ESSAYS ON DEBT AND HETEROGENEOUS AGENT MACROECONOMICS

by

Shujaat A. Khan

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Abstract

In the first essay, I build a heterogeneous agent model of housing default to study how the effectiveness of macroprudential policies changes under different income and house price specifications. When calibrated to match the observed default choices of households during the financial crisis, the model has clear implications for the kind of macroprudential policies that will be more effective in different circumstances. When income shocks are large, restrictions on the loan-to-value ratio are more effective in reducing defaults, while when house price shocks are large, the default rate is more responsive to changes in payment-to-income limits. These results are an implication, filtered through the model, of the well-known double trigger fact: In the Great Recession, defaulting households tended to be those who were both seriously underwater and had experienced a substantial shock to income.

In the second essay, I study whether monetary policy has been less effective since the global financial crisis because of deteriorating household balance sheets. The paper examine the question using household data from the United States. It compares the responsiveness of household consumption to monetary policy shocks in the pre- and post-crisis periods, relating changes in monetary transmission to changes in household indebtedness and liquidity. The results show that the responsiveness of household consumption has diminished since the crisis. However, household balance sheets are not the culprit. More indebted and less liquid households are the most responsive to monetary policy, and their share in the population grew.

In the third essay, I introduce new methods for efficiently solving dynamic optimization problems with both discrete and continuous choices (DC models). These methods extend the Endogenous Gridpoint Method (EGM) by including exogenous outcome probabilities, search frictions, and taste shocks to 'concavify' the value function of the optimization problem. Compared to existing extensions of the EGM for DC models, the methods introduced in this paper have the added advantage of not only providing greater smoothness, but also rationalizing the smoothness into the agent's choice problem.

Keywords: Macroprudential, Default, Heterogeneous Agents, Housing, Household Debt, Monetary Policy, Discrete-Continuous Choice

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Thesis Committee

Christopher D. Carroll (Primary Advisor)

Professor of Economics

Department of Economics

Johns Hopkins University

Gregory R. Duffee

Carl Christ Professor of Economics

Department of Economics

Johns Hopkins University

Olivier Jeanne

Professor of Economics

Department of Economics

Johns Hopkins University

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Dedication

This thesis is dedicated to my parents, Naila A. Khan and Asif A. Khan.

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Chapter 1

Macroprudential Policies in a Heterogeneous Agent Model of Housing Default

1.1 Introduction

In the wake of the housing crisis of 2007-08, several countries have started implementing a variety of macroprudential policies to mitigate systemic risks arising from the housing sector (Lim et al. (2011), Claessens and Kodres (2014))[1]. During the crisis, these risks culminated in the form of higher defaults that had the potential of bringing down the entire financial system (Blinder (2013)). Consequently, a growing

[1] IMF (2018), Mitra (2016), and Darbar and Wu (2015) provide an overview of the macroprudential policies being used by different countries.

number of countries have mandated their central banks, or other regulatory authorities, to design and implement macroprudential policies. However, in spite of their increasing popularity, there is very limited quantitative theory to back the choice of macroprudential policies. This paper's contribution towards filling this gap in the literature is twofold. Firstly, it builds a structural heterogeneous agent model with micro-foundations that can be used to study the outcomes of implementing different macroprudential policies. Secondly, the paper provides analytical results on how the effectiveness of macroprudential policies in reducing defaults changes under different income and house price specifications.

For policymakers to design effective macroprudential policies, it is imperative to have a good understanding of how these policies perform under different income and house price specifications. Depending on the kind of macroprudential policy in place, not only do income and house prices play a central role in determining a household's borrowing decision, they also have an impact on a household's default decision. The macroprudential policies studied in this paper are the loan-to-value (LTV) and payment-to-income (PTI) rules. Both these rules are the predominant macroprudential policies employed by countries to manage overall housing credit in the economy (IMF et al. (2016)); however, each rule operates through different channels (Greenwald (2018)). An LTV rule sets minimum requirements on down payments that depend on the prevalent house prices. In contrast, a PTI rule limits borrowing based on the burden that mortgage payments put on a household's income level.

This paper finds that both the degree of income heterogeneity and house price fluctuations play a non-trivial role in determining the effectiveness of macroprudential policies in reducing defaults. When the size of income shocks increases, the effectiveness of an LTV rule in filtering out households with the highest ex-post probability of default increases. Defaulting households in the model are the ones who are not just seriously underwater, but who also experience a substantial shock to their income. Under an LTV rule, the default risks primarily arise from low income households, who get access to large mortgage balances by meeting the LTV requirement, but the high debt burden makes them susceptible to a bad income and house price shock. Under this household behavior, when the size of income shocks is large, the effectiveness of an LTV rule in reducing the default rate increases. This is because an LTV rule sets minimum downpayment requirements, which become increasingly difficult for low-income households to meet when the size of income shocks is large. The households who can afford the downpayment and become homeowners are better poised to absorb shocks to both their income and house prices.

If instead of the income shocks, house price shocks are large, the default rate is more responsive to changes in PTI limits. Under a PTI rule, the default risks primarily arise from high income households who get access to large mortgage balances by meeting the PTI requirement, but are subsequently hit with a bad income shock. A PTI rule operates by limiting the debt burden that periodic mortgage payments put on a household's income. This means that when the size of the house price shocks is large, for a given income level, a household cannot borrow more than what they would in the baseline. However, they have to put up a larger downpayment if the prices are above average and vice versa. Since under a PTI rule the main risks arise from high income households levering up very high when houses are expensive, a tightening of credit conditions is more effective at reducing defaults by requiring these high risk households to build larger equity buffers.

The key to matching the observed default decisions is the model's ability to reproduce the well-known *double trigger* fact, i.e. in the Great Recession, defaulting households tended to be those who were both seriously underwater and had experienced a substantial shock to income (Bhutta et al. (2010)). The model features that produce these outcomes are built into the micro foundations of homeowners. For homeowners, default is costly. Not only do defaulters face a utility cost upon defaulting, they are also forced out of the housing market for the remainder of their life. In addition, staying a homeowner is appealing to households for multiple reasons. An owner-occupied house provides households higher utility compared to the utility they derive from rental housing. Since households face uninsurable income risk and there is no unsecured borrowing, owner-occupied housing also smoothes a household's housing consumption. Moreover, owned houses are assets that have a potential for capital gains. These factors combined prevent a household from defaulting, even when they are underwater.

While the micro foundations of homeowners determine the characteristics of households with the highest likelihood of default, the micro foundations of renters, in conjunction with the macroprudential policies, determine the characteristics of households who become homeowners. Under different income and house price specifications, macroprudential policies differ from each other in the distribution of homeowners that they filter through into the housing market. These homeowners are consequently exposed to a varying degree of default risks, depending on the mortgage choices they made, the amount of liquid assets they have, and their income level. Renters who intend to buy a house have access to a multitude of long-term mortgages; however, macroprudential policies limit the size of the maximum mortgage balance. With an LTV rule, only the house prices determine the mortgage limit and with a PTI rule, the income level also becomes relevant.

In order to accurately solve for the housing decision rules in a computationally feasible manner, this paper also introduces a new modeling technique for solving optimization problems with both discrete and continuous choices (DC models). Since the housing tenure is a discrete state variable, and liquid resources and mortgage balance are continuous state variables, the model in this paper is a DC model. DC models are highly non-linear and the accuracy of the solution can depend on the density of the grid space on which the model is solved. The modeling technique, which relies on endogenizing housing search effort, minimizes the computational burden arising from the non-linearities in DC models. This allows for a dense grid to be set in regions of the state space with the highest degree of non-linearity, which is essential for accurately solving the homeowners' default and selling decisions. Another positive outcome of the technique is that it allows the model to capture the buyers' and sellers' time-on-market. This is because the housing search effort that households make is endogenously determined by the additional value that they get from switching their housing tenure.

The analysis in this paper is partial equilibrium in nature. Literature uses pecuniary externalities that arise from financial frictions (Bianchi (2011), Davila and Korinek (2018)) to rationalize limits on borrowing. This paper, on the other hand, sets the utility cost from default exogenously and assumes that the macroprudential policymaker's goal is to reduce the default rate. The policymaker can achieve this goal by tightening credit conditions through various macroprudential policies. The simplifying assumptions provide tractability, which leads to very clear implications for the effectiveness of macroprudential policies under various income and house price specifications. In building these insights, this is also the first study to highlight in a structural heterogeneous agent setting the importance of income heterogeneity and house prices in determining the effective of macroprudential policies. Calibrated to micro-level data, the model replicates aggregate household default and homeownership rates very well. This gives the model solid foundations upon which a more complex general equilibrium model can be built.

1.1.1 Related literature

Due to the lack of harmonized data on defaults and limited time-series data on the outcomes of macroprudential policies, the empirical literature has found mixed results on the performance of macroprudential policies. While some studies find LTV and PTI rules to be effective macroprudential tools, others find the opposite or mixed results. Carreras et al. (2018), Akinci and Olmstead-Rumsey (2018), Cerutti et al. (2017) use cross-country evidence to find that macroprudential policies have been effective in containing risks arising from rising housing credit and house prices. In contrast, Ono et al. (2016), using real estate registry data in Japan, find that caps on high LTV ratios are ineffective macroprudential tools in containing risks. Kuttner and Shim (2016), using data from 57 countries, find that PTI rules are effective macroprudential tools, but LTV rules are less effective in times of growing asset values. Adrian and Liang (2018) also note the mixed results for the effectiveness of these tools.

This paper is related to a growing literature that builds structural models of housing with micro foundations. Some of the main distinctive features of the model are that the mortgage contracts are long-term debt contracts and have an option to default. Berger et al. (2018) and Guerrieri and Lorenzoni (2017) construct heterogeneous agent housing models to study household responses to changes in house prices and credit conditions, respectively. Both these studies, however, do not have a default option. Guren et al. (2018) also construct a housing model with long-term mortgages and an option to default to study the housing wealth effect, but the utility cost from default in their model is set so high that the households never actually default. It is also important for mortgages to be long-term contracts, unlike the short-term contracts in Guerrieri and Lorenzoni (2017). This is because short-term mortgage contracts can lead to forced deleveraging in response to short-run fluctuations in house price, which makes it hard for the model to match the observed household behavior.

An important feature of this paper is that it models the implications of household heterogeneity for contract selection. Household characteristics, like wealth and income, have implications for not just contract selection, but also the pool of risky borrowers. A closely related paper to this one is Campbell and Cocco (2015), which constructs a heterogeneous agent model to study the effect of differences in LTV and loan-to-income on households' foreclosure decisions. However, they do not consider the implications of household heterogeneity for contract selection. Similarly, Ganong and Noel (2018) construct a partial equilibrium life-cycle model with housing to study whether a borrower's short-term constrains govern their response to long-term obligations. Households in their model; however, start off as homeowners with a fixed amount of mortgage balance. Greenwald (2018) also builds a general equilibrium model of housing and uses it to study the performance of LTV and PTI rules. The model in that paper though is a representative agent model, without an option to default. Our experience through the housing crisis has shown that representative agent models provide us with a narrow understanding of the risks that could be brewing due to the behavior of households on the tail ends of the income and wealth distribution.

This paper is closely related and complementary to Kaplan et al. (2017) and Garriga and Hedlund (2018). Both these papers are general equilibrium models of housing that are used to study the boom-bust cycle during the housing crisis. Kaplan et al. (2017) focus on the role played by households' expectations during the boom-bust episode. Garriga and Hedlund (2018), on the other hand, study how arrangements in the mortgage market impact the dynamics of the housing boom-bust episode and the economy. Garriga and Hedlund (2018) also study the implications of macroprudential policies on the boom-bust cycle. This study, although partial equilibrium in nature, complements these two studies by providing new insights into how heterogeneity of households could impact the effectiveness of macroprudential policies in reducing default. Moreover, the qualitative results are more broadly applicable, not just to the boom-bust cycle.

The remainder of this paper is organized as follows. Section 1.2 discusses the institutional background that provides the guidelines for building a model of housing default. Section 1.3 describes the key components of the model. Section 1.4 highlights the computational innovations made to solve discrete-continuous choice models efficiently. Section 1.5 outlines the calibration and model fit. Section 1.6 discusses the housing decision rules. Section 1.7 analyzes the performance of alternative macroprudential policies under different income and house price specifications. Section 1.8 concludes. The appendix contains the detailed housing problem and describes the perfect foresight solution.

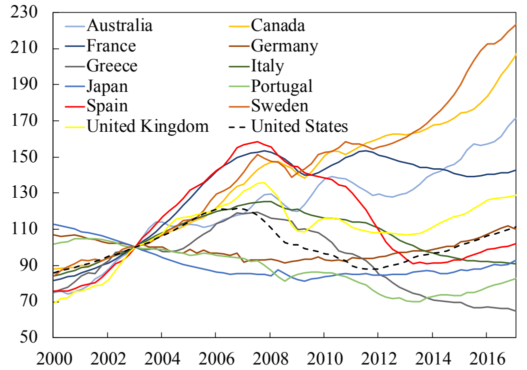
1.2 Background

This section highlights some of the key features of the housing market and provides insights into the behavior of homeowners during the housing crisis. These insights are used to build the model. This section also discusses the institutional arrangements that could give rise to different income and house price specifications.

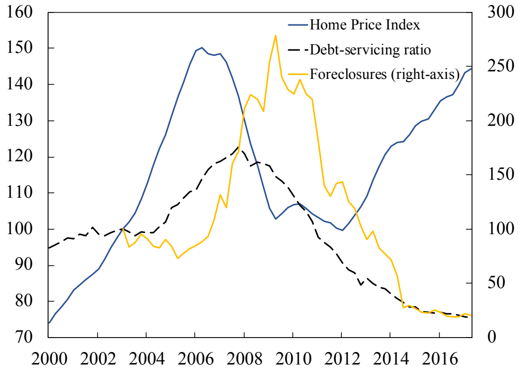
The housing crisis of 2007-08 impacted different regions within the US with a varying degree of intensity (Holly et al. (2010)). Some regions, such as California and Florida, experienced high house price volatility, while others regions, such as Indiana and Montana, did not. These diverse behaviors of house prices were even more pronounced on an international scale (Figure 1.1a). In countries like the US, the UK, and Spain, house prices experienced large swings around 2007. In contrast, in other countries, like Japan and Germany, house prices have been relatively stable over the last two decades. The countries that saw large swings in house prices also witnessed increased default rates. Just looking at the house price dynamics, though, only provides us with a partial picture of the factors that led to the increased default rates.

A boom-bust in house prices alone is not enough to explain the increase in default rates that was observed during the crisis. High leverage is also needed to drive foreclosures in the housing markets when house prices fall (Mian and Sufi (2018), Mian et al. (2017b)). If a household is not highly levered, they can always sell their house rather than default. As households observe a rise in house prices, extrapolated expectations (Bordalo et al. (2018)) lead them to believe that house prices would rise even further. This means that the expected capital gains from homeownership increases, which results in a higher demand for houses. Since houses are expensive, households take up mortgages to buy a house. Eventually, when house prices fall, the highly levered households default. These features can be seen in Figure 1.1b, which shows that a drop in house prices under elevated levels of leverage lead to a rise in foreclosure rates. The foreclosure rates fall when leverage recedes or when house prices begin to rise.

Empirical evidence, however, suggests that being underwater is not a sufficient condition for households to default, they also need to be hit with a bad income shock (Foote et al. (2008), Herkenhoff (2012)). This is referred to as the “double-trigger” that is needed for households to default. Bhutta et al. (2010), using mortgage data for households who purchased homes in 4 different states in 2006, find that 80% of households who default in their sample, default because of negative equity combined with a bad income shock. Thus, income heterogeneity is an essential component that is needed to understand households’ default behavior. Studies find that there is a



(a) Real house price indices for various OECD countries.



(b) US house prices, debt burden, and foreclosures.

Figure 1.1: The left panel shows the house price variation across different countries (series indexed to 100 in 2003). The right panel shows that during the crisis, high leverage and falling house prices led to an increase in housing foreclosure (series indexed to 100 in 2003).

high degree of heterogeneity in income variability across different countries (Acemoglu (1997)). A country with poor social insurance mechanisms in place would lead to a high degree of income heterogeneity and vice versa. Amongst the developed world, the US would correspond to a country with a high degree of income heterogeneity, compared to a country like Denmark, which has a low degree of income heterogeneity.

This study focuses on LTV and PTI rules as alternative macroprudential policies. These are two of the main macroprudential policies that are observed in countries around the world and they hold particular relevance for the US. In order to reform the financial regulation in the US, the Congress passed the Dodd-Frank Act, which became effective in 2010. The law instituted “Ability-to-Repay (ATR)” rules, which were rules that required mortgage lenders to make a good-faith effort to determine that the borrower was likely to be able to pay back the loan. Operationally, the ATR rules imposed limits on loan-to-value (LTV) and payments-to-income (PTI), and also included other measures to reduce the likelihood of a borrower defaulting.

1.3 Model

1.3.1 Households

A home purchase makes up the biggest investment for most households. This is in spite of the fact that a house is an illiquid asset due to both the transaction and search costs associated with buying or selling a house. In addition, house prices, particularly at the individual level, can be highly volatile (Case and Shiller (1989)) and unlike most financial assets, this idiosyncratic risk cannot be diversified, as a house is indivisible. High house price volatility, combined with illiquidity, implies that individual houses as an investment are not very attractive (Piazzesi and Schneider (2016)). Yet, roughly two-thirds of households in the US are homeowners. This can be explained by certain features of an owner-occupied house that make it an appealing asset. Owner-occupied housing provides housing services in excess of those provided by a rental house of similar size and, as an asset, it has the potential of capital gains. For households concerned about income risk, homeownership also allows households to smooth their housing consumption. These features are incorporated into the model.

In the model, a household's housing tenure can take up four different states: renters, homeowners, tenants, and defaulters. Households in each tenure state derive utility from consuming non-durable/non-housing consumption goods, c_t, and housing services, s_t. The aggregate consumption bundle has a Cobb-Douglas form, $\tilde{c}_t = c_t^{\alpha} s_t^{1-\alpha}$. This form is supported by a variety of micro-oriented studies (Berger et al. (2018)). The utility that a household derives from consuming this bundle has a CRRA form

$$u(\tilde{c}_t) = \frac{\left(c_t^{\alpha} s_t^{1-\alpha}\right)^{1-\rho}}{1-\rho}. \tag{1.1}$$

Households start each period with liquid assets, m_t. Renters, in addition to non-housing consumption, also pay for rental services. Rental services can be adjusted costlessly and are assumed to have the same unit cost as a unit of non-housing consumption. This simplifying assumption does not affect our main results, since during most of the pre-crisis years the house price-to-rent ratio has primarily been driven by variations in house prices and rental prices have roughly grown at the same pace as the prices of non-durable consumption goods. Each period, renters decide whether they will stay as renters in the next period **(rr)** or if they will become homeowners **(rh)**. If they choose to become homeowners in the next period, the house must be purchased in the current period; however, it is only made available to the household in the next period. At the time of purchase, buyers chooses how much mortgage debt,

b_t, to take up and, in the baseline model, face a loan-to-value (LTV), or equivalently, a downpayment constraint. Homebuyers also incur lump-sum transaction costs κ_p.

Homeowners, in contrast to renters, only pay for non-durable/non-housing consumption and derive housing services that are proportional to the size of the house, h_t:

$$s_t = \zeta h_t. \tag{1.2}$$

Homeowners in each period decide whether they will stay as homeowners in the next period **(hh)**, sell their house and become tenants in the next period **(ht)**, or default on their mortgage **(hd)**. The reason homeowners who sell their house are called tenants and not renters is because tenancy is a self-absorbing state. Once homeowners leave the housing market, they are not allowed to buy a house again. Similar to tenancy, defaulting is also a self-absorbing state. This assumption is made to simplify the numerical solution (explained in section 1.4). The simplifying assumption, however, does not significantly affect the relevant outcomes of the model, since the average duration of a household's life is 30 years and they are replaced by renters who have the option to become homeowners.

Houses for purchase are only available in one size, h, and their depreciation is offset by maintenance costs, δ_m, which are paid by homeowners who decide to continue being homeowners (**hh**). Continuing homeowners also have to make mortgage payments, which are determined by the size of their mortgage balance at the beginning of the period. Households who sell the house pay off their mortgage balance and receive proceeds from the sale of the house. Even though the household receives the proceeds from the sale of the house in the current period, the house is only made unavailable in the next period. While selling the house, the household also incurs lump-sum transaction cost κ_s. In case the household defaults, they walk away from the house and the mortgage balance, but they incur a utility cost, χ, in the period in which they default. This cost captures the non-financial costs that inhibit a household from defaulting. As noted earlier, if a homeowner decides to discontinue being a homeowner (**ht** or **hd**), they are not allowed to participate in the housing market again in the future.

There are also search frictions in the housing market, which means that renters and homeowners stay in their original housing tenure state, unless they exert an effort to switch the state. Search effort entails convex utility costs and the amount of effort exerted depends on the excess utility flow that the household would receive from successfully making the switch.

Denoting the discount factor by β and the probability of survival by $\not{\mathcal{D}}$, households maximize their infinite horizon expected discounted utility:

$$u(\tilde{c}_t) + \mathbb{E}_t\left\{ \sum_{n=1}^{\infty} (\beta \not{\mathcal{D}})^n u(\tilde{c}_{t+n}) \right\}. \tag{1.3}$$

The infinite horizon problem can also be thought of as a finite horizon problem in which the agents have perfect altruism towards their descendants. In this context, the aggregate utility is called the dynastic utility. The choice of an infinite horizon problem means that the model abstracting away from its life-cycle features, which are studied in detail by Berger et al. (2018), Oswald et al. (2017), Wong (2017), and Yao et al. (2015), among others. The detailed households' problem in Bellman form is outlined in Appendix 1.9.1.

1.3.2 Income and House Prices

Each period, households also face an idiosyncratic risk of getting unemployed. While unemployed, households receive unemployment benefits ν. The transition matrix for the employment state is given by,

$$\pi_{i,t} = \begin{pmatrix} \pi_{e,e} & \pi_{e,u} \\ \pi_{u,e} & \pi_{u,u} \end{pmatrix} \tag{1.4}$$

Conditional on being employed, households face uninsurable income risk. Income, $Y_{i,t}$, follows a process with both a persistent and a transitory component. The process is specified as

$$Y_{i,t} = \exp\{y^p_{i,t} + \theta_{i,t}\}, \tag{1.5}$$

$$y^p_{i,t} = \gamma_y y^p_{i,t-1} + \psi_{i,t}, \tag{1.6}$$

where γ_y is the persistence parameter of the persistent component of income, and $\psi_{i,t}$ and $\theta_{i,t}$ represent the persistent and transitory shocks to income, respectively. The variances associated with these shocks are denoted by σ^2_ψ and σ^2_θ.

House prices, P_t, are assumed to follow a first order autoregressive process:

$$P_t = \exp\{p_t\}, \tag{1.7}$$

$$p_t = \gamma_p p_{t-1} + \xi_t, \tag{1.8}$$

where γ_p governs the persistence of house prices and σ_ξ^2 denotes the variance of the house price shocks.

1.3.3 Financial Markets

Both renters and homeowners can deposit their liquid assets at banks at a fixed interest rate r. Neither renters nor homeowners have access to short-term borrowing, which means that their end of period liquid assets, a_t, cannot fall below 0. Homeowners, however, have access to long-term mortgage debt at a fixed interest rate r_m. Unlike studies that have short term mortgage debt and forced deleveraging (Guerrieri and Lorenzoni (2017), Berger et al. (2018)), households in this model are not forced to delever in response to negative house price shocks. Mortgage balance follows a constant geometric amortization schedule, with a half-life of 15 years (details in Appendix 1.9.4). A constant geometric amortization schedule, instead of a constant amortization schedule, is used since the optimization problem is an infinite horizon problem. Each period's mortgage payments, as a function of the beginning-of-period mortgage balance b_t, are given by

$$\lambda(b_t) = \left(1 - \frac{(1/2)^{1/n}}{1 + r_m}\right) b_t, \tag{1.9}$$

where $n = 15$ at the annual frequency.

At the time of purchase, households face an LTV constraint, which limits the maximum mortgage debt at origination, and is given by

$$b_t \leq \eta^{LTV}\, p_t h. \tag{1.10}$$

An LTV constraint can equivalently be thought of as a downpayment constraint. The minimum downpayment at the time of purchase is $\max\{0, (1-\eta^{LTV})\, p_t h\}$.

The baseline model uses an LTV constraint; however, later in the paper it is replace by a payment-to-income, or PTI, constraint. The PTI constraint imposes a borrowing limit at the time of home purchase based on the debt burden that periodic mortgage payments put on a household's income. The burden is assessed using the household's persistent income in that period, rather than the total income. Lenders typically adjust a loan applicant's income for "special factors", which can be interpreted as an adjustment towards the persistent income (FannieMae (2018), FreddieMac (2016)). The PTI constraint is given by

$$\lambda(b_{t+1}) \leq \eta^{PTI} . y_t^p \tag{1.11}$$

1.3.4 Search frictions

Households have to exert search effort to change their housing tenure. This search effort entails convex utility costs. Normalizing the search effort to equal the probability of a successful search, denote this normalized effort by ε_t^i and the utility cost associated with the effort by $\sigma^i(\varepsilon_t^i)$. For renters choosing to become homeowners, denote the search effort and the utility cost associated with making that search effort by ε_t^h and $\sigma^h(\varepsilon_t^h)$, respectively. The value function and the consumption function for the renter are given by:

$$V_t^r(m_t, y_t^p, p_t) = \max_{\varepsilon_t^h \in [0,1]} (1 - \varepsilon_t^h) V_t^{rr}(m_t, y_t^p, p_t) + \varepsilon_t^h V_t^{rh}(m_t, y_t^p, p_t) - \sigma^h(\varepsilon_t^h) \tag{1.12}$$

and

$$c_t^r(m_t, y_t^p, p_t) = (1 - \varepsilon_t^h) c_t^{rr}(m_t, y_t^p, p_t) + \varepsilon_t^h c_t^{rh}(m_t, y_t^p, p_t). \tag{1.13}$$

The cost function σ^h associated with the renter's search effort ε_t^h is specified as

$$\sigma^h(\varepsilon_t^h) = \mathcal{S}^h \left(\varepsilon_t^h - \log \left(\frac{1}{1 - \varepsilon_t^h} \right)^{(1-\varepsilon_t^h)} \right), \tag{1.14}$$

where $\mathcal{S}^h$ is a smoothness parameter that captures the degree of frictions in the home purchase market. This cost function implies that $\sigma^h(0) = 0$ and $\lim_{\varepsilon_t^h \to 1} \sigma^h(\varepsilon_t^h) = \mathcal{S}^h$. The effort function associated with this cost function is given by

$$\varepsilon_t^h(m_t, y_t^p, p_t) = 1 - e^{-\frac{V_t^{rh} - V_t^{rr}}{\mathcal{S}^h}}. \tag{1.15}$$

This implies that the degree of search effort exerted by the renters is a function of the additional utility flow that the renter gets from successfully making the switch. The greater the utility flow that a renter gets from becoming a homeowner, the greater the effort they exert in the housing search market. Since effort is non-negative, when the value of staying a renter exceeds the value of becoming a homeowner, the effort that a renter exerts is zero.

Unlike renters, homeowners can switch their housing tenure into two states: they can either sell their house and become tenants, or they can default. For better exposition, the homeowner's search problem is split into two sequential search problems. In the first stage, the homeowner faces a choice between staying a homeowner or selling the house. The search effort that a homeowner makes to sell the house, rather than continue occupying it, is denoted by ε_t^t. The utility cost, $\sigma^t(\varepsilon_t^t)$, associated with this effort has the same functional form as the search cost function for the renter, but here the search friction parameter is given by $\mathcal{S}^t$. In the second stage of the search problem for the homeowner, households have to choose between defaulting or facing the search problem described in the first stage of housing search. The search effort is denoted by ε_t^d and the associate search cost and search friction parameter are denoted by $\sigma^d(\varepsilon_t^d)$ and $\mathcal{S}^d$, respectively.

The value function and the consumption function for the homeowner are given by

$$\begin{aligned} V_t^h(m_t, b_t, h_t, y_t^p, p_t) = \max_{\varepsilon_t^d, \varepsilon_t^t \in [0,1]^2} \quad & -\sigma^d(\varepsilon_t^d) + \varepsilon_t^d V_t^{hd}(m_t, b_t, h_t, y_t^p) \\ & + (1 - \varepsilon_t^d)\Big[\varepsilon_t^t V_t^{ht}(m_t, b_t, h_t, y_t^p, p_t) + (1 - \varepsilon_t^t) V_t^{hh}(m_t, b_t, h_t, y_t^p, p_t) \\ & - \sigma^t(\varepsilon_t^t)\Big] \end{aligned} \tag{1.16}$$

and

$$\begin{aligned} c_t^h(m_t, b_t, h_t, y_t^p, p_t) = {} & \varepsilon_t^d c_t^{hd}(m_t, b_t, h_t, y_t^p) \\ & + (1 - \varepsilon_t^d)\Big[\varepsilon_t^t c_t^{ht}(m_t, b_t, h_t, y_t^p, p_t) + (1 - \varepsilon_t^t) c_t^{hh}(m_t, b_t, h_t, y_t^p, p_t)\Big], \end{aligned} \tag{1.17}$$

respectively.

The effort functions for the homeowners are given by:

$$\varepsilon_t^t = 1 - e^{-\frac{V_t^{ht} - V_t^{hh}}{\mathcal{S}^t}} \tag{1.18}$$

and

$$\varepsilon_t^d = 1 - e^{-\frac{V_t^{hd} - \left[\varepsilon_t^t V_t^{ht} + (1-\varepsilon_t^t) V_t^{hh} - \sigma^t(\varepsilon_t^t)\right]}{\mathcal{S}^d}}. \tag{1.19}$$

1.4 Computation

In the housing model outlined above, the beginning-of-period liquid assets m_t and mortgage balance b_t are treated as continuous state variables. This makes it possible to use the Endogenous Gridpoint Method (EGM) (Carroll (2006)), which offers greater computational efficiency compared to traditional root-finding solution methods. Using the EGM, one can feasibly construct a dense grid in the regions of the state space where the degree of non-linearity is the highest. These are also the regions where households typically switch their housing tenure states. Having a dense state space in these regions is particularly important for accurately capturing the households' default behavior. However, the default decision, like the decision to buy or sell a house, is a discrete choice and in models with both discrete and continuous choices (DC models), standard EGM can produce suboptimal solution points. These points need to be identified and removed from the final solution and this additional step deteriorates the computational efficiency of the standard EGM.

To reduce the severity of this problem, Iskhakov et al. (2017) use exogenous taste shocks to smooth out the marginal value functions in their DC problem. In contrast, the model in this paper achieves the smoothness of the marginal value functions through the endogenous housing search mechanism. In the context of housing, this is more economically plausible compared to exogenous taste shocks. The search effort gets endogenously determined by the severity of the kinks in the value function. This makes it possible to smooth out the expected marginal value function enough that the EGM does not produce any suboptimal points at all. Moreover, in contrast to taste shocks, housing search captures a very significant feature of the housing market: the *time on market*. In addition to the transaction costs, the time on market associated with buying and selling a house is another feature of houses that makes them illiquid.

(Details of the computational methods used to solve the model are provided in Appendix 1.9.5).

1.5 Calibration

Following Kaplan et al. (2017) and Garriga and Hedlund (2018) the model is calibrated to match cross-sectional features of the U.S. housing market prior to the housing boom. The model is calibrated at an annual frequency to data from 1998, as it also aligns with a Survey of Consumer Finance release that year. Some of the parameters are calibrated externally, while others are jointly calibrated internally to match key housing data moments.

As is standard in the literature, the coefficient of relative risk aversion ρ is set to 2. In the default model estimated by Campbell and Cocco (2015), the lifespan of a household is set at 20 years. However, in this model the average lifespan of a household is set to 30 years. For the purpose of the modeling exercise in this paper, a lifespan of 30 years is befitting, since 30 years is also the prevalent duration of a mortgage. A lifespan of 30 years implies a survival probability of 0.975.

Following Krueger et al. (2016), the income process is calibrated using annual PSID after-tax earnings data, after removing age, education, and time effects. This yields estimates of 0.9695 for the persistence parameter γ_y, 0.0384 for $\sigma^2_{\psi^y}$ and 0.0522 for σ^2_θ. The transition probabilities of unemployment are calibrated by converting Shimer (2005)'s quarterly estimates to annualized values. This leads to an employment-to-unemployment transition probability, $\pi_{e,u}$, of 0.1 and an unemployment-to-employment transition probability, $\pi_{u,e}$, of 0.99. Also following Shimer (2005), the unemployment benefits, ν, are set to 40% of the average labor income. The housing search parameters, $\mathcal{S}^h$, $\mathcal{S}^t$, and $\mathcal{S}^d$ are set to 0.25, 0.65, and 0.25, respectively. These values provide appropriate smoothing of the expected marginal value functions in the problem[2].

[2] In the annual specification, these parameters do not result in time-on-market moments that match the data well. A quarterly specification would be needed for that.

The generic house size, h, is set to 3.4. This means that the house is 3.4 times the mean labor income (Mitman et al. (2017)). The persistence of house prices, γ_p, is set to 0.988 using the Case-Shiller house price index, and the standard deviation of house prices is set to 0.162 following Campbell and Cocco (2015). The lump-sum transaction costs at the time of purchase, κ_p, and sale, κ_s, of a house are set to 1% of the mean house price. Houses are assumed to not depreciate as homeowners have to incur maintenance costs that offset the depreciation rate. Following Kaplan et al. (2017), these maintenance costs, κ_m, are set to 2% of the mean house price.

The risk-free interest rate r is set to 3%. The fixed mortgage rate r_m is set to 7%, using the average 30-year fixed rate mortgage in the US over 1998. In the baseline model, the LTV limit η^{LTV} is set at 95%, which is 10 percentage points higher than the average CLTV prior to the housing boom (UrbanInstitute (2017)). Since homeowners in this model are not allowed cash-out refinancing, or to borrow against their home equity, this adjustment is made to take into account the fact that in reality, cash-out refinancing can lead to many cases of new mortgages with LTVs greater than 100% (Mitman et al. (2017)). This baseline LTV limit falls between the LTV limits of 90% set by Campbell and Cocco (2015) and 125% set by Mitman et al. (2017). In the baseline model, the PTI constraint is non-binding, however, in the alternate specification in which the LTV rule is replaced with a PTI rule, η^{PTI} is set to 45%[3], following the Seller/Servicer Guidelines provided by Freddie Mac

[3]This is almost equivalent to the PTI limit of 43% set under the Qualifying Mortgage condition of the Ability-to-Repay Rules.

(FreddieMac (2016)).

The remaining parameters of the model are calibrated jointly by targeting the annual foreclosure rate, the homeownership rate, and the median LTV. The Simulated Method of Moments (SMM) is used for this estimation exercise. Using the National Delinquency Survey, the model targets an annual foreclosure rate of 1.6%. The U.S. Census Bureau's data for homeownership provides a target homeownership rate of 67.8%. For the median LTV, the 1998 SCF provides a target of 0.62. Table 1.2 shows the resulting estimates for the discount factor, β, the share of non-housing services in the aggregate consumption bundle, α, the housing services flow from owned housing, ζ, and the default utility cost, χ. All these estimates are within the range of estimates produced by various studies.

Table 1.3 shows that the model fits the key data moments quite well. The model does a very good job of matching the average annual foreclosure rate, the homeownership rate, and the median LTV. Matching the default rate so well is a result of the micro-foundations of households and due to the solution technique employed, which allows the model to capture the behavior of low wealth households very accurately. Not only does the model generated foreclosure rate of 1.62% compare well with the empirical estimate of 1.60%, it also fares better than Kaplan et al. (2017) model-generated annual foreclosure rate of 0.4%. The model also does a fairly reasonable job matching the untargeted moments such as the median net worth (liquid assets + house value - mortgage debt), relative to median after-tax income, and the mean

Table 1.1: Model parameters (external calibration)

Preferences	Description	Value	Source
ρ	Coeff. of relative risk aversion	2	Standard in literature
$\not{D}$	Survival probability	0.975	30yrs lifespan
Income			
γ_y	Persistence of pers. income shock	0.9695	Krueger et al. (2016)
σ^2_ψ	Var. of pers. income shock	0.0384	Krueger et al. (2016)
σ^2_θ	Var. of trans. income shock	0.0522	Krueger et al. (2016)
$\pi_{e,u}$	Probability of unemployment	0.1	Shimer (2005)
$\pi_{u,e}$	Probability of re-employment	0.99	Shimer (2005)
ν	Unemp. benefit (rel. to $\overline{y}$)	0.4	Shimer (2005)
Housing			
h	House size (rel. to $\overline{y}$)	3.4	Mitman et al. (2017)
γ_p	Persistence house price shock	0.9880	Case-Shiller HPI
σ^2_ξ	Var. of house price shock	0.0262	Campbell and Cocco (2015)
κ_p	Purch. trans. cost (rel. to $\overline{p}$)	0.01	
κ_s	Sale trans. cost (rel. to $\overline{p}$)	0.01	
κ_m	Maint. cost (rel. to $\overline{p}$)	0.02	Kaplan et al. (2017)
$\mathcal{S}^h$	Search param. for `rh`	0.25	Author's calculations
$\mathcal{S}^t$	Search param. for `ht`	0.65	Author's calculations
$\mathcal{S}^d$	Search param. for `hd`	0.25	Author's calculations
Fin. conditions			
r	Risk-free rate	3%	Av. 1yr Treasury
r^m	Mortgage rate	7%	Av. 30yr FRM in 1998
η^{LTV}	LTV limit	0.95	UrbanInstitute (2017)
η^{PTI}	PTI limit	0.45	FreddieMac (2016)

Table 1.2: Model parameters (joint calibration)

Variable	Description	Value
β	Discount rate	0.975
α	Share of non-housing	0.70
ζ	Housing services from owned house	1.03
χ	Default utility cost	0.65

Table 1.3: Model fit

Moment	Data	Model
Foreclosure rate (%)	1.60	1.62
Home ownership rate (%)	67.8	71.2
Median LTV	0.62	0.59
Median net worth (rel. to median after-tax income)	1.56	2.78
Median mortgage debt (rel. to median after-tax income)	2.14	2.62

mortgage debt, relative to median after-tax income. The model partly produces a higher median net worth compared to the data, because it generates a slightly higher homeownership rate of 71.2% and a slightly lower median LTV of 0.59, compared to the data.

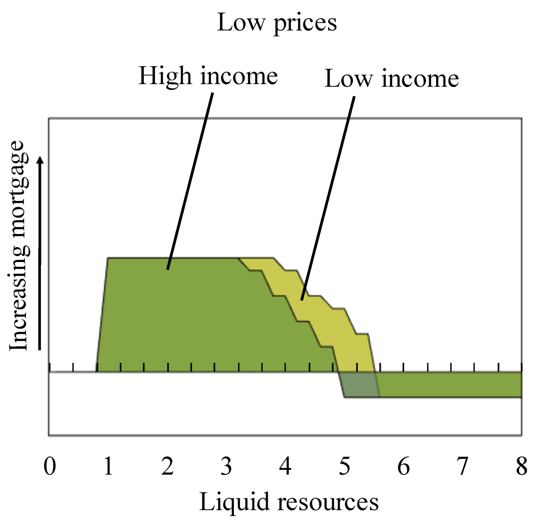
1.6 Housing decisions

Before analyzing the effectiveness of different macroprudential policies under various income and house price specifications, a discussion of the housing decision rules for renters and homeowners in the model is necessary. Figure 1.2 shows the renter's converged housing decision rule under an LTV rule. On the y-axis, the zero-line indicates the renter's decision to stay as a renter. Above the zero-line, a higher value indicates a larger mortgage balance at origination and below the zero-line indicates a renter's decision to buy a house without any mortgage debt. Since the subplots in Fig 1.2 are made conditional on the house price levels, the y-axis normalized by the price level would give the LTV level at mortgage origination.

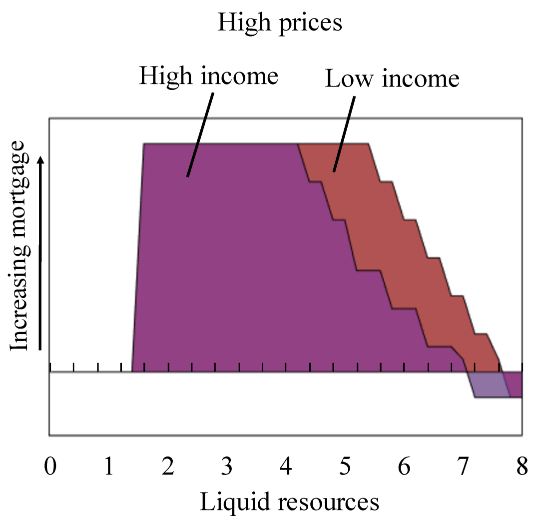
Due to the way in which an LTV rule operates, a household's borrowing limit is entirely determined by the level of house prices. Under low house prices, households have access to lower mortgage levels, regardless of their income, and vice versa. An LTV limit is also equivalent to setting minimum requirements on downpayments that depend on the price of the house. When house prices are low (Fig 1.2a), households need to accumulate fewer liquid assets to buy a house. However, low house prices also limit the maximum available mortgage size at origination. In contrast, at higher house price levels (Fig 1.2b), renters have access to higher levels of mortgage balance. Households with higher liquid assets opt for lower mortgage balance at origination and very rich households choose to buy a house by paying the full price, without any mortgage.

Renters' housing decisions also vary with the level of persistent income. For a given level of house prices and liquid assets, low income homebuyers are levered at least as high as the high income homebuyers, which makes them more vulnerable to bad income or house price shocks. The LTV constraint implies that for a particular house price level, regardless of the income level, all renters have access to similar mortgage contracts. Since renters with low persistent income also have lower prospects of future income, they choose to lever up as much as they can and buy a house to smooth their housing consumption. Even at higher levels of liquid assets, low income households choose to lever up more than high income households, for a given level of liquid assets. This continues to hold even when house prices are high. High leverage, however, makes households a lot more vulnerable to default risks.

Under a PTI rule, since borrowers are constrained by the burden that mortgage payments put on their income, for a given price level, households with lower income have access to lower mortgage balances and vice versa. The behavior of renters is more interesting when we compare low income versus high income households. The



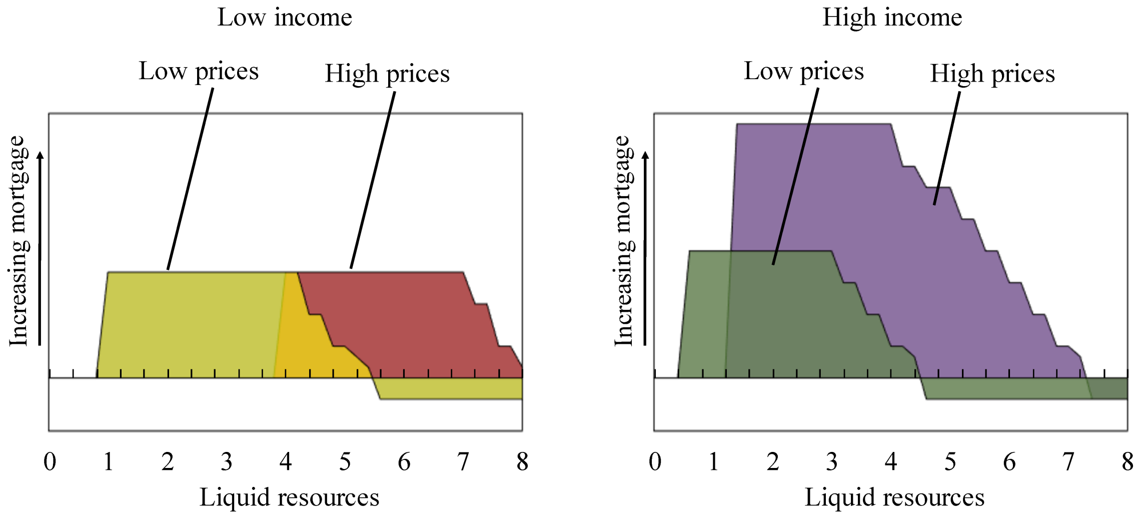
(a) Renter's housing decisions under low prices.



(b) Renter's housing decisions under high prices.

Figure 1.2: Renter's housing decision under an LTV rule, with $\eta^{LTV} = 0.95$. On the y-axis, the 0-line indicates a renter's decision to stay a renter. Above the 0-line, a higher value indicates a larger mortgage balance and origination. Below the 0-line indicates a renter's decision to buy a house without any mortgage debt.

y-axes of the plots in Fig 1.3 normalized by the income level are proportional to the PTI level at mortgage origination, since the interest rate is fixed. Fig 1.3a shows that under a PTI rule of $\eta^{PTI} = 0.45$, low income households, regardless of the price level, borrow up to the same PTI limit. This means that when house prices are high, low income households accumulate a significant equity buffer before buying a house. This is very different from the behavior of low income households under an LTV rule, in which case low income households lever up more when house prices rise. The behavior



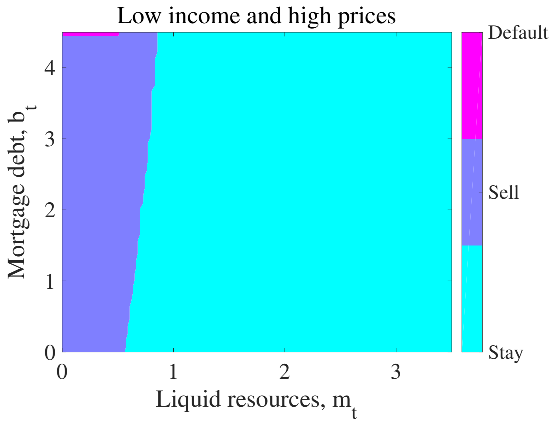
(a) Renter's housing decisions under low income.

(b) Renter's housing decisions under high income.

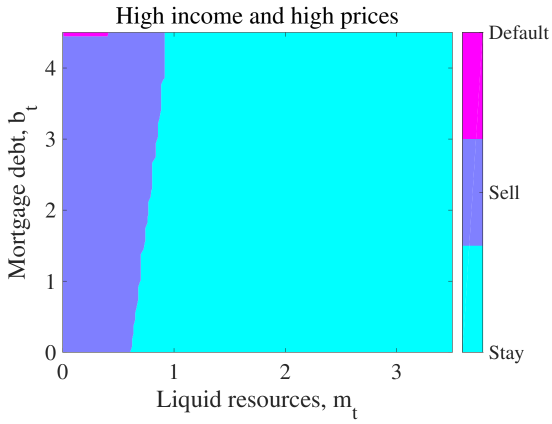
Figure 1.3: Renter's housing decision under a PTI rule, with $\eta^{PTI} = 0.45$. On the y-axis, the 0-line indicates a renter's decision to stay a renter. Above the 0-line, a higher value indicates a larger mortgage balance and origination. Below the 0-line indicates a renter's decision to buy a house without any mortgage debt.

of high income renters remains roughly the same as it is under an LTV rule. The only difference is that high income households, when house prices are low, borrow slightly more under a PTI rule than they borrow under an LTV rule.

Homeowners' housing decisions, in contrast to the renters' decisions, are invariant to macroprudential policies, since macroprudential policies only operate through limiting prospective homebuyers' borrowing. Nonetheless, understanding the homeowners' decision rules is important to identify the environment under which house-



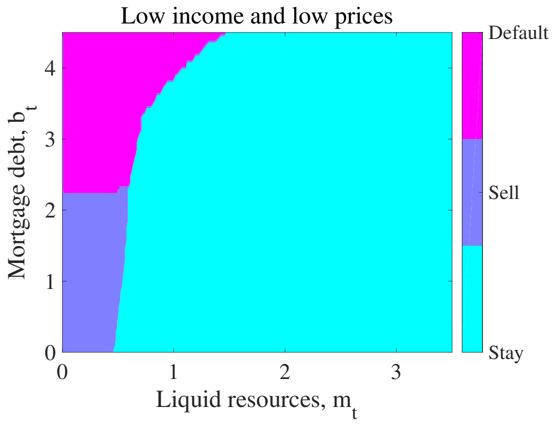
(a) Homeowner's housing decision under low income and high prices.



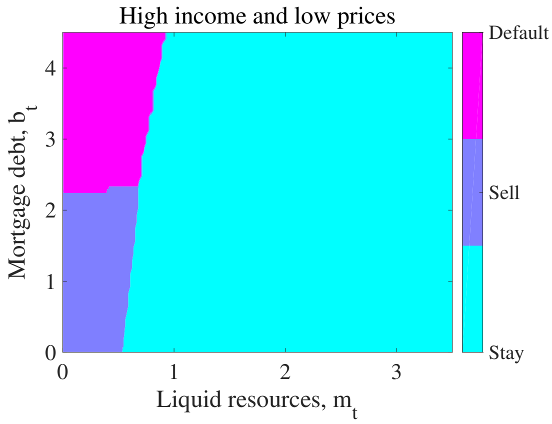
(b) Homeowner's housing decision under high income and high prices.

Figure 1.4: Homeowner's housing decision rule under high house prices. When house prices are high, homeowners do not default. If their are liquidity constrained, they sell their house and extract equity to relax their budget constraint.

holds default. Figure 1.4 shows the housing decision rule for low-income and high income homeowners when house prices are high. These results have two main outcomes. Firstly, when house prices are high, the housing decisions are unaffected by the household's income level. Secondly, when homeowners have ample liquid resources, they choose to stay as homeowners. Homeowners sell their house if their liquid assets are too low. These liquidity constrained households sell rather than default because when house prices are high, homeowners have positive equity for the debt levels considered in this study. Thus, by selling their house, households can extract their equity and relax their budget constraint.



(a) Homeowner's housing decision under low income and low prices.



(b) Homeowner's housing decision under high income and low prices.

Figure 1.5: Homeowner's housing decision rule under low house prices. When house prices are low, homeowners' default if they are highly levered and liquidity constrained. Under these circumstances, their likelihood of default increases significantly if they receive a bad income shock.

When house prices fall and the debt level is low, homeowners' housing decision is still not sensitive to the income level. However, when the debt level is high, the decisions become highly sensitive to the income level. Fig 1.5 shows that when households are not highly levered, at low levels of liquidity, they sell their house. This decision does not change with the level of income. In contrast when debt level is high, under low house prices, the likelihood of default increases significantly when the household income is low. Fig. 1.5a shows that the range of liquid asset wealth along which households default, spreads out when the income level is low. Thus, a homeowner defaults if (i) they are are underwater and (ii) they receive a negative income shock that pushes their liquid assets very low. Moreover, the fewer the liquid resources that a household has, the smaller is the magnitude of the bad income shock that is needed to push the household below the threshold beyond which they default. This is precisely the double-trigger fact that was observed during the Great Recession.

1.7 Effectiveness of macroprudential policies

The baseline income and house price processes are calibrated using the US data. Varying the LTV limit changes the default rate in the economy. This exercise provides the baseline effectiveness of the LTV rule in changing the default rate. In the counterfactuals, the size of the income and house prices are doubled by doubling the variances of these processes, separately, and the LTV limit is varied. The differences in the slopes of the default rate locus produced by this experiment transparently reveal how the effectiveness of an LTV rule changes as the size of the income and house price shocks increases. A similar experiment is repeated by replacing the LTV rule with a PTI rule to evaluate how the size of income and house price shocks affect the performance of a PTI rule.

1.7.1 Baseline performance of LTV rules

Although the focus of this study is defaults, this sub-section also evaluates how some of the other major housing moments change as the LTV limit is varied. This is to provide the reader with deeper insights into the functioning of the model. Figure 1.6 shows that a tightening of the LTV limit (i.e. moving left on the x-axis) leads to a decline in the default rate. This is due to two main reasons. Firstly, a tightening of the LTV limits reduces the overall leverage in the economy and consequently the default rate as well. Secondly, since a reduction in the LTV limit is equivalent to requiring a higher downpayment, a tightening of the LTV rule makes it increasingly difficult for low income households to buy a house. These are households with the highest likelihood of default, as shown in the homeowners' decision rule in Fig 1.5a, and filtering them out of the housing market reduces the default rate.

In the baseline setting, almost all the defaults are concentrated between the LTV limit of 90% and 100%. This is primarily due to the fact that there is only a single-sized house in the model and the households' impatience factor, β, is homogenous. The impatience factor influences the households' borrowing and default decisions. Since in this model it can only take up a single value, the calibration leads to an estimate of β which generates a large density of households concentrated close to the borrowing limit. These are also the households who are the most likely to default if they are hit with bad income and house price shocks. A model with a variety of house

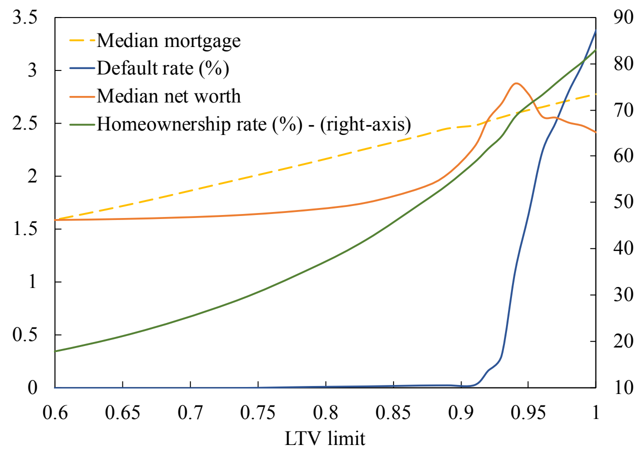


Figure 1.6: Impact of varying the LTV limit on different housing moments.

sizes and heterogeneous βs would not only be able to match the distribution of LTV much better, it would also expand the range of LTV limits over which households default.

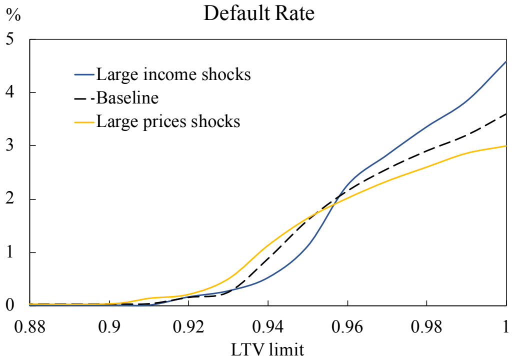
Although a tightening of credit conditions reduces the default rate, since houses become less affordable, the homeownership rate declines in a monotonic manner as well. Unlike the homeownership rate, the relationship between the LTV limit and the net worth of households is non-monotonic. When households default, they lose their home equity and, consequently, their net worth takes a hit. A tightening of the LTV limit means that households now have to accumulate higher levels of liquid assets to pay for the higher down-payment, before they can buy a house. As a result of this, households have a larger equity share in their house. This means that they have a larger capacity to absorb negative income and house price shocks, and their likelihood of default falls. Tightening the LTV limit from a very loose level leads to an increase in the median net worth of households. This is due to both a lower default rate and households accumulating greater liquid assets. A borrowing limit that is too tight, however, can lead to excessive reduction in the homeownership rate, which can lead to a negative impact on the households' median net worth. Figure 1.6 illustrates this point. We can see that the median net worth of households as a function of the LTV limit produces a hump shape. A tightening of the LTV limit, from a very loose value of 100%, initially leads to an increase in the median net worth of households; however, further tightening results in a decline in the median net worth for the reasons outlined above. In the baseline calibration, the LTV limit of 95% is in the region where a moderate tightening of the LTV limit leads to an increase in the median net worth of households.

1.7.2 Performance of LTV rules under alternative income and house price shocks

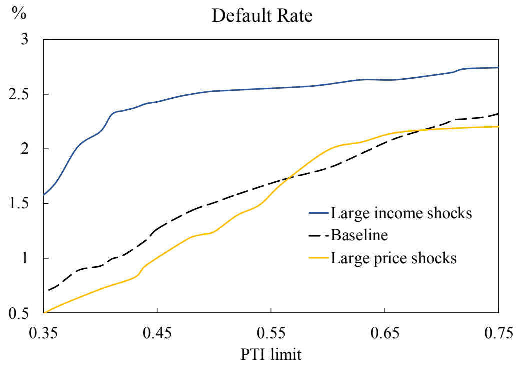
To study the performance of the LTV rule under larger income shocks, the variance of persistent income, σ^2_ψ, is doubled. The blue line in Fig 1.7a plots the performance of the LTV rule in this economy. Fig 1.7a shows that in an economy with larger income shocks, the performance of an LTV rule in reducing the default rate increases. This is demonstrated by the steeper slope of the default rate line under more volatile income. To study the performance of the LTV rule under larger house price shocks, the variance of the house price shocks, σ_ξ^2, is doubled. The yellow line in Fig 1.7a shows that when house price shocks are large, the effectiveness of an LTV rule in reducing defaults declines, as demonstrated by the lower slope of the default rate line.

Under an LTV rule, the default risks primarily arise from low income households, who get access to large mortgage balances by meeting the LTV requirement, but are subsequently hit with a bad house price shock. When income shocks are large, the income distribution becomes wider, which means that low income households have smaller incomes compared to low income households when the shock size is small. Under these circumstances, a tightening of the LTV rule is *more* effective at filtering out from the housing market households with the highest ex-post risk of defaulting. This is because it is relatively harder for low income households to accumulate the higher downpayment when income shocks are large. To put it differently, when income shocks are large, it becomes easier for a tightening in the LTV rule to identify and exclude from the housing market households who have the highest likelihood of defaulting. Consequently, the effectiveness of the LTV rule in reducing defaults improves.

When house price shocks are larger, the ability of a tightening in the LTV rule to exclude high risk households worsens, demonstrated by the more moderate slope of the yellow line in Fig 1.7a. This is because under large house price shocks, when house prices rise, they rise to higher levels, giving both the low and high income households



(a) Performance of LTV rule.



(b) Performance of PTI rule.

Figure 1.7: Effectiveness of LTV and PTI rules in reducing defaults under different income and house price specifications.

access to much larger mortgage balances. Even though the rise in house prices is met with a proportionate rise in the downpayment, the fact that households have access to larger mortgages means that those households who do become homeowners have a much higher debt burden. Compared to high income households, the debt burden is higher for low income households who are able to meet the LTV requirement to become homeowners. Consequently, if house prices drop, even a moderate shock to income can lead them to default. Even though tightening of the LTV limit still reduces the default rate, because homebuyers have to put in additional downpayment, the reduction is more moderate under large house price shocks. The LTV rule does not consider the debt burden that buying a house puts on households and, therefore, cannot identify and exclude from the housing market the households who are able to meet the downpayment requirement but are nonetheless highly burdened by the mortgage payments. In this model, an LTV rule in which η^{LTV} increases with the size of the house prices could potentially minimize this weakness of the standard LTV rule.

1.7.3 Performance of PTI rules

In this section, the LTV rule is replaced with a PTI rule. The dotted line in Fig 1.7b shows that under the baseline income and house price specifications, a tightening of credit conditions that is achieved by decreasing the PTI limit leads to a decline in the default rate. Compared to when an LTV rule is operational, the results show some salient differences under a PTI rule. Another thing to note is that since the income distribution is discretized to finite points, for extremely lax PTI limits, all households have access to the housing market without requiring any downpayment. Unless the PTI limit is tightened aggressively, the default rate is unresponsive to the credit conditions.

Under a PTI rule, the default risks primarily arise from high income households who at the time of mortgage origination get access to large mortgage balances, but are subsequently hit with a bad income shock. When the size of house price shocks is large, the PTI rule becomes more effective at reducing defaults. This is demonstrated by the steeper slope of the yellow line in Fig 1.7b. Under a PTI rule, to maintain the debt burden for a given income level, households have to put up a larger downpayment if house prices are above average and vice versa. Since with a PTI rule in place, the default risks primarily arise from high income households buying expensive houses, when credit conditions are tightened, a PTI rule is more effective at forcing the high risk households to build larger equity buffers. This translates to a higher effectiveness of the PTI rule in reducing defaults when house prices shocks are large.

In contrast, the blue line in Fig 1.7b shows that when the size of income shocks is increased by doubling the variance of the persistence income process, σ_ψ^2, the effectiveness of the PTI rule in reducing the default rates deteriorates. A negative consequence of larger income shocks under a PTI rule is that now high income households have access to much larger mortgage balances, regardless of the price level, and there is no explicit requirement for higher downpayments. Even though a tightening of the PTI rule still reduces the defaults that could potentially arise from low income households entering the housing market, it is unable to effectively filter out risky high income households from the mortgage market. Consequently, a PTI rule's effectiveness in reducing the default rate deteriorates. In this model, a PTI rule in which η^{PTI} increases with the size of the income level could potentially minimize this weakness of the standard PTI rule.

1.8 Conclusion

This paper studies how the effectiveness of LTV and PTI rules in reducing defaults changes under various income and house price specifications. The results suggest that policymakers, when designing macroprudential tools, must also consider the household-level income and house price dynamics of the economy. In an economy with large income shocks, an LTV rule serves as a more effective macroprudential tool in reducing the default rates. In contrast, in an economy with large house price shocks, a PTI rule fares better. A prevalent macroprudential policy, which is not considered in this study, is the debt-to-income (DTI) rule. DTI rules are popular in the European countries, which predominantly have adjustable rate mortgages. A study of whether a DTI rule insulates better against interest rate shocks is left for future research. The modeling framework introduced in this paper can also be extended to address more complex questions, such as the welfare implications of macroprudential policy.

1.9 Appendix

1.9.1 Detailed Problem

In this section, I solve the problem of the different sub-types of households separately.

1.9.1.1 Note on state variables

Since this paper employs the endogenous gridpoint method in solving our model, a distinction needs to be made between the *pre-decision* and *post-decision* state variables. I denote the pre-decision liquid assets by m_t and the post-decision liquid assets by a_t. For homeowner, there are 2 more state variables: the level of housing stock, h_t, and the mortgage balance, b_t. Although there is no uncertainty involved in the dynamic equations linking these pre- and post-decision state variables, just for clarity of thought, I will denote the post-decision housing stock (mortgage debt) variable, which will also be the choice variables for new homeowners, by $\underline{h}_t$ ($\underline{b}_t$), and the pre-decision housing stock (mortgage debt) variable by h_t (b_t).

The dynamic equation for liquid assets is given by

$$m_{t+1} = (1 + r)a_t + y_{t+1}, \tag{1.20}$$

where a_t is the end-of-period assets after all consumption and housing decisions have been made.

The dynamic equation for housing stock is given by

$$h_{t+1} = \underline{h}_t, \tag{1.21}$$

where $\underline{h}_t$ is a choice variable for new homeowners and thereafter it evolves according to

$$\underline{h}_t = h_t, \tag{1.22}$$

unless the homeowner is moving out of the house (**ht, hd**), in which case $\underline{h}_t = 0$.

The dynamic equations for mortgage balance is given by

$$b_{t+1} = (1 + r^m)\underline{b}_t, \tag{1.23}$$

where $\underline{b}_t$ is a choice variable for new homeowners and thereafter it evolves according to

$$\underline{b}_t = b_t - \lambda(b_t), \tag{1.24}$$

where $\lambda(b_t)$ is the mortgage function that gives the mortgage payments for a given beginning-of-period mortgage balance.

Before I proceed to solving the detailed problem, I must note that I will denote the variables that are defined as a function of the end-of-period state variables using a gothic font. That is, the end-of-period value function is denoted by $\mathfrak{v}$, and the end-of-period consumption function (or the consum*ed* function) is denoted by $\mathfrak{c}$.

1.9.1.2 Renter's problem

rr:

The intertemporal optimization problem for **(rr)**, in Bellman form, is given by

$$V_t^{rr}(m_t, y_t^p, p_t) = \max_{c_t, s_t} u(\tilde{c}_t) + \beta \mathop{\mathbb{E}_t}_{\{y,p\}}\{V_{t+1}^r(m_{t+1}, y_{t+1}^p, p_{t+1})\}$$

s.t.

$a_t = m_t - c_t - s_t \geq 0$

$m_{t+1} = (1+r)a_t + y_{t+1}$

Defining total consumption expenditure, $\hat{c}_t$, as $\hat{c}_t = c_t + s_t$ and using the Cobb-Douglas functional form for $\tilde{c}_t$, the intratemporal optimality conditions imply that

$$c_t = \alpha \hat{c}_t \tag{1.25}$$

and

$$s_t = (1-\alpha)\hat{c}_t. \tag{1.26}$$

Moreover,

$$\tilde{c}_t = \alpha^\alpha (1-\alpha)^{1-\alpha} \hat{c}_t \tag{1.27}$$

Hence, for renters, the maximization problem can be written in terms of the total consumption expenditure, $\hat{c}_t$ and the non-durable and housing expenditure can be determined by eqs. 1.25 and 1.26:

$$V_t^{rr}(m_t, y_t^p, p_t) = \max_{\hat{c}_t} u\left(\tilde{c}_t\right) + \beta \underset{\{y,p\}}{\mathbb{E}_t}\{V_{t+1}^r(m_{t+1}, y_{t+1}^p, p_{t+1})\}$$

s.t.

$$a_t = m_t - \hat{c}_t \geq 0$$

$$m_{t+1} = (1+r)a_t + y_{t+1}$$

rh:

The intertemporal optimization problem for **(rh)**, in Bellman form, is given by

$$V_t^{rh}(m_t, y_t^p, p_t) = \max_{\hat{c}_t, \underline{h}_t, \underline{b}_t} u\left(\tilde{c}_t\right) + \beta \underset{\{y,p\}}{\mathbb{E}_t}\{V_{t+1}^h(m_{t+1}, h_{t+1}, b_{t+1}, y_{t+1}^p, p_{t+1})\}$$

s.t.

$$a_t = m_t - \hat{c}_t - [p_t \underline{h}_t - \underline{b}_t] \geq 0$$

$$m_{t+1} = (1+r)a_t + y_{t+1}$$

$$h_{t+1} = \underline{h}_t$$

$$\underline{b}_t \leq \eta^{LTV} . p_t . \underline{h}_t$$

$\lambda_{t+1}(b_{t+1}) \leq \eta^{PTI}.y_t^p$

$b_{t+1} = (1 + r^m)\underline{b}_t$

The renters' problem yields:

$$\boxed{\frac{\partial V_{t+1}^r(m_{t+1}, y_{t+1}^p, p_{t+1})}{\partial m_{t+1}} = \alpha^\alpha (1-\alpha)^{1-\alpha} u'\left(\tilde{c}_{t+1}\right).} \tag{1.28}$$

1.9.1.3 Homeowner's problem

There are 3 sub-types of homeowners: **(hh)**, **(ht)**, and **(hd)**.

hh

The intertemporal optimization problem for **(hh)**, in Bellman form, is given by

$$V_t^{hh}(m_t, h_t, b_t, y_t^p, p_t) = \max_{c_t} u(\tilde{c}_t) + \beta \mathop{\mathbb{E}_t}_{\{y,p\}} \{V_{t+1}^h(m_{t+1}, h_{t+1}, b_{t+1}, y_{t+1}^p, p_{t+1})\}$$

s.t.

$a_t = m_t - c_t - \lambda(b_t) \geq 0$

$m_{t+1} = (1 + r)a_t + y_{t+1}$

$\underline{h}_t = h_t$

$h_{t+1} = \underline{h}_t$

$\underline{b}_t = b_t - \lambda(b_t)$

$b_{t+1} = (1 + r^m).\underline{b}_t$

$s_t = \zeta h_t$

ht:

The intertemporal optimization problem for **(ht)**, in Bellman form, is given by

$$V_t^{ht}(m_t, h_t, b_t, y_t^p, p_t) = \max_{c_t} u(\tilde{c}_t) + \beta \mathop{\mathbb{E}_t}_{\{y\}} \{V_{t+1}^t(m_{t+1}, y_{t+1}^p)\}$$

s.t.

$a_t = m_t - c_t + [p_t h_t - b_t] \geq 0$

$m_{t+1} = (1 + r)a_t + y_{t+1}$

$s_t = \zeta h_t$

hd:

The intertemporal optimization problem for **(hd)**, in Bellman form, is given by

$$V_t^{hd}(m_t, h_t, b_t, y_t^p) = \max_{c_t} u(\tilde{c}_t) - \chi + \beta \mathop{\mathbb{E}_t}_{\{y\}} \{V_{t+1}^d(m_{t+1}, y_{t+1}^p)\}$$

s.t.

$$a_t = m_t - c_t \geq 0$$

$$m_{t+1} = (1+r)a_t + y_{t+1}$$

$$s_t = \zeta h_t$$

The homeowners' problem yields:

$$\boxed{\frac{\partial V^h_{t+1}(m_{t+1}, h_{t+1}, b_{t+1}, y^p_{t+1}, p_{t+1})}{\partial m_{t+1}} = u'(\tilde{c}_{t+1})\frac{\partial \tilde{c}_{t+1}}{\partial c_{t+1}}.} \tag{1.29}$$

1.9.1.4 Tenant and Defaulter's problem

Since the tenant or defaulter always remains a tenant or defaulter, respectively, $V_t^{t,d} = V_t^{tt,dd}$. Their intertemporal optimization problem, in Bellman form, is similar and is given by

$$V_t^{t,d}(m_t, y_t^p) = \max_{\hat{c}_t} u\left(\tilde{c}_t\right) + \beta \underset{\{y\}}{\mathbb{E}_t}\{V^{t,d}_{t+1}(m_{t+1}, y^p_{t+1})\}$$

s.t.

$$a_t = m_t - \hat{c}_t \geq 0$$

$m_{t+1} = (1+r)a_t + y_{t+1}$

The tenants' and defaulters' problem yields:

$$\boxed{\frac{\partial V^{t,d}_{t+1}(m_{t+1}, y^p_{t+1})}{\partial m_{t+1}} = \alpha^{\alpha}(1-\alpha)^{1-\alpha} u'\left(\tilde{c}_{t+1}\right)} \tag{1.30}$$

1.9.1.5 Consumption functions of sub-types

Tenant and Defaulter:

It is possible to define a function

$$\boxed{\mathfrak{v}^{t,d}_t(a_t, y^p_t) = \beta \underset{\{y\}}{\mathbb{E}_t}\{V^{t,d}_{t+1}((1+r)a_t + y_{t+1}, y^p_{t+1})\}} \tag{1.31}$$

that returns the expected $t+1$ value for the tenant or defaulter associated with *ending* period t with assets a_t, having received persistent income y^p_t.

The non-housing consumption function for a tenant or defaulter is then given by:

$$\boxed{\mathfrak{c}_t(a_t, y^p_t) = \alpha \left(\frac{\mathfrak{v}^{t,d\prime}_t(a_t, y^p_t)}{\left(\alpha^{\alpha}(1-\alpha)^{1-\alpha}\right)^{1-\rho}} \right)^{-1/\rho}} \tag{1.32}$$

Renters:

(rr)

It is possible to define a function

$$\boxed{\mathfrak{v}^{r}_t(a_t, y^p_t, p_t) = \beta \underset{\{y,p\}}{\mathbb{E}_t}\{V^{r}_{t+1}((1+r)a_t + y_{t+1}, y^p_{t+1}, p_{t+1})\}} \tag{1.33}$$

that returns the expected $t+1$ value for a household *ending* period t as a renter with assets a_t, having received persistent income y_t^p, and house price shock p_t.

The non-housing consumption function for **rr** is given by:

$$\boxed{\mathfrak{c}_t(a_t, y_t^p, p_t) = \alpha \left(\frac{\mathfrak{v}_t^{r\prime}(a_t, y_t^p, p_t)}{(\alpha^\alpha (1-\alpha)^{1-\alpha})^{1-\rho}} \right)^{-1/\rho}} \tag{1.34}$$

(rh)

It is possible to define a function

$$\boxed{\mathfrak{v}_t^h(a_t, \underline{h}_t, \underline{b}_t, y_t^p, p_t) = \beta \underset{\{y,p\}}{\mathbb{E}_t} \{V_{t+1}^h((1+r)a_t + y_{t+1}, h_{t+1}(\underline{h}_t), b_{t+1}(\underline{b}_t), y_{t+1}^p, p_{t+1})\}} \tag{1.35}$$

that returns the expected $t+1$ value for a household ending period t as a homeowner with assets, a_t, housing stock, $\underline{h}_t$, mortgage balance, $\underline{b}_t$, having received persistent income y_t^p, and house price shock p_t.

The non-housing consumption function for **rh** is given by:

$$\boxed{\mathfrak{c}_t(a_t, \underline{h}_t, \underline{b}_t, y_t^p, p_t) = \alpha \left(\frac{\mathfrak{v}_t^{h\prime}(a_t, \underline{h}_t, \underline{b}_t, y_t^p, p_t)}{(\alpha^\alpha (1-\alpha)^{1-\alpha})^{1-\rho}} \right)^{-1/\rho}} \tag{1.36}$$

Homeowners:

(hh)

The non-housing consumption function for **hh** is given by:

$$\boxed{\mathfrak{c}_t(a_t, \underline{h}_t, \underline{b}_t, y_t^p, p_t) = \left(\frac{\varrho \mathfrak{v}_t^{h\prime}(a_t, \underline{h}_t, \underline{b}_t, y_t^p, p_t)}{(\zeta h_t(\underline{h}_t))^{(1-\alpha)(1-\rho)}} \right)^{\frac{1}{\alpha - \alpha\rho - 1}},} \tag{1.37}$$

where $\varrho = \alpha^{\alpha-\alpha\rho-1}(1-\alpha)^{(1-\alpha)(1-\rho)}$.

(ht, hd)

The non-housing consumption function for **ht** or **hd** is given by:

$$\boxed{\mathfrak{c}_t(a_t, \underline{h}_t(h_t), \underline{b}_t(b_t), y^p_t, p_t) = \left(\frac{\varrho \mathfrak{v}^{t,d\prime}_t(a_t, y^p_t)}{(\zeta h_t)^{(1-\alpha)(1-\rho)}}\right)^{\frac{1}{\alpha-\alpha\rho-1}}.} \tag{1.38}$$

1.9.1.6 The method of endogenous gridpoints

Denote by $\overrightarrow{a_t}$ the grid of end-of-period assets (greater than their lower bound). Each element i of the grid is denoted by $a_{t,i}$. Similarly, a grid is set over persistent income ($\overrightarrow{y^p_t}$), transitory income ($\overrightarrow{\theta_t}$), house prices ($\overrightarrow{p_t}$), housing ($\overrightarrow{\underline{h}_t}$), and mortgage debt ($\overrightarrow{\underline{b}_t}$).

Each $\{a_{t,i}, y^p_{t,j}\}$ pair is associated with some marginal valuation as of the end of period t for tenants, and defaulters, i.e. $\mathfrak{v}^{t,d\prime}_t(a_{t,i}, y^p_{t,j})$. Each $\{a_{t,i}, y^p_{t,j}, p_{t,k}\}$ pair is associated with some marginal valuation as of the end of period t for renters, i.e. $\mathfrak{v}^{r\prime}_t(a_{t,i}, y^p_{t,j}, p_{t,k})$. Similarly, each $\{a_{t,i}, \underline{h}_{t,j}, \underline{b}_{t,k}, y^p_{t,l}, p_{t,m}\}$ pair is associated with some marginal valuation as of the end-of-period t for homeowners, i.e. $\mathfrak{v}^{h\prime}_t(a_{t,i}, \underline{h}_{t,j}, \underline{b}_{t,k}, y^p_{t,l}, p_{t,m})$. Using the expressions for the end-of-period consumption functions solved above, it is then trivial to solve for the value of $\mathfrak{c}^s$ that yields the appropriate marginal valuation for the 7 sub-types of households, s.

With mutually consistent values of $\mathfrak{c}^s_{t,\{i,j\}}$ and $\{a_{t,i}, y^p_{t,j}\}$ for $s \in \{tt, dd\}$, we can find the $m_{t,\{i,j\}}$ that corresponds to them. The $\overrightarrow{m_t}$ gridpoints are endogenous and we can generate a set of $m_{t,\{i,j\}}$ and $c^s_{t,\{i,j\}}$ pairs that can be interpolated in order to yield the consumption interpolation function $c^s(\overrightarrow{m_t}, \overrightarrow{y^p_t})$.

For $s \in \{rr\}$, with mutually consistent values of $\mathfrak{c}^s_{t,\{i,j,k\}}$ and $\{a_{t,i}, y^p_{t,j}, p_{t,k}\}$, we can find the $m_{t,\{i,j,k\}}$ that corresponds to them. This results in the consumption function for continuing renters, given by $c^{rr}(\overrightarrow{m_t}, \overrightarrow{y^p_t}, \overrightarrow{p_t})$. For $s \in \{rh\}$, with mutually consistent values of $\mathfrak{c}^{rh}_{t,\{i,j,k,l,m\}}$ and $\{a_{t,i}, \underline{h}_{t,j}, \underline{b}_{t,k}, y^p_{t,l}, p_{t,m}\}$, we can find the $m_{t,\{i,j,k,l,m\}}$ that corresponds to them. $c^{rh}(\overrightarrow{m_t}, \overrightarrow{y^p_t}, \overrightarrow{p_t})$ then is the consumption function that forms an *upper-envelope* over the different $\{\underline{h}_{t,j}, \underline{b}_{t,k}\}$ pairs.

Similarly, with mutually consistent values of $\mathfrak{c}^{hh}_{t,\{i,j,k,l,m\}}$ and $\{a_{t,i}, \underline{h}_{t,j}, \underline{b}_{t,k}, y^p_{t,l}, p_{t,m}\}$, we can find $\{m_{t,\{i,j,k,l,m\}}, h_{t,\{i,j,k,l,m\}}, b_{t,\{i,j,k,l,m\}}\}$ that correspond to them. This results in the consumption function for continuing homeowners, given by $c^{hh}(\overrightarrow{m_t}, \overrightarrow{h_t}, \overrightarrow{b_t}, \overrightarrow{y^p_t}, \overrightarrow{p_t})$. Finally, for households ceasing to be homeowners, $s \in \{ht, hd\}$, with mutually consistent values of $\mathfrak{c}^s_{t,\{i,j,k,l,m\}}$ and $\{a_{t,i}, \underline{h}_{t,j}(h_{t,j}), \underline{b}_{t,k}(b_{t,k}), y^p_{t,l}, p_{t,m}\}$, we can find the $m_{t,\{i,j,k,l,m\}}$ that corresponds to them. Note that in this case, the exogenous grid is over h_t and b_t rather than $\underline{h}_t$ and $\underline{b}_t$. This gives us the consumption function $c^s(\overrightarrow{m_t}, \overrightarrow{h_t}, \overrightarrow{b_t}, \overrightarrow{y^p_t}, \overrightarrow{p_t})$.

For each of the 7 sub-types, m_t is calculated using

$$m_t = \begin{cases} a_{t,i} + \alpha^{-1}\mathfrak{c}_t^{t,d}(a_{t,i}, y_{t,j}^p) & \text{t, d} \\ a_{t,i} + \alpha^{-1}\mathfrak{c}_t^{rr}(a_{t,i}, y_{t,j}^p, p_{t,k}) & \text{rr} \\ a_{t,i} + \alpha^{-1}\mathfrak{c}_t^{rh}(a_{t,i}, \underline{h}_{t,j}, \underline{b}_{t,k}, y_{t,l}^p, p_{t,m}) + \big(p_{t,m}\underline{h}_{t,j} - \underline{b}_{t,k}\big) + \kappa_p & \text{rh} \\ a_{t,i} + \mathfrak{c}_t^{hh}(a_{t,i}, \underline{h}_{t,j}, \underline{b}_{t,k}, y_{t,l}^p, p_{t,m}) + \lambda(b_{t,k}(\underline{b}_{t,k})) + \kappa_m & \text{hh} \\ a_{t,i} + \mathfrak{c}_t^{ht}(a_{t,i}, h_{t,j}, b_{t,k}, y_{t,l}^p, p_{t,m}) - [p_{t,m}h_{t,j} - b_{t,k}] + \kappa_s & \text{ht} \\ a_{t,i} + \mathfrak{c}_t^{hd}(a_{t,i}, h_{t,j}, b_{t,k}, y_{t,l}^p, p_{t,m}) & \text{hd} \end{cases}$$

For **hh**

$$b_{t,i} = \frac{1 + r_m}{(1/2)^{1/n}}.\underline{b}_{t,i}$$

$$h_{t,i} = \underline{h}_{t,i}.$$

1.9.1.7 Conditional value functions of sub-types

Each of the gridpoint pairs are also associated with some valuation as of the end of period t, i.e. $\mathfrak{v}_t^{t,d}(a_{t,i}, y_{t,j}^p)$, $\mathfrak{v}_t^{r}(a_{t,i}, y_{t,j}^p, p_{t,k})$, and $\mathfrak{v}_t^{h}(a_{t,i}, \underline{h}_{t,j}, \underline{b}_{t,k}, y_{t,l}^p, p_{t,m})$. Given the consumption functions, we can also calculate the conditional value functions v_t. Denoting the $\{\underline{h}_t,\underline{b}_t\}$ pair associated with the upper-envelope for the **rh** problem by $\{\underline{\mathbf{h}}_t,\underline{\mathbf{b}}_t\}$, the expressions for the conditional value functions are given by

$$v_t^s(m_{t,i}, y_{t,j}^p) = u\big(\tilde{c}^s(m_{t,i}, y_{t,j}^p)\big) + \mathfrak{v}_t^{t,d}(a_{t,i}, y_{t,j}^p) \quad s = \text{tt,dd}$$

$$v_t^s(m_{t,i}, y_{t,j}^p, p_{t,k}) = u\big(\tilde{c}^s(m_{t,i}, y_{t,j}^p, p_{t,k})\big) + \mathfrak{v}_t^r(a_{t,i}, y_{t,j}^p, p_{t,k}) \qquad s = \text{rr}$$

$$v_t^s(m_{t,i}, y_{t,l}^p, p_{t,m}) = u\big(\tilde{c}^s(m_{t,i}, y_{t,l}^p, p_{t,m})\big) + \mathfrak{v}_t^h(a_{t,i}, \underline{\mathbf{h}}_{t,j}, \underline{\mathbf{b}}_{t,k}, y_{t,l}^p, p_{t,m}) \qquad s = \text{rh}$$

$$v_t^s(m_{t,i}, h_{t,j}, b_{t,k}, y_{t,l}^p, p_{t,m}) = u\big(\tilde{c}^s(m_{t,i}, h_{t,j}, b_{t,k}, y_{t,l}^p, p_{t,m})\big) + \mathfrak{v}_t^h(a_{t,i}, \underline{h}_{t,j}, \underline{b}_{t,k}, y_{t,l}^p, p_{t,m}) \qquad s = \text{hh}$$

$$v_t^s(m_{t,i}, h_{t,j}, b_{t,k}, y_{t,l}^p, p_{t,m}) = u\big(\tilde{c}^s(m_{t,i}, h_{t,j}, b_{t,k}, y_{t,l}^p, p_{t,m})\big) + \mathfrak{v}_t^t(a_{t,i}, y_{t,l}^p) \qquad s = \text{ht}$$

$$v_t^s(m_{t,i}, h_{t,j}, b_{t,k}, y_{t,l}^p) = u\big(\tilde{c}^s(m_{t,i}, h_{t,j}, b_{t,k}, y_{t,l}^p)\big) - \chi + \mathfrak{v}_t^d(a_{t,i}, y_{t,l}^p) \qquad s = \text{hd}$$

For interpolation, however, we use the transformed value functions, $\Lambda_t^s(...)$, which is given by

$$\Lambda_t^s(...) = u^{-1}(v_t^s(...)).$$

1.9.1.8 Period-T solution and T-1 adjustment

For faster convergence of the solution, we use the converged tenant's solution as the terminal period T's solution. Denote the converged consumption function as $c_\infty^t(m_t, y_t^p)$ and the converged value function as $V_\infty^t(m_t, y_t^p)$. Since we are using the converged tenant's solution as the period T solution for all types of households, we have to make an adjustment to the liquid assets of households who are homeowners at the **end of period T-1** to compensate them for the house that will be revoked from them. One way of making this compensation is by giving them funds that are equivalent to the home equity they would have had at the *beginning of period* T. This adjustment is made between the end of period T-1 and the beginning of period T.

The T-1 problem for rh is given by:

$$V^{rh}_{T-1}(m_{T-1}, y^p_{T-1}, p_{T-1}) = \max_{c_{T-1}, s_{T-1}, \underline{h}_{T-1}, \underline{b}_{T-1}} u\,(c_{T-1}, s_{T-1}) + \mathfrak{v}^h_{T-1}(a_{T-1}, y^p_{T-1}), \tag{1.39}$$

where

$$\mathfrak{v}^h_{T-1}(a_{T-1}, y^p_{T-1}) = \beta \underset{\{y\}}{\mathbb{E}}_{T-1}\{V^t_{\infty}(m_T(a_{T-1}), y^p_T)\}$$

$$a_{T-1} = m_{T-1} - c_{T-1} - s_{T-1} - [p_{T-1}.\underline{h}_{T-1} - \underline{b}_{T-1}] - \kappa_p, \quad a_{T-1} \geq 0$$

$$m_T = R.a_{T-1} + y_T + [p_T.h_T - b_T]$$

$$h_T = \underline{h}_{T-1}$$

$$\underline{b}_{T-1} \leq \eta^{LTV}.p_{T-1}.h_{T-1}$$

$$b_T = R^m.\underline{b}_{T-1}$$

The T-1 problem for hh is given by:

$$V^{hh}_{T-1}(m_{T-1}, h_{T-1}, b_{T-1}, y^p_{T-1}, p_{T-1}) = \max_{c_{T-1}} u\,(c_{T-1}, \zeta h_{T-1}) + \mathfrak{v}^h_{T-1}(a_{T-1}, y^p_{T-1}), \tag{1.40}$$

where

$$\mathfrak{v}^h_{T-1}(a_{T-1}, y^p_{T-1}) = \beta \underset{\{y\}}{\mathbb{E}}_{T-1}\{V^t_{\infty}(m_T(a_{T-1}), y^p_T)\}$$

$$a_{T-1} = m_{T-1} - c_{T-1} - \lambda_{T-1}(b_{T-1}) - \kappa_m, \quad a_{T-1} \geq 0$$

$$m_T = R.a_{T-1} + y_T + [p_T.h_T - b_T]$$

$h_T = h_{T-1}$

$\underline{b}_{T-1} = b_{T-1} - \lambda_{T-1}(b_{T-1})$

$b_T = R^m.\underline{b}_{T-1}$

1.9.2 Perfect Foresight Solution and Method for Extrapolation

1.9.2.1 Tenant's perfect foresight consumption function

I solve for the perfect foresight solution of the tenant's problem without a liquidity constraint. The intertemporal optimization problem for **(tt)**, in Bellman form, is given by

$$V_t^t(m_t, y_t^p) = \max_{\hat{c}_t} u\left(\tilde{c}_t\right) + \beta \underset{\{y\}}{\mathbb{E}_t}\{V_{t+1}^t(m_{t+1}, y_{t+1}^p)\}$$

s.t.

$$a_t = m_t - \hat{c}_t$$

$$m_{t+1} = Ra_t + y_{t+1}$$

The Euler equation for this problem is given by

$$u'(\tilde{c}_t) = R\beta \underset{\{y\}}{\mathbb{E}_t}\{u'(\tilde{c}_{t+1})\} \tag{1.41}$$

With perfect foresight and CRRA utility, this becomes

$$\hat{c}_t{}^{-\rho} = R\beta \;\hat{c}_{t+1}{}^{-\rho} \tag{1.42}$$

This implies that

$$\hat{c}_{t+1} = (R\beta)^{1/\rho} \;\hat{c}_t \tag{1.43}$$

Define, $\mathbb{p} \equiv (R\beta)^{1/\rho}$ and $\mathbb{p}_R \equiv \frac{(R\beta)^{1/\rho}}{R}$.

Then, the present discounted value of total consumption is given by

$$\begin{aligned} PDV(\hat{c}_t) &= \hat{c}_t + \frac{\hat{c}_{t+1}}{R} + \frac{\hat{c}_{t+2}}{R^2} + ... \\ &= \hat{c}_t + \mathbb{p}_R\,\hat{c}_t + \mathbb{p}_R{}^2\,\hat{c}_t + ... \end{aligned} \tag{1.44}$$

For the finite horizon problem, ending at period T,

$$PDV(\hat{c}_t) = \left(\frac{1 - \mathbb{p}_R{}^{T-t+1}}{1 - \mathbb{p}_R}\right)\hat{c}_t \tag{1.45}$$

The lifetime human wealth of the perfect foresight household, $\mathfrak{h}_t$, is given by

$$\mathfrak{h}_t = PDV(y_t) = PDV(\overline{y}) = \left(\frac{1 - (1/R)^{T-t+1}}{1 - (1/R)}\right)\overline{y} \tag{1.46}$$

From the intertemporal budget constraint (IBC) we have that,

$$\begin{aligned} PDV(\hat{c}_t) &= PDV(y_t) + m_t - y_t \\ &= PDV(\overline{y}) + m_t - \overline{y} \\ &= \mathfrak{h}_t + m_t - \overline{y} \end{aligned} \tag{1.47}$$

Substituting from 1.45, the IBC yields

$$\left(\frac{1-\mathbb{p}_R{}^{T-t+1}}{1-\mathbb{p}_R}\right)\hat{c}_t = \mathfrak{h}_t + m_t - \overline{y}$$

This gives the **perfect foresight consumption function** for the tenant,

$$\boxed{\overline{c_t}(m_t) = \alpha\kappa_t\Bigg(\mathfrak{h}_t + (m_t - \overline{y})\Bigg),} \tag{1.48}$$

where α is the fraction of the total expenditure spent on non-durable consumption and $\kappa_t = \left(\frac{1-\mathbb{p}_R}{1-\mathbb{p}_R{}^{T-t+1}}\right)$.

1.9.2.2 Tenant's perfect foresight value function

For tenants

$$\begin{aligned}
\overline{V_t}(m_t) &= \sum_{j=0}^{T-t}\beta^j u\left(\overline{\tilde{c}_{t+j}}\right) \\
&= \sum_{j=0}^{T-t}\beta^j u\left(\mathbb{p}^j\,\overline{\tilde{c}_t}\right) \\
&= \overline{\tilde{c}_t}^{1-\rho}\sum_{j=0}^{T-t}\beta^j u\left(\mathbb{p}^j\right) \\
&= \overline{\tilde{c}_t}^{1-\rho}\sum_{j=0}^{T-t}\frac{\left(\beta\mathbb{p}^{1-\rho}\right)^j}{1-\rho} \\
&= \left(\frac{1-\left(\beta\mathbb{p}^{1-\rho}\right)^{T-t+1}}{1-\left(\beta\mathbb{p}^{1-\rho}\right)}\right)u\left(\overline{\tilde{c}_t}(m_t)\right) = \frac{1}{\kappa_t^V}u\left(\overline{\tilde{c}_t}(m_t)\right),
\end{aligned}$$

where $\kappa_t^V = \left(\frac{1-\left(\beta\mathbb{p}^{1-\rho}\right)}{1-\left(\beta\mathbb{p}^{1-\rho}\right)^{T-t+1}}\right)$.

Applying the $u^{-1}(.)$ operator to this yields:

$$\begin{aligned}
\overline{invV}_t(m_t) &= u^{-1}\left(\overline{V_t}(m_t)\right) \\
&= \left((1-\rho)\overline{V_t}(m_t)\right)^{\frac{1}{1-\rho}}
\end{aligned}$$

1.9.3 Transformation for extrapolation

In the EGM, we need to evaluate the gothic-$\mathfrak{v}$ values for all points on the `a-grid`. For most of the points on the `a-grid` interpolation is used; however, for high values of `a-grid`, extrapolation might be needed. There are two problems involved with extrapolation at high values of `a-grid` that need to be addressed. Firstly, extrapolation at the highest values of `a-grid` can possibly lead to inaccurate results. Secondly, certain multi-dimensional interpolators do not extrapolate. In order to avoid these problems, I will transform the consumption and value functions in a way that allows us to get as close as possible to the true solution at large values of m_t.

In our model, at very high values of liquid assets, m_t, renters choose to stay as renters and homeowners choose to sell their house and become tenants. This is because homeowners only have access to single sized house and at very high values of m_t that house does not provide enough housing services. In contrast to homeowners, renters and tenants can adjust their housing services freely without an upper-bound on the rental services. I use this feature of the model and the fact that the consumption functions are bounded above by the perfection foresight solution to extrapolate a transformed consumption function.

For clarity, I will suppress all the arguments of the consumption functions except for m_t. Denote the consumption function from the optimization problem by $c_t(m_t)$ and the perfect foresight consumption function by $\overline{c}_t(m_t)$. Now define the ratio of the two as

$$\mathbb{c}_t(m_t) = \frac{c_t(m_t)}{\bar{c}_t(m_t)}.$$

We know that

$$\lim_{m_t \to \infty} \mathbb{c}_t(m_t) = 1.$$

Define

$$\mathbb{y}_t(m_t) = \frac{1}{1 + e^{-m_t}}.$$

This implies that

$$m_t = \log\left(\frac{\mathbb{y}_t}{1 - \mathbb{y}_t}\right).$$

$m_t \in (-\infty, \infty)$ and $\mathbb{y}_t \in (0, 1)$.

Now define

$$\tilde{\mathbb{c}}_t(\mathbb{y}_t) = \mathbb{c}_t(m_t(\mathbb{y}_t)).$$

$$\lim_{\mathbb{y}_t \to 1} \tilde{\mathbb{c}}_t(\mathbb{y}_t) = 1.$$

Calling the interpolated version $\overrightarrow{\tilde{\mathbb{c}}}_t$, we can get the consumption function $c_t(m_t)$ by using

$$c_t(m_t) = \overrightarrow{\tilde{\mathbb{c}}}_t(\mathbb{y}_t(m_t))\bar{c}_t(m_t).$$

1.9.4 Geometric mortgage payments

I assume a geometric mortgage payment schedule in which each period's mortgage payment, π_t, is a fixed proportion ρ of the mortgage balance at the beginning of the period, i.e.

$$\pi_t = \rho.b_t \tag{1.49}$$

Mortgage balance evolves according to

$$b_{t+1} = (b_t - \pi_t).(1 + r_m). \tag{1.50}$$

We need to determine the value of ρ such that the half-life of the mortgage balance is 15 years, i.e.

$$b_{t+n} = \frac{1}{2}.b_t, \tag{1.51}$$

where $n = 15$ at an annual frequency and $n = 60$ at a quarterly frequency.

We first substitute eq.(1.49) into eq.(1.50), which yields

$$b_{t+1} = (b_t - \rho.b_t).(1 + r_m)$$

or

$$\frac{b_{t+1}}{b_t} = (1 - \rho).(1 + r_m) \tag{1.52}$$

This can be iterated forward to get

$$\frac{b_{t+n}}{b_t} = \left[(1 - \rho).(1 + r_m)\right]^n \tag{1.53}$$

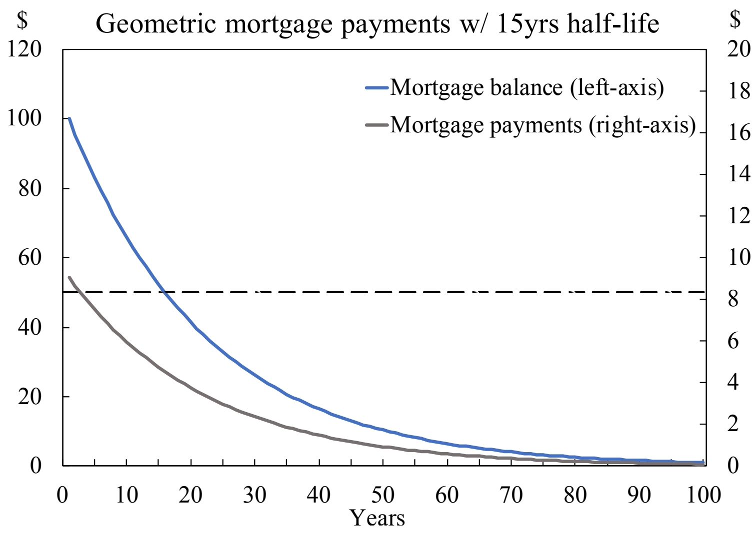


Figure 1.8: Geometric mortgage payment schedule with a half-life of 15 years.

Substituting eq.(1.51) into eq.(1.53), we get

$$\boxed{\rho = 1 - \frac{(1/2)^{1/n}}{1 + r_m}.} \tag{1.54}$$

This expression does not depend on the time since mortgage origination, which means that we do not need an extra state variable that tracks the age of the mortgage. As can be seen in Figure1, the mortgage balance has a half life of 15-years and unlike constant amortization, the annual mortgage payments decline with time.

1.9.5 Computation

If certain conditions are satisfied White (2015), the Endogenous Gridpoint Method (EGM) can be used to solve a variety of models in the consumption-savings class of models. Standard solution techniques rely on numerical root-finding methods which can be computationally intense. The EGM defines the problem in terms of pre- and post-decision state variables – a technique that is used extensively in the Operations Research literature [4]– and bypasses the computationally intense root-finding step altogether. In models with both discrete and continuous choice variables, however, there are discontinuities in the marginal value functions in the regions in the state space where discrete choices are made. The discontinuities in the marginal value functions translate to discontinuities in the consumption functions and consequently the EGM can produce suboptimal solution points. Identifying these suboptimal points imposes another computational challenge (Iskhakov et al. (2017)).

If the problem being solved is an infinite-horizon problem, the computational challenges arising from the discontinuities can become even more troubling. In a standard consumption-saving model with a single discrete choice, each period's solution has kinks in its value functions and discontinuities in its consumption functions at the points around which different discrete choices are made. These are the *primary* kinks or discontinuities. When we iterate backwards and solve for the previous period, not only does the solution have primary kinks from the discrete choices in that period, the solution also has *secondary* kinks due to the kinks that exist in the solution of the next period. These kinks reverberate back through each period and the further back we iterate, the more kinks we get in our solution. This can lead to severe problems in an infinite-horizon problem, as convergence can become virtually

[4] Barnhart, Cynthia and Gilbert Laporte, "Handbooks in Operations Research and Management Science", North-Holland, 2006.

impossible to achieve.

To make using EGM feasible in a discrete-continuous choice model, we need to add 'smoothness' to the problem to reduce the severity of the kinks and the discontinuities. Iskhakov et al. (2017) use exogenous taste shocks to achieve smoothness around the kinks and in our model we use the endogenous search mechanism that is outlined in the text. It must be noted that the smoothness does not remove the primary kink/discontinuity from the period in which the decision is being made. It simply smoothes out the discontinuities in the *expected* marginal value functions that are needed in the EGM step to calculate the contemporaneous consumption function.

Chapter 2

Has Higher Household Indebtedness Weakened Monetary Policy Transmission?

2.1 Introduction

A common perception among many academics and policymakers is that monetary policy in advanced economies has been less effective since the crisis because of higher household debt, and associated credit constraints. Amir Sufi summarized this view in 2015 (Sufi (2015)): "Monetary policy over the past seven years has been ineffective because it has channeled interest savings and additional credit to exactly the households that are least likely to change their spending in response. The households that would normally spend most aggressively out of monetary policy shocks are heavily indebted or have seen their credit scores plummet, rendering them either unwilling or unable to boost spending."

To date, however, the issue has – to our knowledge – not been systematically assessed. While a few studies have examined the role of household balance sheets in monetary transmission, they have focused on the pre-crisis period, and have not directly analyzed whether post-crisis debt levels have impeded transmission[1]. These studies suggest that more indebted and less liquid households react more to monetary policy. The argument is that these households run into collateral and liquidity constraints, which monetary policy directly affects (Aladangady (2014); Cloyne et al. (2018); Di Maggio et al. (2017); Flodén et al. (2017), emphasize households' cash flows; Luo (2017) focuses on households' default risk). Using aggregate data, Hofmann and Peersman (2017) also find a stronger impact of monetary policy in economies with high private debt. One open question, however, is whether at very high debt levels, effects are different. In these cases, monetary easing may do little to alleviate credit constraints, and thus stimulate consumption (Alpanda and Zubairy (forthcoming); Sufi (2015); Beraja et al. (2019)).[2] The responsiveness of households

[1] Without discussing monetary policy effects, Mian et al. (2013) and Kaplan et al. (2014) find that leverage and liquidity significantly affect household's propensity to consume.

[2] Some empirical studies show adverse effects of high debt on consumption, although they do not examine monetary policy effects (such as Alter et al. (2018); Drehmann et al. (2017); Mian et al. (2017a);Melzer (2017); IMF (2017); Dynan (2012)). Many studies highlight the adverse effect on aggregate demand from debt deleveraging caused by the housing crisis during the U.S. Great Recession (such as Eggertsson and Krugman (2012); Mian and Sufi (2014); Guerrieri and Lorenzoni (2011); and Eggertsson et al. (2017)).

to monetary policy may thus display an inverted U-shaped pattern, rising as debt levels grow below a certain threshold, and declining thereafter.

In this paper we compare the transmission of monetary policy through household consumption in the pre- and post-crisis periods, and ask whether changes therein can be explained by the evolution of household balance sheets. To this end, we use quarterly household-level data from the U.S. Consumer Expenditure Survey (CEX) from 1996 to 2014. We first assess average changes in the responsiveness of household consumption to monetary policy shocks, which we identify using exogenous instruments drawn from high-frequency data, in the tradition of Bernanke and Kuttner (2005). We employ both synthetic cohort analysis (which enable us to obtain longer times series and derive local projections), and standard panel data methods that exploit the full micro data set. Next, we explore the role of two household balance-sheet variables in driving cross-sectional differences in the responses to monetary policy shocks: indebtedness (mortgage balance relative to house value), and liquidity (liquid assets to monthly income).[3]

We show that the response of household consumption to monetary policy shocks has diminished since the global financial crisis. We also find that higher-indebted households tend to respond more to monetary policy shocks – particularly relative to durable consumption – in the pre- and post-crisis periods. While effects appear non-linear, they are not U-shaped, as households with the highest indebtedness respond

[3]Recent papers suggest that consumption responses to monetary policy should depend on the distribution of households' liquidity; see Heterogeneous Agent New Keynesian (HANK) models (such as Kaplan et al. (2018), Kaplan and Violante (2018), Hedlund et al. (2016), and Francisco (2018)).

most to monetary policy shocks. This suggests that household debt did not contribute to lessening the effects of monetary policy over time, since the distribution of debt did not change markedly with the crisis, while its average even increased somewhat.[4]

Similar results hold for household liquidity. Households with lower levels of liquid assets react more strongly to monetary policy shocks, both pre- and post-crisis. Again, because the distribution of liquidity across households remained stable over time, liquidity constraints cannot explain the decline in monetary policy effectiveness. The explanation for the lower effectiveness of monetary policy must therefore lie elsewhere, such as in the higher degree of economic uncertainty brought about by the crisis.

2.2 Hypotheses and Data

The main questions we explore in this paper are:

1. Has the response of household consumption to monetary policy shocks declined since the global financial crisis?

2. Do households with greater indebtedness respond more strongly to monetary policy shocks? Is there evidence of nonlinearities – in particular does the responsiveness decline after a certain threshold?

[4] Justiniano et al. (2015) and Yellen (2016) also suggest that debt overhang alone cannot explain the slow recovery from the U.S. Great Recession. Also, Bernanke (2018) does not find strong predictive powers of household balance sheets for economic conditions, although he argues that it does not dismiss the important role of household balance sheets considering the empirical challenges in identifying macro effects.

3. Do households with low levels of liquid assets react more to monetary policy shocks? And again, are non-linear effects discernable?

4. Can shifts in the distribution of household indebtedness and liquidity between the pre- and post-crisis periods explain the observed changes in the average response of household consumption to monetary policy?

2.2.1 Data: Variables of Interest, Sources, and Summary Statistics

We use the Consumer Expenditure Survey (CEX)[5] for household-level consumption, income, and balance-sheet data between 1996Q1 and 2014Q4. The CEX data are well suited for our analysis for three reasons. First, the survey offers rich cross-sectional variation, with about 7,500 households interviewed per quarter. Second, the quarterly frequency is helpful to study the short-run effects of monetary policy on households' consumption. Third, CEX data span a sufficiently long period to compare household behavior before and after the crisis.

We construct measures of durable and non-durable consumption expenditures. This is to allow for the impact of monetary policy to differ across each category of goods since theory and empirics suggest that the marginal propensity to consume for durable and non-durable goods are significantly different (Souleles (1999); Parker

[5] CEX data available at: https://www.bls.gov/cex/pumd_data.htm#stata

et al. (2013); see Appendix 2.7.1 for more details).

We consider two key characteristics of household balance sheets: indebtedness and liquidity. Indebtedness is defined as the ratio of each household's total mortgage balance (summed over all the properties owned by the household) to the value of the houses it owns, as reported by households. We exclude other liabilities like credit card balances, since fewer households report these and because mortgage debt is the most significant liability for most households.[6] We define liquidity as the ratio of liquid assets to monthly income, as reported by households. Liquid assets include the total balance on households' checking and savings accounts, and income is after-tax. Details are provided in Appendix 2.7.1.

Table 2.1 highlights key features of non-durable and durable consumption. On average, households spend four times more on non-durable consumption relative to durable consumption in any given quarter. However, the standard deviation of durable consumption is notably larger than that of non-durable consumption, pointing to the lumpy nature of durable goods purchases (Caballero and Jaffe (1993)). Consumption levels differ across housing tenure, especially for durable consumption (see Appendix 2.7.1).[7] The distribution of consumption quarter-on-quarter growth changes little after the crisis for both durable and non-durable categories, while the distribution of consumption levels shifts slightly to the left after the crisis.

[6]The CEX collects mortgage information in all interviews, while it collects other financial information (such as credit card debt) only in the 2nd and 5th interviews. Therefore, we focus on mortgage debt, the largest component of household debt, in examining the effects of indebtedness.

[7]Housing tenure is a factor that has been found to be correlated with consumption decisions. See, for example Aladangady (2014) and Cloyne et al. (2018).

	Percentiles					
	25th	50th	75th	Mean	Std. Dev.	Obs.
Quarterly consumption, in 2000 Q1 (In U.S. dollars)						
Non-durable	2,358	3,598	5,404	4,320	2,894	354,685
Durable	0	36	306	1,048	3,805	354,685
Growth rate (Q-on-Q) (In percent)						
Non-durable	-18.2	0.0	18.3	0.0	33.1	265,712
Durable	-147.7	-3.6	140.7	-3.6	235.8	114,563

Sources: CEX and IMF staff calculations.
Note: Consumption variables are in constant dollars (2000Q1 = 100) and winsorized at 1 percent of each tail.

Table 2.1: Real Non-Durable and Durable Consumption

		Leverage by Percentile					
	Share of Underwater (In percent)	25th	50th	75th	Mean	Std. Dev.	Obs.
Homeowners w/ mortgage	5.6	29.3	55.0	80.5	59.3	42.3	155,661

Sources: CEX and IMF staff calculations.
Note: Indebtedness is defined as the ratio of mortgage debt to house values and it is winsorized at 1 percent of each tail. "Underwater" households are defined as those with a leverage ratio greater than one (i.e., a negative home equity).

Table 2.2: Household Indebtedness

Homeowners take on varying, though generally high, levels of debt (Table 2.2).[8] On average, 74 percent of households own a house, of which almost two-thirds have mortgage debt (see Appendix 2.7.1).[9] Average indebtedness among households over the entire sample is high, at nearly 60 percent, as is the standard deviation of consumption, at 42 percent. The distribution is skewed to the right, however, and does not change particularly from pre- to post-crisis, as discussed further in Section 4.

[8] As also found in **?** and Hedlund et al. (2016).

[9] More than 80 percent of mortgage contracts in our sample are fixed rate mortgages.

	Share of Hand-to-Mouth (In percent)	Liquidity by Percentile 25th	50th	75th	Mean	Std. Dev.	Obs.
Homeowners	**41.2**	**1.0**	**6.2**	**26.0**	**51.1**	**148.3**	**175,391**
w/ mortgage	28.0	1.1	5.2	17.6	30.1	99.1	111,241
w/o mortgage	13.2	0.8	10.0	57.1	87.6	202.6	64,150
Renters	**18.7**	**0.0**	**0.6**	**5.9**	**19.4**	**92.2**	**70,856**
All	**59.9**	**0.2**	**4.0**	**18.8**	**42.0**	**135.4**	**246,247**

Sources: CEX and IMF staff calculations.
Note: Liquidity is defined as the ratio of liquid assets to monthly income, following Kaplan and others (2014) and it is winsorized at 1 percent of each tail. The "hand-to-mouth" households are defined as those whose liquid assets are less than a half of their monthly income, following Kaplan and others (2014).

Table 2.3: Household Liquidity

Liquidity levels also vary significantly across households (Table 2.3). Median liquidity is lowest for renters, and highest for homeowners without mortgages. The same is true of standard deviations. The distribution is especially skewed towards lower liquidity levels due to "hand-to-mouth" households whose liquid assets are less than a half of their monthly income (Kaplan et al. (2014)). The share of such households is nearly 60 percent, of which about two thirds are homeowners which can thus be considered as "wealthy hand-to-mouth" households.[10] The distribution of liquidity does not change noticeably from pre- to post-crisis, either, as reviewed in Section 4.

[10]The share of hand-to-mouth households is likely overstated due to our narrow definition of liquid assets. Kaplan et al. (2014) find the share to be 31 percent based on a broader definition of liquid assets allowed by granular balance-sheet data from the Survey of Consumer Finances for 1989-2010. However, the paper also finds the share of "wealthy hand-to-mouth" households to be around two-thirds, as in our sample.

2.2.2 Identifying Monetary Policy Shocks

As typical in this literature, we face a tradeoff between overcoming endogeneity and measuring a meaningful relationship between monetary policy and consumption. The former pushes us to seek exogenous monetary policy shocks. However, as these tend to be small, finding a stable and substantial effect on consumption can be difficult.

We identify monetary policy shocks using high frequency data at the time of monetary policy announcements. We do so in the tradition of Bernanke and Kuttner (2005), by capturing changes in asset prices closely correlated with monetary policy expectations. However, unlike Bernanke and Kuttner (2005), we do not use futures on Federal Fund Rates, since these remained little changed (and close to zero) during the post-crisis period, despite repeated steps taken to loosen monetary policy, such as through quantitative easing (QE) programs.

To find a measure that is equally suitable for pre- and post-crisis periods, we resort to changes in 2-year bond yields, taking the cue from Gürkaynak et al. (2005), Gürkaynak et al. (2007), and subsequently, Gilchrist et al. (2015), Ferrari et al. (2017), and Hanson and Stein (2015), among others. The identifying assumption is that 2-year bond yields on the day prior to a scheduled monetary policy announcement capture market expectations of future policy interest rates, as well as perceptions of policy uncertainty as reflected in term premia. Thus, changes in 2-year yields on announcement days reflect the surprise component of monetary policy along both dimensions. We sum monetary policy surprises from all announcements in a given quarter, as in Romer and Romer (2004), to construct measures consistent with our quarterly data on consumption.

2.3 Has the Response of Household Consumption Changed Post-Crisis?

Households are only interviewed by the CEX survey for four consecutive quarters, and subsequently drop out of the dataset. This limits the assessment of consumption reaction to monetary shocks to a time horizon of three quarters. Therefore, for a first analysis of impulse responses to monetary shocks, we construct synthetic cohorts to obtain longer time series.

Constructing synthetic cohorts amounts to categorizing households at any given quarter according to pre-defined buckets, then linking the data between buckets to create longer time-series. The underlying assumption is that households with similar characteristics–belonging to the same bucket–respond similarly to monetary policy shocks. Obviously, such an approach has its limitations, since households can differ along many characteristics which are not controlled for.

We build cohorts using the head of household's birth year and housing tenure. For the grouping by birth year we define 14 groups using 5-birth year intervals, while for the grouping by housing tenure we retain 3 groups: owners with mortgage, owners without mortgage, and renters. As a result, we build 42 representative consumer units with data for the whole sample period. More details on the construction of synthetic cohort panel data are provided in Appendix 2.7.2.

We then use the panel of synthetic cohorts to estimate the response of durable- and non-durable consumption to monetary policy, estimating the impulse response function using Jordà (2005) local projection method:

$$\begin{aligned}\ln\left(\frac{C_{j,t+h}}{C_{j,t-1}}\right) =& \beta_0^{(h)} + \beta_1^{(h)} 2yr_t + \beta_2^{(h)} postGFC + \beta_3^{(h)} postGFC * 2yr_t \\ &+ \beta_4^{(h)} X_{j,t} + \beta_5^{(h)} S_t + \varepsilon_{j,t+h}\end{aligned} \tag{2.1}$$

where $\ln\left(\frac{C_{j,t+h}}{C_{j,t-1}}\right)$ is the cumulative log change in real consumption by the synthetic cohort j between periods t and $t+h$, $2yr_t$ is the 2-year yield, $X_{j,t}$ is a cohort-specific vector of controls that includes age and age squared, S_t is a set of macro controls that includes inflation, GDP growth rate, and quarterly dummies, and $h = 1, ..., 12$.

To test the hypothesis that the effect of monetary policy has changed after the GFC, we include a dummy variable, labeled *postGFC* in the above equation, for the post-crisis period (2009Q1 and onwards) and interact it with the policy rate. The coefficients $\beta_1^{(h)}$ captures the pre-GFC effect of monetary policy and $\beta_3^{(h)}$ captures the additional effect of monetary policy added in the post-GFC. These consumption responses to a contractionary monetary policy are expected to be persistently negative.[11]

[11]Previous studies show that a contractionary monetary policy would generate a hump-shaped

We instrument the 2-year yield ($2yr_t$) to address endogeneity–the possibility that bond yields reflect monetary policy responses to changes in consumption. As instruments, we adopt exogenous monetary policy shocks from high-frequency data, as discussed earlier. We exploit overidentification to overcome weak instrument bias by using the contemporaneous monetary policy shock and its lags as the instruments. We use the Generalized Method of Moments (GMM) to obtain more precise estimates.[12][13]

Turning to the results, Figures 2.1 and 2.2 and plot the GMM estimates for 1996Q1-2014Q4. The dependent variable is the accumulated quarterly growth rate in real consumption. Individual data from CEX are aggregated in 42 synthetic cohorts according to housing status and 5-year birth year intervals. In the first stage regression, the 2-year yield is instrumented by monetary policy shocks. All regressions include a constant, aggregate macroeconomic controls (inflation and real GDP growth), and quarterly seasonal effects. Standard errors are robust to heteroskedasticity and autocorrelation. The full line shows the estimated effect, while the dotted lines show the 90 percent confidence interval. The results show that the pre-GFC effect of monetary policy measured by $\beta_1^{(h)}$ is negative on both durable and non-durable consumption growth, while the additional effect due to the post GFC $\beta_3^{(h)}$ is positive

drop in consumption and investment in the data (e.g., Christiano and Eichenbaum (2005); Cloyne et al. (2018); and Wong (2015)), which could be explained by various frictions (e.g., see Christiano et al. (2010), Iacoviello and Neri (2010), and Alpanda and Zubairy (forthcoming)). These consumption responses to monetary policy are related but different from the intertemporal elasticity of substitution (e.g., see Kaplan et al. (2018)). For a survey of the estimation of the intertemporal elasticity of substitution, see Thimme (2016).

[12]See Ramey (2016) and Stock and Watson (2018).

[13]We also experimented using more than one type of monetary policy shock. Results are robust to instrumenting the policy rate with the signal shock and the risk shock described above.

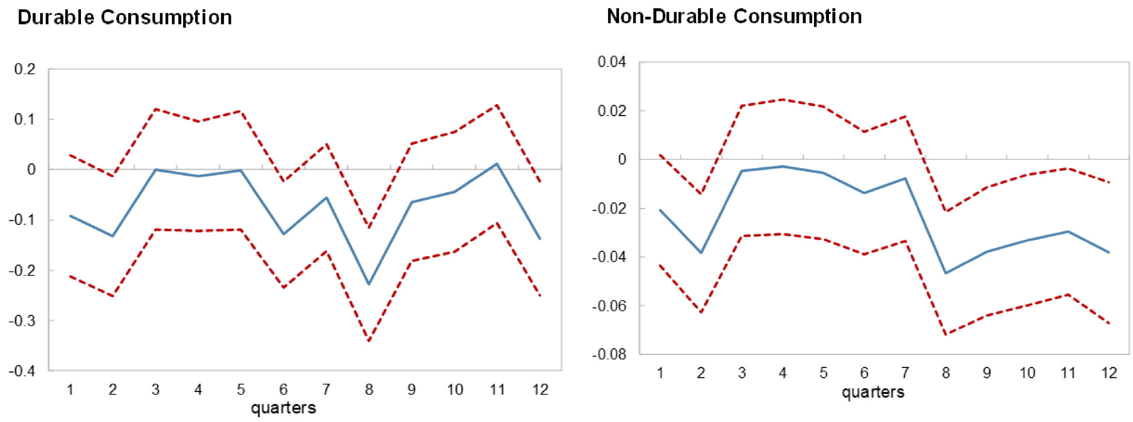


Figure 2.1: Response of Durable and Non-Durable Consumption to Monetary Policy – Direct Effect of Monetary Policy ($\beta_1^{(h)}$)

at most projection horizons.

These results suggest that the responsiveness of consumption to monetary policy has changed and has likely weakened since the crisis. However, the change seems difficult to measure in a precise and robust manner. The size and significance of $\beta_3^{(h)}$ varies as the specification of equation (2.1) is modified by, for example, changing the set of controls to include more lags. The message we therefore take from this exercise is that it offers suggestive, but not conclusive and precise evidence for a weakening of monetary policy effects on household consumption in the post-crisis period.

We thus tackle the same question using the full micro-data set, without aggregating households in cohorts–at the expense of only observing consumption growth for three consecutive quarters for any single household. We run the following regression:

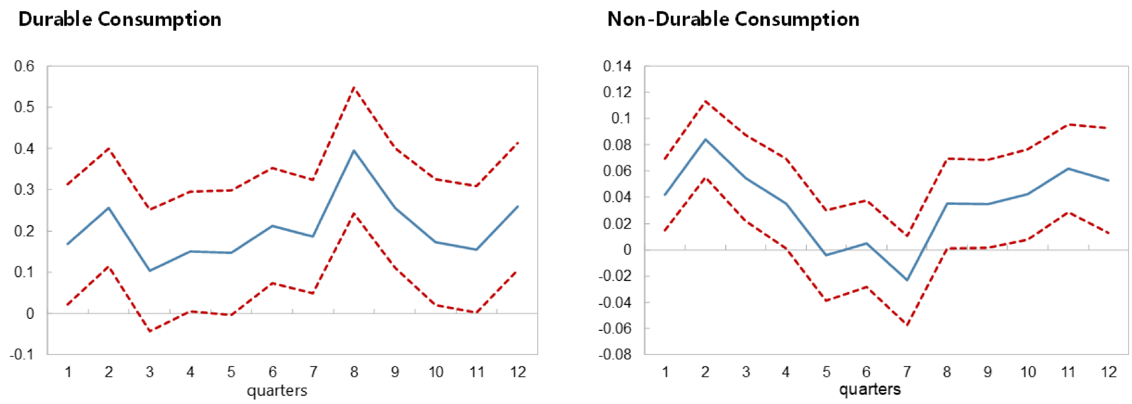


Figure 2.2: Response of Durable and Non-Durable Consumption to Monetary Policy – Marginal Effect of Monetary Policy Interacted with Post-GFC Dummy ($\beta_3^{(h)}$)

$$\begin{aligned} \ln\left(\frac{C_{i,t+1}}{C_{i,t-1}}\right) =& \beta_0 + \beta_1 2yr_t + \beta_2(postGFC.2yr_t) + \beta_3 postGFC \\ & + \beta_4 Z_{i,t} + \lambda_{s(t)} + u_{i,t} \end{aligned} \tag{2.2}$$

where $\ln\left(\frac{C_{j,t+1}}{C_{j,t-1}}\right)$ is the cumulative log change in real consumption for individual household i (instead of a synthetic cohort as above). We focus on two-quarter growth rates in consumption to allow for lags in monetary policy transmission.[14] $postGFC$ again denotes a dummy variable that takes the value of one since 2009Q1, $Z_{i,t}$ denotes household level controls, which include race, education level, age, family size, and marital status. $\lambda_{s(t)}$ stands for seasonal fixed effects.[15]

[14]The rotating nature of data does not allow for a more dynamic analysis of consumption response to monetary policy, an issue we explore using synthetic cohorts. We use 2-quarter ahead consumption growth in the panel analysis to strike a balance between allowing for a transmission lag and not losing too many observations. The results are broadly robust to the choice of 1,2, or 3 quarters.

[15]See Table A.1 for correlations among consumption growth, household characteristics. and balance sheet variables. Households' balance sheet variables (liquidity and leverage) are not found to be highly correlated with household level characteristics (family size, education, ethnicity, marital status, etc.).

We follow the same procedure as before in instrumenting 2-year yields ($2yr_t$) using high-frequency monetary policy shocks and their lags as instruments to overcome weak instrument bias, and using GMM estimation.

The results confirm the earlier findings of a weaker impact of monetary policy after the crisis. Overall, we find the expected response of both durable and non-durable consumption to monetary policy shocks. In the pre-crisis period, an expansionary monetary policy shock (a 10-basis point reduction in the 2-year yield) increases non-durable and durable consumption by about 3 percent and 2 percent, respectively (Table 2.4, columns 3 and 4). In the post-crisis period, the response of durable- and non-durable consumption to monetary policy is clearly weaker (as seen by positive and significant values of β_2). For durable consumption, the effect is only marginally statistically significant (Table 2.4, columns 4 and 6).[16]

Household-level controls have a significant and expected impact on households' non-durable consumption. College-educated, white, married, and older households display higher consumption growth following looser monetary policy. However, these characteristics are not found to be important determinants of durable consumption.

When we estimate equation (2.2) over the full sample, by removing the post-GFC dummy and its interaction with monetary policy, results show that expansionary monetary policy boosts both durable and non-durable consumption, as expected (Figure 2.3 and Table 2.4, column 1 and 2). The estimated effect is stronger for nondurables;

[16]Our main results are robust to adding income change as an additional control variable (Table 2.4 (columns 5 and 6)).

Variables	(1) Non-durables	(2) Durables	(3) Non-Durables	(4) Durables	(5) Non-Durables	(6) Durables
2-yr yield	-24.24***	-4.37*	-28.60***	-17.81**	-31.00***	-16.55*
	(4.46)	(2.27)	(5.20)	(8.57)	(5.92)	(8.76)
Family size	-0.13	0.06	-0.13	0.07	-0.17**	0.04
	(0.08)	(0.74)	(0.09)	(0.74)	(0.09)	(0.75)
College education	0.85***	0.41	0.88***	1.33	0.95***	1.25
	(0.24)	(1.97)	(0.24)	(1.88)	(0.24)	(1.91)
White	0.74**	2.49	0.72**	2.15	0.71**	1.86
	(0.29)	(2.87)	(0.29)	(2.84)	(0.30)	(2.87)
Married	0.63***	0.90	0.59**	0.55	0.54**	0.53
	(0.25)	(2.20)	(0.25)	(2.17)	(0.25)	(2.20)
Reference age	0.02***	0.05	0.02***	0.08	0.02***	0.07
	(0.01)	(0.07)	(0.01)	(0.07)	(0.01)	(0.07)
postGFC			-7.69***	-0.22	-8.44***	0.33
			(1.47)	(5.56)	(1.65)	(5.74)
postGFC*2-yr yield			3.31***	11.60*	3.64***	11.46*
			(0.92)	(6.50)	(0.99)	(6.79)
Income growth					0.04***	0.08**
					(0.00)	(0.03)
Observations	166,921	69,781	166,921	69,781	161,449	68,008

Table 2.4: Household Liquidity

a 10 basis-point increase in the 2-year yield reduces non-durable consumption by 2.5 percent and durable consumption by 0.5 percent.[17] Results for durable consumption are in general less robust, partly reflecting the diminished response of durables consumption to monetary policy shocks post-crisis, as shown above.[18]

[17] These results are not directly comparable to those in the literature because this full sample includes the post-GFC period. Moreover, to our knowledge, our novel approach, which uses monetary policy shocks as instruments, has not been used in other studies with micro-level data; the literature tends to use monetary policy shocks as regressors (e.g., Wong (2015)) or use other variables to instrument a change in a relevant interest rate (e.g., Aladangady (2017)). Further, while Wong (2015) estimates consumption elasticity to monetary policy shocks, we focus on the consumption response to exogenous changes in policy-relevant interest rates, allowing for making more meaningful and policy relevant conclusions.

[18] Another reason may be the lumpy nature of durables consumption, which implies lags in responses to monetary policy shocks. To partly account for potential lags, we use current and lagged monetary policy shocks as instruments for the 2-year yield when estimating equation 2 for durable consumption growth.

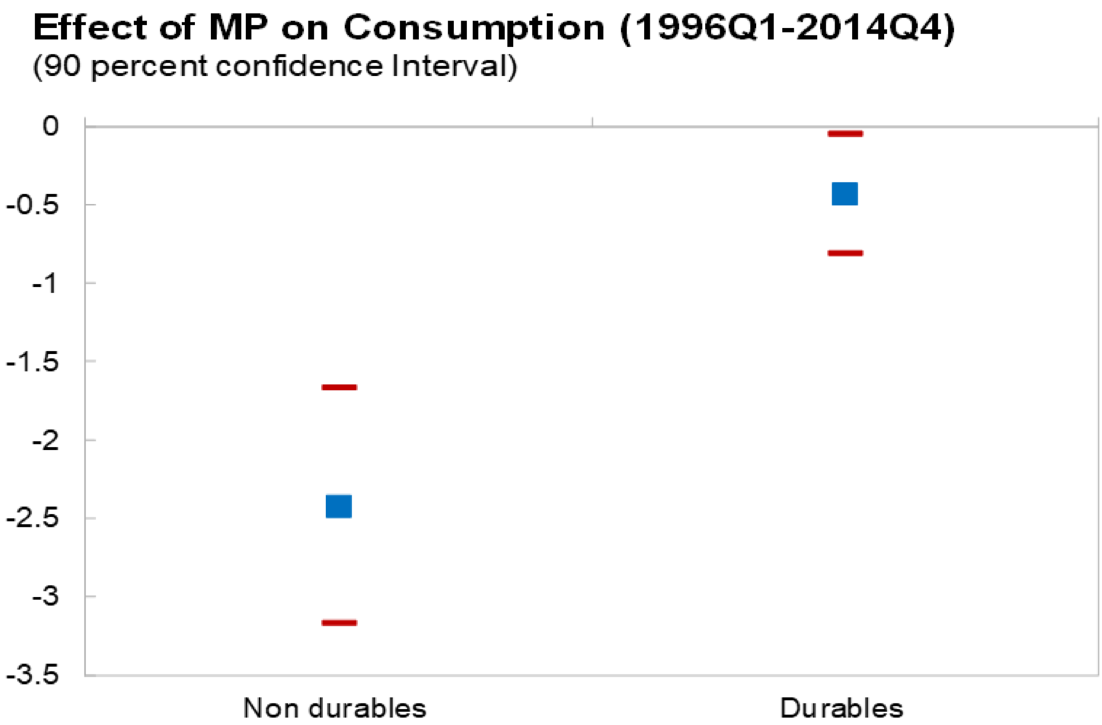


Figure 2.3: Consumption Response to 10-basis point Increase in 2-year Yield

2.4 Does Household Indebtedness Matter?

In this section we ask whether household indebtedness affects the response of consumption to monetary policy impulses. Next, we explore the role of non-linearities, and ask whether the change in the distribution of household indebtedness post-crisis might help explain the lower monetary policy impact on consumption detected earlier.

To tackle the first question, we estimate an equation of the form:

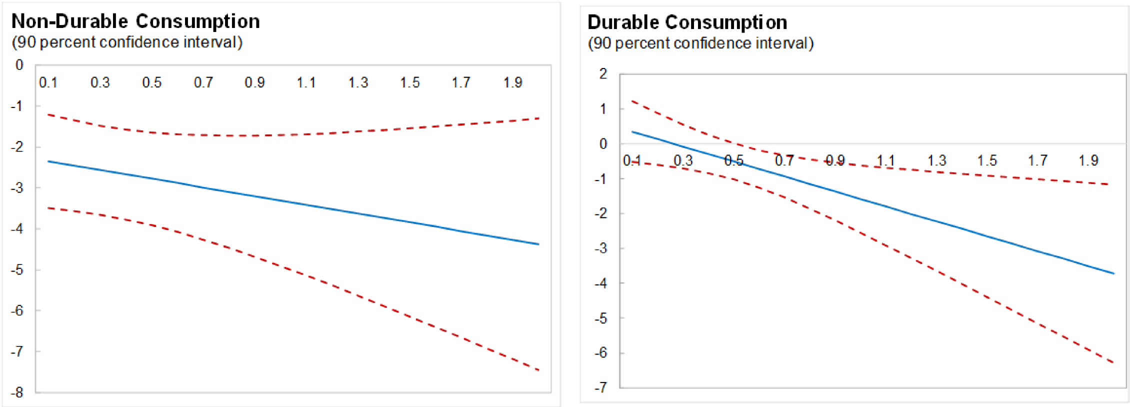
$$\begin{aligned}\ln\left(\frac{C_{i,t+1}}{C_{i,t-1}}\right) =&\beta_0 + \beta_1 2yr_t + \beta_2(LTV_{i,t-1}.2yr_t) + \beta_3 LTV_{i,t-1} \\ &+ \beta_4 Z_{i,t} + \lambda_{s(t)} + u_{i,t}\end{aligned} \tag{2.3}$$

As earlier, the model is estimated using GMM, where the 2-year yield is instrumented by monetary policy shocks. A negative value of β_2 supports the hypothesis that households with higher indebtedness respond more to monetary policy shocks. However, the total effect of monetary policy loosening on consumption growth must be read from $\beta_1 + \beta_2 * LTV$.

The results show that β_2 has a negative sign, in line with the notion of a higher responsiveness of more indebted households. The estimated coefficient is, however, only significant for durable consumption. To understand further whether and how the responsiveness of consumption to monetary policy shocks varies with household indebtedness, we check for the joint significance of β_1 and β_2 along the spectrum of possible values for indebtedness (Figure 2.4).

Variables	(1) Non-Durables	(2) Durables	(3) Non-Durables	(4) Durables	(5) Non-Durables	(6) Durables
2-yr yield	-22.41***	5.66	-8.73***	-27.33	-18.93***	-41.46*
	(7.19)	(6.02)	(2.81)	(22.05)	(4.30)	(23.41)
Family size	-0.15	-0.16	-0.07	-0.23	-0.10	-1.36
	(0.14)	(1.02)	(0.12)	(1.01)	(0.14)	(1.21)
College education	0.77**	0.81	0.97***	1.87	0.90**	2.45
	(0.35)	(2.71)	(0.32)	(2.55)	(0.39)	(3.06)
White	0.89*	0.31	1.19***	-0.63	0.72	-3.49
	(0.53)	(4.23)	(0.45)	(4.17)	(0.56)	(5.23)
Married	0.57	0.16	0.41	-1.26	0.39	-0.93
	(0.42)	(3.35)	(0.38)	(3.28)	(0.48)	(4.01)
Reference age	0.01	-0.15	0.03**	-0.10	0.03	-0.13
	(0.02)	(0.14)	(0.01)	(0.11)	(0.02)	(0.14)
LTV	29.48	69.79**				
	(28.66)	(29.68)				
LTV*2-yr yield	-10.67	-21.43**				
	(10.11)	(10.04)				
I.(LTV < 0.9)			-6.69***	-6.43	-40.22***	-78.48
			(1.78)	(14.65)	(8.66)	(48.50)
I.(LTV < 0.9)*2-yr yield			2.44***	5.02	9.74***	19.79*
			(0.67)	(5.15)	(2.10)	(11.31)
Observations	73,184	36,866	73,307	37,225	51,272	26,713
No. of Households	37,576	24,351	37,644	24,517	26,075	17,346
Sample	Full	Full	Full	Full	Pre-crisis	Pre-crisis

Table 2.5: Impact of Monetary Policy on Consumption: The Role of Indebtedness



Source: IMF staff estimates.
Note: This graph shows the response of consumption to a 10-basis point increase in 2-year yield at different levels of household indebtedness. X-axis denotes LTV ratio and Y-axis is the consumption response measured by $\widehat{\beta_1} + \widehat{\beta_2} * LTV$ (based on equation 3), estimated at different LTV levels and corresponding 90 percent confidence intervals.

Figure 2.4: Effect of Monetary Policy on Consumption Growth at Different LTV Levels

The confidence intervals widen at higher LTV levels. Furthermore, in the case of durable consumption, the overall impact of monetary policy is found to be significant only at LTV levels higher than 0.5, suggesting that monetary policy may be effective only beyond a certain threshold. These results indicate that the response of consumption growth to monetary policy may potentially be non-linear.

As discussed earlier, looser monetary policy would be expected to strengthen balance sheets and reduce indebtedness by boosting house prices and reducing the net present value of mortgage payments. In turn, these effects should relax credit constraints and favor higher consumption. However, when indebtedness is especially high, the marginal improvement in balance sheets may not be sufficient to restore access to credit or allow for debt refinancing. We refer to this as the debt-overhang hypothesis.[19]

To study whether consumption growth responds non-linearly to household indebtedness, we estimate a threshold regression of the form:

$$\begin{aligned}\ln\left(\frac{C_{i,t+1}}{C_{i,t-1}}\right) =& \beta_0 + \beta_1 2yr_t + \beta_2(I_{LTV<0.9}.2yr_t) + \beta_3 I_{LTV<0.9} \\ &+ \beta_4 Z_{i,t} + \lambda_{s(t)} + u_{i,t}\end{aligned} \quad (2.4)$$

where $I_{LTV<0.9}$ is an indicator function that takes a value of 1 when indebtedness is less than the 90th percentile over the sample period 1996Q1–2014Q4. Therefore, a significant value of β_2 implies that transmission is different across households with high and low indebtedness. We find that higher indebtedness increases responsiveness to monetary policy shocks for non-durable consumption over the full and pre-crisis samples (Table 2.5, columns 3 and 5, respectively). Thus, effects of indebtedness appear to be non-linear. The results for durable consumption are comparable over the pre-crisis sample (Table 2.5, column 6), and have the expected sign though are not significant for the full sample (Table 2.5, column 4).

We further explore the responsiveness of consumption at other thresholds, namely at 70, 80, 95, and 99th percentiles. Results corroborate the above findings: the response to monetary policy shocks increases with indebtedness, but there is no evidence

[19]Moreover, as discussed in Alpanda and Zubairy (forthcoming) high levels of debt may dampen the eectiveness of monetary policy because highly indebted households may be less willing, or less able, to borrow further in response to a rate cut, especially during recessionary periods when agents are facing higher job insecurity and income uncertainty. After a shock, households may need to rebuild wealth and increase precautionary savings (Mian and Sufi (2014), Carroll and Kimball (1996)). A more specific channel refers to the mechanism by which under-the-water-households may not invest in their homes in response to a monetary easing (Melzer (2017)).

	LTV < 70	LTV > 70	LTV < 80	LTV > 80	LTV < 95	LTV > 95	LTV < 99	LTV > 99
				Full - sample				
Non-durables	-0.63***	-0.72***	-0.63***	-0.76***	-0.63***	-1.09***	-0.64***	-2.03***
	(0.004)	*(0.002)*	*(0.004)*	*(0.002)*	*(0.004)*	*(0.002)*	*(0.004)*	*(0.001)*
Durables	-2.19	-2.35	-2.22	-2.55	-2.24	-3.49	-2.25	-6.55
	(0.201)	*(0.203)*	*(0.198)*	*(0.185)*	*(0.193)*	*(0.204)*	*(0.202)*	*(0.21)*
				Pre - crisis				
Non-durables	-0.91***	-1.19***	-0.92***	-1.30***	-0.94***	-2.99***	-0.93***	-6.64***
	(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*
Durables	-2.21*	-2.50	-2.19*	-2.93*	-2.14*	-6.77*	-2.14	-15.67*
	(0.085)	*(0.102)*	*(0.087)*	*(0.083)*	*(0.085)*	*(0.052)*	*(0.1)*	*(0.05)*

Source: IMF staff estimates

[1] Figures in parenthesis are P-values.

Table 2.6: Response of Consumption Growth to a 10-basis-point Increase in the 2-year Yield at Different LTV Thresholds

that it diminishes above a very high threshold (Table 2.6).[20] This is particularly evident for durable consumption which shows a monotonically increasing coefficient on indebtedness as the threshold is raised.

In summary, our results suggest that more indebted households respond more to monetary policy impulses. However, we do not find evidence of a debt overhang effect–that is, of a weakened response at very high levels of indebtedness. Thus, for indebtedness to explain the decrease in the average response of household consumption to monetary policy shocks, the overall distribution of indebtedness must have shifted leftward post-crisis, toward less indebted households. However, if anything, the distribution of indebtedness shifted to the right, though its mean declined somewhat, as shown in Figure 2.5. Using estimated coefficients from equation 2.3 (Figure 2.4), the

[20]The results are robust to alternative specifications. For example, in a quadratic specification responsiveness of consumption to monetary policy shocks is found to increase non-linearly with leverage.

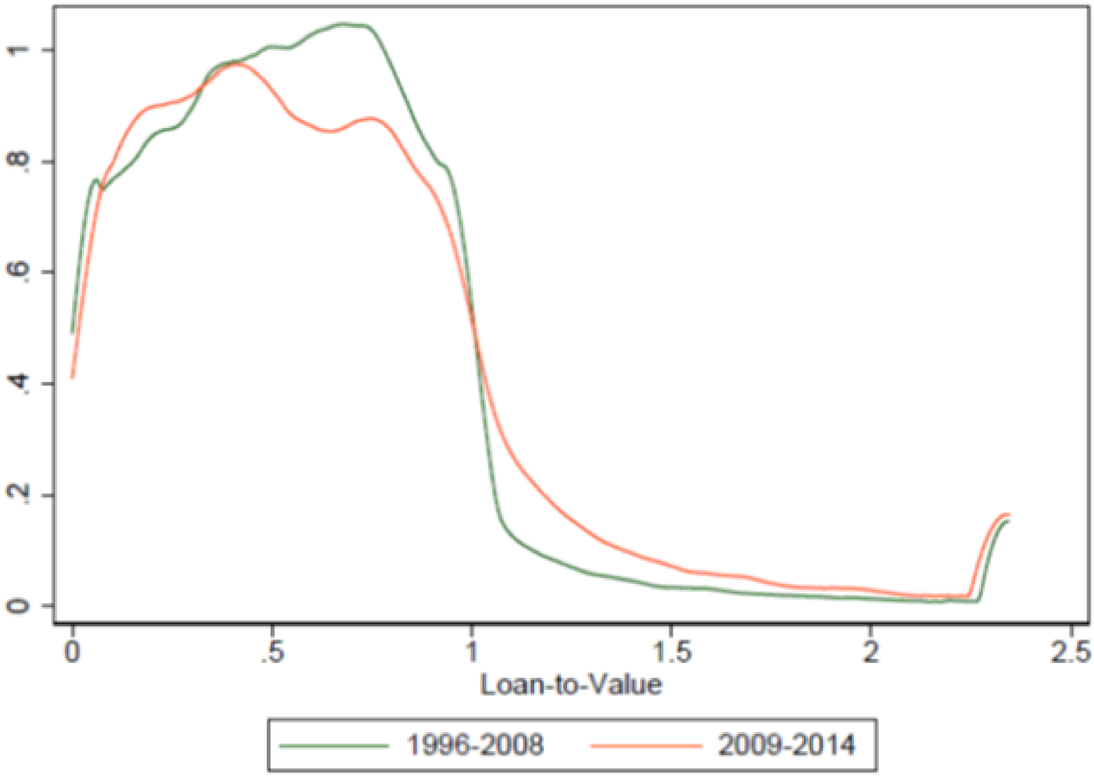


Figure 2.5: Density of Loan-to-Value

responsiveness of non-durable and durable consumption to a 10-basis-point rise in the 2-year yield is found to increase by 2 and 4 basis points, respectively, due to shift in the distribution of household indebtedness post-crisis. The proportion of households in the top 10 percentile of LTV distribution grew from 5 percent before crisis to 8 percent in the post-crisis period. According to equation 2.4, this implies a 3- and 6-basis-point increase in the responsiveness of non-durable and durable consumption, respectively, to a 10-basis-point hike in the 2-year yield. Thus, both specifications indicate that changes in the LTV distribution have *per se* contributed to a higher responsiveness of consumption to monetary policy in the post-crisis period. We must therefore look elsewhere to seek a plausible explanation for the drop in monetary policy effectiveness relative to consumption.

2.5 Does Household Liquidity Matter?

We proceed in much the same way as in the earlier section. We ask whether the liquidity position of households affects their consumption response to monetary policy impulses. We further ask whether there are non-linearities, and whether the change in the distribution of household liquidity post-crisis might help explain the lower monetary policy impact on consumption detected earlier in this paper.

We begin by estimating the following equation:

$$\ln\left(\frac{C_{i,t+1}}{C_{i,t-1}}\right) = \beta_0 + \beta_1 2yr_t + \beta_2(LIQ_{i,t-1}.2yr_t) + \beta_3 LIQ_{i,t-1} + \beta_4 Z_{i,t} + \lambda_{s(t)} + u_{i,t} \tag{2.5}$$

In this specification, a positive value of β_2 supports the hypothesis that households with low liquidity respond more to monetary policy shocks.

Estimates of β_2 are however found to be insignificant (Table 2.7, columns 1 and 2). To investigate the issue further, we examine whether the responsiveness of consumption to monetary policy shocks varies with liquidity levels. For this purpose, we check for the joint significance of β_1 and β_2 along the spectrum of liquidity values (Figure 2.6). The results show that the responsiveness of non-durable consumption is only significant at relatively low liquidity values (with liquid-assets-to-monthly income ratios of up to around one).

We explore the possibility that only households with liquidity below a certain threshold respond more to interest rate shocks in a nonlinear setting. Specifically, we consider the following threshold regressions:

$$\ln\left(\frac{C_{i,t+1}}{C_{i,t-1}}\right) = \beta_0 + \beta_1 2yr_t + \beta_2(I_{LIQ>.25}.2yr_t) + \beta_3 I_{LIQ>.25} + \beta_4 Z_{i,t} + \lambda_{s(t)} + u_{i,t} \tag{2.6}$$

where $I_{LIQ>.25}$ is an indicator function that takes a value of 1 when a household's liquid-assets-to-income-ratio is greater than the 25^{th} percentile over the sample period 1996Q1–2014Q4.

The results indicate that non-durable consumption responds most strongly when households are liquidity constrained. We find qualitatively similar, but not statistically significant results for durable consumption (Table 2.7, column 3 and 4).[21] Table 2.8 offers an interpretation of results, listing the extent of the consumption response to a 10-basis point surprise hike in the 2-year interest rate. The response of non-durable consumption increases monotonically as liquidity is lowered from the 20^{th} to the 10^{th} and 5^{th} percentiles. In the first case, consumption of non-durables rises by 2.3 percentage points, while in the last it increases by 2.5 percentage points–not an innocuous difference.

Overall, our results provide some support for the findings of Kaplan and Violante (2014) that non-durable consumption of wealthy hand-to-mouth households (namely those with limited liquid assets) responds more strongly to interest rate changes.

Lastly, we ask whether the change in the distribution of liquidity from pre- to post-crisis times might help explain the decline in monetary policy effects on consumption. For liquidity to be relevant, the distribution should have moved rightward, toward a lower share of liquidity constrained and highly responsive households.

However, the distribution of liquidity has hardly changed over time, or, if anything, has shifted to the left (Figure 2.7). Based on coefficient estimates from equation 5, the responsiveness of non-durable consumption is found to marginally strengthen after the crisis due to the observed shift in the liquidity distribution (a 10-basis-point

[21] As in the case of leverage, we also estimated equation 6 for durable consumption for the pre-crisis period. Liquidity continues to not matter for transmission of monetary policy to durable consumption even in the pre-crisis period.

Variables	(1) Non-durables	(2) Durables	(3) Non-Durables	(4) Durables	(5) Non-Durables	(6) Durables
2-yr yield	-25.58**	-1.65	-23.41***	-14.98	-24.92***	-18.62
	(10.39)	(6.12)	(8.58)	(18.26)	(9.27)	(21.55)
Family size	-0.12	-0.84	0.01	0.16	-0.06	-0.18
	(0.16)	(1.36)	(0.14)	(1.15)	(0.14)	(1.15)
College education	1.56**	4.16	0.88**	3.21	1.08***	4.17
	(0.73)	(3.48)	(0.38)	(2.92)	(0.38)	(2.90)
White	0.80	-1.99	0.90	1.11	0.91	1.61
	(0.89)	(5.98)	(0.55)	(4.84)	(0.56)	(4.84)
Married	0.56	-1.32	0.46	-1.19	0.54	-0.45
	(0.55)	(4.45)	(0.45)	(3.69)	(0.46)	(3.70)
Reference age	0.03	-0.17	0.02*	-0.12	0.03*	-0.12
	(0.02)	(0.16)	(0.01)	(0.13)	(0.02)	(0.13)
Liquid assets/Income	-23.82	-1.85				
	(35.52)	(14.13)				
Liq*2-yr yield	7.39	0.99				
	(10.56)	(3.15)				
I.(liq > 0.25)			-4.00*	4.28		
			(2.25)	(9.04)		
I.(liq > 0.25)*2-yr yield			2.27***	1.21		
			(0.75)	(2.61)		
I.(liq > 0.10)					-9.16**	-18.45
					(4.55)	(19.10)
I.(liq > 0.10)*2-yr yield					3.93***	5.36
					(1.51)	(5.81)
Observations	52,344	20,874	52,345	28,892	52,345	28,892
No. of Households	26,885	18,820	26,885	18,821	26,885	18,821

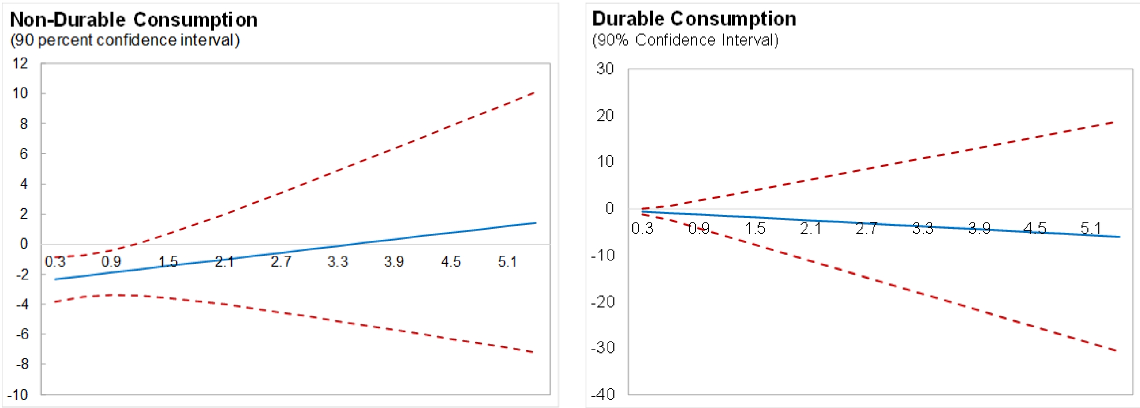
Table 2.7: Impact of Monetary Policy on Consumption: The Role of Liquidity

	LIQ < 25	LIQ > 25	LIQ < 20	LIQ > 20	LIQ < 10	LIQ > 10	LIQ < 5	LIQ > 5
Non-durables	-2.34***	-2.11***	-2.39***	-2.11***	-2.49***	-2.10***	-2.49***	-2.10***
	(0.006)	*(0.007)*	*(0.006)*	*(0.007)*	*(0.007)*	*(0.007)*	*(0.007)*	*(0.007)*
Durables	-1.50	-1.38	-1.72	-1.36	-1.86	-1.33	-1.86	-1.33
	(0.412)	*(0.399)*	*(0.364)*	*(0.408)*	*(0.387)*	*(0.419)*	*(0.387)*	*(0.419)*

Source: IMF staff estimates

[1] Figures in parenthesis are P-values.

Table 2.8: Response of Consumption Growth to a 10 Basis Point Increase in the 2-year Yield at Different Liquid-Asset-to-Income Thresholds



Source: IMF staff estimates.
Note: This graph shows the response of consumption to a 10-basis point increase in 2-year yield at different levels of household liquidity. X-axis denotes liquid-assets-to-income ratio and Y-axis is the consumption response, measured by $(\widehat{\beta_1} + \widehat{\beta_2} * LIQ$ (based on equation 5), estimated at different liquidity levels and corresponding 90 percent confidence intervals.

Figure 2.6: Effect of Monetary Policy on Consumption Growth at Different Liquidity Levels

increase in 2-year yields leads to an additional 0.1-basis-point decline in non-durable consumption in the post-crisis period).

The share of households in the lower 25^{th} percentile of liquidity distribution rose from 24 percent pre-crisis to 28 percent after the crisis, which according to the estimates from equation 2.6 should also strengthen the responsiveness of non-durable consumption by 0.1 basis point (to a 10-basis-point increase in 2-year yield).

Hence, liquidity cannot explain the weakened response of consumption observed after the crisis either. The widespread concern that a deterioration of household balance sheets after the crisis hampered monetary policy effectiveness thus does not seem to hold. The explanation for the lower effectiveness of monetary policy must

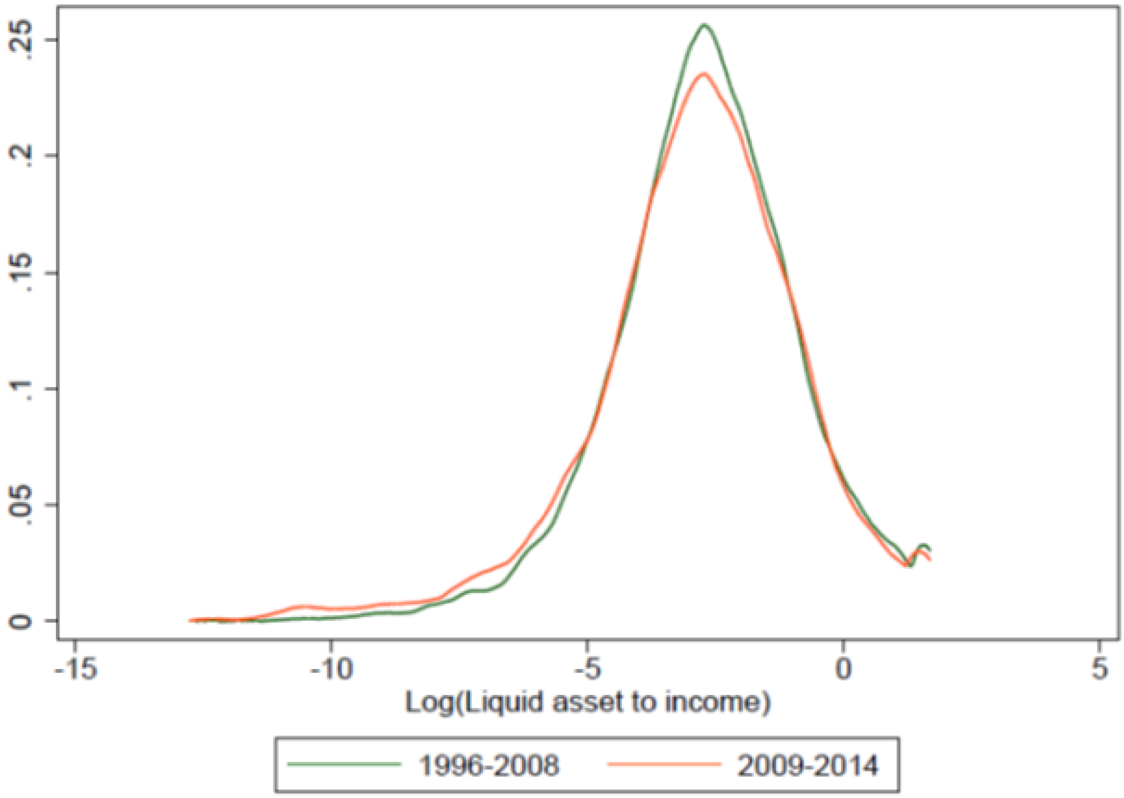


Figure 2.7: Density of Liquid-Asset-to-Income

therefore lie elsewhere, such as in the higher degree of economic uncertainty brought about by the crisis.[22]

2.6 Conclusion

We find that the average responsiveness of U.S. household consumption to well-identified monetary policy shocks has declined since the global financial crisis. However, this result cannot be explained by higher indebtedness or lower liquidity levels. Households with higher debt levels and lower shares of liquid assets are the most responsive to monetary policy, and the share of these households in the population grew. The common notion that a deterioration of household balance sheets after the crisis hampered monetary policy effectiveness is not validated in the data.

Nevertheless, household balance sheets do matter for the strength of monetary policy transmission, and our results underscore the notion that monetary policy makers need to pay close attention to them. Moreover, given the presence of nonlinearities (the responsiveness of more indebted households rises non-linearly with indebtedness), monitoring the distribution of household balance sheet characteristics is important.

[22] Aastveit et al. (2017) find that U.S. monetary policy shocks affect economic activity less when uncertainty is high, in line with "real-option" effects from theory (e.g., Bloom (2009)). While not reported here, we explored whether the higher uncertainty in the post-crisis period accounts for the lower effectiveness of monetary policy, by interacting monetary policy shocks in our estimation with the index of economic policy uncertainty developed by Baker et al. (2016). While this preliminary investigation provides only suggestive evidence for such effects, further research on this issue seems worthwhile.

2.7 Appendix

2.7.1 Data

The Consumer Expenditure Survey (CEX) is a survey conducted by the Census Bureau and is primarily used by the Bureau of Labor Statistics to determine the weights assigned to different goods and services in calculating the consumer price index (CPI). The CEX is a rotating panel survey and each household is interviewed once per every three months for, at most, 15 consecutive months. In addition, the survey sample is designed to be representative of the U.S. civilian non-institutional population.

2.7.1.1 Data cleanup

We take several steps to clean up the raw CEX data. We drop observations in which the CEX records negative consumption for households. We also drop observations for households with more than one consumption unit and households with less than four interview observations. This cleanup results in roughly 5,000 quarterly household observations, of which 74 percent are homeowners and 45 percent are homeowners with outstanding mortgage balance. On average, households spend \$4,320 on non-durable goods and \$1,048 on durable goods. Some summary statistics for housing tenure and consumption for the data are shown in Appendix I Table 2.9 below.

One peculiar feature of the CEX survey is that the interview quarter and the

		Average Level		Average Growth	
	Share (percent)	Non-Durable	Durable	Non-Durable	Durable
Homeowners	**74**	4,803	1,204	0.14	-3.44
w/ mortgage	45	5,286	1,397	0.02	-4.02
w/o mortgage	29	4,044	900	0.32	-2.25
Renters	**26**	2,935	601	-0.23	-4.26
All	100	4,320	1,048	0.04	-3.60

Sources: CEX and authors' calculations.

Table 2.9: Housing Tenure and Real Consumption

consumption quarters may not align perfectly. Each time a household is interviewed, they are inquired about their consumption expenditures over the three months prior to the month of interview. Since households may be interviewed during any month within a given quarter, the interview quarter does not necessarily correspond with the months for which the consumption data are acquired. We make the appropriate adjustments to the consumption data so that they align with their respective calendar quarters.

2.7.1.2 Definitions of consumption variables

Following Aladangady (2014), non-durable consumption consists of food, alcohol, tobacco, housing operations, utilities, gasoline, public transportation, personal care, reading, entertainment, apparel, healthcare, and education expenses. Durable consumption consists of expenditure on cars (new and used), furniture, and equipment.

Appendix I Table 1 outlines the CEX variables used to construct non-durable and durable consumption variables.

Details of the CEX variables used in constructing non-durable and durable consumption variables are mentioned in Appendix I Table 2.10.

2.7.1.3 Leverage and Liquidity

Most of the household balance sheet data are only available in the 5th interview, while mortgage information is asked in every interview. Leverage is proxied by the ratio of mortgage balance to the reported house value. We aggregate the mortgage balances reported on all the properties owned by the household. The CEX variables used for constructing this are QBLNCM1X or QBLNCM2X, which report the household's mortgage balance at the beginning of the month, three months prior to the interview or two months prior to the interview, respectively. Our choice over which of the two variables to use depends on which month corresponds to the first month in the consumption quarter. If a household refinances its mortgage on a property, we adjust the household's mortgage balance such that the mortgage balances before and after refinancing are not double-counted. For property value we use PROPVALX. We construct a house price index using this variable and it matches well with the Case-Shiller Home Price Index, particularly the boom-bust in the house prices although it is not shown here.

Liquid assets include the total balance a household has in their checking and savings accounts. From 2013 onwards, liquid assets also include money market accounts and certificates of deposits. The CEX variables used in constructing the liquid assets variable are LIQUIDX for the period covering 2013–14 and CKBKACTX + SAVACCTX for 1994–2012. Unlike balance-sheet variables, income is reported in both the second and the fifth interview. We use the imputed after-tax income, FINCATXM from 2004 onwards. For the prior years, we use the reported after-tax income, FINCATAX, and replace invalid missing entries with imputed income data. Appendix Table 2.11 shows the correlation matrix among key variables.

2.7.1.4 Cohorts and control variables

We construct the synthetic cohorts using housing tenure (CUTENURE) and the household head's birth year, which is determined by the interview date and the household head's age (AGE_REF) at the time of the interview. The control variables used in the panel analysis include race (REF_RACE), education (EDUC_REF), age (AGE_REF), family size (FAM_SIZE), and marital status (MARITAL1).

2.7.2 Synthetic Cohort Panel Data

2.7.2.1 Construction of Synthetic Cohort Panel

To measure the responsiveness of households' consumption to monetary policy over time, we need a panel data, although the CEX is designed as repeated cross-section

Variable	Details	CEX Name
C	Total Expenditure	TOTEXP
	Non-durable Expenditure	
	Food	FOOD
	Alcohol	ALCBEV
	Tobacco	TOBACC
	Housing operations	HOUSOP
	Utilities	UTIL
	Gasoline	GASMO
	Public transportation	PUBTRA
	Personal care	PERSCA
	Reading	READ
	Entertainment	ENTERT
	Apparel	APPAR
	Healthcare	HEALTH
	Educational expenses	EDUCA
	Durable Expenditure	
	Cars & trucks, new	CARTKN
	Cars & trucks, used	CARTKU
	Other vehicles	OTHVEH
	Furnishing & equipment	HOUSEQ

Table 2.10: Definitions of Key Consumption Variables

	Non Durable Consumption	Durable Consumption	LTV	Liquidity	Family Size	College Education	Ethnicity (white = 1)	Marital Status	Reference Age
Non durable consumption	1								
Durable consumption	0.045	1							
LTV	-0.0068	-0.0068	1						
Liquidity	0.0098	0.0039	-0.1774	1					
Family size	0.0001	0.0004	0.2209	-0.1502	1				
College education	0.0055	0.004	0.0839	0.0879	-0.0121	1			
Ethnicity (white = 1)	0.0056	0.0028	-0.0481	0.0802	-0.0351	0.0374	1		
Marital status	0.0055	0.0027	0.0821	-0.0056	0.4539	0.1051	0.1163	1	
Reference age	0.0078	0.003	-0.4468	0.2449	-0.3801	-0.088	0.0589	-0.0947	1

Table 2.11: Correlation Matrix

data (Appendix I). Therefore, we construct a synthetic panel, as in Attanasio and Davis (1996), Narita and Narita (2011) and Cloyne et al. (2018). We construct synthetic cohorts based on the arguably time-invariant household characteristics, which are the birth year and housing tenure of the household head. That is, we construct a panel data set of each representative consumer unit with one of the combination of these characteristics.

The birth cohorts are defined by a 5-year band. The oldest cohort consists of people who were born between January 1910 and December 1914. We focus on household heads of age 25 to 75. The housing status is categorized into three levels: owners without mortgage, owners with positive mortgage balance, and renters. This procedure yields an unbalanced panel of 42 synthetic cohorts with a minimum of 20 CUs in each of them.

Our choice of a small set of characteristics is driven by the objectives of avoiding having few CUs in some synthetic cohorts, and avoiding short time series. The number of CUs in a synthetic cohort varies across cohorts. This variation in the number of CUs in synthetic cohorts can be problematic. The time-series data of synthetic cohorts with few CUs tend to be much volatile than that of synthetic cohorts with many CUs, because household-specific changes in consumption are not averaged out. This leads to high standard errors for synthetic cohorts with few CUs. Also, if the time-series of consumption and income are too short, estimation may suffer from a small sample bias.

2.7.2.2 Estimation of Cohort-Level Variablesl

Given the definition of synthetic cohorts, we estimate durable and non-durable consumption paths for each cohort. We consider a reduced form relationship between cohort-level consumption and individual household-level consumption in the cohort as follows:

$$\log(c_{j,i,t}) = \log(c_{i,t}) + \varepsilon_{j,i,t},$$

where $\varepsilon_{j,i,t} \sim$ i.i.d.$(0, \sigma_{i,t}^2)$

where $c_{j,i,t}$ is consumption level of household j in cohort i at time t, $c_{i,t}$ is cohort-level consumption for cohort i at time t, and $\varepsilon_{j,i,t}$ is a household-specific idiosyncratic shock at time t, which has mean zero and variance $\sigma_{i,t}^2$. That is, we model log of individual consumption as a random draw from a distribution with mean $\log(c_{i,t})$ and variance $\sigma_{i,t}^2$.

In this reduced form model, the simple average of $\log(c_{j,i,t})$ over households in cohort i at time t is a consistent estimate of $log(c_{i,t})$ by the law of large numbers. Since the CEX is a random sample from U.S. population, we use the CEX sample weights in taking the average. We interpret the CEX sample weights as the number of off-sample households who are represented by the consumer unit in the sample. Namely, we consider that there are $\omega_{j,i,t}$ households who are similar to household i, and hence whose consumptions are equal to $c_{j,i,t}$. Therefore, our estimate of cohort-level logged consumption is the weighted average of logged consumption expenditures over households in the cohort, using the CEX sample weights. That is,

$$\log(\hat{c_{i,t}}) := \frac{1}{\omega_{i,t}} \sum_{j \in I_{i,t}} \omega_{j,i,t} \log(c_{j,i,t})$$

$$\log(c_{i,t}) = 1$$

where $I_{i,t}$ is the set of households in cohort i at time t, $\omega_{j,i,t}$ is the CEX sample weights, and $\omega_{i,t} = \sum_{j \in I_{i,t}} \omega_{j,i,t}$.

Chapter 3

Search Smooths Discontinuities in Discrete/Continuous Problems

3.1 Introduction

This paper introduces new methods for efficiently solving dynamic optimization problems with both discrete and continuous choices (DC models). These methods extend the Endogenous Gridpoint Method (EGM) (Carroll (2006)) by including exogenous outcome probabilities, search frictions, and taste shocks to '*concavify*' the value function of the optimization problem. Compared to existing extensions of the EGM for DC models, the methods introduced in this paper have the added advantage of not only providing greater smoothness, but also rationalizing the smoothness into the agent's choice problem.

For dynamic stochastic optimization problems with continuous choice variables, the Endogeneous Gridpoint Method (EGM) is significantly more efficient compared to standard root-finding methods in finding the optimal decision rules. In these problems, the value functions are typically concave and the Euler equation is a necessary and sufficient condition for the optimal decision rule. In contrast, when the choice variables include both continuous and discrete choices (DC models), the Euler equation has multiple solutions for the continuous choice. Therefore, the Euler equation is only a necessary, but not sufficient, condition for the optimal decision rule.

The complication in solving a dynamic optimization problem arises due to the kinks in the value function and discontinuities in the decision rule at the points in the state space where discrete choices are made. These discontinuities make the decision rule non-monotonic, which leads to the Euler equation producing suboptimal points. In addition, each period's kinks and discontinuities are propagated back in time as the solution is solved by backward iteration, thus, exacerbating the problem. Fella (2014), Druedahl and Jørgensen (2017), and Iskhakov et al. (2017) provide methods for finding and discarding these suboptimal points from the final solution.

In order to mitigate the complications arising from the accumulation of the discontinuities, Iskhakov et al. (2017) introduce Extreme Value Type I taste shocks that affect the likelihood of the discrete choice outcomes. These taste shocks have the advantage of smoothing out the expected value function and the expected marginal utility function. With larger taste shocks, the value function can be 'concavified' to a greater degree. Such taste shocks, however, are in fact 'behavioral' shocks that add a degree of randomization to the discrete choice outcomes.

In contrast, this paper introduces search frictions whereby agents exert search effort that determines the likelihood of changing the discrete state. In addition, it includes log-normally distributed taste shocks that affect the utility that agents derive from making discrete choices. Combined, these two features not only lead to greater smoothness of the decision rule, they also rationalize the smoothness arising from the taste shocks, as these shocks determine the search effort that the agents exert.

To illustrate the properties of search as a smoothing mechanism and to compare it to other modeling features that can also provide smoothness, we use a standard consumption-saving model in which the agent can also receive the 'option' to make a binary retirement decision, depending on the search effort that they make. If the agent makes no search effort, they receive the option to retire with exogenous probability p and if they make the maximum effort, they receive the option to retire with certainty. Effort, however, is costly and the cost function is assumed to be a convex function of effort. In addition, households receive taste shocks that affect the relative value they derive from working and are also exposed to income uncertainty.

All four of the aforementioned modeling features, i.e. search, exogenous switching probability, taste shocks, and income shocks, serve to smooth out the decision rule in the DC model with varying degrees of effectiveness. While search provides the highest degree of smoothness, in terms of reducing the size of the discontinuities in the decision rule, it has to be combined with taste shocks to yield uniform smoothness around the discontinuities. Moreover, each of the smoothing mechanisms have different economic interpretations and affect the decision rules differently. While greater smoothness from search and exogenous switching probability shifts the consumption function upwards, greater smoothness from taste shocks and income shocks shifts the consumption function downwards.

3.2 Model

In this section, we build a consumption-saving model and extend it to include the option to retire and search frictions. Agents in this model derive utility from consuming non-durable goods c_t and get disutility from working. The disutility parameter is denoted by δ. Each period, there is a possibility for the agents to receive an 'option' to retire. The likelihood of receiving the option depends on the amount of search effort ε_{t-1} that the agent exerted in the previous period. Search effort, however, is costly and we assume that the search cost is a convex function of effort, with a functional form given by $\theta(\varepsilon_t) = \sigma^s \left(\varepsilon_t + (1-\varepsilon_t)\log\left(1-\varepsilon_t\right)\right)$. This cost function implies that the initial marginal effort is cheap; however, greater effort that increases the likelihood of receiving the option to switch the discrete state leads to increasingly larger costs. Assuming logarithmic utility from consumption, and denoting the choice to retire and the choice to work by $d_t = 0$ and $d_t = 1$, respectively, the utility function for the agent is given by

$$u(c_t, d_t, \varepsilon_t) = \log(c_t) - \delta d_t - \theta(\varepsilon_t). \tag{3.1}$$

At the end of each period, households can save their end-of-period liquid assets A_t at a risk free rate R. Working in the current period determines whether the agent receives income y_t in the following period. Therefore, the beginning-of-period assets are given by $M_t = RA_t + y_t d_{t-1}$. Furthermore, we assume that $y_t = y\eta_t$, where η_t follows a mean-one log-normal distribution, $\eta_t \sim \mathcal{N}(-\sigma_\eta^2/2, \sigma_\eta^2)$.

3.2.1 Retiree's sub-problem

Retirement is assumed to be self-absorbing, so the problem reduces to the simple consumption-saving problem, which in Bellman form is given by:

$$V_t^{Ret}(M_t) = \max_{0 \leq c_t \leq M_t} \{u(c_t, 0, 0) + \beta V_{t+1}^{Ret}(M_{t+1})\}, \tag{3.2}$$

where $M_{t+1} = A_t R$ and the end-of-period assets $A_t = M_t - c_t$. Using the gothic script V ($\mathfrak{V}$) to represent the value of ending the period with assets A_t, the end-of-period value for a retired agent is, by definition

$$\mathfrak{V}_t^{Ret}(A_t) \equiv V_{t+1}^{Ret}(M_{t+1}(A_t)) = V_{t+1}^{Ret}(A_t R) \tag{3.3}$$

Thus, the problem in terms of the end-of-period assets can be restated as

$$\mathfrak{V}_{t-1}^{Ret}(A_{t-1}) = \max_{0 \leq c_t \leq A_{t-1} R} \{\log(c_t) + \beta \mathfrak{V}_t^{Ret}(A_t)\}. \tag{3.4}$$

3.2.2 Worker's sub-problem

Each period, a worker either receives the option to retire or has to continue working. There is an exogenous probability p of receiving the option to retire. In addition, a worker can increase the likelihood of receiving the option to retire by exerting search effort. The search effort is made at the end of a period and affects the likelihood of receiving the option to retire in the following period. When period t starts, an income shock is realized. This is followed by the realization of a taste shock, ξ_t, which is a multiplicative shock that affects the utility that a worker derives from working. The taste shocks are assumed to follow a mean-one log-normal distribution, i.e. $\xi_t \sim \mathcal{N}(-\sigma_\xi^2/2, \sigma_\xi^2)$. These taste shocks are different from the Extreme Value Type I shocks in Iskhakov et al. (2017) in two major ways. Firstly, the assumption of log-normal taste shocks means that these shocks can take up multiple distinct values in the discrete approximation of the distribution. Secondly, these shocks are not the purely behavioral shocks that randomize the discrete choice outcome. Instead, these taste shocks affect the likelihood of the discrete outcome by determining the search effort that the agent exerts.

Based on the search effort made in the previous period, if the agent receives the option to switch, the agent decides whether to retire or continue working. If the agent does not receive the option to switch, then they continue to work. Having made the discrete choice, the agent makes a consumption decision. If the agent chooses to retire,

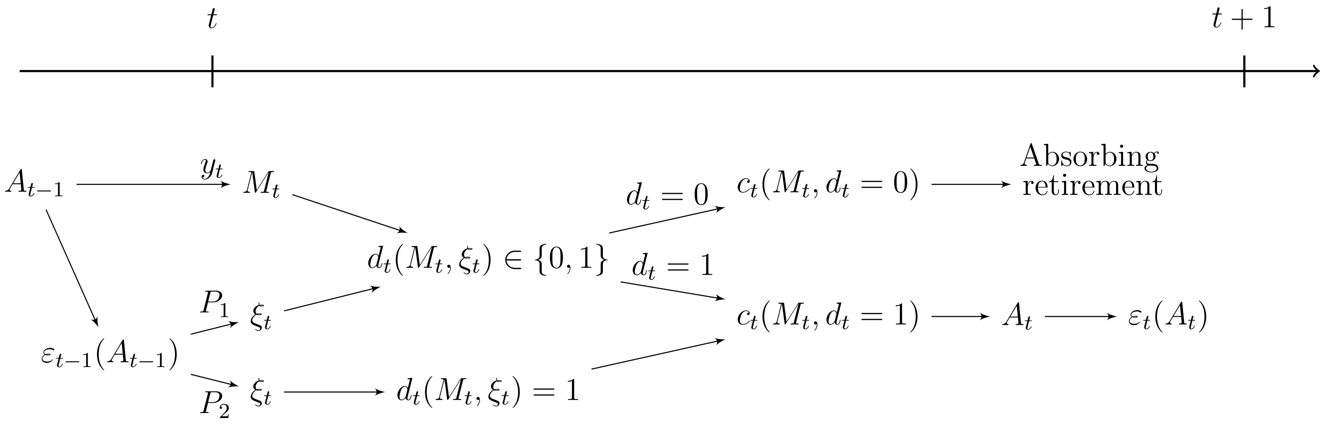


Figure 3.1: Timing of shock realizations and decisions in the worker's problem.

they stay as a retiree, because the assumption that retirement is self-absorbing. If the agent chooses to continue working, they exert search effort, which then determines the likelihood of receiving the option to switch in the next period. Fig.3.1 outlines the timeline of the realization of various shocks and the decisions in the worker's problem.

Let the set of possible choice sets for the worker's discrete choice be given by $\mathscr{D}_t \in \big\{\{\text{Retire}, \text{Work}\}, \{\text{Work}\}\big\} \equiv \big\{\{d_t = 0, d_t = 1\}, \{d_t = 1\}\big\}$. The first element of this set is the case when the agent has the option to switch and the second element is the case when the agent does not have the option to switch and must continue working. For the case when $\mathscr{D}_t = \{W\}$, i.e. the worker does not have the option to switch, the Bellman equation is given by:

$$V_t(M_t, \{W\}, \xi_t) = \xi_t. \max_{\substack{0 \leq c_t \leq M_t \\ 0 \leq \varepsilon_t \leq 1}} \big\{u(c_t, 1, \varepsilon_t) + \beta P_1(\varepsilon_t)\mathfrak{V}^1_t(A_t) + \beta P_2(\varepsilon_t)\mathfrak{V}^2_t(A_t)\big\} \quad (3.5)$$

where $\mathfrak{V}^1_t(A_t)$ is the expected value of saving A_t when having the option to retire in $t+1$, $P_1(\varepsilon_t) = (1-\varepsilon_t)p + \varepsilon_t$ is the probability receiving the option to retire in $t+1$, given the search effort ε_t in the current period, $\mathfrak{V}_t^2(A_t)$ is the expected value of saving A_t when not having the option to retire in $t+1$, and $P_2(\varepsilon_t) = (1-\varepsilon_t)(1-p)$ is the probability of not receiving the option to retire in $t+1$, given the search effort ε_t in the current period.

For the case when $\mathscr{D}_t = \{R, W\}$, i.e. the case when the worker has the option to switch, the Bellman equation is given by:

$$V_t(M_t, \{R, W\}, \xi_t) = \max_{d_t \in \{0,1\}} \Big[V_t^{Ret}(M_t); V_t(M_t, \{W\}, \xi_t) \Big], \tag{3.6}$$

or

$$\begin{aligned} V_t(M_t, \{R, W\}, \xi_t) = \max_{d_t \in \{0,1\}} \Big[& \max_{0 \leq c_t \leq M_t} \big\{ u(c_t, 0, 0) + \beta \mathfrak{V}_t^{Ret}(A_t) \big\}; \\ & \xi_t \cdot \max_{\substack{0 \leq c_t \leq M_t \\ 0 \leq \varepsilon_t \leq 1}} \big\{ u(c_t, 1, \varepsilon_t) + \beta P_1(\varepsilon_t) \mathfrak{V}_t^1(A_t) + \beta P_2(\varepsilon_t) \mathfrak{V}_t^2(A_t) \big\} \Big]. \end{aligned} \tag{3.7}$$

The expected value of saving A_t when having the option to retire in $t+1$, i.e. $\mathfrak{V}_t^1(A_t)$, is given by

$$\mathfrak{V}_t^1(A_t) = \int_{y_{t+1}} \int_{\xi_{t+1}} V_{t+1}(M_{t+1}(A_t), \{R, W\}, \xi_{t+1}) dF(y_{t+1}) dF(\xi_{t+1}), \tag{3.8}$$

where $M_{t+1} = A_t R + d_t y$. Similarly, the expected value of saving A_t when not having the option to retire in $t+1$, i.e. $\mathfrak{V}_t^2(A_t)$, is given by

$$\mathfrak{V}_t^2(A_t) = \int_{y_{t+1}} \int_{\xi_{t+1}} V_{t+1}(M_{t+1}(A_t), \{W\}, \xi_{t+1}) dF(y_{t+1}) dF(\xi_{t+1}). \tag{3.9}$$

3.2.3 Optimal effort and consumption functions

Since we assume that the consumption decision is made before the effort decision, (3.5) can be split into two sequential sub-period problems and rewritten as:

$$V_t(M_t, \{W\}, \xi_t) =_{\xi_t} \max_{0 \le c_t \le M_t} \Big[\mathcal{V}_t(A_t) \Big] \tag{3.10}$$

where

$$\mathcal{V}_t(A_t) = \max_{0 \le \varepsilon_t \le 1} \big\{ u(c_t, 1, \varepsilon_t) + \beta P_1(\varepsilon_t) \mathfrak{V}_t^1(A_t) + \beta P_2(\varepsilon_t) \mathfrak{V}_t^2(A_t) \big\} \tag{3.11}$$

Thus, the first order condition of (3.11) gives the optimal effort as a function of the end-of-period assets that are determined after the optimal consumption decision has been made in the previous sub-period. The optimal effort is given by

$$\frac{\partial \theta(\varepsilon_t)}{\partial \varepsilon_t} = \beta \mathfrak{V}_t^1(A_t) \frac{\partial P_1(\varepsilon_t)}{\partial \varepsilon_t} + \beta \mathfrak{V}_t^2(A_t) \frac{\partial P_2(\varepsilon_t)}{\partial \varepsilon_t} \tag{3.12}$$

$$-\sigma^s \log(1 - \varepsilon_t^*) = \beta(1-p)\mathfrak{V}_t^1(A_t) - \beta(1-p)\mathfrak{V}_t^2(A_t) \tag{3.13}$$

$$\varepsilon_t^* = 1 - \exp\left(-\frac{\beta(1-p)}{\sigma^s} \big(\mathfrak{V}_t^1(A_t) - \mathfrak{V}_t^2(A_t) \big) \right). \tag{3.14}$$

This expression states that optimal effort is an increasing function of the excess value of receiving the option to switch in the next period over not receiving the option to switch.

Next, we derives the expressions for the optimal consumption functions for the retiree and the worker. Since retirement is self-absorbing, the retiree's consumption function is relatively straightforward to derive and is given by:

$$u'(c_t, 0, 0) = R\beta u'(c_{t+1}, 0, 0) \tag{3.15}$$

or

$$c_t^{Ret} = \left(R\beta \left(c_{t+1}^{Ret}(A_t R)\right)^{-1}\right)^{-1}. \tag{3.16}$$

To derive the consumption function for the worker, we start with the solution to the problem of a worker who does not have the option to retire, i.e. $\mathscr{D}_t = \{W\}$. The first-order-condition for problem (3.10), with respect to c_t, is given by

$$u'(c_t, 1, \varepsilon_t^*) = \beta\Big[P_1(\varepsilon_t^*)\frac{\partial \mathfrak{V}_t^1(A_t)}{\partial A_t} + P_2(\varepsilon_t^*)\frac{\partial \mathfrak{V}_t^2(A_t)}{\partial A_t}\Big], \tag{3.17}$$

where ε_t^* is the optimal effort function, which is determined in (3.14), and the derivates of the end-of-period values are given by

$$\frac{\partial \mathfrak{V}_t^1(A_t)}{\partial A_t} = \mathbb{E}_t\Big\{\frac{\partial V_{t+1}^{R,W}(M_{t+1}, \xi_{t+1})}{\partial M_{t+1}}\frac{\partial M_{t+1}}{\partial A_t}\Big\} = R\mathbb{E}_t\Big\{\frac{\partial V_{t+1}^{R,W}(M_{t+1}, \xi_{t+1})}{\partial M_{t+1}}\Big\} \tag{3.18}$$

and

$$\frac{\partial \mathfrak{V}_t^2(A_t)}{\partial A_t} = R\mathbb{E}_t\Big\{\frac{\partial V_{t+1}^{W}(M_{t+1}, \xi_{t+1})}{\partial M_{t+1}}\Big\}, \tag{3.19}$$

where the expectation is over next period income and taste shocks, and $V_{t+1}^W(M_{t+1}, \xi_{t+1})$ and $V_{t+1}^{R,W}(M_{t+1}, \xi_{t+1})$, are shorthands for $V_{t+1}(M_{t+1}, \{W\}, \xi_{t+1})$ and $V_{t+1}(M_{t+1}, \{R, W\}, \xi_{t+1})$, respectively. Substituting these expression into (3.17) yields

$$u'(c_t, 1, \varepsilon_t^*) = R\beta\Big[P_1(\varepsilon_t^*)\mathbb{E}_t\Big\{\frac{\partial V_{t+1}^{R,W}(M_{t+1}, \xi_{t+1})}{\partial M_{t+1}}\Big\} + P_2(\varepsilon_t^*)\mathbb{E}_t\Big\{\frac{\partial V_{t+1}^{W}(M_{t+1}, \xi_{t+1})}{\partial M_{t+1}}\Big\}\Big]. \tag{3.20}$$

The Envelope Condition for (3.10) is given by

$$\frac{\partial V_t^W(M_t,\xi_t)}{\partial M_t} = \xi_t\beta\Big[P_1(\varepsilon_t^*)\frac{\partial\mathfrak{V}_t^1(A_t)}{\partial A_t} + P_2(\varepsilon_t^*)\frac{\partial\mathfrak{V}_t^2(A_t)}{\partial A_t}\Big] \tag{3.21}$$

or

$$\frac{\partial V_t^W(M_t,\xi_t)}{\partial M_t} = \xi_t u'(c_t,1,\varepsilon_t^*) \tag{3.22}$$

which can be iterated forward to yield

$$\frac{\partial V_{t+1}^W(M_{t+1},\xi_{t+1})}{\partial M_{t+1}} = \xi_{t+1}u'(c_{t+1},1,\varepsilon_{t+1}^*). \tag{3.23}$$

To derive a similar expression for the derivative of $V_{t+1}^{R,W}$, we note that for the worker who has the option to retire, i.e. $\mathscr{D}_t = \{R,W\}$, the value function is simply the maximum of the value function of the retiree and the value function of the worker who continues to work

$$V_t^{R,W}(M_t,\xi_t) = \max\Big\{V_t^{Ret}(M_t); V_t^W(M_t,\xi_t)\Big\}. \tag{3.24}$$

Therefore, it can be shown that

$$\frac{\partial V_{t+1}^{R,W}(M_{t+1},\xi_{t+1})}{\partial M_{t+1}} = \mathbb{1}_{t+1}^0.u'(c_{t+1},0,0) + \mathbb{1}_{t+1}^1.\xi_{t+1}u'(c_{t+1},1,\varepsilon_{t+1}^*), \tag{3.25}$$

where $\mathbb{1}_{t+1}^0 \equiv \mathbb{1}_{t+1}^0(M_{t+1},\xi_{t+1}) = 1$ if $V_{t+1}^{Ret}(M_{t+1}) \geq V_{t+1}^W(M_{t+1},\xi_{t+1})$ and 0 otherwise, and $\mathbb{1}_{t+1}^1 \equiv \mathbb{1}_{t+1}^1(M_{t+1},\xi_{t+1}) = 1$ if $V_{t+1}^{Ret}(M_{t+1}) < V_{t+1}^W(M_{t+1},\xi_{t+1})$ and 0 otherwise.

Substituting (3.23) and (3.25) into (3.20) yields the consumption function for the worker who continues to work

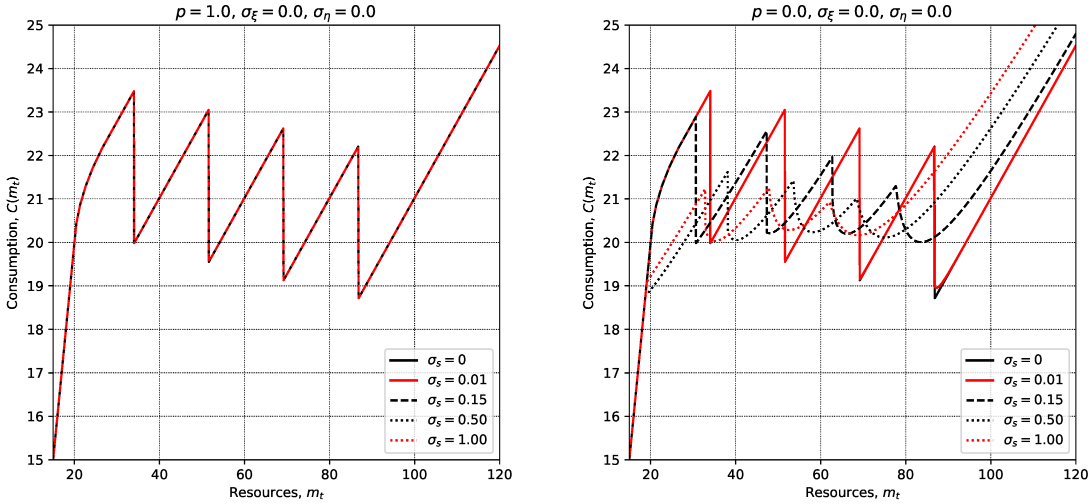
$$\begin{aligned} u'(c_t, 1, \varepsilon_t^*) = R\beta\Big[P_1(\varepsilon_t^*) \int_{y_{t+1}} \int_{\xi_{t+1}} & \left[\mathbb{1}_{t+1}^0 . u'(c_{t+1}, 0, 0) + \mathbb{1}_{t+1}^1 . \xi_{t+1} . u'(c_{t+1}, 1, \varepsilon_{t+1}^*)\right] \\ & dF(y_{t+1}) dF(\xi_{t+1}) + P_2(\varepsilon_t^*) \int_{y_{t+1}} \int_{\xi_{t+1}} \left[\xi_{t+1} u'(c_{t+1}, 1, \varepsilon_{t+1}^*)\right] \\ & dF(y_{t+1}) dF(\xi_{t+1})\Big] \end{aligned} \tag{3.26}$$

or

$$\begin{aligned} c_t^W = \Bigg(R\beta\Big[P_1(\varepsilon_t^*) \int_{y_{t+1}} \int_{\xi_{t+1}} & \left[\mathbb{1}_{t+1}^0 . \left(c_{t+1}^{Ret}(A_t R)\right)^{-1} + \mathbb{1}_{t+1}^1 . \xi_{t+1} \left(c_{t+1}^W (A_t R + y_{t+1})\right)^{-1}\right] \\ & dF(y_{t+1}) dF(\xi_{t+1}) + P_2(\varepsilon_t^*) \int_{y_{t+1}} \int_{\xi_{t+1}} \left[\xi_{t+1} \left(c_{t+1}^W (A_t R + y_{t+1})\right)^{-1}\right] \\ & dF(y_{t+1}) dF(\xi_{t+1})\Big]\Bigg)^{-1} . \end{aligned} \tag{3.27}$$

3.3 Results

Having established the solution of the retirement problem, we demonstrate how varying the different smoothness parameters affects the solution of the problem. Fig. 3.2a shows the optimal consumption rule of the worker who continues to work in period $T-5$, for the case in which $p = 1.0$. The plot shows the consumption rule for a set of search cost scales, in the absence of income uncertainty and taste shocks. In this scenario, there is no smoothing and the discontinuities that arise from discrete choices being made in periods $T-4$ through $T-1$ are distinctly visible. This is because

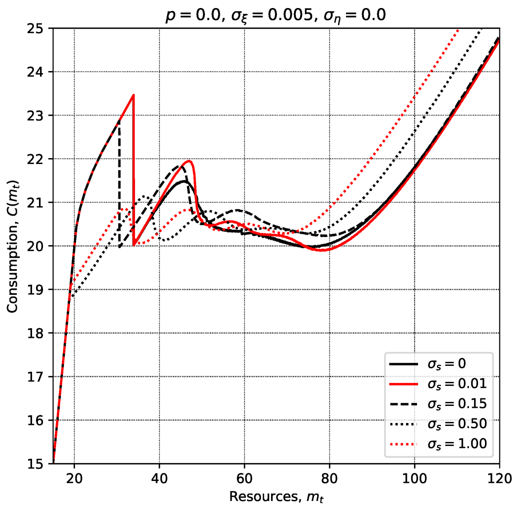


(a) Receive switching option with certainty, $p = 1$. (b) Stay as worker with certainty without search effort, $p = 0$.

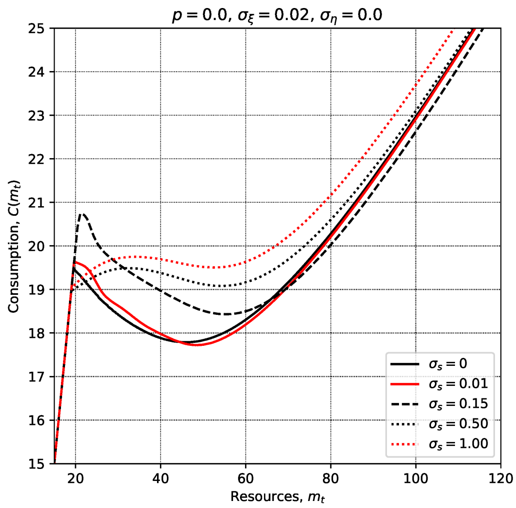
Figure 3.2: The plots show optimal consumption rules of the workers who decides to continue working in period $t = T - 5$ for a set of search cost scales σ_s in the absence of income uncertainty or taste shocks, for the case with $p = 1.0$ (left panel) with $p = 0.0$ (right panel). The rest of the model parameters are $R = 1$, $\beta = 0.98$, $y = 20$.

when $p = 1.0$, the worker receives the option to switch with certainty. In such a case the worker does not exert any search effort, regardless of the search frictions. Consequently, in the case with $p = 1.0$, for all levels of search frictions, we get the same optimal consumption rule, without any smoothing.

In contrast, when $p = 0.0$ (Fig. 3.2b), the worker who does not exert search effort, continues to stay as a worker with certainty. This gives the worker a motive to exert search effort, which determines the likelihood of receiving the option to switch. For a



(a) Small taste shocks, $\sigma_\xi = 0.005$.



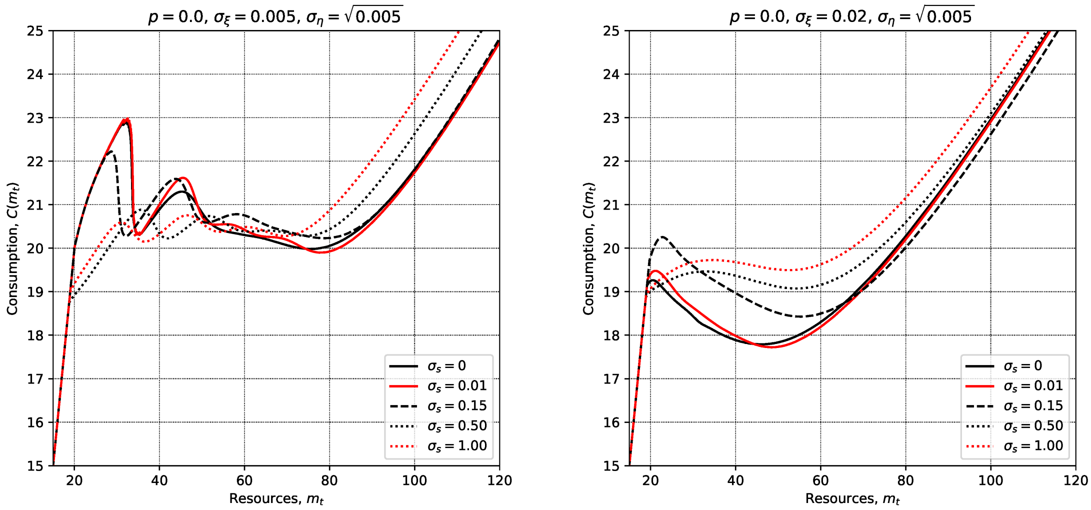
(b) Large taste shocks, $\sigma_\xi = 0.02$.

Figure 3.3: The plots show optimal consumption rules of the workers who decides to continue working in period $t = T - 5$ for the case with $p = 0$, in the presence of small taste shocks (left panel) and large taste shocks (right panel), without income uncertainty.

given level of search frictions, decreasing p increases the smoothness of the solution. Moreover, for a given level of p, increasing the search frictions (higher value of σ_s) also increases smoothness. Fig. 3.2b also shows the asymmetric smoothness offered by search, when there are no taste or income shocks. The smoothness happens to the right of the discontinuities. This is because for low liquid resources, workers optimally choose to continue working and hence do not exert any search effort. Workers only exert positive effort to receive the option to retire when they have ample liquid resources. Therefore, the smoothness from search frictions only happens to the right of the discontinuities.

To address this asymmetric smoothness, we need to introduce taste shocks. Since the taste shocks can take a range of different values, there are certain realizations of the taste shock that make the worker want to have the option to switch, even at levels of liquid assets at which, in the absence of a taste shock, the worker would not want to switch. In other words, with the introduction of taste shocks, the range of liquid assets around the discontinuities over which the worker exerts search effort expands. Fig.3.3a shows that with the incorporation of taste shocks, the consumption function now smoothes out more uniformly around the discontinuities. Moreover, introducing taste shocks allows the search frictions to significantly smooths out most of the discontinuities in the optimal consumption function. This holds even for a very small smoothness parameter for the taste shocks, $\sigma_\xi = 0.005$. This is because now there is a positive likelihood of a worker receiving a taste shock that makes them exert search effort even for values of M_t for which the worker would not exert any search effort in the absence of taste shocks. Increasing the magnitude of the taste shocks to $\sigma_\xi = 0.02$ (Fig.3.3b) significantly reduces the size of all the discontinuities in the consumption function. Moreover, only moderately sized search frictions are need to virtually completely smooth out the consumption function.

The results discussed so far do not include any income uncertainty. Adding income uncertainty even into standard consumption-saving models adds curvature and smoothness to the consumption rules. We study the impact of incorporating income uncertainty to the retirement problem on the smoothness of the decision rules in



(a) Small taste shocks, $\sigma_\xi = 0.005$.

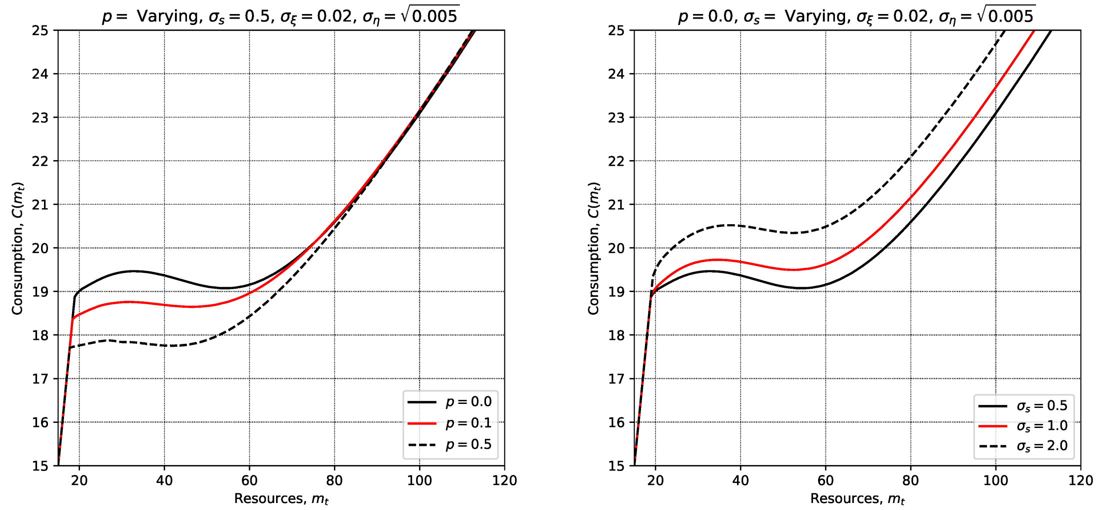
(b) Large taste shocks, $\sigma_\xi = 0.02$.

Figure 3.4: The plots show optimal consumption rules of the workers who decides to continue working in period $t = T - 5$ for the case with $p = 0$, in the presence of small taste shocks (left panel) and large taste shocks (right panel), with income uncertainty.

Figs.3.4a and 3.4b. These figures show that while income uncertainty adds smoothness, it does not play a significant role in reducing the size of the discontinuities. Therefore, search frictions and taste shocks play the dominant role in smoothing out the problem. An appropriate combination of search frictions, taste shocks, and income uncertainty can yield a monotonically increasing outcome-specific consumption function by smoothing out the discontinuities that propagate through each period while iterating backwards.

It is important to note that although each of these smoothness mechanisms serve to smooth out the solution for the worker's problem, they have different economic interpretations and affect the optimal consumption rule differently. While lowering the exogenous probability of receive the option to switch, p, and increasing the search frictions, σ_s, shift the consumption function upwards, large taste shocks, σ_ξ, and income shocks, σ_η, shift the consumption function downwards. Ceteris paribus, decreasing p or increasing σ_s, reduce the likelihood of the worker switching their discrete state successfully. This means that the likelihood of the worker retiring in a given period decreases. As a result, the expected number of years that an agent works increases and they save less in a given period. In contrast, when the size of the taste shocks is increased, the uncertainty surrounding the likelihood of a worker's retirement decision increases. Consequently, the agents save more in a given period, because unexpectedly retiring early would limit the resources that the agent can consume for the remainder of their life. Similarly, when the size of the income shocks increases, households increase their buffer stock savings, and the consumption function shifts down. Therefore, although each of the smoothness mechanisms achieve the common goal of 'concavifying' the solution, they have different interpretations and different impacts on the optimal consumption rule.

To illustrate these points graphically, we start from the baseline consumption function that has the discontinuities completely smoothed out using smoothness parameters $p = 0$, $\sigma_\xi = 0.02$, $\sigma_s = 0.5$, and $\sigma_\eta = \sqrt{0.005}$. Figs.3.5a shows the impact of lowering p from the baseline level to $p = 0.1$ and $p = 0.5$, which decreases smoothness, and Fig.3.5b shows the impact of increasing the search frictions from the baseline

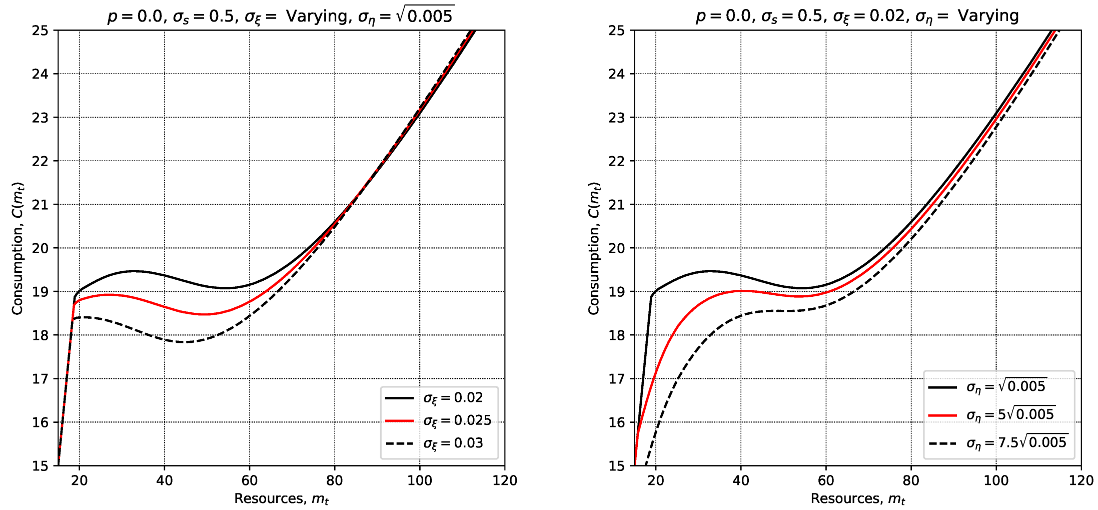


(a) Varying the exogenous probability of receiving the option to switch.

(b) Varying search friction intensity.

Figure 3.5: The plots show the impact of increasing the smoothness parameters for the exogenous option probability (left panel) and increasing the magnitude of the search frictions (right panel).

level to $\sigma_s = 1.0$ and $\sigma_s = 2.0$, which increases smoothness. As mentioned above, increasing smoothness by adjusting either of the two mechanisms shifts the optimal consumption rule upwards. On the other hand, Fig.3.6a shows the impact of increasing the size of the taste shocks from the baseline level to $\sigma_\xi = 0.025$ and $\sigma_\xi = 0.03$, and Fig.3.6b shows the impact of increasing the size of the income shocks from the baseline level to $\sigma_\eta = 5\sqrt{0.005}$ and $\sigma_\eta = 7.5\sqrt{0.005}$. These figures demonstrate that, as expected, increasing the magnitude of the taste or income shocks shifts the optimal consumption function downwards. Therefore, while a combination of these smoothing mechanisms will smooth out the discontinuities, it is also important to note how each



(a) Varying the magnitude of the taste shocks. (b) Varying the magnitude of income shocks.

Figure 3.6: The plots show the impact of increasing the smoothness parameters for the taste shocks (left panel) and increasing the magnitude of the income shocks (right panel).

of them shift the decision rule differently.

3.4 Algorithm

This section outlines the algorithm for solving the problem of the worker who continues to work in the current period. The problem for the retiree can be pre-solved relatively easily. With the solution of the worker who continues to work and the retiree at hand, it is straightforward to compute the solution of the worker with the option to switch. With ample smoothness, the EGM-step should not produce any suboptimal points. However, if there is not enough smoothness, then an additional 'upper envelope' step is required to remove the suboptimal points. The upper envelope refinement step in Iskhakov et al. (2017) is an example of an algorithm that can be employed at the end of the EGM-step to achieve this.

Algorithm 1: EGM-step with search for the worker who continues to work

Let $\overrightarrow{\eta} = \{\eta^1, ..., \eta^J\}$ and $\overrightarrow{\xi} = \{\xi^1, ..., \xi^K\}$ be vectors of quadrature points with associated weights $\overrightarrow{\omega} = \{\omega^1, ..., \omega^J\}$ and $\overrightarrow{\mu} = \{\mu^1, ..., \mu^K\}$, respectively.

Form an ascending grid over end-of-period wealth, $\overrightarrow{A}_t = \{A_t^1, ..., A_t^I\}$

for $i = 1, ..., I$ **do**

  for $j = 1, ..., J$ **do**

    Compute $M_{t+1}^j(A^i) = RA^i + y\eta_{t+1}^j$

    for $d_{t+1} = 0, 1$ **do**

      Compute $c_{t+1}^{Ret}(M_{t+1}^j(A^i))$ by interpolating $c_{t+1}^{Ret}(\overrightarrow{M}_{t+1})$ at the point $M_{t+1}^j(A^i)$

      Compute $c_{t+1}^{W}(M_{t+1}^j(A^i))$ by interpolating $c_{t+1}^{W}(\overrightarrow{M}_{t+1})$ at the point $M_{t+1}^j(A^i)$

      Compute $V_{t+1}^{Ret}(M_{t+1}^j(A^i))$ by interpolating $V_{t+1}^{Ret}(\overrightarrow{M}_{t+1})$ at the point $M_{t+1}^j(A^i)$

      for $k = 1, ..., K$ **do**

        Compute $V_{t+1}^{W}(M_{t+1}^j(A^i), \xi^k)$ by interpolating $V_{t+1}^{W}(\overrightarrow{M}_{t+1})$ at the point $M_{t+1}^j(A^i)$ and multiplying it by ξ^k

        Compute $V_{t+1}^{RW}(M_{t+1}^j(A^i), \xi^k) = \max\{V_{t+1}^{Ret}(M_{t+1}^j(A^i)), V_{t+1}^{W}(M_{t+1}^j(A^i), \xi^k)\}$

        Compute $\mathbb{1}_{t+1}^0(M_{t+1}^j(A^i), \xi^k) = 1$ if $V_{t+1}^{Ret}(M_{t+1}^j(A^i)) \geq V_{t+1}^{W}(M_{t+1}^j(A^i), \xi^k)$ and 0 otherwise.

        Compute $\mathbb{1}_{t+1}^1(M_{t+1}^j(A^i), \xi^k) = 1 - \mathbb{1}_{t+1}^0(M_{t+1}^j(A^i), \xi^k)$

        Compute $RHS_1^{pre}(M_{t+1}^j(A^i), \xi^k) = \mathbb{1}_{t+1}^0(M_{t+1}^j(A^i), \xi^k).u'(c_{t+1}^{Ret}(M_{t+1}^j(A^i))) + \xi^k.\mathbb{1}_{t+1}^1(M_{t+1}^j(A^i), \xi^k).u'(c_{t+1}^{W}(M_{t+1}^j(A^i)))$

        Compute $RHS_2^{pre}(M_{t+1}^j(A^i), \xi^k) = \xi^k.u'(c_{t+1}^{W}(M_{t+1}^j(A^i)))$

      end

    end

  end

  Compute $\mathfrak{V}_t^1(A^i) = \sum_{j=1}^J \sum_{k=1}^K \omega^j \mu^k.V_{t+1}^{RW}(M_{t+1}^j(A^i), \xi^k)$

  Compute $\mathfrak{V}_t^2(A^i) = \sum_{j=1}^J \sum_{k=1}^K \omega^j \mu^k.V_{t+1}^{W}(M_{t+1}^j(A^i), \xi^k)$

  Compute $\varepsilon_t(A^i) = 1 - \exp\left(-\beta(1-p)(\mathfrak{V}_t^1(A^i) - \mathfrak{V}_t^2(A^i))/\sigma^s\right)$

  Compute $\theta(\varepsilon_t(A^i)) = \sigma^s\left(\varepsilon_t(A^i) + (1-\varepsilon_t(A^i))\log(1-\varepsilon_t(A^i))\right)$

  Compute $P_1(\varepsilon_t(A^i)) = (1-\varepsilon_t(A^i)).p + \varepsilon_t(A^i)$

  Compute $P_2(\varepsilon_t(A^i)) = (1-\varepsilon_t(A^i)).(1-p)$

  Compute $RHS_1(A^i) = \sum_{j=1}^J \sum_{k=1}^K \omega^j \mu^k.RHS_1^{pre}(M_{t+1}^j(A^i), \xi^k)$

  Compute $RHS_2(A^i) = \sum_{j=1}^J \sum_{k=1}^K \omega^j \mu^k.RHS_2^{pre}(M_{t+1}^j(A^i), \xi^k)$

  Compute $RHS(A^i) = R\beta\left(P_1(\varepsilon_t(A^i))RHS_1(A^i) + P_2(\varepsilon_t(A^i))RHS_2(A^i)\right)$

  Compute expected value function $EV_{t+1}(M_{t+1}(A^i)) = P_1(\varepsilon_t(A^i)).\mathfrak{V}_t^1(A^i) + P_2(\varepsilon_t(A^i)).\mathfrak{V}_t^2(A^i)$

  Compute current consumption $c_t^W(A^i) = u'^{-1}\left(RHS(A^i)\right)$

  Compute value function $V_t^W(M_t(A^i)) = u(c_t^W(A^i), 1, \varepsilon_t(A^i)) + \beta EV_{t+1}(M_{t+1}(A^i))$

  Compute endogenous grid point $M_t(A^i) = c_t^W(A^i) + A^i$

end

Collect the points $M_t(A^i)$, $c_t^W(A^i)$, and $V_t^W(M_t(A^i))$ to form the consumption function, $c_t^W(\overrightarrow{M}_t)$, and value function, $V_t^W(\overrightarrow{M}_t)$.

3.5 Conclusion

This paper introduces search as a new method for smoothing out discrete-continuous choice (DC) problems. While search can significantly reduce the size of the discontinuities in the decision rule of a DC problem, to get uniform smoothness around the discontinuities, we need to incorporate taste shocks as well. Together, search and taste shocks offer modelers greater control over the degree of smoothness in DC models, while rationalizing the smoothness into the agent's choice problem. With ample smoothness, discontinuities in the decision rule can be smoothed out entirely; thus, eliminating the need for incorporating an additional, computationally costly, step of identifying and removing suboptimal points generated in the EGM step for DC models. This paper studies search in the context of a consumption-saving-retirement problem; however, the framework can be easily applied to models of housing purchase, lumpy firm investments, and a wide array of other DC models.

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Vita

Shujaat A. Khan received the Bachelor of Arts degree in Physics and Economics from Middlebury College in 2010. After graduating from college, he worked as a Research Associate in the Macroeconomics and Monetary Policy Section of the Economic Research Department at the Federal Reserve Bank of Kansas City. Shujaat enrolled in the Economics Ph.D. program at Johns Hopkins University in 2013. His research interests are in Macroeconomics, Monetary Economics, Macro-Finance, and Computational Economics. During the Ph.D., he also worked for the International Monetary Fund, the World Bank, and the New Zealand Treasury.

Starting in September 2019, Shujaat will be joining the International Monetary Fund. He will be working in the Strategy, Policy, and Review Department's External Policy Division.