

**THE INTERPLAY BETWEEN NATURAL AND
ACCIDENTAL SUPERSYMMETRY**

by

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Abstract

In this thesis, we will explore the subject of the little hierarchy problem which plagues solutions to the big hierarchy problem of the Standard Model of particle physics.

In the first half of this thesis, we study the theoretical framework for a supersymmetric resolution of the little hierarchy problem, known as natural supersymmetry, and argue that regions of the parameter space of this model have been missed by search strategies employed at the large hadron collider, but could be searched for with new search strategies.

In the second half of this thesis, we explore the possibility of embedding natural supersymmetry in models of warped extra dimensions in order to UV-complete them by utilizing a mechanism known as accidental supersymmetry. We study the mechanism of accidental supersymmetry in the Randall-Sundrum framework by focusing on a toy model, and argue that accidental supersymmetry is capable solving the little hierarchy problem in that toy model. Finally, as models in the Randall-Sundrum framework themselves require UV completions, we demonstrate that it is possible to

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realize the mechanism of accidental supersymmetry within the UV-complete framework of type IIB superstring theory.

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Dedication

This thesis is dedicated to April, for everything she has done for me.

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Chapter 1

Introduction and Background

This thesis concerns the subjects of natural and accidental supersymmetry (SUSY), and the potential for interplay between them. *Natural supersymmetry* is a mechanism proposed to solve the *little hierarchy problem*, exacerbated by recent results from the Large Hadron Collider (LHC). *Accidental supersymmetry* is a mechanism which generates radiatively stable little hierarchies, to be discussed much more below, and has been proposed as a partial UV-completion of natural SUSY [3]. We will study the consequences of both of these mechanisms, how accidental SUSY could give rise to natural SUSY, and how accidental SUSY could itself be realized in type IIB superstring theory.

In this chapter, we will review the Standard Model (SM) of particle physics and its successes. We then turn to a discussion of the (big) *hierarchy problem*, related to the cancellation of various contributions to the Higgs mass parameter which come

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from energy scales of very different orders of magnitude. We discuss two popular resolutions of the hierarchy problem: supersymmetry and warped extra dimensions. We will also review the *little hierarchy problem* (LHP), which is the subject of this thesis; it is a problem due to the non-observation of *any* popular resolution to the big hierarchy problem in any experiment performed thus far. In this thesis, we will explore how accidental SUSY gives rise to natural SUSY at low energies, and how natural SUSY solves the little hierarchy problem by stabilizing contributions to the Higgs mass parameter.

There are, of course, a proliferation of different *models* of supersymmetry, many of which resolve the (big) hierarchy problem. In contrast, a common theme in this thesis is the notion of *minimality*; the notion of minimality simultaneously goes hand-in-hand with exploring the *mechanism* which underlies this array of models, while also minimizing the risk of being in tension with experiment without forgoing all opportunities to discover new physics.

In chapters two and three, we will discuss the interpretation of “more minimal SUSY” as “natural SUSY”, the *minimal* module of supersymmetric physics required to stabilize the little hierarchy. We will investigate theoretical motivations and constraints for such a scenario by taking a bottom-up approach. We will also study the constraints on natural SUSY models coming from 1/fb’s worth of data at the LHC, and propose a search strategy intended to facilitate the discovery of sbottoms decaying through a particular topology. These chapters are based off of the author’s work

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in [4, 5], which were performed in collaboration with Raman Sundrum, Andrey Katz and Scott Lawrence. These chapters are to be read with a historical perspective, as although the general theme and theory of these chapters is the same today as when the works were completed, more experimental data is available now which we do not consider here.

In chapter four, we will discuss the mechanism of accidental SUSY in models of *warped extra dimensions*, and tie them in to their gauge duals. We will explore a toy model of accidental SUSY, and discuss ingredients necessary to build a full Beyond the Standard Model (BSM) model utilizing the mechanism of accidental SUSY. This chapter is based off of work to be published in [6] in collaboration with Raman Sundrum.

In chapter five, we will discuss how accidental SUSY might itself be UV-completed by string theory, by virtue of offering an explicit example of accidental SUSY in type IIB superstring theory. This chapter is based off of the author's work to be published in [7].

1.1 The Standard Model and Motivations for its Extension

1.1.1 The Standard Model

The Standard Model of particle physics is an effective field theory (EFT) which describes a wide array of physical phenomena quite accurately below several hundred giga-electron volts (GeV). It is a gauge theory based on the group $SU(3)_c \times SU(2)_L \times U(1)_Y$, spontaneously broken down to $SU(3)_c \times U(1)_{E\&M}$ by the vacuum expectation value (VEV) of a fundamental scalar field known as the Higgs field. The gauge couplings of the original gauge groups are referred to as g_3 , g_2 and g_1 , respectively, whereas the $E\&M$ gauge coupling is called e .

In addition to the gauge and Higgs bosons, there are three generations of matter fields which are Weyl fermions. They form representations of the gauge groups; those representations are labelled by three numbers (x, y, z) . x and y tell us the representation of $SU(3)_c$ and $SU(2)_L$, respectively, and z tells us the charge under the $U(1)_Y$. The field content of the Standard Model is presented in table 1.1.1.

The Standard Model possesses a perturbatively exact global $U(1)_B$ baryon number symmetry, where all of the quark doublets transform with charge 1 and all of the right-handed quarks transform with charge -1 . In addition, there is a perturbatively exact global $U(1)_L$ lepton number symmetry which behaves the same way as the

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Name	Field	Lorentz	SM Gauge
Gluon	G	Vector	$(8, 1, 0)$
W	W	Vector	$(1, 3, 0)$
B	B	Vector	$(1, 1, 0)$
Higgs	h	Scalar	$(1, 2, \frac{1}{2})$
Up-type Quarks	u_R^c, c_R^c and t_R^c	RH Spinor	$(\bar{3}, 1, -\frac{2}{3})$
Down-type Quarks	d_R^c, s_R^c and b_R^c	RH Spinor	$(\bar{3}, 1, \frac{1}{3})$
Quark Doublets	q_L	LH Spinor	$(3, 2, \frac{1}{6})$
Right-handed Leptons	e_R^c, μ_R^c and τ_R^c	RH Spinor	$(1, 1, 1)$
Lepton Doublets	ℓ_L	LH Spinor	$(1, 2, -\frac{1}{2})$

Table 1.1: The fields in the Standard Model.

baryon number symmetry. There is also a global custodial $SU(2)$ symmetry which is preserved by the Higgsing of $SU(2)_L \times U(1)_Y$.

There are three generations of all of the fermions, meaning that there are three identical copies of all of the fermions in the Standard Model with the exception of their Yukawa couplings to the Higgs field. In the absence of these Yukawa couplings, there would be an additional $SU(3)^3$ global “flavor” symmetry, under which the three generations of each of the types of fields formed triplets.

The Lagrangian of the Standard Model is as follows:

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$$\begin{aligned}
\mathcal{L} = & i q_L^\dagger \not{D} q_L + i u_R^{c\dagger} \not{D} u_R^c + i d_R^{c\dagger} \not{D} u_R^c + i \ell_L^\dagger \not{D} \ell_L + i e_R^{c\dagger} \not{D} e_R^c \\
& - \frac{1}{4} (\text{Tr}(G_{\mu\nu} G^{\mu\nu}) + \text{Tr}(W_{\mu\nu} W^{\mu\nu}) + B_{\mu\nu} B^{\mu\nu}) - \text{Tr} \left(\frac{\theta}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu} \right) \\
& + |D_\mu h|^2 + m_h^2 |h|^2 - \frac{\lambda}{2} |h|^4 - u_R^c Y_u h q_L - d_R^c Y_d h' q_L - e_R^c Y_e h' \ell_L
\end{aligned} \tag{1.1}$$

where $h' = i\sigma_2 h^*$ and $Y_{u,d,e}$ are 3×3 matrices in flavor-space, and $\tilde{G}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}$.

The higgs mass-squared is negative at $h = 0$, indicating the need to perturb around a different value of h . The potential is minimized at

$$|h| \equiv \frac{v}{\sqrt{2}} = \frac{m_h}{\sqrt{\lambda}} \tag{1.2}$$

We can use the $SU(2)$ symmetry to place this VEV entirely in the second component, breaking $SU(2)_L \times U(1)_Y$ down to $U(1)_{E\&M}$. We are left with the massless gauge boson of $U(1)_{E\&M}$, the photon, as well as massive gauge bosons W^\pm and the Z . There is a residual real scalar field known as the physical Higgs h_{phys} defined by $h_2 = \frac{1}{\sqrt{2}}(v + h_{phys})$ with a potential

$$V = \frac{1}{2} \lambda v^2 h_{phys}^2 + \frac{1}{2} \lambda v h_{phys}^3 + \frac{1}{8} \lambda h_{phys}^4 \tag{1.3}$$

The W and Z receive masses through the Higgs mechanism, and with the knowledge of their masses along with the physical Higgs mass $m_{h,phys} = \sqrt{\lambda} v = 125.9$ GeV [8], we can conclude that $v \approx 246$ GeV and $\lambda \approx .26$. We also note for reference the value of the Higgs mass parameter $m_h \approx 89$ GeV. The gauge couplings measured

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at M_Z are $g_1 \approx .352$, $g_2 \approx .652$ and $g_3 \approx 1.1$. The top and bottom yukawa couplings are fixed by measurements of their mass, and are measured to be $y_t \approx 1$ and $y_b \approx .025$. All other yukawa couplings are even smaller yet, and therefore their magnitudes are irrelevant for the purposes of this thesis.

In order to work in the mass eigenbasis for the quarks, one must diagonalize the yukawa matrices by acting on the u , d and l sector with unitary transformations. We refer to the pre-diagonalized basis as the “gauge eigenbasis” and the diagonalized basis as the “mass eigenbasis”. Crucially, the unitary matrices U_u and U_d which contribute to the diagonalization of Y_u and Y_d from left-handed quarks are not equal. Therefore, in the coupling of the W to the up- and down-type quarks, we obtain off-diagonal flavor couplings parameterized by the unitary CKM matrix U^{CKM} :

$$\begin{aligned} g\delta_{ij}\bar{d}_{\text{gauge}}^i W u_{\text{gauge}}^j &= g\delta_{kl}\bar{d}_{\text{mass}}^k (U_d^\dagger)_i^k W (U_u)_j^l u_{\text{mass}}^j \\ &\equiv gU_{ij}^{CKM}\bar{d}_{\text{mass}}^i W u_{\text{mass}}^j \end{aligned} \quad (1.4)$$

Note that gauge indices are suppressed. The CKM matrix has been experimentally measured very precisely, and the magnitudes of its numerical entries are known to be of order

$$U^{CKM} \sim \begin{pmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix} \quad (1.5)$$

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where ε is the sine of the Cabibbo angle $\sin \theta_c \approx .225$.

The Z boson is known to not mediate flavor-changing neutral currents (FCNCs). FCNCs occur in the SM at loop-level, but are suppressed by the GIM mechanism. FCNCs occur generically in BSM physics models, and therefore FCNCs will provide a powerful constraint as well as a powerful probe of new physics.

CP violation in the SM is due to the presence of a CP-violating phase in the CKM matrix. However, this phase can be moved into any given column of the CKM matrix by residual unitary transformations, and so any CP violation that occurs as a result must be sensitive to interference of all three generations of quarks. This involves using many off-diagonal elements in the CKM matrix, and so therefore transitions such as $K - \bar{K}$ happen with a very small amplitude in the SM. Again, these occur frequently in BSM physics models, and so CP violation will prove a powerful probe of new physics. Note that the parameter θ in the SM Lagrangian violates CP as well; it has been measured to be zero to rather high precision. Generic values of θ would lead to much larger CP-violation than is observed.

1.1.2 The Hierarchy Problem

The SM should be viewed as an effective field theory below some high-energy cutoff Λ , the scale of new physics. As an absolute maximum, this scale should be $M_{pl} \approx 1.2 \cdot 10^{19}$ GeV, the scale of quantum gravity. If Λ is up at the GUT or Planck scale, then there exists a large hierarchy between Λ and the electroweak scale

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v . This scale is unstable against radiative corrections coming from the standard model particles. For example, the top loop shown in fig. 1.1 induces a quadratically divergent contribution to the Higgs mass parameter:

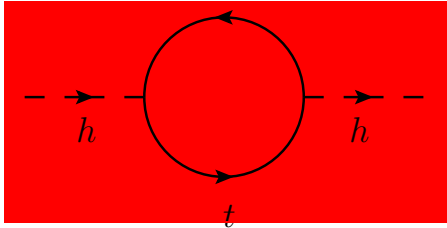


Figure 1.1: The SM top-loop contribution to the Higgs mass.

$$\begin{aligned} \delta m_h^2 &= i(-iy_t)^2 (-1)N_c \int \bar{d}^4l \frac{\text{Tr}(i(\not{l})i(\not{l}))}{(l^2 + i\varepsilon)^2} \\ &= -\frac{6y_t^2}{16\pi^2}\Lambda^2 \end{aligned} \tag{1.6}$$

Note that this computation was performed pre-electroweak symmetry breaking (EWSB), and that we work with two-component spinor notation, not four, as the SM is chiral. In the low-energy theory, one should introduce a counterterm $V \supset -\delta_{c.t.}|\phi|^2$ such that $m_h^2 = m_{h,0}^2 - \delta_{c.t.}$ to parameterize the effects of UV physics that have been integrated out. Therefore, $\delta_{c.t.}$ and therefore m_h^2 could naturally be $O(\Lambda^2)$. However, to get the correct physical Higgs mass, one must cancel the correction of all SM loops nearly entirely (but not completely) against the counterterm. In other words, it must be that m_h is related to Λ by

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$$m_{h,ren}^2 = (89 \text{ GeV})^2 = m_h^2 - \frac{6}{16\pi^2} y_t^2 \Lambda^2 + \dots \quad (1.7)$$

where Λ may be as large as 10^{19} GeV. This implies that one must “fine-tune” the UV-sensitive mass parameter against quadratic corrections arising from IR physics. Strictly speaking, this is not an experimental problem, but it is quite distasteful, and we have no reason to suspect that nature should have chosen such a particular value for the bare parameter. So this issue is dubbed the “hierarchy problem”; it is occasionally called the big hierarchy problem to distinguish it from the little hierarchy problem, to be discussed below.

The hierarchy problem is considered appalling in light of the postulate of *naturalness*, which asserts that the size of quantum corrections should not be larger than the physical values of the parameters; e.g. $|\frac{\delta m_h^2}{m_{h,phys}^2}| \lesssim 1$. One should not view naturalness as a *requirement* of a theory, but rather as a postulate to be tested, as it has frequently been a good guiding principle in the past. Indeed, credence is lent to the idea of the hierarchy problem being taken seriously by previous observations of new physics stepping in to resolve other hierarchy problems. There are many proposed non-tuned resolutions of the hierarchy problem, *all* of which involve introducing new physics at the TeV-scale in order to remove quadratic sensitivity to scales above the TeV-scale. We will discuss two of them further below.

1.1.3 Other Concerns with the SM

The SM has been tested very thoroughly on a variety of frontiers; on all accounts, the SM predictions have been verified and so one concludes that the SM is indeed very successful at describing the physics of our universe. Before we proceed with a discussion of the hierarchy problem, we pause to note features of our universe which seem to indicate the need for physics beyond the SM.

1: We have observed that we are surrounded by an approximately thermal bath of photons with a temperature $T_{CMB} = 2.726$ K. These photons are known as the “cosmic microwave background” (CMB) and are the result of recombination; protons and electrons settling down into hydrogen at $T_{rec} \sim .25$ eV. At this point, the free-streaming length of photons increased tremendously, allowing photons to free-stream to us today from all over the universe. Photons coming from opposite directions have been separated by many billions of light-years, and because they were not in causal contact, we have no reason to suspect that they should have the same temperature; yet they do. It has been proposed that if the universe were once very small and everything was in thermal equilibrium, then something known as “inflation” happened, increasing the scale factor of the universe by many orders of magnitude, then photons from opposite sides of the universe would remain at approximately equal temperatures. Minimal models of inflation involve introducing at least one new scalar field known as the “inflaton”, which is new physics.

2: There is now rather conclusive evidence that there exists “dark matter” (DM); a

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gravitating form of matter which is quite a bit more prevalent in our universe than SM particles. We know that it was “cold” (meaning $T \ll m$) when the temperature of our universe was comparable to the temperature of recombination due to measurements inferred from the CMB. The energy density stored in DM is known to be around five times that which is stored in SM fields.

3: Neutrinos in the SM are massless; observations of neutrino oscillations indicate the need for neutrino masses. In the absence of new fields, the lowest-dimension gauge-invariant operator one can write down which accomodates this is $hlhl$, which is dimension-five. The scale which is suppressed by is the scale at which the EFT containing this operator breaks down, indicating the need for new physics at this scale. Alternatively, one can introduce right-handed neutrinos ν_R^c to write down the dimension-four operator $\nu_R^c h \ell$, but the introduction of new fields is already new physics.

4: Recently, it was measured that our universe is accelerating. The vacuum energy of our universe V today is $\sim 10^{-120} M_{pl}^4$. The simplest explanation for this is a cosmological constant (CC) $\Lambda \sim 10^{-120} M_{pl}^2$. However, there is also a hierarchy problem associated with the CC. One might expect that its resolution involves new physics, much as the proposed resolutions of the Higgs hierarchy problem below involve new physics. Note that this problem is not to be treated on the same footing as the other problems discussed in this subsection, as the smallness of the CC is a theoretical rather than an observational problem, in that the measured value of the CC is also

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unnatural.

5: In order for us to exist in the universe, there must have been more matter than antimatter at the time of structure formation. The mechanism for changing the number densities of baryons and antibaryons to be unequal is known as baryogenesis. It is known that for baryogenesis to occur, three “Sakharov conditions” must be satisfied by the interactions responsible: baryon number violation, C and CP violation and the interactions must occur out of thermal equilibrium. Baryon number is a perturbatively exact symmetry in the SM, but nonperturbative processes such as “electroweak sphalerons” only respect $B - L$. However, it is believed that these nonperturbative effects are not large enough to generate the observed baryon asymmetry, indicating the need for a BSM physics explanation of baryogenesis.

6: Attempts to write down an action for a quantized theory of gravity that reproduces GR at long distances necessarily include terms which are nonrenormalizable by power-counting. Therefore, there is reason to suspect that new physics might come in at the Planck scale.

7: The measured value of the CP-violating parameter θ is consistent with 0, essentially due to a non-observation of CP-violating strong interactions. It is not clear why the parameter should be so small when it could have been anything $0 \leq \theta < 2\pi$. This is known as the “strong CP problem”, and explanations for it, such as the QCD axion, involve the introduction of new physics.

In short, there are many reasons to suspect that there may be physics beyond

the SM, and the question of whether nature has fine-tuned the electroweak scale is a paramount question to address.

1.2 Extensions of the SM

As alluded to in the previous section, there are a number of mechanisms which resolve the hierarchy problem. We describe two of them in somewhat more detail as the discussion is relevant for the content of this thesis.

1.2.1 The Minimal Supersymmetric Standard Model

Supersymmetry is a spacetime symmetry which relates bosons to fermions. The representation theory of the supersymmetric algebra leads one to discuss superfields on superspace, as discussed in the appendix. Supersymmetry leads one to a nonrenormalization theorem stating that the superpotential is not perturbatively renormalized, due to a degeneracy between bosonic and fermionic states, which in turn removes the quadratic sensitivity to higher scales. As we will explore shortly, supersymmetry is at best an approximate symmetry of nature at low energies; however, in theories with *softly* and *spontaneously* broken supersymmetry, quadratic corrections are cut off at the scale of SUSY-breaking, as above that scale, supersymmetry is restored and the nonrenormalization theorem ensures the cancellation of quadratic divergences. Therefore, if supersymmetry were softly and spontaneously broken at the TeV-scale, the

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electroweak scale would be natural. Note, however, that logarithmic divergences may still be present in such theories, but in the cases of interest, these do not constitute a fine-tuning of the parameters of the theory.

The hierarchy problem is therefore solved in these sorts of supersymmetric models; the mass parameters of bosons are related by supersymmetry to the mass parameters of fermions, but the latter do not suffer from a hierarchy problem as quantum corrections give rise to logarithmic rather than quadratic divergences. However, in order to implement supersymmetry as a resolution to the hierarchy problem, one must first add in the appropriate fields such that the SM admits a representation on superspace. This process of “supersymmetrization” introduces supersymmetric partners of the gauge bosons (“gauginos”) and fermions (“sfermions”), as well as the partner of the Higgs (the “Higgsino”). In addition, because the Higgsino is not vectorlike, it contributes to a number of gauge anomalies, causing the model to be sick. To resolve this issue, one adds in another Higgs multiplet with the opposite hypercharge to cancel the gauge anomalies. These multiplets are known as the Higgs-up (H_u) and Higgs-down (H_d) multiplets, where the Higgs boson that has been observed at the LHC is to be identified with a linear combination of these. In doing so, one obtains the matter content for the Minimal Supersymmetric Standard Model (MSSM). The MSSM is reviewed in, e.g. [9, 10].

The MSSM contains the following superfields, omitting generational indices:

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$$\begin{aligned}
Q &\equiv \begin{pmatrix} U \\ D \end{pmatrix} \equiv (\tilde{q}_L, q_L) \equiv \left(\begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}, \begin{pmatrix} u_L \\ d_L \end{pmatrix} \right) \\
U^c &\equiv (\tilde{u}_R^c, u_R^c) \\
D^c &\equiv (\tilde{d}_R^c, d_R^c) \\
L &\equiv \begin{pmatrix} N \\ E \end{pmatrix} \equiv (\tilde{\ell}_L, \ell_L) \equiv \left(\begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}, \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \right) \\
E^c &\equiv (\tilde{e}_R^c, e_R^c) \\
H_u &\equiv (h_u, \tilde{h}_u) \\
H_d &\equiv (h_d, \tilde{h}_d) \\
V_1 &\equiv (B_\mu, \lambda_1) \\
V_2 &\equiv (W_\mu, \lambda_2) \\
V_3 &\equiv (G_\mu, \lambda_3)
\end{aligned} \tag{1.8}$$

The superpotential of matter superfields in the MSSM can be written as

$$W = Y_u U^c H_u Q - Y_d D^c H_d Q - Y_e E^c H_d L + \mu H_u H_d \tag{1.9}$$

where again, flavor indices are implicit. Since SUSY is not respected in nature (e.g. there is no selectron with $m_{\tilde{e}} = 511 \text{ keV}$), it must be spontaneously broken. Such a breaking introduces $O(100)$ free parameters (“soft terms”) into the Lagrangian. As a general notation point, terms that involve three scalars are known as A -terms

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(e.g. $\mathcal{L} \supset -A_t \tilde{t}_R^c h_u \tilde{q}_L$). Terms with two non-identical scalars are known as B -terms. The most important B -term for our purposes will be $B\mu h_u h_d$. Sociologically speaking, people often prefer to treat “ $B\mu$ ” as one dimension-two variable, rather than a product of two dimension-one variables. Gaugino masses are written m_i , and there are soft mass terms as well.

We will see in detail in the next chapter how spontaneously broken SUSY can solve the hierarchy problem; for now we suffice it to note that when one also includes the supersymmetric partner of the top loop shown above in fig. 1.1 (the “stop loop” in fig. 1.2), one obtains instead the following result for the loop correction to the Higgs mass parameter:

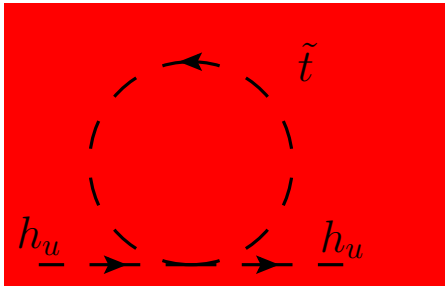


Figure 1.2: The stop-loop contribution to the Higgs mass.

$$\delta m_{h_u}^2 = -\frac{3}{4\pi^2} y_t^2 m_{\tilde{t}}^2 \ln \left(\frac{\Lambda}{m_{\tilde{t}}} \right) \quad (1.10)$$

We see that the quadratic divergence has been cut off at the scale above which SUSY has been restored; $m_{\tilde{t}}$. Provided $m_{\tilde{t}}$ is $O(v)$, there is no fine-tuning associated with one-loop top corrections to the Higgs mass parameter. We refer to this scenario

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as “weak-” or “low-scale SUSY breaking”.

Of course, a resolution to the hierarchy problem is not the only reason to study supersymmetry in nature; the MSSM gauge coupling unification is much more accurate than in the SM. The MSSM offers a natural dark-matter candidate, the lightest supersymmetric partner (LSP), stable due to R -parity (to be discussed below). A popular candidate for a self-consistent theory of quantum gravity, superstring theory, is supersymmetric, offering the idea that perhaps somewhere in the UV, SUSY is a good symmetry of nature. All of these reasons and more motivate the study of the MSSM.

Supersymmetric theories generally possess an R -symmetry; in the MSSM only a \mathbb{Z}_2 subgroup of it is preserved (“ R -parity”). The action of the \mathbb{Z}_2 is to send all SM fields to themselves and all superpartners to minus themselves (treating both h_u and h_d as SM fields). In the MSSM, R -parity is imposed because those supersymmetric terms which violate R -parity,

$$W_{RPV} = \lambda LLE^c + \lambda' QLD^c + \kappa LH_u + \lambda'' U^c D^c D^c \quad (1.11)$$

lead to rapid proton decay when all terms are turned on. The first three terms break the lepton number symmetry of the MSSM and are called “lepton-number violating” (LNV) terms, whereas the last term breaks the baryon number symmetry and is called the “baryon-number violating” (BNV) term. These three λ s have suppressed flavor indices which are antisymmetric about superfields related by a flavor transformation

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(e.g. $\lambda'' = \lambda''_{ijk} = \lambda''_{i[jk]}$). Protons can decay to SM particles when both baryon and lepton number are violated, and so turning on either BNV *or* LNV terms is safe. Note that the residual $U(1)_L$ in the case of BNV or $U(1)_B$ in the case of LNV symmetry as well as the nonrenormalization theorem ensure that these couplings are not generated in the Lagrangian, supersymmetrically or nonsupersymmetrically, at loop level. Generally speaking, the R -parity violating (RPV) MSSM is not considered as relevant to our universe due to the loss of the dark matter candidate (because all superpartners become unstable), difficulties with embedding the RPV terms in simple GUT models and the compounding of the MSSM flavor problems, to be discussed below.

Although the MSSM addresses the hierarchy problem superbly, it comes with its own set of problems; given the proliferation of flavor-breaking and CP-violating parameters, generic SUSY models tend to come with large FCNCs and strong CP-violation, in gross disagreement with experiment. Another issue is that to be phenomenologically viable, $|\mu|^2$ (a SUSY-preserving parameter-squared) and $B\mu$ (a SUSY-breaking parameter) must be of comparable sizes. Generically there is no good reason to suspect that this would happen, and explaining that they are the same size by coincidence is known as the $\mu - B\mu$ problem.

Supposing we lived in a world where the hierarchy problem were solved by supersymmetry, then one would expect a plethora of superpartners at the SUSY-breaking scale, as well as lots of other exciting phenomenological consequences. However, no

such evidence for supersymmetry has been found thus far in nature.

1.2.2 Warped Extra Dimensions

Warped extra dimensions can provide another solution to the hierarchy problem [11]. The crucial idea is that we might “live” on a 3-brane (like the sort that arises in string theory), but the universe is actually higher-dimensional, with strong gravitational fields in the “bulk” of the space. Gravity famously redshifts light leaving a massive body, lowering the energy of the photons. By analogy, the strong gravitational fields of the bulk might be responsible for “redshifting” very high, natural energy scales *exponentially quickly*, potentially down many orders of magnitude. By imagining that the Higgs boson lived a small ways down the gravitational field, it is possible to make its (Planck-scale) physics instead play out at $O(100)$ GeV, as we will see in more detail below.

Suppose the universe were five-dimensional (described by coordinates x^μ and z) with 5d Planck mass M , but there are two branes located at $z = 0$ and $z = \ell$, called the UV and IR branes, respectively, for reasons to become clear soon. The topology here is taken to be that of $\mathbb{R}^{1,3} \times S^1/\mathbb{Z}_2$, so this is an orbifold. The action for this reads

$$S = \int d^4x dz \sqrt{g} (2M^3 R - \Lambda - T_{UV} \delta(z) - T_{IR} \delta(z - \pi r)) \quad (1.12)$$

Provided the tensions and bulk CC are related by $\Lambda = kT_{UV} = -kT_{IR} = 24M^3 k^2$

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with some $k < M$ (so that we trust the classical approximation), there exists a static solution to the Einstein equations with the AdS_5 metric:

$$ds^2 = e^{-2k|z|} \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \quad (1.13)$$

where the 4d Planck mass M_{pl} is related to M and k by

$$M_{pl}^2 \approx \frac{M^3}{k} \quad (1.14)$$

so that we might expect $M \sim k \sim M_{pl}$. This is known as the Randall-Sundrum 1 (RS1, or simply RS) framework. Warped phenomenology is reviewed, for example, in [12–14].

How does this help with the hierarchy problem? Consider the action for a “Higgs” scalar field that is confined to the IR brane:

$$S = \int d^4x dz \sqrt{g} \delta(z - \ell) \left(g^{\mu\nu} D_\mu \bar{h} D_\nu h + m_h^2 |h|^2 - \frac{\lambda}{2} |h|^4 \right) \quad (1.15)$$

The effective 4d action is obtained by integrating over the extra dimension:

$$S_{eff} = \int d^4x e^{-4k\ell} \left(e^{2k\ell} D_\mu \bar{h} D^\mu h + m_h^2 |h|^2 - \frac{\lambda}{2} |h|^4 \right) \quad (1.16)$$

The kinetic term is not properly normalized, so we redefine $h \rightarrow e^{k\ell} h$ so that the action becomes

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$$S_{eff} = \int d^4x \left(D_\mu \bar{h} D^\mu h + e^{-2k\ell} m_h^2 |h|^2 - \frac{\lambda}{2} |h|^4 \right) \quad (1.17)$$

so that the effective mass-squared of the higgs is not m_h^2 but rather $e^{-2k\ell} m_h^2$! For $m_h \sim M_{pl}$ and $k\ell \sim 40$, we can easily generate the weak scale. This is referred to as a warping or redshifting of scales; the further “down the throat” we live (the larger z we’re at), the more redshifted energy scales are.

One of the most important aspects of RS models are that because they are models in AdS_5 , they are related by the AdS/CFT correspondence to four-dimensional models with approximately conformal fixed points. Specifically, these are strongly-coupled models that go by the name of “composite Higgs models”, as the Higgs we just described is AdS/CFT dual to a scalar which is a “pion” of the strongly coupled confining gauge group. Spontaneous conformal symmetry breaking, and therefore the presence of confinement in the dual theory is related to the presence of an IR-brane in RS; therefore we expect the confinement scale to be related to the position of the IR-brane by $\Lambda_{comp} \sim e^{-k\ell} M$. Being able to utilize the power of AdS/CFT is greatly advantageous to understanding what is happening in such models.

Attempting to embed the SM in RS is feasible with the assistance of the AdS/CFT dictionary. However, one does run into a number of problems. First, fluctuations in the 5d metric lead to the size of the extra dimension ℓ being dynamical. It is a “modulus” of the theory; i.e. $\ell(x)$ has no potential, and thus there is nothing forcing its VEV to be $\sim 40/k$. One needs to “stabilize” the size of the extra dimension in

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an untuned fashion. Secondly, gauge theory in $5d$ is naïvely nonrenormalizable; the covariant derivative $D \sim \partial - igA$ has mass-dimension 1, and inspecting the kinetic term $(\partial A)^2$ for gauge bosons tells us that they have mass-dimension $\frac{3}{2}$, implying that g has mass-dimension $-\frac{1}{2}$. As we need 5d gauge theory as an ingredient to write down the warped SM, and we require $g \sim O(1)/\sqrt{k}$, we consequently need a UV-completion right away in 5d, implying new 5d physics states which can themselves redshift down to the compositeness scale. It is known that nonsupersymmetric RS can indeed be realized in type IIB superstring theory [15]. Finally, one must consider a nonminimal 5d gauge sector in order to prevent against large corrections to electroweak precision observables.

Supposing we indeed did live in a universe with warped extra dimensions, we would expect to see lots of exciting new physics around the compositeness scale; KK excitations of all SM fields, the KK graviton, and perhaps even the excitations of the fields (or strings) which UV-complete RS. However, much like with the MSSM, we have seen no evidence for any of these exciting new physics observables.

1.2.3 The Little Hierarchy Problem

In the past few years, we have been collecting evidence that if one believes firmly that the hierarchy problem has a solution, then there is in fact a “little hierarchy problem” (LHP). This problem is essentially the non-observation of any evidence that the hierarchy problem has indeed been solved by nature. In order to eliminate

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fine-tuning, one generically expects new physics to have shown up by $O(100 \text{ GeV})$, and certainly no later than $O(1 \text{ TeV})$. However, despite the exhaustive set of experimental searches that have been performed, we have yet to find compelling evidence for the existence of new particles which solve the hierarchy problem. We have probed the SM on many frontiers; we have tested its CP physics, its flavor physics; we've performed electroweak precision measurements and performed direct searches at colliders, and yet none of these have turned up any evidence for new physics.

One might suppose that the new physics is right around the corner ($O(10 \text{ TeV})$), but pushing the SUSY-breaking scale or the compositeness scale reintroduces fine-tuning at the percent-level or worse. The LHP is shared by all solutions to the hierarchy problem, and none offer resolutions of the LHP by themselves. Various solutions to the LHP exist; Twin Higgs, Little Higgs and superlatives and natural SUSY are a few examples of solutions. We will focus on natural SUSY; in this context, the solution is the following: take the *minimal* amount of supersymmetric physics required to stabilize the little hierarchy below 10 TeV, and then push all of the *other* states up to higher energies. This strategy crucially allows for the little hierarchy to be stabilized, while removing various experimental constraints on supersymmetric models, as we will see in great detail in this thesis. However, within this framework there are still various nontrivial opportunities to discover even this minimal set of new physics.

In this thesis, we will explore phenomenological consequences and self-consistency

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of natural SUSY as a solution to the LHP. There is no reason a priori to suspect that perturbative SUSY must be *the* solution to the *big* hierarchy problem, and so we work in a bottom-up framework in our study of natural SUSY, asking what physics we absolutely *need* for the stabilization of the little hierarchy. We then move on to the possibility that the aforementioned natural SUSY spectrum might fall into our laps *naturally* when studying supersymmetric RS models exhibiting a mechanism known as “accidental SUSY”. In this mechanism, one breaks SUSY at high scales, but a part of the SUSY “accidentally” redshifts down the throat and is present in the low-energy spectrum. Finally, although it is known that warped throats similar to RS can be embedded in string theory, it is not clear ahead of time that we can find UV-completions of RS models which *also* exhibit accidental SUSY. Therefore, we offer a proof-of-principle that accidental SUSY can be realized in noncompact IIB string models.

Chapter 2

Natural SUSY: The Story Below

10 TeV

In this chapter, we discuss the mechanism of natural SUSY, and its implications for the little hierarchy problem. As mentioned in the previous chapter, the goal is to work from the bottom up; impose a moderate cutoff of 10 TeV on the theory, and ask what *minimal* amount of supersymmetric physics is *needed* to ensure radiative stability of the theory up to the cutoff. This constitutes a resolution to the little hierarchy problem as the minimal amount of new physics, lacking as spectacular of signatures as other points in MSSM parameter space, can elude searches at the LHC and in other experiments. Ref. [16] dubbed this kind of structure “effective SUSY”; in this thesis we will refer to it as “natural SUSY”, as we shall explain later. Since [16, 17], a number of quite different approaches to far-UV dynamics have converged on such

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a “more minimal” spectrum at accessible energies [3, 18–20]. The significance of the 10 TeV scale is that almost all experiments, up to and including the LHC, only have sensitivity to new physics $\lesssim 10$ TeV, be it through direct searches or virtual effects. In this regard, flavor physics tests are exceptional in probing vastly higher scales and consequently they require special consideration.

What we will find, as mentioned in [21, 22] and developed in [17], is that superpartners with $O(1)$ coupling to the SM are required for naturalness up to 10 TeV. In natural SUSY, these superpartners are the stops, sbottoms, gauginos and the Higgs sector, but with other squarks and sleptons heavy and beyond reach of the LHC. This particle content satisfies the criterion of being natural up to 10 TeV. The omitted superpartners may play a crucial role in weak scale stability up to much higher inscales, but this is outside the scope of the effective theory and outside the grasp of the LHC. We will investigate the range of squark, gaugino and Higgsino masses for which the electroweak scale is natural, as well as explore indirect constraints on this scenario, in particular from flavor and CP tests. We will see that from a bottom-up perspective, R-parity violation is very well-motivated. Finally, we will explore the phenomenological differences between Majorana and Dirac gauginos.

This chapter is borrowed heavily from the author’s work in [4], and the research was performed in collaboration with Raman Sundrum, Andrey Katz and Scott Lawrence.

2.1 Introduction and General Concerns

Naturalness is our driving concern when building our BSM framework. However, one must weigh naturalness against those general concerns of the SUSY paradigm which at least partially relate to very high energies, such as:

- The SUSY flavor problem
- Grand unification
- Proton stability and R-parity
- Superpartner dark matter candidates
- SUSY-breaking dynamics
- The Higgs mass

In this chapter, we study minimal effective theories that arise from insisting on naturalness of the low-energy theory. They are “minimal” in terms of the particle content and parameter space of \mathcal{L}_{eff} . This does *not* imply, however, that their UV-completions are also minimal in some way. Conversely, the MSSM is a minimal visible sector from the high-energy perspective, but is non-minimal in the sense that matters to the LHC effective theory and phenomenology, as we will review.

Of course, there is no guarantee that at accessible energies new SUSY physics will be turn out to be minimal. Rather, we study minimal LHC-effective theories for three reasons:

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- They represent possible SUSY phenomenology, and there do exist UV SUSY dynamics that match onto them
- A part of the natural parameter space remains open, and yet is discoverable by the LHC
- Minimal models in any arena of exploration represent an important departure point for thinking more broadly.

In this chapter, we will take a relatively UV-agnostic approach to the minimal effective theory at LHC energies than has been previously considered. We do not do this blindly, but only after discussion of the general SUSY concerns listed above. We will argue that modern developments in model-building and SUSY field theory have proliferated the range of UV options that relate to these issues, and it is precisely for this reason that we advocate thinking more modularly about them, and with less commitment to any one UV plot. Our goal will be to use electroweak naturalness, flavor constraints, minimality, and earlier searches as a guide to the LHC phenomenology, to be discussed in chapter 3. We will use this platform to study the LHC phenomenology in more detail, and in chapter 3 and in future work to broaden and help optimize experimental search strategies. We will adopt the name “natural SUSY” to refer to this minimalist and UV-agnostic approach to the LHC-effective theory. Our study of natural SUSY coincides with the accumulation of significant LHC data. However, there are earlier collider studies relevant to natural SUSY on

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which our work expands, such as [23–29].

The chapter is organized as follows: in section 2.2, we derive the minimal natural SUSY Lagrangian subject to electroweak naturalness with a cutoff of ~ 10 TeV. Here we impose R-parity and make the useful idealization that the third generation does not mix with the first two generations. We also make the standard assumption that the Higgsino mass arises from a supersymmetric μ term. In section 2.3, we perform the same exercise but with a cutoff of only ~ 1 TeV, in a sense increasing our agnosticism towards what lies above the early 7 and 8 TeV LHC reach. One possibility, but not the only one, is that this 1 TeV effective theory derives straightforwardly from the 10 TeV effective theory of section 2.2. In sections 2.4 and 2.5, we study the possibility that Higgsinos obtain mass from soft SUSY breaking rather than a μ term, and we write an even more minimal set of effective Lagrangians with 10 TeV and 1 TeV cutoffs. In section 2.6 we put back consideration of third-generation mixing, and review and extend the constraints provided by low-energy flavor and CP tests. We emphasize the considerable safety of the natural SUSY scenario. In section 2.7, we make the case for R-parity violation as a very plausible option, write the natural SUSY R-parity violating interactions, and discuss some of the low-energy constraints. In section 2.8, we discuss the interesting possibility of Dirac gauginos and how this can considerably affect the collider phenomenology and low-energy constraints.

While the work this chapter was derived from was being completed, we became aware of three other groups pursuing partially overlapping work [30–32].

2.2 Natural SUSY $\lesssim 10$ TeV with Light Higgsinos

Let us start with the MSSM field content and ask which superpartners are minimally needed in order to maintain electroweak naturalness below 10 TeV, roughly the collider reach in the years to come. We will not ask here what physics lies above this scale. Therefore at the technical level, $\Lambda_{UV} \equiv 10$ TeV provides the cutoff for any UV divergences encountered in the effective theory, and this allows us to estimate electroweak fine-tuning and check where in parameter space natural SUSY solves the little hierarchy problem of the SM.

SM particles with order one couplings to the Higgs boson must certainly have superpartners in the effective theory because they would otherwise give rise to quadratically divergent Higgs mass-squared contributions at one loop, $\sim \Lambda_{UV}^2/(16\pi^2)$, big enough to require significant fine-tuning. In order to supersymmetrically cancel these divergences, the effective theory must therefore include the left-handed top and bottom squarks, $\tilde{q}_L \equiv (\tilde{t}_L, \tilde{b}_L)$, and the right-handed top squark, \tilde{t}_R , as well as the up-type Higgsino, \tilde{h}_u , and electroweak gauginos, $\lambda_{1,2}$.

Considerations beyond SUSY itself imply that we need to retain even more superpartners. Electroweak gauge anomaly cancellation implies that \tilde{h}_u must be accompanied by \tilde{h}_d in the effective theory. Indeed, one might have anticipated that down-type Higgs bosons, h_d , are required anyway to give masses to the down-type fermions, and

that \tilde{h}_d provide the required superpartners.¹ With the h_d bosons present in the effective theory, there is a new quadratic divergence, even in the supersymmetric limit, in the form of a (supersymmetric) hypercharge D -term. It is associated by supersymmetry with the mixed hypercharge-gravity triangle anomaly. The quadratic divergence vanishes only if $\text{Tr}(Y) = 0$, where Y is the hypercharge charge matrix over the scalar fields of the effective theory. With the field content described, including h_d , this condition is not satisfied, and the theory remains unnatural despite superpartners for the main players in the SM. Vanishing $\text{Tr}(Y)$ can be arranged by retaining the right-handed bottom squark, \tilde{b}_R^c , within the effective theory.

For the most part, two-loop quadratic divergences $\sim \Lambda_{UV}^2/(16\pi^2)^2$ are not important for Higgs naturalness, with a cutoff as low as 10 TeV. But the QCD coupling is an exception. In particular, the $\tilde{q}_L, \tilde{t}_R^c$ masses must themselves be so light in order to protect Higgs naturalness at one loop order, that they suffer from their own naturalness problem due to one-loop mass corrections from QCD. This one loop QCD destabilization of the squarks, hence two-loop destabilization of the Higgs, requires the gluino, λ_3 , to be in the effective theory.

In this way, the effective theory has complete supermultiplets,

¹We proceed with this logic in this section, although there is a loop-hole whereby h_u can provide down-type fermion masses in the effective theory, and h_d bosons are not needed. We discuss this option in sections 2.4 and 2.5.

$$\begin{aligned}
 Q &\equiv \begin{pmatrix} T \\ B \end{pmatrix} \equiv (\tilde{q}_L, q_L) \equiv \left(\begin{pmatrix} \tilde{t}_L \\ \tilde{b}_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \right) \\
 \bar{T} &\equiv (\tilde{t}_R^c, t_R^c) \\
 \bar{B} &\equiv (\tilde{b}_R^c, b_R^c) \\
 H_u &\equiv (h_u, \tilde{h}_u) \\
 H_d &\equiv (h_d, \tilde{h}_d) \\
 V_1 &\equiv (B_\mu, \lambda_1) \\
 V_2 &\equiv (W_\mu, \lambda_2) \\
 V_3 &\equiv (G_\mu, \lambda_3)
 \end{aligned} \tag{2.1}$$

2.2.1 Effective Lagrangian, Neglecting Third-Generation Mixing

Above, we have introduced squarks belonging to only the “third generation”, and yet this notion is slightly ambiguous because generation-numbers are not conserved, even in the SM. However, CKM mixing involving the third generation is at least highly suppressed, so we will begin by considering the “zeroth order” approximation in which third-generation number is exactly conserved. For most purposes in LHC studies of the new physics, this approximation is sufficient. But for complete realism and to check the viability of the theory in the face of very sensitive low-energy flavor

constraints, the extra subtlety of third-generation mixing must be taken into account. We defer this discussion until section 2.6. For now, this mixing is formally “switched off”. Further, we will impose R-parity on natural SUSY, and defer the discussion of possible R-parity violating (RPV) couplings to section 2.7.

With the field content described above, the effective Lagrangian is given by

$$\begin{aligned}
 \mathcal{L}_{eff} = & \int d^4\theta K + \left(\int d^2\theta \left(\frac{1}{4} \mathcal{W}_\alpha^2 + y_t \bar{T} H_u Q + y_b \bar{B} H_d Q + \mu H_u H_d \right) + \text{h.c.} \right) \\
 & + \mathcal{L}_{kin}^{light} - \left(\bar{u}_R Y_u^{light} h_u \psi_L + \bar{d}_R Y_d^{light} h_d \psi_L + \text{h.c.} \right) + \mathcal{L}_{lepton} \\
 & - m_{\tilde{q}_L}^2 |\tilde{q}_L|^2 - m_{\tilde{t}_R^c}^2 |\tilde{t}_R^c|^2 - m_{\tilde{b}_R^c}^2 |\tilde{b}_R^c|^2 - m_{h_u}^2 |h_u|^2 - m_{h_d}^2 |h_d|^2 \\
 & - \left(m_{i=1,2,3} \lambda_i \lambda_i + B \mu h_u h_d + A_t \tilde{t}_R^c h_u \tilde{q}_L + A_b \tilde{b}_R^c h_d \tilde{q}_L + \text{h.c.} \right) \\
 & + \mathcal{L}_{hard} + \mathcal{L}_{non-ren.}
 \end{aligned} \tag{2.2}$$

where the first line is in superspace/superfield notation, while the remaining lines are in components. Here, K is the standard gauge-invariant Kähler potential for the chiral superfields of Eq. (2.1), and $\mathcal{L}_{kin}^{light}$ denotes the standard gauge-invariant kinetic terms for the light SM quarks (that is, not the top and bottom), $u_R, d_R, \psi_L \equiv (u_L, d_L)$. \mathcal{L}_{lepton} denotes all terms involving leptons, with Yukawa couplings to h_d (neglecting neutrino mass terms). The super-field strength tensors are implicitly summed over all three gauge groups of the standard model, both here and throughout the chapter. Even the second line can be thought of as the result of starting from the supersymmetric MSSM, but then deleting all superpartners for light SM fermions. As

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mentioned above, we ignore the third generation mixing with the first two generations (until section 2.6). The third and fourth lines are soft SUSY breaking terms for the superfields of the effective theory.

The absence of superpartners for the light fermions will necessarily induce *hard* SUSY-breaking divergences at one-loop order. To renormalize these, we must include hard SUSY breaking couplings into the effective Lagrangian, and naturalness dictates that the renormalized couplings be at least of one-loop strength, $\gtrsim 1/(16\pi^2)$. These couplings are included in the last line, in \mathcal{L}_{hard} . Such couplings can then appear within one-loop Higgs self-energy diagrams, yielding two-loop sized quadratic divergences, $\gtrsim \Lambda_{UV}^2/(16\pi^2)^2$. While this is acceptable from the viewpoint of naturalness, we see that we cannot tolerate order one hard breaking couplings. UV completions of natural SUSY theory can contain mechanisms to naturally yield such non-vanishing, but suppressed, hard breaking terms, for example [3, 19]. Because the hard breaking is necessarily small, it is largely negligible for early LHC phenomenology. On the other hand, at a later stage of exploration, measuring hard SUSY breaking such as a difference between gauge and gaugino couplings may provide a valuable diagnostic.

Natural SUSY is expected to arise from integrating out heavy physics above 10 TeV, some of which is crucial in solving the hierarchy problem to much higher scales. It should therefore be a non-renormalizable effective theory, with higher-dimension interactions suppressed by ~ 10 TeV or more. These are contained in $\mathcal{L}_{non-ren}$ on the last line. Again, these will be largely irrelevant for early LHC phenomenology,

but can be very important in precision low-energy experiments, such as CP or flavor tests. The most stringent of such tests imply that at least some non-renormalizable interactions have to be suppressed by effective scales much beyond 10 TeV. Again, there are UV completions of natural SUSY which possess natural mechanisms to explain this required structure.

2.2.2 Higgs Mass

The experimental measurement of the physical Higgs scalar provides some of the most stringent constraints on weak scale SUSY. The dominant couplings of our effective Lagrangian are just those of the MSSM, so the electroweak symmetry-breaking and Higgs-mass predictions are essentially the same. This is problematic because naturalness dictates stops lighter than a few hundred GeV, while the physical Higgs mass constraints require higher stop masses. One difference with the high-scale MSSM is that in natural SUSY we have hard SUSY breaking couplings, among which can be Higgs quartic couplings which ultimately contribute to the physical Higgs mass. However, these contributions are modest, just a few GeV, since the hard SUSY-breaking couplings must be suppressed for electroweak naturalness. Instead, sizeable upward contributions to physical Higgs mass require new particle content beyond the MSSM (see e.g. [33] and references therein). For example, this is readily accomplished by adding a chiral superfield gauge singlet S to the effective theory [34–37],

$$\begin{aligned} \delta\mathcal{L}_{eff} = & \int d^4\theta |S|^2 + \int d^2\theta \left(\kappa S H_u H_d + \frac{1}{2} \sigma S^2 \right) + \text{h.c.} \\ & - m_s^2 |s|^2 + \text{other soft terms} \end{aligned} \quad (2.3)$$

which contains a new contribution to the Higgs quartic couplings, $\sim \kappa^2$. The soft scalar mass-squared term m_s^2 can be $O(\text{TeV}^2)$ without destabilizing EWSB. It can also ensure that the singlet does not acquire a vacuum expectation. In principle, in natural SUSY with a 10 TeV cutoff, we must commit to which type of physics, $\delta\mathcal{L}_{eff}$, accounts for an acceptable physical Higgs mass. But for early LHC superpartner searches, the details of $\delta\mathcal{L}_{eff}$ need not be relevant, as the new particles can lie above 1 TeV. In such cases, the new physics is just a “black box” which gives viable physical Higgs masses. Indeed, in writing natural SUSY theories with a lower ~ 1 TeV cutoff, we will see that we can formally imagine having integrated out the new physics responsible for new Higgs quartic couplings.

2.2.3 Naturalness in Natural SUSY

Here, we assemble the electroweak naturalness constraints on natural SUSY, thereby giving a rough idea of the motivated regions of its parameter space. For this purpose, we will compute various independent corrections to the h_u mass-squared, and simply ask them to be $\lesssim (200 \text{ GeV})^2$ for naturalness. We will compute these corrections *before* EWSB. Contributions sensitive to EWSB are typically $\sim O((100 \text{ GeV})^2)$,

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and therefore typically do not compromise naturalness. Given the intrinsically crude nature of naturalness arguments, we see no merit in a more refined analysis.

We begin with a classical “tuning” issue. The μ term gives a supersymmetric $|\mu|^2$ contribution to the Higgs mass-squareds. While the soft terms also contribute to Higgs mass-squareds, naturalness forbids any fine cancellations, so therefore by the criterion stated above,

$$|\mu| \lesssim 200 \text{ GeV} \quad (2.4)$$

This same parameter then also plays the role of the Higgsino mass parameter, ensuring relatively light charginos and neutralinos in the superpartner spectrum. (Of course, after EWSB, these physical states may also contain admixtures of electroweak gauginos.)

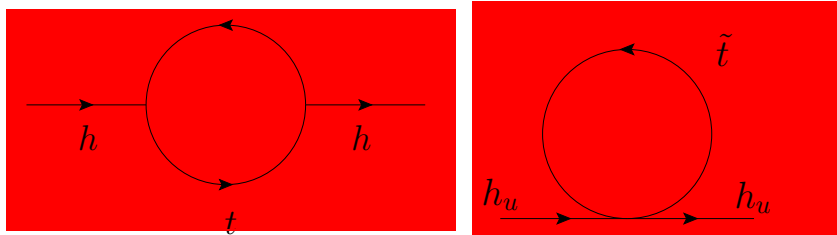


Figure 2.1: Higgs mass corrections

Next, we turn to quantum loops. We assume that \tilde{q}_L, \tilde{t}_R have approximately the same mass, $m_{\tilde{t}}$, for simplicity, and we also neglect the μ and A -terms. We work pre-EWSB since we are concerned with sensitivity to parametrically higher scales. We now evaluate the diagrams in figure 2.1. Note that we ignore finite terms, assume the stops are the same mass for simplicity and assume $m_{\tilde{t}} \ll \Lambda$.

$$\begin{aligned}
 \delta m_{h,t}^2 &= i(-iy_t)^2(-1)N_c \int \bar{d}^4l \frac{\text{Tr}(i(\not{l})i(\not{l}))}{(l^2 + i\varepsilon)^2} \\
 &= -\frac{6y_t^2}{16\pi^2}\Lambda^2
 \end{aligned} \tag{2.5}$$

$$\begin{aligned}
 \delta m_{h,\tilde{t}}^2 &= i(2)(-iy_t^2)N_c \int \bar{d}^4l \frac{i}{l^2 - m_{\tilde{t}}^2 + i\varepsilon} \\
 &= \frac{6y_t^2}{16\pi^2} \left(\Lambda^2 - 2m_{\tilde{t}}^2 \ln \left(\frac{\Lambda}{m_{\tilde{t}}} \right) \right)
 \end{aligned} \tag{2.6}$$

Summing these, we find that the $m_{h_u}^2$ parameter receives the following correction:

$$\delta m_{h_u}^2 = -\frac{3y_t^2}{4\pi^2} m_{\tilde{t}}^2 \ln \left(\frac{\Lambda}{m_{\tilde{t}}} \right) \tag{2.7}$$

Naturalness therefore requires, very roughly,

$$m_{\tilde{t}} \lesssim 400\text{GeV} \tag{2.8}$$

There are also electroweak gauge/gaugino/Higgsino one-loop contributions to Higgs mass-squared. Again, working before electroweak symmetry breaking (gaugino-Higgsino mixing) and just looking at the stronger $SU(2)_L$ coupling, the Higgs self-energy diagrams are in figure 2.2. There are nontrivial gauge index contractions in these diagrams; in all four, they contract to return the quadratic Casimir C_2 .

$$\begin{aligned}
 \delta m_{h,\tilde{h}-\tilde{W}}^2 &= i(-i\sqrt{2}g_2)^2 C_2(-1) \int \bar{d}^4l \text{Tr} \left(\frac{\not{l}}{l^2 - m_{\tilde{h}}^2 + i\varepsilon} \frac{\not{l}}{l^2 - m_{\tilde{W}}^2 + i\varepsilon} \right) \\
 &= -\frac{4g_2^2 C_2}{16\pi^2} \left(\Lambda^2 - 2(m_{\tilde{h}}^2 + m_{\tilde{W}}^2) \ln \left(\frac{\Lambda}{m_{\tilde{W}}} \right) \right)
 \end{aligned} \tag{2.9}$$

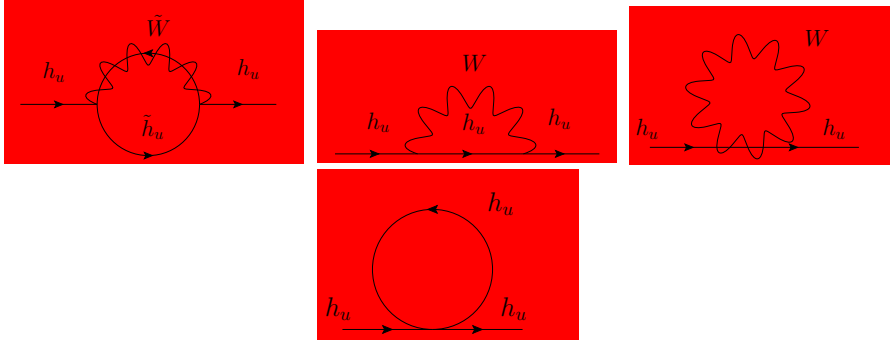


Figure 2.2: Higgs mass correction

$$\begin{aligned}
 \delta m_{h,h-W}^2 &= i(-ig_2)^2 C_2 \int \bar{d}^4 l \, l^\mu \frac{-i\eta_{\mu\nu}}{l^2 + i\varepsilon} l^\nu \frac{i}{l^2 + i\varepsilon} \\
 &= -\frac{g_2^2 C_2}{16\pi^2} \Lambda^2
 \end{aligned} \tag{2.10}$$

$$\begin{aligned}
 \delta m_{h,W}^2 &= i(2ig_2^2) \frac{1}{2} C_2 \int \bar{d}^4 l \frac{-i\delta_\mu^\mu}{l^2 + i\varepsilon} \\
 &= \frac{4g_2^2 C_2}{16\pi^2} \Lambda^2
 \end{aligned} \tag{2.11}$$

$$\begin{aligned}
 \delta m_{h,h}^2 &= i(-ig_2^2) C_2 \int \bar{d}^4 l \frac{i}{l^2 + i\varepsilon} \\
 &= \frac{g_2^2 C_2}{16\pi^2} \Lambda^2
 \end{aligned} \tag{2.12}$$

For $SU(2)$, the quadratic Casimir is $C_2 = \frac{3}{4}$, and so the Higgs mass correction is then given by

$$\delta m_{h_u}^2 = \frac{3g_2^2}{8\pi^2} (m_{\tilde{W}}^2 + m_h^2) \ln \frac{\Lambda}{m_{\tilde{W}}} \tag{2.13}$$

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We identify the Higgsino mass with μ . Because we are already taking $\mu \lesssim 200$ GeV, this translates into a roughly natural wino mass range of

$$m_{\tilde{W}} \lesssim \text{TeV} \quad (2.14)$$

Next, in general, there are quartic scalar terms in the Lagrangian that would give rise to quadratic contributions to the Higgs mass, arising from the D -terms of the various groups. In general, though, they are proportional to $\text{Tr}T^a$ and therefore vanish for $SU(N)$. We are therefore led to consider the contributions from $U(1)_Y$. We compute the hypercharge D -term loop contribution to Higgs mass-squared, in figure 2.3:

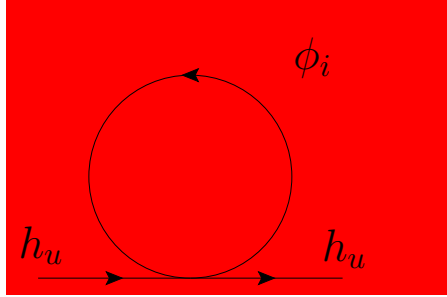


Figure 2.3: Higgs mass correction

This gives rise to a higgs mass correction:

$$\begin{aligned} \delta m_{h_u}^2 &= i \sum_{\text{scalars } j} (-iY_{h_u}Y_j) \int \bar{d}^4l \frac{i}{l^2 - m_j^2 + i\epsilon} \\ &= \sum_{\text{scalars } j} \frac{g_2^2 Y_j Y_{h_u}}{16\pi^2} \left(\Lambda^2 - m_j^2 \ln \frac{\Lambda^2 + m_j^2}{m_j^2} \right) \end{aligned} \quad (2.15)$$

Including both the right-handed sbottom and the down-type higgs, as we do in this section, ensures that the quadratic divergence cancels, but there is still a residual

correction to the higgs mass. Given that other scalars have already been argued to be relatively light, we can use this correction to estimate the natural range for the mass of \tilde{b}_R ,

$$m_{\tilde{b}_R} \lesssim 3\text{TeV} \quad (2.16)$$

Finally, \tilde{q}_L, \tilde{t}_R also being relatively light scalars, suffer from their own naturalness problem, with mass corrections dominated by the diagrams in figure 2.4:

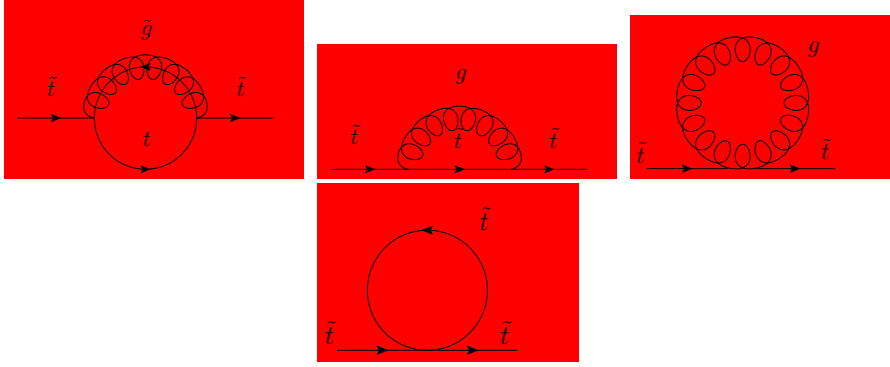


Figure 2.4: Stop mass correction

$$\begin{aligned} \delta m_{\tilde{t}, t-\tilde{g}}^2 &= i(-i\sqrt{2}g_3)^2 C_2(-1) \int \bar{d}^4 l \text{Tr} \left(\frac{\not{l}}{l^2 - m_{\tilde{g}}^2 + i\epsilon} \frac{\not{l}}{l^2 + i\epsilon} \right) \\ &= -\frac{4g_3^2 C_2}{16\pi^2} \left(\Lambda^2 - 2m_{\tilde{g}}^2 \ln \left(\frac{\Lambda}{m_{\tilde{g}}} \right) \right) \end{aligned} \quad (2.17)$$

$$\begin{aligned} \delta m_{\tilde{t}, \tilde{t}-g}^2 &= i(-ig_3)^2 C_2 \int \bar{d}^4 l \, l^\mu \frac{-i\eta_{\mu\nu}}{l^2 + i\epsilon} l^\nu \frac{i}{l^2 - m_{\tilde{t}}^2 + i\epsilon} \\ &= -\frac{g_3^2 C_2}{16\pi^2} \left(\Lambda^2 - 2m_{\tilde{t}}^2 \ln \left(\frac{\Lambda}{m_{\tilde{t}}} \right) \right) \end{aligned} \quad (2.18)$$

$$\begin{aligned}
 \delta m_{t,g}^2 &= i(2ig_3^2)\frac{1}{2}C_2 \int \bar{d}^4l \frac{-i\delta_\mu^\mu}{l^2 + i\varepsilon} \\
 &= \frac{4g_3^2 C_2}{16\pi^2} \Lambda^2
 \end{aligned} \tag{2.19}$$

$$\begin{aligned}
 \delta m_{t,\bar{t}}^2 &= i(-ig_3^2)C_2 \int \bar{d}^4l \frac{i}{l^2 - m_{\bar{t}}^2 + i\varepsilon} \\
 &= \frac{g_3^2 C_2}{16\pi^2} \left(\Lambda^2 - 2m_{\bar{t}}^2 \ln \left(\frac{\Lambda}{m_{\bar{t}}} \right) \right)
 \end{aligned} \tag{2.20}$$

The quadratic Casimir for $SU(3)$ is $C_2 = \frac{4}{3}$, giving rise to a stop mass correction:

$$\delta m_{\bar{t}}^2 = \frac{2g_3^2}{3\pi^2} m_{\bar{g}}^2 \ln \frac{\Lambda}{m_{\bar{g}}} \tag{2.21}$$

For squark masses \sim few hundred GeV, naturalness requires

$$m_{\bar{g}} \lesssim 2m_{\bar{t}} \tag{2.22}$$

2.3 Natural SUSY $\lesssim 1$ TeV with Light Higgsinos

Although the LHC has a multi-TeV reach in principle, parton distribution functions fall so rapidly at high energies that most parton collisions have sub-TeV momentum transfers. In the early LHC era, statistically significant natural SUSY signals would be in this regime. For example, in natural SUSY, gluino production would

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have a cross-section of just a few fb for TeV gluino mass. We can therefore focus our attention on just the early accessible physics by constructing a rough natural SUSY theory with a cutoff $\Lambda_{UV} \sim \text{TeV}$, while not committing strongly to the physics above this scale. With such a low cutoff, only top quark loops in the SM destabilize Higgs naturalness. This is cured by SUSY cancellation upon including the squarks, \tilde{q}_L, \tilde{t}_R , to form complete supermultiplets, $Q \equiv (\tilde{q}_L, q_L), \bar{T} \equiv (\tilde{t}_R^c, t_R^c)$, as before. Even hypercharge D -term divergences from the uncanceled $\text{Tr}(Y)$ are not quantitatively significant. It therefore appears that we can dispense with Higgsinos, \tilde{b}_R , and the gauginos in the effective theory. However, if Higgsino mass arises from a supersymmetric μ term, as discussed in section 2.2.3, then electroweak naturalness also forces the Higgsinos to be light. We will continue with this assumption in this section, and therefore retain complete supermultiplets, $H_{u,d} \equiv (h_{u,d}, \tilde{h}_{u,d})$.

Even though we do not commit here to the structure of the theory above 1 TeV, one possibility is that it is just that of the last section.² But in that case, by eqn. (2.21), we should include the gluino in the sub-TeV effective theory. However, non-minimal physics in the 1 – 10 TeV window can change this conclusion, and indeed the gluino might naturally be considerably heavier than 1 TeV. We illustrate such

²While the LHC might be dominated by sub-TeV physics, as explained above, electroweak precision tests at lower energy machines are famously sensitive to multi-TeV scales via virtual processes. In the 1 TeV effective theory, this translates to precision test sensitivity to higher dimensional operators. In the case, where this effective theory merely originates from our 10 TeV effective theory, such higher-dimensional operators are suppressed by the 10 TeV scale and are safe from electroweak precision tests. We take this as an existence proof that multi-TeV physics of the sort we contemplate can easily yield sufficiently suppressed higher-dimensional operators in the TeV effective theory to be safe, and make it an assumption for our consideration of TeV effective theory in general. We do not further specify the structure of such operators, given their lower relevance for LHC processes.

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new physics in section 2.8, with the example of a Dirac gluino. It exemplifies the general theme that non-minimal UV physics can lead to more minimal IR physics, while still being compatible with naturalness. Here, we merely check within the TeV effective theory that naturalness indeed requires stops, but that these stops *do not require* gluinos. The first statement follows from eqn. (2.7), where naturalness up to $\Lambda_{UV} \sim 1$ TeV then implies

$$m_{\tilde{t}} \lesssim 700\text{GeV} \tag{2.23}$$

The second statement follows from eqn. (2.21), where we see that with the logarithm of order one and gluino mass ~ 1 TeV, we can naturally have stops as light as 300 GeV. In our phenomenological studies of section 3.1, we mostly keep in mind lighter stops, $m_{\tilde{t}} \lesssim 400\text{GeV}$, compatible with either 1 or 10 TeV cutoffs as discussed in section 2.2.

2.3.1 Effective Lagrangian, Neglecting Third-Generation

Mixing

Given the light superpartner content described above, the R-parity conserving effective theory below a TeV is given by

$$\begin{aligned}
\mathcal{L}_{eff} = & \int d^4\theta K + \left(\int d^2\theta \left(\frac{1}{4} W_\alpha^2 + y_t \bar{T} H_u Q + y_b \bar{B} H_d Q + \mu H_u H_d \right) + \text{h.c.} \right) \\
& + \mathcal{L}_{kin}^{light} - \left(\bar{u}_R Y_u^{light} h_u \psi_L + \bar{d}_R Y_d^{light} h_d \psi_L + \text{h.c.} \right) + \mathcal{L}_{lepton} \\
& - m_{\tilde{q}_L}^2 |\tilde{q}_L|^2 - m_{\tilde{t}_R^c}^2 |\tilde{t}_R^c|^2 - m_{h_u}^2 |h_u|^2 - m_{h_d}^2 |h_d|^2 \\
& - (B\mu h_u h_d + A_t \tilde{t}_R^c h_u \tilde{q}_L + \text{h.c.}) \\
& + \mathcal{L}_{hard} + \mathcal{L}_{non-ren.}
\end{aligned} \tag{2.24}$$

which is to be interpreted as in eqn. (2.2) except that all *terms involving gauginos or \tilde{b}_R^c are to be thrown away* after expanding the superspace expressions in components.

With the cutoff as low as 1 TeV, the hard SUSY-breaking can now include $|h_u|^4$ couplings strong enough to give contributions to the physical Higgs mass of tens of GeV without making EWSB scale unnatural. One can think of these terms as arising from new fields, such as discussed in section 2.2.2, heavier than 1 TeV, which have therefore been integrated out. One virtue of this sub-TeV theory is that we do not have to commit to just what UV physics contributed to Higgs mass; whatever it might be is parametrized by the effective hard couplings.

2.3.2 Dark Matter Considerations

In our TeV effective theory, we must take the Higgsinos as the lightest superpartners in order to avoid phenomenologically dangerous colored (collider-)stable particles in the form of stops or sbottom. Such Higgsinos will then form charginos and neutralinos at the ends of superpartner decay chains. Higgsino neutralinos would have a thermal relic abundance smaller than needed to fully account for all of dark matter. This is not an issue if dark matter is dominated by other physics not accessible to the LHC. Another possibility is that the wino and bino, $\lambda_{1,2}$, which are not required to be light by naturalness, are nevertheless light and in the effective theory, and a linear combination of gaugino-Higgsino forms a neutralino LSP. It is possible then that such a hybrid LSP has the correct thermal relic abundance to account for dark matter. This computation still remains to be checked in the natural SUSY context however. Even in this case, our minimal effective theory is still useful, in that for the purposes of *early* LHC phenomenology the details of charginos/neutralinos are not as important as their existence and the LSP mass. The Higgsino LSP in our effective theory can therefore serve as a toy model of whatever the real chargino/neutralino degrees of freedom are. More refined modeling can wait until the new physics is discovered.

2.4 Natural SUSY $\lesssim 10$ TeV with Heavy Higgsinos

As alluded to earlier, given that we necessarily have hard SUSY breaking couplings in natural SUSY, we can reduce the particle content even further by eliminating h_d bosons and the right-handed bottom squark \tilde{b}_R^c from the effective theory. See references [38–40] for earlier related works. This move maintains the vanishing of $\text{Tr}(Y)$ required for naturalness with 10 TeV cutoff, but forces us to obtain Yukawa-couplings for down-type fermions by coupling them to

$$h_u^* \equiv i\sigma_2 h_u^\dagger \quad (2.25)$$

where σ_2 is the second weak-isospin Pauli matrix. This is the usual approach to getting down-type fermion masses in the SM with a single Higgs doublet. In the SUSY context, such a coupling cannot arise from a superpotential, which can only depend on H_u , not H_u^\dagger . Instead, it represents a hard SUSY breaking effect (though it may arise from soft SUSY breaking from the vantage of a UV completion). It poses no threat to naturalness if the couplings are $\ll 1$. This is certainly the case for all the down-type Yukawa couplings.

2.4.1 Effective Lagrangian, Neglecting Third-Generation

Mixing

With the particle content described above, the R-parity conserving effective Lagrangian is given by

$$\begin{aligned}
 \mathcal{L}_{eff} = & \int d^4\theta K + \left(\int d^2\theta \left(\frac{1}{4} W_\alpha^2 + y_t \bar{T} H_u Q \right) + \text{h.c.} \right) \\
 & + \mathcal{L}_{kin} - \left(\bar{u} Y_u^{light} h_u q_L + y_b \bar{b} h_u^* q_L + \bar{d} Y_d^{light} h_u^* q_L + \text{h.c.} \right) + \mathcal{L}_{lepton} \\
 & - m_{\tilde{q}_L}^2 |\tilde{q}_L|^2 - m_{\tilde{t}_R^c}^2 |\tilde{t}_R^c|^2 - m_{h_u}^2 |h_u|^2 \\
 & - \left(m_{i=1,2,3} \lambda_i \lambda_i + A \tilde{t}_R^c h_u \tilde{q}_L + m_{\tilde{h}} \tilde{h}_u \tilde{h}_d + \text{h.c.} \right) \\
 & + \mathcal{L}_{hard} + \mathcal{L}_{non-ren.}
 \end{aligned} \tag{2.26}$$

The Kahler potential K consists of the gauge-invariant kinetic terms for the chiral superfields, \bar{T}, Q, H_u , while compared with eqn. (2.2), the kinetic terms for the (now un-superpartnered) fermions b_R and \tilde{h}_d have now been added to \mathcal{L}_{kin} . The second to fourth lines still follow from the MSSM after deleting fields that are absent in our effective theory, except for the small Yukawa couplings of h_u^* to down-type fermions, which we pointed out above are a form of hard SUSY breaking. Other hard breaking as well as non-renormalizable couplings appear on the last line. Our discussion of the physical Higgs mass, and contributions to it, is similar to section 2.2.2. However a singlet coupling to $h_u h_d$ is not possible since we have removed h_d , but in an electroweak triplet coupled to $H_u H_u$ is possible and results in a $|h_u|^4$ terms in the potential [41–43].

2.4.2 Higgsino Mass

Note that the Higgsino mass now takes the form of a soft SUSY-breaking mass term, $m_{\tilde{h}}$, as opposed to a supersymmetric μ term as in section 2.2. In this way, it is uncorrelated with any contribution to Higgs boson mass-squared. Therefore, there is only one modification to the bounds obtained in section 2.2.3; namely, that now $m_{\tilde{h}}$ is only constrained by eqn. (2.13), so that

$$m_{\tilde{h}} \lesssim \text{TeV} \tag{2.27}$$

2.5 Natural SUSY $\lesssim 1$ TeV with Heavy Higgsinos

In the most minimal of our effective theories, all gauginos and Higgsinos can naturally be heavier than a TeV and thus integrated out of the sub-TeV effective theory. If we identify h_u with the SM Higgs doublet, the only new particles are $\tilde{t}_L, \tilde{b}_L, \tilde{t}_R$.

2.5.1 Effective Lagrangian, Neglecting Third-Generation

Mixing

The effective Lagrangian with R-parity is then given by

$$\begin{aligned}
 \mathcal{L}_{eff} = & \mathcal{L}_{SM} + \mathcal{L}_{kin}^{squarks} - V_{D\text{-terms}} - y_t^2 (|h_u \tilde{q}_L|^2 + |\tilde{t}_R^c \tilde{q}_L|^2 + |\tilde{t}_R^c h_u|^2) \\
 & - m_{h_u}^2 |h_u|^2 - m_{\tilde{q}_L}^2 |\tilde{q}_L|^2 - m_{\tilde{t}_R^c}^2 |\tilde{t}_R^c|^2 - (A \tilde{t}_R^c h_u \tilde{q}_L + \text{h.c.}) \\
 & + \mathcal{L}_{hard} + \mathcal{L}_{non-ren.}
 \end{aligned} \tag{2.28}$$

\mathcal{L}_{SM} is the SM Lagrangian with h_u playing the role of the SM Higgs doublet, but with no Higgs potential. The Higgs potential is a combination of the soft Higgs mass term in the second line, the D -term potential and possible hard SUSY-breaking couplings $\sim |h_u|^4$. As discussed in section 2.2.2, these hard SUSY breaking couplings can be large enough to easily satisfy the Higgs mass bound without spoiling naturalness.

With exact R-parity, one of the colored superpartners would necessarily be stable and phenomenologically dangerous. However, we can use the above effective Lagrangian as the minimal departure point for adding R-parity violating corrections. We take this up in section 2.7.

2.5.2 Effective Lagrangian with Neutralino LSP

Another possibility is that R-parity is exact but there is a neutralino LSP in the spectrum, even though it is not required by electroweak naturalness. It may or

may not be the dominant constituent of dark matter. Since we cannot determine its identity by theoretical considerations alone, we will just add a temporary “placeholder”, that allows the squarks to decay promptly while preserving R-parity. We choose this to be the bino, λ_1 , even though taken literally, it would predict too large a thermal relic abundance of dark matter. A more refined description of the neutralino would not add much to the early LHC search strategy. In this option, as compared to that of section 2.3.2 and eqn. (2.24), we do not have a chargino.

The effective Lagrangian then takes the form

$$\begin{aligned}
 \mathcal{L}_{eff} = & \mathcal{L}_{SM} + \mathcal{L}_{kin}^{squarks} - V_{D-terms} - y_t^2 (|h_u \tilde{q}_L|^2 + |\tilde{t}_R^c \tilde{q}_L|^2 + |\tilde{t}_R^c h_u|^2) \\
 & - m_{h_u}^2 |h_u|^2 - m_{\tilde{q}_L}^2 |\tilde{q}_L|^2 - m_{\tilde{t}_R^c}^2 |\tilde{t}_R^c|^2 - (A \tilde{t}_R^c h_u \tilde{q}_L + \text{h.c.}) \\
 & + i \bar{\lambda}_1 \partial \cdot \sigma \lambda_1 - (m_1 \lambda_1 \lambda_1 + \text{h.c.}) - \sqrt{2} g_2 \left(\frac{1}{6} \bar{q}_L \bar{\lambda}_1 \tilde{q}_L + \frac{1}{6} \tilde{q}_L \lambda_1 q_L - \frac{2}{3} \bar{t}_R^c \bar{\lambda}_1 \tilde{t}_R^c - \frac{2}{3} \tilde{t}_R^c \lambda_1 t_R^c \right) \\
 & + \mathcal{L}_{hard} + \mathcal{L}_{non-ren.}
 \end{aligned} \tag{2.29}$$

2.6 Flavor-Changing Neutral Currents and CP Violation

Above, we have worked in the drastic approximation that the mixing between the third generation with the first two generations vanishes, so that the meaning of “third generation” squarks, $\tilde{q}_L, \tilde{t}_R^c, \tilde{b}_R^c$, is completely unambiguous. In this limit, there

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is a conserved third-generation (s)quark number. In the real world, third generation mixing is non-zero but small. In the Wolfenstein parametrization, mixing with the second generation is of order ϵ^2 and mixing with the first generation is of order ϵ^3 , where $\epsilon \sim 0.22$ corresponds to Cabibbo mixing. Given this fact, it is more natural to have comparable levels of violation of third-generation (s)quark number in the physics we have added beyond the SM.

In practice this means that for every interaction term in which the squarks currently appear, where third-generation number is conserved by the presence of t or b quarks (in electroweak gauge basis), we now allow more general couplings, with the third generation quarks replaced by quarks of the first and second generations. The associated couplings with second generation quarks are taken to be of order ϵ^2 , while those with first generation quarks are taken to be of order ϵ^3 , all in electroweak gauge basis. All these couplings involving the squarks are technically hard breaking of SUSY, but $\epsilon^{2(3)}$ is so small that, like other hard breaking in the effective theory, they do not spoil Higgs naturalness below 10 TeV. For most, but not all, of the LHC collider phenomenology the small $\epsilon^{2(3)}$ effects are negligible and we can proceed with our earlier effective Lagrangians. (We must of course keep SM third generation mixing effects, so that, for example, the bottom quark decays.) But in the more realistic setting with third-generational mixing, we must confront the SUSY flavor problem. In natural SUSY, this problem has two faces, IR and UV.

The UV face of the problem is contained in the non-renormalizable interactions of

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eqn. (2.2). For example, they can include flavor-violating interactions such as $\bar{s}d\bar{s}d$. If such a non-renormalizable interaction were suppressed only by $(10 \text{ TeV})^2$, it would lead to FCNCs in kaon mixing, orders of magnitude greater than observed. It is therefore vital for the non-renormalizable interactions to have a much more benign flavor structure. Whether this is the case or not is determined by matching to the full theory above 10 TeV, IR natural SUSY considerations alone cannot decide the issue. References [3, 19] are examples of UV theories which reduce to natural SUSY at accessible energies and automatically come with the kind of benign UV flavor structure we require. In this thesis, we simply assume that the UV-sensitive non-renormalizable interactions are sufficiently flavor-conserving to avoid conflict with FCNC constraints.

There remain FCNC effects that are UV-insensitive but are assembled in the IR of the effective theory through the small $\epsilon^{2(3)}$ flavor-violating couplings. Many of these have been studied in [44] and are small enough to satisfy current constraints. Indeed this feature is one of the selling points of natural SUSY. Here, we illustrate one such FCNC effective interaction for (CP-violating) $K - \bar{K}$ mixing arising as a SUSY “box” diagram. Similar processes were studied in [45–48], with minor adaptations needed in our case.

While the effect is suppressed by $O(\epsilon^{10})$ in natural SUSY, it is more stringently constraining than $B_d - \bar{B}_d$ mixing or $B_s - \bar{B}_s$ mixing, even though these are suppressed by just $O(\epsilon^6)$ and $O(\epsilon^4)$ respectively. We show that with our rough flavor-changing

power-counting the \tilde{b}_R squark is constrained to lie above several TeV in the absence of flavor-parameter tuning.

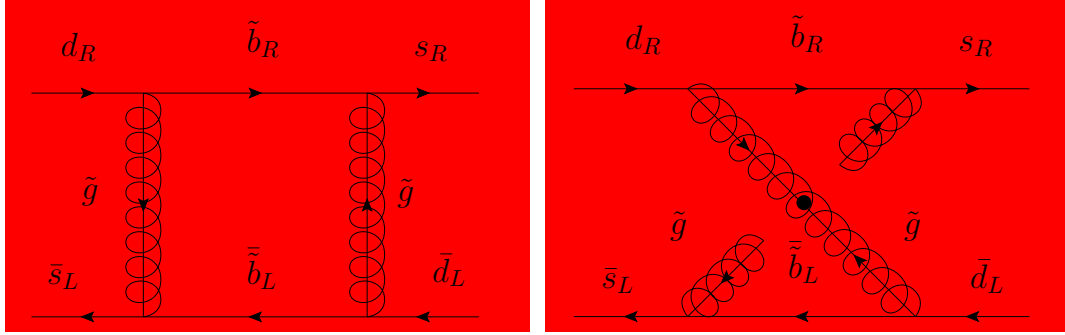


Figure 2.5: Contributions to $K - \bar{K}$ mixing

In a low-energy effective Lagrangian to be run down to the hadronic scale, we match onto effective operators of the form

$$\mathcal{L}_{eff} \supset \kappa (\bar{s}_L d_R) (\bar{s}_R d_L) \quad (2.30)$$

Strictly speaking there are two different operators depending on color contraction. As shown in [45] an operator $\mathcal{O}_5 \propto \bar{d}_R^i s_L^j \bar{d}_L^j s_R^i$ (where i, j are color indices) is not enhanced by QCD running and has $1/N_c$ -suppressed QCD matrix element. Therefore we concentrate on $\mathcal{O}_4 \propto \bar{d}_R^i s_L^i \bar{d}_L^j s_R^j$, which has enhanced QCD running and large hadronic matrix element. Therefore, for the purpose of our simple estimate, in (2.30) we only study the case where each bilinear is a color singlet.

Integrating out the superpartners yields:

$$\begin{aligned}
 \kappa \sim & -\frac{g_3^4 \epsilon^{10}}{4\pi^2} \frac{2}{3} \frac{m_3^2}{(m_3^2 - m_{\tilde{q}_L}^2)^2 (m_3^2 - m_{\tilde{b}_R^c}^2)^2 (m_{\tilde{q}_L}^2 - m_{\tilde{b}_R^c}^2)} \\
 & \times ((m_{\tilde{q}_L}^2 - m_{\tilde{b}_R^c}^2)(m_3^2 - m_{\tilde{b}_R^c}^2)(m_3^2 - m_{\tilde{q}_L}^2) + m_{\tilde{q}_L}^2 (m_3^4 + m_{\tilde{b}_R^c}^4) \ln \frac{m_{\tilde{q}_L}^2}{m_3^2} \\
 & + 2m_3^2 m_{\tilde{q}_L}^2 m_{\tilde{b}_R^c}^2 \ln \frac{m_{\tilde{b}_R^c}^2}{m_{\tilde{q}_L}^2} + m_{\tilde{b}_R^c}^2 (m_{\tilde{q}_L}^4 + m_3^4) \ln \frac{m_3^2}{m_{\tilde{b}_R^c}^2}) \\
 & - \frac{g_3^4 \epsilon^{10}}{8\pi^2} \frac{1}{12} \frac{1}{(m_3^2 - m_{\tilde{q}_L}^2)^2 (m_3^2 - m_{\tilde{b}_R^c}^2)^2 (m_{\tilde{q}_L}^2 - m_{\tilde{b}_R^c}^2)} \\
 & \times ((2m_3^2 m_{\tilde{q}_L}^4 m_{\tilde{b}_R^c}^2 - m_3^4 m_{\tilde{q}_L}^4) \ln \frac{m_3^2}{m_{\tilde{q}_L}^2} + (2m_3^2 m_{\tilde{q}_L}^2 m_{\tilde{b}_R^c}^4 - m_3^4 m_{\tilde{b}_R^c}^4) \ln \frac{m_{\tilde{b}_R^c}^2}{m_3^2} \\
 & + m_{\tilde{q}_L}^4 m_{\tilde{b}_R^c}^4 \ln \frac{m_{\tilde{q}_L}^2}{m_{\tilde{b}_R^c}^2} + m_3^2 (m_3^2 - m_{\tilde{q}_L}^2)(m_3^2 - m_{\tilde{b}_R^c}^2)(m_{\tilde{q}_L}^2 - m_{\tilde{b}_R^c}^2)) \quad (2.31)
 \end{aligned}$$

where, as discussed above, the squark couplings to second generation quarks are assigned strength $\sim g_3 \epsilon^2$, while squark couplings to the first generation are $\sim g_3 \epsilon^3$. We neglect \tilde{b}_L - \tilde{b}_R mixing (after EWSB). Note that our result contains large logarithms of the form $\ln m_{\tilde{b}_R^c}^2 / m_{\text{squark}}^2$, which in principle should be resummed (for example, see [48]). However, we do not do this since, again, we only seek an estimate for κ .

Current constraints on ϵ_K require that [49]

$$(\text{Im}(\kappa)) \lesssim \left(\frac{1}{3 \times 10^5 \text{ TeV}} \right)^2 \quad (2.32)$$

For $m_3 \sim \text{TeV}$ and $m_{\tilde{q}_L} \sim 350 \text{ GeV}$, this translates into a bound on \tilde{b}_R mass of roughly $m_{\tilde{b}_R^c} \gtrsim 17 \text{ TeV}$.

Of course, this bound is extremely sensitive to our estimates for the flavor-changing vertices. For example, if each flavor-changing vertex were only half as strong as our

above estimates, the bound would be relaxed to $m_{\tilde{b}_R^c} \gtrsim 4$ TeV, roughly consistent with the requirements of naturalness in section 2.2.3. Alternatively, there may be small phases present in the vertices that further suppress κ . In the even more minimal natural supersymmetry structure of sections 2.4 and 2.5, \tilde{b}_R^c is completely absent and there is no robust infrared contribution to κ at one-loop order to worry about.

There are also CP-violating effects unrelated to flavor-changing, in particular electric dipole moment (EDM) constraints. From [50, 51] (see also references therein), we see that again natural SUSY has a relatively safe IR structure, with large regions of viable parameter space.³

For example, see case II of Table III in [51] and the surrounding discussion. We show that these constraints are even more relaxed in the case of Dirac gauginos, in section 2.8.

2.7 R-Parity versus R-Parity Violation

R-parity plays a central role in theory and phenomenology within the weak scale SUSY paradigm. We will review some of the reasons for this, and argue that in light of several modern theoretical developments, the case for R-parity conservation in natural SUSY is less compelling. We are therefore more strongly motivated to take seriously an R-parity violating phenomenology. Quite apart from these theoretical

³Here, we are discussing the supersymmetric CP problem as opposed to the Strong CP problem. We assume for concreteness that the Peccei-Quinn mechanism with an axion resolves the Strong CP problem.

considerations, we believe that this RPV phenomenology of natural SUSY is quite distinctive, and has so far not received enough attention. We will take up its study in chapter 3.

2.7.1 Proton Decay

The standard motivation for R-parity is that it leads to conserved baryon number. But it does not follow in complete generality. In the MSSM, baryon-number conservation only follows from R-parity after restricting to renormalizable interactions. For example, R-parity conserving but non-renormalizable superpotential interactions of the general form $W \propto \bar{U}\bar{U}\bar{D}\bar{E}$ give rise to proton decay. If the MSSM is taken as valid up to an extremely high scale, such a non-renormalizable term, and the resulting proton decay rate, would be suppressed by that high scale. However, if the MSSM is an effective theory emerging only below some lower threshold, then the non-renormalizable operator can be suppressed by just this lower threshold scale, leading to excessive proton decay. This is precisely the issue in many SUSY GUT theories, where such an effective interaction arises in the effective MSSM after integrating out a color-triplet GUT-partner of the Higgs. The moral only gains strength in natural SUSY, with a 10 TeV cutoff. For example, a dimension-6 R-parity conserving operator such as $u_L d_L u_R e_R$ can be viewed as a remnant of a supersymmetric non-renormalizable Kahler potential term. It gives rise to extremely rapid proton decay if suppressed by just $(10 \text{ TeV})^2$. Such an operator might well arise upon integrating

out new thresholds above 10 TeV.

We conclude that R-parity is not by itself enough to protect against proton decay in natural SUSY; in general we need some other symmetry, such as baryon-number or lepton-number symmetry.⁴ Clearly then, the proton-stability motivation for R-parity is gone.

2.7.2 Unification

Traditionally, the reason for arguing against new physics thresholds between the GUT and weak scales is because such new physics generally spoil the success of gauge coupling unification. But this is evaded if the new physics comes in complete GUT multiplets. For example, this is what is typically assumed for the messenger threshold of gauge-mediated SUSY-breaking models. In the model-building of recent years, we have seen that even quite radically new intermediate structure can maintain the success of gauge coupling unification by following this basic rule of GUT-degenerate thresholds [52]. There also exist new unification mechanisms that improve on the imperfect unification of SM via strong coupling effects over intermediate scales [53]. Therefore, we cannot have confidence that there is a Weak-GUT desert, as is often assumed. There may well be important new physics (not far) above 10 TeV, and in this context R-parity does not save us from excessive proton decay, as discussed

⁴While baryon number (lepton number) is broken by anomalies, just as in the SM, this need only imply baryon number violation via non-perturbatively small interactions, which can easily be well below any experimental bounds.

above.

Another GUT-related reason in favor of R-parity is that in the context of traditional GUT models, imposing baryon- or lepton-number symmetry conflicts with the unification of quarks and leptons, whereas imposing R-parity does not. However, such traditional GUT models also suffer from other difficulties such as the notorious doublet-triplet splitting problem. In more recent years, it has been understood that some of the successes attributed to SUSY GUTs can arise more generally, in particular in the context of Orbifold GUT models (see [54, 55] and references therein). Such models employ “split multiplets”, in which quarks and leptons can naturally arise as incomplete parts of *separate* GUT multiplets, and the Higgs doublet and triplet are also neatly split in the same manner. In this orbifold unification context, one can straightforwardly impose baryon- or lepton-number symmetry, safeguarding proton stability without requiring R-parity.

In this way, the unification considerations that originally favored R-parity over baryon- or lepton-number symmetry are less compelling.

2.7.3 Dark Matter

There is a second traditional motivation for R-parity, namely that the lightest R-odd superpartner is stable, and therefore may account for the dark matter of the Universe, enjoying the rough quantitative success known as the “WIMP-miracle”. RPV interactions spoil this stability and seem to rob us of such a dark matter can-

didate. However, it is entirely possible that dark matter does consist of weak scale WIMPs, but these WIMPs are stabilized by carrying a different symmetry than R-parity, under which the SM is inert. This natural possibility leads us to separate the question of modeling dark matter from the questions of electroweak and Higgs naturalness, at least for the immediate purpose of pursuing collider phenomenology. In the traditional view, every superpartner produced cascade decays down to the dark matter particle. But more generally, we can have R-parity violation and dark matter may or may not be at the end of superpartner decay chains.

2.7.4 RPV and FCNCs

A final reason for favoring R-parity is that in standard weak scale SUSY, large parts of RPV parameter space lead to excessive FCNCs, only exacerbating the usual SUSY Flavor Problem. However, this point is mitigated, though not completely evaded, in natural SUSY, because of the greatly reduced squark content, as discussed below. Again, this makes RPV a more motivated possibility in the natural SUSY context.

In the end, we think that both R-parity and RPV alternatives are plausible in the natural SUSY context, and make for very different phenomenological features and search strategies. Below we discuss RPV with proton decay protected by lepton number symmetry, and alternatively by baryon number conservation.

2.7.5 RPV with Lepton Number Conservation

The standard renormalizable RPV SUSY couplings preserving lepton number are of the superpotential form $W \propto \bar{U}_I \bar{D}_J \bar{D}_K$, with generational indices I, J, K . Such couplings give rise to a variety of RPV Yukawa couplings and (after SUSY breaking) RPV A-terms which can decisively affect superpartner decays and flavor physics. Here, we specialize to the most minimal particle content of natural SUSY, as discussed in section 2.4 and 2.5, with beyond-SM field content given by $\tilde{q}_L, \tilde{t}_R^c, \tilde{h}_{u,d}, \lambda_i$. While there is the up-type scalar singlet \tilde{t}_R^c , there is no down-type scalar singlet, and therefore no RPV A-terms are possible in the natural SUSY theory. The only RPV Yukawa couplings that come from truncating the above type of superpotential to natural SUSY are of the form

$$\mathcal{L}_{RPV} = \kappa_{IJ} \tilde{t}_R^c d_R^{cI} d_R^{cJ} \quad (2.33)$$

We will consider this to be added to the minimal 10 TeV effective Lagrangian of eqn. (2.26), or the 1 TeV effective Lagrangian of eqn. (2.28).

Flavor constraints on these couplings, reviewed in [56], easily allow RPV coupling strengths that lead to prompt squark decays into quarks at colliders. But while lepton-number conservation is sufficient to protect against proton decay (assuming the gravitino or other non-minimal fermions are heavier than the proton), it does not forbid neutron-antineutron oscillations. This is because (accidental) $U(1)$ baryon-number symmetry is incompatible with the combination of RPV couplings, gaugino-squark-

quark coupling, and Majorana gaugino masses. The bounds on neutron-antineutron oscillations are stringent (see [57] for review), even in natural SUSY where CKM suppressions are incurred in mediating such effects via the third generation squarks and gauginos. Again, RPV couplings can straightforwardly be strong enough to lead to prompt squark decays to quarks at colliders. And yet, they cannot be order one in strength. Theoretically, having RPV couplings $\ll 1$ is plausible enough, related perhaps to the smallness of ordinary Yukawa couplings. Experimentally, small RPV couplings imply that squarks cannot be *singly* produced at colliders.

Remarkably, there is a way of recovering $U(1)$ baryon number symmetry consistent with *order one* RPV couplings of the form of eqn. (2.33), but it requires realizing gauginos as components of Dirac fermions. Observing single squark production can then be an interesting diagnostic of supersymmetry breaking, even those parts out of direct reach of the 7 TeV LHC. We will show how this works in section 2.8.

2.7.6 R-Parity Violation with Baryon Number Conservation

The standard renormalizable RPV SUSY couplings preserving baryon number are superpotential terms of the form, $W \sim LL\bar{E}, QL\bar{D}, LH_u$. Let us again consider truncating to the minimal beyond-SM field content described in sections 2.4 and 2.5, $\tilde{q}_L, \tilde{t}_R^c, \tilde{h}_{u,d}, \lambda_i$. Again, there are no A-terms of the forms of these superpotentials

possible, and the $LL\bar{E}$ completely vanishes. The bilinear superpotential turns into a mixing mass term $\ell\tilde{h}_u$. Since \tilde{h}_d and the left-handed leptons, ℓ , share the same gauge quantum numbers, we can choose a new basis for them such that there are no $\ell\tilde{h}_u$ terms. The only surviving RPV Yukawa couplings are then of the form,

$$\mathcal{L}_{RPV} = \kappa'_{IJ} d_R^{cI} \ell_L^J \tilde{q}_L \quad (2.34)$$

We defer the study of the flavor constraints and the LHC implications of this type of baryon-number conserving RPV interactions within natural SUSY to future work. Reference [56] reviews such interactions in the more general SUSY context.

2.8 Dirac Gauginos

We have argued in the context of our 10 TeV natural SUSY theories that naturalness requires sub-TeV gluinos, which provides a very significant and visible SUSY production channel at the LHC. Yet, if we remain uncommitted to the structure of physics above 1 TeV, we have argued that the gluino need not be present in the sub-TeV effective theory. At first sight, these two statements might seem in conflict, but in fact they merely exemplify a general theme in SUSY models: a very minimal field content in the far IR often *requires* a less minimal field content at higher energies. This is the case with regard to gauginos, and gluinos in particular due to their stronger couplings. The idea of Dirac gauginos [58–60] is to have extra field content

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in the form of a chiral superfield, Φ_i , in the adjoint representation of each SM gauge group, with soft SUSY breaking such that the Φ_i fermion, χ_i , and the gaugino, λ_i , get a Dirac mass with each other, $m_{\lambda_i}\lambda_i\chi_i$. With such non-minimal field content below 10 TeV we will see that it is natural to have the Dirac gauginos heavier than 1 TeV.

The 10 TeV effective theory with Dirac gauginos, analogous to the construction of eqn. (2.2), is given by

$$\begin{aligned}
\mathcal{L}_{eff} = & \int d^4\theta K + \left(\int d^2\theta \left(\frac{1}{4}W_\alpha^2 + y_t\bar{T}H_uQ + y_b\bar{B}H_dQ + \mu H_uH_d + (\sqrt{2}m_i\theta^\alpha)W_{i\alpha}\Phi_i \right) + \text{h.c.} \right) \\
& + \mathcal{L}_{kin}^{light} - \left(\bar{u}Y_u^{light}h_uq_L + \bar{d}Y_d^{light}h_dq_L + \text{h.c.} \right) + \mathcal{L}_{lepton} \\
& - m_{\tilde{q}_L}^2 |\tilde{q}_L|^2 - m_{\tilde{t}_R^c}^2 |\tilde{t}_R^c|^2 - m_{\tilde{b}_R^c}^2 |\tilde{b}_R^c|^2 - m_{h_u}^2 |h_u|^2 - m_{h_d}^2 |h_d|^2 - m_{\phi_i}^2 |\phi_i|^2 \\
& - B\mu h_u h_d - A_t \tilde{t}_R^c h_u \tilde{q}_L - A_b \tilde{b}_R^c h_d \tilde{q}_L + \text{h.c.} \\
& + \mathcal{L}_{hard} + \mathcal{L}_{non-ren.}
\end{aligned} \tag{2.35}$$

where the explicit Grassmann θ^α dependence parametrizes the soft SUSY breaking Dirac gaugino mass term in superspace notation, and m_ϕ^2 in the third line gives soft mass-squared to the scalars in the adjoint superfield Φ . The remaining terms are as discussed below eqn. (2.2).

Similarly, the 10 TeV effective theory with Dirac gauginos, analogous to the construction of Eq. (2.26), is given by

$$\begin{aligned}
 \mathcal{L}_{eff} = & \int d^4\theta K + \left(\int d^2\theta \left(\frac{1}{4} W_\alpha^2 + y_t \bar{T} H_u Q + (\sqrt{2} m_i \theta^\alpha) W_{i\alpha} \Phi_i \right) + \text{h.c.} \right) \\
 & + \mathcal{L}_{kin} - \left(\bar{u} Y_u^{light} h_u q_L + y_b \bar{b} h_u^* q_L + \bar{d} Y_d^{light} h_u^* q_L + \text{h.c.} \right) + \mathcal{L}_{lepton} \\
 & - m_{\tilde{q}_L}^2 |\tilde{q}_L|^2 - m_{\tilde{t}_R}^2 |\tilde{t}_R|^2 - m_{h_u}^2 |h_u|^2 - m_{\phi_i}^2 |\phi_i|^2 \\
 & - \left(A \tilde{t}_R^c h_u \tilde{q}_L + m_{\tilde{h}} \tilde{h}_u \tilde{h}_d + \text{h.c.} \right) \\
 & + \mathcal{L}_{hard} + \mathcal{L}_{non-ren.} \tag{2.36}
 \end{aligned}$$

This scenario was first emphasized and studied in detail in the context of full supersymmetry in [40].

2.8.1 Naturalness

Expanding the soft gaugino mass term from superspace into components yields couplings,

$$\mathcal{L} \supset \sqrt{2} m_{\lambda_i} D^i (\phi_i + \bar{\phi}_i) - m_{\lambda_i} (\chi^i \lambda_i + \bar{\chi}^i \bar{\chi}_i) \tag{2.37}$$

The D -term contributes mass to the *real* part of ϕ_i so that the total mass-squared is $m_{R_i}^2 = 2(m_{\lambda_i}^2 + m_{\phi_i}^2)$, while the imaginary part has mass-squared of just m_ϕ^2 . In addition, the D -term generates a coupling of the real part of ϕ to the other scalars charged under the related gauge group. For the case of Dirac gluinos, we obtain the coupling $\mathcal{L} \supset -\sqrt{2} m_{\lambda_3} g_3 (\phi_3^a + \bar{\phi}_3^a) (\bar{q} T^a q)$, where T^a are the Gell-Mann color matrices. This provides a new correction to the stop mass-squared at one loop which cancels

the logarithmic divergence found in eqn. (2.21) [60]. eqn. (2.21) is then replaced by a UV-finite total correction,

$$\delta m_{\tilde{t}}^2 = \frac{2g_3^2 m_{\tilde{g}}^2}{3\pi^2} \ln \frac{m_{R_3}}{m_{\tilde{g}}} \quad (2.38)$$

Taking the stop much lighter than the gluino and the scalar gluon (“sgluon”) to be comparable to the gluino mass (the above logarithm ~ 1), and requiring naturalness of the stop mass, yields

$$m_{\tilde{g}} \lesssim 4m_{\tilde{t}} \quad (2.39)$$

This implies it is natural to have gluinos above a TeV for stops as light as ~ 300 GeV. In such cases, it is sensible to remove the gluino and sgluons from the sub-TeV effective theory, and from early LHC phenomenology.

2.8.2 R-Parity Violation

As advertized in section 2.7.5, Dirac gauginos are also important for the case of lepton-number conserving RPV because they completely relax the stringent constraints from neutron-antineutron oscillations by allowing one to have a $U(1)$ baryon number symmetry. The trick is that this symmetry is realized as an R-symmetry in the sense that different fields in a supermultiplet carry different charges. The charges of the fields are given in table 2.1. One can then check that eqn. (2.36) and the RPV

Boson	q	Fermion	q
h_u	0	\tilde{h}_u	-1
		\tilde{h}_d	1
\tilde{q}_L	$\frac{4}{3}$	q_L	$\frac{1}{3}$
		$(u_L, d_L), (c_L, s_L)$	$\frac{1}{3}$
\tilde{t}_R^c	$\frac{2}{3}$	t_R^c	$-\frac{1}{3}$
		$u_R^c, d_R^c, s_R^c, c_R^c, b_R^c$	$-\frac{1}{3}$
		leptons	0
A_μ	0	λ	1
ϕ	0	χ	-1

Table 2.1: R-charges of particles in theory with eqn. (2.33) and Dirac gaugino masses.

couplings of eqn. (2.33) respect such a baryon number R -symmetry in the absence of the A term.

With baryon R -symmetry, neutron-antineutron oscillations are forbidden, even when RPV couplings are sizeable, which raises the possibility that stops can be singly produced at colliders.⁵ But we first have to ask if this is plausible in light of flavor physics and CP constraints. A useful way to think of the new flavor structure of RPV couplings of \tilde{t}_R^c in natural SUSY is that they effectively make this antisquark

⁵Reference [61] discusses a model in which it is \tilde{b}_R that is singly produced (at the Tevatron), and in which neutron-antineutron oscillation placed important constraints. Dirac gauginos would also loosen these constraints in this context. (Our flavor estimates suggest that \tilde{b}_R lighter than TeV is disfavored, but perhaps this is possible with a more special flavor structure.)

a “diquark”, even up to its baryon number. In this way, the general discussion and constraints of flavor structure for scalars with $d_R d_R$ diquark couplings given in [62] applies to the natural SUSY setting here. In particular, [62] discusses the different plausible hierarchical structures for such couplings and the mechanisms underlying their safety from FCNC and CP-violating constraints. As is shown there, it is indeed plausible for the \tilde{t}_R^c to have order one couplings to light quarks, and therefore be singly produced.⁶

Baryon-number R-symmetry, by forbidding the A-term, also makes for an interesting signature for pair-production of \tilde{q}_L since they can no longer mix with \tilde{t}_R^c after electroweak symmetry breaking. These squarks do not directly couple to quark pairs, unlike \tilde{t}_R^c , which means that each \tilde{q}_L will decay into two third generation quarks *plus* a quark pair.

2.8.3 Electric Dipole Moments

With the baryon R-symmetry as described above, it is straightforward to check that all the soft SUSY breaking parameters can be made real by appropriate rephasing of fields in eqn. (2.36). Therefore there are no new CP-violating contributions to electric dipole moments from this Lagrangian. However, as discussed in section 2.6, we should more realistically add third-generation flavor-changing corrections to any such Lagrangian, which can contain new CP-violating phases. However, as discussed

⁶A similar analysis is possible for (non-R-symmetry) baryon-number preserving RPV and loosening the constraints from lepton-number violation tests such as neutrinoless double- β decay.

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there these new terms will be suppressed by $O(\epsilon^2)$. In this way, we expect non-vanishing but highly suppressed new contributions to EDMs. These observations for natural SUSY are closely related to the observations made in [60, 63, 64].

Chapter 3

Natural SUSY at the LHC

In this chapter, we will discuss the phenomenology of natural SUSY at the 7 and 8 TeV LHC, and argue that some part of the parameter space has thus far eluded detection. In section 3.1, we focus on the light subsystem of stops, a sbottom, and a neutralino with R-parity, in order to probe collider bounds as of 1/fb's worth of data. We find LHC bounds as of 1/fb at 7 TeV are mild and large parts of the motivated parameter space remain open. We also make brief remarks about other phenomenological regimes of natural SUSY.

In sections 3.2 through 3.5, we discuss collider signatures of natural supersymmetry with baryon-number violating R-parity violation. We argue that this is one of the few remaining viable incarnations of weak scale supersymmetry consistent with full electroweak naturalness. We show that this intriguing and challenging scenario contains distinctive LHC signals, resonances of hard jets in conjunction with rela-

tively soft leptons and missing energy, which are easily overlooked by existing LHC searches. We propose novel strategies for distinguishing these signals above background, and estimate their potential reach at the 8 TeV LHC. We show that other multi-lepton signals of this scenario can be seen by currently existing searches with increased statistics, but these opportunities are more spectrum-dependent.

The work in this chapter is borrowed from the author’s work in [4, 5], and the research was performed in collaboration with Raman Sundrum, Andrey Katz and Scott Lawrence. The reader should further note that this chapter is to be read from a historical perspective, and so more modern searches and search strategies performed on 20/fb’s worth of 8 TeV data are absent from this thesis.

3.1 Collider Phenomenology of R-Parity Conserving Natural SUSY

In this section we will demonstrate three things:

1. After $\sim 1/\text{fb}$ LHC running, there are analyses that put non-trivial constraints on the motivated parameter space of natural SUSY.
2. Nevertheless, very large parts of the parameter space, fully consistent with electroweak naturalness, are still alive.
3. The most constraining searches for natural SUSY, so far, are not always those

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optimized for more standard SUSY scenarios.

While natural SUSY has many interesting experimental regimes, we will not attempt a complete study in this chapter. Rather, we will focus on the simplest natural setting, and do enough of the related phenomenology to make the points (1 – 3) above.

The central consideration for natural SUSY phenomenology is the great reduction in new colored particles, squarks, compared with standard SUSY scenarios. In natural SUSY we keep just the minimal set of superpartners below TeV needed to stabilize the electroweak hierarchy. This has the effect of lowering the new physics cross-sections substantially. Furthermore, in standard SUSY settings one typically entertains higher superpartner masses than is technically natural, partly a result of renormalization group running of super-spectra from very high scales, and partly in order to radiatively raise the physical Higgs boson mass to the experimentally measured value. In our bottom-up natural SUSY, with less UV prejudice, we have only tried to constrain the spectrum from the viewpoint of naturalness and the little hierarchy problem. As we have seen, other mechanisms for raising the physical Higgs mass work well within natural SUSY. Therefore, we favor the regime where stops are lighter than 500 GeV, while gluinos may be so heavy as to be irrelevant in the early LHC. The decay products of lighter stops in natural SUSY can easily fail to pass the harsher cuts on missing energy and jet energies used in searches optimized for heavier superpartners.

In the following subsection, we will study in detail collider constraints which one can put on the most minimal scenario, namely light stops and sbottom (predominantly left-handed) with a neutralino at the bottom of the spectrum. We will briefly review the Tevatron constraints on this scenario and further analyze the constraints arising from LHC data at $\mathcal{L} \sim 1 \text{ fb}^{-1}$. In the subsequent subsection we will survey other variations, but will not go into details. We leave this to later in the chapter and to future work.

3.1.1 Neutralino and Squarks

In this section we will simplify considerations even further to the effective theory of eqn. (2.29), where we have just a bino LSP lighter than the squarks. The neutralino might more generally be an admixture of several neutral gauge eigenstates, but phenomenologically this is not very relevant; the neutralino is simply a way of invisibly carrying off odd R-parity from colored superpartner decays. The bino is a good proxy for such a general neutralino. In the remainder of this section, we focus on the collider phenomenology of eqn. (2.29).

One further simplification we make is to take the stops and sbottom to be roughly degenerate. If there is no substantial left-right mixing, this is a very good approximation in the left-handed (LH) sector. The mass difference between the LH stop and

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sbottom is given by

$$\Delta m^2 \approx \frac{m_W^2 \sin^2 \beta}{2m_{\tilde{q}_L}} \quad (3.1)$$

Since this splitting comes from $SU(2) \times U(1)$ D-terms, it is proportional to the mass of the W . Usually if the splitting is dominated by D-terms, one gets that $m_{\tilde{b}} > m_{\tilde{t}}$. This might suggest that one should also consider a decay mode $\tilde{b} \rightarrow W^{(*)}\tilde{t}$. However this would imply a three-body decay, which is therefore highly suppressed. More importantly, stop decay modes $\tilde{t} \rightarrow W^{(*)}\tilde{b}$ can become competitive with other stop decay modes, if it is forced to proceed through an off-shell top. However this can happen only if the left-right mixing between the stops is large, and we will neglect this possibility further.

Before considering the LHC, we should note several D0 searches which directly address this scenario. The first relevant search looks for b-jets + \cancel{E}_T [65]. This search constrains the sbottom mass to be higher than 247 GeV if the neutralino is massless. The constraints become weaker if the neutralino is heavier, but unless there is an accidental degeneracy, the lower bounds on the sbottom are still around 200 GeV. Another search of D0 looks for stops, which are pair-produced and further decay into $b l + \cancel{E}_T$ (where this decay mode is assumed to have 100 % branching fraction). The most updated search used events with opposite flavor pairs [66]. This search also bounds the stop mass at 240 GeV if the neutralino is massless and for massive neutralino (without any accidental degeneracy with the stop) the bound is of order 200 GeV, depending on the neutralino mass.

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CDF has a more elaborate search, where it looks for $t\bar{t} + \cancel{E}_T$. This search was performed in monoleptonic [67] and hadronic [68] channels. The bounds one can put on production cross sections from these two measurements are comparable to each other, but too weak to constrain natural SUSY with its small squark cross section.¹

Now let us turn our attention to the LHC searches. As we will see, the bounds from the LHC are not very stringent (partly due to an insufficient number of dedicated searches). This is in part because, with the exception of an ATLAS top-group search for $t\bar{t} + \cancel{E}_T$ (which we will discuss later), there are no dedicated searches for this scenario. However there are several general searches, which can be sensitive to the stop/sbottom/neutralino subsystem we are studying here. We explicitly considered the following list of searches:

1. jets + \cancel{E}_T (including simple \cancel{H}_T search and an α_T search) [70, 71]
2. jets + \cancel{E}_T with b-tag [72, 73]
3. lepton + jets + \cancel{E}_T [74]
4. OS dileptons + jets + \cancel{E}_T [75]
5. lepton + jets with b-tag + \cancel{E}_T [76]

In order to estimate the bounds on our scenario, we simulated events and checked

¹Hereafter we do not consider a mass range of stop below 200 GeV, where the stop mostly decays off-shell. This intriguing possibility is not yet excluded, and the reader is referred to [30, 69].

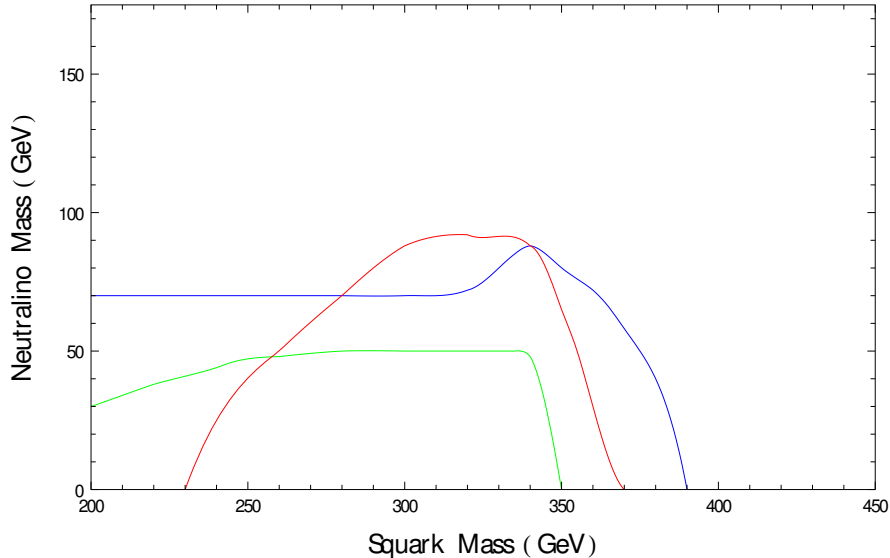


Figure 3.1: Exclusion curves for our minimal model, eqn. (2.29), from three relevant searches as a function of masses for squarks and neutralino. We assume roughly equal masses for all three squark species, two stops and a sbottom. The green line represents exclusion by α_T search, the blue line is an exclusion by \cancel{H}_T search and the red one is exclusion by $t\bar{t} + \cancel{E}_T$ search.

the acceptances within the channels listed above.² The events were generated and decayed with `MadGraph 5` [80] and further showered and hadronized with `Pythia 6` [81]. The events were reconstructed with `FastJet-2.4.4` [82]. We calculated all the NLO cross-sections with `Prospino 2` [83] and reweighted all the events appropriately. We ran each spectrum assuming that the mass difference between the stops and sbottom are negligible. Given the mass difference, eqn. (3.1), this is not a bad approximation. (One can of course play with the mass difference between \tilde{t}_L and \tilde{t}_R , still keeping the spectrum natural, but we did not perform this study.)

²Whenever both ATLAS and CMS have performed closely overlapping searches, we have considered just the CMS representative. The relevant ATLAS searches are [77, 78]. We also did not explicitly simulate an additional CMS jets + \cancel{E}_T search which takes advantage of the m_{t2} variable [79], since it is not expected to have a good acceptance in our case.

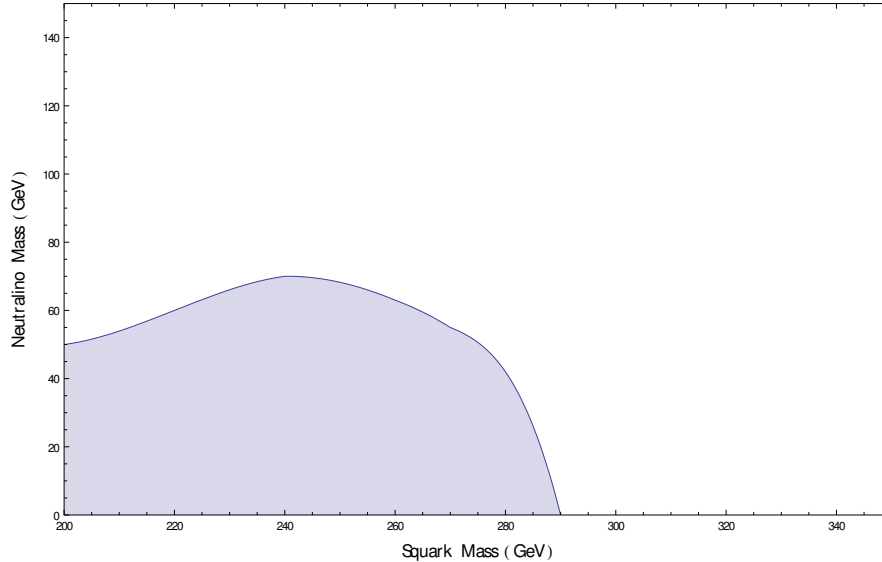


Figure 3.2: Exclusion of a single sbottom due to jets + \cancel{H}_T search as a function of a sbottom and neutralino masses.

We find that all the searches listed above, except searches for jets + \cancel{E}_T , do not put any interesting bounds on the subsystem that we are discussing here. The searches in leptonic modes put extremely harsh cuts on the H_T of the entire event, and therefore easily miss the stops in the range between 200 and 400 GeV, while the cross sections in the higher mass range are far too small. Unfortunately, the ATLAS search for jets + $l + b$ -tag + \cancel{E}_T [76] also does not add interesting constraints, mostly because it is tuned to detect (or exclude) gluinos above 400 GeV which further cascade-decay to bottom, top and neutralino.³ The jets + \cancel{E}_T searches indeed put interesting constraints on our stop/sbottom/neutralino subsystem and we show our bounds in Fig. 3.1. We found that more than half of all the relevant events which contribute

³This search claims that it looks for events with 4 b-jets with lepton and \cancel{E}_T , however demands only a single b-tag in the event selection. One can probably put more interesting bounds by demanding more than one b-tag.

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to the exclusion come from sbottom production and decays. In fact, even a single sbottom without any stops would be excluded all the way to 300 GeV with the same searches for massless neutralino. For more general neutralino mass the single-sbottom exclusion plot appears in Fig. 3.2. By comparison, the same searches put no bounds on a single stop (or even both stop species), due to extremely bad acceptance in this range of masses.

This, however, does not conclude the full list of searches. There is an additional search by ATLAS, which looks precisely for $t\bar{t} + \cancel{E}_T$ in a monoleptonic channel [2]. This particular search puts almost no bound for production of a single species of stop, but the picture is different when we have both stops roughly degenerate (with double the production cross sections). We show the final exclusion plots on Fig. 3.1, where the exclusion due to $t\bar{t} + \cancel{E}_T$ search is given by the red curve. On Fig. 3.3 we show the ranges excluded by this search if we split the masses of the stops (neutralino mass is assumed to be zero). Note that this exclusion is comparable to the exclusion one gets with the jets + \cancel{E}_T search.

3.1.2 Overview of Some Other Possibilities

3.1.2.1 Gluinos

Because of their large color charge and the high multiplicity of their decay products, the biggest phenomenological consideration for the 7 TeV LHC is the presence

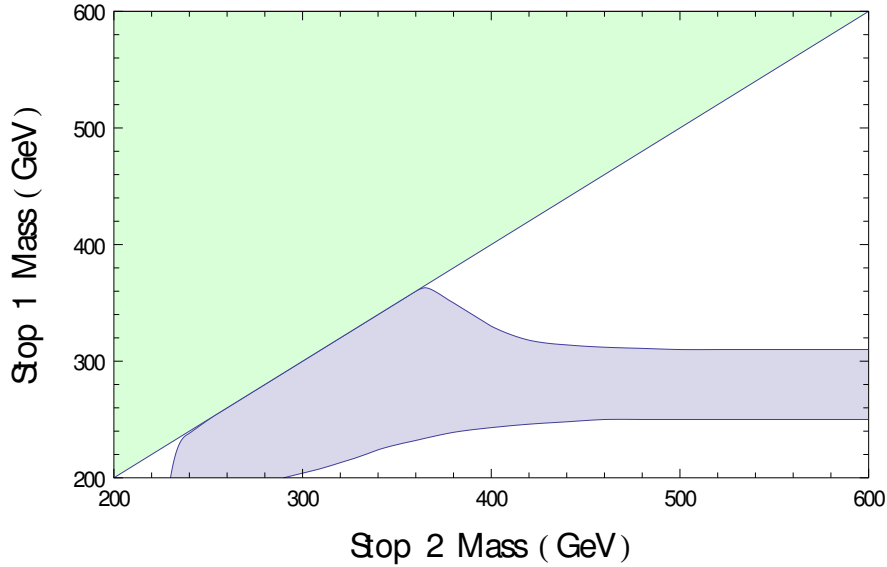


Figure 3.3: Exclusion curves for two stops with different masses from the ATLAS search for $t\bar{t} + \cancel{E}_T$ in monoleptonic channel [2]. The neutralino mass is assumed to be zero. Note a narrow band between 250 and 290 GeV for the first stop which is excluded even when the second stop is very heavy. This is the region where the sensitivity of the search is maximized.

or absence of gluinos below a TeV. Production cross-section grows significantly as gluinos are taken below 1 TeV in mass, and gluinos decay exclusively into the third generation squarks. This scenario has been studied both in cases when the gluino decays into a sbottom (see abovementioned searches for jets plus \cancel{E}_T with a b-tag) or into a stop [76]. However, there are reasons to believe that a monoleptonic channel with one b-tag, which was used in the ATLAS search is not optimal. The model of gluinos decaying exclusively to stops was carefully studied in [26] and it was found that with luminosity of 1 fb^{-1} gluinos up to 650 GeV can be discovered, if one takes advantage of a few competitive channels, like same-sign dileptons, multileptons with or without b-tags (and sometimes multiple b-tags).

3.1.2.2 Collider-Stable Squarks

One can also consider the very simple scenario with \tilde{t}_L , \tilde{t}_R and \tilde{b}_L at the bottom of the superpartner spectrum. With R-parity, the lightest scalar (either stop or sbottom) is stable. We should of course assume that it decays at some point (for example it can decay into a gravitino, or through some tiny R-parity violating coupling) in order to avoid constraints from searches for ultra-heavy hydrogen atoms [84], but this still allows squarks with cosmological lifetimes [85]. If this is the case, \tilde{t} or \tilde{b} should show up as R-hadrons at the LHC. Recent bounds from CMS impose severe constraints on this scenario if the lightest superpartner is a stop [86].⁴ Results of these searches imply that a stable stop in the mass range between 100 and 800 GeV is excluded if its production cross section is of order 10^{-2} pb. Comparing these results to theoretically expected production cross sections [87], we find that these cross-sections are expected for a single stop with mass up to 600 GeV. However in our case, we should at least multiply the cross sections by a factor of three (we have two stops and at least one sbottom), rendering the bound to somewhat higher than 600 GeV. Therefore, if one takes the little hierarchy problem seriously up to ~ 10 TeV, this scenario is disfavored.⁵

⁴Even though the authors of this paper do not interpret their results in terms of stable \tilde{b} , there is no reason to believe that this bound would be dramatically different.

⁵However, as noted in section 2.5, the effective theory of eqn. (2.28) is a useful departure point for adding in RPV phenomenology.

3.1.2.3 Neutralino and Chargino LSPs

A safer option is to consider the effective theory of eqn. (2.29), where we see the Higgsinos providing natural neutralino/chargino candidates. If the neutralino is the LSP, bounds on stable charged or colored particles are evaded. Of course, the neutralinos and charginos may more generally be an admixture of several electroweak gauge eigenstates.

In detail, the presence of a chargino as an NLSP makes a phenomenological difference, but we believe that it is less decisive in the present context. The difference from the scenario described in section 3.1.1 is that on top of the decay modes $\tilde{t} \rightarrow t\tilde{\chi}^0$ and $\tilde{b} \rightarrow b\tilde{\chi}^0$ we have already considered, we will have competing modes $\tilde{b} \rightarrow t\tilde{\chi}^\pm$ and $\tilde{t} \rightarrow b\tilde{\chi}^\pm$. Since we are mostly interested in the region of mass parameters where the top-quark mass is far from negligible, we conclude that the decay mode $\tilde{b} \rightarrow t\tilde{\chi}^\pm$ will be mostly suppressed due to the phase space. Therefore, introducing the chargino at the bottom of the spectrum will usually have a mild effect on sbottom decay modes and the constraints which come from these decays (mostly jets plus \cancel{E}_T). However the stops decay modes will be altered compared to our discussion in section 3.1.1, since the decay mode $\tilde{t} \rightarrow b\tilde{\chi}^\pm$ is now phase space unsuppressed. The chargino will consequently decay to the neutralino and W (maybe off-shell). Therefore, this will look roughly similar to the decay modes of a regular stop, even though the kinematics might be different. If the chargino and neutralino are quasi-degenerate, then the decay modes of stops very much resemble those of sbottoms, thereby effectively

increasing the production cross sections for jets plus \cancel{E}_T and making the constraints somewhat more stringent than what we find in section 3.1.1.

While the above are reasonable deductions, explicit simulation is still required when charginos are light. We again leave this to future work.

3.2 R-Parity Violating Phenomenology

In light of the current LHC bounds on R-parity conserving SUSY, it is difficult to envision any other viable version of SUSY which is consistent with full electroweak naturalness (that is, absence of EW fine-tuning). By contrast, we argued above that natural SUSY easily evades the most stringent LHC constraints with integrated luminosity $\mathcal{L} \sim 1 \text{ fb}^{-1}$, as also demonstrated in [30–32]. Later dedicated searches for R-parity conserving natural supersymmetry have appeared, better constraining particular spectra with [76] or without [88] light gluinos, however the natural parameter space is still quite open.

Natural SUSY has an even wider significance, in that it beautifully illustrates the general theme of how a “top-partner” can algebraically cancel destabilizing top-quark radiative corrections to the Higgs potential. This relates it to the theory and phenomenology of fermionic top-partners [89], appearing in non-supersymmetric Little Higgs (see for review [90, 91]) and Twin-Higgs models [92, 93]. In this sense, light stop searches fit into the broader program of testing whether *any* top-partner

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is helping to stabilize the weak scale. The LHC is the first experiment in history that can test naturalness on such a broad front, in a relatively comprehensive and well-defined way. *Either a discovery of such top-partners at the natural scale of a few hundred GeV, or even their exclusion to high confidence, would constitute a significant scientific finding.* In the previous chapter, we have argued that for this grand and challenging experimental undertaking, one should free the natural SUSY setting from too much UV prejudice and anticipation, lest this lead to overlooking experimental opportunities *now* and because UV considerations have not led to any sharp no-go “theorem”. We have carved out a simple theoretical framework that facilitates this. The remainder of this chapter will discuss important but widely overlooked signals that follow straightforwardly from this perspective, as well as the combination of methods that can separate them from SM background.

In particular, although most experimental searches for natural supersymmetry have concentrated on the R-parity conserving case, we consider here the case of (baryon-number violating) R-parity violation (RPV). The plausibility and attractiveness of this scenario was argued in the previous chapter from a number of viewpoints and considerations (also see [94] and references therein for recent models of spontaneous RPV). The spectrum of RPV and R-parity conserving natural SUSY can be quite similar. If the theory is completely natural we expect both species of stops and at least the left-handed sbottom with masses of order ~ 400 GeV or lighter. Gluinos can be naturally twice as heavy if they are Majorana fermions, and even heavier if

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they are (part of) Dirac states [60]. That is, we cannot guarantee the gluino to be experimentally accessible in the near future. However, if we are lucky and Majorana gluinos are light enough in RPV natural SUSY, they can produce spectacular signatures in same-sign dileptons [95]. Here, we assume more minimally that the stops and sbottom mandated by naturalness are the only accessible colored superpartners.

RPV is distinct from R-symmetry conservation because phenomenological viability does not require a neutral superpartner to be at the bottom of the SUSY spectrum, since superpartners are allowed to decay into SM particles. Therefore EW gauginos can easily be heavier than the stops and the sbottom, having no significant impact on the phenomenology.

Non-minimal Higgs degrees of freedom are subtler. Higgsinos are usually assumed to acquire mass from the same “ μ -term” that also contributes to Higgs scalar potential. Higgs naturalness then requires Higgsinos not much heavier than ~ 200 GeV. (However, in the previous chapter we argued for a bottom-up description in which Higgsinos can be much heavier.) We will show in section 3.3 that light Higgsinos can remain relatively well-hidden in the RPV context. On the other hand, extra Higgs *boson* degrees of freedom of SUSY can be heavier without compromising naturalness. We therefore only keep the SM Higgs scalar in our study of stop/sbottom phenomenology. Finally, there are by now stringent bounds on the SM Higgs mass, and even tentative hints of its presence at ~ 125 GeV. Theoretically accommodating the Higgs mass in *high-energy* models has been an increasing challenge ever since

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LEP2, but there are certainly interesting ideas for doing this. By contrast, a 125 GeV Higgs mass is straightforwardly accommodated within our bottom-up natural SUSY framework, deferring the full UV description of physics lying outside experimental reach. In particular, we do not restrict stop/sbottom masses by their radiative contributions to the physical Higgs mass, since there may well be other contributions from unknown heavier sources.

Collider signatures of RPV SUSY are largely dictated by the detailed structure of RPV interactions, which cannot be anarchical (for a review see [56]). Either baryon number or lepton number should be conserved to avoid prompt single proton decay. While lepton number violation (LNV) is interesting by itself and deserves more study, it has already meaningful constraints from the LHC, since it mostly leads to leptons and taus in the final states, which are relatively easy to spot. Baryon number violation (BNV) is experimentally more challenging than LNV, resulting in jetty final states and suffering from enormous QCD and $t\bar{t}$ backgrounds. We will focus on this challenging scenario of two stops and a sbottom at the bottom of a BNV SUSY spectrum. Not only does this fill a gap in SUSY searches, but it also shares several features with, and insights into, other top-partner searches. This spectrum was considered earlier in [61] in the context of the CDF “ Wjj anomaly” [96]. Although the anomaly was later refuted by D0 [97], as well as by a similar CMS search [98], this paper was an important step in understanding of collider signatures of the minimal spectrum. Here, we will elaborate on several points briefly touched on in [61], broaden the motivations

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and scope, and detail new search strategies.

A particular difference with [61] is that we will assume that BNV is governed by small couplings and therefore we will neglect single-resonance production of superpartners, concentrating on pair-production. Such smaller couplings make the theory more straightforwardly safe from low-energy precision data. We will argue that it is theoretically very plausible that a stop is the lightest superpartner, which can decay into a pair of jets. Needless to say, by itself this is an extremely challenging signature, given relatively small production cross sections and absence of any “interesting” features in the event, e.g. leptons or missing transverse energy (MET). However, one can take advantage of production of the heavier sbottom and stop which further cascade decay into the lightest stop, emitting W , Z and/or higgs (on- or off-shell) along the way. These events are more promising, because they can potentially contain leptons and MET. Nonetheless, existing cut-and-count searches are not optimized for signatures like this and generally overlook them. They do not take advantage of the most important qualities of these events: hard jets reconstructing a pair of *resonances*, in conjunction with leptons and/or MET which is relatively soft compared to the top quark background.

We will substantiate these claims, and use them to craft a search strategy for the most promising and robust of these cascades. We will show that the backgrounds are under control and $\sqrt{s} = 8$ TeV LHC can have a good reach for these events. We will also discuss other channels, which can be promising, but where the back-

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grounds are not easy to estimate with theoretical tools. The alternative possibilities for the lightest superpartner, a sbottom or Higgsino, are also plausible but even more phenomenologically challenging, and we defer their consideration from this thesis.

The chapter is organized as follows. In the next section we review the BNV RPV natural SUSY scenario and reduce it to its most LHC-relevant features, thereby arriving at a useful “simplified model”. In section 3.4 we study the various “charged current” (W) channels and relevant backgrounds, and roughly estimate which of these channels is viable. In section 3.5 we perform explicit simulations of signal and background in the most promising of these channels and discuss cuts (which are quite different from standard SUSY searches) in greater detail. In section 3.6 we briefly discuss “neutral current” (Z, h) cascade decays between stops. While getting a substantial number of these events is only possible in a subset of stop/sbottom/higgsino spectra, they can be quite spectacular, and indeed the multi-lepton CMS search [1] already has an appreciable sensitivity. They remain an exciting discovery channel for the future with more statistics. Finally in section 3.7 we conclude.

3.3 Reduction to the RPV Simplified Model

3.3.1 Spectrum

As we argued in the previous chapter, the only superpartners robustly required by naturalness to lie under 500 GeV are (in EW gauge basis) the $\tilde{t}_R, \tilde{q}_L \equiv (\tilde{t}_L, \tilde{b}_L)$ stops

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and sbottom, and \tilde{h}_u, \tilde{h}_d higgsinos (if their mass arises from a μ term). Along with a SM Higgs boson h , we shall consider these the only new particles substantially accessible to the 7 – 8 TeV LHC with moderate luminosity.

Colored superpartners have strong production cross-sections, which suggests that we focus our searches on them. As can be seen in table 3.1, even these strong cross-sections peter out for squark masses above 500 GeV, so that the natural regime for the spectrum is also our only hope for direct visibility. We will argue that light higgsinos are typically a complication in these searches, either mild or major depending on the spectrum, but rarely do they present a spectacular new opportunity. For now we simply neglect them, but return at the end of this section to better justify this position.

The gauge-basis squark states are non-trivially related to the mass-eigenstates after EW symmetry breaking, due to two effects, the splitting of \tilde{t}_L from \tilde{b}_L , and the mixing of \tilde{t}_L and \tilde{t}_R . The first of these effects is given by the sum of F and D term potentials,

$$m_{\tilde{t}_L} - m_{\tilde{b}_L} \approx \frac{m_t^2}{m_{\tilde{b}_L} + m_{\tilde{t}_L}} - \frac{m_W^2 \sin^2 \beta}{m_{\tilde{b}_L} + m_{\tilde{t}_L}}. \quad (3.2)$$

The $\tilde{t}_L - \tilde{t}_R$ mixing arises from a possible SUSY-breaking A -term, $A\tilde{t}_R^c h_u \tilde{q}_L$. This results in mass eigenstates \tilde{t}_1 and \tilde{t}_2 (with $m_{\tilde{t}_1} < m_{\tilde{t}_2}$ by convention) related to the gauge-eigenstates by an angle $\theta_{\tilde{t}}$. Combining these effects, the sbottom is either the middle or the lightest of our squarks. We focus on the former case, with a spectrum

$\tilde{t}_2 - \tilde{b} - \tilde{t}_1$. Note also, that even though the mass splitting between \tilde{b} and \tilde{t}_1 is essentially a free parameter, it cannot be too big if we play by two rules: (a) we keep the lightest squark heavier than the top quark, which would otherwise change the phenomenological possibilities, and (b) keep the spectrum natural and production cross-sections for \tilde{b} appreciable by not making it too heavy.

3.3.2 Couplings

The central novel interaction being considered is the BNV RPV coupling,

$$\mathcal{L} \supset \frac{\lambda''_{3IJ}}{2} \tilde{t}_R^c d_R^I d_R^{cJ} + \text{h.c.} \quad (3.3)$$

enabling stop decay to SM quarks.

The exact flavor structure of these couplings is constrained by a variety of low-energy flavor and precision tests. From the viewpoint of LHC visibility the central issue is whether the quarks in the dominant BNV couplings carry heavy flavor (b_R quarks) or not. We have argued in the previous chapter that in either case, low-energy constraints can be satisfied within quite plausible UV flavor paradigms. Relatedly, in the case where all three generations of squarks are present, suitable flavor paradigms have also been studied in [99, 100]. We shall therefore consider two cases for stop decay to quarks via BNV: (a) one b quark and one light quark, and (b) two light quarks.

We will always consider the generic possibility that there is at least modest non-

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vanishing A -induced mixing, such that both $\tilde{t}_{1,2}$ inherit BNV couplings to quarks via their \tilde{t}_R component. Similarly, they both have weak couplings to the sbottom via their \tilde{t}_L component. Furthermore, mixing also leads to $\tilde{t}_1^c \tilde{t}_2 Z$ and $\tilde{t}_1^c \tilde{t}_2 h$ couplings, determined by the A -terms and mixing angles. We assume small BNV couplings, $\lambda''_{3ij} \ll 1$, so that in general squarks will only decay through such interactions if decay by W, Z or h emission is kinematically suppressed, as for example is obviously the case for \tilde{t}_1 (which, recall, we are considering as the lightest superpartner).

Since $\lambda''_{3ij} \ll 1$ is the most straightforward way to comply with low-energy constraints,⁶ it is important to ask how small these couplings can be without resulting in displaced vertices at the LHC from a long \tilde{t}_1 lifetime. In order to not have a displaced vertex, we need c/Γ to be less than about 1 mm. The expression for the distance traveled before decay by a pure \tilde{t}_R particle (ignoring mixing) is

$$L \sim (1 \text{ mm}) \left(\frac{300 \text{ GeV}}{m_{\tilde{t}_R}} \right) \left(\frac{(2.5 \cdot 10^{-7})^2}{\sum \lambda''_{3ij}{}^2} \right) \quad (3.4)$$

Thus, for 300 GeV squarks, we need λ'' to be roughly bigger than $2.5 \cdot 10^{-7}$. If the BNV couplings are smaller than this bound, we will have events with *jets* emerging from displaced vertices, which can further help discriminate against background. We will tackle the more challenging case in this chapter, by assuming that the BNV couplings are strong enough that \tilde{t}_1 decays are prompt.

⁶Although one can easily satisfy low-energy constraints with $\lambda''_{3ij} \ll 1$, there is still a concern that BNV can wash out the cosmological baryon asymmetry, if this is generated in the early Universe. One can try to turn this into a mechanism for actually generating the baryon asymmetry below the EW scale [101–104]. A new robust approach can be found in [105].

3.3.3 Higgsinos

We are now in a position to understand how higgsinos might affect the LHC physics. We have discussed how the spectrum of squarks can be produced and then cascade decay by EW boson emission, with a final prompt BNV decay to quarks. Higgsinos of comparable mass to the squarks allow these steps to potentially be bypassed, by opening up alternative squark decays to higgsinos.

The simplest case would be if the higgsinos were even a little heavier than the stops and sbottom. Since direct EW production has substantially lower cross-section, such higgsinos would be phenomenologically irrelevant. But if the higgsinos are lighter than the heaviest stop, then \tilde{t}_2 decays via EW emission or BNV can be substantially degraded by decay to \tilde{H}^+b . (The alternate decay to \tilde{H}^0t is likely to be phase-space suppressed.) In turn, \tilde{H}^+ will decay (via \tilde{t}_1 and BNV) to three jets. In this way, the higgsinos will degrade events with leptons from (possibly off-shell) W, Z , and add events with extra b jets. This is the basic complication we alluded to earlier: higgsinos can force us to look in multi-jet events, without spectacularly high p_T , with resonances obscured by combinatorial background, and with only the handle of several b jets.

But fortunately, the higgsinos can easily *not* degrade sbottom decays even if they happen to be lighter than the sbottom but heavier than the lightest stop, because the only sbottom decay to higgsinos (for small bottom Yukawa coupling) is to \tilde{H}^-t , which is likely highly phase-space suppressed. Unfortunately if the Higgsinos are at the bottom of the spectrum they will be produced in abundance in $\tilde{t}_1 \rightarrow b\tilde{H}^+$ decays.

This decay mode does not affect the W production but complicates the resonance reconstruction from the jets. This is one of the reasons that we focus on sbottom charged current decays in this chapter: the phenomenology of sbottom $\rightarrow W^{(*)}\tilde{t}_1$ is largely “immune” to higgsinos if they are *not* the lightest SUSY particles.

We proceed by dropping higgsinos from the discussion as part of arriving at our simplified model of stops and sbottoms in sections 3.4 and 3.5. As discussed above, this will only modestly affect our central channel of sbottom production and cascade decay, and is the best case for the other channels (but dependent on the spectrum). We will return however to the possibility of Higgsinos in the spectrum in section 3.6 since higgsinos can easily dramatically reduce the contributions of the neutral current decay $\tilde{t}_2 \rightarrow Z^{(*)}\tilde{t}_1$. In particular the higgsinos can suppress a yield of neutral current decays in an otherwise spectacular multilepton channel.

3.4 Signals and Strategy of Search

One finds the highest production cross sections for the lightest particles, which would imply in our case a search for pair-production of the lightest stop with subsequent decays into four jets. However a search for resonances in 4-jet events is very challenging at the LHC [106], because it has to deal with a big uncertain QCD background, and even the multijet trigger is probably not 100% efficient in this case.⁷

⁷An analogous search for resonances in 6-jet events [107] has a better reach, but it is relevant only for light gluinos in an RPV spectrum.

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Therefore it is fruitful to concentrate on longer cascades, which involve Higgs or EW boson emission (either on- or off-shell). This means we consider production of heavier states, \tilde{b} , \tilde{t}_2 and subsequent transitions

$$\tilde{b} \rightarrow W^{(*)}\tilde{t}_1, \quad \tilde{t}_2 \rightarrow Z^{(*)}\tilde{t}_1, \quad \tilde{t}_2 \rightarrow h^{(*)}\tilde{t}_1 . \quad (3.5)$$

While the first process in eqn. (3.5) is fairly robust, the branching ratio of the two other processes is model-dependent. The relative rate between the second and the third process in eqn. (3.5) is determined by the couplings of the stops to Z and h and by phase space effects. While *the neutral current decays can have spectacular multi-lepton signature* (see Sec. 3.6 for a detailed discussion) it might also happen that *the second stop's mass is between 400 and 500 GeV, rendering the production cross-section tiny* (see Table 3.1). Moreover most of the spectacular signatures come from the decays into Z rather than the Higgs, though it would be very nice to eventually observe the Higgs in these new physics processes. We might however find ourselves in the situation that the mixing angle between the stops, $\theta_{\tilde{t}}$ is large and Higgs transitions are preferred. Therefore, it is fair to say that the charged current transition is the robust and spectacular channel at the 7-8 TeV LHC and we will give it most of our attention.

Before we continue with a detailed analysis of the cascade decays, we note that the table 3.1 pair-production cross sections for stops were calculated at the NLO with **Prospino 2.1** [83]. Sbottom production cross sections are usually slightly bigger due to electroweak corrections. These numbers will further help us in our numerical

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$m = 230 \text{ GeV}$	$m = 250 \text{ GeV}$	$m = 270 \text{ GeV}$	$m = 300 \text{ GeV}$	$m = 400 \text{ GeV}$	$m = 500 \text{ GeV}$
8.3 pb	5.3 pb	3.4 pb	1.9 pb	0.34 pb	0.08 pb

Table 3.1: Pair production cross sections for stop at NLO for center of mass energy $\sqrt{s} = 8 \text{ TeV}$. Sbottom production cross-sections are very similar, slightly bigger though due to electro-weak effects.

estimates.

Charged-current $\tilde{b} \rightarrow W\tilde{t}$ transitions with a subsequent RPV decay of the stop into two jets were first addressed in [61] in the case of resonant production of sbottom. (Pair-production was also briefly considered, but was not the primary focus.) Although resonant production is not categorically excluded by the bounds on RPV, it requires strong enough BNV to raise FCNC and $n - \bar{n}$ oscillation concerns. However we focus on pair production with RPV decay mediated by couplings which can be much smaller than one, and therefore safer from low-energy tests.

In this case the most spectacular signature shows up when both W s decay leptonically, leading to a signature $l^+l^-jjjj + \cancel{E}_T$, where the jets reconstruct two resonances with equal masses (see diagram in Fig 3.4). What should be our search strategy for these events? Performing cut-and-count search on events which reconstruct resonances is probably not ideal. However we can try to reconstruct resonances with the following steps:

- Find events with 2 isolated leptons and moderate \cancel{E}_T (the latter should be non-zero to remove the background from DY dilepton production).

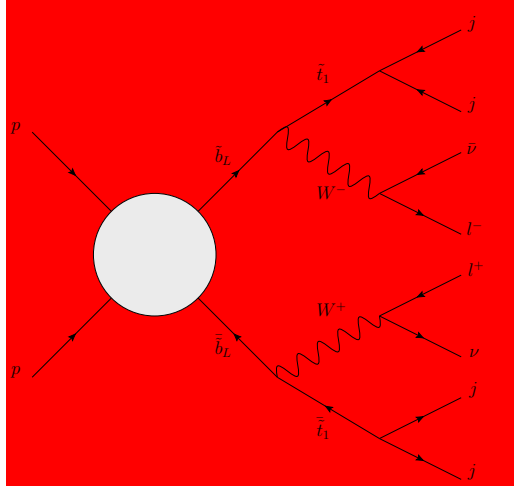


Figure 3.4: The sbottoms are pair-produced and undergo charged-current decay. When both W s (either on- or off-shell) decay leptonically, they leave a spectacular signature of two leptons + jets, which reconstruct two equal-mass resonances. We analyze this signal in Sec. 3.4 and 3.5.

- Cluster the jets with sufficiently big radius (otherwise there is a danger that we lose the hadronic activity which reconstructs the resonance and thereby get edges instead of peaks).
- If the event contains 4 jets (or more), try all possible pairings between the jets, and pick up the combination which minimizes the difference between the reconstructed invariant masses. Discard the event if the minimal possible mass difference is too big. This step is essentially identical to the standard multi-jet resonances search [108].

Unfortunately our events with 2 leptons, MET and multijets have an appreciable background, on top of which we are looking for our bumps. This background is heavily dominated by dileptonic $t\bar{t}$ (including $l\bar{l}$ decay modes). One can show that

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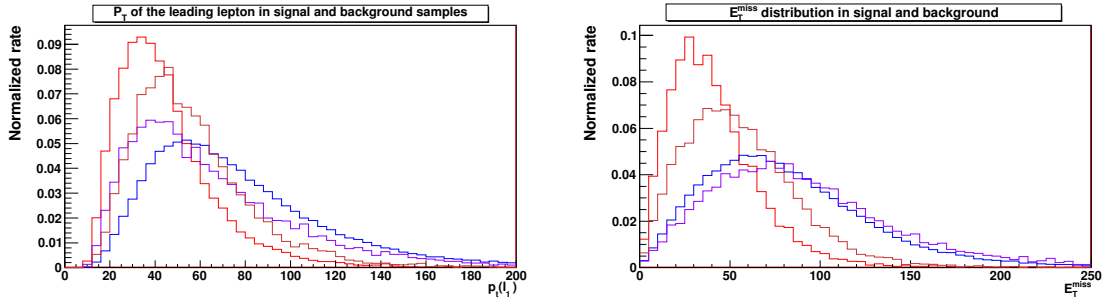


Figure 3.5: Leading lepton p_T and E_T^{miss} distribution in signal and background events. Light red and dark red curve stand for the signal spectra 1 and 2 (see Table 3.2). The blue line represents the distribution in dileptonic $t\bar{t}$ background and the violet line represents the $l\tau$ background (which we simulate separately since it has slightly different kinematics). See Sec. 3.5 for details of simulations.

with an adequate choice of cuts all other backgrounds ($Z \rightarrow \tau_l \tau_l + \text{jets}$, DY dileptonic production with jets, $WW + \text{jets}$) are highly subdominant to $t\bar{t}$, and we will discuss it in more detail in the next section. Production cross section for dileptonic $t\bar{t}$ exceeds our signal by two orders of magnitude, and even though the extra jets in these events do not come from resonances, reconstructing “by accident” two pairs of jets with similar invariant masses is common. The above mentioned steps, plus standard cuts for the overall hardness of the event, are still not enough in order to see clear bumps on top of this continuous $t\bar{t}$ background after $\sqrt{s} = 8$ TeV run. We therefore use other, less standard discriminators to distinguish the signal from the background.

There are two additional important features which distinguish our signal from the background. Usually in a dileptonic $t\bar{t}$ event, hardness of the entire event correlates with the hardness of the leptons and the E_T^{miss} . This happens because the W is often boosted in the rest frame of the decaying top. However it is not the case in the signal.

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As we have explained in Sec. 3.3, naturalness and visibility motivate mild splittings between the stop and the sbottom, usually so small that they do not allow emission of the *on-shell* W . Even if emission of the on-shell W is allowed it typically has little boost in the rest frame of the decaying sbottom. This results in relatively small $p_T(l)$ and E_T even if the event overall is very hard. We demonstrate the distribution of E_T and the transverse momentum of the leading lepton in signal and background events on Fig. 3.5. This immediately suggest that just cutting on the tail of high E_T and high $p_T(l_1)$ should be a decent discriminator between the signal and the background. We checked it explicitly and it indeed removes a fair portion of the background. We will use a refined version of this discriminator below.

It turns out one can do even better than just cutting on a high E_T and high $p_T(l_1)$ events. As we explained, the key feature of the $t\bar{t}$ events is that usually the leptons and the E_T are correlated with the hardness of the event, or S_T defined as

$$S_T \equiv \sum_i p_T(j_i) + \sum_k p_T(l_k) + E_T . \quad (3.6)$$

On the other hand in the signal events these quantities are mostly uncorrelated. For this purpose we define the following variables:

$$r_l \equiv \frac{p_T(l_1)}{S_T}, \quad r_{E_T} \equiv \frac{E_T}{S_T} . \quad (3.7)$$

One should also prefer using these variables rather than E_T , $p_T(l_1)$ because they are dimensionless and therefore cutting on them we do not introduce an explicit scale to the problem. We expect these quantities in the signal events to be in general small.

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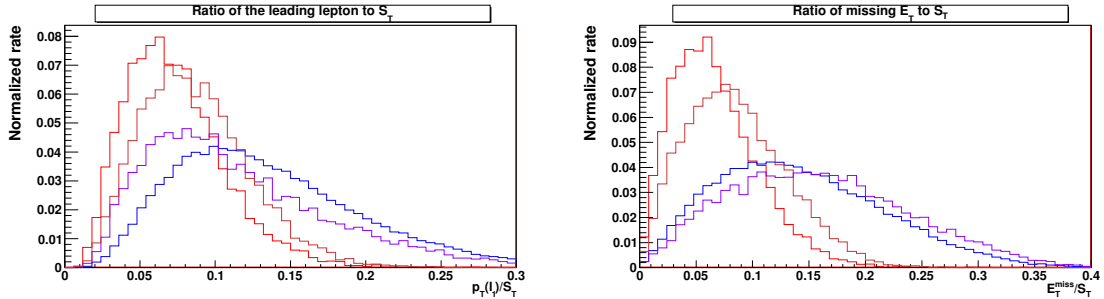


Figure 3.6: Distribution of r_l (on the left) and r_{E_T} (on the right) as defined in eqn. (3.7). Light red and dark red curves stand for the signal spectra 1 and 2 (see Table 3.2), blue curve represents the dileptonic background and violet represents $l\tau_l$ background. See Sec. 3.5 for details of simulations.

We plot these variables for signal and background events in Fig. 3.6 and it follows this expectation. Moreover, we see that r_l and r_{E_T} are slightly less dependent on the particular spectrum than $p_T(l_1)$ and E_T . In the next section we show that using this strategy together with the cuts on variables (3.7) we will have an excellent reach after the $\sqrt{s} = 8$ TeV, $\mathcal{L} = 20 \text{ fb}^{-1}$ run.

To summarize, the dileptonic channel is an excellent channel for the charged current decays. We will elaborate on a feasibility of this search explicitly and make more comments on the background behavior and shapes in section 3.5.

Finally we briefly comment on semileptonic and all-hadronic decay modes. The latter will probably be very hard to utilize, since it just results in multijets (up to 8 or even more) events without any evident handles like isolated leptons or missing E_T . In the semileptonic search one has signal events with isolated lepton, moderate E_T and at least 6 jets, typically resulting in small E_T , high H_T events. We will not try to elaborate on the feasibility of the cut-and-count search in this channel, because

	$m_{\tilde{b}}$	$m_{\tilde{t}_1}$	$\sigma(\tilde{b}\tilde{b}^*)$
1	250 GeV	186 GeV	5.7 pb
2	270 GeV	189 GeV	3.7 pb
3	300 GeV	217 GeV	2.0 pb

Table 3.2: Benchmark points for the charged-current decay search. The production cross sections are given only for sbottoms .

evidently these searches are not optimal (they cannot reconstruct the resonances in jetty channels, taking advantage of the most interesting feature of the RPV signal) and basically already exist in some form both in ATLAS and CMS collaborations (they do not yield any interesting bounds though). It would be interesting to see though how the variables (3.7) can be used in these searches to improve further the reach and suppress the backgrounds.

3.5 Details of Event Simulations, Backgrounds and Reach

To estimate the feasibility in dileptonic channel as explained in section 3.4 we analyzed three benchmark points with masses presented in Table 3.2. For the signal we simulated parton-level events with `MadGraph 5` [80] for three signal benchmark points given in Table 3.2 and showered and hadronized them with `Pythia 8` [109].

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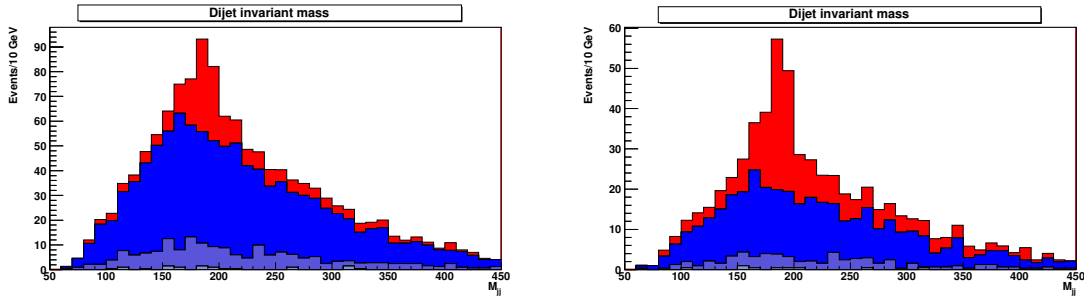


Figure 3.7: Signal and background events for the benchmark point 1 after $\mathcal{L} = 20 \text{ fb}^{-1}$. Red represents the signal, blue the dileptonic $t\bar{t}$ background, violet is $t\bar{t}$, $l\tau_l$ background and grey is $t\bar{t}$, $\tau_l\tau_l$ background. On the LH side plot we do not impose b-veto, while on the RH side plot we do. We conservatively assume b-tag efficiency $\sim 40\%$.

We wrote down a tailored model in `Feynrules` [110] for `MadGraph 5` to capture the effects of the simplified model described in Sec. 3.3. For the background we simulated the events in `MadGraph 5` and showered with `Pythia 6` [81]. In order to capture correctly the effects of extra-jets (which are crucial for our analysis) we matched our samples up to two jets with the MLM procedure at 55 GeV. Events were clustered with `FastJet 3` [82, 111].⁸

Following the discussion in section 3.4 we reconstruct our events and impose the following cuts:

1. Cluster all the hadronic activity with anti- k_T algorithm, clustering radius $R = 0.7$. Relatively large clustering radius is dictated by the fact that we are looking for the resonances, and smaller radius usually leads to losing relevant hadronic activity. The clustering radius is not optimized, but radii of order $R \sim 1.0$ are likely to be the most adequate.

⁸Detector effects are neglected, but the results are sharp enough to survive full treatment.

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2. Demand precisely two isolated leptons (carrying more than 85% of the p_T in the cone around the lepton with radius $R = 0.3$) in each event. We demand $p_T(l_1) > 20$ GeV and $p_T(l_2) > 10$ GeV.⁹ The leptons should have $|\eta| < 2.5$. We discard the event if the leptons have same flavor *and* 81 GeV $< m_{ll} < 101$ GeV to remove the background from Z + jets events.
3. Demand that the event is sufficiently hard, $S_T > 400$ GeV as defined in eqn. (3.6) and $E_T > 35$ GeV.
4. Require four or more hard jets in the event with $p_T(j_4) > 30$ GeV. This requirement is natural since we are trying to reconstruct two resonances of \tilde{t}_1 , which both decay into two quarks.
5. Using the variables in eqn. (3.7), demand $r_{E_T} < 0.15$ and $r_l < 0.15$.
6. Try all possible pairings between four leading jets, and pick up the combination which minimizes the difference between the reconstructed invariant masses. Discard the event if the minimal possible mass difference is bigger than 10 GeV.¹⁰

⁹The logic of the cut on the p_T of these leptons is dictated by trigger demands. Unfortunately the trigger information is not public. However relying on the logic of $\sqrt{s} = 7$ TeV run, we hope that the events with these leptons should be triggered on with sufficiently high efficiency, namely more than 90% [1]. Parenthetically we notice that if the threshold on the p_T of the leading lepton can be lowered, the results that we performed can be further improved. Moreover, some of the events can be triggered on because they have sufficient H_T or 4 or more sufficiently high- p_T jets. We do not try to take into account the events which do not pass these lepton requirement, however lots of them can be “salvaged” since they pass other triggers and the ideal search will have to combine several different triggers.

¹⁰These cuts are not optimized, but it is also not very different from 7.5% of the resonance mass which was used in [106]. We explicitly checked our results with respect to variation of this cut. The results are rather stable as long as this cut does not exceed $\sim 25 - 30$ GeV. We leave further optimization of these cuts to the experimentalists as it is also going to be affected by jet energy resolution.

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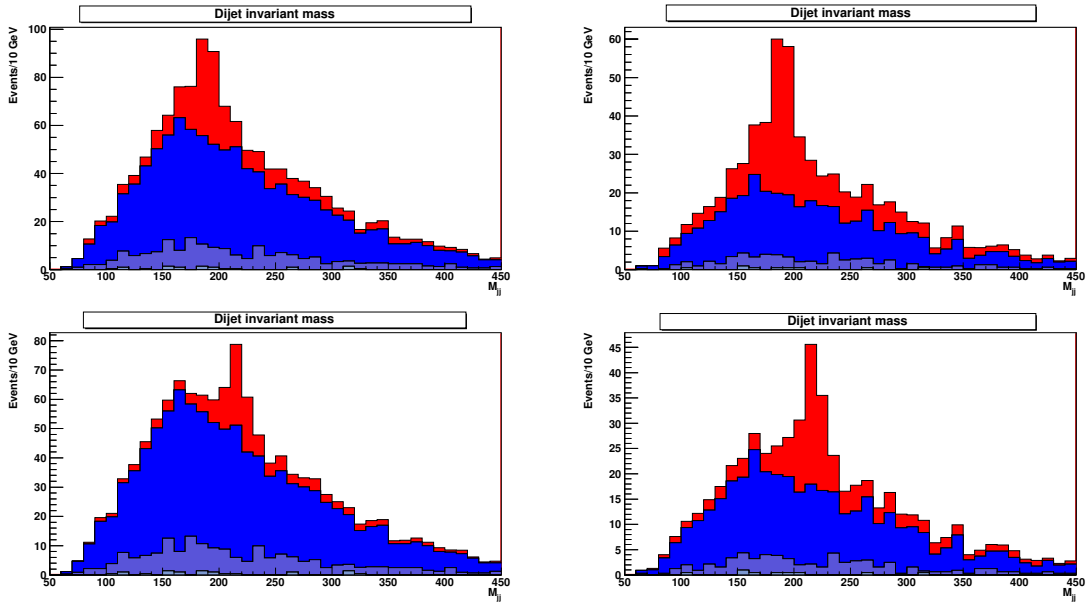


Figure 3.8: Signal and background events for the benchmark points 2 (up) and 3 (down) after $\mathcal{L} = 20 \text{ fb}^{-1}$. Red represents the signal, blue the dileptonic $t\bar{t}$ background, violet is $t\bar{t}, l\tau_l$ background and grey is $t\bar{t}, \tau_l\tau_l$ background. On the LH side plot we do not impose b-veto, while on the RH side plot we do. We conservatively assume b-tag efficiency $\sim 40\%$.

If the event has five or more jets with $p_T > 25 \text{ GeV}$, try all possible pairings of two and three jets. If we get better results when taking the fifth jet into account, use the best combination which minimizes the mass difference between the reconstructed objects.

7. Look for resonances in the reconstructed dijet invariant mass.

Before we present the results of our simulations we discuss the backgrounds to our analysis. Clearly the most formidable background is dileptonic $t\bar{t}$ (also including the leptons coming from leptonic τ decays). Naively, one could also worry about $Z \rightarrow (\tau_l\tau_l) + \text{jets}$, as well as DY l^+l^- production and $W^+W^- + \text{jets}$. We do not

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simulate these backgrounds and we rely on experimental results which found these backgrounds negligible to $t\bar{t}$ with the cuts which were very similar to ours. First, it was shown in [112] that the background $Z \rightarrow (\tau_l\tau_l) + \text{jets}$ becomes completely negligible to $t\bar{t}$ when the third hard jet is required in the event. We also see from Table 1 in [112] that the DY background is subdominant to $t\bar{t}$ at least by factor of 5 after requiring at least two hard jets and $E_T > 35$ GeV. However we demand four hard jets in our events, which is supposed to decimate the DY dileptonic production and render it completely negligible to $t\bar{t}$. Therefore we will further concentrate on $t\bar{t} + \text{jets}$ as the dominant background to our signal, and neglect the subdominant channels.

Since we are looking for bumps in the dijets invariant-mass distribution, it would first be helpful to understand what effects our cuts have on the backgrounds and how they shape the background distribution. Not surprisingly, before all the cuts m_{jj} in the background is a smoothly falling distribution which is peaked around 50 GeV (this peak is carved by our demand from each jet to have $p_T > 25$ GeV. Further demands on hardness of the event move this peak to significantly higher values of masses. For example a cut on $H_T \equiv \sum_i p_T(j_i) > 400$ GeV (which does a reasonable job with suppressing the backgrounds) moves this peak to the vicinity of 200 GeV, which is uncomfortably close to the mass scale where we are looking for our resonances. Cuts on the S_T and the p_T of the softest necessary jet have a similar effect, therefore we choose the cut in points (3) and (4) discussed above to be

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relatively moderate (one could choose way harder cut which would still remove more background than signal events, for the price of moving the peak of the background distribution to higher masses). On the other hand, cuts on r_{E_T} and r_l in point (5) do not have this effect, they relatively uniformly discriminate against background from all the invariant masses (as this distribution have already been shaped by H_T , S_T and $p_T(j_4)$). Note also, that this terrain of models is still relatively unexplored and the new physics can hide at very low masses. It is very natural to expect that the lightest stop has a mass of $\mathcal{O}(200 \text{ GeV})$ or even lighter. Therefore we emphasize that given a choice of cuts, one should always prefer harsher cuts on r_{E_T} and r_l , preferring as mild as possible cuts on “hardness variables”, namely $E_T, H_T, S_T, p_T(j)$. This approach is ultimately dictated by our attempt to avoid carving spurious bumps on the background distributions.

Armed with this understanding of the background behavior we turn to the actual analysis. For this purposes we assume the NLO $t\bar{t}$ production cross section at $\sqrt{s} = 8 \text{ TeV}$ to be 205 pb (this result is taken from MC@NLO [113,114] with default scale choice). Practically, the cross section for $t\bar{t}$ production is going to be slightly bigger, tallying up to 230 pb (from NNLL resummation) with $\mathcal{O}(10\%)$ uncertainty [115,116]. However we take NLO results to be consistent in our estimates comparing the signal, which we know at the NLO, to the background. As we will see our results are strong enough, that increasing the background moderately without changing the signal cross section by no mean changes our conclusions. We present the results of our simulations

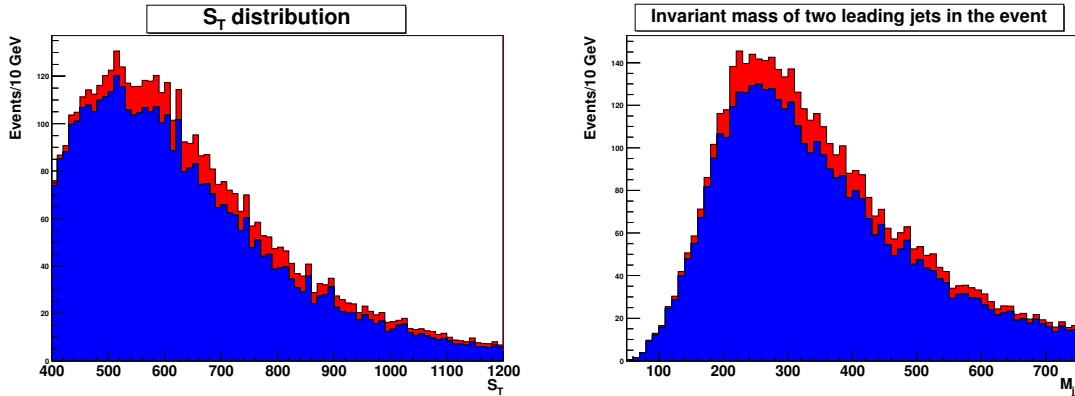


Figure 3.9: Distribution of S_T (left) and invariant mass of two leading jets invariant mass (right) in signal and background events. We use the benchmark point (3) for these plots. Only dileptonic $t\bar{t}$ background is plotted. We use all the same cuts as in resonance search *except the point (6)*. Using these distributions as discriminator in a cut-and-count search will be at least very challenging.

in Figs. 3.7 and 3.8. In both cases we present the results with and without b-veto. As explained in Sec. 3.3 the lightest stop can have decay modes which either include a b-jet or not. Since the dominant background is $t\bar{t}$, one can achieve much better reach if the signal events contain only light quarks. In this case we perform b-veto which further reduces the background, leaving the signal intact. The plots on the RH side of Figs. 3.7 and 3.8 correspond to this picture. On the other hand, if the stop decays to a b-quark + a light flavor, we cannot perform the b-veto (LH side plots), but the backgrounds are still under very good control and one has a reasonable discovery reach for all three benchmark points.

Finally, we briefly explain why our resonance search in this channel is much more efficient than simple cut-and-count searches. One could naively expect that the discriminators that we use to separate the signal from the background are sufficient for

an easier cut-and-count search. This naive expectation is not true. To illustrate this we plot in Fig. 3.9 distributions of S_T and the invariant mass of the two leading jets. We use all the same cuts as in our resonance-searching analysis except the step (6). We see in Fig. 3.9 that using a cut-and-count strategy will be, in the best case, extremely challenging and will require detailed understanding of the *normalization of the background*. Therefore we conclude that di-resonance search is the optimal strategy here.

3.6 Brief Comments on Neutral Current Decays

Until now we were very detailed in describing the searches for charged-current decays. However the charged current decays reveal only part of the full picture. As we explained in Sec. 3.3, in certain spectra the charge-current decay will dominate the collider signatures, while in other spectra, the neutral current decays will leave the most spectacular signatures (see processes (2) and (3) in eqn. 3.5). We will briefly comment on these processes (see Fig. 3.10 for a summary of diagrams) in the current section, but we will be less detailed because as we will see the most important potential discovery channel (the multilepton channel) is already considered by the CMS. Other channels are not expected to be as strong as multileptons and will mostly favor cut-and-count strategy rather than resonance reconstruction due to the high multiplicity

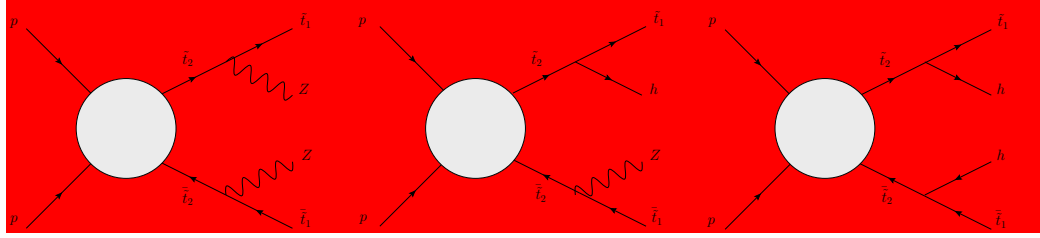


Figure 3.10: Neutral current decay processes which can be relevant for the most minimal spectrum.

of jets.

In neutral-current decay events we have pair-produced heavy stops decaying into their light partners emitting two Z 's, two Higgses, or one Z and one Higgs. Z 's can be emitted either on- or off-shell, while Higgs decays are very unlikely to proceed off-shell due to very strong bottom Yukawa suppression. Higgs decays are rarely spectacular, the most important Higgs decay mode (assuming that its mass is ~ 125 GeV, as current experimental hints suggest) is $h \rightarrow b\bar{b}$ and the third important is $h \rightarrow gg$. If the heavy stop decays to the light one emitting two additional jets (b-quarks or gluons) we get a very challenging event without any obvious handles, i.e. without MET and/or leptons. The second important Higgs decay mode $h \rightarrow WW^*$ (BR bigger than 20%) and the fourth important ($h \rightarrow \tau\tau$) are more distinctive, but the resulting going rate of the higgs into two leptons (either through W or through τ) is smaller than the rate of Z . Therefore we will mostly concentrate on neutral current decays with the Z in final state.

Both Z 's decaying leptonically can probably be considered a “golden” channel, even though the branching fractions are very small, $\sim 0.5\%$. These rare but very

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spectacular events with up to four isolated leptons and lots of energetic jet activity can be probed by multilepton searches. The backgrounds for these events from the SM are extremely small and therefore even observation of few events can be considered discovery (or, alternatively, even the benchmark points which are expected to yield very few events in certain channels can be excluded).

To illustrate this point we explicitly compared a yield of three different benchmark points with the results of analysis [1]. This CMS analysis is very special because it has a very low p_T threshold for the leptons (leptons, which are as soft as $p_T = 8$ GeV are considered, higher p_T s are required for trigger leptons though). The expected yields of all three different points are presented in Table 3.3. We also compare the yields with the theoretical expectations and experimental results, which are quoted from [1]. In all these points we assumed, for simplicity of estimation, 100% branching ratio for $\tilde{t}_2 \rightarrow \tilde{t}_1 Z$ decay. However, more realistically, such decays would account for only an order one (but highly spectrum-dependent) fraction of \tilde{t}_2 decays. If higgsinos lie between the two stops, than \tilde{t}_2 decay will likely be dominated by $\tilde{t}_2 \rightarrow b\tilde{H}^+$ (mediated by top Yukawa coupling). Moreover, as discussed in section 3.3, the sbottom should lie between the stops, so that \tilde{t}_2 should also undergo charged current decays to sbottom. For squeezed enough spectrum such decays may produce even softer leptons than the neutral current decays, and could be more challenging to detect. For the estimates below, we ignore such subtleties and study the ansatz of neutral-current dominance. We will see that with enough statistics, even with significant depletion

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to other channels, neutral current decays might provide spectacular evidence for supersymmetry.

Clearly with this assumption of 100% branching fraction to Z the first two benchmark points in Table 3.3 are excluded. However full exclusion plots are beyond the scope of this chapter, because they would demand more refined simulations and more well defined assumptions about the Higgsinos, sbottoms and the mixing angle between the stops. Note that if the mass splitting between the stops is smaller than 125 GeV the decay will almost never proceed through h^* due to smallness of bottom Yukawa. Alternatively, pushing the heavy stop mass above 300 GeV, we begin to lose the sensitivity in this channel, therefore we find that there is a big part of parameter space which is far from exclusion and probably is not expected to make a significant contribution to the multilepton channel even after $\mathcal{L} = 20 \text{ fb}^{-1}$ run.

The yields are still smaller than one would naively expect. Most of the events either do not have an isolated leading lepton harder than 20 GeV, or the next to leading lepton harder than 10 GeV, which renders them unsuitable for a dileptonic trigger. As expected the most populated bins are those with low E_T and high H_T , the H_T in our events comes from $\tilde{t}_1 \rightarrow jj$ decays, while the MET is merely instrumental. The prediction in $4l$, high H_T low MET channel are already in tension with the experiment because of extremely low (essentially non-existent) SM background.

It is also expected that there is relatively high chance to lose at least one of the leptons due to isolation criteria, or due to high rapidity, so the bins with three

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leptons turn out to be even more informative than the bins with four isolated leptons. Interestingly, the only bin where we could predict a significant excess, low MET and high H_T without Z , is precisely the bin where CMS observes a non-negligible excess of events, recording 11 events where 4.5 ± 1.5 events are expected. Again, the yield of our benchmark points is too high to explain the excess, unless non-vanishing (but also not overwhelming) BR for $\tilde{t}_2 \rightarrow \tilde{H}^+ b$ is assumed. We do not try to claim that it is an anomaly, or that we try to explain this excess, however it is probably an interesting channel to watch when $\sqrt{s} = 8$ TeV data is analyzed. We do not try to estimate the reach of the multilepton channels for $\sqrt{s} = 8$ TeV since the event yield both of the signal and the background is very low, and the backgrounds are very hard to estimate. Nonetheless it is clear that the LHC right now is on the edge of probing an interesting region, and high H_T low MET channels are of particular interest to watch.

The second important channel in this category, is one where one of the Z 's decays leptonically, while its counterpart decays invisibly. This channel has higher branching ratio of order 3% but has bigger backgrounds. It also has signatures which naively resemble R-parity conserving SUSY - namely opposite-sign dileptons with jets and E_T . If the mass gap between the stops is sufficiently big, such that Z decays on-shell, the signature is leptonic $Z + \text{jets} + E_T$, naively resembling one of the well-known signatures of R-parity conserving gauge-mediation [117]. Alternatively, if the Z decays off-shell we find opposite-sign same-flavor pairs and E_T , again very similar

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Spectrum	$m_{\tilde{t}_1} = 180$ GeV	$m_{\tilde{t}_1} = 185$ GeV	$m_{\tilde{t}_1} = 189$ GeV	Exp.	Err.	Obs.
Selection	$m_{\tilde{t}_2} = 245$ GeV	$m_{\tilde{t}_2} = 260$ GeV	$m_{\tilde{t}_2} = 277$ GeV	[1]	[1]	[1]
4l $E_T > 50$, $H_T > 200$	< 0.1	< 0.1	< 0.1	0.018	0.005	0
4l $E_T > 50$, $H_T < 200$	< 0.1	< 0.1	< 0.1	0.2	0.07	1
4l $E_T < 50$, $H_T > 200$	3.6	3.5	1.8	0.006	0.001	0
4l $E_T < 50$, $H_T < 200$	1.0	0.6	0.4	2.6	1.1	1
4l $E_T > 50$, $H_T > 200$, Z	< 0.1	< 0.1	< 0.1	0.22	0.05	0
4l $E_T > 50$, $H_T < 200$, Z	< 0.1	< 0.1	< 0.1	0.79	0.21	1
4l $E_T < 50$, $H_T > 200$, Z	1.0	1.0	2.5	0.83	0.33	1
4l $E_T < 50$, $H_T < 200$, Z	0.3	0.2	0.4	37	15	33
3l $E_T > 50$, $H_T > 200$	0.3	0.4	0.7	5.0	1.3	8
3l $E_T > 50$, $H_T < 200$	< 0.1	< 0.1	< 0.1	27.0	7.6	30
3l $E_T < 50$, $H_T > 200$	16.9	14.1	8.8	4.5	1.5	11
3l $E_T < 50$, $H_T < 200$	< 0.1	< 0.1	< 0.1	144	36	123
3l $E_T > 50$, $H_T > 200$, Z	< 0.1	< 0.1	0.2	18.9	6.4	20
3l $E_T > 50$, $H_T < 200$, Z	< 0.1	< 0.1	< 0.1	134	50	141
3l $E_T < 50$, $H_T > 200$, Z	2.5	2.3	4.5	19.2	4.8	15
3l $E_T < 50$, $H_T < 200$, Z	< 0.1	< 0.1	< 0.1	764	183	657

Table 3.3: Expected yields of events in the “golden” channel in the multilepton search of [1], $\sqrt{s} = 7$ TeV with assumption $\text{BR}(\tilde{t}_2 \rightarrow Z^* \tilde{t}_1) = 100\%$. Channels with high H_T , low MET are the most informative. All possible leptonic decays of Z^* have been simulated including leptonic τ s. We define the Z window such that the invariant mass of the OSSF pair is $76 \text{ GeV} < m_{ll} < 106 \text{ GeV}$. We do not simulate channels with hadronic τ s due to difficulties to mimic one-prong τ_h detection with our theoretical tools. Three right columns cite the results of [1] where Exp. stands for the expected yield of the SM, Err. is the systematic error as it was estimated by the experimentalists, and Obs. stands for the observed number of events at $\mathcal{L} = 4.98 \text{ fb}^{-1}$.

to a standard R-parity conserving signature with decay chain proceeding through a low mass slepton.

Unfortunately these resemblances are not close enough to be useful. For example, we explicitly checked the event yield for all three reference points in table 3.3 in analysis [118] and found that the yields are far below values which one needs in order to have exclusion (usually yielding one event or even less in each of the signal

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regions). These channels are so different because they typically have very low missing E_T and relatively soft leptons, which makes the discovery very difficult with simple cut-and-count experiments.

Nonetheless one can exploit these events if different strategies are used. If the Z goes off-shell we get precisely a signature which is identical to the charge-current decay in the dileptonic channel (see Fig. 3.4) and was analyzed in details in Secs. 3.4 and 3.5. One can use precisely the same techniques and if an excess in multilepton is found and confirmed in the dilepton + MET channel, this can be an excellent cross-check to establish if the excess indeed comes from $\tilde{t}_2 \rightarrow Z^* \tilde{t}_1$ decay chains.

One can also suggest similar searches when Z decays on-shell. In this case the search should be modified, because removing the events in the Z -window would wash our signal out. Maybe this search is also feasible, however with theory tools it will be hard to estimate reliably the background which comes from $(Z \rightarrow l^+ l^-) +$ jets with instrumental E_T . Therefore, we point out that this search can be tried, but we do not make any conclusions about the backgrounds.

Although all other channels of Z -decays have much bigger branching ratios, we do not see any clear strategies for how these can be utilized. The case where one Z decays leptonically and the second hadronically would suffer from an enormous $l^+ l^-$ DY background (if Z is on-shell, we get $Z +$ jets, which is even worse), without even modest MET. One faces a similar problem if one of the Z 's decays invisibly and the second one decays hadronically.

3.7 Outlook

In this chapter, we analysed some of the phenomenological aspects of both R-parity conserving and violating natural SUSY. As we have shown, the constraints on natural SUSY, even with the most conservative approach, R-parity with neutralino at the bottom of the spectrum, are very mild. With these assumptions, the 1/fb data still allow a spectrum consistent with electroweak naturalness.

This conclusion strongly suggests the future research program in this direction. Evidently, current LHC searches are not optimized for this scenario. It would be interesting to see how one can increase the sensitivity of the current searches and vary the cuts so as to allow better acceptance for natural SUSY. We expect that there is a strong opportunity for searches optimized to natural SUSY to make great inroads into discovery or exclusions with the full 8 TeV data set.

Another promising avenue one can take has to do with R-parity violation. As we emphasized in sections 3.2 through 3.5, RPV is highly motivated if natural SUSY indeed describes the physics immediately beyond the SM. Even the signals of RPV SUSY with lepton-number violation can be quite challenging if squark decays into leptons involve τ . The signals of RPV SUSY with baryon-number violation are even more challenging, because the decays of the squarks will mostly results in jets. However, as pointed out for the case with baryon R-symmetry, squarks can have more spectacular decays into several jets, including two with heavy flavor. Current exotica searches [119] put very mild bounds on these RPV scenarios and it is very interesting

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if one can improve these search strategies to get better sensitivity to the new physics.

The main results of this chapter are new searches that we propose for the 8 TeV LHC and the novel techniques that we find useful to discriminate the new physics signal from the background. These searches are motivated by natural SUSY with renormalizable baryon-number violating RPV interactions. This yields a set experimental signatures which are not efficiently captured by current LHC searches. We pointed out that the signatures of charged-current decays can be discovered by a di-resonance search with two additional leptons. The leptons, being soft and accompanied by modest MET are essentially useless for cut-and-count search, but provide us an excellent handle and allow us to see the dijet resonances despite small production cross section. It is in fact surprising how efficient these searches can be. Moreover, there is a good reason to believe that one can do even better than our estimates. Although we tried to choose an adequate clustering radius, we neither optimized it nor used “grooming” techniques. Simple optimization and using “trimming” [120], which is the most adequate grooming technique for these purposes, can further improve the sensitivity.

To efficiently discriminate the signal from the background (which is almost completely composed of $t\bar{t}$ events) we propose to use a set of rather novel cuts combined with more standard tools. On one hand we are cutting on the hardness of the MET, leptons and the entire event, which is a standard tool, but we also emphasize that these cuts should not be too harsh. On the other hand we propose to put an upper

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cut on r_l and r_{E_T} variables, which is a novel way to discriminate signal events where the hardness of the special objects in the event (leptons and E_T) is uncorrelated with the overall hardness of the event.

The discriminators that we propose to use are not completely unknown, for example CMS was using a much weaker version of r_l as one of the variables in artificial neural network in their dileptonic analysis [121] (whose reach is hard to estimate because it uses a cumbersome multi-variate approach). The variable r_{E_T} has not yet been use in any analysis that we are aware of. We point out that use of these tools can go much beyond the particular analyses that we propose, and can be used in cut-and-count experiments as well as in resonance searches. We point out that these techniques are suitable in any new physics scenario where one finds transitions between the states with small mass splitting (see e.g. [122, 123]).

Finally we point out that the searches that we propose form one more important step in the program to map the collider signatures of RPV natural SUSY (for previous works see [61, 95]). This is in general a challenging subject, and even R-parity conserving signatures often demand non-standard approaches [124–128], because regular jets+MET searches simply fail. The subject of RPV natural SUSY has received little attention thus far, and its collider signatures are still largely unexplored (see however searches by CMS where very little or no MET in the signal region is required [129, 130]). It would be interesting to study more signatures characterizing natural SUSY with baryon-number violation or lepton-number violation, as well as

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the more challenging spectrum orderings for the system introduced in the previous chapter (lightest superpartner being sbottom or Higgsino).

We are very hopeful that searches along the lines described above will soon be performed at the LHC, and will help further our understanding of the grand hypothesis of naturalness.

Chapter 4

Accidental SUSY and RS: The Story Above 10 TeV

The strongly coupled regimes of gauge theories are the home of many diverse phenomena in quantum field theory which are often missed in perturbative studies of those theories, such as confinement and the growth of extra dimensions. Strong coupling has been used as a tool for realizing several mechanisms in field theory, and such lines of inquiry have suggested new solutions to the hierarchy problem [11, 131, 132]. However, with the recent discovery of a SM-like Higgs boson [133, 134] and the absence of any discoveries of new physics to stabilize the electroweak scale, this motivates even more strongly the need to test the postulate of naturalness very thoroughly. Recently, there have been many models which add in a minimal module of new physics capable of stabilizing the little hierarchy from the electroweak scale

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up to a new physics scale above the reach of the LHC. The supersymmetric version of this resolution to the little hierarchy problem, natural SUSY, has been the subject of chapters 2 and 3 of this thesis, and LHC search strategies capable of probing such a scenario have been studied extensively both here and elsewhere [135–143].

However, with such a low cutoff for such models, one very rapidly requires a UV completion in order to study the viability of the model. Supersymmetric extensions are certainly possible, but can themselves come with UV tuning issues [144], motivating one to consider alternative UV completions. It seems natural for us to consider UV completions to natural SUSY which involve the ingredients of those mechanisms which solve the big hierarchy problem. Strong coupling is such an ingredient, and indeed, using it to solve the little hierarchy problem has been proposed [3] and studied through the use of the AdS/CFT correspondence [145]. The mechanism in play responsible for the stabilization of the little hierarchy and the big hierarchy simultaneously is known as accidental supersymmetry, and will be the subject of this and the following chapters.

In this chapter, we study the mechanism of accidental SUSY as a potential UV-completion of natural SUSY by virtue of studying a toy model in RS. In section 4.1, we review the mechanism of accidental SUSY as discussed in [3]. In section 4.2, we explore a toy model of accidental SUSY in RS and lay the foundation for BSM model-building. A crucial tool exploited in this chapter is the AdS/CFT correspondence, reviewed in, for example, [146, 147].

The work in this chapter is based off of unpublished research performed in collaboration with Raman Sundrum [7].

4.1 Intro to Accidental SUSY

Accidental SUSY is a mechanism for composite Higgs models which has been proposed to stabilize the little hierarchy. One realization of this idea is to couple a sector which breaks SUSY at high energies to an approximate CFT in such a way that the SUSY-breaking terms flow to 0; in other words, use the strongly coupled sector to give all relevant SUSY-breaking operators a large positive anomalous dimension [148]. We therefore consider the spectrum to be accidentally supersymmetric if there are no global singlet relevant operators (GSROs). Note that in full generality, accidental SUSY is *more* than this; subleading terms push us in the right direction to restore SUSY *faster* in the IR [3]. This is a sort of “focusing” effect. Therefore, in the theory below the compositeness scale, the composite states appear to have an only mildly broken supersymmetry. In fact, accidental supersymmetry can provide approximately supersymmetric states a loop factor down from the compositeness scale, $m^2 \sim \frac{1}{16\pi^2} \Lambda^2$ as shown from the CFT perspective in [3], and from the AdS perspective below.

Suppose we had a 4d theory containing a supersymmetric perturbative visible sector V which weakly gauges a strongly-coupled approximate SCFT with a very low compositeness scale Λ_{comp} , which in turn communicates with a SUSY-breaking sector

with some high-scale SUSY-breaking operators at characteristic scale Λ . Suppose further that V contains some set of scalar global singlet operators \mathcal{O}_i with associated scaling dimensions Δ_i . One generically would expect that at Λ , the SCFT would mediate SUSY-breaking to V , generating any allowable terms in the action for V . Integrating out the SUSY-breaking sector modifies the visible-sector Lagrangian to

$$\mathcal{L} = \mathcal{L}_V(\Lambda) + \mathcal{L}_{SCFT} + g_i(\Lambda)\mathcal{O}_i \quad (4.1)$$

where \mathcal{O}_i are generally all allowed operators and g_i may naturally contain various loop factors. Running down to the compositeness scale yields the Lagrangian

$$\mathcal{L} = \mathcal{L}_V(\Lambda_{comp}) + \mathcal{L}_{SCFT} + g_i(\Lambda) \left(\frac{\Lambda_{comp}}{\Lambda} \right)^{\Delta_i-4} \mathcal{O}_i \quad (4.2)$$

where $\Delta_i \equiv \Delta_{\mathcal{O}_i}$. We see that for $\Delta_i < 4$, the effects of SUSY-breaking become more important in the IR, whereas for $\Delta_i \geq 4$, the effects can naturally be parametrically small. Therefore, we conclude that the mechanism of accidental SUSY can be at play in this scenario when all global singlet operators \mathcal{O}_i of V are marginal or irrelevant; in other words, there are no global singlet relevant operators.

The above discussion is a weak statement of accidental SUSY; clearly, SUSY-breaking could be inherently large, as parameterized by g_i , and operators with $\Delta = 4$ do not ensure that we flow to a supersymmetric fixed point. A stronger statement of accidental SUSY, as used in [3], is to say that there exists a nonsupersymmetric flow to a supersymmetric fixed point, without even needing to assume UV SUSY.

This could be true if there were, in addition to not having GSROs, no global singlet marginal operators (GSMOs). This is true of the toy model we discuss below, as we shall verify from the RS perspective. However, in the next chapter, we will consider a weaker form of accidental SUSY where there are GSMOs. As these can be suppressed by loop factors and spurion insertions, we see that even in the presence of GSMOs, SUSY can be approximately present and natural below the compositeness scale.

4.2 A Toy Model of Accidental SUSY

We would like to exhibit a composite Higgs model with accidental SUSY, in order to demonstrate that it is possible to stabilize a little hierarchy from $O(100)$ GeV to $O(10)$ TeV. One set of composite Higgs models are dual to models in RS with a Higgs field localized to the IR brane. As we will have UV SUSY and are again plagued with anomaly cancellation concerns from the Higgsinos on the IR brane, we must introduce a second Higgs doublet to cancel anomalies. In this chapter, we are not concerned with the model being realistic, and so we just focus on the necessary ingredients in order to pursue model-building at a later date.

We embed the minimal accidental SUSY model in a slice of the Poincaré patch of AdS_5 with a metric

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \quad (4.3)$$

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with η the mostly-minus Minkowski metric. k is the inverse AdS radius of curvature, relating the 4d and 5d Planck scales via $M_4^2 \approx M_5^3/k$. The UV brane is located at $y = 0$ and the IR brane is located at $y = \ell$, and we do not consider the space outside of the branes (we treat the space as the orbifold S^1/\mathbb{Z}_2). The KK scale is roughly $ke^{-k\ell}$, which we take to be $O(10\text{TeV})$. The scale at which gravity becomes strong in the IR and consequently the scale at which this model requires a UV completion is $M_5e^{-k\ell}$. Therefore, we take $M_5 > k$ by a modest bit to allow for a reasonable regime in which the 5d EFT is valid. We begin at first ignoring gravitational and radius stabilization effects, and will argue later that they do not alter this story significantly.

We define the minimal model as follows:

- There exists $5d \mathcal{N} = 1$ SUSY, broken to $4d \mathcal{N} = 1$ SUSY explicitly on the boundaries.
- In the bulk lives an $SU(2)$ SYM with $5d$ gauge coupling g . It will be called the weak group, its gauge bosons W , etc. The associated superfields are a vector superfield V and a chiral superfield Σ . The scalar component of Σ is $w = \frac{1}{\sqrt{2}}(\sigma + iW_5)$, with σ the real scalar from the $5d$ vector multiplet and W_5 being the fifth component of W_M . These are both even under the orbifold.
- On the UV brane lives a SUSY-breaking spurion $\mathcal{V}_S = \vartheta^4 D_S$, which we assume to be D -type SUSY breaking so as to preserve the R-symmetry, coupled to the $SU(2)$ gauge sector through physics which has been integrated out. We

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imagine $D_S \sim 10000$ TeV so that any UV-localized fields beyond the minimal model will receive large SUSY-breaking soft masses. We treat \mathcal{V}_S as a gauge-invariant spurion. As we will discuss in the next chapter, one might imagine that the UV brane is a proxy for the full dynamics of the bulk of a Calabi-Yau manifold that the underlying string theory is compactified on. We would like to couple this to the 5d gauge scalar σ and allow for the transmission of SUSY-breaking to the Higgs sector.

- On the IR brane live three chiral superfields; S , H_u and H_d . S is an $SU(2)$ singlet whereas H_u and H_d are doublets. They couple with superpotential $\lambda S(H_u H_d - v^2)$ on the IR brane in order to break the electroweak symmetry.

Putting all of this together, the most minimal of actions is as follows:

$$S = \int d^5x \det(e) \left(\int d^4\vartheta \mathcal{K} + \int d^2\vartheta \mathcal{W} + \text{h.c.} \right) \quad (4.4)$$

$$\begin{aligned} \mathcal{K} = & \frac{1}{g^2} \left(1 - 2\delta(y) \frac{\mathcal{V}_S}{k} \right) \text{Tr} \left(\left(\sqrt{2}\mathcal{D}_5 + \Sigma \right) e^{-V} \left(-\sqrt{2}\mathcal{D}_5 + \bar{\Sigma} \right) e^V + \mathcal{D}_5 e^{-V} \mathcal{D}_5 e^V \right) \\ & + \delta(y - \ell) \left(\bar{H}_d e^V H_d + \bar{H}_u e^{-V} H_u \right) \end{aligned} \quad (4.5)$$

$$\mathcal{W} = \frac{1}{4g^2} \text{Tr} (\mathcal{W}^\alpha \mathcal{W}_\alpha) + \delta(y - \ell) \lambda S (H_u H_d - v^2) \quad (4.6)$$

where everything is explicitly AdS diff-invariant. Note that in these conventions, g has mass dimension $-\frac{1}{2}$, and W_M and Σ has mass dimension 1. M is the messenger scale which communicates SUSY-breaking to the vector multiplet. The 5d $SU(2)$ supergauge transformations are:

$$e^V \rightarrow e^{\bar{\Lambda}} e^V e^{\Lambda} \quad (4.7)$$

$$e^{-V} \rightarrow e^{-\Lambda} e^{-V} e^{-\bar{\Lambda}} \quad (4.8)$$

$$\mathcal{W}_\alpha \rightarrow e^{-\Lambda} \mathcal{W}_\alpha e^{\Lambda} \quad (4.9)$$

$$\Sigma \rightarrow e^{-\Lambda} \left(\Sigma - \sqrt{2} \mathcal{D}_5 \right) e^{\Lambda} \quad (4.10)$$

$$\bar{\Sigma} \rightarrow e^{\bar{\Lambda}} \left(\bar{\Sigma} + \sqrt{2} \mathcal{D}_5 \right) e^{-\bar{\Lambda}} \quad (4.11)$$

$$H_u \rightarrow e^{\Lambda} H_u \quad (4.12)$$

$$H_d \rightarrow e^{-\Lambda} H_d \quad (4.13)$$

Note that the $H_d \Phi H_u$ coupling is not allowed by gauge invariance, explicitly breaking the $4d \mathcal{N} = 2$ supersymmetry down to $\mathcal{N} = 1$ on the brane.

We are interested in the effects of SUSY-breaking on the Higgs sector. Supersymmetrically, there is no mass term for H_u or H_d , and so both the Higgses and Higgsinos are 4d-massless. The underlying dynamics are supersymmetric, and so we expect by nonrenormalization theorems that the only nontrivial contributions to the 4d Higgs masses will come from SUSY-breaking effects. As there is no coupling of Σ , the field coupled to SUSY-breaking, to the Higgses at tree-level, the effect must proceed through a 5d loop. We would like to compute that loop in order to understand the parametric scaling of the SUSY-breaking Higgs mass. We begin by passing to the supertangent space in order to have canonical Feynman rules, and then set up the appropriate loop diagram and evaluate it.

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We can pass to flat superspace/supertangent space coordinates as follows:

$$\mathcal{D}_\mu = e^{k|y|} D_\mu \quad (4.14)$$

$$\mathcal{D}_5 = \partial_5 \quad (4.15)$$

$$\vartheta^\alpha = e^{-\frac{1}{2}k|y|} \theta^\alpha \quad (4.16)$$

$$d\vartheta_\alpha = e^{\frac{1}{2}k|y|} d\theta_\alpha \quad (4.17)$$

$$e_\mu^a = e^{-k|y|} \delta_\mu^a \quad (4.18)$$

$$e_5^5 = \delta_5^5 = 1 \quad (4.19)$$

$$\det(e) = e^{-4k|y|} \quad (4.20)$$

$$\mathcal{W}_\alpha = -\frac{1}{4} \bar{\mathcal{D}}^2 (e^{-V} \mathcal{D}_\alpha e^V) = -\frac{1}{4} e^{\frac{3}{2}k|y|} \bar{D}^2 (e^{-V} D_\alpha e^V) = e^{\frac{3}{2}k|y|} W_\alpha \quad (4.21)$$

$$\mathcal{V}_S = e^{2k|y|} V_S \quad (4.22)$$

This gives rise to the following 5d flat-space action:

$$S = \int d^5x \left(\int d^4\theta K + \int d^2\theta W + \text{h.c.} \right) \quad (4.23)$$

$$\begin{aligned} K &= \frac{1}{g^2} e^{-2k|y|} \left(1 - 2\delta(y) e^{2k|y|} \frac{V_S}{k} \right) \\ &\quad \times \text{Tr} \left(\left(\sqrt{2}\partial_5 + \Sigma \right) e^{-V} \left(-\sqrt{2}\partial_5 + \bar{\Sigma} \right) e^V + \partial_5 e^{-V} \partial_5 e^V \right) \\ &\quad + e^{-2k|y|} \delta(y - \ell) \left(\bar{H}_d e^V H_d + \bar{H}_u e^{-V} H_u \right) \end{aligned} \quad (4.24)$$

$$W = \frac{1}{4g^2} \text{Tr} (W^\alpha W_\alpha) + e^{-3k|y|} \delta(y - \ell) \lambda S (H_u H_d - v^2) \quad (4.25)$$

Note here that the kinetic term for H_u and H_d is not canonically normalized upon reduction to 4d. This can be fixed by sending $H_i \rightarrow e^{k|y|} H_i$. This shifts the Kahler and superpotentials to

$$\begin{aligned}
 K = & \frac{1}{g^2} e^{-2k|y|} \left(1 - 2\delta(y) e^{2k|y|} \frac{V_S}{k} \right) \\
 & \times \text{Tr} \left(\left(\sqrt{2} \partial_5 + \Sigma \right) e^{-V} \left(-\sqrt{2} \partial_5 + \bar{\Sigma} \right) e^V + \partial_5 e^{-V} \partial_5 e^V \right) \\
 & + \delta(y - \ell) \left(\bar{H}_d e^V H_d + \bar{H}_u e^{-V} H_u \right)
 \end{aligned} \tag{4.26}$$

$$W = \frac{1}{4g^2} \text{Tr} (W^\alpha W_\alpha) + e^{-k|y|} \delta(y - \ell) \lambda S (H_u H_d - e^{-2k|y|} v^2) \tag{4.27}$$

The only effect of the SUSY-breaking spurion is contained in the expansion of the Kahler potential. It is the term

$$S \supset \int d^5x \delta(y) \left(-\frac{1}{g^2 k} D_S |w|^2 \right) \tag{4.28}$$

Therefore, 4d SUSY is broken, and can be mediated to the Higgs fields via w . The presence of 4d couplings of the w to the scalar higgses is ensured by the $SU(2)$ D -equation of motion:

$$D = -D_5 (e^{-2k|y|} \sigma) - \frac{g^2}{2} \delta(y - \ell) (h_d \bar{h}_d - h_u \bar{h}_u) \tag{4.29}$$

where $D_5 = \partial_5 + iW_5$, as usual. In the higgsless, non-warped limit this correctly reproduces the result of [149] of $D = -D_5 \sigma$. Consequently, there is a diagram for

the higgs mass correction as shown in 4.1. In such a fashion, SUSY breaking can be transmitted to the higgs sector.

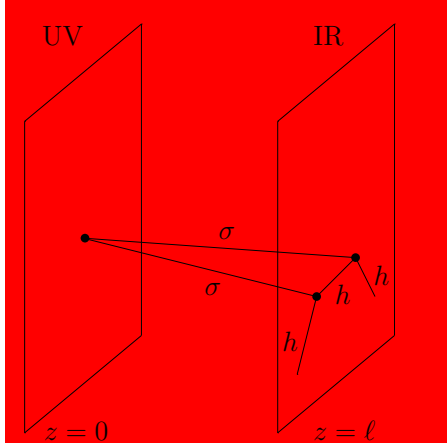


Figure 4.1: The 5d loop diagram responsible for transmitting SUSY-breaking to the h .

The σ propagator currently has mass dimension 1; in order to use a canonical $5d$ propagator we should properly normalize by sending $\sigma \rightarrow g\sigma$. After this normalization, the mass-term for σ is

$$\mathcal{L} \supset -\delta(y) \frac{D_s}{2k} \sigma^2 \tag{4.30}$$

The operator in question comes from the cross-term of D_s . In the basis where both σ and h have canonical $5d$ and $4d$ propagators, respectively, is

$$\frac{g}{2} e^{-2k|y|} \delta(y - \ell) \bar{h} (2k\sigma - \partial_5 \sigma) h \tag{4.31}$$

which is dimension 5 as desired. The metric for which we have a form for the scalar propagator is

$$ds^2 = \frac{1}{k^2 z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \quad (4.32)$$

This is related to our y -metric by $z = \frac{e^{ky}}{k}$. Being sure to be careful about converting from y to z , we can easily use Feynman rules for the non-boundary terms in the Feynman diagram to evaluate the loop integrand. The answer is

$$\begin{aligned} \delta m_h^2 &= \frac{1}{4(2\pi)^4} \int d^4 q \frac{1}{q^2} \int_{y=0}^{y=\ell} dz_1 dz_2 dz_3 \frac{\partial y_1}{\partial z_1} \frac{\partial y_2}{\partial z_2} \frac{\partial y_3}{\partial z_3} \delta(y_2) \frac{D_s}{k} \\ &\quad \times \frac{g}{2} \delta(y_1 - \ell) e^{-2ky_1} \left(2k - \frac{\partial z_1}{\partial y_1} \partial_{z_1} \right) G_q(z_1, z_2) \\ &\quad \times \frac{g}{2} \delta(y_3 - \ell) e^{-2ky_3} \left(2k - \frac{\partial z_3}{\partial y_3} \partial_{z_3} \right) G_q(z_2, z_3) \end{aligned} \quad (4.33)$$

where q is the 4d loop-momentum, $G_q(z, z')$ is the 5d mixed p - z scalar propagator with 4d momentum q and z -beginning and ending points z and z' . We convert fully to z -space by $\frac{\partial z}{\partial y} = kz$ and $\frac{\partial y}{\partial z} = \frac{1}{kz}$. The boundaries are $y = 0$ and ℓ , making them $z = \frac{1}{k}$ and $\frac{e^{k\ell}}{k}$. Finally, $\delta(y - y_0) = \delta(z - z_0) / \frac{\partial y}{\partial z} |_{z_0}$, where $y(z_0) = y_0$.

The propagator G for scalars was found in [150]. For our specific case, we use their result, plugging in the following: $s = 4$, $\hat{M}^2 = ak^2$. We want a flat profile for the right-handed gaugino, which we obtain by picking the gaugino mass-parameter to be $c = -1/2$. By supersymmetry, $a = -4$. Finally, $\alpha = 0$, $r = b$, and $b = 2$. We define the auxiliary Bessel functions $\tilde{J}_\alpha = (-r + s/2)J_\alpha + zJ'_\alpha = zJ'_\alpha$. As $\alpha = 0$, $J'_0 = -J_1$. Therefore, $\tilde{J}_0 = -zJ_1$.

H_α is a Hankel function of the first kind, called $H_\alpha^{(1)} = J_\alpha + iY_\alpha$. We can define

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\tilde{H} in a similar fashion to \tilde{J} . Since $Y'_0 = -Y_1$ as well, $\tilde{H}_0 = -zH_1$.

The conventions of [150] call for $z = e^{ky}/k$. Since their Planck brane is at $y = 0$ and their TeV brane at $y = \pi R$, we see that $z = 1/k$ for the Planck brane and $z = e^{\pi kR}/k$ for the TeV brane. The explicit form for the propagator is

$$G_p(z, z') = \frac{i\pi}{2} k z z' \left(\frac{\tilde{J}_\alpha(p e^{\pi kR}/k) H_\alpha(p z_>) - \tilde{H}_\alpha(p e^{\pi kR}/k) J_\alpha(p z_>)}{\tilde{J}_\alpha(p e^{\pi kR}/k) H_\alpha(p/k) - \tilde{H}_\alpha(p e^{\pi kR}/k) J_\alpha(p/k)} \right) \times \left(\tilde{J}_\alpha(p/k) H_\alpha(p z_<) - \tilde{H}_\alpha(p/k) J_\alpha(p z_<) \right) \quad (4.34)$$

where $z_>$ and $z_<$ are the greater and lesser of z and z' , respectively. We match conventions by setting $\ell = \pi R$. The integral is

$$\delta m_h^2 = \frac{1}{4(2\pi)^4} \frac{g^2}{4} \int d^4 q \frac{1}{q^2} \int_{1/k}^{e^{k\ell}/k} dz_1 dz_2 dz_3 \frac{1}{k^3 z_1 z_2 z_3} \delta(z_2 - 1/k) \frac{D_S}{k} \times e^{k\ell} \delta(z_1 - e^{k\ell}/k) \frac{1}{k^2 z_1^2} (2k - k z_1 \partial_{z_1}) G_q(z_1, z_2) \times e^{k\ell} \delta(z_3 - e^{k\ell}/k) \frac{1}{k^2 z_3^2} (2k - k z_3 \partial_{z_3}) G_q(z_2, z_3) \quad (4.35)$$

which simplifies to:

$$\delta m_h^2 = \frac{g^2 D_S e^{2k\ell}}{256 k^6 \pi^4} \int d^4 q \frac{1}{q^2} \int_{1/k}^{e^{k\ell}/k} dz_1 dz_2 dz_3 \frac{1}{z_1^3 z_2 z_3^3} \delta(z_2 - 1/k) \times \delta(z_1 - e^{k\ell}/k) (2 - z_1 \partial_{z_1}) G_q(z_1, z_2) \times \delta(z_3 - e^{k\ell}/k) (2 - z_3 \partial_{z_3}) G_q(z_2, z_3) \quad (4.36)$$

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After doing the z -integrals and Wick-rotating, this reduces to

$$\delta m_h^2 = \frac{g^2 D_S e^{2k\ell}}{64k^6 \pi^4} k^7 e^{-6k\ell} 2\pi^2 \int_0^\infty dq q (2 - z_1 \partial_{z_1}) G_q(z_1, z_2) (2 - z_3 \partial_{z_3}) G_q(z_2, z_3) \Big|_{z_1=z_3=e^{k\ell}/k, z_2=1/k} \quad (4.37)$$

As G_q has mass-dimension -1 , we can surmise that the q -integral has mass dimension 0. The resulting integral can be done numerically, and then fit to an analytic form. Performing this procedure yields that the solution scales as $\frac{1}{k\ell}$, where the coefficient is nearly 1. Putting all of this together, we obtain

$$\delta m_h^2 = \frac{(g^2 k) D_S}{32\pi^2 (k\ell)} e^{-4k\ell} \quad (4.38)$$

This is the central result of this chapter. We have demonstrated that SUSY-breaking D_S is warped down to the compositeness scale by four powers of the warp factor $e^{-k\ell}$, and then is suppressed further by a loop factor. Furthermore, all other mass-scales (such as the electroweak scale v) are only redshifting with one power of the warp factor per mass dimension; here we are redshifting the higgs mass by two powers. Therefore, this toy model exhibits accidental SUSY, since as we flow further and further into the IR, dual to moving the IR brane further and further away, we flow to a supersymmetric fixed point, as the SUSY-breaking effects are flowing to 0 faster than SUSY-preserving effects.

4.3 Further Considerations

Although this toy model does indeed exhibit a supersymmetric fixed point, there are a number of other issues to be addressed. First, as with nonsupersymmetric RS, the radius needs to be stabilized. As has been shown in [151], the radius can be stabilized in 5d SUSY RS models by adding a hypermultiplet $(\Phi, \tilde{\Phi})$ with a 5d mass $(\Delta - 4)k$, where Δ is the scaling dimension of the dual stabilization operator, as well as constant superpotentials C_{IR}^3 and C_{UV}^3 on the IR and UV branes, respectively. This is a supersymmetric generalization of the Goldberger-Wise mechanism [152]. We imagine that $C_{IR} < M_5$, which can easily have the effect of stabilizing the radion at $k\ell \sim 40$ for an adequate hierarchy, keeping the norm of Φ and $\tilde{\Phi}$ small enough to not gravitationally backreact on the geometry significantly, and also making the radion-mediated contributions to composite soft masses adequately small.

Furthermore, anomaly-mediated contributions are small enough because $m_{AMSB} \sim \frac{D_S}{16\pi^2 M_5} \ll \text{GeV}$.

Finally, gravity-mediated contributions to the higgs 4d mass are determined by the value of the stabilizer field F -terms with the IR brane. As argued in [151], these are related to the hierarchy between C_{IR} and M_5 , and so we can get rid of any of these contributions by turning down the IR constant superpotential.

There are a number of other details to address, such as the focusing effect in the RG equations discussed in [3]. This is dual to additional loop diagrams in RS, which will be the study of future research. There is a great deal of opportunity for

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constructing a BSM model exhibiting the mechanism present in this toy model.

Chapter 5

Accidental SUSY and String Theory: The Story Above The Story Above 10 TeV

The toy model in the previous chapter is indeed accidentally supersymmetric, but is itself only an effective theory, because the mass dimension of the gauge coupling is $-\frac{1}{2}$. Therefore, new physics must appear imminently at higher energies. It is therefore paramount to ascertain whether UV completions of mechanisms in RS exist. In this chapter, we will explore the question of whether it is possible to realize even a single example of accidental supersymmetry in a concrete UV-complete framework as proof-of-principle for studying the mechanism in further detail in the RS framework. We choose to focus on type IIB string theory as our UV-complete framework, though it

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would be interesting to pursue accidental SUSY in other frameworks in the future.

The structure of the chapter is as follows: in section 5.1, we review accidental SUSY and explore what ingredients must be present in our string background to ensure accidental SUSY. In section 5.2, we review those details of the Klebanov-Witten and Klebanov-Strassler geometries which will be relevant to our chosen background, as well as their gauge duals. In section 5.3, we study string perturbations to the theory on the deformed complex cone over \mathbb{F}_0 , the zeroth Hirzebruch surface, in order to demonstrate that its dual theory possesses accidental SUSY.

The work in this chapter is based on unpublished work of the author [6].

5.1 Accidental SUSY Ingredients

Our goal is to write down a UV-complete string model exhibiting accidental SUSY. A good starting place would be having a model that preserved SUSY in the first place, then breaking SUSY by hand at high energies and watching it be restored as we flow to the compositeness scale. Such a model would require a strongly coupled SCFT, which is a warped throat in the dual string description. However, there is a no-go theorem [153] stating that the only way to obtain a warped throat in type IIB string theory on a *compact* M_6 is through the presence of O3-planes or D7-branes. In the absence of local objects in the theory, the only solution to the equations of motion are vanishing G_3 and constant fiveform flux and warp factor, giving an unwarped

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solution. Consequently, in order to have a warped solution in string theory, one needs to add local objects such as O3-planes or D7-branes which violate the assumptions of the no-go theorem. Therefore, one would hope that one could find warped models of accidental SUSY in F-theory compactifications [154], or in the orientifold limit of F-theory [155].

However, these constraints do not apply to warped, noncompact geometries. Although these models are perfectly satisfactory in their own right, they lack dynamical four-dimensional gravity, and therefore are unsatisfactory for attempts to recreate BSM physics models. Nevertheless, these geometries act as local, effective descriptions of full compact solutions. The bulk of these compact manifolds act as “UV branes” on the EFTs living in the throat. Therefore, one can study the physics of a throat alone, and incorporate fluctuations sourced by physics in the bulk of the compact manifold in the effective description. This methodology is described and utilized in, e.g., [156, 157] to study the potentials of D3-branes for the purposes of studying inflation in string theory.

We are led, therefore, to consider a warped space which is of the form

$$ds^2 = e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(r)} ds_{M_6}^2 \quad (5.1)$$

with

$$ds_{M_6}^2 = dr^2 + r^2 ds_{X_5}^2 \quad (5.2)$$

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where $ds_{X_5}^2$ is the metric on a five-dimensional horizon manifold. In order for the noncompact space $dr^2 + r^2 ds_{X_5}^2$ to be Calabi-Yau (thereby preserving bulk supersymmetry), X_5 should be Sasaki-Einstein [158]. In order to find accidental SUSY, therefore, the more modest goal is to look for noncompact solutions to supergravity with brane sources, bulk supersymmetry and with this metric ansatz which have no GSROs, deferring the question of realizations on compact manifolds until future work.

Supposing we had such a background to work with, classical perturbations of the various supergravity fields by non-normalizable solutions to the linearized equations of motion are dual to deformations of the Lagrangian of the dual SCFT. Of course, such solutions would be normalizable upon compactification of the manifold. The masses of the linear supergravity perturbations determine the scaling dimensions of the dual operators, and the quantum numbers of the global symmetry group follow as well. By the AdS/CFT correspondence, those objects which are dual to Lagrangian perturbations of the CFT are scalar fields on AdS_5 . Therefore, one can classify all scalar single trace primary operators, their scaling dimensions (to leading order in $\frac{1}{N}$) and their global quantum numbers by studying the UV perturbation theory of 5d scalars about the background in question. Note that since these supergravity fields will propagate on a supersymmetric background, they will be sourced by SUSY-breaking effects in the compact part of the manifold and therefore do not need to respect supersymmetry.

5.2 The Conifold and its Deformation

There is a famous example of a noncompact, finite warped throat in string theory; this is the Klebanov-Strassler (KS) solution [159], which describes IIB string theory on a warped deformed conifold. It is a perturbation away from the fixed point of the warped throat of Klebanov-Witten (KW) [160]. KW describes string theory on the conifold, which is a real cone over the space $T^{1,1}$. A brief discussion of $T^{1,1}$ is given in the appendix. KW describes a theory with N D3-branes placed at the tip of the conifold, whose worldvolumes coincide with $\mathbb{R}^{1,3}$. The Klebanov-Witten solution to IIB supergravity is

$$ds^2 = e^{2A} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A} (dr^2 + r^2 ds_{T^{1,1}}^2) \quad (5.3)$$

$$A = \ln \frac{r}{R} \quad R = \left(\frac{27\pi g_s N}{4} \right)^{1/4} \quad \tau = \frac{i}{g_s} \quad G_3 = 0 \quad (5.4)$$

$$\tilde{F}_5 = (1 + *) d\alpha \wedge d^4x \quad \alpha = e^{4A} = \frac{r^4}{R^4} \quad (5.5)$$

where R is the AdS radius of curvature measured in units of the string length, $l_s = 1$. Note that $T^{1,1}$ is Sasaki-Einstein, implying the existence of $4d \mathcal{N} = 1$ SUSY. The dual SCFT is described by gauge groups $SU(N) \times SU(N)$ along with bifundamental and antibifundamental fields A^i and B^j , respectively, each transforming as a doublet under their own $SU(2)$ flavor symmetry group. At the strongly-coupled conformal

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fixed point, the scaling dimension of the matter fields is $\Delta = \frac{3}{4}$, and the theory has a superpotential

$$W = \lambda \epsilon_{ik} \epsilon_{jl} \text{Tr}(A^i B^j A^k B^l) \quad (5.6)$$

In the perturbative regime, the theory has a Kähler potential

$$K = \text{Tr}(\bar{A}e^{V_1} A e^{-V_2} + \bar{B}e^{V_2} B e^{-V_1}) \quad (5.7)$$

The associated quiver diagrams for KW is shown on the left in fig. 5.1.

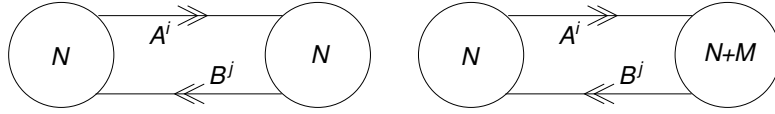


Figure 5.1: The quiver diagrams associated with the Klebanov-Witten (left) and Klebanov-Strassler (right) theories.

We use the following conventions for supergauge transformations of the fields:

$$e^{V_1} \rightarrow e^{-i\bar{\Lambda}_1} e^{V_1} e^{i\Lambda_1} \quad e^{V_2} \rightarrow e^{-i\bar{\Lambda}_2} e^{V_2} e^{i\Lambda_2} \quad (5.8)$$

$$W_1 \rightarrow e^{-i\Lambda_1} W_1 e^{i\Lambda_1} \quad W_2 \rightarrow e^{-i\Lambda_2} W_2 e^{i\Lambda_2} \quad (5.9)$$

$$A \rightarrow e^{-i\Lambda_1} A e^{i\Lambda_2} \quad B \rightarrow e^{-i\Lambda_2} B e^{i\Lambda_1} \quad (5.10)$$

The KS solution is obtained by modifying the KW solution by wrapping M D5-branes around the nonvanishing twocycle at the base of the conifold, and allowing the remaining three directions to be parallel to the D3-branes. This action famously

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deforms the conifold and generates a logarithmic dependence of the total flux through $T^{1,1}$ on the radial coordinate, which is dual to a cascading gauge theory which undergoes repeated Seiberg dualities. After a change of radial variables from r to ρ , defined implicitly through $r^3 = \sqrt{\frac{27}{32}}\epsilon^2 e^\rho$ with $\epsilon^{2/3}$ being a mass parameter, the IIB supergravity solution that describes KS is governed by the following equations:

$$\tau = \frac{i}{g_s} \quad (5.11)$$

$$ds^2 = \frac{1}{\sqrt{h(\rho)}}\eta_{\mu\nu}dx^\mu dx^\nu + \sqrt{h(\rho)}ds_6^2 \quad (5.12)$$

$$ds_6^2 = \frac{\epsilon^{4/3}}{2}K(\rho) \left(\sinh^2\left(\frac{\rho}{2}\right)(g_1^2 + g_2^2) + \cosh^2\left(\frac{\rho}{2}\right)(g_3^2 + g_4^2) + \frac{1}{3K(\rho)^3}(g_5^2 + d\rho^2) \right) \quad (5.13)$$

$$\tilde{F}_5 = \frac{g_s M^2}{16}\ell(\rho) \left(g_1 \wedge g_2 \wedge g_3 \wedge g_4 \wedge g_5 + \frac{16}{K(\rho)^2 h(\rho)^2 \sinh^2 \rho \epsilon^{8/3}} d^4 x \wedge d\rho \right) \quad (5.14)$$

$$G_3 = \frac{M}{4} (g_5 \wedge g_3 \wedge g_4 + d(F(\rho)(g_1 \wedge g_3 + g_2 \wedge g_4)) - id(f(\rho)g_1 \wedge g_2 + k(\rho)g_3 \wedge g_4)) \quad (5.15)$$

where the solution has been specified in terms of the following auxiliary functions:

$$K(\rho) = \frac{(\sinh(2\rho) - 2\rho)^{1/3}}{2^{1/3} \sinh(\rho)} \quad F(\rho) = \frac{\sinh \rho - \rho}{2 \sinh \rho} \quad (5.16)$$

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$$h(\rho) = \frac{(g_s M)^2}{2^{4/3} \epsilon^{8/3}} \int_{\rho}^{\infty} dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh(2x) - 2x)^{1/3} \quad (5.17)$$

$$f(\rho) = \frac{\rho \coth \rho - 1}{2 \sinh \rho} (\cosh \rho - 1) \quad k(\rho) = \frac{\rho \coth \rho - 1}{2 \sinh \rho} (\cosh \rho + 1) \quad (5.18)$$

$$\ell(\rho) = \frac{\rho \coth \rho - 1}{4 \sinh^2 \rho} (\sinh 2\rho - 2\rho) \quad (5.19)$$

The dual gauge theory of the above solution is similar to the KW gauge theory, but with gauge groups $SU(N) \times SU(N + M)$, as shown in the quiver on the right of fig. 5.1. However, due to nonvanishing beta functions, the dual gauge theory runs as we flow into the IR, undergoing repeated Seiberg dualities returning to a self-similar state but with lower-rank gauge groups, finally ending when we've reached $\mu \sim \epsilon^{2/3}$.

KS contains a warped throat with a nontrivial supersymmetric field content in the IR, making it a promising starting point for a study of accidental SUSY in string theory. However, compactifying and breaking SUSY in the UV will lead to a violent disruption of IR SUSY, due to the presence of GSROs such as $|\text{Tr}(AB)|^2$ ($\Delta = 3$) in the spectrum of operators [15]. Consequently, KS does not exhibit accidental SUSY. Therefore, we will be interested in an orbifold of KS that removes this operator from the spectrum, as under our orbifold, $\text{Tr}(AB)$ is odd.

5.3 The Complex Cone Over \mathbb{F}_0

There is a close cousin of KW which describes string theory on a complex cone¹ over \mathbb{F}_0 ; this is just a \mathbb{Z}_2 orbifold of the conifold² [158]. The orbifold can be described on the supergravity side by taking the KW solution and modding out by $z_i \sim -z_i$. In terms of $T^{1,1}$, this operation can be described as identifying $\psi \sim \psi + 2\pi$. Note that this is nontrivial because the coordinate range of ψ is $0 \leq \psi < 4\pi$ on the conifold. This is dual to orbifolding the gauge group and $A \sim -A$ in the gauge dual, removing the dangerous $\text{Tr}(AB)$ from the spectrum. This solution can also be deformed and exhibits KS-like running. In terms of the basis of oneforms described in the appendix, all g_i are invariant under this operation, and therefore so are the fluxes in KS. This theory has been studied before, but in a different context [161–163]. Also note that this is to be contrasted with the SUSY-breaking orbifold described in [15], as our orbifold preserves SUSY. As discussed in section 5.1, the idea is to categorize all non-normalizable perturbations of the supergravity solution, and therefore gain an understanding of the scaling dimensions of possible Lagrangian deformations of the dual gauge theory. We first discuss the supergravity perturbation theory in subsection 5.3.1, then turn to a categorization of operators in the dual gauge theory in subsection 5.3.2, before finally returning to an AdS/CFT matching in subsection 5.3.3.

¹More precisely, we consider the noncompact total space of the line bundle fibered over \mathbb{F}_0 , where the line bundle is the anticanonical bundle of \mathbb{F}_0 .

²This space can also be viewed as a real cone over $T^{1,1,2}$.

5.3.1 The Supergravity Side

Our goal is to obtain the effective AdS_5 masses of all non-normalizable perturbations of the string theory on the deformed complex cone over \mathbb{F}_0 due to compactifications which behave as AdS_5 scalars, as these are the perturbations which will be dual to scalar perturbations of the dual gauge theory Lagrangian. Studying the perturbation theory around the deformed cone would in general be a true feat, but fortunately, since we're interested in UV Lagrangian perturbations, dual to perturbations at large r , we can utilize the fact that the geometry becomes asymptotically that of $AdS_5 \times T^{1,1}/\mathbb{Z}_2$, and study perturbations on that geometry instead. Furthermore, since the action of the orbifold becomes much more transparent on the gauge side, it is sufficient³ to classify perturbations in KW and study the action of the orbifold after AdS/CFT matching. This procedure has been carried out extensively before for KW/KS [157, 164, 165], so we merely review relevant details.

Before proceeding with the perturbation equations, we pause to introduce helpful notation. We introduce projection operators P_{\pm} which project threeforms onto their imaginary self-dual and imaginary anti-self dual (ISD and IASD, respectively) components. These operators are defined by

$$P_{\pm} = \pm \frac{1}{2i} (*_6^{(0)} \pm i) \tag{5.20}$$

³There is a subtlety regarding this approach; there can in general be torsion in the homology and cohomology of the orbifold, and so effects sensitive to topology must be considered independently. These are dual to anomalies in global symmetries, as we will discuss later.

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where $*_6$ only dualizes with respect to M_6 and g_{mn} . These operators satisfy

$$P_+^2 = P_+ \quad P_-^2 = P_- \quad P_+P_- = P_-P_+ = 0 \quad P_+ + P_- = 1 \quad (5.21)$$

We consider a supergravity perturbation theory similar to that described in [157], allowing all fields to be systematically expanded in a formally small parameter⁴ ε about their background values; e.g.

$$\tilde{F}_5 = \tilde{F}_5^{(0)} + \varepsilon \tilde{F}_5^{(1)} + \dots \quad (5.22)$$

In this case we want to allow all supergravity fields to be expanded, but insist that the resulting modes be AdS_5 -scalar perturbations. Consequently, we ignore all fermion equations of motion. Furthermore, we would like the fluxes associated with the various gauge fields to not break 4d Poincaré-invariance. With respect to the KW background solution, the linearized perturbation equations become

$$\square_{(0)}\tau^{(1)} = 0 \quad dG_3^{(1)} = 0 \quad d\tilde{F}_5^{(1)} = 0 \quad \square_{6(0)}\Phi_-^{(1)} \quad (5.23)$$

$$d(r^4 P_- G_3^{(1)}) = 0 \quad \square_{6(0)}\Phi_+^{(1)} = \frac{16}{r}\partial_r\Phi_+^{(1)} \quad (5.24)$$

In addition, there is the Einstein equation describing symmetric tensor perturbations of $T^{1,1}$; however, having found the spectrum of scalar operators in the CFT,

⁴From the point of view of the dual gauge theory, this is a natural expansion to make; powers of ε can be treated as powers of spurions.

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we can deduce that any operator not matched to one of the above supergravity fields must be dual to a symmetric tensor perturbation, and consequently not solve the Einstein equation.

The process of solving the perturbation equations is greatly simplified by knowing the radial scaling of the scalar harmonics on $T^{1,1}$, which are known to be [164]

$$\delta = -2 + \sqrt{H(j, l, r) + 4} \quad (5.25)$$

where (j, l, r) describe the representation of the harmonic with respect to the isometry group of $T^{1,1}$, which is $SU(2) \times SU(2) \times U(1)$. The lowest values of δ are shown in table 5.1. Note that for every nonvanishing value of r , there is another representation $(j, l, -r)$ with the same scaling dimension.

The $\tau^{(1)}$ perturbation equation is independent of the rest and is satisfied by any harmonic scalar on $T^{1,1}$. We use coordinates x for AdS_5 and y for $T^{1,1}$; then we expand $\tau^{(1)}(x, y) = \sum_{jlr} \phi_{jlr}(x) Y^{jlr}(y)$. The 10d Laplacian becomes $\square_{AdS} - \square_{T^{1,1}}$, where $\square_{T^{1,1}} Y^{jlr}(y) = H(j, l, r) Y^{jlr}(y)$. Therefore, the AdS equation of motion for each term τ_{jlr} in the expansion becomes

$$\square_{AdS} \tau_{jlr}(x, y) = H(j, l, r) \tau_{jlr}(x, y) \quad (5.26)$$

These have mass-squareds beginning at $m^2 = H(0, 0, 0) = 0$ in AdS_5 , and so with the exception of the constant mode, the rest cannot transmit SUSY-breaking from the UV as they're dual to operators with $\Delta > 4$. The constant mode is dual to an operator

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δ	j	l	r
0	0	0	0
1.5	$\frac{1}{2}$	$\frac{1}{2}$	1
2	1	0	0
2	0	1	0
3	1	1	2
3.29	1	1	0
3.5	$\frac{3}{2}$	$\frac{1}{2}$	1
3.5	$\frac{1}{2}$	$\frac{3}{2}$	1

Table 5.1: The scaling dimensions and quantum numbers of the first few scalar harmonics on $T^{1,1}$.

with $\Delta = 0$ or 4, where these two choices are related by a Legendre transform [166]; the $\Delta = 0$ mode is a modulus which is the sum of the inverse-squared gauge couplings $1/g_1^2 + 1/g_2^2$ [160].

Δ	j	l	r
4	0	0	0

Table 5.2: The scaling dimensions and quantum numbers of scalar harmonics of τ with $\Delta \leq 4$.

By the Bianchi identity, $G_3^{(1)}$ is a closed threeform; therefore it is either exact or a representative of a cohomology class. We consider these in turn. As $\tau^{(0)}$ is constant,

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we can write G_3 as dA_2 , where A_2 is the complex twoform $C_2 - \frac{i}{g_s}B_2$. Perturbations of A_2 must be AdS_5 -scalars; therefore A_2 can be decomposed in terms of twoform harmonics on $T^{1,1}$. We write this decomposition as

$$G_3^{(1)}(x, y) = \sum_i d(f_i(x)\Omega_2^i(y)) \quad (5.27)$$

where Ω_2^i are all harmonic two-forms on $T^{1,1}$, satisfying $*_5 d\Omega_2 = i\delta\Omega_2$, where $*_5$ is the Hodge star operator with respect to $T^{1,1}$. δ are ($-i$ times) the eigenvalues of the Laplace-Beltrami operator. However, f_i must be only a function of r and not of 4d Minkowski coordinates; otherwise we would have a nontrivial flux along $\mathbb{R}^{1,3}$ in the CFT, breaking the Poincaré invariance of the CFT vacuum. We use the ansatz $f_i(x) = r^{\alpha_i}$, allowing for potentially multiple values of α for a given i . As we can solve the perturbation equation term-by-term, we must solve

$$d(r^4 P_-(r^\alpha \Omega_2)) = 0 \quad (5.28)$$

The solution to these was found in [157]; the claim is that our ansatz solves the perturbation equations when $\alpha = \delta - 4$ or $-\delta$ (for $\delta \neq 0$). In these cases, the scaling dimension of the dual operator⁵ is $\Delta = \max(4 - \delta, \delta)$. The eigenvalues of the twoform harmonics were worked out in [164]; the answers can again be expressed in terms of the quantum numbers of the isometry group (j, l, r) . There are six eigenvalues of the

⁵This rule is used to enforce unitarity of the dual operator ($\Delta \geq 1$). In the cases where both $4 - \delta$ and δ are allowed by unitarity, either scaling dimension is acceptable in the dual CFT, the two choices being related by a Legendre transform [166]. We choose the larger scaling dimension for the duals of G_3 -flux for ease in matching to the CFT spectrum.

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Laplace-Beltrami operator for each (j, l, r) , and they are⁶

$$\delta_{(j,l,r+2)} = 1 \pm \sqrt{H(j, l, r) + 4} \quad (5.29)$$

$$\delta_{(j,l,r-2)} = -1 \pm \sqrt{H(j, l, r) + 4} \quad (5.30)$$

$$\delta_{(j,l,r)} = \pm \sqrt{H(j, l, r) + 4} \quad (5.31)$$

where the allowed values of (j, l, r) and the definition of $H(j, l, r)$ are the same as in table 5.1, but now the physical value of r of the perturbation may not be r , but rather $r \pm 2$, as indicated in the subscript of δ . This occurs because the twoform harmonics can be built out of scalar harmonics, but some of the solutions depend on the holomorphic threeform Ω on the conifold, which carries an R -charge of 2.

The de Rham cohomology representative twoform ω_2 of $T^{1,1}$ is an acceptable harmonic perturbation of the twoform gauge fields, corresponding to vanishing G_3 . ω_2 is therefore another modulus of the dual CFT; in this case it is the difference in the inverse gauge-squared couplings $\frac{1}{g_1^2} - \frac{1}{g_2^2}$. Again, this is the $\Delta = 0$ choice related to the $\Delta = 4$ choice by a Legendre transform.

The second singular cohomology of $T^{1,1}/\mathbb{Z}_2$ contains a \mathbb{Z}_2 torsion subgroup, related by Poincaré-duality to the presence of torsion threecycle in homology. However, although the torsion threecycle plays an important role in AdS/CFT, the associated twoform is not present in the de Rham cohomology; $H_{\text{dR}}^2(T^{1,1}/\mathbb{Z}_2, \mathbb{R}) = \mathbb{R}$. Consequently, there is no A_2 -perturbation associated with the torsion twoform.

⁶Note that we disagree with the authors of [164] with regard to a minus sign.

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Finally, one can ask about G_3 -flux in $H^3(T^{1,1})$. However, the cohomology representative $\omega_3 = g_5 \wedge \omega_2$ of $T^{1,1}$ does not satisfy $d(r^4 P_- \omega_3) = 0$, and so it does not constitute an allowable perturbation of the solution.

Series	Δ	j	l	r
I	2.5	$\frac{1}{2}$	$\frac{1}{2}$	-1
I	3	0	0	-2
I	3	1	0	-2
I	3	0	1	-2
I	4	1	1	0
II	2	0	0	0
II	3.5	$\frac{1}{2}$	$\frac{1}{2}$	1
II	4	1	0	0
II	4	0	1	0
III	3	0	0	2
ω_2	4	0	0	0

Table 5.3: The scaling dimensions and quantum numbers of scalar harmonics of A_2 with $\Delta \leq 4$.

We turn to Φ_- , satisfying $\square_6 \Phi_-^{(1)} = 0$. We expand again as $\Phi_-^{(1)} = \sum_i r^{\alpha_i} Y_i$, and solve term by term. The solutions are

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$$\alpha = -2 \pm \sqrt{4 + H} \tag{5.32}$$

where $\square_5 Y = HY$. The dual operators' scaling dimensions are therefore $\Delta = -2 + \sqrt{4 + H}$, and we list those modes in table 5.4.

Δ	j	l	r
1.5	$\frac{1}{2}$	$\frac{1}{2}$	1
2	1	0	0
2	0	1	0
3	1	1	2
3.29	1	1	0
3.5	$\frac{3}{2}$	$\frac{1}{2}$	1
3.5	$\frac{1}{2}$	$\frac{3}{2}$	1

Table 5.4: The scaling dimensions and quantum numbers of harmonics of Φ_- with $\Delta \leq 4$.

The solutions for Φ_+ have scaling dimensions beginning at 6 [164] and so do not contribute to the transmission of SUSY-breaking.

Finally, perturbations of the fiveform flux which are in the cohomology of $T^{1,1}$ clearly satisfy the equations of motion as $d\omega_5 = 0$ and $d*\omega_5 = 0$; however, perturbing the fiveform flux in KW simply changes the number of D3-branes one has in the solution, dual to changing the rank of the gauge group.

5.3.2 The Gauge Side

In this section, we study the gauge dual of the supersymmetry-preserving \mathbb{Z}_2 -orbifold of Klebanov-Strassler. We denote $n = \frac{1}{2}N$ and $m = \frac{1}{2}M$, implicitly assuming that N and M are even. Recall that the orbifold action in the supergravity was $z_i \rightarrow -z_i$, where $z \sim AB$. A and B are bifundamentals under the gauge group, and can therefore be written as $N \times (N + M)$ and $(N + M) \times N$ matrices, respectively. The action of the orbifold can therefore be embedded on the gauge side as follows:

- $SU(N + M)$ is orbifolded by $\text{diag}(I_{n+m}, -I_{n+m})$. The resulting groups are two copies of $SU(n + m)$, and these are called groups G_1 and G_3 , respectively.
- $SU(N)$ is orbifolded by $\text{diag}(I_n, -I_n)$. The resulting groups are two copies of $SU(n)$, and these are called groups G_2 and G_4 , respectively.
- The gauginos are embedded the same way as the gauge bosons so as to preserve supersymmetry.
- The two superfields A^i are odd under the orbifold. The resulting superfields are embedded in A^i as:

$$\begin{pmatrix} 0 & Q_1^i \\ Q_3^i & 0 \end{pmatrix} \tag{5.33}$$

- The two superfields B^j are even under the orbifold. The resulting superfields

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are embedded in B^j as:

$$\begin{pmatrix} Q_2^i & 0 \\ 0 & Q_4^i \end{pmatrix} \quad (5.34)$$

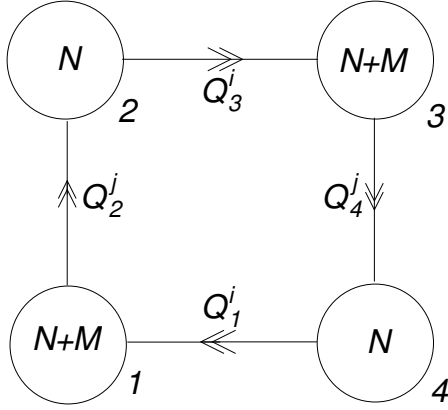


Figure 5.2: The quiver diagram associated with the theory dual to the complex cone over \mathbb{F}_0 .

Under the action of the orbifold, the superpotential can be written as

$$W = \lambda \varepsilon_{ik} \varepsilon_{jl} \text{Tr} (Q_1^i Q_2^l Q_3^k Q_4^j) \quad (5.35)$$

where $\varepsilon_{12} = 1$. The representations under the various groups are listed in table 5.5.

The $U(1)_A$ is a spurious symmetry⁷, with the superpotential coupling λ acting as the spurion. Furthermore, the $U(1)_R$ symmetry is anomalous; the exact symmetry is \mathbb{Z}_{2m} . We also record the charges of the holomorphic intrinsic scales Λ_i , which we introduce shortly.

⁷This statement is only true perturbatively; instantons would break $U(1)_A$ even in the absence of a superpotential.

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Field	G_1	G_2	G_3	G_4	$SU(2)_1$	$SU(2)_2$	B	B_A	B_B	A	R
Q_1	\square	1	1	$\bar{\square}$	2	1	1	1	0	1	$\frac{1}{2}$
Q_2	$\bar{\square}$	\square	1	1	1	2	-1	0	1	1	$\frac{1}{2}$
Q_3	1	$\bar{\square}$	\square	1	2	1	1	-1	0	1	$\frac{1}{2}$
Q_4	1	1	$\bar{\square}$	\square	1	2	-1	0	-1	1	$\frac{1}{2}$
λ	1	1	1	1	1	1	0	0	0	-4	0
$\Lambda_{1,3}^{b_{1,3}}$	1	1	1	1	1	1	0	0	0	$4n$	$2m$
$\Lambda_{2,4}^{b_{2,4}}$	1	1	1	1	1	1	0	0	0	$4(n+m)$	$-2m$

Table 5.5: The representations of the fields in the orbifold of KS. G_1 and G_3 are $SU(n+m)$, G_2 and G_4 are $SU(n)$, and B , B_A , B_B , A and R are all $U(1)$ s.

There are also two additional baryon numbers B_A and B_B we can define in the orbifolded theory. These baryon numbers rotate the daughters of A and the daughters of B into each other, respectively. However, these symmetries are both anomalous with respect to the gauge groups; both $U(1)_{B_A}$ and $U(1)_{B_B}$ are broken by instantons down to the intersection of \mathbb{Z}_{2n} and $\mathbb{Z}_{2(n+m)}$. Notice that this would be true even in the conformal ($m \rightarrow 0$) limit. As this is the limit we match to on the supergravity side, we study it further.

There are baryons associated with the symmetries; we define

$$\mathcal{B}_A^{i\dots k} = \det(Q_1) - \det(Q_3) \quad (5.36)$$

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$$\mathcal{B}_B^{j\dots l} = \det(Q_2) - \det(Q_4) \quad (5.37)$$

\mathcal{B}_A and \mathcal{B}_B are chiral operators and so they have protected scaling dimension $\frac{3}{4}n$. They transform in the spin- $\frac{n}{2}$ rep of their respective $SU(2)$ flavor groups. They have baryon charge n under their respective $U(1)$ s, and so under their respective residual exact symmetries each transform as $\mathcal{B} \rightarrow -\mathcal{B}$. We expect that since D3-branes wrapping the threecycle in homology in KW are dual to baryons in the CFT, then this \mathbb{Z}_2 is related to the nonvanishing torsion threecycle in homology, which generates a \mathbb{Z}_2 [158]. We expect that the currents J_{B_A} and J_{B_B} associated with these to not have protected scaling dimension 2 due to instanton-induced $O(1)$ anomalous dimensions.

There is a residual \mathbb{Z}_2 symmetry of the quiver that exchanges groups 1 and 3 and also groups 2 and 4, along with an associated swapping of matter fields. We opt to impose that symmetry on all of our perturbations as deviations will lead to a different kind of IR. This will place a restriction on the allowed sorts of supersymmetry-breaking in the compactification. Crucially, this forces the equality of gauge couplings g_1 and g_3 , as well as g_2 and g_4 .

Furthermore, there is a \mathbb{Z}_2 outer automorphism of KW that exchanges the two gauge groups as well as A and B . The orbifold inherits this automorphism when $m = 0$; in fact, it combines with the \mathbb{Z}_2 of the previous paragraph to form a full D_4 symmetry group [158], though we will not need the full D_4 for the purposes of this thesis. The \mathbb{Z}_2 needs to be respected by SUSY-breaking in the compactification, as this prevents potentially dangerous operators.

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The NSVZ exact beta function for SQCD with N colors and F vector-like flavors is [167]:

$$\beta_g \equiv \mu \frac{dg}{d\mu} = -\frac{g^3}{16\pi^2} \frac{(3N - F(1 - \gamma))}{1 - \frac{g^2 N}{8\pi^2}} \equiv \frac{b^e g^3}{16\pi^2} \frac{1}{1 - \frac{g^2 N}{8\pi^2}} \quad (5.38)$$

where γ is the anomalous dimension of a quark field, related to the scaling dimension Δ of a field with engineering dimension d by $\Delta = d + \frac{1}{2}\gamma$. As with KS, there is a UV fixed point when $m = 0$. There, the anomalous dimension of the quark superfields are $\gamma = -\frac{1}{2}$ [158, 161], and there are two marginal couplings corresponding to a two-parameter family of fixed points. Here, we choose to sit near the fixed point, so $\gamma_i = -\frac{1}{2} + O\left(\frac{m^2}{n^2}\right)$. The vanishing of the first-order term arises because of a symmetry $m \rightarrow -m, n \rightarrow n + m$ of the theory. For groups 1 and 3, we have $n + m$ colors, and we effectively have $2n$ vector-like flavors. For groups 2 and 4, we have n colors and $2(n + m)$ vector-like flavors. To leading order in $\frac{m}{n}$, then, we have $b_1^e = b_3^e = -3m$ and $b_2^e = b_4^e = 3m$. Thus, gauge groups 1 and 3 are UV-free, meaning that as we flow into the IR, they confine at some scale $|\Lambda_1|, |\Lambda_3|$ respectively. Groups 2 and 4, on the other hand, are IR-free, so the gauge couplings flow towards zero as we flow into the IR. As with SQCD, we introduce the holomorphic confinement scales

$$\Lambda_i = |\Lambda_i| e^{\frac{i\theta_i}{b_i}} \quad (5.39)$$

where θ_i is the theta-angle of the i -th group and $b_i = -3N + F$ is the one-loop beta function coefficient. In our model, we have $b_1 = b_3 = n + 3m$ and $b_2 = b_4 = n - 2m$.

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If we impose for our UV-free gauge groups $g_{1,3} = g_0$ at $\mu = \mu_0$, we reach a Landau pole as we flow into the IR at $\mu = |\Lambda|$, where

$$|\Lambda| = \mu_0 e^{-\frac{8\pi^2}{3mg_0^2}} \quad (5.40)$$

The parent KS theory has an RG cascade, where one performs a Seiberg dual on the confining gauge group and continue flowing into the IR [168]. The orbifolded theory we've constructed inherits this RG cascade when $g_1 = g_3$ and $g_2 = g_4$. The dual theory is identical to the original theory, with $n + m$ replaced with $n - m$, the $U(1)_B$ and $U(1)_A$ charges rescaled, and the coupling λ inverted. The case where the gauge couplings do not start out equal results in a dual theory which is not self-similar [161–163], but this is not relevant for our purposes.

We list the field content of the dual in table 5.6, where $G'_{1,3}$ are $SU(n - m)$ and B , A , etc. are again $U(1)$ s. The charges under the $U(1)$ s were determined by anomaly matching. The $SU(2)_1$, $SU(2)_2$ and $U(1)_B$ anomalies should match exactly. From the point of view of groups 1 and 3, groups 2 and 4 are flavor symmetries, and therefore we should match those anomalies as well. However, the $U(1)_{BA}$, $U(1)_{BB}$, $U(1)_A$ and $U(1)_R$ symmetries are anomalous⁸; therefore, we only require the anomaly coefficients to match up to actual symmetry transformations⁹ [169]. The $U(1)_R$ is broken to \mathbb{Z}_{2m} , and $U(1)_A$ is broken to the intersection of $\mathbb{Z}_{4(n+m)}$ and \mathbb{Z}_{4n} . That intersection is \mathbb{Z}_4

⁸Of course, $U(1)_A$ isn't even a symmetry in the first place; regardless, it's a useful check to match anomaly coefficients.

⁹In order to utilize the results of [169], one needs to rescale $U(1)$ charges to be all integers.

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Field	G'_1	G_2	G'_3	G_4	$SU(2)_1$	$SU(2)_2$	B	B_A	B_B	A	R
Q'_1	$\bar{\square}$	1	1	\square	2	1	$\frac{n+m}{n-m}$	$\frac{n+m}{n-m}$	0	$\frac{n+m}{n-m}$	$\frac{1}{2}$
Q'_2	\square	$\bar{\square}$	1	1	1	2	$-\frac{n+m}{n-m}$	0	$\frac{n+m}{n-m}$	$\frac{n+m}{n-m}$	$\frac{1}{2}$
Q'_3	1	\square	$\bar{\square}$	1	2	1	$\frac{n+m}{n-m}$	$-\frac{n+m}{n-m}$	0	$\frac{n+m}{n-m}$	$\frac{1}{2}$
Q'_4	1	1	\square	$\bar{\square}$	1	2	$-\frac{n+m}{n-m}$	0	$-\frac{n+m}{n-m}$	$\frac{n+m}{n-m}$	$\frac{1}{2}$
M'_{12}	1	\square	1	$\bar{\square}$	2	2	0	0	0	2	1
M'_{34}	1	$\bar{\square}$	1	\square	2	2	0	0	0	2	1
λ'	1	1	1	1	1	1	0	0	0	$-4\frac{n+m}{n-m}$	0
$\Lambda'_{1,3}{}^{b'_{1,3}}$	1	1	1	1	1	1	0	0	0	$4n\frac{n+m}{n-m}$	$-2m$
$\Lambda'_{2,4}{}^{b'_{2,4}}$	1	1	1	1	1	1	0	0	0	$4(n+m)$	$2m$

Table 5.6: The representations of the fields in the orbifold of KS after a Seiberg dual. G'_1 and G'_3 are $SU(n-m)$, G_2 and G_4 are $SU(n)$, and B , B_A , B_B , A and R are all $U(1)$ s.

when $n+m$ and n are coprime, but is \mathbb{Z}_{4k} for some natural number k when $n+m$ and n are not coprime. Finally, note that we don't need to match anomalies with gauge groups 1 or 3, because the anomaly matching trick would require the addition of spectators that were charged under those groups, which changes the details of the confinement. Up to the above caveats, all anomaly coefficients match, lending credence to the idea of a duality cascade.

In the dual theory, there is a superpotential inherited from the original theory, which is self-similar after integrating out the mesons:

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$$W = \lambda' \varepsilon_{ik} \varepsilon_{jl} \text{Tr} (Q_1'^i Q_2'^l Q_3'^k Q_4'^j) \quad (5.41)$$

Now, groups 2 and 4 are UV-free and groups 1 and 3 are IR-free, and so the cascade continues until we cannot dualize any more, at which point the story plays out in a similar fashion to KS, with the deformation of the complex cone and the introduction of an ADS superpotential.

Such a duality cascade, complete with chiral symmetry breaking in the IR, is a fantastically rich IR, with lots of opportunities for model-building.

As with supergravity, we can classify operators by setting $m = 0$ and studying their quantum numbers in the high-energy theory. The allowed operators fall into representations of the superconformal symmetry algebra $su(2, 2|1)$ plus the global symmetry algebra $su(2) + su(2) + u(1)_R$. Note that we ignore the global baryon number B , as all combinations of the fields which are gauge-invariant are automatically B -singlets. Chiral primary operators formed from various gauge and matter fields can be determined at weak coupling¹⁰ and are subject to the following constraints:

- They must be gauge-invariant.
- The F -term equations of motion kill various potential flavor-singlet operators:

$$\varepsilon_{ik} A^i B^j A^k = 0 \quad \varepsilon_{jl} B^j A^i B^l = 0 \quad (5.42)$$

¹⁰We are concerned with the spectrum at strong coupling; many of the operators we consider have protected scaling dimension and therefore their fixed-point scaling dimension can be determined. This approach misses possible mixing between various primaries and descendants, but these are irrelevant for the purposes of spectroscopy.

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- The D -term equations of motion are

$$\begin{aligned} 0 &= \text{Tr} (T_1^a (A^i e^{-V_2} \bar{A}_i - \bar{B}^j e^{V_2} B_j)) \\ 0 &= \text{Tr} (T_2^b (\bar{A}^i e^{V_1} A_i - B^j e^{-V_1} \bar{B}_j)) \end{aligned} \quad (5.43)$$

telling us that the adjoint operators $A^i e^{-V_2} \bar{A}_i - \bar{B}^j e^{V_2} B_j$ and $\bar{A}^i e^{V_1} A_i - B^j e^{-V_1} \bar{B}_j$ vanish in the supersymmetric vacuum, and only the associated singlets can be used to build operators. However, other than these operators themselves, any other operator we could build from them will necessarily be double-trace, and therefore irrelevant to our discussion.

- The super-equations of motion reduce the number of chiral primaries. They are

$$\bar{D}^2 (e^{-V_2} \bar{A} e^{V_1}) = 0 \quad \bar{D}^2 (e^{-V_1} \bar{B} e^{V_2}) = 0 \quad (5.44)$$

- One must consider that various “commutator” operators may not, in fact, be chiral primaries¹¹ due to superspace identities such as

$$W_{1\alpha} AB - AW_{2\alpha} B = -\frac{1}{4} \bar{D}^2 (e^{-V_1} D_\alpha (e^{V_1} A e^{-V_2}) e^{V_2} B) \quad (5.45)$$

- The one- θ components of the W are real; therefore they equal the one- $\bar{\theta}$ components of the \bar{W} :

$$D^\alpha W_\alpha = \bar{D}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \quad (5.46)$$

Superfields have protected scaling dimensions if they fall into one of the following categories:

¹¹A more accurate way to phrase this would be to say that these operators are \bar{D} -cohomologous to zero, or that they vanish in the chiral ring.

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- They are chiral; $\bar{D}_{\dot{\alpha}}X = 0$.
- They are semichiral; $\bar{D}_{(\dot{\alpha}}X_{\beta_1\dots\beta_n)\beta_1\dots} = 0$.
- They are conserved; $D^\alpha X_{\alpha\dots\dot{\alpha}\dots} = \bar{D}^{\dot{\alpha}} X_{\alpha\dots\dot{\alpha}\dots} = 0$ or $\bar{D}^2 X_{\alpha\dots} = 0$.
- They are semiconserved; $\bar{D}^{\dot{\alpha}} X_{\alpha\dots\dot{\alpha}\dots} = 0$.

The classification of the operators in KW was carried out in [160, 164, 165]; we list the results for the chiral primaries with scalar components which have scaling dimension $\Delta \leq 4$, using the convention (j, l, r) for the global quantum numbers to match the supergravity solutions. Note that non-real operators have hermitian conjugates with $(j, l, -r)$ and the same Δ which are excluded from the below list for brevity. This list can be found in table 5.3.3, matched to their dual supergravity modes.

5.3.3 The AdS/CFT Matching

We have explored the matching of all SUGRA perturbations and SCFT primaries with $\Delta \leq 4$ in table 5.3.3. In order for an operator to be dangerous to us, it must be a GSRO and furthermore be even under the orbifold action $A \rightarrow -A$, $B \rightarrow B$. Furthermore, it must be even under the two \mathbb{Z}_2 symmetries described above. Such a GSRO could be a singlet relevant single-trace operator or a double-trace operator which is the square of a (non-singlet) single-trace operator¹² with $\Delta < 2$. We see that

¹²The double-trace operator will have an anomalous dimension, but it will be $O(\frac{1}{n})$ and so we ignore the correction.

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there are no such operators of either kind in the spectrum, and therefore the complex cone over \mathbb{F}_0 exhibits accidental SUSY.

Note that in KW and its orbifold, the R-symmetry is exact. However, instantons in KS break this symmetry, and therefore could presumably induce dangerous operators. We treat operators in the UV, KW limit, and so we should not expect to be able to generate operators which violate the R-symmetry. Even if we were to discuss operator perturbations to KS, though, for $m \geq 2$, a group larger than R-parity is preserved, and so instantons would not be able to generate any operators in the superpotential that would threaten to destabilize the hierarchy.

5.4 Conclusions

In this work, we have demonstrated that a SUSY-preserving \mathbb{Z}_2 orbifold of the noncompact Klebanov-Strassler theory admits accidental supersymmetry. This theory can also be described as type IIB string theory propagating on a deformed complex cone over \mathbb{F}_0 . We were able to describe the space of UV Lagrangian deformations of the dual gauge theory by studying classical perturbations to the KW theory, and using the AdS/CFT correspondence to match those perturbations to operators in the gauge theory.

The theory discussed in this chapter is over a noncompact Calabi-Yau, meaning that the theory does not have dynamical four-dimensional gravity. This is not a

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problem from the point of view of attempting to exhibit any UV-complete model of accidental SUSY, but one might hope to incorporate 4d gravity in more realistic string models which reproduced the MSSM or the natural SUSY spectrum. Due to a no-go theorem, a compactification to four dimensions is best described in the language of F-theory. In that language, the model described in this paper is an effective, local description of the warped throat present in the full compactification. By allowing for non-normalizable perturbations in supergravity, we systematically allow any and all operators respecting the \mathbb{Z}_2 symmetries described in 5.3.2, knowing that all such operators cannot effectively transmit SUSY-breaking.

In future work, we plan to pursue the question of constructing an F-theory model which reproduces the model in this paper in some local patch of the base of the Calabi-Yau fourfold. It is not immediately obvious that such a fourfold should exist, but the existence of F-theory models such as described in [15, 153] gives us hope that it is feasible. One would like the F-theory compactification to satisfy the following checklist:

- It should contain a warped throat which supports an adequately large hierarchy, meaning that the Euler number of the fourfold should be sufficiently large.
- It must respect the two \mathbb{Z}_2 symmetries described in section 5.3.2, which are necessary to protect against GSROs.
- Compact Calabi-Yaus do not possess continuous isometries, and so we must pre-

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serve a sufficiently large discrete subgroup of $SU(2) \times SU(2) \times U(1)_R$ to prevent those non-global singlet operators described in 5.3.3 from being generated.

- We must break SUSY in the bulk of the Calabi-Yau.

There are, of course, the usual worries about moduli stabilization; we assume that the F-theory fluxes stabilizes the moduli of the compactification. It would be interesting to explore general string corrections to this model; in particular, it would be interesting to see how the no-scale structure is broken in relation to the length of the throat that can be generated.

Another interesting direction for future work is related to the presence of a massless scalar glueball in the field theory on the baryonic branch of the moduli space in KS [170], dual to axionic D-strings. More generally, this is related to the existence of a one-parameter family of solutions that KS belongs to [171]. It would be interesting to explore the orbifolds of this family and see how well SUSY-breaking can be transmitted.

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Scalar \mathcal{O}	Type	\mathbb{Z}_2	Orb.	Δ	SUGRA	(j, l, r)
I	-	E	Y	0	Vac.	$(0, 0, 0)$
$\text{Tr}(AB)$	Chiral	E	N	1.5	Φ_-	$(\frac{1}{2}, \frac{1}{2}, 1)$
$\text{Tr}(AB)_{\theta 2}$	Chiral	E	N	2.5	G_3^I	$(\frac{1}{2}, \frac{1}{2}, -1)$
$\text{Tr}(\bar{A}e^{V_1} A e^{-V_2} - \bar{B}e^{V_2} B e^{-V_1})$	Cons.	O	Y	2	G_3^{II}	$(0, 0, 0)$
$\text{Tr}(\bar{A}e^{V_1} A e^{-V_2} - \bar{B}e^{V_2} B e^{-V_1})_{\theta 2}$	Cons.	O	Y	3	G_3^I	$(0, 0, -2)$
$\text{Tr}(\bar{A}e^{V_1} A e^{-V_2})$	Cons.	-	Y	2	Φ_-	$(1, 0, 0)$
$\text{Tr}(\bar{A}e^{V_1} A e^{-V_2})_{\theta 2}$	Cons.	-	Y	3	G_3^I	$(1, 0, -2)$
$\text{Tr}(\bar{B}e^{V_2} B e^{-V_1})$	Cons.	-	Y	2	Φ_-	$(0, 1, 0)$
$\text{Tr}(\bar{B}e^{V_2} B e^{-V_1})_{\theta 2}$	Cons.	-	Y	3	G_3^I	$(0, 1, -2)$
$\text{Tr}(ABAB)$	Chiral	E	Y	3	Φ_-	$(1, 1, 2)$
$\text{Tr}(ABAB)_{\theta 2}$	Chiral	E	Y	4	G_3^I	$(1, 1, 0)$
$\sim \text{Tr}(A B e^{-V_1} \bar{B} \bar{A} e^{V_1})$	Long	E	Y	3.29	Φ_-	$(1, 1, 0)$
$\text{Tr}(\bar{A}e^{V_1} A B A e^{-V_2})$	Cons.	-	Y	3.5	Φ_-	$(\frac{3}{2}, \frac{1}{2}, 1)$
$\text{Tr}(\bar{B}e^{V_2} B A B e^{-V_1})$	Cons.	-	Y	3.5	Φ_-	$(\frac{1}{2}, \frac{3}{2}, 1)$
$\text{Tr}(W_{1\alpha} A B + A W_{2\alpha} B)_{\theta}$	Chiral	E	N	3.5	G_3^{II}	$(\frac{1}{2}, \frac{1}{2}, 1)$
$\text{Tr}(W_{1\alpha} A e^{-V_2} \bar{A} e^{V_1} + W_{2\alpha} e^{-V_2} \bar{A} e^{V_1} A)_{\theta}$	Semicons.	-	Y	4	G_3^{II}	$(1, 0, 0)$
$\text{Tr}(W_{1\alpha} e^{-V_1} \bar{B} e^{V_2} B + W_{2\alpha} B e^{-V_1} \bar{B} e^{V_2})_{\theta}$	Semicons.	-	Y	4	G_3^{II}	$(0, 1, 0)$
$\text{Tr}(W_1^2 + W_2^2)$	Chiral	E	Y	3	G_3^{III}	$(0, 0, 2)$
$\mathcal{L} \left(\frac{1}{g_1^2} + \frac{1}{g_2^2} \right) = \text{Tr}(W_1^2 + W_2^2)_{\theta 2}$	Chiral	E	Y	4	τ	$(0, 0, 0)$
$\text{Tr}(W_1^2 - W_2^2)$	Chiral	O	Y	3	h	$(0, 0, 2)$
$\mathcal{L} \left(\frac{1}{g_1^2} - \frac{1}{g_2^2} \right) = \text{Tr}(W_1^2 - W_2^2)_{\theta 2}$	Chiral	O	Y	4	$A_2 = \omega_2$	$(0, 0, 0)$

Table 5.7: A matching of SUGRA and SCFT scalar modes with $\Delta \leq 4$. The columns include: scalars in their respective superoperators, why their dimensions might be protected, whether they are even under the outer \mathbb{Z}_2 -automorphism of KW, their symmetry structure under our orbifold, their scaling dimension, what mode they are dual to in SUGRA, and their quantum numbers under $SU(2) \times SU(2) \times U(1)_R$.

Chapter 6

Looking Forward

This thesis has concerned itself with the subjects of natural and accidental supersymmetry (SUSY), and the potential for interplay between them. This interplay goes hand in hand with the interplay between the little and big hierarchy problems. Natural supersymmetry solves the little hierarchy problem, and accidental supersymmetry generates radiatively stable little hierarchies while also solving the big hierarchy problem. We have studied the consequences of both of these mechanisms, how accidental SUSY could give rise to natural SUSY, and how accidental SUSY could itself be realized in type IIB superstring theory.

There are many reasons to look forward, and a great deal still to be done. Some possibilities for future research directions include the following:

- 1: There are many corners of the natural SUSY parameter space, especially in the BNV framework, which CMS and ATLAS are currently blind to. One example of

CHAPTER 6. LOOKING FORWARD

a robust signal would be the chain decay $\tilde{t} \rightarrow b(\tilde{\chi}^+ \rightarrow jjj)$. As the chargino is a color singlet and has the potential to be at least mildly boosted, this decay is ideally suited for a study of new soft substructure variables tracking color flow. Furthermore, decays such as $\tilde{b} \rightarrow b(\tilde{\chi}^0 \rightarrow \tilde{t}^*t^* \rightarrow jjbW)$ have the potential to be long-lived for even moderate values of $\lambda'' \sim 10^{-2}$, opening up the exciting possibility of using displaced vertices to search for such events. Exploring the phenomenology of these signals and more at the LHC would be a promising and exciting way to search for natural SUSY.

2: On the RS front, there are a number of issues to be made more concrete, such as radius stabilization and a parameterization of graviton and graviphoton contributions to the Higgs mass. Furthermore, it would be very exciting to study accidental SUSY in BSM models, as this could potentially imply exciting new signatures at the LHC as well, perhaps due to an interplay of supersymmetric and extra-dimensional dynamics.

3: It would be exciting to search for a realization of accidental SUSY in F-theory, as discussed in the previous chapter. Furthermore, it would be interesting to see what we can learn about the possibility of accidental SUSY by considering complex cones over del Pezzo surfaces, which admit a warped throat.

4: Finally, it would be very exciting to have a sense of how “generic” features such as warped throats or accidental SUSY are on the landscape, defined in relation to the number of critical points with high-scale SUSY-breaking and a finely-tuned electroweak scale. We hope that tests of warped throats or accidental SUSY can be developed and utilized to scan the landscape, in a similar spirit to the surveys of [172–174].

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This could serve as inspiration to pursue the idea of naturalness further at the 14 TeV LHC and beyond, as well as open new chapters in the field of string phenomenology.

It would, of course, be fantastic to discover signs of natural SUSY, KK modes or even strings at the LHC14, and the author looks forward to what the future holds.

Appendix A

Notations and Conventions

We set $\hbar = c = k_B = 1$. We use $v \approx 246$ GeV and $M_{pl} \approx 1.2 \cdot 10^{19}$ GeV.

In 4d and 5d, we use the mostly-minus metric signature $\text{sign}(g_{00}) = 1$, $\text{sign}(g_{ii}) = -1$ (no summation). In 10d, we use the mostly-plus metric signature $\text{sign}(g_{00}) = -1$, $\text{sign}(g_{ii} = 1)$ to allow for a positive-signature metric on internal spaces. We always sum unless otherwise specified. Consequently, we use P&S conventions for canonical kinetic terms.

Groups are capital (e.g. $SU(2)$), whereas the corresponding algebras are lower-case (e.g. $\mathfrak{su}(2)$). We abuse notation and denote the representation with the same letters as the basis elements of the algebra, and we use numbers denoting the size of representations to refer to the entire representation. We use both the spin label (e.g. $\frac{1}{2}$) and the dimension label (e.g. 2) to refer to $SU(2)$ representations, as which one we're using is clear from context.

APPENDIX A. NOTATIONS AND CONVENTIONS

When giving nonvanishing Christoffel symbols, we give half of them; e.g. $\Gamma_{23}^1 = z$ implies Γ_{32}^1 is also z . We do not write vanishing Christoffels.

We always conjugate fermions so that all Weyl fermions are left-handed.

We often use the following conventions for indices: μ is usually a 4d Lorentz index. M is a 10d index. m is an internal index, sometimes spanning the five-dimensional horizon and sometimes the entire threefold. m can also refer to indices on a general manifold. r is not an index and refers to the radial direction. α and $\dot{\alpha}$ refer to 4d spinor indices, generally on superspace. We use Wess and Bagger conventions for superfields, up to our choice of metric.

In four- and five-dimensional theories, all gauge bosons A_μ have associated field strengths with the same letter but an additional index; e.g. $A_{\mu\nu}$.

d^4x is shorthand for $dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$. κ , α' and g_s are related by $2\kappa^2 = (2\pi)^7 \alpha'^4 g_s^2$. We set the string length $l_s = 1$ ($\alpha' = \frac{1}{2}$) at times in chapter 5, and other times set $R_{AdS} = 1$, depending on which leads to cleaner formulae. $g_s = \langle e^\phi \rangle = \langle \frac{1}{\text{Im}\tau} \rangle$.

Appendix B

Relevant Mathematical Results

B.1 Groups, Algebras and Representations

There are two important numbers that we can associate with an irreducible representation (“irrep”) of a Lie algebra; these are the quadratic Casimir C_2 and the index of the representation T . These are defined in terms of representation matrices T by

$$T_{ij}^a T_{jk}^a = C_2 \delta_{ik} \tag{B.1}$$

$$\text{Tr}(T^a T^b) = T \delta^{ab} \tag{B.2}$$

These are conventions which can be changed by simply rescaling generators. Of course, these numbers may differ from irrep to irrep. We list some important values of these for important irreps of $\mathfrak{su}(N)$:

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Irrep	$\dim(r)$	$C_2(r)$	$T(r)$
1	1	0	0
\square	N	$\frac{N^2-1}{2N}$	$\frac{1}{2}$
$\bar{\square}$	N	$\frac{N^2-1}{2N}$	$\frac{1}{2}$
Ad	$N^2 - 1$	N	N

B.2 Differential Geometry

The N -*bien* (where N is the German word for the number which is the dimension of the space) is defined by

$$g_{\mu\nu} = \eta_{ab} e_\mu^a e_\nu^b \quad (\text{B.3})$$

The Levi-Civita connection coefficients are defined as

$$\Gamma_{\mu\nu}^\lambda = \frac{1}{2} g^{\lambda\alpha} (\partial_\mu g_{\nu\alpha} + \partial_\nu g_{\mu\alpha} - \partial_\alpha g_{\mu\nu}) = \Gamma_{\nu\mu}^\lambda \quad (\text{B.4})$$

where $g^{\mu\nu}$ is the inverse metric, defined by $g^{\mu\nu} g_{\nu\lambda} = \delta_\lambda^\mu$.

The covariant derivative acting on covariant vectors can be expressed in components as

$$\nabla_\mu v^\nu = \partial_\mu v^\nu + \Gamma_{\mu\alpha}^\nu v^\alpha \quad (\text{B.5})$$

and on oneforms as

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$$\nabla_{\mu} v_{\nu} = \partial_{\mu} v_{\nu} - \Gamma_{\mu\nu}^{\alpha} v_{\alpha} \quad (\text{B.6})$$

The covariant derivative acts in a similar way on tensors, picking up a + for every swapped upstairs/covariant index and picking up a – for every swapped downstairs/contravariant index.

The Riemann tensor provides a measure of the intrinsic curvature of the space. It is defined by

$$R^{\rho}{}_{\mu\lambda\nu} = \partial_{\lambda}\Gamma_{\nu\mu}^{\rho} + \Gamma_{\lambda\sigma}^{\rho}\Gamma_{\nu\mu}^{\sigma} - \partial_{\nu}\Gamma_{\lambda\mu}^{\rho} - \Gamma_{\nu\sigma}^{\rho}\Gamma_{\lambda\mu}^{\sigma} \quad (\text{B.7})$$

It is a measure of the failure to close of two parallel transports along different paths.

It has a natural contraction, the Ricci tensor,

$$R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu} \quad (\text{B.8})$$

which appears in the Einstein equation. It can also be contracted to give a Lorentz scalar, the Ricci scalar,

$$R = g^{\mu\nu} R_{\mu\nu} \quad (\text{B.9})$$

which makes its appearance in the Einstein-Hilbert action.

Spaces can have isometries; these are defined by a particular kind of vector field. These vector fields satisfy the property that when you drag the metric along them,

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the metric remains invariant. Such a vector field is called a *Killing vector field*; the precise statement is that the *Lie derivative* of the metric along v , the Killing vector field, vanishes;

$$\mathcal{L}_v g = 0 \tag{B.10}$$

In components, this becomes the *Killing equation*:

$$2\nabla_{(\mu} v_{\nu)} = \nabla_{\mu} v_{\nu} + \nabla_{\nu} v_{\mu} = 0 \tag{B.11}$$

Vector fields which satisfy the Killing equation are isometries of the space.

The wedge product on p -forms is

$$\alpha \wedge \beta \equiv \frac{p!q!}{(p+q)!} (\alpha \otimes \beta - \beta \otimes \alpha) \tag{B.12}$$

We define p_{α} to be the number p on a p -form α . The wedge product on an n -dimensional space satisfies the useful identity

$$\alpha \wedge \beta = (-1)^{p_{\alpha} p_{\beta}} \beta \wedge \alpha \tag{B.13}$$

We define the privileged top-form

$$\omega = \sqrt{|\det g|} dx^1 \wedge \dots \wedge dx^n \tag{B.14}$$

The exterior derivative d satisfies the identity

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$$d(\alpha \wedge \beta) = d\alpha \wedge \beta + (-1)^{p_\alpha p_\beta} \alpha \wedge d\beta \quad (\text{B.15})$$

Suppose we have a p -form with components $\alpha_{a_1 \dots a_p}$. Then the components of the Hodge dual form $*\alpha$ are

$$(*\alpha)_{c_1 \dots c_{n-p}} = \frac{\sqrt{|\det g|}}{p!} \alpha_{a_1 \dots a_p} g^{a_1 b_1} \dots g^{a_p b_p} \varepsilon_{b_1 \dots b_{n-p} c_1 \dots c_p} \quad (\text{B.16})$$

where ε is the Levi-Civita symbol. Note the following properties of $*$:

$$* 1 = \omega \quad (\text{B.17})$$

$$* * \alpha = (-1)^{s+p_\alpha(n-p_\alpha)} \alpha \quad (\text{B.18})$$

where s is the sign of the determinant; in other words, it is 0 if there are an even number of minus signs in the signature and 1 otherwise.

$$\alpha \wedge * \beta = \beta \wedge * \alpha \quad (\text{B.19})$$

The *codifferential* d^\dagger which takes p -forms to $(p-1)$ -forms is

$$d^\dagger = *^{-1} d * \quad (\text{B.20})$$

The *Laplacian* takes p -forms to p -forms:

APPENDIX B. RELEVANT MATHEMATICAL RESULTS

$$\square = dd^\dagger + d^\dagger d \quad (\text{B.21})$$

p -forms which satisfy $\square\alpha = 0$ are called *harmonic* forms.

B.3 Grassmann Variables

We follow Wess and Bagger conventions for fermionic coordinates on superspace:

$$\theta^2 = \theta\theta = \theta^\alpha\theta_\alpha \quad (\text{B.22})$$

$$\theta^\alpha\theta^\beta = -\frac{1}{2}\varepsilon^{\alpha\beta}\theta^2 \quad (\text{B.23})$$

$$\bar{\theta}^2 = \bar{\theta}\bar{\theta} = \bar{\theta}_{\dot{\alpha}}\bar{\theta}_{\dot{\alpha}} \quad (\text{B.24})$$

$$\bar{\theta}_{\dot{\alpha}}\bar{\theta}_{\dot{\beta}} = -\frac{1}{2}\varepsilon_{\dot{\alpha}\dot{\beta}} \quad (\text{B.25})$$

$$\varepsilon^{12} = 1 \quad \varepsilon_{i\dot{j}} = 1 \quad (\text{B.26})$$

$$d^2\theta = -\frac{1}{4}\varepsilon_{\alpha\beta}d\theta^\alpha d\theta^\beta \quad (\text{B.27})$$

$$d^2\bar{\theta} = -\frac{1}{4}\varepsilon^{\dot{\alpha}\dot{\beta}}d\bar{\theta}_{\dot{\alpha}}d\bar{\theta}_{\dot{\beta}} \quad (\text{B.28})$$

$$\int d^2\theta \theta^2 = 1 \quad \int d^2\bar{\theta} \bar{\theta}^2 = 1 \quad (\text{B.29})$$

$$d^4\theta \equiv d^2\theta d^2\bar{\theta} \quad (\text{B.30})$$

Appendix C

Relevant Physics Results

C.1 Introduction to SUSY

A supersymmetry is a spacetime symmetry of a manifold which is generated not by a killing vector field, but rather by a killing spinor field. Consequently, a supersymmetry is a fermionic symmetry of the theory. A killing spinor is a spinor ψ which, when Lie transported along any vector field X , is invariant ($\mathcal{L}_X\psi = 0$). In components, this means

$$\nabla_m\psi = 0 = \partial_m\psi - \frac{1}{4}\omega_{mab}\gamma^{ab}\psi \tag{C.1}$$

where $\omega_{mab} = -\frac{1}{2}(\Omega_{mab} - \Omega_{abm} + \Omega_{bma})$ is the spin connection and $\Omega_{mn}{}^a$ is $\partial_n e_m^a - \partial_m e_n^a$. e is the N -bien, a and b are tangent-space indices, and m and n are indices of vectors on the manifold.

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In $\mathbb{R}^{1,3}$, $\Omega = 0$ and so $\nabla_\mu \psi$ turns into $\partial_\mu \psi$, solved by any constant ψ . As with killing vectors, the generators of isometries close into an algebra. However, as the killing spinors are fermionic, the algebra must be graded. The basis elements of the superalgebra transform in the spinor representation on the manifold, and so on $\mathbb{R}^{1,3}$ we have left-handed and right-handed Weyl fermion generators Q_α and $\bar{Q}^{\dot{\alpha}}$.

$\nabla_m \psi = 0$ implies $[\nabla_m, \nabla_n] \psi = 0$. However, the latter is $R_{mnpq} \gamma^{pq} \psi$, where $\gamma^{pq} = [\gamma^p, \gamma^q]$ is the commutator of elements of the Clifford algebra. It can be verified that $R_{mnpq} \gamma^{pq} \psi = 0$ implies $R_{mn} = 0$, or the Ricci curvature of M vanishes. This follows from the Clifford algebra identity

$$\gamma^n \gamma^{pq} = \gamma^{npq} + g^{np} \gamma^q - g^{nq} \gamma^p \tag{C.2}$$

and the Bianchi identity for the Riemann tensor. Note that on Kähler manifolds, the statement that the manifold is Ricci flat is equivalent to the statement that it is Calabi-Yau by Calabi's conjecture, proven by Yau.

A theorem by Haag, Lopuszanski and Sohnius argues that the only consistent superalgebra which acts on a manifold that admits a consistent quantum field theory is known as the supersymmetry algebra,

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$$\begin{aligned} \{Q_\alpha^M, \bar{Q}_{\dot{\alpha}}^N\} &= 2\sigma_{\alpha\dot{\alpha}}^\mu P_\mu \delta^{MN} \\ \{Q_\alpha^M, Q_\beta^N\} &= \varepsilon_{\alpha\beta} Z^{MN} \\ \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} &= \varepsilon_{\dot{\alpha}\dot{\beta}} \bar{Z}^{MN} \end{aligned} \tag{C.3}$$

with all other supercommutators except those defining the Poincaré algebra vanishing. M and N run over the number of supersymmetries \mathcal{N} . Z is an antisymmetric matrix of central charges. In practice, \mathcal{N} must be less than 9 for the spectrum to not need to contain particles of spin greater than 2 or less than 5 for the spectrum to not need to contain particles of spin greater than 1. We will primarily focus on the $\mathcal{N} = 1$ algebra in this thesis, and so there is no central charge.

The $\mathcal{N} = 1$ supersymmetry algebra admits a representation on the superspace $\mathbb{R}^{1,3|4}$. The representation is

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu \tag{C.4}$$

$$\bar{Q}^{\dot{\alpha}} = \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha \sigma_{\alpha\dot{\beta}}^\mu \varepsilon^{\dot{\beta}\dot{\alpha}} \partial_\mu \tag{C.5}$$

These satisfy $\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2i\sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu$ as required. In addition, we can introduce the covariant derivatives

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu \tag{C.6}$$

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu \tag{C.7}$$

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which anticommute with Q and \bar{Q} .

In order to have a supersymmetric quantum field theory, we obviously need supersymmetry to be a symmetry of the ground state $|\Omega\rangle$. This is the statement that finite supersymmetry transformations leave $|\Omega\rangle$ invariant, or equivalently, that Q and \bar{Q} annihilate $|\Omega\rangle$:

$$e^{i(\varepsilon Q + \bar{\varepsilon} \bar{Q})} |\Omega\rangle = |\Omega\rangle \quad \Leftrightarrow \quad Q|\Omega\rangle = \bar{Q}|\Omega\rangle = 0 \quad (\text{C.8})$$

Since there is a natural representation of the supersymmetry algebra on superspace, one might hope that in order to develop $\mathcal{N} = 1$ supersymmetric QFTs, one should consider fields which themselves *live* on superspace. Such fields are known as *superfields*. Because of the finiteness of the Taylor expansion, we can write out the expansion of a scalar superfield in terms of *component* scalar, spinor and vector fields on $\mathbb{R}^{1,3}$. The two important kinds of superfields for us are the *chiral superfield* Φ satisfying $\bar{D}_{\dot{\alpha}}\Phi = 0$ and the *vector superfield* V satisfying $V = V^\dagger$. Chiral superfields are appropriate for holding the matter of a theory, and vector superfields contain the gauge sector of a theory. These have the following Taylor expansions:

$$\Phi = \phi + \sqrt{2}\theta\psi + \theta\theta F \quad (\text{C.9})$$

$$V = \theta\sigma^\mu\bar{\theta}A_\mu + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D \quad (\text{C.10})$$

where we have used supergauge transformations to gauge away unphysical degrees of freedom in the vector superfield. λ is the “gaugino”. The top component of Φ and V

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are respectively called the F - and D -terms; if either has a VEV then supersymmetry is spontaneously broken.

In addition, there can be spinor and vector superfields; for example, the graviton in supergravity lives in a vector superfield. We will not need this formalism much in this thesis, except for the (chiral) field strength superfield, which is defined differently for Abelian (A) and non-Abelian (NA) gauge groups:

$$W_\alpha^A = -\frac{1}{4}\bar{D}\bar{D}D_\alpha V = -i\lambda_\alpha + \left(\delta_\alpha^\beta D - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu)_\alpha{}^\beta F_{\mu\nu} \right) \theta_\beta + \theta\theta\sigma_{\alpha\dot{\alpha}}^\mu\partial_\mu\bar{\lambda}^{\dot{\alpha}} \quad (\text{C.11})$$

$$W_\alpha^{NA} = -\frac{1}{4}\bar{D}\bar{D}(e^{-V}D_\alpha e^V) = -i\lambda_\alpha + \left(\delta_\alpha^\beta D - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu)_\alpha{}^\beta F_{\mu\nu} \right) \theta_\beta + \theta\theta\sigma_{\alpha\dot{\alpha}}^\mu D_\mu\bar{\lambda}^{\dot{\alpha}} \quad (\text{C.12})$$

where in the second equation, fields are matrix-valued and derivatives are now gauge-covariant.

Superfields are closed under multiplication, and products of chiral superfields give another chiral superfield.

By acting with a finite supersymmetry transformation on our superfields, we translate θ to $\theta + \varepsilon$. By collecting terms again by powers of θ , we can instead say that each component transforms under a supersymmetry transformation. It can be verified that the top component of a general superfield is supersymmetry-invariant up to total derivatives and equations of motion. This is true because $d^4\theta$ -integration is defined

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to be supertranslation-invariant, and this integration picks out the top θ -component of a superfield. Similarly, a change of spacetime variables $x^\mu \rightarrow y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ leaves chiral superfields $\bar{\theta}$ -independent, and so $d^2\theta$ -integration again picks out a supersymmetry-invariant piece of a chiral superfield.

We demand that the Hamiltonian of our QFT be hermitian so that the time-evolution operator is unitary; with these restrictions in mind the most general Lagrangian for a supersymmetric QFT is

$$\mathcal{L} = \int d^4\theta K + \left(\int d^2\theta W + \text{h.c.} \right) \quad (\text{C.13})$$

where K is a real function of the superfields known as the *Kähler potential* and W is a holomorphic (chiral) function of the superfields known as the *superpotential*.

Supersymmetric models often come with an R -symmetry; this is a global symmetry $U(1)_R$ which is gauged in supergravity where Q and \bar{Q} themselves have charge -1 and 1 , respectively. This gives θ and $\bar{\theta}$ charge 1 and -1 , and therefore $d\theta$ and $d\bar{\theta}$ charge -1 and charge 1 . Therefore, if a superfield has charge r , its bottom component will have charge r , its $1-\theta$ component will have charge $r + 1$, etc. The superpotential W has R -charge 2 .

It is possible to spontaneously break SUSY in one sector of the theory and transmit it to another sector of the theory. In these cases, SUSY-breaking can be parameterized by spurions which only possess constant F or D terms, and no propagating degrees of freedom. SUSY-breaking is considered to be *soft* if the coefficients in front of

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the SUSY-breaking terms in the Lagrangian have positive mass dimension. Since in perturbation theory, SUSY-breaking operator insertions place the coefficient in the numerator instead of the denominator, as we take the coefficient to zero, we recover the supersymmetric result. This is what we want, because as we zero our F and D terms, our spurions must vanish as well. The converse is *hard* SUSY-breaking; an example would be a SUSY-breaking scalar quartic, which may not decouple as SUSY-breaking is turned off. An example of a hard SUSY-breaking is explicit SUSY-breaking in the Lagrangian.

If Lorentz symmetry is gauged by the graviton, then SUSY, being a spacetime symmetry, is also gauged. Gauged supersymmetry is known as supergravity (“SUGRA”). The graviton has a spin- $\frac{3}{2}$ superpartner, the “gravitino”, which becomes massive after eating the “Goldstino” of spontaneous SUSY-breaking.

C.2 Anti-de Sitter Space

The Einstein-Hilbert action for general relativity in five dimensions with cosmological constant Λ is

$$S = \frac{1}{16\pi G} \int d^5x \sqrt{g} (R - 2\Lambda) \tag{C.14}$$

Recall

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$$R = g^{\mu\nu} R_{\mu\nu} = g^{\mu\nu} R^\lambda{}_{\mu\lambda\nu} \quad (\text{C.15})$$

$$R^\rho{}_{\mu\lambda\nu} = \partial_\lambda \Gamma^\rho{}_{\nu\mu} + \Gamma^\rho{}_{\lambda\sigma} \Gamma^\sigma{}_{\nu\mu} - \partial_\nu \Gamma^\rho{}_{\lambda\mu} - \Gamma^\rho{}_{\nu\sigma} \Gamma^\sigma{}_{\lambda\mu} \quad (\text{C.16})$$

$$\Gamma^\delta{}_{\alpha\beta} = \frac{1}{2} g^{\gamma\delta} (\partial_\beta g_{\gamma\alpha} + \partial_\alpha g_{\beta\gamma} - \partial_\gamma g_{\alpha\beta}) = \Gamma^\delta{}_{\beta\alpha} \quad (\text{C.17})$$

The Einstein equation is:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - k^2 g_{\mu\nu} = 0 \quad (\text{C.18})$$

These have a static solution given by AdS_5 :

$$ds^2 = (1 + k^2 r^2) dt^2 - (1 + k^2 r^2)^{-1} dr^2 - r^2 d\Omega^2 \quad (\text{C.19})$$

We will be doing physics on the Poincaré patch of AdS_5 , which is a coordinate patch which covers one causal diamond of AdS_5 . The reason for this is that we would like a dual CFT which is defined on 4d Minkowski space, whereas the spatial boundary ($r \rightarrow \infty$) of the full AdS_5 is S^3 . On this patch, which I will call $PAdS_5$, we can write the metric in the following two useful forms:

$$ds^2 = \frac{1}{k^2 z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \quad (\text{C.20})$$

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \quad (\text{C.21})$$

where they are conformally related by $z = \frac{1}{k} e^{ky}$. These relate to the original metric by

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$$\frac{1}{k^2 z^2} = 1 + k^2 r^2 \tag{C.22}$$

which gives the right metric in the $z \ll l$ limit. We list for completeness the nonvanishing Christoffel symbols on eqn. C.20, with 0, 1, 2, 3 referring to x_0, x_1, x_2, x_3 :

$$\begin{aligned} \Gamma_{0z}^0 = \Gamma_{1z}^1 = \Gamma_{2z}^2 = \Gamma_{3z}^3 = \Gamma_{0z}^z = \Gamma_{zz}^z = -\frac{1}{z} \\ \Gamma_{1z}^z = \Gamma_{2z}^z = \Gamma_{3z}^z = \frac{1}{z} \end{aligned} \tag{C.23}$$

The *Randall-Sundrum framework* is a framework where one studies physics on the Poincaré patch of AdS_5 , viewing it as a sort of linearization or approximation of a more sophisticated geometry. One considers placing a “UV-brane” in the space, possibly allowing fields and interactions localized to the brane, at $z = \varepsilon$ (oftentimes treating $\varepsilon = 0$). Furthermore, we can consider placing an “IR-brane” at $z = \ell$, effectively compactifying the space. We study physics on this slice of AdS_5 , and do not consider the rest of the space. As reviewed in chapter 1, fields living on the IR brane experience a redshifting of energy scales. The presence of both branes allows for the existence of a normalizable 4d graviton mode in the KK reduction of the 5d graviton, something true more generally of compactifications of string theory. Removing the UV brane and considering the infinite space all the way out to $z = 0$ prevents the 4d graviton mode from being normalizable.

Let’s study a free scalar in RS. The action is

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$$S = \int d^4x dz \sqrt{g} (|\partial_M \Phi|^2 - M^2 |\Phi|^2) \quad (\text{C.24})$$

As a quick aside, the mass-squared of a scalar Φ can actually be negative! It turns out that this is okay; the theory does not have a vacuum instability so long as $M^2 > -\frac{4}{l^2}$.

This is known as the *Breitenlohner-Freedman bound*.

Varying the action yields

$$\square \Phi + e^{2kz} \partial_z (e^{-4kz} \partial_z \Phi) + M^2 e^{-2ky} \Phi = 0 \quad (\text{C.25})$$

The action as stated has either Dirichlet or Neumann boundary conditions, $(\delta \Phi \partial_z \Phi)_{0, \pi r} = 0$. At this point, there are two options; we can treat Φ as a 5d field and derive its 5d propagator. This can be done, and yields the propagator discussed in chapter 4. Alternatively, we could KK reduce the theory to 4d and study the physics of the infinite number of 4d fields. We proceed to do the latter. We assume a separation of variables

$$\Phi(x, z) = \sum_{n=0}^{\infty} \phi^{(n)}(x) f^{(n)}(z) \quad (\text{C.26})$$

where $\phi^{(n)}$ satisfies $\square \phi^{(n)} = m_n^2 \phi^{(n)}$, and \square is now the 4d Laplacian. f is referred to the wavefunction or profile in the fifth dimension. One can justify using onshell wavefunctions in 5d right at or below around the scale of the first KK mode. Plugging in, we get

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$$m_n^2 e^{-2kz} f^{(n)} + \partial_z (e^{-4kz} \partial_z f^{(n)}) + M^2 e^{-4ky} f^{(n)} = 0 \quad (\text{C.27})$$

This is a Sturm-Liouville differential equation; such an equation has infinite number of eigenvalues m_n^2 which are real and well-ordered, and in addition, we learn that the eigenfunctions are orthogonal. We choose to rescale modes such that

$$\int_0^{\pi r} dz e^{-2kz} f^{(n)}(z) f^{(m)}(z) = \delta_{nm} \quad (\text{C.28})$$

We write $M^2 = ak^2$. The general solution for the 0th KK mode with $m_n = 0$ of eqn. C.27 is

$$f^{(0)}(z) = c_1 e^{(2-\sqrt{4+a})z} + c_2 e^{(2+\sqrt{4+a})z} \quad (\text{C.29})$$

This does not satisfy either Dirichlet or Neumann boundary conditions for any choice of a . Therefore, we add boundary terms to the action

$$S' = \int d^4x dz \sqrt{g} (2bk) (\delta(z) - \delta(z - \pi r)) |\Phi|^2 \quad (\text{C.30})$$

with $b = 2 \pm \sqrt{4+a}$. The bottom mode now goes like $e^{(b-1)z}$ (where the proportionality constant can be determined by the normalization constraint eqn. C.28). We see that for $b > 1$, the scalar is localized towards the IR brane, $b < 1$, the UV brane, and $b = 1$, flat. For $m_n \neq 0$, the solution looks like

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$$f^{(n)}(z) = N^{(n)} e^{2kz} \left(J_{2-b} \left(\frac{m_n}{k e^{-kz}} \right) + b^{(n)} Y_{2-b} \left(\frac{m_n}{k e^{-kz}} \right) \right) \quad (\text{C.31})$$

where $b^{(n)}$ is determined by boundary conditions, $N^{(n)}$ is determined by eqn. C.28, and in the $kr \gg 1$ limit, the m_n are given by

$$m_n \approx \left(n + \frac{\sqrt{4+a}}{2} - \frac{3}{4} \right) \pi k e^{-\pi k r} \quad (\text{C.32})$$

The fact that the masses of the KK modes are redshifted down implies that they should be localized at the IR brane, which is indeed the case. Suppose in the 5d picture, we had a coupling that looked like $\lambda_5 |\Phi|^4$. In order to determine the low-energy couplings of, say, 2 0th and 2 1st KK modes, one integrates out the extra dimension;

$$\lambda_4 = \lambda_5 \int_0^{\pi r} dz \sqrt{g} f^{(0)}(z) f^{(0)}(z) f^{(1)}(z) f^{(1)}(z) \quad (\text{C.33})$$

Note that Φ has mass dimension $\frac{3}{2}$, ϕ has mass dimension 1, so f has mass dimension $\frac{1}{2}$. λ_5 has mass dimension -1 so that $\int d^4 x dz \lambda_5 |\Phi|^4$ has mass dimension 0; this implies that λ_4 will have mass dimension 0, as it should. The fact that we need a coupling of negative mass dimension in 5d to generate a coupling which is marginal in 4d tells us that we need a UV completion at the scale at which the EFT breaks down. That scale will be, at most, the 5d Planck scale M_5 , implying the existence of new KK modes at 4d scales no larger than $\sim M_5 e^{-k\ell}$.

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A similar story exists for fermions and gauge bosons. However, spinors in 5d start with 4 components; we must therefore use boundary conditions to project out the massless states that we don't want. The fermion mass is given by $M = ck$; upon doing the same steps as with the scalar, one concludes that for $c > \frac{1}{2}$, the fermion is localized near the UV brane, and $c < \frac{1}{2}$, it's localized near the IR brane. The massless mode of a gauge boson has a flat profile. What this means is that it will have the same overlap with all states, since the coupling will always be bilinear in fermions or scalars; pulling the same trick as above, we see that a flat profile can be pulled out of the gauge boson-fermion-fermion or whichever coupling, and so we end up with just eqn. C.28 as the overlap. Finally, there is a massless graviton mode in 4d; it is localized near the UV brane (as it should so that gravity is weak!). The KK graviton is localized near the IR brane, and so it's an exciting resonance to look for at the LHC.

The SM in RS is interesting to explore; one could imagine wanting the top to have a large yukawa coupling, and so you enforce a large wavefunction overlap between the top and the higgs by localizing the top near the IR brane; conversely, if you want very small neutrino masses, you could strongly localize the neutrinos near the UV brane so that their masses are very suppressed. This is a rich and interesting tool for model-buildling. There exists an RS-GIM mechanism in which FCNCs are suppressed simply by wavefunction overlaps.

C.3 Conformal Field Theory

The conformal transformation generators are spanned by the usual Poincaré generators P_μ and $M_{\mu\nu}$, and in addition, there are two other generators D and K_μ , called the dilation or dilatation and special conformal generators, respectively. The generators are

$$\begin{aligned}
P_\mu &= i\partial_\mu \\
M_{\mu\nu} &= i(x_\mu\partial_\nu - x_\nu\partial_\mu) \\
D &= ix^\mu\partial_\mu \\
K_\mu &= i(x^2\partial_\mu - 2x_\mu x_\nu\partial^\nu)
\end{aligned} \tag{C.34}$$

Computing commutators gives the algebra

$$\begin{aligned}
[P_\mu, M_{\nu\rho}] &= i(\eta_{\mu\nu}P_\rho - \eta_{\mu\rho}P_\nu) \\
[M_{\mu\nu}, M_{\rho\sigma}] &= i(\eta_{\nu\rho}M_{\mu\sigma} - \eta_{\mu\rho}M_{\nu\sigma} - \eta_{\nu\sigma}M_{\mu\rho} + \eta_{\mu\sigma}M_{\nu\rho}) \\
[K_\mu, M_{\nu\rho}] &= i(\eta_{\mu\nu}K_\rho - \eta_{\mu\rho}K_\nu) \\
[P_\mu, K_\nu] &= -2i\eta_{\mu\nu}D - 2iM_{\mu\nu} \\
[D, P_\mu] &= -iP_\mu \\
[D, K_\mu] &= iK_\mu
\end{aligned} \tag{C.35}$$

with all other commutators vanishing. This algebra is $\mathfrak{so}(2, 4)$.

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The action of generators on dilatation eigenoperators is

$$\begin{aligned}
 [D, \mathcal{O}] &= i(-\Delta + x \cdot \partial)\mathcal{O} \\
 [P_\mu, \mathcal{O}] &= i\partial_\mu\mathcal{O} \\
 [J_{\mu\nu}, \mathcal{O}] &= i(x_\mu\partial_\nu - x_\nu\partial_\mu + \Sigma_{\mu\nu})\mathcal{O} \\
 [K_\mu, \mathcal{O}] &= i(x^2\partial_\mu - 2x_\mu x \cdot \partial + 2x_\mu\Delta - 2x^\nu\Sigma_{\mu\nu})\mathcal{O}
 \end{aligned} \tag{C.36}$$

Here, the $\Sigma_{\mu\nu}$ s are the representations of the $\mu\nu$ -th generator of the Lorentz group. Each Σ is a matrix acting on the appropriate index of \mathcal{O} , be it a spinor, vector, or tensor.

Conserved currents in quantum field theories have protected scaling dimension $\Delta = 3$, and the stress tensor has protected scaling dimension $\Delta = 4$.

C.4 The AdS/CFT correspondence

The AdS/CFT correspondence, as originally formulated, is the following statement: *$SU(N)$ $\mathcal{N} = 4$ super-Yang Mills in $4d$ is the same theory as type IIB superstring theory propagating on $AdS_5 \times S^5$ in the vicinity of N D3-branes.* This equivalence is quite shocking; it tells us that there are two descriptions of the same theory; one is a four-dimensional conformal field theory (without gravity), and the other is a five-dimensional string theory in AdS coupled to gravity, in the vicinity of five additional dimensions. More specifically, there is a one-to-one map between states in the two

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descriptions of the theory. In particular, one expects there to be a map between building blocks of the CFT (single-trace primaries, or STPs) and building blocks of the AdS theory (strings, or in the low-energy limit, fields). The 5d masses of fields are related to the scaling dimensions of the 4d operators. For example, for scalars, $l^2 M^2 = \Delta(\Delta - 4)$, where the LHS refers to the AdS theory and the RHS refers to the CFT. Correlators in the CFT can be recovered from correlators in AdS by a certain limiting procedure where we drag correlators of fields to the boundary of the space.

Over the years, it has become apparent that this magic applies to many other pairs of descriptions as well. Furthermore, there are many theories in which there is an effective field theory description of the string theory (giving rise to something akin to RS), and these have dual CFT descriptions as well. This effective language involves there being a mass gap in the AdS description of the theory, dual to a scaling dimension gap in the CFT. Furthermore, we can leave the fixed point in the CFT, dual to deforming geometry to be not quite AdS anymore. This can be tracked down by comparing the isometry groups of the theory; leaving a fixed point removes the action of the CFT algebra on the operators, and so we should not expect to find a dual theory with AdS isometries anymore.

Distances in AdS are generically dual to energy scales in the CFT; $e^{-ky} \leftrightarrow \mu$. The IR brane in RS therefore indicates the end of some AdS-like region, dual to the end of a CFT-like flow. Therefore, we conclude that the dual theory sits at an approximate fixed point. The energy scale that the IR-brane position is dual to indicates the

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compositeness or confinement scale of the dual quasi-CFT. BSM models in RS are dual to composite Higgs models in 4d. Finally, general classical perturbations of the AdS solution in AdS tell us information about deformations of the Lagrangian of the dual theory.

It is understood that if one considers perturbative gauge symmetries in AdS, then correlators of gauge bosons dragged to the boundary produce operators in the CFT which are understood as global symmetry currents. Note that for full AdS, there is no normalizable zero-mode in the 4d theory arising from the KK reduction of the theory, but if we inserted UV and IR branes in AdS, we would indeed have a normalizable zero mode, meaning the dual CFT would have a weakly gauged global symmetry. The higher KK modes in the AdS description would be dual to the single-trace primary conserved current. The protection of the scaling dimension of the conserved current is dual to the gauge protection of the AdS gauge boson mass.

In a similar fashion, the 5d graviton in full AdS is dual to the stress tensor in the CFT. Deforming the theory with an IR and a UV brane gives the graviton a normalizable zero mode in the KK reduction, and so the resulting theory has dynamical 4d gravity.

C.5 IIB Supergravity

The low-energy bosonic part of the 10d supergravity action can be written as

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$$\begin{aligned}
S = & \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left(e^{-2\phi} \left(R + 4\partial_\mu \phi \partial^\mu \phi - \frac{1}{2 * 3!} |H_3|^2 \right) \right. \\
& \left. - \frac{1}{2} \sqrt{-g} \left(|F_1|^2 + \frac{1}{3!} |F_3 - C_0 H_3|^2 + \frac{1}{2 * 5!} |F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3|^2 \right) \right) \\
& - \frac{1}{4\kappa^2} \int C_4 \wedge H_3 \wedge F_3 \tag{C.37}
\end{aligned}$$

in what is known as the *string frame*. We can perform a transformation to the *Einstein frame*, where the Ricci scalar is properly normalized, by a transformation $g \rightarrow e^{-\frac{\phi}{2}} g$. In this frame, and using the language of forms, the action becomes [175]:

$$\begin{aligned}
S = & \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} R - \frac{1}{4\kappa^2} \int \left(d\phi \wedge *d\phi + e^{2\phi} dC_0 \wedge *dC_0 + e^{-\phi} H_3 \wedge *H_3 \right. \\
& \left. + e^\phi (F_3 - C_0 H_3) \wedge *(F_3 - C_0 H_3) + \frac{1}{2} \tilde{F}_5 \wedge *\tilde{F}_5 + C_4 \wedge H_3 \wedge F_3 \right) \tag{C.38}
\end{aligned}$$

where we have introduced the fiveform $\tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$, and expressed things in forms. Note that the fiveform F_5 is supposed to be self-dual, but this constraint is to be placed on the theory at the level of the equations of motion, rather than at the level of the action. The action is invariant under 10d diffeomorphisms as well as the three gauge transformations

$$C_2 \rightarrow C_2 + d\Lambda_1, \quad C_4 \rightarrow C_4 + \frac{1}{2} \Lambda_1 \wedge H_3 \tag{C.39}$$

$$B_2 \rightarrow B_2 + d\Lambda'_1, \quad C_4 \rightarrow C_4 - \frac{1}{2} \Lambda'_1 \wedge F_3 \tag{C.40}$$

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$$C_4 \rightarrow C_4 + d\Lambda_3 \quad (\text{C.41})$$

The equations of motion can be obtained through a straightforward application of the variational principle on the Einstein frame action, and they are [176]

$$\nabla^2 \phi = e^{2\phi} \partial_M C_0 \partial^M C_0 - \frac{g_s e^{-\phi}}{12} |H_3|^2 + \frac{g_s e^\phi}{12} |F_3 - C_0 H_3|^2 \quad (\text{C.42})$$

$$\nabla^M (e^{2\phi} \partial_M C_0) = -\frac{g_s e^\phi}{6} H_3 \cdot \tilde{F}_3 \quad (\text{C.43})$$

$$d(*e^\phi (F_3 - C_0 H_3)) = g_s F_5 \wedge H_3 \quad (\text{C.44})$$

$$d*(e^{-\phi} - C_0^2 e^\phi) H_3 - C_0 e^\phi F_3 = -g F_5 \wedge F_3 \quad (\text{C.45})$$

$$d*\tilde{F}_5 = -F_3 \wedge H_3 \quad (\text{C.46})$$

$$\begin{aligned} R_{MN} = & \frac{1}{2} \partial_M \phi \partial_N \phi + \frac{e^{2\phi}}{2} \partial_M C_0 \partial_N C_0 + \frac{g_s^2}{96} \tilde{F}_{MPQRS} \tilde{F}_N{}^{PQRS} \\ & + \frac{g}{4} \left(e^{-\phi} H_{MPQ} H_N{}^{PQ} + e^\phi \tilde{F}_{MPQ} \tilde{F}_N{}^{PQ} \right) - \frac{g}{48} g_{MN} \left(e^{-\phi} |H_3|^2 + e^\phi |\tilde{F}_3|^2 \right) \end{aligned} \quad (\text{C.47})$$

Here, we must impose self-duality of \tilde{F}_5 . The difference between self-duality of F_5 and of \tilde{F}_5 amounts to the presence of additional boundary terms in the action, which do not affect the equations of motion. Although these equations of motion and the Bianchi identities are consistent with self-duality, they do not imply it.

The Bianchi identities are simply the statement that exact forms are closed in the absence of magnetic sources; $dF_3 = dH_3 = dF_5 = 0$. However, these seemingly trivial statements help in looking for gauge-invariant solutions to supergravity, because they allow us to work directly with the field strengths.

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Finally, the action possesses an $SL(2, \mathbb{R})$ symmetry, of which only a $SL(2, \mathbb{Z})$ subgroup is respected by the full string theory. The symmetry is best noticed by defining the field combinations

$$\tau = C_0 + ie^{-\phi} \quad G_3 = F_3 - \tau H_3 \quad (\text{C.48})$$

Noticing that $*1 = \sqrt{-g}d^{10}x$, we can write the action in a very compact form

$$S = \frac{1}{2\kappa^2} \int \left(R * 1 - \frac{d\tau \wedge *d\bar{\tau}}{2(\text{Im}\tau)^2} - \frac{G_3 \wedge *\bar{G}_3}{2\text{Im}\tau} - \frac{\tilde{F}_5 \wedge *\tilde{F}_5}{4} - \frac{C_4 \wedge G_3 \wedge \bar{G}_3}{4i\text{Im}\tau} \right) \quad (\text{C.49})$$

In this form, we cannot simply vary the action to obtain equations of motion, but we can see the $SL(2, \mathbb{Z})$ symmetry easily. The symmetry acts on fields as

$$\tau \rightarrow \frac{a\tau + b}{c\tau + e} \quad (\text{C.50})$$

$$\begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & e \end{pmatrix} \begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \quad (\text{C.51})$$

$$G_3 \rightarrow \frac{G_3}{c\tau + e} \quad (\text{C.52})$$

$$\tilde{F}_5 \rightarrow \tilde{F}_5 \quad (\text{C.53})$$

where $\begin{pmatrix} a & b \\ c & e \end{pmatrix} \in SL(2, \mathbb{Z})$. For convenience, we record various other useful transformation properties:

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$$\bar{\tau} \rightarrow \frac{a\bar{\tau} + b}{c\bar{\tau} + e} \quad (\text{C.54})$$

$$\text{Im}\tau \rightarrow \frac{\text{Im}\tau}{|c\tau + e|^2} \quad (\text{C.55})$$

$$d\tau \rightarrow \frac{d\tau}{(c\tau + e)^2} \quad (\text{C.56})$$

$$d\bar{\tau} \rightarrow \frac{d\bar{\tau}}{(c\bar{\tau} + e)^2} \quad (\text{C.57})$$

$$d * d\tau \rightarrow = \frac{d * d\tau}{(c\tau + e)^2} - \frac{2cd\tau \wedge *d\tau}{(c\tau + e)^3} \quad (\text{C.58})$$

$$\begin{pmatrix} F_3 \\ H_3 \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & e \end{pmatrix} \begin{pmatrix} F_3 \\ H_3 \end{pmatrix} \quad (\text{C.59})$$

$$\bar{G}_3 \rightarrow \frac{\bar{G}_3}{c\bar{\tau} + e} \quad (\text{C.60})$$

$$d * G_3 \rightarrow \frac{d * G_3}{c\tau + e} - \frac{cd\tau \wedge *G_3}{(c\tau + e)^2} \quad (\text{C.61})$$

We would like to pass from the equations of motion for IIB supergravity in terms of the fundamental fields to equations utilizing τ , G_3 , \tilde{F}_5 and h_{MN} . This can be done, and yields

$$-\frac{2}{\text{Im}\tau} d\tau \wedge *d\tau + 2id * d\tau = G_3 \wedge *G_3 \quad (\text{C.62})$$

$$\frac{d\tau \wedge *(G_3 + \bar{G}_3)}{2\text{Im}\tau} - id * G_3 = \tilde{F}_5 \wedge G_3 \quad (\text{C.63})$$

$$d * \tilde{F}_5 = -\frac{G_3 \wedge \bar{G}_3}{2i\text{Im}\tau} \quad (\text{C.64})$$

$$R_{MN} = \frac{\partial_M \tau \partial_N \bar{\tau}}{2(\text{Im}\tau)^2} + \frac{G_{3MPQ} G_{3N}{}^{PQ}}{4\text{Im}\tau} + \frac{\tilde{F}_{5MPQRS} \tilde{F}_{5N}{}^{PQRS}}{96} - g_{MN} \frac{|G_3|^2}{48\text{Im}\tau} \quad (\text{C.65})$$

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Under a modular transformation, the first of these equations transforms by multiplying the entire equation by $\frac{1}{(c\tau+e)^2}$. The second equation becomes multiplied by $\frac{1}{c\tau+e}$. The third and fourth are invariant under modular transformations. The rewritten Bianchi identities are:

$$dG_3 = -d\tau \wedge H_3 \tag{C.66}$$

$$d\tilde{F}_5 = -\frac{G_3 \wedge \tilde{G}_3}{2i\text{Im}\tau} \tag{C.67}$$

We again must impose by hand that $\tilde{F}_5 = *\tilde{F}_5$. We often work with the metric and fiveform flux ansätze

$$ds^2 = e^{2A}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A}(dr^2 + r^2 ds_{T^{1,1}}^2) \tag{C.68}$$

$$\tilde{F}_5 = (1 + *)d\alpha \wedge d^4x \tag{C.69}$$

C.6 The Conifold

The conifold is a noncompact Calabi-Yau threefold, given by the real cone over $T^{1,1}$. The conifold is the subspace of \mathbb{C}^4 satisfying

$$\sum_{i=1}^4 z_i^2 = 0 \tag{C.70}$$

After a linear change of variables

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$$Z_1 = z_1 + iz_2 \quad Z_2 = z_1 - iz_2 \quad Z_3 = iz_3 - z_4 \quad Z_4 = iz_3 + z_4 \quad (\text{C.71})$$

this is brought to $Z_1 Z_2 = Z_3 Z_4$, which can be solved by

$$x_0 y_0 = Z_1 \quad x_0 y_1 = Z_4 \quad x_1 y_0 = Z_3 \quad x_1 y_1 = Z_2 \quad (\text{C.72})$$

where however we must mod out by points in x and y which describe the same Z coordinates; this corresponds to the equivalence relations

$$x_i \sim \lambda x_i \quad y_i \sim \lambda^{-1} y_i \quad (\text{C.73})$$

$$x_i \sim e^{i\alpha} x_i \quad y_i \sim e^{-i\alpha} y_i \quad (\text{C.74})$$

where λ is positive. In order to determine the geometry of the space, we recognize that the space is a cone with a finite 5d space as its transverse space. We study that by picking some distance out on the cone and studying the resulting space; we pick

$$\sum_i |z_i|^2 = \frac{1}{2} \quad (\text{C.75})$$

In terms of the x_i, y_i this condition reads

$$(|x_0|^2 + |x_1|^2)(|y_0|^2 + |y_1|^2) = 1 \quad (\text{C.76})$$

Since we have a rescaling equivalence between x and y , we can fix both x and y to be of length 1;

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$$|x_0|^2 + |x_1|^2 = 1 \quad |y_0|^2 + |y_1|^2 = 1 \quad (\text{C.77})$$

modulo the phase equivalence relation. In terms of the real coordinates

$$x_0 = a_1 + ib_1 \quad x_1 = c_1 + id_1 \quad y_0 = a_2 + ib_2 \quad y_1 = c_2 + id_2 \quad (\text{C.78})$$

this is the two equations

$$a_i^2 + b_i^2 + c_i^2 + d_i^2 = 1 \quad (\text{C.79})$$

The horizon in question is called $T^{1,1}$, and is therefore the space of $S^3 \times S^3$, modded out by an equivalence relation describing a $U(1)$, which we will ultimately orbifold by a \mathbb{Z}_2 . Each S^3 is readily solved and parameterized by

$$a_i = \cos \frac{\theta_i}{2} \cos \beta_i \quad (\text{C.80})$$

$$b_i = \cos \frac{\theta_i}{2} \sin \beta_i \quad (\text{C.81})$$

$$c_i = \sin \frac{\theta_i}{2} \cos \gamma_i \quad (\text{C.82})$$

$$d_i = \sin \frac{\theta_i}{2} \sin \gamma_i \quad (\text{C.83})$$

where each θ runs from 0 to π and β runs from 0 to 2π and we choose the range of γ to be from $-\pi$ to π . These are Hopf coordinates on the S^3 , and the corresponding induced metric from \mathbb{R}^4 is

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$$ds_{i,Ind}^2 = \frac{1}{4}d\theta_i^2 + \cos^2 \frac{\theta_i}{2} d\beta_i^2 + \sin^2 \frac{\theta_i}{2} d\gamma_i^2 \quad (\text{C.84})$$

where i is 1 or 2 and Ind indicates that we're looking at the induced metric from \mathbb{R}^4 .

In order to describe the modding out by $U(1)$, it's more convenient to switch to coordinates $\beta_i = \frac{1}{2}(\psi_i + \phi_i)$ and $\gamma_i = \frac{1}{2}(\psi_i - \phi_i)$, where ψ and ϕ run from 0 to 2π .

The $U(1)$ -equivalence is

$$(\beta_1, \gamma_1, \beta_2, \gamma_2) \sim (\beta_1 + \alpha, \gamma_1 + \alpha, \beta_2 - \alpha, \gamma_2 - \alpha) \quad (\text{C.85})$$

or, in terms of ψ_i and ϕ_i , that $\psi_1 \sim \psi_1 + \alpha$, $\psi_2 \sim \psi_2 - \alpha$ and ϕ_i are invariant. Therefore, it's helpful to introduce two coordinates $\psi_+ \equiv \psi = \psi_1 + \psi_2$ and $\psi_- = \psi_1 - \psi_2$. Therefore, ψ is invariant under the equivalence and has a range from 0 to 4π and $\psi_- \equiv \psi_- + 2\alpha$. However, the metric we will use on this 6d space has ∂_{ψ_-} as a killing vector, and so we are free to use the equivalence to gauge-fix ψ_- to 0 and study the resulting space.

The induced metric in these coordinates that one would like to study can be written out explicitly by using the above equations:

$$\begin{aligned} ds_{Ind}^2 &= \sum_i \frac{1}{4}d\theta_i^2 + \frac{1}{4} \cos^2 \frac{\theta_i}{2} (d\psi_i + d\phi_i)^2 + \frac{1}{4} \sin^2 \frac{\theta_i}{2} (d\psi_i - d\phi_i)^2 \\ &=_{GF} \frac{1}{4} \left(\frac{1}{2}d\psi^2 + d\theta_1^2 + d\theta_2^2 + d\phi_1^2 + d\phi_2^2 + (\cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2) d\psi \right) \quad (\text{C.86}) \end{aligned}$$

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where the subscript GF indicates that gauge-fixing ψ_- happened between the two equalities.

However, in order for the cone over the horizon (the 6d space) to be Calabi-Yau and admit a Ricci-flat metric

$$ds^2 = dr^2 + r^2 ds_H^2 \quad (\text{C.87})$$

it must be the case that ds_H^2 is an Sasaki-Einstein metric, meaning $R_{MN} = (d-2)g_{MN}$.

Therefore, we'd like to not use the induced metric on this space, but a different metric with however the same topology. This can be accomplished by rescaling ds_{Ind}^2 by $\frac{2}{3}$ and adding a term which does not change the topology to the metric; we'll call this Δds^2 . The resulting metric for the horizon is

$$ds_H^2 = \frac{2}{3} ds_{Ind}^2 + \frac{2}{9} \Delta ds^2 \quad (\text{C.88})$$

$$\Delta ds^2 = -\frac{1}{4} (d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2)^2 \quad (\text{C.89})$$

The resulting Sasaki-Einstein metric can be written in terms of ψ , ϕ_i and θ_i as

$$ds_H^2 = \frac{1}{9} (d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2)^2 + \frac{1}{6} (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2 + d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2) \quad (\text{C.90})$$

making manifest the S^1 fibration across $S^2 \times S^2$. The metric can be rewritten diagonally in terms of globally-defined one-forms $g_{1\dots 5}$, related to the coordinates above

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by

$$\begin{aligned}
 g_1 &= \frac{1}{\sqrt{2}} (-\sin \theta_1 d\phi_1 - \cos \psi \sin \theta_2 d\phi_2 + \sin \psi d\theta_2) \\
 g_2 &= \frac{1}{\sqrt{2}} (d\theta_1 - \sin \psi \sin \theta_2 d\phi_2 - \cos \psi d\theta_2) \\
 g_3 &= \frac{1}{\sqrt{2}} (-\sin \theta_1 d\phi_1 + \cos \psi \sin \theta_2 d\phi_2 - \sin \psi d\theta_2) \\
 g_4 &= \frac{1}{\sqrt{2}} (d\theta_1 + \sin \psi \sin \theta_2 d\phi_2 + \cos \psi d\theta_2) \\
 g_5 &= d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2
 \end{aligned} \tag{C.91}$$

The metric in these coordinates is

$$ds^2 = \frac{1}{6} (g_1^2 + g_2^2 + g_3^2 + g_4^2) + \frac{1}{9} g_5^2 \tag{C.92}$$

The orbifold we study is $z_i \sim -z_i$, which maps to $Z_i \sim -Z_i$. This can be solved by either $x_i \sim -x_i$ or $y_i \sim -y_i$; the free \mathbb{Z}_2 orbifold of either S^3 . Clearly, as $\alpha = \pi$ is the equivalence relation sending $x_i \rightarrow -x_i$ and $y_i \rightarrow -y_i$, we can always shift it to the other by means of the equivalence relation. This means sending one vector (a_i, b_i, c_i, d_i) in \mathbb{R}^4 to minus itself, while keeping the other intact. Without loss of generality, let's choose this to be x_i ; i.e. the first S^3 . This can be accomplished by sending $(a_1, b_1, c_1, d_1) \rightarrow -(a_1, b_1, c_1, d_1)$, or in terms of the angles, $\beta_1 \rightarrow \beta_1 + \pi$ and $\gamma_1 \rightarrow \gamma_1 + \pi$. Both of these can be accomplished by sending $\psi_1 \rightarrow \psi_1 + 2\pi$ and leaving ϕ invariant. Therefore, we're identifying $\psi \sim \psi + 2\pi$ and $\psi_- \sim \psi_- + 2\pi$, but the latter is absorbed by a gauge identification, meaning nothing happens. Therefore, the only

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effect in terms of our final parameterization is the shrinking of the range of ψ from $0 \leq \psi < 4\pi$ to $0 \leq \psi < 2\pi$, halving the volume of the manifold.

Finally, as all of the one-forms g_i on the manifold are functions of $\cos \psi$, $\sin \psi$ and $d\psi$, and all of these are even under the orbifold action, it follows that all one-forms g_i are even/invariant under the orbifold action, and any flux such as F_3 , H_3 or F_5 constructed from these g_i are also even under the orbifold action, preserving them under the orbifold. Consequently, the KS solution is even under the orbifold.

Topologically, $T^{1,1}$ is $S^2 \times S^3$; therefore we expect its homology groups to be $H_0 = H_2 = H_3 = H_5 = \mathbb{Z}$, $H_1 = H_4 = 0$. By Poincaré duality, we expect the cohomology groups to be $H^0 = H^2 = H^3 = H^5 = \mathbb{Z}$, $H^1 = H^4 = 0$. The second and third cohomology generators of $T^{1,1}$ are

$$\omega_2 = g_1 \wedge g_2 + g_3 \wedge g_4 = \frac{1}{2} (\sin \theta_1 d\theta_1 \wedge d\phi_1 - \sin \theta_2 d\theta_2 \wedge d\phi_2) \quad (\text{C.93})$$

$$\omega_3 = g_5 \wedge \omega_2 \quad (\text{C.94})$$

Other elements are not in the cohomology because of identities such as

$$\sin \theta_1 d\theta_1 \wedge d\phi_1 + \sin \theta_2 d\theta_2 \wedge d\phi_2 = -dg_5 \quad (\text{C.95})$$

$$g_5 \wedge (g_1 \wedge g_2 - g_3 \wedge g_4) = d(g_1 \wedge g_3 + g_2 \wedge g_4) \quad (\text{C.96})$$

$$g_5 \wedge (g_1 \wedge g_3 + g_2 \wedge g_4) = -d(g_1 \wedge g_2 - g_3 \wedge g_4) \quad (\text{C.97})$$

Finally, the fifth cohomology generator is the volume form of $T^{1,1}$, which is

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$$\omega_5 = \frac{1}{54}\omega_2 \wedge \omega_3 = \frac{1}{108} \sin \theta_1 \sin \theta_2 d\psi \wedge d\theta_1 \wedge d\theta_2 \wedge d\phi_1 \wedge d\phi_2 \quad (\text{C.98})$$

Integrating this over $T^{1,1}$ gives the volume $V = \frac{16\pi^3}{27}$.

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