Comparative analysis of MCDM methods for the assessment of sustainable housing affordability

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Abstract

While affordability is traditionally assessed in economic terms, this paper tests a new assessment method that draws closer links with sustainability by consideringeconomic, social and environmental criteriathat impact on a household's quality of life. The paper presents an empirical application and comparison of six different multiple criteria decision making (MCDM) approaches for the purpose of assessing sustainable housing affordability.

The comparative performance of theweighted product model (WPM), the weighted sum model (WSM),the revised AHP, TOPSIS and COPRAS, is investigated.The purpose of the comparative analysis is to determinehow different MCDM methods compare when used for a sustainable housing affordability assessment model. 20 evaluative criteria and 10 alternativeareas in Liverpool, England,were considered.The applicability of different MCDM methods for the focused decisionproblem was investigated.The paper discusses the similarities in MCDM methods,evaluates their robustness and contrasts the resulting rankings.

1. Introduction

It is imperative that both affordability and sustainability issues are simultaneously tackled in order to create successful housing and communities. Affordable housing alone is not enough to achieve community and family wellbeing; households need decent quality affordable housing that is well located within good quality environments that are clean, safe and have good access to jobs, key services and public transport [1-3]. There is both an efficiency and equity imperative to ensure that affordable housing is environmentally sustainable and socially equitable [4]. Accordingly, it may not only be the cost of housing that needs to be addressed in order to improve housing affordability; access to amenities, facilities and the energy efficiency of housing may need to be improved to create successful and sustainable living environments [5, 6];. However, traditional measures of affordability are one dimensional and continue to focus solely on economic criteria as the basis of assessment [7-10].

Researchers suggest that the traditional way of defining and measuring housing affordability - the relationship between household's income and expenditure - is too limited [11-13]. Accordingly, in order to assist in achieve successful housing outcomes, there is a need to develop a more holistic housing affordability assessment tool that is better aligned with sustainability concerns and household wellbeing.

Limitations in the assessment of affordability can be eliminated by the use of methods which are able to take into account a wider range of criteria than traditional methods do. The paper aims to test a housing affordability assessment methodology that is more holistic and capable of considering such a broad spectrum of criteria that affect the wellbeing of households - including economic, environmental and social aspects. Here, a number of widely used MCDM methods – the Weighted Sum Model (WSM), the Weighted Product Model (WPM), the revised Analytic Hierarchy Process (AHP), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Complex Proportional Assessment (COPRAS) –applied for the assessment of sustainable housing affordability. The rankingsof the alternatives and their tolerance to change in criterion weightsare comparedamongst selected MCDM methods. The comparative analysis of

these different methods will aid in establishing the most appropriate and compatible methodology for the purpose of sustainable housing affordability assessment.

2. Housing affordability

Housingaffordability has received considerable attention across the globe for a number of years [13-20]. However, the concept and measurement of housing affordability remains a challenging and contested issue. Affordability measures generally focus on the financial burden of housing costs, such as the house price to income ratio approach [20] , the residual measure (income remaining after housing costs) [21] and, since the impact of the latest recession, purchase and repayment affordability measures [7]. The most commonly referred to and internationally recognised method of measuring affordability is the ratio method, which determines the proportion of income spent on housing costs [10]. This is not surprising since it has the advantage of being easy to compute as it only relies on a few, usually easily accessible, variables. Nevertheless, this simplicity is precisely what limits its effectiveness since it does not incorporate a number of factors that affect housing affordability and the household situation. This traditional approach is one-dimensional and researchers [5,11-13,22-24] are increasingly documenting its limitations. In particular, the ratio measure fails to account for differences in housing costs that are the result of perceived higher neighbourhood quality [23]. Belsky*et al*. [22] suggest that an ideal affordability appraisal would account for the trade-offs that households make to lower housing costs, such as transportation, access to public services, health and safety. Stone *et al*. [25] also emphasise a growing concern that standard affordability measures do not recognise the trade-offs between cheap or affordable housing; just because a household has an 'affordable dwelling' does not necessarily mean it has 'affordable living', owing to trade-offs. Likewise, Rowley and Ong [13] recognise that, in reality, housing affordability encompasses quality and location trade-offs. Additional costs may be imposed on households as a result of such trade-offs, both monetary and socioeconomic costs, which are disguised by traditional measures of affordability.

Housing affordability is a complex and multi-dimensional issue. Accordingly, to gain a better insight into the problem, it should not be analysed using just one concept, measure or definition [26,27]. It is clearly difficult, perhaps impossible, to address all concerns related to affordability within one simple measure. Issues such as housing adequacy, e.g. physical quality, location and access to services and appropriateness may need to be addressed by additional complementary indicators [12]. McCord *et al.* [27] elucidate that a one measure fits all approach to assessing affordability is problematic and policy makers must consider more than one measure when reforming policy instruments. Despite these findings, research often continue to focus on economic criteria alone as the basis of housing affordability assessments [7-10], with little regard for what households get in return for what they spend on housing in terms of housing location and neighbourhood characteristics. There is a specified need for the criteria by which housing is judged as affordable to be refined [11].

The literature highlights the need for innovations in the assessment of housing affordability. The researchers postulate that housing affordability must be defined and assessed in a more meaningful way, requiring a new paradigm of thinking that goes beyond the financial implications experienced by households.An international desire to create more affordable and more sustainable communities means that closer links must be drawn between economic, environmental and social concerns. Housing affordability and sustainability issues are increasingly being discussed mutually and are recognised as being interlinked. Affordable housing clearly has a fundamental role to play in contributing to the improved economic, environmental, social and physical health of communities [28,29]. While at the same time, a sustainable living environment has an essential role to play in contributing to the success of affordable housing [2,3]. It is important that such issues are tackled simultaneouslyand accordingly a broader range ofcriteria ought to be considered in the assessment of housing affordability [30]).Limitations in the assessment of affordability can be eliminated by the use of methods which are able to take into account a wider range of criteria than traditional methods do.

Methods such as cost benefit analysis (CBA)and hedonic modelling were considered for this purpose. CBA seeks to quantify the benefits and costs associated with a particular alternative. However, critics claim that CBA is of limited use in complex situations because all criteria must be measured in monetary terms [31]. A monetary value cannot be assigned to all factors related to housing affordability, such as social and environmental considerations, including individuals' welfare.Hedonic modelling is based on the fact that prices of goods in a market are affected by their characteristics and does not consider sustainability related features. This helps to estimate the value of a commodity based on people's willingness to pay for the commodity as and when its characteristics change. However, if consumers are unaware of the relationship between certain characteristics and the benefits they may have on them or their housing, then the value will not be reflected in the property price. Once more, this method focuses on obtaining economic values for characteristics and this may be difficult to ascertain for some environmental and social factors. Moreover, the amount of data that needs to be collected for hedonic modelling is extremely large.Given the presence of numerous conflicting factors, multiple criteria decision making (MCDM) methods were deemed particularly suitable for this issue and are utilised as the basis of the sustainable housing affordability assessment.

3. Overview of multiple criteria decision making methods

MCDM is a set of methods which deal with the evaluation of a set of alternatives in terms of numerous, often conflicting, decision criteria [32,33]. Thus, given a set of alternatives (options) and a number of decision criteria, the goal of MCDM is to provide a choice, ranking, description, classification, sorting and in a majority of cases an order of alternatives, from the most preferred to the least preferred option [34-36]. There are three stages that all MCDM techniques follow [32]:

1. Determine relevant criteria and alternatives;

2. Attach numerical measures to the relative importance of the criteria and to the impacts of the alternative on these criteria;

3. Process the numerical values to determine a ranking of each alternative.

MCDM can consider qualitative and quantitative criteria. While criteria based on quantitative variables are expert independent, qualitative criteria (variables) are expert dependent and may be subjective, since different approaches such as ranking, point or other systems can be used to transform qualitative variable into quantitative units compatible with MCDM methodology. Thus, in decision making, qualitative variables (criteria) are transformed into quantitative variables using expert-designed indicators and units.

This paper is concerned with the processing of the numerical values in the final decision matrix and the determination of the ranking of the alternatives; i.e. the weights of the decision criteria and the performance of the alternatives in terms of each criterion are predetermined by the expert method.

The literature presents an array of MCDM methodologies, each with their own characteristics, varying levels of sophistication and diverse scope of application [37-44]. There are different classifications of MCDM problems and methods. MCDM problems are frequently categorised according to the nature of the alternatives; either discrete or continuous [33,45-47]. A discrete problem can be described as a multi attribute discrete option, which often consists of a modest collection of alternatives (Multi Attribute Decision Making (MADM)), whereas a continuous problem usually consists of a vast or infinite amount of decision alternatives (Multi Objective Decision Making (MODM))[33,45]. MCDM methods may also be classified depending on their compensatory or non-compensatory nature. Compensatory methods allow explicit tradeoffs among criteria, whereas non-compensatory methods are principally based on the comparison of alternatives with respect to individual criteria.The objective of this study is to assess different housing locations based on an established set of sustainable housing affordability assessment criteria. The decision making situation is thus a ranking problem where alternatives need to be ranked from best to worst.The problem has a discrete nature, that is to say the alternatives (housing locations) will be pre-specified, and therefore a MADM method will be suitable in this instance.Consequently, the paper focuses on MADM methods. For MADM

problems there are generally two Schools of thought; those based on multi-attribute value functions and multi-attribute utility theory (MAUT) (the American School) [48] and those based on outranking methods (the French School) [49]. The methods based on MAUT (e.g. WSM, WPM, AHP, TOPSIS, COPRAS)commonly have a compensatory nature and mainly consist of aggregating the criteria into a function which has to be maximised [36] In contrast, the outranking methods allow for incomparability between alternativesand hence have a non-compensatory nature.ELECTRE [49] and PROMETHEE [50] are the most widely used outranking methods. However, it has been suggested that ELECTRE and PROMETHEE are not always able to give a complete ranking of the alternatives [32,50,51].Accordingly, such methods may be unsuitable for the type of decision problem in hand, which requires a complete ordering of alternatives,and have thereforenot been considered in this study.

4. Multiple criteria assessment of sustainable housing affordability

Numerous MCDM methods have been applied in housing and sustainability studies. For example, AHP has been used to aid house selection for buyers [52], to analyse the environmental preferences of homeowners [53], to examine housing location attributes [54] and in the assessment of urban quality of life in Iran [55]. Johnson has applied a number of MCDM methods, including AHP [56] and PROMETHEE [57], to housing choice problems.COPRAS has been utilisedto determine the most rational housing investment instruments and lenders in Lithuania [58], to evaluate the sustainability of residential areas [59] and to define the utility and market value of real estate [60].PROMETHEE has been used to assess land-use suitability for residential housing construction [61]. The WSM, WPM, AHP, revised AHP, TOPSIS and COPRAS have aided in the process of building maintenance [62]. SAW, TOPSIS and ELECTRE were utilised to assist stakeholders in making better decisions on housing evaluation [63]. Furthermore, COPRAS, SAW and multiplicative exponential weighting (MEW) were applied for the purpose of selecting an appropriate dwelling, taking into account the environmental impact of its construction, financial and qualitative criteria [64].

MCDM methods have become increasingly popular in decision-making for sustainability given the multi-dimensionality of the concept [51].MCDM methods are suitable for the evaluation presented in this paper since affordability and sustainability issues aremulti-dimensional and involve multiple conflicting criteria. MCDM methods can incorporate these various aspects into one evaluation process; MCDM is capable of considering criteria of incommensurable units of measure (e.g. ratios, points, percentages) and those of both benefit(positive) and cost (negative)influence.

The initial data collection process (in this case) for the basis of the MCDM methods includes the following stages:

- determine criteria for the comprehensive assessment of sustainable housing affordability (achieved via literature review and interviews with professionals);

- determine criteria weights to reflect their importance (achieved via questionnaires surveys conducted with professionals);

- select decision alternatives for comparison;

- calculate criteria values for each alternative (see measurement examples in Mulliner and Maliene [65]).

A total of 20 decision criteria were identified for the basis of the sustainable housing affordability assessment and weights were introduced in order to express the relative importance of the criteria (Table 1). The criteria were identified via interviews with housing and planning professionals in the UK and a supplementary extensive literature review[65]. A questionnaire survey was distributed to housing and planning professionals across all regions of the UK in order to further verify the criteria and elicit data on the importance of the criteria.Over 300 experts from different regions of the UK ranked the criteria on a scale of importance ranging from 1 to 10, where a ranking of 1 meant "not important" and a ranking of 10 meant "most important". In order to calculate criteria weights, the mean ranking of importance obtained for each criterion was divided by the sum of the mean scores, as such it ensures the total of all weights is equal to one.

Liverpool, UK, was chosen as the location for the empirical case study. Although it has experienced relatively fast economic grow in recent decade, this city still contains some of the most deprived areas (housing wards) in the UK and thus is an excellent example for this type of study. However,the MCDM methodology couldbe applied to any city or region within the UK or potentially worldwide.

Tenhousing wards in Liverpool wererandomly selected for comparison purposes. The alternatives were: A_1 (Everton), A_2 (Childwall), A_3 (West Derby), A_4 (Cressington), A_5 (Allerton and Hunts Cross),A_6 (Yew Tree), A_7 (Belle Vale), A_8 (Princes Park), A_9 (Fazakerley) andA_{10} (St Michaels) (Figure 1). The alternative areas were each measured against the 20 decision criteria and the values obtained are shown in table 1. Succeeding the initial data collection, a varietyof MCDM methodscan be applied to the data in order to process the values and prioritise the alternative areas.

<Figure 1 here>

5. Comparative analysis of MCDM methods

Despite the large quantity of MCDM methods available, no single method is considered the most suitable for all types of decision-making situation[66,67].This generates the paradox that the selection of an appropriate method for a given problem leads to an MCDM problem itself [32]. A major criticism of MCDM is the reality that different methods can yield different results when applied to the same problem [36]. The identification and selection of an appropriate MCDM method is thus not a simple task and considerable consideration must be given to the choice of method. The literature presents a number of practical applicationscomparative analyses of different MCDM methods [47,68-70].Furthermore, a number of authors have developed guidelines facilitating the choice of an appropriateMCDM method [66,67].However, it has also been acknowledged that several methods can be potentially valid for a particular decision

making situation; there is not always an overwhelming reason to adopt one technique over another [71].It seems that one of the most important criteria in selecting a MCDM method is its compatibility with the problem's objective [49].

The problem proposed in this study is to assess the sustainable housing affordability of a number of alternative areas. To achieve this, a ranking of alternatives needs to be identified. Therefore, the objective of this problem is to rank alternatives. Consequently, a MCDM method that has the ability to provide a complete ranking of alternatives (indicating the position of each alternative) is required. Additionally, the method must have the ability to handle both benefit and costcriteria and those of a quantitative and qualitative nature. Furthermore, ease of use and understanding of the MCDM technique is important so that any interested parties can easily adopt the proposed method.

The comparative performance of severalappropriate MCDM methods - the WSM, WPM, the revised AHP, TOPSIS and COPRAS - is investigated in this paper. These techniques are applied to the practical case study data contained in the initial decision making matrix (Table 1). Using each method, the aim is to determine the relative significance of each alternative under assessment, as well as establishing the priority order of the alternatives in respect of one another. The selected methods for the comparative analysis differ in their basic principles, the type of data normalization process and the way they combine the criteria values and the criteria weights into the evaluation procedure.Since criteria generally have different units of measurement,MCDM methods use a form of normalization to eliminate the units of criterion values (e.g. ratio, points, percentage, price)so that all the criteria are non-dimensional[36].There are different techniques of normalization but in many cases this stage is essential to the consistent and correct application of the method.The WSM, WPM, revised AHP and COPRAS methods are fairly similar in their normalisation procedure, although TOPSIS is somewhat different.

<Table 1 here>

5.1.Weighted Sum Model (WSM)

The WSM (also known as simple additive weighting (SAW) method)[72] is one of the simplest and most commonly used MCDM methods. The method involves adding together criteria values for each alternative and applying the individual criteria weights. Generally, the WSM only deals with benefit criteria. Accordingly, it was necessary for cost (minimizing) criteria to be transformed into benefit (maximizing) ones prior to normalization.The transformation of cost criteria into benefit ones can be achieved by a simple process: for each cost criterion, add the maximum criterion value to the minimum criterion value and then subtract the criterion value under consideration.Succeeding such a transformation, the lowest-cost criterion value becomes the largest and the largest-cost value becomes the lowest. Following this transformation on cost criteria, a new initial matrix was created using only benefit values (Table 2). The normalized matrix can then be created by dividing each criterion value by the sum of its row. Then each criterion value is multiplied by its corresponding weight. Once values for all alternatives have been aggregated, the alternative with the highest value is selected as the best solution [72]:

$$A^*_{WSM} = max_j \sum_{i=1}^{M} w_i \ a_{ij}$$

(2)

Here the $M{\times}N$ matrix A has data entries a_{ij} corresponding to the value of the jth (of N) alternatives in terms of the ith (of M) decision criterion. A^* is the WSM score of the optimal alternative and w_i is the weight (importance) of the ith criterion.

<Table2 here>

5.2.Weighted Product Model (WPM)

The WPM[73,74]is akin to the simple WSM method. The principal difference is that in the main mathematical process there is multiplication instead of addition, where each alternative is compared with the others by multiplying a number of ratios, one for each criterion and each ratio is raised to the power equivalent of the relative weight of the corresponding criterion[75]. This eliminates any units of measure and thus allows for dimensionless analysis so that the method can be used in single- and multi-dimensional decision problems. Like for use of the WSM, the WPM also requires cost criteria to be transformed into benefit ones prior to normalization. From the normalised matrix, in order to compare the alternativesA_K and $A_L$we calculate [73,74]:

$$R(A_K/A_L) = \prod_{j=1}^{N}(a_{Kj}/a_{Lj})^{w_j,} \qquad (3)$$

Here *N* is the number of criteria, $a_{ij}$is the actual value of the *i-th*alternative in terms of the *j-th*criterion, and $w_j$is the weight of importance of the *j-th*criterion. If the ratio $R(A_K/A_L)$ is ≥ 1, then alternative $A_K$is more desirable than alternative A_L(in the maximization case). The best alternative is the one that is better than or equal to all the other alternatives.

5.3.The revised Analytic Hierarchy Process (revised AHP)

The AHP is based on the use of pair-wise comparisons, both to estimate criteria weights and to compare the alternatives with regard to the decision criteria [76]. If criteria values and weights cannot be obtained directly then a method based on the pair-wise comparisons must be employed.In this instance, criteria weights were pre-determined by the expert method and not using AHP. Only the final stages of the AHP, i.e. the processing of the numerical values, were required in this study. The final step in the AHP deals with the construction of an *M* × *N* matrix (where *M* is the number of alternatives and *N* is the number of criteria) that is made using the relative importance of the alternatives in terms of each criterion [32]. Although this final stage of AHP is similar to WSM, a central difference with the AHP method is that the values of the decision matrix are normalized to sum to 1. This allows values with units of measurement to be

transformed into dimensionless ones. The best alternative (when all the criteria are maximizing) is indicated by the following additive formula:

$$A^*_{AHP} = \max_{i} \sum_{j=1}^{N} a_{ij} w_{j}, \; for \; i = 1, \; 2, \; 3,..., \; M.$$

(4)

A^*_{AHP} is the score of the optimal alternative. The entry a_{ij}, in the $M \times N$ matrix, represents the actual value of the i-th alternative in terms of the j-th criterion, w_j is the weight of importance of the j-th criterion and N is the number of decision criteria.

Belton and Gear [77] observed a problem with the original AHP method; they noted that AHP can reverse the ranking of the alternatives when an alternative identical to one already existing is introduced. Accordingly, they proposed a revised version where, instead of having the relative values of the alternatives sum up to one, each relative value is divided by the maximum value of the relative values [32,77]. This revision was subsequently accepted as a variation of the original AHP and is also referred to as 'ideal mode AHP' [78]. Triantaphyllou and Mann [75] advocate that the revised version appears to be more powerful than the original AHP approach. The revised AHP method was tested in two different ways:

1. RevisedAHP 1 – The first approach uses only benefit criteria values within the assessment. Thus, as with the WSM and WPM, cost criteria were transformed into benefit ones prior to normalization of the matrix (Table 2). This is the standard way of handling cost criteria with the AHP methods [79].

2. RevisedAHP 2 – The second approach uses both benefit and cost criteria values. Cost criteria were kept within the analysis by incorporating them as negative weights within the initial matrix. In order to do so, weights for cost criteria were multiplied by -1.

The remaining stages of the revised AHP process were the same for both approaches. The normalisation procedure of the revised AHP involves dividing each relative criterion value in the decision matrix by the maximum value (largest entry) in each column. Subsequently, each normalised value is multiplied by its weight. Then, the sum of all the weighted normalised

criteria values for each alternative is computed to obtain a final score for the alternative.The best alternative (when all the criteria are maximizing) is indicated again by the additive formula(4), but now the normalization is different.

5.4.COPRAS (ComplexProportional Assessment)

COPRAS [80] acts in a similar way to the WSM. However, COPRAS allows for both benefit and cost criteria to be considered within the matrix and the data are normalized so that different measurement units can be used and compared.

The procedure of the COPRAS method is generally carried out in the following stages [56]. The first step is the normalisation of the decision-making matrix:

$$d_{ij} = \frac{q_i}{\sum_{j=1}^{n} x_{ij}} . x_{ij}$$

(1)

Where $x_{ij}$is the value of the i-th criterion of the j-th alternative, and $q_i$is the weight of the i-th criterion.

The second stage calculates the sums of weighted normalised criteria describing the j-th alternative. The alternatives are described by benefit (maximising) criteria S_{+j} and cost (minimising) criteria S_{-j}. Sums are calculated according to the formulae:

$$S_{+j} = \sum_{z_i=+} d_{ij}$$

$$S_{-j} = \sum_{z_i=-} d_{ij}$$

(6)

The significance of the comparative alternatives is determined in the third stage on the basis of describing benefit (+) and cost (-) qualities that characterise the alternatives. The relative significance Q_j of each alternative A_j is determined according to:

$$Q_j = S_{+j} + \frac{S_{-\min} \cdot \sum_{j=1}^{n} S_{-j}}{S_{-j} \cdot \sum_{j=1}^{n} \frac{S_{-\min}}{S_{-j}}}, j = \overline{1,n}.$$

(7)

The first term of Q_j increases for higher positive criteria S^+_j, whilst the second term of Q_j increases with lower negative criteria S^-_j. The fourth stage is the prioritisation Q_j of the alternatives. The greater the value Q_j, the higher the priority (significance) of the alternative. In this case, the significance $Q_{\max}$ of the most rational alternative will always be the highest. The method also estimates the utility degree of the alternatives, showing, as a percentage, the extent to which one alternative is better or worse than the others being compared [68].With the increase/decrease of the priority of the analysed alternative, its degree of utility also increases/decreases. The degree of utility is determined by comparing each analysed alternative with the most efficient one. The optimal alternative is expressed by the highest degree of utility N_j equalling 100%. All utility values related to the considered alternatives will range from 0% to 100%, between the worst and best alternative out of those under consideration. The degree of utility N_j of the alternative A_j is determined according to the following formula:

$$N_j = \frac{Q_j}{Q_{max}} .100\%$$

(8)

Where Q_j and Q_{max} are significances of the alternatives calculated at the previous stage.

5.5.TOPSIS

TOPSISis based on an aggregating function representingcloseness to reference points [45]. TOPSIS approaches a MCDM problem by considering that the optimal alternative should have

the shortestdistance from the ideal solution and the farthestdistance from the negative-ideal solution.TOPSIS can be applied both to maximizing (benefit) and minimizing (cost) criteria [80]. TOPSIS begins with the normalization of criteria values, using vector normalisation. The normalized value r_{ij} is calculated as[32]:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{M} x_{ij}^2}}$$

(9)

Where x_{ij} represents the value of j-attribute for i-alternative, r_{ij}represents the value of the new normalized decision-making matrix.

The next step is to calculate the weighted normalized decision matrix V.A set of weights $W = (w_1, w_2, \ldots, w_n)$with $\sum w_i = 1$is used in combination with the previous normalised decision matrix to determine the weighted normalized matrix V, defined as:

$$v_{ij} = w_{ij}r_{ij,}$$

(10)

The ideal/best (A*) solution and the negative-ideal/worst (A-) solution are then determined:

$$A^* = \{(\max_i v_{ij} \,|j \in J), (\min_i v_{ij} \,|j \in J')|i = 1,2,3,\ldots,M\} =$$
$$= \{v_{1^*}, v_{2^*}, \ldots, v_{N^*}\}.$$

(11)

$$A^- = \{(\min_i v_{ij} \,|j \in J), (\max_i v_{ij} \,|j \in J')|i = 1,2,3,\ldots,M\} =$$
$$= \{v_{1^-}, v_{2^-}, \ldots, v_{N^-}\}.$$

(12)

Where$J = \{ j = 1, 2, \ldots, N$ and j is associated with benefit criteria$\}$; and $J' = \{ j = 1, 2, \ldots, N$ and j is associated with cost/loss criteria$\}$.

The ideal solution represents a hypothetical option that consists of the most desirable level of each criterion across the options under consideration. Whereas the negative-ideal solution represents a hypothetical option that consists of the least desirable level of each criterion across the options under consideration. The separation measure (distance) of each alternative from the ideal-solution and negative-ideal solution using the n-dimensional Euclidean distance method is then calculated:

$$S_i^* = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^*)^2} \; , i = 1, \ldots, M.$$

(13)

Where S_{i*} is the separation (in the Euclidean sense) of each alternative from the ideal solution.

$$S_i^- = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^-)^2} \; , i = 1, \ldots, M.$$

(14)

Where S_{i-} is the separation (in the Euclidean sense) of each alternative from the negative-ideal solution.

The relative closeness of each alternative A_j to the ideal solution A^* can be calculated:

$$C_{i^*} = \frac{S_{i-}}{S_{i^*} + S_{i-}}, 0 \leq C_{i^*} \leq 1, i = 1,2,3, \ldots, M$$

(15)

If $C_i=1$ then $a_i = A^*$ (ideal solution) and if $C_i=0$, then $a_i = A-$ (anti-ideal solution). Therefore, the conclusion is that the alternative a_i is closer to A^* if C_i is closer to the value of 1.

Finally, the preference order is ranked according to C_i. The best alternative will be the one that is closest to the ideal solution and the maximum distance away from the anti-ideal solution [45,81].Thus, the optimal alternative should be the one that best maximises the beneficial criteria and minimises the unbeneficial criteria. However, while these two reference points (ideal and anti-ideal) are identified, TOPSIS does not consider the relative importance of

the distances from such points [41]. Differently from other methods that uses a linear normalisation, in TOPSISthe vector normalisation is applied to eliminate the units of criterion functions {41}.As a result,squared terms in the evaluation of criteria are used and this should be highlighted. The consequence of this is that very good and very bad data points (criteria values) can be exaggerated, having more of an impact on the final outcome, whereas average data points will not have as much of an impact (in comparison with methods that do not utilise squared terms). AccordinglyTOPSIS may have better distinguishing capability where criteria values for each alternative are otherwise similar.

6. Comparison of alternative rankingsusing different MCDM methods

The MCDM methods (WSM, WPM, revised AHP (approach 1 and 2), TOPSIS and COPRAS) were applied to the case study data. TOPSIS, COPRAS and Revised AHP 2 were applied to the initial decision making matrix in Table 1, while it was necessary forWSM, WPM and revised AHP 1to be applied to the initial matrix containingonly benefit values (Table 2).The obtained ranking results are presented in Table 3. The priority order of the alternatives is compared in Table 4; highlighting/shading has been used in order to easily demonstrate where different methods have acted in the same way with regard to the prioritisation of alternatives.All tested methods concluded that the optimal alternative was A_{10} (St Michaels). All methods rankedA_4 (Cressington)in 2nd position. Threeof the approaches, all except TOPSIS and WPM, concluded that A_7 (Belle Vale) was the worst performing alternative, followed by A_9 (Fazakerley),ranking 10th and 9th consecutively, whereas TOPSIS and WPM ranked A_7(Belle Vale) as9th priority.Revised AHP acted rather similarly to WSM, with both methods ranking six of the alternatives (60%) in identical positions.COPRASalso acted rather similarly to WSM, with both methods ranking five of the alternatives (50%) in identical positions. As an example, the ranking of housing wards using COPRAS method is highlighted with different colour circles, green (high), yellow (medium) and red (low) in Figure 1. TOPSIS acted most correspondingly to the revised AHP, with the two methods prioritising four of the alternatives (40%) in identical positions.

However, the two methods also produced some rather contrasting results, for example, in relation the prioritisation of A_2 (Childwall).In fact,A_2 produced rather unstable rankings by the different methods tested, along with A_1 and A_8.Although it is not usual to adopt the second approach within the revised AHP method, i.e. incorporating cost criteria as negative weights, the final priority order of the alternatives was actually equivalent using both approaches (Table 3). Accordingly, this approach could be a valid option for future studies that wish to incorporate cost criteria within AHP methods. The WPM was the most inconsistent with the other methods tested, in terms of the prioritisation of alternatives. It should be noted that the use of the WPM proved problematic owing to the '0' (zero) value assigned to C_{20}/A_5 within the initial matrix(Table 1)/ C_{20}/A_1 within the 'all benefit' criteria matrix (Table 2). This method does not seem to function well where criterion values of zero are used and this may have contributed to the dissimilar rankings achieved by the method.

<Table3 here>

<Table4 here>

The similarity in the rankings obtained by different methods can be further demonstrated by analysis of pairwise correlation. Pairwise correlationbetween the MCDM methods showed that five methods (COPRAS, TOPSIS, WSM, revised AHP 1 and 2) out of six methods perform very similarly (Pearson correlation coefficient of 0.831 to 0.995) with revised AHP1 and revised AHP2 methods delivering the same rankings of alternatives (Table 5). The overall similarity of one MCDM method to all other methods used in the analysis compared as follows (with average correlation coefficient shown in brackets): COPRAS (0.786) >TOPSIS (0.762) >WSM (0.745) >revised AHP1/2 (0.735) >WPM (0.266). COPRAS, WSM, revised AHP 1 and 2are highly correlated amongst themselves. Interestingly, TOPSIS, which differssignificantly from other MCDM methodson the basis that the optimal alternative should have the shortestdistance from the ideal solution and the farthestdistance from the negative-ideal

solution,showed very high similarity to COPRAS (Pearson correlation coefficient of 0.969) and was highly correlated with WSM, revised AHP 1 and 2 methods.These findings are fairly consistentwith a number of other studies comparing the results obtained by applying several MCDM methods. For example, Banaitiene*et al*. [68] found that SAW (also known as WSM), TOPSIS and COPRAS produced equal rankings of alternatives. Ginevicius and Podvezko[82] also used SAW, TOPSIS and COPRAS and found similarity, albeit not entirely equal, in the ranking of alternatives. Rao [83] found similarity in the rankings given by TOPSIS, COPRAS and AHP. Zanakis*et al*. [47]concluded that all version of AHP behave similarly to SAW, while they found that TOPSIS behaves closer to AHP.

<Table5 here>

7. Sensitivity analysis

Ranking results in the MCDM depends heavily on the nature of criteria that are used in the analysis and most notablyon a distribution of the weighting amongst criteria. Also, it has to be taken into consideration thatthe criteria weights are usually established on the basis of professional perception, which can be to some extent subjective and may varyaccordingly. Therefore, the effect of a possible deviation of the weight value should be evaluated.

The professional opinion-determined values of criteria weights and values of alternatives were combined in the mathematical models of MCDM methods described in subsections 5.1-5.5.A sensitivity analysis was performed to quantify the level of crosstalk between criteria and ranking, through revealing how the ranking of alternatives change due to variation of criteria weights. Results of the sensitivity analysis for each individual criterion werecompared in Figure 2. It should be noted that any change in onecriterion weight (increase or decrease) was reflected inremaining criteria weights by adjusting proportionally criteria weights and ensuring that the total criteria weight is equal to unity.

<Figure 2 here>

Figure 3 representssensitivity coefficients*SC**, which were calculated for each criterion usingall six MCDM methods. The specific value of the sensitivity coefficient SC_j^* for criterion C_j, indicates that a 5% or 50% increase or decrease of the criterion weight leads to a single, double or multiple changes in the ranking of alternatives. The distribution of sensitivity coefficients is summarised in Table 6. Results revealed that criteria C3, C8, C12, C13, C15 and C19 were robust for all six MCDM methods used in the analysis.

<Table 6 here>

<Figure 3 here>

The comparative analysis of the distribution of sensitivity coefficientsrevealed that thesimulated 5% change in the criterion weight (increase and decrease)did not have any influence on the ranking of alternatives using WPM and COPRAS methods and had some effect on theranking withother methods. The WPM method-based ranking was least affected by the 50% change in the criterion weight, while other methods tolerated such change within acceptable limits for a majority of criteria (Figure 2, Table 6).

Next, we investigated what was the most critical criterion in each MCDM model. The "most criticalcriterion" was defined as the criterion C_j for which the smallest relative change (in percentage), denoted as D_j, in its weight value W_j must occur to alter the existingranking of thealternatives. The sensitivity coefficient of criterion C_j, denoted as SC_j, can be used as a measure of sensitivity to the change of criterion weight and is given as follows:

$$SC_j = \frac{1}{D_j}, \quad j = \overline{1, n}. \tag{17}$$

As shown in the Figure 4, criteria C4 (WSM), C20 (WPM, TOPSIS and COPRAS) and C16 (Revised AHP 1 and 2) were identified as most critical criteria for any alternative, and C16 was the most critical criterion for best alternative in case of all MCDM methods.

<Figure 4 here>

8. Discussion

The comparative analysis of the MCDM methods - WSM, WPM, revised AHP(approach 1 and approach 2), TOPSISand COPRAS - highlighted that the WSM, revised AHP methods and COPRAS are relatively simple to use. The WPM also appeared straightforward, although it was problematic with the use of zero values within the analysis. However, a drawback of the WSM, WPM and revised AHP is that benefit and cost criteria should not generally be used at the same time within the analysis. Cost criteria ought to be transformed into benefit criteria prior to normalisation. However, Millet and Schoner[79] discussed this transformation in relation to the AHP methods and suggest that it can cause computational complexity and elicit inconsistent results. There is an option, mathematically, to incorporate cost criteria as negative weights within methods, as demonstrated within the comparative analysiswith the revised AHP (approach 2). However, such a way of dealing with cost criteria is not generally adopted in practice and thus the results may not always be acceptable. In contrast, the TOPSIS and COPRAS methods allow for both benefit and cost criteria to be incorporated with one analysis without difficulty or question. However, the TOPSIS method was more complex and time consuming to apply in comparison to COPRAS. Dyer *et al*. [84] warn that the complexity of many MCDM methods can prevent their application in practice. Moreover, the findings of several comparative studies actually suggest that simpler evaluation techniques are often superior [47,69,70].

All methods produced somewhat different ranking results. COPRAS, TOPSIS, WSM, revised AHP 1 and 2showed most consistency amongst themselves. Although none of these five methods outclassed others considerably, the correlationanalysis showed that COPRAS would be an optimal choice if one method to be used for alternative's ranking purpose.The sensitivity analysis also revealed that COPRAS (together with WPM) tolerated best the 5% change in criterion weight (increase and decrease), which did not have any influence on ranking alternatives using these two methods. COPRASalso has the ability to account for both benefit (maximizing) and cost (minimizing) evaluation criteria, which can be assessed separately within one evaluation process. Contrastingly, the WSMand revised AHP methodsrequire transformation of cost criteria into benefit ones. This makes the procedure more complicated and time consuming for potential users and can elicit inconsistent results. The COPRAS method is transparent, simple to use and has a low calculation time in comparison with other MCDM methods, such as the AHP and TOPSIS [85]. This was confirmed during the comparative analysis. Therefore, the COPRAS method can more easily be adopted by any interested parties in the future. An important feature that makes the COPRAS method superior to other available MCDM methods is that it estimates the utility degree of alternatives, showing, as a percentage, the extent to which one alternative is better or worse than other alternatives taken for comparison. Visually, this can further aid the decision maker and would be particularly useful for the presented sustainable housing affordability assessment method if results are utilised by, for example, policy makers and planners. Furthermore, recentresearch shows that decisions yielded by the COPRAS method are more efficient and less biased than those yielded by TOPSIS and SAW (also known as WSM) [86].

The sensitivity analysis showed that if criterion weights are subjected to a higher level of change (50% increase or decrease), other MCDM methods such as TOPSIS, WSM and WPM should be considered as their tolerance to criterion change in some instances can outperform the COPRAS method.In particular, WPM showed exceptional tolerance to the high level of uncertainty in criterion weight. This can be explained by the peculiarity of the mathematical

process of this method, involving multiplication instead of addition in the course of alternative comparison.

9. Conclusions

This paper has shown that MCDM is applicable for the assessment of sustainable housing affordability, in part due to the ability of such methods to deal with the multidimensionality of the issue and the numerous conflicting decision criteria present. However it was stressed that, frequently, different MCDM methods can yield different results when applied to the same decision problem. Accordingly, in this paper we examined the application of several MCDM methods for sustainable housing affordability assessment in a comparative study. 20 decision criteria, weighted by experts, were the basis of the sustainable housing affordability assessment and a case example of 10 alternative areas (housing wards) within Liverpool, England was used to illustrate the results. Five widely applied MCDM methods – WSM, WPM, revised AHP, TOPSIS and COPRAS - were used and evaluated in order to identify differences and similarities in the methods and the ranking results obtained, and also to consider their applicability in aiding the assessment of sustainable housing affordability.

In the presented decision problem the 'best' (and second best) alternative obtained by all examined methods was equal but the overall ranking of all alternatives varied between methods. We found that some of the methods were in strong agreement with high correlations found amongst rankings, but certain differences also arose. The comparative analysis demonstrates that none of the MCDM methods are considered to be 'perfect'. We suggest that ideally, and where possible, more than one method should be applied to the same problem in order to provide a more comprehensive decision basis. However, when this is not possible we recommend the use of the COPRAS method since it exhibited the highest potential in sustainable housing affordability decision analysis. Nonetheless, considering the results of the sensitivity analysis, in cases of higher levels of uncertainty with regard to criteria importance (weighting),

TOPSIS, WSM and WPM can also be considered owing to their better tolerance to a higher level of change in criterion weight.

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Figure legends

Figure 1. Liverpool housing wards used for comparison purpose in this study. Alternative numbers are provided in brackets. Different colour circles highlight ranking output computed using the COPRAS method: green (high), yellow (medium) and red (low).

Figure 2. Criteria sensitivity to the change. Chart represents sensitivity analysis results demonstrating how the change of criterion weight affects the ranking of alternatives. Dark green rectangles indicate tolerable change of criteria weight (as shown in the top panel), to which the alternative ranking is not sensitive, while light green rectangles represent the range that contributes to the single change of alternatives. In principle, the length of the horizontal bar indicates the criteria sensitivity to the change, where the shorter the barimplies the higher level of sensitivity. Abbreviations of criteria are shown on the left side of panel. Results for six MSDM methods in each criterion panel are displayed in the following order: WSM (top), WPM, revised AHP 1, revised AHP 2, TOPSIS and COPRAS (bottom).

Figure 3. Diagram of sensitivity coefficients *SC**s for each criterion. Multiple bars for each criterion show sensitivity coefficients calculated for all six MCDM methods allowing for -5%, -50%, +5%, and +50% changes of the criterion weight.

Figure 4. Most critical criteria for any and best alternatives. Bar chart compares sensitivity coefficients *SC*s of most critical criteria established using different MCDM models.