

# CHASSIS DYNAMOMETER TORQUE CONTROL SYSTEM DESIGN BY DIRECT INVERSE COMPENSATION

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## Abstract:

This paper presents a methodology for the design of a robust torque control system for a transient 1.2m (48in) dia, 120 kW, DC Chassis Dynamometer. The method includes system identification of the nonlinear dynamometer torque supply system, linearisation by direct inverse compensation, and linear identification of both the compensated and uncompensated plants. A combined feedforward-feedback control structure is proposed and robust feedback controllers are designed using a fixed-order parameter space method.

Keywords: Chassis Dynamometer, Direct Inverse Control, Feedback, Feedforward, Identification, Parameter Space, Multiplicative Uncertainty, Nonlinear, Road-Load Simulation.

## 1. INTRODUCTION

This paper presents a methodology for the design of a combined feedforward-feedback torque controller (Figure 1) to be implemented in a chassis dynamometer road-load simulation system. The method proposed makes use of direct inverse control similar to that presented in (Petridis and Shenton, 2002). The introduction of a direct nonlinear inverse compensator provides the combined advantage of linearising the systems' nonlinear behaviour, and providing a unity path. The technique is evaluated by application to the 1.2m dia, 120 kW, DC chassis dynamometer system of the Powertrain Control Group, University of Liverpool.

The nonlinear compensated system is identified to generate a number of LTI models, gathered under different operating conditions. For comparison here a set of LTI models are also identified for the system without the compensator in place. For each set of models, circular uncertainty templates

are defined over the important range of frequencies, to model both system nonlinearity and intrinsic uncertainty. The multiplicative uncertainty in both compensated and uncompensated plants can thus be obtained and used to evaluate the robustness of the compensator and to determine the frequency domain controller specifications.

The identified LTI models are used for the purpose of robust feedback controller design. Solutions are presented using a parameter space design method (Besson and Shenton, 1997) which derives a low order controller element for compact implementation. The performance of the controllers is assessed through simulation.

## 2. NONLINEAR INVERSE COMPENSATOR

In the proposed scheme the nonlinear compensator is an inverse system model which is identified using inverted input-output data, that is, data with the input and output causality switched. The

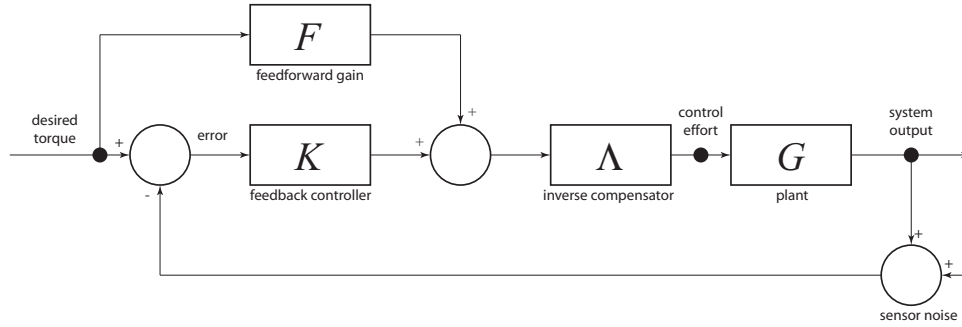


Fig. 1. The proposed closed-loop system with feedforward action

system input is varied through its full range of  $\pm 10$  volts while the dynamometer torque response is recorded.

In the initial implementation of the Liverpool dynamometer, tested here, it was found that a non-linear non-dynamic gain element provided results as good as a dynamic compensator in this case since the performance of an identified dynamic inverse model was compromised by the oscillatory nature of the load-cell signal.

The system is accordingly modelled as a polynomial which is fitted using a least squares algorithm and for which an appropriate model was found to be 7th order. The resulting function is shown in Figure 2.

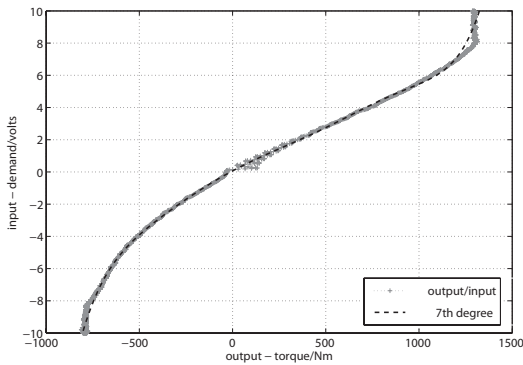


Fig. 2. Polynomial fit to inverse input/output data

### 3. LINEAR SYSTEM IDENTIFICATION

A set of linear models are identified for the compensated plant  $G_c$  and, for comparison purposes, for the uncompensated plant  $G_u$ , with each set of models represented as a collection of frequency response models about a centered nominal model. The purpose of this comparative process is to determine any beneficial effect of compensating the plant in reducing model multiplicative uncertainty.

#### 3.1 Uncompensated System

Initially, the uncompensated chassis dynamometer system is identified as a black box model.

A random-walk excitation signal is applied at the system input, and the unfiltered system torque (Nm) response is recorded, as shown in Figure 3. Both input and output signals are logged at an interval of 5ms. System identification is carried out using multiple sets of input-output data, collected concurrently. For each set of data, the system is identified as an ARX model using the Matlab System Identification Toolbox (Ljung, 2005).

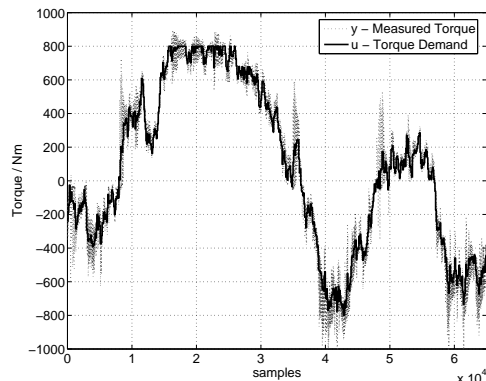


Fig. 3. An example of input/output data collected from the chassis dynamometer.

In order to identify a set of models which adequately represents the range of dynamics seen in the system, the maximum amplitude of the input signal is varied for each set of identification data collected.

In application to the test dynamometer the random-walk input signal was varied at a rate of 10Hz. The DC drive was operated in a basic torque control mode, requiring an analogue control input of  $\pm 10$  volts full-range. The dynamometer torque output was measured through the reaction force measured by a load-cell mounted between the dynamometer base and a calibrated torque arm. The load-cell output signal was sampled every 5ms.

It was noted that in addition to sensor noise, some structural dynamics were also detected. It has been shown (Suzuki and et al, 1994) that for the standard torque measurement arrangement as used here, the structural dynamics of the load measurement arrangement including the torque arm itself, can be superimposed on the measured system response. This gives the torque measurement an oscillatory nature which is difficult to filter without introducing unacceptable lag to the system. The removal of these unwanted and uncontrollable measurement disturbances would provide significant improvement to the fidelity of torque measurement.

A parametric ARX structure was selected for the models with a discrete transfer function of the form:

$$\frac{y}{u} = G(z) = \frac{\theta_1 z^4}{z^6 + \theta_2 z^5 + \theta_3 z^4 + \theta_4 z^3 \dots + \theta_7}$$

The discrete system models were converted to continuous models using a bilinear Tustin approximation.

The nominal model  $G_o$  was fitted through the centre point of the uncertainty circles. The continuous nominal model and its parameters are given in Eqn.1 and Table 1.

$$\frac{y}{u} = G_o(s) = \frac{\phi_1 s^8 + \phi_2 s^7 + \phi_3 s^6 \dots + \phi_8 s + \phi_9}{s^8 + \phi_{10} s^7 + \phi_{11} s^6 \dots + \phi_{16} s + \phi_{17}} \quad (1)$$

Table 1. Uncompensated System Model Parameters

Parameter	Value	Parameter	Value
$\phi_1$	0.1538	$\phi_{10}$	92.73
$\phi_2$	123.4	$\phi_{11}$	$1.602 \times 10^6$
$\phi_3$	$-2.341 \times 10^4$	$\phi_{12}$	$4.093 \times 10^7$
$\phi_4$	$-3.929 \times 10^7$	$\phi_{13}$	$1.286 \times 10^{11}$
$\phi_5$	$-4.231 \times 10^9$	$\phi_{14}$	$1.561 \times 10^{12}$
$\phi_6$	$-3.092 \times 10^{12}$	$\phi_{15}$	$1.774 \times 10^{13}$
$\phi_7$	$6.511 \times 10^{14}$	$\phi_{16}$	$7.456 \times 10^{13}$
$\phi_8$	$5.366 \times 10^{15}$	$\phi_{17}$	$3.049 \times 10^{14}$
$\phi_9$	$4.571 \times 10^{16}$		

The frequency response for the set of continuous identified models, including complex uncertainty templates, is shown in the Nyquist plot of Figure 4.

### 3.2 Compensated System

The nonlinear compensated system is identified using the same order ARX models as for the uncompensated system. A range of system models are identified using 5 sets of data, each with a maximum input amplitude varying from 1000Nm to 600Nm in increments of 100Nm. The model parameters are given in Table.2.

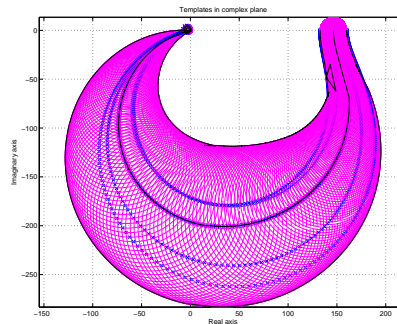


Fig. 4. Uncompensated System Frequency Response

The system input is Torque demand (in Nm) and the system output is measured Torque response (Nm).

The continuous nominal plant model  $G_o$  for the compensated system is again described by the locus which passes through the centre of all uncertainty circles. The continuous nominal system is taken to be the same structure as that of the uncompensated plant (Eqn.1), and its model parameters are given in Table 2.

Table 2. Compensated System Model Parameters

Parameter	Value	Parameter	Value
$\phi_1$	0.01442	$\phi_{10}$	389.6
$\phi_2$	10.8	$\phi_{11}$	$1.644 \times 10^6$
$\phi_3$	-1448	$\phi_{12}$	$1.587 \times 10^8$
$\phi_4$	$-3.475 \times 10^6$	$\phi_{13}$	$1.574 \times 10^{11}$
$\phi_5$	$-4.168 \times 10^8$	$\phi_{14}$	$7.085 \times 10^{12}$
$\phi_6$	$2.074 \times 10^{11}$	$\phi_{15}$	$1.918 \times 10^{15}$
$\phi_7$	$6.075 \times 10^{13}$	$\phi_{16}$	$2.549 \times 10^{16}$
$\phi_8$	$5.235 \times 10^{15}$	$\phi_{17}$	$7.667 \times 10^{17}$
$\phi_9$	$7.701 \times 10^{17}$		

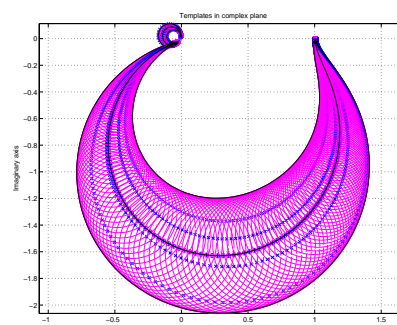


Fig. 5. Compensated System Frequency Response

Figure 5 shows the frequency response for the set of compensated plant models, with complex uncertainty circles plotted over a range of frequencies.

### 3.3 Multiplicative Uncertainty

Directly comparing Figure 4 and Figure 5, where individual frequencies are not indicated, yields lit-

the objective information about the system uncertainty since the plot axes are necessarily different, and comparison is only useful if it can be made on a frequency, by frequency basis. Indeed, for the inverse compensator to be effective, the uncertainty need only be reduced around the crossover frequency.

Multiplicative uncertainty  $|\Delta(s)|$  can be defined at each frequency at which the plant model is defined by the form

$$G(s) = G_o(s) + \Delta(s)G_o(s)$$

Then  $|\Delta(s)|$  gives a measure of how much the system deviates from its nominal behavior  $G_o(s)$ .

Figure 6 shows the multiplicative uncertainty for both the compensated, and uncompensated plants over a range of frequencies. It can be seen that the compensated plant has reduced multiplicative uncertainty over all frequencies up to the Nyquist frequency. This reduction in system uncertainty provides a strong justification for the addition of an inverse compensator since the controller performance may be improved whilst maintaining robust stability margins. The multiplicative uncertainty is next used to shape the Complimentary Sensitivity weighting function for robust controller design.

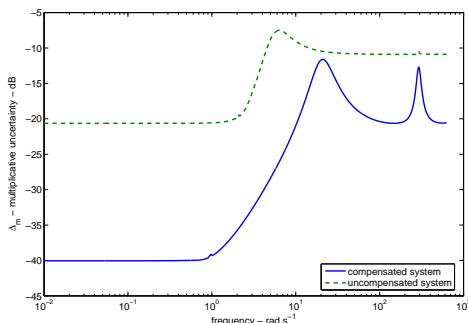


Fig. 6. Multiplicative Uncertainty

#### 4. CONTROLLER STRUCTURE

The control structure chosen is the combined feedforward-feedback arrangement of Figure 1 with a direct inverse compensator in place.

The feedback controller element implemented in the combined system does not need to be as aggressive as in a pure feedback system. For the same stability margins, the combination of feedforward and feedback allows a faster system response to be achieved than with feedback alone.

#### 5. FEEDFORWARD GAIN SELECTION

The feedforward gain ( $F$ ) determines the percentage of the control demand which is fed forward

to the inverse compensator. For the compensated system, which has a unity path, the feedforward gain ( $F_c$ ) determines how much of the desired torque demand is fed forward directly. This value will normally be close to unity if fast response is to be achieved, but can be modified to tune the overshoot and response time. In the case of the uncompensated plant, the feedforward gain ( $F_u$ ) is a tuned fraction of the inverse system gain.

#### 6. FEEDBACK CONTROLLER DESIGN

In a chassis dynamometer control system, the torque controller is required to provide fast tracking of a transient torque demand signal which is generated by the road-load algorithm. For satisfactory inertia simulation, fast response with limited overshoot and rapid settling are important. Controller performance must be achieved in the face of system uncertainty, due to nonlinearities and variations in system behavior due to environmental effects, as well as sensor noise and unwanted sensor dynamics. In the light of these factors, significant stability margins must be obtained.

In this study feedback controllers are designed for both the compensated and uncompensated system, using the nominal plant models which were found through system identification.

The proposed design method uses mixed sensitivity functions in order to provide the required levels of both nominal performance and robust stability.

##### 6.1 Weighting Functions

The transient response performance of the system is primarily determined by the feedforward aspect of the control system. The primary sensitivity function is shaped to ensure the system tracks despite any errors remaining from this feedforward action or any disturbances.

The primary sensitivity transfer function

$$S(s) = \frac{1}{1+G(s)K(s)}$$

is accordingly shaped by a weighting  $W_S$

$$\|W_S(s)S(s)\|_\infty < 1, \quad \forall \omega \in [0; +\infty)$$

The weighting function is chosen to obtain the required integral action in the low frequencies for tracking and to obtain roll off at high frequencies for adequate noise attenuation. An appropriate selection of the primary sensitivity weightings was thus:

$$W_S = \frac{0.6s + 0.9}{s}$$

for the compensated plant and

$$W_S = \frac{0.3s + 3}{s}$$

for the uncompensated plant.

The complementary sensitivity function is also used to obtain an additional level of robustness to plant uncertainty. The complementary sensitivity transfer function

$$T(s) = \frac{G(s)K(s)}{1 + G(s)K(s)}$$

is accordingly shaped by a weighting function  $W_T$  such that

$$\|W_T(s)T(s)\|_\infty < 1, \quad \forall \omega \in [0; +\infty)$$

General guidelines (Skogestad and Postlethwaite, 1996) are used together with the multiplicative uncertainty identified to obtain appropriate complementary sensitivity weighting functions for the compensated and uncompensated systems. For the test dynamometer these were chosen respectively as:

$$W_T = \frac{0.5s + 1}{1}$$

for the compensated plant and

$$W_T = \frac{0.5s + 1}{10}$$

for the uncompensated plant.

## 6.2 Parameter Space Design

The feedback controllers for both nonlinear compensated and uncompensated systems is designed with the fixed structure:

$$K(s) = \frac{b_2s^2 + b_1s + b_0}{a_2s^2 + a_1s + a_0}$$

The feedback controller is designed using the nominal plant models and weighting functions  $W_S$  and  $W_T$  selected in each case. The design method adopted is detailed in (Besson and Shenton, 1997) and has the advantages of being an interactive method which allows the time response of the controlled system to be tuned, while simultaneously meeting robust stability margins. Tuning the controller in this manner has the advantage that the important effect of the feedforward in the overall control system can be taken into account directly. It is desirable in this situation, where model uncertainty is great, to use as little feedback control action as is possible while still meeting the time response specifications.

The following controllers were respectively selected for the compensated and uncompensated systems:

$$K_c(s) = \frac{0.0014s^2 - 0.0135s + 0.9983}{0.07s^2 + s}$$

$$K_u(s) = \frac{0.0009s^2 + 0.0048s + 0.0267}{0.07s^2 + s}$$

The time response performance of the closed loop systems was checked through simulation. Feedforward gains ( $F_c$  and  $F_u$ ) were tuned to either reduce overshoot or to reduce response time depending upon the basic performance of the feedback controller and initial feedforward gain selected ( $F_c = 0.99$  and  $F_u = 0.0050$ ). Figure 7 shows the simulated closed-loop system response for both systems. It can be seen that the compensated system has a significantly faster response time (82.5ms), and settling time (565ms) than the uncompensated system (214ms and 1.15s respectively), while the compensated system displays almost the same overshoot (37 percent compared with 33 percent) during initial response.

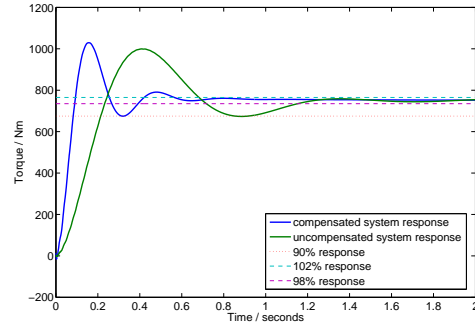


Fig. 7. Controlled system response

In order to conservatively establish system robustness of the controller on the nonlinear plant, robustness to possible rapid switching between the component LTI models can be established by a critical disk (Petridis and Shenton, 2002) analysis. This provides an additional level of conservative robustness over that required for a purely linear uncertain system by representing a region around the -1 point in the Nyquist plot of the loop functions  $L_c = K_c \Lambda G_c$  and  $L_u = K_u G_u$ . The circular disks represent a conservative region outside which system stability can be guaranteed. The disks are each defined by two points,  $\alpha = \frac{|G|_{min}}{|G|_{nom}}$  and  $\beta = \frac{|G|_{max}}{|G|_{nom}}$  which are centred on the real axis. Figures 8 and 9 show the Nyquist plot (including uncertainty disks), with a critical disk plotted around the -1 point.

Comparison of Figure 8 and Figure 9 shows that the compensated plant, with its superior time response performance, also guarantees a superior stability margin.

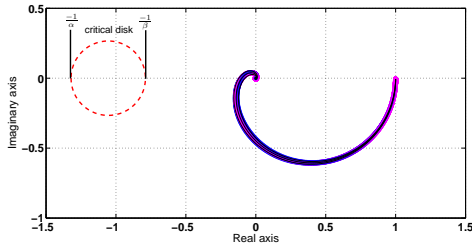


Fig. 8. Nyquist plot of the loop function for the compensated system

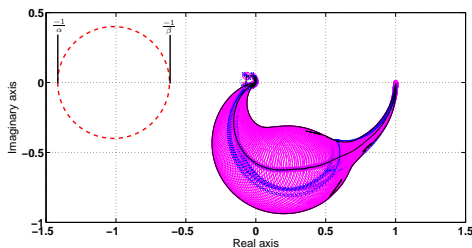


Fig. 9. Nyquist plot of the loop function for the un-compensated system.

## 7. CONCLUSIONS

A robust nonlinear direct inverse compensation design technique is proposed for a chassis dynamometer torque control system.

Multiplicative uncertainty in both compensated and uncompensated models was evaluated, and it was shown that the compensated system displays reduced uncertainty over all frequencies.

Robust feedback controllers were designed with a fixed order, fixed structure parameter space method, using mixed sensitivity weighting function specifications.

Additional conservative robustness to account for application of the linear design on the original nonlinear dynamometer plant was established by evaluating the stability margins using the boundary of a critical disk centered around the  $-1$  point in the Nyquist plots.

The system time response performance was assessed through simulation. Performance was found to be significantly better for the compensated system than for the uncompensated system.

Future work should implement the control system designed here in the control hardware and software of the chassis dynamometer, and should verify its performance through further experimental testing.

Improvements in the quality of the measured torque response, including noise reduction and filtering of the sensor dynamics should be attempted to improve both the quality of the identified system models and the controller performance.

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