x are in turn passed through to the weight function from which the results are recorded. The fourth column calculates the error given by e . This is simply the absolute difference between the degree of significance we know to be true γ , against the values returned by the whitenisation weight function. The main goal of the weight function is encapsulate the returned degrees of significance such that they stay true to the original values and that the error e is as small as possible. The error rate for this example has been set to $e = 0.01$, so if any value after the absolute difference has been calculated has exceeded this threshold, the weight function parameters will need to be readjusted and recalculated accordingly.

According to fourth column in Table 2, the error value for the membership value 0.57, returned an error of 0.3300, highlighted in red, this far exceeds the error threshold of 0.01. The fifth column indicates a change in the parameter values for the weight function, as such the new error values are presented in the sixth column. These new parameter values have reduced the error for the membership value to 0.1633, still unacceptable according to our threshold of 0.01. But more alarming, the membership value of 0.71 has now also registered as a viable candidate, even though the returned degree of significance was an absolute 0. Based on the proximity of the membership values to one another, a triangular based whitenisation function will not be able to effectively encapsulate the membership values according to their associated returned degrees of significance. Therefore, the seventh column indicates that a trapezoidal membership function be used with the parameters set at $[0, 0.43, 0.54, 0.71]$. With these new values the returned errors are below the error threshold

TABLE II. WHITENISATION OF AC

J_x	γ	Weight Function	Error	Weight Function	Error	Weight Function	Error
		[0, 0.43, 0.71]	e	[0, 0.43, 0.85]	e	[0, 0.43, 0.54, 0.71]	e
0.00	0	0	0.0000	0	0.0000	0	0
0.14	0.33	0.3256	0.0044	0.3256	0.0044	0.3256	0.0044
0.29	0.67	0.6744	0.0044	0.6744	0.0044	0.6744	0.0044
0.43	1	1	0.0000	1	0	1	0
0.57	0.83	0.5	0.3300	0.6667	0.1633	0.8235	0.0065
0.71	0	0	0.0000	0.3333	0.3333	0	0
0.86	0	0	0.0000	0	0.0000	0	0
1.00	0	0	0.0000	0	0.0000	0	0

TABLE III. WHITENISATION OF AN

J_x	γ	Weight Function	Error	Weight Function	Error	Weight Function	Error
		[0.43, 0.71, 1.00]	e	[0.54, 0.71, 1.00]	e	[0.54, 0.71, 0.80, 0.98]	e
0.00	0	0	0.0000	0	0.0000	0	0.0000
0.14	0	0	0.0000	0	0.0000	0	0.0000
0.29	0	0	0.0000	0	0.0000	0	0.0000
0.43	0	0	0.0000	0	0.0000	0	0.0000
0.57	0.17	0.5	0.3300	0.1765	0.0065	0.1765	0.0065
0.71	1	1	0.0000	1	0.0000	1	0.0000
0.86	0.67	0.4828	0.1872	0.4828	0.1872	0.6667	0.0033
1.00	0	0	0.0000	0	0.0000	0	0.0000

TABLE IV. WHITENISATION OF VN

J_x	γ	Weight Function	Error	Weight Function	Error	Weight Function	Error
		[0.71, 1.00, 1.00]	e	[0.78, 1.00, 1.00]	e	[0.81, 0.96, 1.00, 1.00]	e
0.00	0	0	0.0000	0	0.0000	0	0.0000
0.14	0	0	0.0000	0	0.0000	0	0.0000
0.29	0	0	0.0000	0	0.0000	0	0.0000
0.43	0	0	0.0000	0	0.0000	0	0.0000
0.57	0	0	0.0000	0	0.0065	0	0.0000
0.71	0	0	0.0000	0	0.0000	0	0.0000
0.86	0.33	0.5172	0.1872	0.3636	0.0336	0.3333	0.0033
1.00	1.00	1	0.0000	1	0.0000	1	0.0000

of $e = 0.01$, making this configuration the final configuration. The continuous representation of AC is presented in Fig. 2.

Table 3, Table 4 and Table 5, present the findings for the R-fuzzy sets; AN , VN and NN , respectively. The highlighted values in red indicate where an error has exceeded the threshold. The parameter values for the functions are changed accordingly until all errors for all membership values are as minimal as possible. Simply put, if a parameter value is not allowing for the correct expected response, increase it. If the use of a triangular whitenisation function does not unequivocally encapsulate the returned degree of significance with the correct intersections, then make use of a trapezoidal membership function instead, and repeat the process.

IV. CONCLUSION

As was the case with the original work, using grey whitenisation weight functions has allowed for a continuous representation, where the degrees of significance are also the degrees of membership, akin to a fuzzy perspective. The use of grey whitenisation weight functions allow for a more simplistic membership set using the minimal number of parameter points. A continuous visualisation of *Example*

TABLE V. WHITENISATION OF NN

J_x	γ	Weight Function	Error
		[0, 0, 0.43]	e
0.00	1.0	1	0
0.14	0.67	0.6744	0.0044
0.29	0.33	0.3256	0.0044
0.43	0	0	0
0.57	0	0	0
0.71	0	0	0
0.86	0	0	0
1.00	0	0	0

I using the grey whitenisation method can be seen in Fig. 3. For your convenience, the original continuous plot for *Example I* using arbitrary values is presented in Fig. 4. By visually inspecting these two very different plots, one can see that the membership functions given by the grey whitenisation functions, are smoother and use far less parameters in their construction as compared to the arbitrary values given for Fig. 4.

In the original work by khuman et al. in [2], [3], it was

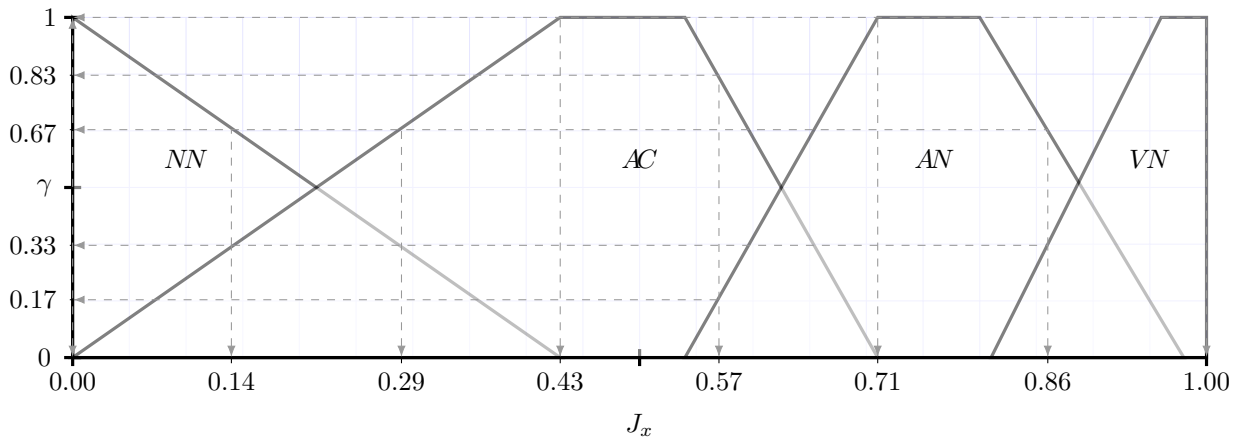


Fig. 3. A continuous visualisation for *Example 1*, based on all the generated significance measures for the R-fuzzy sets of *NN*, *AC*, *AN* and *VN*

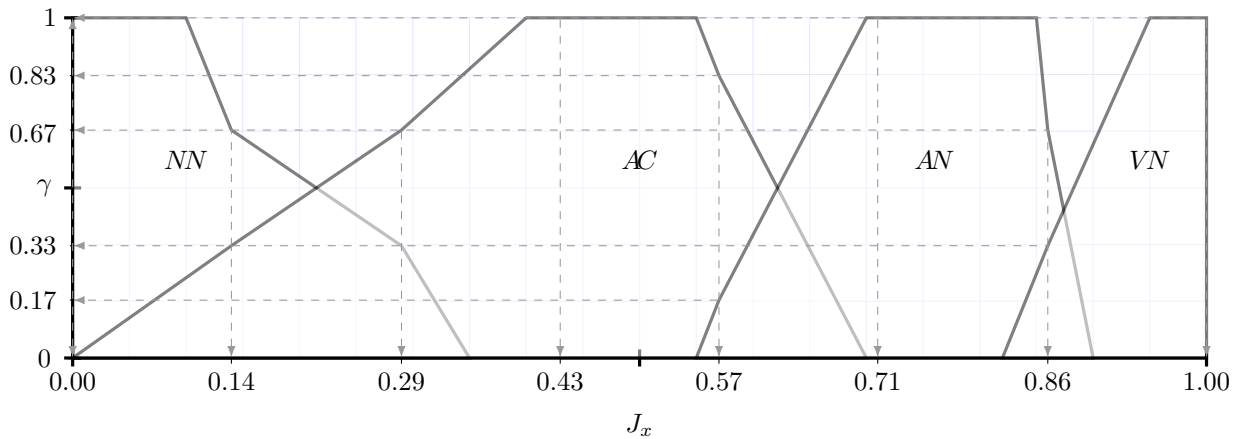


Fig. 4. The original continuous visualisation based on arbitrary values for *Example 1* by Khuman et al. [3], based on all the generated significance measures for the R-fuzzy sets of *NN*, *AC*, *AN* and *VN*

shown that the combination of the R-fuzzy framework and the analysis of the significance measure allows for a harmonic pairing. One which replicates in part the high detail of uncertainty representation from a type-2 fuzzy approach, with the relative ease and objectiveness of a type-1 fuzzy approach. As a result, this new research method allows for a practical means for domains where ideally a generalised type-2 fuzzy set is more favourable, but ultimately unfeasible due to the subjectiveness of type-2 fuzzy membership values. By securing a higher degree of accuracy by using the significance measure, problem domains can then be entirely encapsulated using grey whitenisation weight functions. This not only provides a basis to infer from, but is completely dynamic allowing for the addition of extra criterion C , additional descriptors $d(x_i)$.

The use of grey theory was done to demonstrate the effectiveness of marrying together of grey, R-fuzzy and the significance measure. The originally intended use of whitenisation from a purely grey perspective is mainly with regards to clustering [5], [7]. Using a grey whitenisation weight function allows for the classification of observations or objective indices, into definable classes. This is the intended future research of this framework. By making use of R-fuzzy sets to encapsulate the individual and general consensus of a problem domain, the significance measure can then allow for a type-

2 fuzzy degree of detail to be obtained. The classification of this higher ordered detail can then be analysed using the whitenisation weight functions from grey theory, to provide for a thorough and more detailed approach in garnering and inference. The more detailed one can harness from the domain the more informed the solution.

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