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Dynamic Decision Making under Ambiguity: A Portfolio Choice Experiment

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Abstract

Neoclassical economic theory assumes that when agents tackle dynamic decisions under ambiguity, preferences are represented by Expected Utility and prior beliefs are updated according to Bayes rule, upon the arrival of partial information. Nevertheless, when one considers non-neutral ambiguity attitudes, either the axiom of *dynamic consistency* or of *consequentialism* should be relaxed. We report the results of a new experiment, designed to investigate how people behave in a dynamic choice problem under ambiguity, where decisions are made both before and after the resolution of some uncertainty. We study which of the two rationality axioms people violate, along with the question of whether this violation is part of a conscious planning strategy or not. The combination of the two, allows us to classify subjects to three behavioural types: resolute, naïve and sophisticated. Using data from a portfolio choice experiment where ambiguity is represented in a transparent and non-manipulable way, we cannot reject the hypothesis of Bayesian updating for half of our experimental population. For ambiguity non-neutral subjects, we find that the majority are sophisticated, a few are naïve and few are resolute.

JEL classification: C91, D81, D83, D90

Keywords: Ambiguity, Subjective Beliefs, Dynamic Consistency, Consequentialism, Portfolio Choice, Experiment

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1 Introduction

Underlying much of economic theory are three key assumptions. These are that economic agents: (1) use probabilities to describe risky and ambiguous situations; (2) behave in a dynamically consistent way; and (3) update probabilities according to Bayes rule, upon the arrival of partial information. Subjective Expected Utility theory (SEU, Savage (1954)) binds these three assumptions together in a logically and intellectually satisfying manner. Nevertheless, since the seminal thought experiments proposed by Ellsberg (1961), challenging the first assumption, a vast literature of theoretical models emerged, aiming to accommodate Ellsberg-type preferences[1]. The direct consequence of this, was the rapid development of a large body of experimental work, that either tests the attitudes towards ambiguity or performs *horse-race* comparisons to identify the model that best describes data[2].

However, as it is highlighted in Gilboa and Schmeidler (1993), if one wants to confirm the theoretical validity of any model of decision making under uncertainty, this model should be able to successfully cope with the dynamic aspect of the choices. When SEU is extended to its dynamic dimension, the independence axiom (often called the "sure thing principle") is equivalent to two rationality axioms, namely *dynamic consistency* (DC) and *consequentialism* (C), along with other conventional assumptions. DC requires that the *ex-ante* preferences coincide with the *ex-post* ones, while C dictates that past decisions play no role and only available options matter[3]. Ghirardato (2002), provides the elegant result that when both DC and C are satisfied, preferences are represented by SEU and the agent's beliefs are updated according to Bayes rule[4]. However, given that most of the non-SEU models relax the independence axiom, modelling dynamic choice requires the theoretician to abandon either DC or C and consequently, to abandon Bayes rule. Al-Najjar and Weistein (2009) classify the literature into four broad categories, dealing with inconsistencies in dynamic choice, namely naïveté, sophistication, distortion of updating rules and restriction of information structures, where in almost all cases one of the two axioms is relaxed[5].

[1]See among others Gilboa and Schmeidler (1989), Schmeidler (1989), Tversky and Kahneman (1992), Ghirardato et al. (2004), Klibanoff et al. (2005), Maccheroni et al. (2006), Gajdos et al. (2008), Siniscalchi (2009). For an extensive review of the models see Etner et al. (2012).

[2]See Halevy (2007), Hayashi and Wada (2010), Hey et al. (2010), Abdellaoui et al. (2011), Charness et al. (2013), Hey and Pace (2014), Ahn et al. (2014), Stahl (2014), Baillon and Bleichrodt (2015). Trautmann and van de Kuilen (2015).

[3]Consequentialism was first proposed in Hammond (1988) and it requires that the conditional preferences to remain unaffected by the outcomes outside the conditional events. Representing the dynamic problem with a decision tree, consequentialism is satisfied when the decision maker (DM) does not take into account states that are not available anymore and thinks of the rest of the decision tree as being a new problem.

[4]Klibanoff and Hanany (2007) claim that dynamic consistency is the primary justification for Bayesian updating and under the view that Bayesian updating should be taken as given, DC comes "for free" under Expected Utility.

[5]We briefly expand on these notions in section 2.

Even though there exist plenty of experimental studies that report extensive deviations from Bayesian updating (El-Gamal (1995), Charness and Levin (2005), Charness et al. (2007), Holt and Smith (2009)), all focus on choice in risky environments (existence of objective probabilities) and their results are usually based on the conventional assumption of risk *neutrality*. In a recent study by Antoniou et al. (2015), where the authors investigate how accounting for risk attitudes alters inferences on deviations from Bayes rule, they conclude that "Previous analyses of subjective Bayesian decision-making, including our own here, have assumed that the subject is neutral towards the uncertainty that is involved in the use of an inferred posterior probability. To address this hypothesis one would need theoretical, experimental and econometric extensions of our approach". In the present study we apply the extensions indicated in Antoniou et al. (2015) so that we can test for deviations from Bayes rule, when the DMs are characterised by non-neutral ambiguity attitudes and have non-SEU preferences. Our general aim is to provide insights on how people behave in a dynamic decision problem under ambiguity and decisions are made before and after the resolution of some uncertainty. Decomposing the above idea and using an experimental design (a portfolio choice problem) that diverges from the traditional Ellsberg-urn type experiments, we aspire to investigate three main questions. First, do people behave according to the predictions of the SEU model and therefore, update beliefs in a Bayesian way? Second, when people deviate from SEU, which of the two rationality axioms do they violate? Third, when subjects violate the axioms of SEU, are they aware of this violation? In other words, is this violation the consequence of a conscious planning strategy? To address the first question, we propose a new experimental design that allows to directly test for violations of the SEU model. Regarding the second question, there is already evidence of extensive violation of DC (Dominiak et al. (2012)), in the framework of the dynamic Ellsberg urn[6]. We provide new evidence of violation of the two axioms, using alternative decision tasks and ways to represent ambiguity in the lab. Furthermore, previous studies of dynamic choice under ambiguity were constrained in answering whether subjects violate DC or C. Although this distinction is useful to inform theory and provide future directions, it does not clarify whether divergence from SEU is intended or not. We extend this analysis and test if this violation is part of the subjects' planning strategy by assuming three behavioural types, the resolute, the naïve and the sophisticated. We assume a particular model of decision making under ambiguity, the α-Maxmin Expected Utility preferences (α-MEU, Ghirardato et al. (2004)) and by making appropriate assumptions, we fit preference functionals to our data and

[6]To save space, we do not describe the dynamic Ellsberg urn problem here. The interested reader can consult Epstein and Schneider (2003), Klibanoff and Hanany (2007) and Dominiak et al. (2012). An illustration of this extension is also provided in the supplementary material.

compare decisions to the predictions of the SEU model that we use as a benchmark. Moreover, we further investigate issues that studies of static choice under ambiguity usually focus on (i.e. ambiguity attitudes and correlation between risk and ambiguity attitudes). Overall, we find substantial heterogeneity in behaviour. Almost half of our experimental population behaves according to SEU. For the ambiguity non-neutral subjects, the majority are best described by the sophisticated type, few by the naïve and the remaining by the resolute. To the best of our knowledge, this is the first study that experimentally investigates dynamic decision making under ambiguity in a portfolio choice experiment and considers behavioural heterogeneity in planning strategies.

2 Relevant Literature

Al-Najjar and Weistein (2009), classify theories to four different categories, depending the assumptions they make on how DMs tackle dynamic problems and update their beliefs upon the reception of partial information. The first category includes theories that abandon DC and are labeled as "naïve updating" theories since it is not necessary for the decisions at the present to take into consideration future preferences. This includes Gilboa and Schmeidler (1993), Pires (2002), Wang (2003), Eichberger et al. (2007) and Eichberger et al. (2010)[7]. In this approach, each of the stages is faced independently of the other, strategy that may lead to dynamic inconsistencies and dominated results. The second category, includes theories that require the DM to behave in a sophisticated way, violating DC This approach is mainly represented by Siniscalchi (2011), who does not assume any particular preference functional or update rule. The idea is based on the notion of *consistent planning*, where the *ex-post* preferences are taken into account when the *ex-ante* choices are made. An alternative way to model dynamic choice, involves the relaxation of C. This family of models proposes the use of a set of distorting updating rules that ensure DC. This includes Klibanoff and Hanany (2007), Hanany and Klibanoff (2009) and Klibanoff et al. (2009), who have axiomatised and extended few of the most commonly used ambiguity models to their dynamic version. Finally, there is a category of models in the literature that maintains both C and DC in the framework of multiple-priors representation of beliefs. To this end, these models require the restriction of information sets and allow the updating only of the set of beliefs that do not reverse the *ex-ante* choices based on the *rectangularity* condition. A representative model of this approach is presented in Epstein and Schneider (2003)[8].

[7]Ozdenoren and Peck (2008) in a game theoretical framework, show that violating DC is the rational course of action, when suspicion is perceived regarding the composition of the Ellsberg urn.

[8]An exhaustive review of the theoretical literature is beyond the scope of this study. Al-Najjar and Weistein

Early experimental evidence on violations of DC in a risky framework includes Tversky and Kahneman (1981), Cubitt et al. (1998), Busemeyer et al. (2000) and Nebout and Dubois (2014). More recently, Hey and Panaccione (2011) and Nebout and Willinger (2014), categorise their subjects to behavioural types according to the planning strategies and the axioms they satisfy when they tackle dynamic problems[9]. As far as ambiguity is concerned, although there is a rich experimental literature on static ambiguity preferences, this is not the case when one considers dynamic choice and updating where the literature is surpisingly limited. Our work is closer to the studies by Cohen et al. (2000), Dominiak et al. (2012) and Corgnet et al. (2013), which all focus on dynamic choice under ambiguity. Cohen et al. (2000) study the descriptive validity of the main two updating rules that have been axiomatised for the multiple-priors family, the *Maximum Likelihood Updating* (MLU) rule and the *Full Bayesian Updating* (FBU) rule[10]. Using a design based on the dynamic Ellsberg urn, they confirm the Ellsberg type behaviour and show that the FBU rule is applied more often. They assume *separability* (an assumption close to C), fact that does not allow for a direct test of which axiom subjects satisfy. In addition, the experiment was not incentivised in monetary terms. Dominiak et al. (2012), use a similar design as in Cohen et al. (2000). They test whether subjects satisfy DC or C, providing evidence of extensive violation of C and they also find supporting evidence for the FBU rule. Finally, Corgnet et al. (2013) study trader reaction to ambiguity when dividend information is sequentially revealed in an experimental asset market. They find that the role of ambiguity cannot explain financial anomalies.

The experiment we report here differs from the aforementioned studies in various ways. Both Cohen et al. (2000) and Dominiak et al. (2012) use the same experimental design, while we use a dynamic portfolio task along with an alternative device to represent ambiguity (a Bingo Blower) in a way that can potentially generate less suspicion *vis-à-vis* the Ellsberg urn (see Charness et al. (2013, p. 3), Ozdenoren and Peck (2008)). Furthermore, in the previous studies, the inference is based on a constrained set of four pairwise choice questions per participant. In order to eliminate possible confusion, but also to estimate preference functionals, we ask our participants a large set of allocation questions that allows us to gather potentially more informative data, since the choice variable now is continuous rather than binary ((Loomes and Pogrebna, 2014)). An additional aspect of our study is that we do not constraint the analysis to the question of which axiom is violated by ambiguity non-neutral agents, but instead, we estimate structural ambiguity models which permit the identification of various behavioural

(2009), Klibanoff et al. (2009) and Siniscalchi (2011), provide excellent reviews of the various approaches on modelling dynamic preferences under ambiguity.

[9]For an overview see Hammond and Zank (2014).

[10]We present the two update rules in Section 4.

types regarding their planning strategies. In other words, we do not only test for violations of DC, but we extend the analysis and try to understand whether subjects are aware of this violation and take it into consideration when they make choices. We also account for the stochastic part of decision making, allowing for heterogeneity in choices within subjects by adopting an appropriate error story. Finally, Corgnet et al. (2013), focus on experimental asset markets and are interested in aggregated decisions while we are interested in behaviour at the individual level and the heterogeneity in choices between the participants. Moreover they do not test any particular decision model or updating rule neither do they take into consideration heterogeneity in planning strategies. To summarise, our study contributes to the ambiguity literature and more specifically, to the literature of dynamic choice under ambiguity and updating of ambiguous beliefs. Using a new experimental design that diverges from the standard Ellsberg urn, and asking subjects a series of portfolio allocations, we provide new evidence of violation of SEU and Bayesian updating.

3 The Experimental Design and the Portfolio Choice Problem

In this paper we use experimental data to estimate models of decision making under ambiguity in a dynamic framework. We need to jointly elicit risk and ambiguity attitudes of the participants, as well as subjective probabilities (beliefs). In the literature there have been proposed various ways to elicit and measure subjective beliefs (scoring rules, matching probabilities, stated beliefs) which either require several assumptions regarding the risk attitudes of the DM or are difficult to explain to subjects. Instead, we elicit beliefs and strategies based on a *revealed preference* argument. To achieve this, we ask our subjects a series of 2-stage portfolio allocation questions, an experimental design inspired by Loomes (1991), who used allocation type questions and simply asked subjects to allocate experimental income between two risky alternatives. This allocation procedure, seems to provide more informative data, and it has been generally applied to the literature in various contexts[11]. Using allocation data allows us to parametrically estimate latent specifications of theoretical decision making models and to further test for deviations from Bayesian updating, as well as for the existence of different planning strategies.

Our design shares similarities with Ahn et al. (2014) and extends it in two ways. We use

[11] Studies that use allocation problems include Choi et al. (2007) in a portfolio choice experiment under risk, Charness and Gneezy (2010) studying portfolio choices, Hey and Panaccione (2011) on dynamic decision making under risk, Ahn et al. (2014) in a portfolio choice experiment under ambiguity, Hey and Pace (2014) comparing different static models of choice under ambiguity and Loomes and Pogrebna (2014) studying individual risk attitudes. See Loomes and Pogrebna (2014) for an extensive discussion on the allocation procedure.

a simple two-stage portfolio allocation task, with three possible states of nature. At each state s, corresponds a state-contingent *Arrow* security, the return of which equals e_s if state s occurs and 0 otherwise, where e_s is the rate of return of asset s (henceforth exchange rate) for every unit of income allocated to this asset. Contrary to Ahn et al. (2014) who represent ambiguity using the 3-colour Ellsberg urn, we are representing ambiguity with the aid of a transparent and non-manipulable device, a Bingo Blower[12]. The Bingo Blower consists of a transparent box that contains colourful table tennis balls. At the bottom of the box, there is a motor that generates a stream of air, which makes the balls to continuously move inside the box. The advantage of this device, is that when the number of the balls is sufficiently high, one is not able to count their exact number, distinguishing this environment from one with objective probabilities. What is possible to do, is to distinguish that there is at least one ball of each colour (lower bound probabilities) and to obtain a rough idea of the maximum number of the balls (upper bound) preserving always some ambiguity.[13] In other words, while there exist objective probabilities (known only to the experimenter), the subjects are not able to precisely construct an objective probability distribution. Inside the Bingo Blower there are balls of three different colours blue (B), red (R) and yellow (Y), to represent the three different Arrow assets. The use of the Bingo Blower helps us avoid two main drawbacks of the Ellsberg urn. The first, as was mentioned earlier, is related to the suspicion that the subjects raise regarding the actual composition of the urn. Another important drawback, as is explained in Ahn et al. (2014, p. 209), is that in this particular framework, it is not possible to identify the ambiguity attitude parameter separately from the set of priors. As we are interested in both the attitude an the set of priors, using three ambiguous states rather two permits, as we explain later, the joint identification of the parameters.

The most important difference with Ahn et al. (2014) is that we extend this framework to its dynamic version. At time $t = 0$, an agent is endowed with experimental income m and is asked to allocate it between the three assets, given the vector of exchange rates e and satisfying the budget constraint. The demand for the assets is a function of the preferences of the DM, the exchange rates, the available income and the beliefs of the DM. At $t = 1$, nature moves and a state of the world is realised (a ball is drawn from the Bingo Blower). At this point, the actual state of the world is not yet revealed to the subject. Instead, partial information is provided to the agent that one of the states of the world has not occurred in the form "the ball is not s". The DM is consequently loosing the proportion of the income that has been allocated to

[12] A similar Bingo cage has been used by Andersen et al. (2012) and the Bingo Blower has been used by Hey et al. (2010) and Hey and Pace (2014), all in *static* choice problems.

[13] Roughly, when the number of balls is more than 10, the environment becomes ambiguous enough, otherwise it is possible that subjects might be able to count the exact number of balls, transforming the problem to a risky one.

that state and at the second stage, she is asked to allocate the remaining experimental income to the two available assets based on her preferences, the exchange rates and her now updated beliefs. At $t = 2$, all ambiguity is resolved, the actual state of the world is revealed and the DM is paid the state-contingent dividend. We ask our subjects a series of 60 two-stage allocation questions, where an allocation question consists of an amount m of experimental income and a vector e containing the exchange rates for the three assets. Every question involves different levels of income and exchange rates. During the experiment, the draws from the Bingo Blower were hypothetical[14] for the simple reason that we wanted to focus purely on updating[15]. Allowing continuous sampling from the Bingo Blower, could potentially generate *learning effects* regarding the actual probability distribution, that would transform the problem to a risky one (objective probabilities)[16]. A drawback of this experimental design is that we need to assume that the subjects are either risk averse or risk neutral. A risk neutral person would allocate everything to the asset with the highest expected payoff whereas, a risk seeking person, would be willing to allocate negative amounts to some of the assets[17]. As a result, it is not possible to distinguish behaviour between a risk seeking and a risk neutral subject. Nevertheless, the analysis shows that the number of seeking or neutral subjects was very limited.[18]

4 Theoretical Framework and the Different Types

In this section we present the latent structural models of decision making that we fit to our data, as well as the various behavioural types of DMs that we assume. First we provide few definitions needed to characterise the different types, namely dynamic consistency, consequentialism and consistent planning.

Dynamic Consistency. *An agent satisfies dynamic consistency (DC) whenever her ex-ante choices coincide with her ex-post.*

While in a pairwise choice context, DC dictates the lack of reversals, in the allocation con-

[14]This form of hypothetical signals has been previously applied in the literature in Griffin and Tversky (1992) and Kraemer and Weber (2004).

[15]To generate the appropriate signals, we adopted the following procedure. The software was programmed to perform i.i.d. draws for every allocation problem, based on the actual probability distribution of balls inside the Bingo Blower. For each problem a virtual ball was drawn. Imagine, that for a given problem, this ball is red. Then, a signal was sent to the participants, where with probability p=0.5 they were informed that the ball is not yellow, otherwise they received the signal that the ball is not blue. To ensure credibility, the virtual draw did not define the winning colour of the experiment. We return to this point in section 5.

[16]See for example Trautmann and Zeckhauser (2013), Ert and Trautmann (2014) and Baillon et al. (2015)

[17]Allowing subjects to do so, would require to consider negative payoffs, something that we would like to skip at this point and investigate in future research.

[18]Gneezy et al. (2015) have recently designed an experiment that allows joint estimation of risk and ambiguity attitudes and allows for risk seeking behaviour. Their results confirm that a large proportion of subjects can be characterised as risk averse.

text, we need to slightly adapt this definition. Let $u : \mathbb{R} \to \mathbb{R}$ a standard von Neumann-Morgenstern utility function, that satisfies the usual assumptions of being twice differentiable, strictly increasing and strictly concave and X a non-negative portfolio allocation $X = (x_R, x_B, x_Y$. Then DC should guarantee that the unconditional *marginal rate of substitution* between two assets (e.g. R and B) is equal to the conditional one, given the information that Y has not happened. Thus, DC requires that:[19]

$$\frac{\partial u_0/\partial x_R}{\partial u_0/\partial x_B} = \frac{\partial u_1/\partial x_R{}^{\neg Y}}{\partial u_1/\partial x_B{}^{\neg Y}} \tag{1}$$

Consequentialism. *An agent satisfies consequentialism (C) when her preferences conditional on a non-null event E are not affected by the outcomes outside the conditional event.*

More specifically, this axiom requires that no weight is placed on the consequences of acts that are not available any more and the conditional preferences depend only on the information provided by the conditioning event E[20]. It naturally follows that an ambiguity non-neutral, consequentialist DM will not always satisfy Equation 1. Finally, the last definition is needed for the DMs who although violate DC, they are aware of this violation. It is based on the notion of *consistent planning*, first introduced in Strotz (1955-56) in the context of deterministic dynamic choice.

Consistent Planning. *An agent adopts the consistent planning (CP) strategy, if at each decision node, the best plan among those that will be actually followed is chosen.*

This concept borrows elements from the game theoretical literature, where the dynamic problem is represented by a game played by multiple selves of the same individual. The DM applies *backward induction* and her planning strategy requires to first consider the terminal choice node of a decision tree and choose the optimal course of action at this point. Then, by "folding back", she calculates the optimal choice in the previous nodes, taking into consideration her future preferences. Siniscalchi (2011) formally axiomatises this concept for dynamic choice under ambiguity by deriving *ex-ante* conditional preferences over decision *trees* rather than over *acts*[21]. We next derive the behaviour for each of the specifications that we consider. We start by presenting the benchmark model of SEU with Bayesian updating and then, we subsequently relax the assumption of ambiguity neutral attitudes. We present the strategies assuming a generic form regarding the utility representation[22].

[19]It is easy to show that Bayesian updating requires Equation 1 to be satisfied.
[20]Al-Najjar and Weistein (2009) refer to this type of updated preferences as *fact-based* updated preferences.
[21]We expand on this notion in Section 4.4.3.
[22]Since the problem is restricted to a 2-period model, it is possible to obtain solutions in a closed form. In the supplementary material, we provide the analytical solutions we used for the analysis, for all the specifications.

4.1 Subjective Expected Utility

The DM is assumed to hold a unique set of subjective, additive priors $\pi = \{\pi(R), \pi(B), \pi(Y)\}$ regarding the three possible states of the world such that $\pi(R) + \pi(B) + \pi(Y) = 1$. As already highlighted, a convenient feature of the SEU model is that the DM satisfies both DC and C and consequently, the beliefs of the agent are updated according to the Bayes rule which ensures that the *ex-ante* allocation coincides with the *ex-post*. Hence, it suffices to solve the problem as if it was a static one with three possible states of the world. Assuming a utility function $u(.)$, the objective of the DM is to calculate the optimal portfolio X, based on her subjective beliefs, that maximises the expected utility, subject to the budget and the non-negativity constraints. The optimal allocation is given by solving:

$$\max_{X} \pi(R)u(e_R x_R) + \pi(B)u(e_B x_B) + \pi(Y)u(e_Y x_Y)$$

$$\text{s.t. } x_R + x_B + x_Y = m$$

The first order conditions of this optimisation program require that:

$$\pi_R e_R \frac{\partial u(e_R x_R)}{\partial x_R} = \pi_B e_B \frac{\partial u(e_B x_B)}{\partial x_B} = \pi_Y e_Y \frac{\partial u(e_Y x_Y)}{\partial x_Y}$$

Assuming a particular form of the utility function, we obtain closed-form solutions regarding the demand of each asset of the form $x_s^* = f(\pi, m, e, l)$, where π is the set of subjective beliefs, m is the experimental income, e is the vector of exchange rates, l is a vector of individual characteristics (e.g. risk aversion) and $s \in \{R, B, Y\}$. By definition, a DM that holds additive subjective beliefs is characterised by a *neutral* attitude towards ambiguity.

4.2 The α-Maxmin Model

In this section we relax the assumption of additive beliefs and we introduce non-neutral ambiguity attitudes assuming that the DM has α-Maxmin preferences (α-MEU, Ghirardato et al. (2004)). In this model the agent believes that the true probabilities over the state space lie within a continuous, closed and convex set of subjective priors Π (multiple-priors representation). This set includes all the possible scenarios regarding the future states of the world, in the form of subjective probability distributions (beliefs). Figure 1 illustrates this set Π using a two-dimensional unit simplex (known as the Marschak-Machina Triangle (MMT)[23]) where the probability that the state of the world is R (Y) is represented in the horizontal (vertical) axis. Assuming that there exist non-zero low bounds of the DM's subjective beliefs ($\underline{\pi}(R), \underline{\pi}(B), \underline{\pi}(Y)$), we are able to draw the interior triangle, the size of which illustrates the degree of *ambiguity*

[23] This representation of prior beliefs in the MEU model first appeared in Hey et al. (2010) and then broadly used in the ambiguity literature (see Kothiyal et al. (2014), Burghart et al. (2015)).

perception of the agent. When this interior triangle coincides with the simplex, the DM perceives ambiguity at its maximum level whereas, when it shrinks to a single point inside the simplex, then all ambiguity vanishes, the set Π is a singleton and the model reduces to SEU. In the general case, a portfolio $X = (x_R, x_G, x_B)$ is evaluated as a convex combination of its minimal and its maximum expected utilities over this set Π:

$$U(X) = \alpha \min_{\pi \in \Pi} \left[\sum_{s \in S} \pi(s) u(x_s) \right] + (1 - \alpha) \max_{\pi \in \Pi} \left[\sum_{s \in S} \pi(s) u(x_s) \right] \tag{2}$$

with $\Pi = \{\pi(s) : \pi(s) \geq \underline{\pi}(s)\}$ and $s \in \{B, R, Y\}$[24]. The α coefficient can be interpreted as a measure of the agent's *aversion* to this perceived ambiguity. When $\alpha = 1$ the model collapses to the MEU preferences (Gilboa and Schmeidler, 1989) where maximal aversion to ambiguity is expressed. In contrast, when $\alpha = 0$, all the weight is put to the optimistic outcome. Intuitively, $\alpha > 0.5$ implies that the DM is ambiguity averse, whereas $\alpha < 0.5$ implies ambiguity seeking attitude. Notice that in the particular framework of our study, $\alpha = 0.5$ does not imply ambiguity neutral attitudes and the model does not collapse to SEU as is the case in Ahn et al. (2014). Neutral attitudes are expressed by the uniqueness of the set Π. When this set is a singleton, the model is equivalent to the SEU and the parameter α cannot be identified.

Before presenting the different types of the DMs we present how this model can be extended to its dynamic form. As is common in the ambiguity literature, this model satisfies the property of separating subjective beliefs from tastes (ambiguity attitudes). Therefore, when updating takes place, only the belief part of the preferences' representation is affected, while utility remains intact.

4.3 Updating Beliefs in Multiple-priors Models

We first present the updating rules for MEU, the special case of α-MEU when $\alpha = 1$. Then these rules can be naturally extended for the Hurwicz α criteria family. In the literature there have been suggested two ways to update beliefs in multiple-priors models, one that satisfies DC (Epstein and Schneider (2003), Hanany and Klibanoff (2009), Hanany et al. (2011)) and one that satisfies C (Gilboa and Schmeidler (1993), Pires (2002), Eichberger et al. (2007)). In the former case, it suffices to solve the problem as a static one and the allocation in the first period will determine the conditional allocation of the second period, respecting always the MEU preferences of the DM. The interesting case is when C is assumed which allows the agent to behave in a dynamically inconsistent manner. The two most commonly updating

[24] We summarise the various sets of priors in Table 1.

rules include the *Maximum Likelihood Update* (MLU) and the Full Bayesian Update (FBU)[25] rule. According to the MLU rule, only the set of priors that maximise the probability of the conditional event are updated according to the Bayes rule. In the FBU rule , all the sets of priors are updated in a Bayesian way and the set of posteriors is used to evaluate the different acts. In the supplementary material, we show that in our framework with three ambiguous assets, MLU and FBU coincide. Therefore, in our analysis we assume that beliefs are updated according to the MLE rule. We now define the three behavioural types we consider and we describe how the update rules are extended to accommodate α-MEU type of preferences (when updating takes place).

4.4 Taxonomy of the Types

We classify DMs based on two criteria: (1) which axiom do they satisfy and (2) if they are time inconsistent, whether they are aware of this inconsistency or not. We follow Machina (1989) who defines four different types of DMs in dynamic choice under risk: the so called *α-people*, the dynamically consistent agents that maximise EU preferences, the *β-people* that are non-EU agents and apply consequentialism, acting in a dynamically inconsistent way (myopic behaviour), the *γ-people* who are non-EU agents but are dynamically consistent and finally, the *δ-people* who are characterised as *sophisticated* and satisfy consistent planning[26]. We adopt the terminology of Hey and Panaccione (2011) and we define the *naïve*, the *resolute* and the *sophisticated* type that correspond to the *β-people*, *γ-people* and *δ-people* respectively[27].

4.4.1 The Resolute Type

The resolute type, first introduced in Hammond (1988), and later formalised in McClennen (1990) and Machina (1989) in risky contexts, embraces the simplest strategy. A resolute DM, satisfies DC and the allocations at both stages coincide. This may happen for two reasons, as either the DM is dedicated to somehow commit to the first stage allocations regardless the available information at $t = 1$[28], or one can assume that beliefs are updated in a dynamically consistent manner as in Epstein and Schneider (2003). In either case, the resolute strategy with commitment is behaviourally equivalent to the dynamically consistent updating of beliefs, and

[25] See Gilboa and Schmeidler (1993) for an axiomatisation of the rules and for references. They refer to these rules as *pseudo-Bayesian* rules.

[26] A similar classification has been also applied first in hyperbolic discounting contexts (O'Donoghue and Rabin (1999)) and later in Hey and Panaccione (2011) and Barberis (2012), in contexts of dynamic decision making under risk. Houser et al. (2004) study heterogeneity in planning strategies in a dynamic stochastic decision problem under certainty.

[27] Barberis (2012) considers two types of sophistication, one without commitment and one with. These types correspond to our sophisticated and resolute respectively.

[28] This strategy is also known as *aversion to information*.

to find the optimal solution, it suffices to solve the first stage problem. The optimal allocation is calculated by optimising Equation 2 subject to the budget constraint, given the DM's individual characteristics and subjective beliefs. We denote with z_s the *return* of asset s which is defined as the product between the exchange rate of the asset (e_s) and the amount of income that has been allocated to this asset (x_s): $z_s = e_s \times x_s$ with $s \in \{R, B, Y\}$. Then, in order to calculate the optimal allocation, one needs to take into consideration the relative ranking between the returns of the three assets. Take for example the ranking $z_R \geq z_B \geq z_Y$[29] where red is the *best* possible outcome and yellow the *worst*. The maximum expected utility occurs at the point where the probability of the best outcome to happen is maximised (point A in Figure 1). Similarly, the minimum expected utility is obtained at the point where the probability of the best outcome R is minimised or stating it differently, where the probability of the worst outcome Y is maximised (point C). Then the α-Maxmin utility from a portfolio X is:

$$\begin{aligned} U(X) =& \alpha[(1 - \underline{\pi}(B) - \underline{\pi}(Y))u(e_R x_R) + \underline{\pi}(B)u(e_B x_B) + \underline{\pi}(Y)u(e_Y x_Y)] \\ &+ (1-\alpha)[\underline{\pi}(R)u(e_R x_R) + \underline{\pi}(B)u(e_B x_B) + (1 - \underline{\pi}(R) - \underline{\pi}(B))u(e_Y x_Y)] \end{aligned}$$

and writing the utility from the portfolio in its general form, the objective of the DM is to find an allocation X that optimises $U(X) = \sum_{s \in S} \pi(s)u(e_s x_s)$, subject to the budget and the non-negativity constraints. Here $\pi(s)$ is defined as $\pi(s) = \alpha\pi^{\min}(s) + (1-\alpha)\pi^{\max}(s)$ where $\pi^{\min}$ ($\pi^{\max}$) stands for the set of priors where the probability of the best outcome to happen is minimised (maximised). The solution of this program will provide the optimal demand for the three assets in the form $x_s^* = f(\pi, m, e, l)$, where π are now the non-additive subjective beliefs and l includes both the risk and the ambiguity attitude, which will coincide with the optimal conditional demand.

4.4.2 The Naïve Type

The *naïve* or *myopic* behaviour was first introduced in the literature in Strotz (1955-56) and later in Pollak (1968) indicating an agent who fails to understand the sequential nature of the problem. As a consequence, each of the stages is faced independently of the other, strategy that may lead to dynamic inconsistencies and dominated results. The allocation at each stage is based on the optimisation of the objective function at the current stage, or stating in a different way, the DM solves a series of static problems and maximises utility at present. A naïve DM ignores that she is time inconsistent and as a result, the decisions that are made can potentially differ from those that had been planned. At the first stage, this type behaves in the same way

[29] In the supplementary material, we enumerate all the possible 13 rankings including both weak and strict inequalities. We also describe how our algorithm calculates the optimal allocation.

as a resolute does and solves the problem as if it is a static one, leading to the unconditional portfolio $X = (x_R^*, x_B^*, x_Y^*)$. Then, at stage 2, she receives the partial information that one of the states did not occur, updates her prior beliefs, and based on these posteriors, she solves the maximisation problem that now involves the two remaining states subject to the available income. Consider again the ranking $z_R \geq z_B \geq z_Y$. The DM chooses a portfolio allocation for the first period. Now assume that the partial information that the ball is not yellow ($\neg Y$) is revealed. Using the MLU rule, the DM updates those priors that maximise the probability of the event $\pi(\neg Y)$ (or $\pi(R \cup B)$). In Figure 1, this occurs in both the prior sets A and B. In addition, since the ranking of the outcomes requires that $z_R \geq z_B$[30], for the evaluation of the α-Maxmin utility it holds that $\pi^{\max} = \pi_A$ and $\pi^{\min} = \pi_B$. We denote with ${x_R}^{\neg Y}, {x_B}^{\neg Y}$ the allocations to assets R and B respectively, conditional on the information that the state is not Y. The utility of the DM of this conditional portfolio is:

$$U(X) = \pi(R|\neg Y)u(e_R {x_R}^{\neg Y}) + \pi(B|\neg Y)u(e_B {x_B}^{\neg Y}) \tag{3}$$

with $\pi(R|\neg Y) = \alpha\pi_B(R|\neg Y) + (1-\alpha)\pi_A(R|\neg Y)$ ($\pi(B|\neg Y)$ is defined in a similar way[31]). The problem now requires to find the conditional allocation that optimises this α-MEU, subject to both the non-negativity constraint and the new budget constraint $\hat{m}^{\neg Y} = m - x_Y^*$ where m is the initial endowed income and x_Y^* is unconditional allocation to asset Y. The conditional demand will be of the form $x_s^{*\neg q} = f(\hat{\pi}, \hat{m}^{\neg Y}, e, l)$ for $s \in S \backslash q$ and $s \neq q$, where $\hat{\pi}$ is now the set of the updated beliefs.

4.4.3 The Sophisticated Type

Strotz (1955-56) and later Pollak (1968) were among the first to recognise that pre-commitment (resolute type) is not always the optimal strategy. More specifically, the idea is that a DM who is not able to commit to her future behaviour, would prefer to adopt a strategy of *consistent planning* and then pick up the optimal plan that will be actually followed, sketching the profile of a *sophisticated* type. A sophisticated DM applies *backward induction* in order to figure out the optimal strategy for every given problem. As Hammond and Zank (2014) describe, sophistication is like the sub-game perfect Nash equilibrium of an extensive form game, between the future and present self of the DM, as in Selten (1975). Starting from the final decision nodes of a decision tree (the last period) a DM anticipates an event E to occur and therefore, the future course of action is determined by the conditional preferences of the *ex-post* self. Working back-

[30]Notice that for the naïve DM, it is not necessary for the ranking between the returns of two assets to be the same in both stages. Our estimation algorithm takes this possibility into consideration.

[31]The interested reader can consult the supplementary material where we extensively present how all the updated beliefs are calculated.

wards and applying the same principle to all the previous decision nodes, always satisfying the preferences of the *ex-post* self, she can define the optimal path that will lead her from the start of the tree to the most preferable node. In this way, an optimal plan of action for the whole problem is chosen. Following this process, the DM will violate DC, as the second period optimal allocation is based on the conditional beliefs which have been updated in a dynamically inconsistent way[32]. Nevertheless, the agent is aware of this inconsistency and as is described in Pollak (1968), "succeeds to adopt a strategy of *consistent planning* and choose the best plan among those that he will actually follow." The above idea has been axiomatised and applied in dynamic choice under ambiguity in Siniscalchi (2011).

The optimal solution for the sophisticated type requires two steps. Let again the same ordering of the outcomes $z_R \geq z_B \geq z_Y$. As the solution requires the DM to work backwards, she first solves all the three conditional problems $(\neg R, \neg B, \neg Y)$, using her conditional beliefs and satisfying the budget and non-negativity constraints. For instance, when the information is $\neg Y$, the optimisation problem is to find the conditional allocation for assets R and B, taking into consideration the conditional beliefs, that they have now been updated based on the relevant information, and always satisfying the outcome ranking ($z_R \geq z_B$) and the conditional budget constraint $\hat{m}^{\neg Y} = m - x_Y^*$. The conditional allocations $x_R^{*\neg Y}$ and $x_B^{*\neg Y}$ can be written in the general form $x_s^{*\neg q} = f(\hat{\pi}, e, \hat{m}, l)$ for $s \in S \backslash q$ and $s \neq q$ (similarly we solve for the conditional allocations for $\neg B$ and $\neg Y$). These demands are calculated in the same way as the second-stage decisions of the naïve DM (see section 4.4.2) and they indicate to the agent what would be the optimal course of action for each of the conditional states (last stage of the decision problem). The second step of the solution, requires to solve the first stage unconditional problem taking into consideration the optimal conditional allocations, the non-negativity and budget constraint, as well as the relevant ranking constraint between the outcomes. The α-Maxmin utility of this two-stage portfolio is given by:

$$
\begin{aligned}
U(X) = {} & \frac{1}{2}\pi(\neg Y)\left[\pi(R|\neg Y)u(e_R x_R^{*\neg Y}) + \pi(B|\neg Y)u(e_B x_B^{*\neg Y})\right] \\
& + \frac{1}{2}\pi(\neg B)\left[\pi(R|\neg B)u(e_R x_R^{*\neg B}) + \pi(Y|\neg B)u(e_Y x_Y^{*\neg B})\right] \\
& + \frac{1}{2}\pi(\neg R)\left[\pi(B|\neg R)u(e_B x_B^{*\neg R}) + \pi(Y|\neg R)u(e_Y x_Y^{*\neg R})\right]
\end{aligned} \tag{4}
$$

with $\pi(R|\neg Y) = \alpha\pi_B(R|\neg Y) + (1-\alpha)\pi_A(R|\neg Y)$ and $\pi(\neg Y) = \alpha\pi_B(\neg Y) + (1-\alpha)\pi_A(\neg Y)$ where $\pi_A = \pi^{\max}$ and $\pi_B = \pi^{\min}$ are the sets of priors that satisfy the ranking of the outcomes (similarly we define the probabilities for the cases $\neg R$ and $\neg B$). The probability of each conditional event is multiplied by $1/2$ since the subjects were informed in advance that the par-

[32] Otherwise the sophisticated strategy is identical to the resolute one.

tial information to be revealed, is randomly chosen from the two available states with equal chances[33]. This also ensures that $\pi(S) = 1$. Notice that the conditional demands are a function of the conditional income $\hat{m}^{\neg s} = m - x_s^*$ which in turn is a function of the unconditional optimal demand for the asset s at stage 1. To calculate the unconditional demand for the three assets, it suffices to substitute the conditional income to Equation 4 and optimise with respect to the unconditional demands x_s^*. Given these demands and plugging-in to the formulas of conditional income, we derive the conditional demands.

Suffice to say that if the agent is *probabilistically sophisticated* (holds additive beliefs) then the three types are behaviourally indistinguishable compared to SEU.

5 Experimental Procedures

Upon arrival to the lab, subjects were randomly assigned to a computer terminal and were provided with written instructions[34]. After reading the instructions, the participants were able to go through a slide-show presentation which was available at each computer terminal and contained simplified instructions and examples, which could navigate at their own pace. Then, they were free to go near the Bingo Blower, which was located in the middle of the lab, and observe its composition regarding the three assets. During the experiment a live image of the Bingo Blower was projected through two large screens in the lab and in addition, the subjects were free to walk around and physically observe it at all times. The actual composition of the Bingo Blower consisted of 4 blue (20%), 6 red (30%) and 10 yellow (50%) balls out of the total 20[35]. The participants were then presented with the 60 2-stage allocation questions[36]. As mentioned before, an allocation problem consisted of a specific amount of experimental income and the exchange rates of the three assets. The income ranged from 9 to 110 units, expressed in tokens, and the exchange rates between experimental income and money, ranged from 0.1 to 1.8. All subjects faced the same set of allocation questions, presented in a randomised order to each participant in an effort to eliminate any potential order effects.

The experimental interface was developed in *Python*.[37] Each allocation question that the

[33]This was defined by the experimental software by an independent, random draw from a uniform distribution for each of the allocation problems.

[34]The instructions are available in the supplementary material.

[35]From two pilot studies we ran, it seems that changing the colour with the highest likelihood, does not affect the results.

[36]The questions have been chosen after extensive Monte Carlo simulations that would ensure three issues: (1) that for a simulated dataset using a given set of parameter values, it is possible to estimate (recover) the value of the actual parameters; (2) that it is possible to identify between the different specifications and; (3) that our estimation programs work efficiently. See Section 6 for details on the econometric analysis.

[37]Python Software Foundation. Python Language Reference, version 2.7. Available at http://www.python.org. Screenshots of the experimental framework can be found in the supplementary material.

subjects were required to answer had two stages. In the first stage screen, the subjects could see three sliders, one for each respective asset, and information on the allocation question (the total income to allocate and the exchange rates of the three assets). The sliders were programmed to be inter-connected with each other, so at any time, the budget constraint would be satisfied with strict equality (there was no possibility to allocate money to a safe asset or to allocate negative amounts). In addition, information was provided regarding the allocated income to each asset and the respective second-stage potential income that this allocation implied, depending on the conditional state of the world. Subjects were required to spend at least 10 seconds before submitting their preferred allocation and they had 90 seconds available for each stage[38]. Pushing the "next" button at stage 1, the software was programmed to reveal some hypothetical partial information based on a uniform distribution. In the second stage, the subjects could only see two sliders for the remaining states of the word, along with all the relevant information (available conditional income, exchange rates and expected payoffs). The choices were recorded in integer steps in the range [0,m].

The experiment was conducted at an experimental economic lab in the UK, known to prohibit deception between May and June 2013. 58 subjects were recruited from a standard student experimental population using the ORSEE system (Greiner (2004)). The majority of the subjects were undergraduate students from several different majors and 52% were females. The experiment lasted for less than one hour and the subjects were paid privately and in cash directly after the end of the experiment. The average payment was £14.16 including a *show-up* fee of £3. The maximum payment was £25.5. The payment was determined by applying the *random incentive mechanism*, where one out of the 60 problems was randomly chosen (different for each participant) to be played for real. The computer then recovered the actual choices of the participant at that problem, as well as the partial information that was revealed (i.e. the state is not s). Then, the subject was continuously drawing balls, till a ball that is not s came out. That ball determined the actual state of the world and the participant was paid the amount that was allocated at this state, at the specific problem.

6 Econometric Analysis

We estimate the specifications presented in section 4, based on a subject-level analysis. Adopting this approach, allows to introduce between and within subjects heterogeneity in three different dimensions. First, instead of assuming a representative agent, we individually estimate

[38]Subjects were informed that if they have not submitted their allocation before the 90 seconds, the computer would allocate zero amounts to the three assets. This happened only three times out of the total 6960 observations.

the values of the preference parameters (risk and ambiguity attitudes, beliefs, precision) for each of our subjects without requiring any uniform pattern of behaviour. In addition to the parameter heterogeneity, we account for heterogeneity regarding the planning strategies of the agents by fitting the individual data to both the SEU model and the three different type specifications. Finally, we allow for heterogeneity within the participants by incorporating a random (stochastic) part in choices so as to capture within-person variability (noise).

We need to make few structural assumptions regarding the shape of the utility function, the ambiguity model and the stochastic structure of our data in order to be able to *jointly* estimate all the parameters of interest. We assume that the subjects receive utility from a power utility function that is characterised by Constant Relative Risk Aversion (CRRA) of the following form:

$$u(x) = \begin{cases} \frac{x^{1-r}}{1-r} & \text{if } r \neq 1 \\ \ln(x) & \text{if } r = 1 \end{cases}$$

where x is the respective payoff and r is the coefficient of risk aversion. The reasons why we favour the power form of utility are twofold. On the one hand, there is extensive evidence that the CRRA utility function provides a good fit to experimental data (Wakker (2008), Stott (2006), Balcombe and Fraser (2015)). Then, assuming a power utility naturally satisfies the non-negativity constraint that was imposed by our experimental protocol, as the CRRA representation does not allow for boundary portfolios (allocating everything or nothing to one asset)[39]. Regarding the ambiguity model, we adopt the α-MEU specification mainly for five reasons: (1) it provides a parsimonious way to capture *perceived ambiguity*; (2) in contrast to the MEU model which is characterised by pessimism, the α-MEU takes into consideration both the worst and the best case scenario, providing a measure of attitude towards ambiguity; (3) there have been well-established updating rules for the multiple-priors family of models, for both the dynamically consistent and inconsistent DM; (4) combining α-MEU with power utility provides elegant, *closed-form* solutions for the optimal allocations and; (5) kinked specifications have been shown to fit experimental data better compared to smooth ones (Ahn et al. (2014)).

Finally, we need to adopt an appropriate model to capture the stochastic part of decisions. In the literature there have been proposed various ways to model noise and variability in choices (see Wilcox (2008), Bardsley et al. (2009, chap. 7)). Since our data are continuous and constrained by definition to the interval $[0, m]$, a convenient way to model noise in choices is to follow Hey and Panaccione (2011) and consider the ratio x_s/m at a specific allocation question. This amount is constrained to the *unit* interval and therefore, we can assume that

[39]Only a risk neutral or risk loving agent would choose a boundary portfolio in this particular framework.

the ratio $\frac{x_s}{m}$ is distributed according to a *Beta* distribution, a continuous probability distribution defined in the interval $[0, 1]$. A Beta distributed variable, is characterised by two positive shape parameters α and β and the moments (mean and variance) of this distribution are given by $E_x = \frac{\alpha}{\alpha+\beta}$ and $Var(x) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ respectively. By setting $\alpha = \frac{x_s^*}{m}(\sigma-1)$ and $\beta = (1 - \frac{x_s^*}{m})(\sigma-1)$, it is guaranteed that $E(\frac{x_s}{m}) = \frac{x_s^*}{m}$ and $\frac{\frac{x_s^*}{m}(1-\frac{x_s^*}{m})}{\sigma}$, where x_s is the actual allocation observed in the experiment, x_s^* is the optimal constrained allocation for asset s, for a given allocation problem and σ is an indicator of the precision of the distribution of the random variable $\frac{x_s}{m}$.[40] The former expression implies that the random variable $\frac{x_s}{m}$ is centered to $\frac{x_s^*}{m}$, while the latter, implies that the variance becomes smaller at the bounds 0 and 1 and larger near 0.5. We then need to specify the likelihood function that will be maximised. As there are two stages, with 3-way allocations in the first stage and 2-way in the second, it suffices to consider two of the allocations in the first stage and one at the second. For instance, let for a given allocation problem the unconditional allocation x_R, x_B, x_Y to red, blue and yellow and assume the conditional state $\neg Y$ that will lead to the conditional allocation $x_R{}^{\neg Y}, x_B{}^{\neg Y}$ and the conditional income $\hat{m}^{\neg Y}$. Using the allocations at the first stage, we assume that $\frac{x_R}{m}$ is Beta distributed, with the appropriate shape parameters that satisfy the properties above. As the actual allocations are recorded in integer values during the experiment, we approximate the contribution to the likelihood function from a continuous Beta distribution. Thus, for the above example and for a given allocation problem i, the contribution to the likelihood function by the allocation to the red asset is given by:

$$g_1 = \begin{cases} \text{if } x_R = 0, & \text{Prob}(\frac{x_R}{m} = 0) = \ln(\Phi(\frac{0.5}{m}, \alpha, \beta)); \\ \text{if } x_R = m, & \text{Prob}(\frac{x_R}{m} = 1) = \ln(1 - \Phi(\frac{m-0.5}{m}, \alpha, \beta)); \\ \text{if } 0 < x_R < m, & \text{Prob}(\frac{x_R}{m} = \frac{x_R^*}{m}) = \ln(\Phi(\frac{x_R^*+0.5}{m}, \alpha, \beta) - \Phi(\frac{x_R^*-0.5}{m}, \alpha, \beta)); \end{cases}$$

Then, we consider the remaining available income and we assume that $\frac{x_B}{m-x_R}$ is also Beta distributed. Again the contribution to the likelihood function by the blue asset is given by:

$$g_2 = \begin{cases} \text{if } x_B = 0, & \text{Prob}(\frac{x_B}{m-x_R} = 0) = \ln(\Phi(\frac{0.5}{m-x_R^*}, \alpha, \beta)); \\ \text{if } x_B = m - x_R, & \text{Prob}(\frac{x_B}{m-x_R} = 1) = \ln(1 - \Phi(\frac{m-x_R^*-0.5}{m-x_R^*}, \alpha, \beta)); \\ \text{if } 0 < x_B < m - x_R, & \text{Prob}(\frac{x_B}{m-x_R} = \frac{x_B^*}{m-x_R^*}) = \ln(\Phi(\frac{x_B^*+0.5}{m-x_R^*}, \alpha, \beta) - \Phi(\frac{x_B^*-0.5}{m-x_R^*}, \alpha, \beta)); \end{cases}$$

Finally, at the conditional stage, we assume the same for the conditional allocation to red with respect to the conditional income, $\frac{x_R{}^{\neg Y}}{\hat{m}^{\neg Y}}$.

$$g_3 = \begin{cases} \text{if } x_R{}^{\neg Y} = 0, & \text{Prob}(\frac{x_R{}^{\neg Y}}{m-x_Y} = 0) = \ln(\Phi(\frac{0.5}{m-x_Y^*}, \alpha, \beta)); \\ \text{if } x_R{}^{\neg Y} = m - x_Y, & \text{Prob}(\frac{x_R{}^{\neg Y}}{m-x_Y} = 1) = \ln(1 - \Phi(\frac{m-x_Y^*-0.5}{m-x_Y^*}, \alpha, \beta)); \\ \text{if } 0 < x_R{}^{\neg Y} < m - x_Y, & \text{Prob}(\frac{x_R{}^{\neg Y}}{m-x_Y} = \frac{x_R^{*\neg Y}}{m-x_Y^*}) = \ln(\Phi(\frac{x_R^{*\neg Y}+0.5}{m-x_Y^*}, \alpha, \beta) - \Phi(\frac{x_R^{*\neg Y}-0.5}{m-x_Y^*}, \alpha, \beta)); \end{cases}$$

[40]The higher the value of σ, the more precise are the choices.

where x_s is the actual observed choice, x_s^* is the optimal allocation for a given allocation question i and Φ stands for the cumulative distribution function (cdf) of the Beta distribution. We consider the remaining two conditional states in a symmetric way. The likelihood function to maximise is defined as:

$$\ln(\mathcal{L}(r,\alpha,\pi,\sigma,X)) = \sum_{i=1}^{60}\sum_{j=1}^{3} g_i(r,\alpha,\pi,\sigma,X)) \tag{5}$$

We then jointly estimate the values of the parameters by maximising Equation 5 using Maximum Likelihood Estimation techniques. As the problem is quite complex in nature, it is expected that the likelihood surface will not be smooth and consequently, a global maximum will not be easy to reach. To ensure that the solution is not trapped to a local optimum, and that we instead reach a global one, we use a general nonlinear *augmented Lagrange multiplier* optimisation routine that allows for *random initialisation* of the starting parameters as well as multiple restarts of the solver.[41] We conclude this section by commenting on the number of parameters for all the specifications, as well as on the lower and upper bounds that we apply in our estimation codes. For the SEU specification there are four parameters to estimate, the coefficient of risk attitude r, the subjective beliefs for two out of the three states π_R and π_B and the precision parameter σ. For the α-MEU specifications, we need to estimate on top of r and σ, the set of non-additive priors π (the lower bounds) for the three states and the coefficient of ambiguity attitude α, giving in total 6 parameters. As was mentioned before, we assume either risk aversion or risk neutrality therefore, $r \geq 0$. The set of non-additive beliefs should satisfy the constraint $\underline{\pi}(R) + \underline{\pi}(B) + \underline{\pi}(Y) \leq 1$ and α is constrained to the interval [0,1], with 0 expressing extreme ambiguity seeking and 1 extreme ambiguity aversion.

7 Results

Before presenting the results, it is important to stress the fact that all the analysis, directly depends on the structural assumptions concerning the functional forms and the stochastics, as those presented in section 6. In addition, an assumption that is implicitly made is that the type of the subjects remains stable during the experimental session and the same holds for their preferences.

To obtain a general idea of our results, we first plotted the portfolios of the subjects for each of the conditional states. Figure 2 illustrates the choices for three[42] subjects for all the questions

[41] The estimation was conducted using the *R* programming language for statistical computing (The *R* Manuals, version 3.0.2. Available at: http://www.r-project.org/). For the multiple-restarts routine, the package *rsolnp* (Ghalanos and Theussl (2012)) was used. The estimation codes are available upon request.

[42] The full set of scatter plots, for all the conditional states and all the subjects is available upon request.

where the information $\neg B$ was revealed. The horizontal (vertical) axis represents the payoff if the ball is yellow (red). The 45° line stands for all the portfolio allocations that guarantee the same payoff, regardless the actual state of the world[43]. The hollow (solid) dots correspond to portfolios at period 0 (1). Two points in this Figure worth noticing. First, it is apparent that there is extensive violation of DC, since for a dynamically consistent agent, the portfolio allocations for the two periods should coincide. On top of that, these violations do not seem to follow a uniform pattern, indicating the existence of a variety of planning types.[44] Along all the datasets, we find extensive heterogeneity, with subjects' choices sharing similarities with one of the three types shown in Figure 2, fact that calls for further structural investigation.

For each of the 58 participants, we fitted all the possible types that we described in section 4.[45] For each subject and for each type, we have estimates of their subjective beliefs, the coefficient of risk and ambiguity attitudes (r and α), the precision parameter σ and the value of the *maximised log-likelihood*. Based on the value of the maximised log-likelihood, we can detect which type best explains data (provides the best fit to the data) for each subject and therefore, classify subjects to different types. The first column in Table 2 reports the mean and the standard deviation of the fitted log-likelihoods across all subjects. On average, the likelihood is highest for the sophisticated type. Nevertheless, since SEU has 2 degrees of freedom less compared to α-MEU, every α-MEU specification is bound to perform at least as well as SEU due to *overfitting*. Thus, this comparison is meaningful only when we compare across the three types. To take this difference into consideration, we correct for the degrees of freedom by calculating both the Bayesian Information Criterion (BIC), that controls for the different number of parameters and the Akaike Information Criterion (AIC), that accounts for both the number of the parameters and the size of the dataset[46]. The values of AIC and BIC are reported in columns 2 and 3 respectively, in Table 2. On average, it seems that the sophisticated type is the best, followed by the naïve, the resolute and then the SEU.

In Table 3 we use the values of the maximised log-likelihood, the AIC and the BIC, to classify types at the individual level. The first column of the Table reports the classification based on the fitted log-likelihood. As expected, SEU has always the worst performance. For the rest of the types, the sophisticated is the best for 50% of our subjects, followed by the naïve and the resolute with roughly the same proportions. Columns 2 and 3 report the same

[43] An extremely risk averse agent would always choose portfolios along this line.

[44] Indeed, subsequent econometric analysis confirmed that the left panel in Figure 2 belongs to a resolute subject (subject 13), the middle to a naïve (subject 17) and the right to a sophisticated one (subject 27).

[45] The subject-level analysis created a large dataset that contains the estimated parameters for each individual and for all the specifications. We do not report it here, but full details are available upon request.

[46] $BIC = -2\ln(L(\hat{\theta}|x)) + k\ln(n)$, $AIC = -2\ln(L(\hat{\theta}|x)) + 2k$ where $ln(L(\hat{\theta}|x))$ is the value of the maximised log-likelihood, k is the number of the free parameters in the model and n the number of observations. As is the case with the value of the log-likelihood, a lower value indicates a better fitting.

information based on the corrected log-likelihoods. When AIC is used to interpret the data, the prominent type is the sophisticated one (34%), followed by the SEU (28%), the naïve (21%) and the resolute (17%). However, when BIC is used, the majority of the subjects are classified as SEU (53%), followed by the sophisticated type (24%) and a minority of naïve (12%) and resolute (10%) DMs. Depending on the two information criteria, it seems impossible to make a safe inference regarding the classification of the types. Hence, we proceed by testing the statistical significance of the difference between the values of the fitted log-likelihoods of the different types. In other words, we test whether the maximised log-likelihood for the best-fitting type, is significantly higher compared to SEU. To this end, we conduct *likelihood ratio tests*. These tests have been used to compare two *nested* models where the null model is a special case of the alternative model[47]. The test statistic is given by the ratio of the two fitted likelihood functions

$$LRT = -2\ln\left(\frac{\mathcal{L}_s(\hat{\theta})}{\mathcal{L}_g(\hat{\theta})}\right)$$

where $\mathcal{L}_s$ is the maximised likelihood of the simpler model (the nested model) while $\mathcal{L}_s$ is the maximised likelihood of the general model (the nesting model). The LRT statistic follows a *Chi-square* distribution with degrees of freedom $df_g - df_s$, with df_g and df_s being the number of free parameters for the nesting and the nested model respectively. With 4 parameters of the SEU and 6 of the α-MEU, the test statistic is distributed with 2 degrees of freedom. Table 4 reports the classification of the subjects to types, based on the significance of the LRT. SEU best describes behaviour for 53.4% (46.5%) of our experimental population at 1% (5%) level of significance. For the remaining non-SEU population, the majority can be classified as sophisticated 51.9% (52.6%), followed by the naïve DMs 25.9% (25.8%) and the resolute 22.2% (22.6%). This finding contradicts Hey and Panaccione (2011) who find that a significant proportion of DMs are resolute. Nevertheless, they study dynamic choice in risky environments, using a different decision task, so it is not possible to directly compare the results.

Finding 1. *For the majority of the subjects, we cannot reject the null hypothesis at 1% significance level, that they are behaving according to the SEU model and therefore comply with Bayesian updating. Focusing on the non-SEU subjects, the sophisticated type best explains behaviour for more than half of the population, followed by the naïve and the resolute type.*

The above classification provides evidence to the question of which axiom do the subjects satisfy when they tackle dynamic decisions under ambiguity. When we consider the non-SEU

[47]Two models are nested, if the first model can be transformed into the second model by imposing constraints on the parameters of the first model. In our framework, when the beliefs are additive, the α-meu is transformed to the SEU, so the SEU model is nested within the α-MEU.

subjects, 77.4% of the subjects satisfy C, while only 22.6% satisfy DC[48], which confirms the results in Dominiak et al. (2012). In the total population, 53.4% (46.5%) satisfy both DC and C, 10.3% (12.1%) satisfy only DC and 36.2% (41.4%) satisfy C.

Finding 2. *Less than 1/4 of the experimental population with non-neutral ambiguity attitudes satisfies DC, while the vast majority satisfies C. In the total experimental population, more than half of the participants satisfy both axioms.*

We now turn to the estimates of our structural models. Table 5 reports a summary of the mean and the standard deviation of the estimated values of the parameters, for all the specifications and types. We also report the median, especially for the risk aversion and the precision parameter, as the existence of extremely risk averse subjects or subjects with high levels of precision, inflates the value of the average[49]. On aggregate, there is extensive heterogeneity regarding the values of the parameters. We illustrate this by providing the density plots for the parameters under investigation. Figure 3 shows the distribution of the risk aversion coefficient which confirms the lack of a uniform level of risk aversion, a commonly observed pattern in experiments of choice under risk. Figures 4, 5 and 6 show the distribution of the estimated subjective probabilities for the blue, red and the yellow states respectively, for all the types (the vertical dashed line indicates the objective probability of each state). Two aspects in these distributions worth noticing. First, it seems that the distribution of the estimated beliefs when SEU is assumed, is characterised by less fat tails compared to the non-SEU types. Then, when the value of subjective beliefs is compared to the objective probabilities that were actually applied during the experiment, it seems that subjects over-estimate low probability events and under-estimate high probability events. Evidence for this finding is provided by both Table 5 and Figures 4-6. In all four cases, both the median and the average of the low probability event (B) is significantly higher compared to the actual one (the distribution in Figure 4 is skewed to the right). Similarly, the estimates for the high probability event (Y) are significantly lower compared to the objective probability (the distribution in Figure 6 is skewed to the left). This result is in line with similar findings with a commonly observed over (under)-weighting of low (high) probability events, confirming the existence of *likelihood insensitivity*[50]. Various experiments have demonstrated the existence of this insensitivity in both student and general

[48] The results are identical at both 1 and 5% level of significance.

[49] In theory, the value of risk aversion for an extremely risk averse subject tends to infinity. In our estimation program, we set the upper bound for risk aversion equal to 10 and for the precision parameter 100. The logic behind this choice is that when risk aversion is significantly high, it is impossible to behaviourally distinguish choices (allocations tend to equalise payoffs at every state). For the precision parameter, we set it sufficiently high, so that it can accommodate behaviour for most of the subjects.

[50] As is explained in Trautmann and van de Kuilen (2015), likelihood insensitivity appears when people cannot distinguish between events bounded away from zero and one and transform subjective likelihoods towards fifty-fifty, resulting to an over-weighting of unlikely events and under-weighting of highly likelihood events.

populations, all in static frameworks (see among others Wakker (2010) and Abdellaoui et al. (2011)). The present study, verifies the existence of this component of ambiguity attitudes, in dynamic choice frameworks. Finally, Figure 7 illustrates the distribution of the sum of subjective beliefs for each of the three types[51] since the family of multiple-prior models is based on the assumption of non-additive beliefs. The Figure confirms the existence of non-additivity showing that for the majority of the subjects, the sum of beliefs is distributed in the interval [0.80-1). This finding is in line with the results in Baillon and Bleichrodt (2015) who report extensive violation of probabilistic sophistication.

Finding 3. *There is a systematic over-weighting of the low probability event and similarly, an under-weighting of the high probability event.*

Based on the above estimations, we can classify subjects according to their attitudes towards ambiguity. Notice that all SEU subjects are automatically classified as ambiguity neutral. For the non-SEU subjects, the classification is based on the value of the α parameter, where for $\alpha > 0.5$ the subject exhibits ambiguity averse attitude, while for $\alpha < 0.5$ ambiguity seeking. For the classification of the non-SEU subjects, we consider the estimated value of α for the best fitting type. 53.4% (46.6%) of the subjects are characterised as ambiguity neutral, 24.1% (27.6%) ambiguity seeking and 22.4% (25.9%) ambiguity averse at 1% (5%) level of significance respectively. These results are in line with Charness et al. (2013), Hey and Pace (2014), Ahn et al. (2014) and Stahl (2014), all accounting for ambiguity attitudes in static frameworks.

Finding 4. *For more than half of the population we cannot reject the null hypothesis of neutral ambiguity attitudes and therefore, SEU preferences. For the non-SEU agents, ambiguity seeking and averse attitudes are observed in roughly equal proportions.*

At this point we have classified the subjects based on the significance of the LRT. In order to provide an indication of how much better the best-fitting type is, relative to the others, we adopt a *Bayesian* approach and we calculate the *posterior* probabilities of the resolute, naïve and sophisticated types being the actual ones, assuming that the *ex-ante* probability was equal to 1/3 for each type. Denoting by $ll(r), ll(n)$ and $ll(s)$ the fitted log-likelihood values for the resolute, naïve and sophisticated type respectively, the posterior probability that type i is the correct one, is given by

$$P(i) = \frac{\exp(ll(i))}{\exp(ll(r)) + \exp(ll(n)) + \exp(ll(s))} \tag{6}$$

with $i \in \{$r,n,s$\}$. To illustrate this with an example, for subject 8 we obtained the following values for the log-likelihoods: $ll(r) = -424.84, ll(n) = -426.39$ and $ll(s) = -423.59$.

[51] By definition, in the SEU model the beliefs add up to 1.

Substituting these values to Equation 6, the posterior probabilities for the three types are $P(r) = 0.21, P(n) = 0.04$ and $P(s) = 0.74$ indicating that the probability of the subject being sophisticated is almost three times higher than being resolute, while it is quite unlikely that this subject is naïve. Figure 8 presents these posteriors graphically. Since we consider three types, it is possible to represent the probabilities with a triangle. The horizontal (vertical) axis represents the probability of the sophisticated (resolute) being the correct type while the probability of the naïve is simply the residual. The triangle is divided in three equally-sized areas, with the top one being where the resolute type is most probable, the bottom-left being where the naïve is the most probable and the bottom-right indicating the area where the sophisticated is the most probable. Figure 8a illustrates the posteriors for all the participants, while Figure 8b only for the subjects with non-neutral attitudes. Notice that in the first case (all subjects), there is both concentration towards the vertices of the triangle, where the probability of being type i is maximised, and concentration around the middle point which corresponds to being one of the types with equal probability. When we focus only on the ambiguity non-neutral subjects, the posteriors clearly tend towards the vertices, confirming the robustness of our results. We conclude by noting that there is a significant concentration of posterior probabilities in the neighborhood of 1 for the naïve and the sophisticated type, indicating that these types perform significantly better compared to the resolute type.

Finally, since we have estimated parameters at the individual level, we investigate whether there is any kind of correlation between risk and ambiguity attitudes. A similar test is meaningful only for the subset of subjects with non-neutral attitude towards ambiguity. For this set, we use the estimated parameters of risk and ambiguity attitude of the best fitted type for each individual. Using a Pearson product-moment correlation test, we find that there is virtually no correlation between the two measures (ρ=-0.065, p-value = 0.727). This result confirms the findings in Cohen et al. (2011) who find no correlation between the risk and ambiguity attitudes. Moreover, this finding raises interesting methodological issues that call for further investigation. In the experimental literature of ambiguity preferences, there is still no consensus of whether there is correlation or not between the two measures of attitude. Trautmann and van de Kuilen (2015) provide a review of the various studies that test for the existence of correlation and conclude that the majority of the studies that report positive correlation, are based on elicitation methods that measure risk and ambiguity attitudes separately. Recent experimental evidence suggests that risk elicitation procedures are likely to be highly context-specific (Loomes and Pogrebna, 2014) and therefore, joint elicitation of risk and ambiguity attitudes may be more informative regarding the actual relation between the two.

Finding 5. *There is no significant correlation between risk and ambiguity attitudes.*

8 Conclusion

In this study we report the results of a simple two-period portfolio allocation experiment, where we study heterogeneity in dynamic decision making under ambiguity. Based on which rationality axiom people satisfy in combination with assumptions of their planning strategy, we categorise subjects to resolute, naïve and sophisticated. Our results are summarised as: (1) almost half of the subjects behave according to the SEU model and comply with Bayesian updating; (2) there is extensive violation of dynamic consistency by the non-SEU subjects; (3) the majority of the non-SEU subjects are sophisticated, few are naïve and a few are resolute; (4) ambiguity neutrality is prevalent while ambiguity seeking and aversion are observed in roughly the same proportions and; (5) there is no correlation between risk and ambiguity attitudes.

Our results provide support to those theories that promote the idea of sophistication and therefore, reject the axiom of dynamic consistency as in Siniscalchi (2011). Then support is provided, to a lower degree, to theories that assume naïve updating (Gilboa and Schmeidler (1993), Pires (2002), Eichberger et al. (2007), Eichberger et al. (2010)) and very little evidence is provided in favour of theories that assume dynamic consistency (Epstein and Schneider (2003), Klibanoff et al. (2009)). The importance of the above needs to be highlighted for two reasons. Recent empirical research in dynamic financial decision making based on field data[52], models behaviour by implicitly assuming dynamic consistency (the model that the minority of our subjects comply with) whereas, recent theoretical studies on dynamic asset markets under ambiguity[53], assume heterogeneity in planning strategies and behaviour. Hence, accounting for heterogeneity could potentially provide better insights of how people actually behave in dynamic, ambiguous environments, fact that calls for further empirical investigation. Our paper is a first step towards studying behavioural heterogeneity regarding planning strategies in dynamic environments under ambiguity. Extensions are needed in order to capture behaviour in more complicated environments that include different representations of ambiguity (e.g. natural events), longer time horizons, effects of social interaction or connect it with the *decision from experience* literature[54] as well as with the time preferences literature.

[52] See Thimme and Völkert (2015), Jeong et al. (2015).
[53] See Easley and O' Hara (2009), Mele and Sangiorgi (2015).
[54] See Ert and Trautmann (2014).

Table 1: Prior beliefs in the MMT

π	$\pi(R)$	$\pi(B)$	$\pi(Y)$
A	$1-\underline{\pi}(B)-\underline{\pi}(Y)$	$\underline{\pi}(B)$	$\underline{\pi}(Y)$
B	$\underline{\pi}(R)$	$1-\underline{\pi}(R)-\underline{\pi}(Y)$	$\underline{\pi}(Y)$
C	$\underline{\pi}(R)$	$\underline{\pi}(B)$	$1-\underline{\pi}(R)-\underline{\pi}(B)$

Table 2: Average values of goodness of fit

Type	LL	AIC	BIC
SEU	-451.06	910.11	921.26
	(108.49)	(216.99)	(216.99)
Resolute	-448.31	908.62	925.35
	(108.07)	(216.14)	(216.14)
Naïve	-446.91	905.81	922.54
	(108.6)	(217.21)	(217.21)
Sophisticated	**-444.04**	**900.08**	**916.80**
	(107.5)	(214.99)	(214.99)
Obs	58	58	58

Table 3: Classification based on goodness of fit

Type	LL	AIC	BIC
SEU	0	16	31
%	(0)	(0.28)	(0.53)
Resolute	14	10	6
%	(0.24)	(0.17)	(0.10)
Naïve	15	12	7
%	(0.26)	(0.21)	(0.12)
Sophisticated	**29**	**20**	**14**
%	(0.50)	(0.34)	(0.24)
Total	58	58	58

Table 4: Classification based on LRT significance

Type	Number of subjects with highest LL	Significantly different from SEU at 1%	Significantly different from SEU at 5%
SEU	0	31	27
Resolute	14	6	7
Naïve	15	7	8
Sophisticated	**29**	**14**	**16**
Non-EU	58	27	31
Total	58	58	58

Table 5: Summary of Estimates

Parameter		SEU	Resolute	Nave	Sophisticated
	Mean	0.279	0.255	0.242	0.225
π_B	Median	0.295	0.283	0.272	0.257
	St. Dev	(0.084)	(0.092)	(0.107)	(0.104)
	Mean	0.372	0.347	0.335	0.337
π_Y	Median	0.357	0.338	0.334	0.339
	St. Dev	(0.068)	(0.058)	(0.081)	(0.102)
	Mean	0.348	0.321	0.298	0.329
π_R	Median	0.335	0.324	0.317	0.320
	St. Dev	(0.07)	(0.087)	(0.107)	(0.082)
	Mean	1.486	1.428	1.422	1.268
r	Median	0.882	0.841	0.849	0.915
	St. Dev	(1.94)	(2.021)	(1.966)	(1.596)
	Mean	-	0.437	0.491	0.527
a	Median	-	0.208	0.475	0.530
	St. Dev	-	0.411	0.408	0.379
	Mean	31.854	32.187	32.557	32.783
s	Median	15.773	15.816	15.370	16.370
	St. Dev	(35.074)	(35.073)	(35.199)	(35.107)

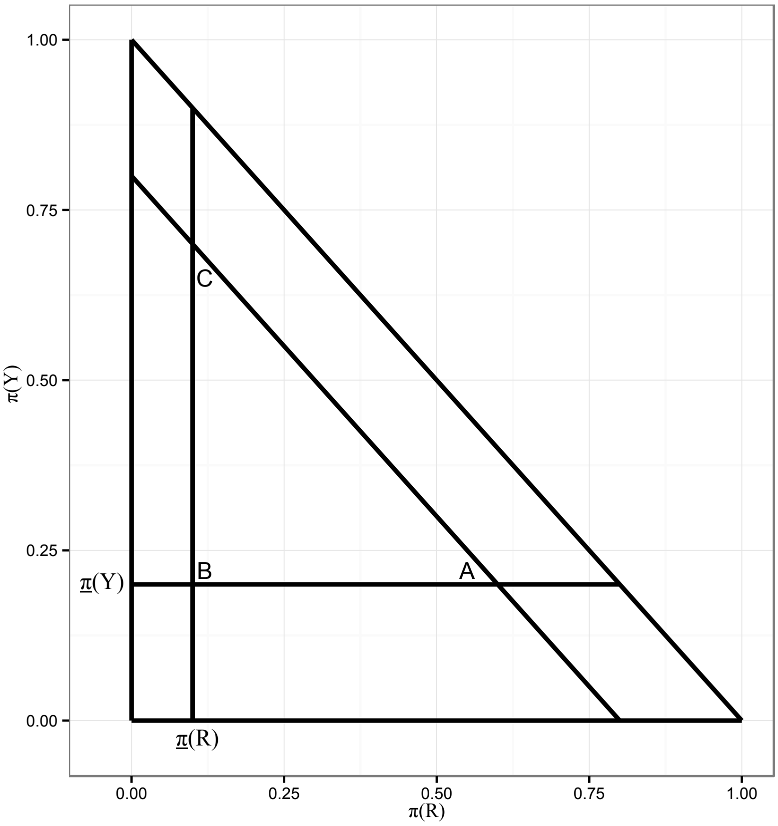
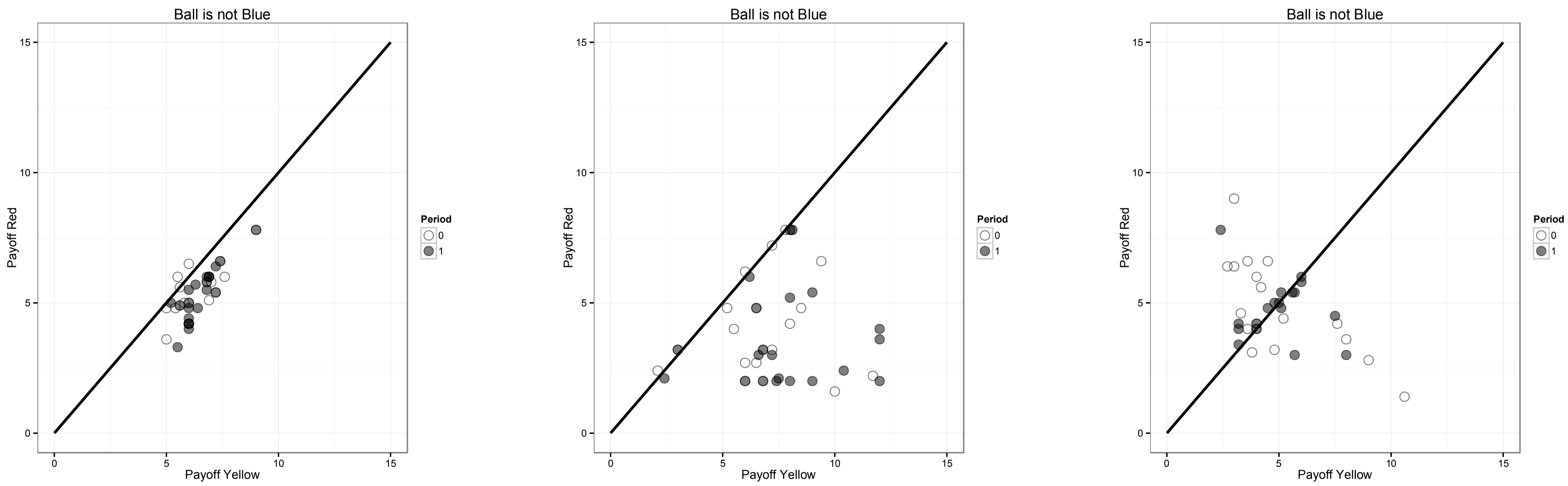


Figure 1: Prior Beliefs



(a) Subject 13 (b) Subject 17 (c) Subject 27

Figure 2: Scatter-plots of Portfolios

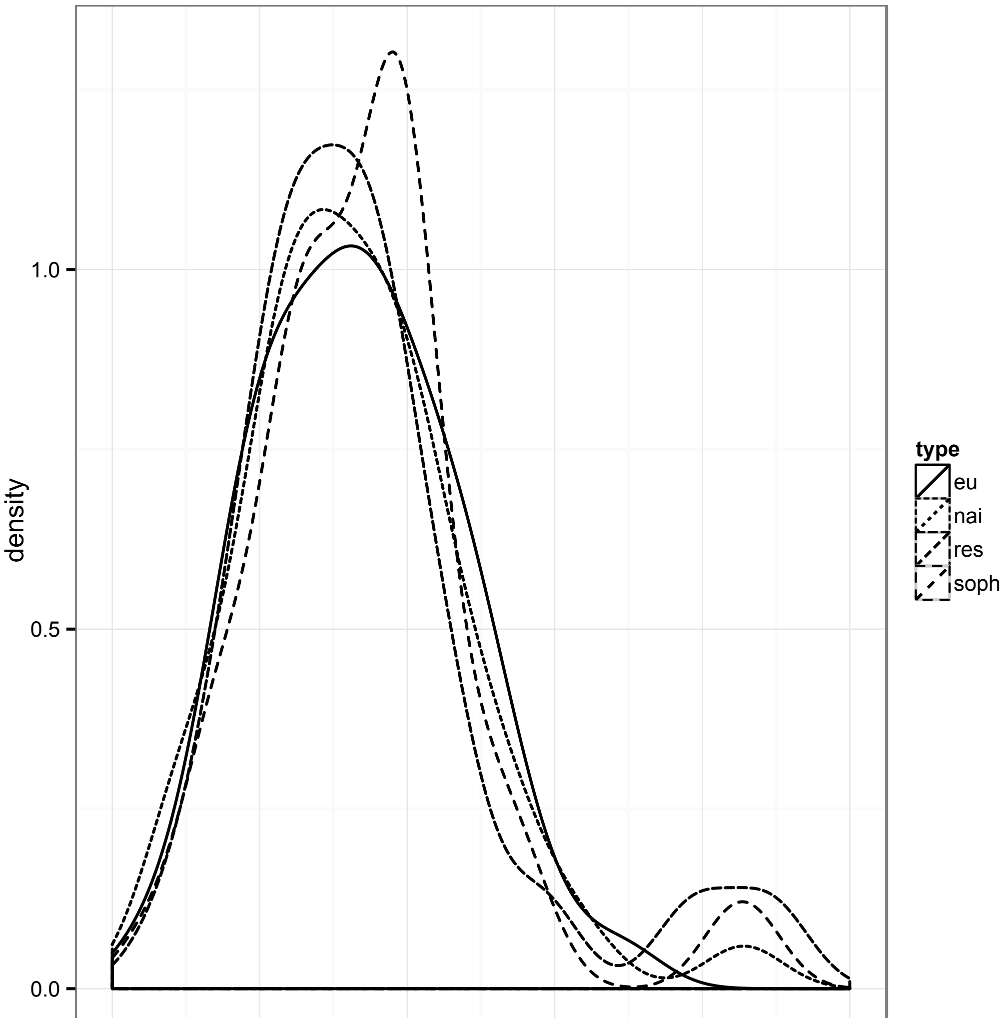


Figure 3: Kernel Distribution of the Estimated Risk Aversion Parameter

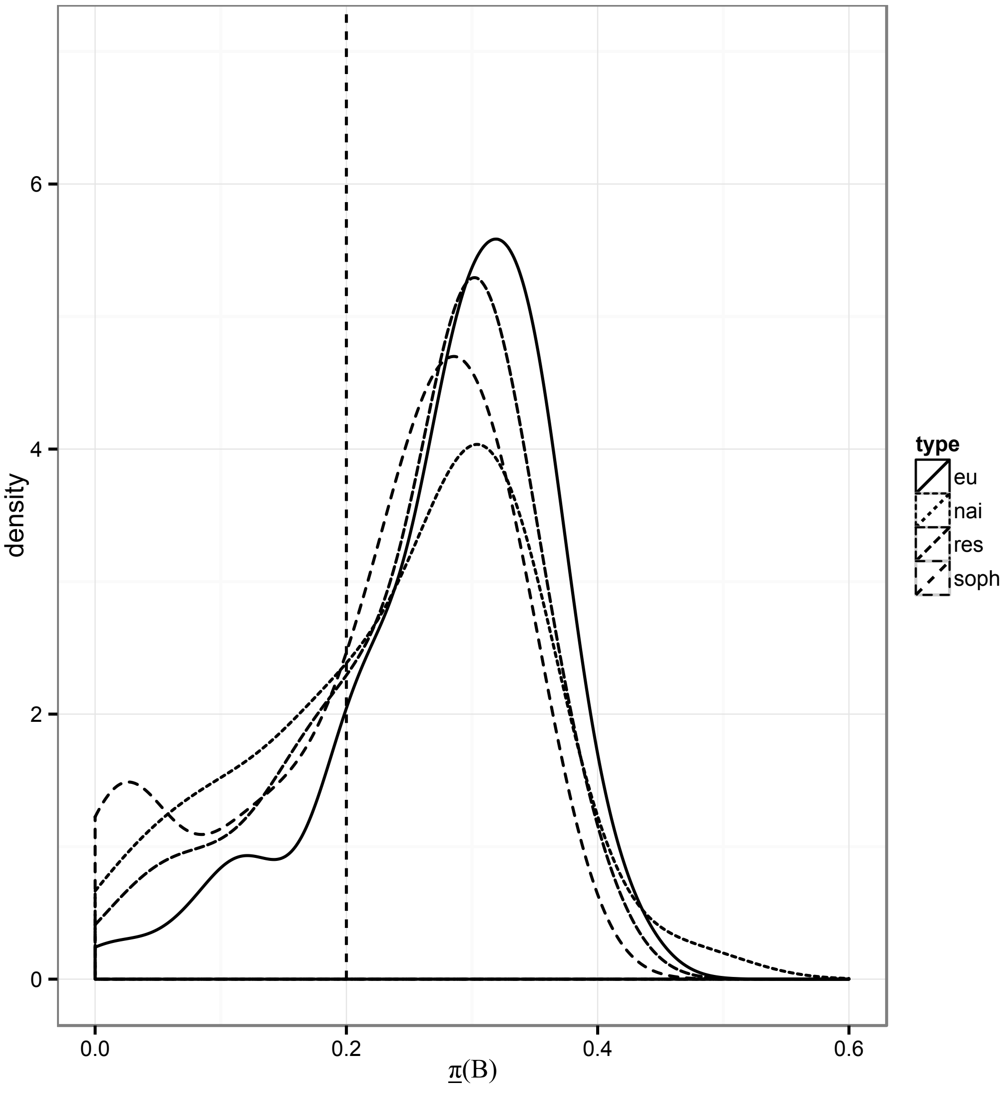


Figure 4: Kernel Density of Estimated Beliefs for the blue Asset (actual=0.2)

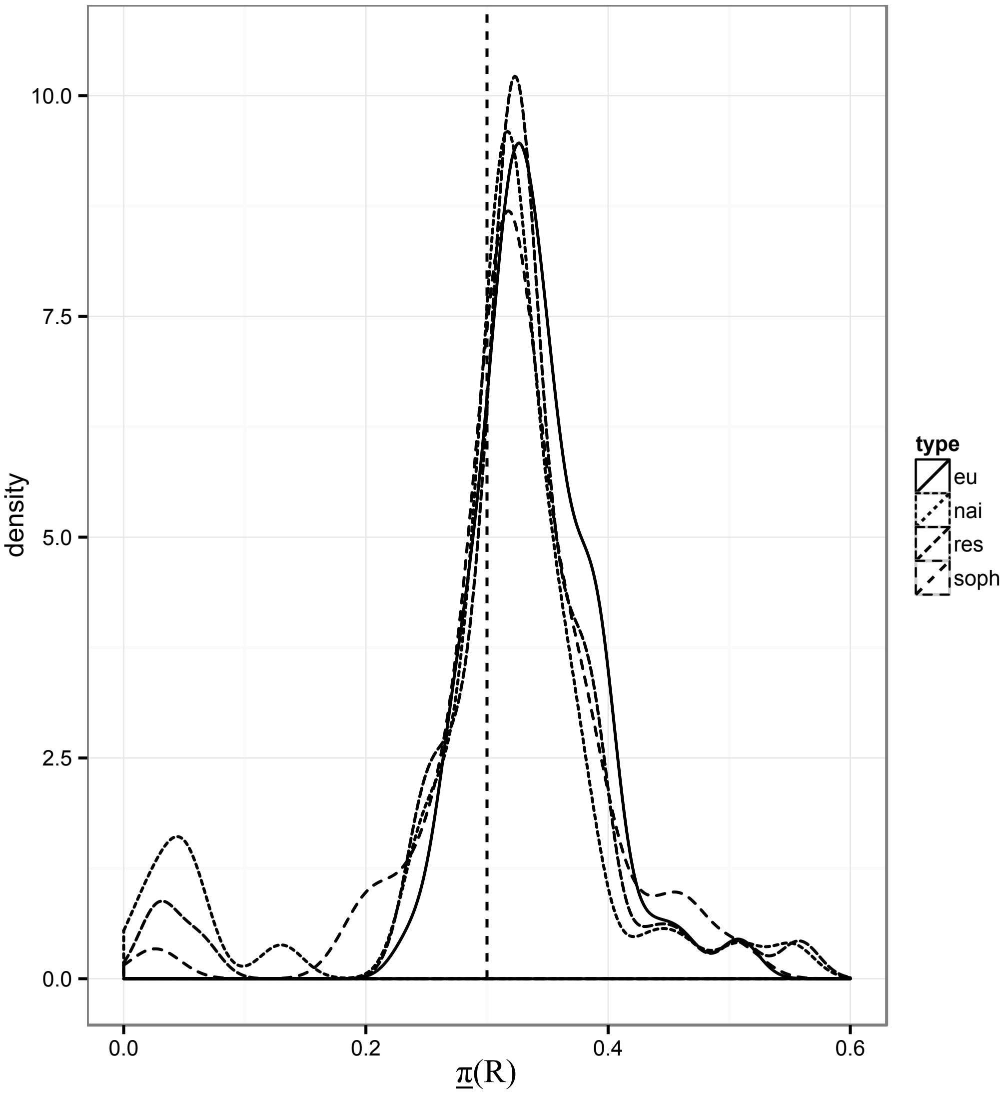


Figure 5: Kernel Density of Estimated Beliefs for the red Asset (actual=0.3)

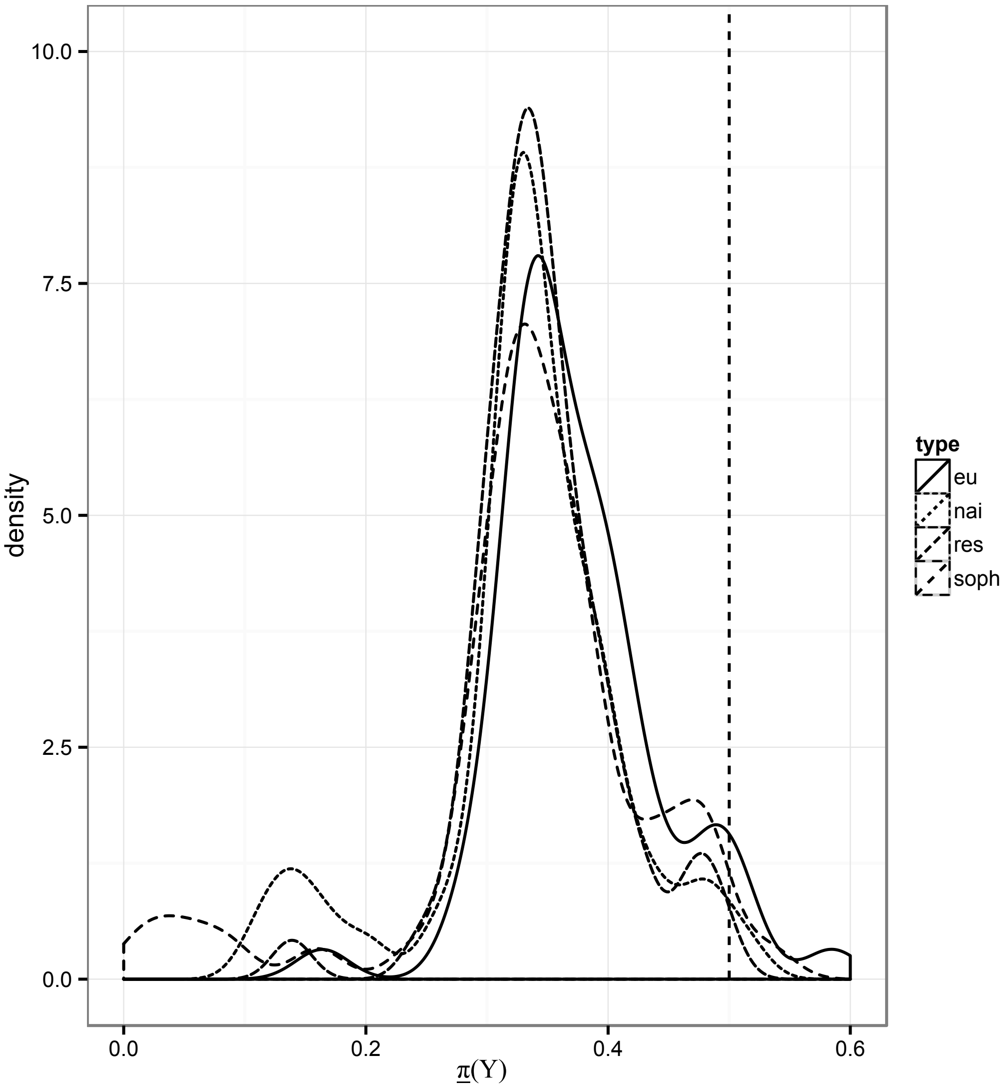


Figure 6: Kernel Density of Estimated Beliefs for the yellow Asset (actual=0.5)

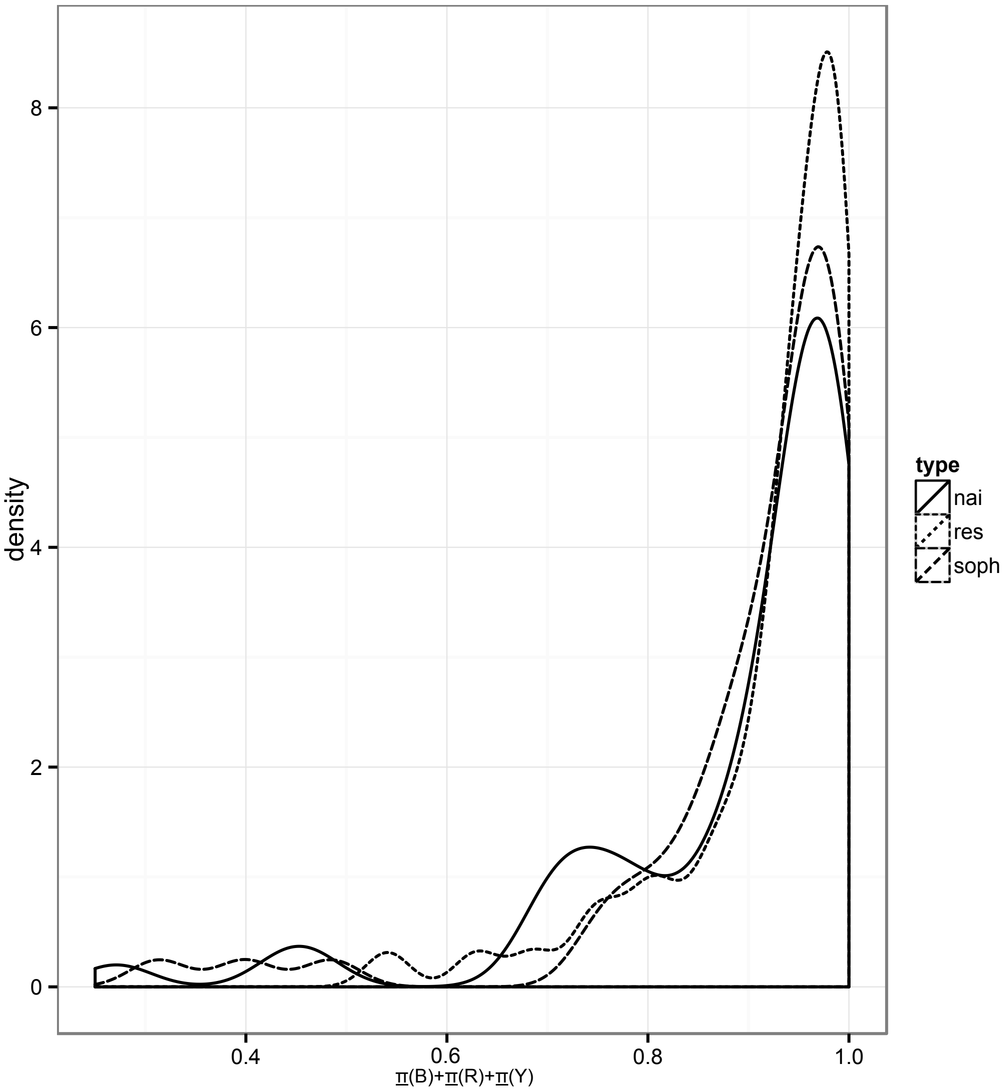
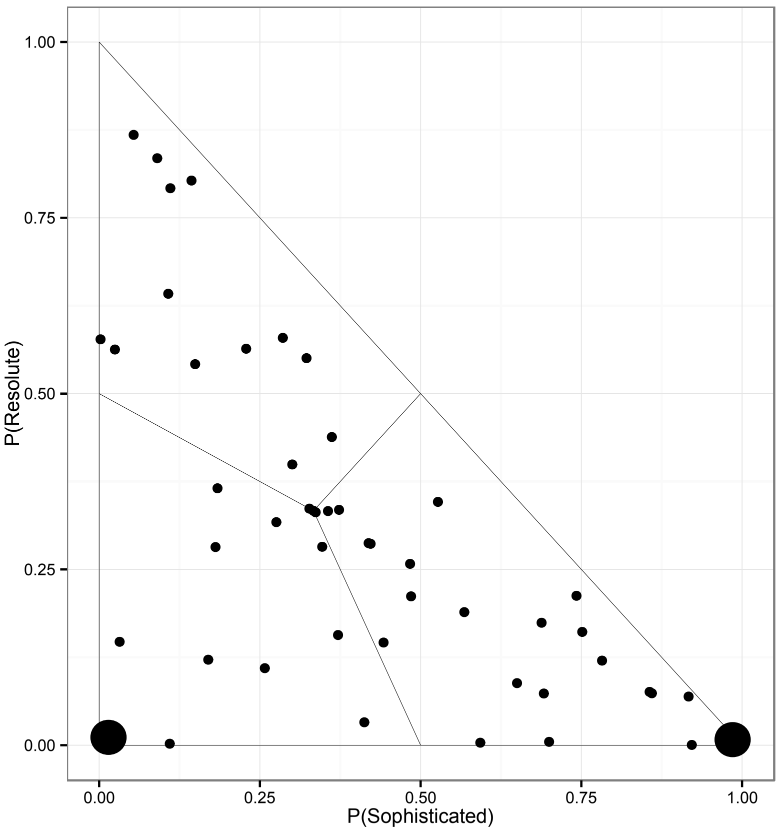
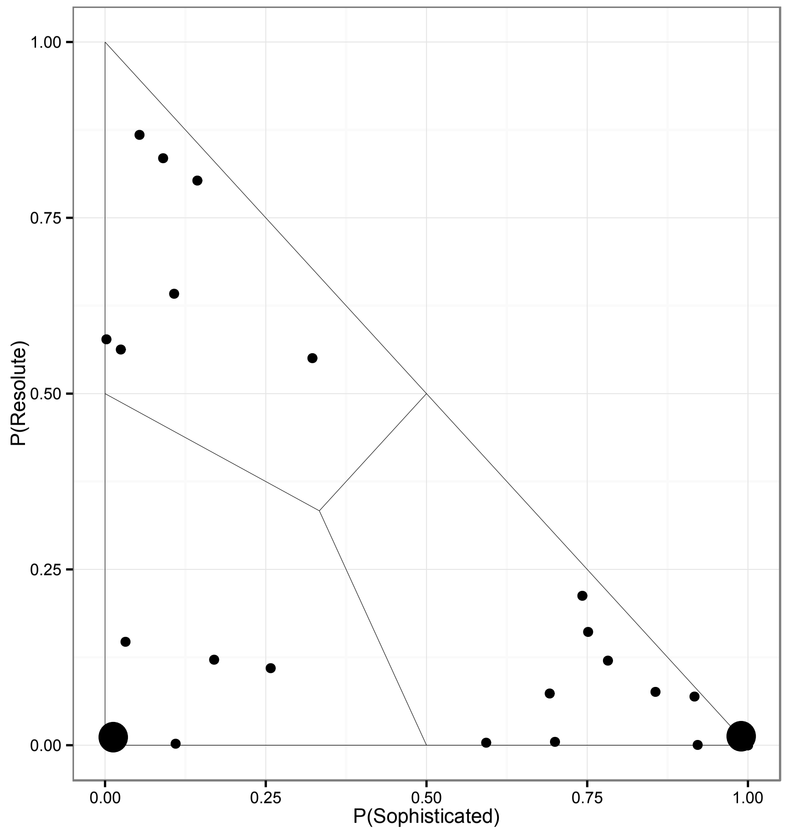


Figure 7: Kernel Density of the Sum of the Estimated Beliefs



(a) (all subjects)



(b) (non-EU subjects)

Figure 8: Posterior Probabilities

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