Confidence intervals for a spatially generalized, continuous simulation flood frequency model for Great Britain

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[1]   There is growing interest in the application of "continuous simulation" conceptual rainfall-runoff models for flood frequency estimation as an adjunct to event-based or statistical design methodology. The approach has advantages that stem from the use of models with continuous water balance accounting. Conceptual rainfall-runoff models usually require calibration, which in turn requires gauged rainfall and flow data. One of the key challenges is therefore to develop ways of generalizing models for use at ungauged sites. Recent work has produced a prototype scheme for achieving this aim in Great Britain for two catchment models by relating model parameters to spatial catchment properties, such as soils, topography, and geology. In this paper we present an analysis of the uncertainty associated with one of the generalized models (the "probability distributed model") in terms of confidence intervals for simulations at test sites that are treated as if they were ungauged. This is done by fitting regression relationships between hydrological model parameters and catchment properties so as to estimate the parameters as distribution functions for the ungauged site case. Flood flow outputs are then simulated from the parameter distributions and used to construct approximate confidence intervals. Comparison with gauged data suggests that the generalized model may be tentatively accepted. Uncertainty in the modeled flood flows is often of a similar order to the uncertainty surrounding a more conventional statistical model, in this case a single-site generalized Pareto distribution fitted to the gauged data.   *INDEX TERMS:* 1821 Hydrology: Floods; 1860 Hydrology: Runoff and streamflow; 9335 Information Related to Geographic Region: Europe; *KEYWORDS:* Britain, continuous simulation, flood frequency, uncertainty

Citation: Lamb, R., and A. L. Kay (2004), Confidence intervals for a spatially generalized, continuous simulation flood frequency model for Great Britain, *Water Resour. Res.*, *40*, W07501, doi:10.1029/2003WR002428.

1.   Introduction

[2]   This paper explores the calculation of uncertainty in modeled estimates of river flows, and, in particular, of flood frequencies. The "continuous simulation" (CS) approach to modeling flood frequency is being researched in the UK as a future method for routine flood risk assessment [*Calver et al.*, 1999]. Continuous simulation is based on modeling long time series of river flow data, typically using a discrete time step conceptual rainfall-runoff model. Flood peaks may then be extracted and used, in effect, as a proxy data set to develop derived flood frequency curves.

[3]   The CS method has a number of advantageous hydrological features. In comparison with the event-based design hydrograph approach that has been applied in engineering practice for many years, continuous simulation does not require arbitrary base flow separation, there is no need for calculation of storm antecedent conditions (because of adopting continuous water balance accounting), and

rainfall design storms do not have to be constructed for particular return periods associated with a design flood estimate. The CS approach is also flexible, in that any chosen quantity can potentially be derived from the modeled flow series, for example peak flows averaged over different time step lengths, accumulated volumes, periods exceeding a certain flow threshold, or, indeed, measures of interest for low-flow analysis as well as for flood risk.

[4]   Early work demonstrating the potential to use continuous simulation for flood frequency analysis focused on the outputs of calibrated rainfall-runoff models at gauged catchments [*Bras et al.*, 1985; *Beven*, 1987]. However, a key requirement for a general method is the ability to model a range of catchments of different scale and type. Crucially, practical applications may also include ungauged sites, where measurements of river flows are not available and model calibration is therefore not possible. Parameterizing a model to meet these requirements can be thought of as "spatial generalization".

[5]   Progress in spatial generalization of the CS method in Great Britain has been reported by *Calver et al.* [1999] and

Lamb et al. [2000a, 2000b], who used linear regression to fit relationships between CS model parameters (as calibrated at gauged sites) and physically based catchment properties (e.g., indices describing soils, topography and vegetation). The work reported in this paper adopts the same basis for spatial generalization, but extends the analysis by investigating uncertainty in the generalized modeling. Estimates of uncertainty, to be characterized in this case by approximate confidence intervals about derived flood frequency curves, are motivated by three main factors: There is an imperative for knowledge of uncertainty in planning and policy guidance (for example, *Department for Environment, Food and Rural Affairs (DEFRA)* [2001] in the UK), there is scope to use confidence intervals as a measure of precision of the generalized CS method, and such intervals have the potential to be interpreted heuristically as analogues for significance tests to guide acceptance or rejection of the generalized CS parameterizations.

2. Approach

[6] Attempts have been made to generalize conceptual hydrological models in a number of studies, not necessarily motivated by flood estimation [e.g., *Abdulla and Lettenmaier*, 1997; *Post et al.*, 1998; *Sefton and Howarth*, 1998]. Spatial generalization of model parameters remains a difficult problem for which there is, arguably, no standard methodology. However, a typical approach is as follows.

[7] 1. Select a sample of n gauged catchments representing a range of physiographic characteristics and with good quality rainfall and flow records extending over several years.

[8] 2. For each catchment k ($k = 1, \ldots, n$), collate a range of p catchment properties, $\mathbf{x}_k = (x_{1,k}, x_{2,k}, \ldots, x_{p,k})$, that describe fixed or relatively stationary characteristics, such as topography, geology, soils and stream network topology. Selected properties should be available geographically as widely as possible.

[9] 3. Calibrate the vector of m parameters, $\boldsymbol{\theta}_k$, of a chosen CS hydrological model at each catchment using the observed rainfall and flow data.

[10] 4. Treat the calibrated parameters as if they are "observations" and fit, for each parameter individually, a regression relationship of the form $\theta = M(\mathbf{X}) + \varepsilon$ between the calibrated parameters and a subset of catchment properties, $\mathbf{X}$.

[11] 5. Use the fitted regression equations to estimate model parameters at ungauged locations, nationally or within specified regions.

[12] Using an empirical technique (in this case least squares regression) to estimate hydrological model parameters from catchment properties will introduce a degree of uncertainty. The main focus of this paper is on implications for flood estimates of the uncertainty associated with step 4 and, consequently, expected in step 5.

[13] There is also uncertainty involved in the calibration of hydrological model parameters against gauged flow data. Reasons for this include the difficulty of measuring what constitutes the "best fit" and the numerical interaction between parameters, such that sets of different values can often provide very similar hydrological simulations [e.g., *Beven*, 1993; *Duan et al.*, 1992]. The problems of calibration uncertainty have received considerable attention

in the literature; see, for example, *Sorooshian and Dracup* [1980], *Kuczera* [1983], *Beven and Binley* [1992]. *Lamb* [1999] investigated the calibration of the Probability distributed model (PDM) hydrological model, also used in this paper, and found that combining measures of the model fit to flood peaks and flow duration curves could help to constrain the calibration. To enable us to concentrate in this paper on the analysis of "generalization uncertainty", the issue of calibration uncertainty has not been examined in detail. It has instead been assumed for simplicity that model parameters can be fitted to flow data at gauged sites to produce point estimates that have some practical utility. Calibration uncertainty has been reflected only in terms of the variance of the calibrated values within the regression model for each parameter. However, suggestions are also made about the extension of the approach to incorporate calibration uncertainty more explicitly.

[14] Although the parameters of conceptual hydrological models are mostly designed to be physically meaningful, relating them to corresponding physical properties can be very difficult in large part because of nonuniqueness reflected in the calibration uncertainties. This issue has been noted by *Fernandez et al.* [2000], who proposed a regional calibration approach to simultaneously fit a monthly water balance model to gauged data and catchment properties. *Cameron et al.* [1999] also investigated uncertainty in the calibration of the conceptual rainfall-runoff model TOPMODEL [*Beven and Kirkby*, 1979; *Beven et al.*, 1995] for use in flood frequency estimation. In their work, TOPMODEL was combined with a stochastic model for area-averaged rainfall, parameterized at each study catchment using rainfall data derived by *Lamb and Gannon* [1996], and the combined models were run within the GLUE [*Beven and Binley*, 1992] Bayes Monte Carlo framework.

[15] The use of a rainfall model allows long synthetic simulations to be run, from which estimates of rare floods can be derived. Estimates of the 100-year flood have been derived in this way by *Calver and Lamb* [2001] for the set of catchments used in this paper. In this study, we concentrate solely on the spatial generalization of hydrological model parameters rather than extension of the continuous simulation approach to model rare events using simulated rainfall. The main reasons for this are to avoid having to incorporate potential errors from rainfall modeling and because there are very few (if any) gauged flow records long enough to support objective testing of simulated flood frequencies at the longer return periods.

3. Data and Study Sites

[16] Rainfall and river flow data from 40 gauged catchments in Great Britain were used for model calibration. The calibration data are at an hourly time resolution. This was selected as a practical compromise for use over the set of catchments ranging in scale from 1 km^2 to 532 km^2 (mean 156 km^2) and with typical response times of 3 to 24 hours (mean 8 h), as indicated by the unit hydrograph time to peak (defined for British catchments according to *Institute of Hydrology* [1999] variable $T_p(0)$). The location of the calibration sites is shown in Figure 1.

[17] There are on average 9 years of continuous hourly rainfall and river flow data at each site. In total, the data set comprises more than 300 station years of continuous hourly
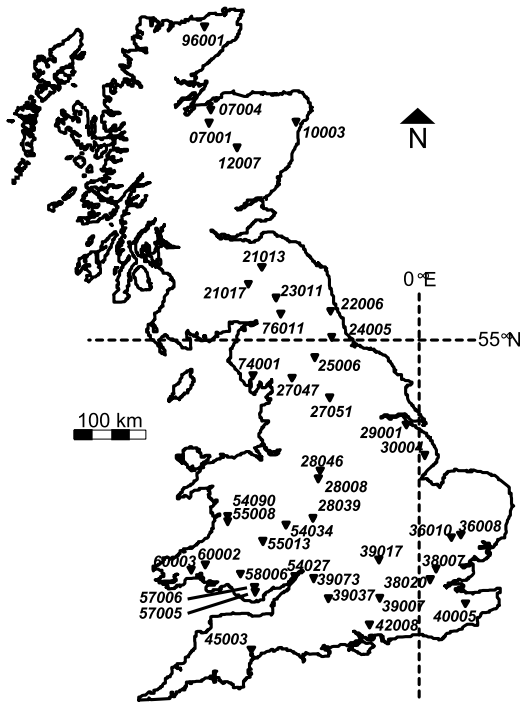
Figure 1. Location of hourly gauged catchments used in this study.

data. Daily average potential evaporation (PE) series were derived from UK MORECS [*Thompson et al.*, 1981] synoptic site data.

[18] Spatial data describing catchment properties were derived from a number of sources, and included variables available digitally on a 1 km grid over Britain. Some of the catchment properties were derived from the hydrology of soil types (HOST) soils classification scheme [*Boorman et al.*, 1995] and a comprehensive integrated hydrological digital terrain model (IHDTM) of the UK developed at the Institute of Hydrology, Wallingford. Other properties were derived from geological maps, river network properties and topography. In total, 58 catchment property variables were

available to use as a basis for seeking relationships with calibrated model parameters.

4. Hydrological Model

[19] The hydrological model we have used is the probability distributed model (PDM), developed by *Moore and Clarke* [1981]. The PDM was chosen as typical of the relatively simple model structures that have the potential to be applied effectively for many parts of the UK. It is based on conceptual stores and transfers, as shown in Figure 2, and attempts to represent nonlinearity in the transformation of rainfall to runoff using a probability distribution of soil moisture storage capacity. This distributed soil moisture store determines the time-varying proportion of the catchment contributing to runoff. Runoff is split into "fast" and "slow" pathways, which are routed via parallel storage components. The two pathways are then combined to determine the final simulated streamflow.

[20] It is worth noting that the generic continuous simulation approach does not require any particular rainfall-runoff model, although model complexity may have implications for the ability to generalize successfully to ungauged sites. The PDM is one of two models for which particular experience has been gained in spatially generalized CS flood frequency estimation in Great Britain. The other model used in this context has been the time-area topographic extension (TATE) model of *Calver* [1993, 1996], which is of a similar basic structure. Various other CS hydrological models have also been applied to model flood frequency [see, e.g., *Beven*, 1987; *Bradley and Potter*, 1992; *Bras et al.*, 1985; *Hashemi et al.*, 2000; *Goel et al.*, 2000].

[21] The PDM and its variants have been widely used in many studies and only a brief algorithm description is given here (for a full description of the original theory of the PDM, see *Moore* [1985]). Rain falling during each time step accumulates in the soil moisture store. At any time, there will then be a critical depth (shown in Figure 2) at which the storage capacity, c, is equal to the accumulated depth of water. Parts of the catchment where the storage capacity is less than this critical depth will only be able to store accumulated rainfall to a depth c, and the remainder spills
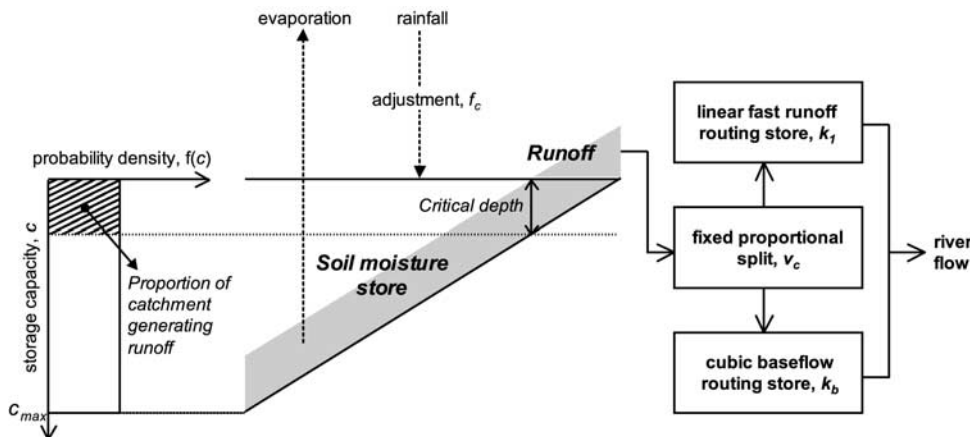

Figure 2. Structure of the simplified PDM catchment model used in this study. The five parameters are a volume adjustment f_c, the base flow routing constant k_b, the maximum soil moisture storage capacity c_{max}, the fast flow routing constant k_1, and the proportional split (v_c) between the base flow and fast flow pathways.

over as runoff. The area of the catchment generating this runoff can be computed by integrating under the probability distribution of storage capacity, as illustrated in Figure 2. For this study, a uniform distribution was assumed because this is a simple form that requires only one parameter to be fitted.

[22] The soil moisture storage is depleted by evaporation as a function of the potential rate and the volume in storage. In the current study, a linear relationship was assumed. The soil moisture store is updated at each time step according to rainfall inputs and evaporation losses. In the current model, the slow response (or "base flow") has been calculated as a fixed proportion of the runoff, which is a simplification introduced to keep the number of parameters small. The slow and fast runoff volumes are routed to the catchment outlet using simple storage-based routing. After initial trials, best results were obtained by using a linear storage function for fast flow routing and a cubic store for slow flow routing.

[23] The PDM, as configured for this study, has five parameters. The water balance parameters are f_c, a constant adjustment factor applied to rainfall inputs and c_{max}, the maximum depth in mm of the uniform soil moisture storage capacity distribution. Parameter v_c is the constant proportional split between runoff entering the fast and slow routing paths. The final two parameters are k_1 and k_b, the constants of the fast flow and slow flow routing stores. Units for k_1 and k_b are hours and (mm^2), respectively.

[24] The formulation is a somewhat simplified version of a more general 7-parameter PDM structure used for flood frequency estimation as described in greater detail by *Lamb* [1999]. The simplification was driven by potential benefit for spatial generalization of reducing the number of parameters in order to produce a robust model. *Wheater* [2002] has noted that this parsimonious modeling approach has allowed progress to be made in regionalization of continuous simulation modeling. While accuracy in some details of hydrograph simulation may be sacrificed, the simpler model has been found [*Lamb et al.*, 2000a, 2000b] to produce improved generalized flood frequency simulations when combined with a spatial generalization technique as described below.

[25] As a further simplification, the split between fast and slow runoff paths, v_c, was specified to be proportional to catchment property HOSTSPR, which is a catchment-averaged estimate of standard percentage runoff derived from soils information. Since v_c was specified in this way as a fixed quantity, rather than being fitted to catchment properties by a regression relationship, it was treated as being known without uncertainty. The four parameters f_c, c_{max}, k_1 and k_b therefore remained to be estimated from relationships fitted to catchment properties.

5. Spatial Generalization Method

[26] The basis of the method we have used for spatial generalization is, as outlined above, to fit regression relationships between calibrated parameters of the PDM and catchment properties using ordinary least squares (OLS). Calibration of the PDM was carried out at each gauged site using a combination of computationally intensive uniform random sampling of a wide parameter space with manual "fine-tuning." The criteria for calibration were a combination of two objective functions. The first objective function measured the summed absolute differences between ranked flood peaks extracted from simulated and observed flow data,

$$O_P = \sum_{i=1}^{n} |q_i - \hat{q}_i|, \qquad (1)$$

where q_i is the magnitude of the ith-ranking extracted peak in the observed flow record, and $\hat{q}_i$ is the ith-ranking peak in the simulated flows. Peaks were extracted as a partial duration series with an implicit threshold such that a total of $3L$ peaks would be available for an L-year period of record. The second objective function O_M was the *Nash and Sutcliffe* [1970] efficiency measure, applied to series of 30-day averages calculated from the modeled and gauged flows.

[27] There is a degree of arbitrariness in deciding how to combine measures. In this case, the overall aim was to maximize the agreement between observed and simulated flood peaks for each gauged site, with the fitting of monthly flows being used as a measure of overall hydrological consistency. However, the volume adjustment factor f_c was first calibrated solely by fitting to monthly flows. For calibration of the remaining three parameters, the two objective functions were weighted equally and the set of Pareto-optimal solutions was identified to indicate the range of parameter values best suited as constraints on the calibration. Pareto-optimal solutions are those for which no other parameter set θ_k can be found to have better performance in terms of two or more objective functions (see, for example, *Gupta et al.* [1998] for further discussion).

[28] The trade-offs between the multiple possible solutions were then considered by selecting conceptually reasonable members of the Pareto set and carrying out further calibration based on visual inspection of simulated and observed data. Although it introduces a subjective element, this manual calibration also allowed flexibility in applying hydrological judgment rather than adopting an automated rule for the final selection of the parameter values. This was considered to be particularly important because of the need to nominate a single "optimum" parameter set from what may in some cases be a relatively wide range of possible parameter values. *Lamb* [1999] discussed some of the issues raised by calibration of the PDM when used for flood frequency estimation, in particular the limitations of inferences drawn from typical objective functions.

[29] A further constraint was placed on the calibration procedure in an attempt to account, at least in part, for the problem of functional interaction between model parameters. Examination of profile plots of O_P and O_M produced by uniform random sampling showed that the ability of the PDM to fit the observed flow data was generally more sensitive to some of the PDM parameters than to others. Profile plots for f_c were by far the most likely to indicate a clear single optimum value (which is not surprising, given the critical role f_c plays in controlling the overall modeled responses). The parameter f_c was therefore chosen to be the first for which a regression relationship was fitted to catchment properties.

[30] For each catchment, the fitted relationship was used to calculate an "as-ungauged" estimate $f_{c,k}^* = M_1(\mathbf{x_k})$, where

Table 1. Regression Equations Used to Relate PDM Parameters to Catchment Properties[a]

Equation	R^2
$f_c = 0.71 + 6.6 * 10^{-4} * DPSBAR + 0.0016 * MEDWET - 0.40 * HOSTP$	0.7
$c_{max} = -96.6 + 10.6 * SKEW + 4.97 * DPLBAR + 0.056 * S6190 - 1175.3 * URBFRAC$	0.7
$k_1 = -42.7 + 62.4 * HOSTBFI + 14.8 * SDIST + 1.1 * RESIDM - 19.9 * SUBFRAC$	0.8
$k_b = 32.2 - 224.5 * HOSTBFI + 0.33 * PORO + 25.5 * GEOLP + 524.6 * HOSTP$	0.6
$v_f = 0.01 * HOSTSPR$	fixed

[a]Descriptions of the catchment properties are given in Table 2.

the asterisk is used to denote a regression estimate and $\mathbf{x}_k$ is the vector of catchment properties for catchment k. The remaining PDM parameters were then recalibrated for each catchment, conditional upon the fixed, as-ungauged, values f_c^*. Again, the parameter judged to have produced greatest sensitivity in the objective function response surfaces was chosen, and the process repeated until all four parameters had been calibrated and used to fit regression relationships against catchment properties. This process has been referred to as the "sequential generalization" procedure [*Lamb et al.*, 2000a, 2000b].

[31] It is important to note that the sequential generalization procedure effectively combines model "calibration" (i.e., fitting θ_k to local gauged flow data) with what is often referred to as "regionalization" (i.e., transferring information, in this case to estimate θ_k^* from catchment properties). Instead of two separate steps, our sequential procedure attempts to fit a spatially generalized model that incorporates both local fit to gauged data and spatial fit to catchment properties. For convenience, we refer to "calibrated" model parameters where these have been fitted to flow data, but it should be noted that the calibration for parameter θ_i ($i \neq 1$) is conditional upon catchment properties used to predict θ_{i-1}.

[32] Regression equations were fitted at each step by an exhaustive search of catchment properties based on explained variance, R^2. This resulted in a number of alternatives with similar R^2 but with different combinations of catchment properties and regression coefficients. The best alternatives were compared and a final regression equation selected for each PDM parameter based on

judgments about the hydrological explanation offered by the selected catchment properties, although it has to be accepted that there will not always be a simple physical link between catchment properties and conceptual model parameters. It is also likely that catchment properties may act as surrogates in some cases; for example location and altitude attributes will, to a degree, reflect geology, soil type and rainfall regimes in Britain.

[33] The resulting equations are summarized in Table 1. Definitions of the relevant catchment properties (i.e., the explanatory variables in the regression models) are given in Table 2. Many of these properties are widely available for the UK as part of the *Flood Estimation Handbook* "Catchment Descriptors" data set. The explanatory variables are significant to at least 88% probability, with the exception of the regression for k_b. Here, there is some correlation between base flow index (HOSTBFI) and effective porosity (HOSTP). It might be expected that the uncertainty for regression predictions of k_b based on the four-variable equation would therefore be inflated. However, the equation shown in Table 1 provided a visually better prediction of the calibrated values of k_b and tests showed that the effect on the regression model uncertainty of removing one of the correlated variables was negligible. The Table 1 equation was therefore accepted for the present study, although further work would be recommended to seek a potentially more robust regression relationship for wider application of the generalized-parameter model.

[34] Figure 3 compares the values of PDM parameters obtained by calibration with estimates generated from catchment properties data via the fitted regression equa-

Table 2. Catchment Descriptors Used for Estimating Parameters of the PDM Hydrological Model

Name	Units	Source[a]	Description
DPLBAR	km	FEH	mean drainage path length
DPSBAR	m/km	FEH	mean slope of drainage paths to the site
GEOLP		Geol	assessment of relative groundwater permeability
HOSTBFI		FEH	base flow index, calculated from weighted average of hydrology of soil types (HOST) soils classes covering each catchment
HOSTP		HOST	index of porosity as weighted average of values inferred from HOST soils classes covering each catchment
HOSTSPR	%	FEH	standard percent runoff calculated from weighted average of soils classes over the catchment
MEDWET	days	FEH*	median length of periods of soil moisture deficit (SMD) less than 6 mm between 1961 and 1990
PORO	%	Soils	estimated total soil porosity
RESIDM	%	Soils	estimated residual soil moisture
S6190	mm	FEH	standard average annual rainfall, 1961–1990
SDIST		DTM	distance from gauge at which number of channels is a maximum, measuring along the channel network
SKEW		DTM	skew of distribution of the log(area/slope) index [*Beven and Kirkby*, 1979]
SUBFRAC		FEH*	suburban fraction of total catchment area
URBFRAC		FEH*	urbanized fraction of total catchment area

[a]Data sources are as follows: FEH, properties appearing in digital form from *Institute of Hydrology* [1999]; FEH*, properties derived from FEH data, but not included in the current FEH CD-ROM database; DTM, properties derived from the CEH-Wallingford integrated hydrological digital terrain model (IHDTM); Geol, derived by interpretation of geological map and aquifer properties information; Soils, soil physical parameters derived from the UK SEISMIC soils database; HOST, properties derived from the HOST soils classification system [*Boorman et al.*, 1995].
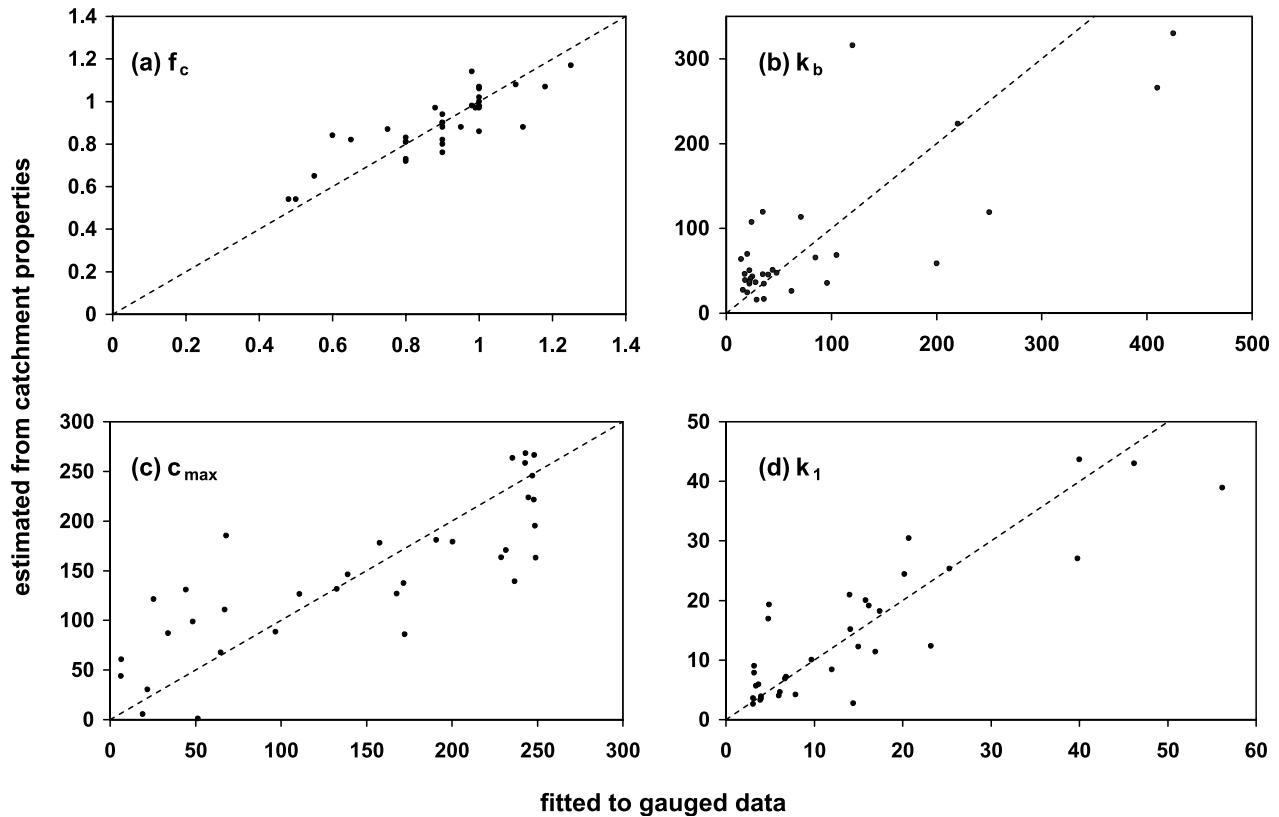
Figure 3. Regression estimates of four PDM catchment model parameters: (a) volume adjustment f_c, (b) slow flow routing constant k_b, (c) maximum soil moisture storage capacity c_{max}, and (d) fast flow routing constant k_1. Horizontal axes are calibrated values (based on model fit to gauged flow data), and vertical axes are corresponding regression estimates (based on catchment properties).

tions. Clearly there is a degree of scatter in these plots, but relatively little indication of bias. Given the difficulty of attempting to relate hydrological model parameters to catchment properties data, and the relatively small sample size available here, the regression relationships are considered to be a useful first step toward a national application.

6. Uncertainty Estimation Method

[35] The regression relationships developed using the sequential fitting procedure are the basis for parameterizing the PDM for ungauged catchments. However, our interest here is the distribution of values surrounding the regression lines, as well the mean estimate. Expressing the estimated PDM parameters as distributions for each catchment, we used Monte Carlo simulation to generate corresponding model output distributions of simulated river flows and hence flood frequency curves. Approximate confidence intervals were then computed from the simulated output distributions. The overall procedure is illustrated in Figure 4 and described in more detail below.

[36] For any catchment k, each estimated PDM parameter θ_k^* is the mean of a distribution for which an estimate of the variance is $\sigma_d^2 = SS_r/d$, where SS_r is the sum of squares of the residuals in the regression relationship and d is the corresponding number of degrees of freedom. The quantity θ_k^* is the "as-ungauged" parameter estimate, and, following

standard statistical theory [see, e.g., *Draper and Smith*, 1998], the $100(1 - \alpha)\%$ confidence limits may be written

$$\theta_k^* \pm t\left(d, 1 - \frac{1}{2}\alpha\right)\sigma_d\sqrt{\mathbf{x}_k^T\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{x}_k} = \theta_k^* \pm g_k(\alpha; d). \quad (2)$$

In equation (2), $t(d, 1 - \alpha/2)$ is the value from the t distribution with d degrees of freedom with area $(1 - \alpha/2)$ to its left and $\alpha/2$ to its right, $\mathbf{X}$ is the matrix with row i consisting of relevant catchment properties for the ith gauged catchment (and including a row for catchment k, if this is ungauged) and $\mathbf{x}_k$ is the vector of catchment properties at catchment k.

[37] Equation (2) was used to produce a cumulative distribution function for the as-ungauged estimate for each PDM parameter for each study catchment by plotting $(1 - \alpha/2)$ against $\theta_k^* + g_k(\alpha; d)$ for α in [0, 2]. Figure 5 shows two examples. In the case of the River Taff at Pontypridd (57005) the median estimate is in close agreement with the calibration value for all parameters except c_{max}. In contrast, the as-ungauged estimates for the Dove at Izaak Walton (28046) seem to demonstrate a failure of the regression-based parameter generalization. The Dove has been chosen as an example here because, contrary to the relatively poor as-ungauged parameter estimates illustrated in Figure 5, the simulated flood frequency confidence intervals will be seen to be much better than expected.
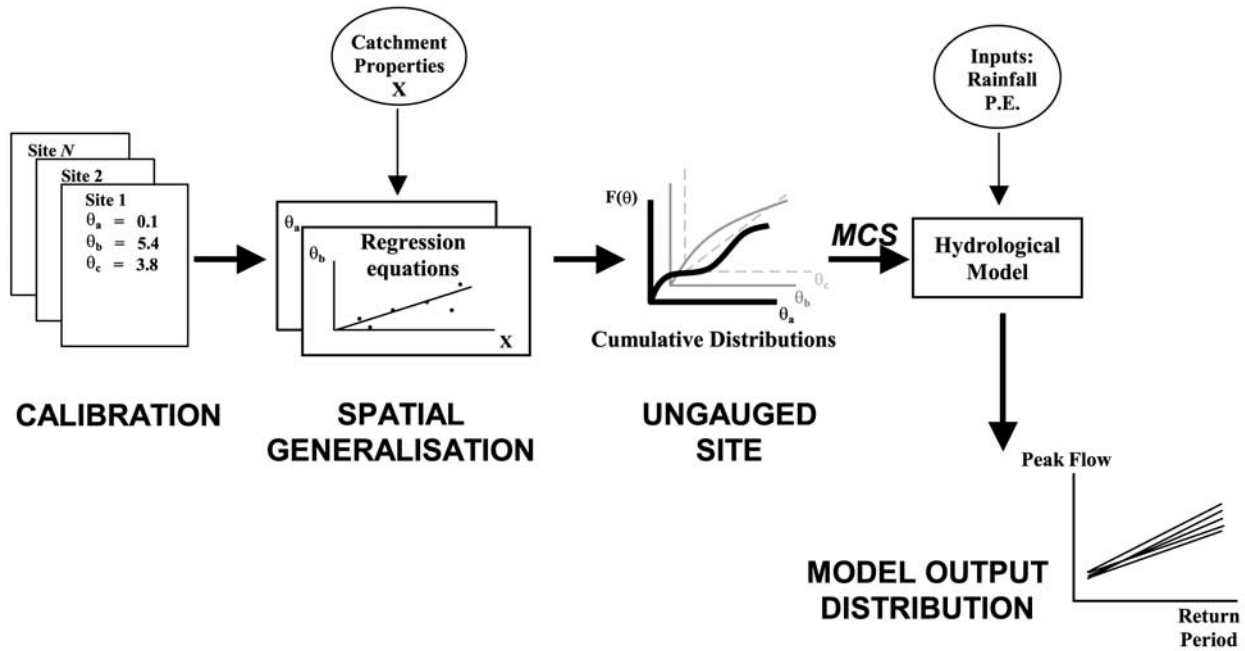
Figure 4. Schematic of the method used to calculate approximate confidence intervals using Monte Carlo simulation (MCS) for catchments treated as ungauged.

[38] For each of the study catchments, we then generated a distribution of hydrological model outputs (i.e., flow data) by running $R = 1000$ realizations of the PDM, randomly drawing values from the regression-estimate cdf for each model parameter. The choice of $R = 1000$ was made after tests with values as large as 10,000 revealed very little difference in the outputs. For simplicity, we assumed in this experiment that the forcing (rainfall and PE) data are known
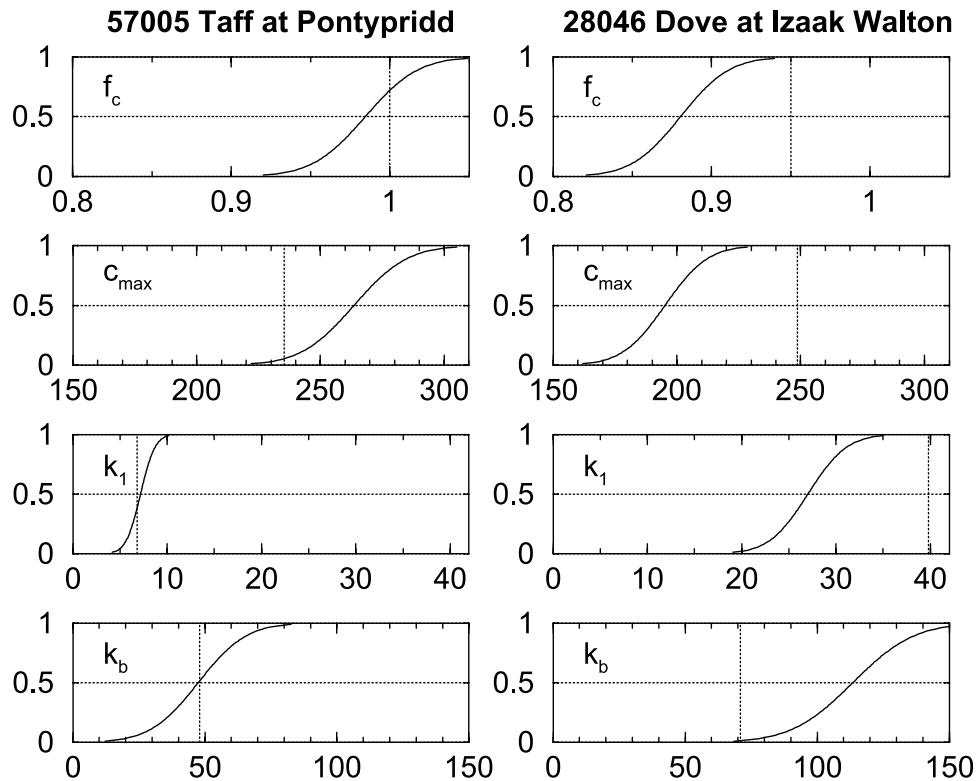

Figure 5. Cumulative distribution functions of PDM parameters for two example catchments treated as ungauged. Vertical bars indicate calibration values. "57005" and "28046" are U.K. National Water Archive catchment index numbers.

with negligible uncertainty, at least when compared to the uncertainty about parameterization of the hydrological model. The same, fixed, rainfall and PE data were therefore used to drive the hydrological model in each realization of the PDM.

[39] A peaks-over-threshold (POT) series was extracted from each simulation, adopting an extraction rate of three peaks per year. This results in a total of $(1000 \times 3 \times L)$ peaks being extracted in rank order of magnitude for each catchment, where L is the length of record for the catchment in years. The extraction rules stated by the *Natural Environment Research Council* (*NERC*) [1975, vol. 1] were followed. Approximate 90% confidence intervals were then constructed as follows for each catchment: For a given rank l ($l = 1, \ldots, 3L$), the 1000 simulated POT data were arranged in a series in order of magnitude and the 50th and 950th values were recorded (i.e., for $z\%$ confidence intervals take the $\{(R/100)(100 - z)/2\}$th values, counting in from each end of the series). Note that this procedure provides intervals only at the plotting position corresponding to each value in the extracted POT series. Curves have been plotted by piecewise linear interpolation between each series of simulated points.

7. Results

[40] Previously reported work [*Lamb et al.*, 2000a, 2000b] gave an indication of the uncertainty about spatially generalized flood estimates by simulating flood quantiles (using as-ungauged rainfall-runoff model parameters) and then plotting histograms of the magnitudes of differences between the simulations and corresponding observations for a group of catchments. The standard deviations of errors in simulated flood quantiles, expressed as a percentage of the corresponding value on the "observed" flood frequency curve, were 18 for a return period of $T = 2$ years, 21 for $T = 10$ years and 23 for $T = 20$ years. However, when it is considered that the uncertainty arising from spatial generalization is expressed in the distributions of regression estimates of (in this case) four parameters, and that the parameters interact in a very nonlinear fashion to produce simulated flow data, then it seems likely that the location and coverage of the calculated approximate confidence intervals may vary between catchments. It will be seen that our results confirm this.

[41] Figure 6 shows approximate 90% confidence intervals for as-ungauged simulation, calculated using the methods described in the preceding section. The confidence intervals are accompanied by a flood frequency curve corresponding to using the mean parameter estimates θ^* (i.e., the best single estimate for the ungauged case). Also plotted are peaks extracted from the observed flow series along with a curve showing the generalized Pareto distribution (GPD), fitted to the observed peaks using probability weighted moments [*Hosking and Wallis*, 1987]. We will refer to this curve as the "empirical flood frequency curve." The GPD is used because it has been found to be a suitable distribution for fitting peaks-over-threshold data for many UK catchments [*Naden*, 1992].

[42] In most cases, the empirical flood data shown in Figure 6 lie within the approximate 90% intervals constructed from the generalized-parameter PDM simulations. We interpret this result as a partial validation of the spatially generalized modeling approach. It can only be partial, however, because the empirical flood frequency curves cross the simulated 90% intervals in other cases. Taking return periods of 2, 5 and 10 years for reference, the empirical curve plotted outside of the 90% intervals at one or more of these return periods for 14 of the study catchments. Figure 6 includes two examples, catchments 30004 (Partney Lymn at Partney Mill) and 54034 (Dowles Brook at Oak Cottage) where the observed flood peak data lie entirely outside of the 90% intervals. There were four catchments in the study set of 40 where this type of failure was noted.

[43] The aim of this study was to investigate a new method for fitting a spatially generalized rainfall-runoff model, and assessing its performance. For this reason, we were interested to observe both successful and unsuccessful cases. For future applications, where the aim is to fit the best possible generalized model, a greater number of catchments would be used. In this case, catchments where the flow data could not be modeled successfully would be removed from the generalized fitting procedure.

[44] For catchments such as 30004 and 54034, where the generalized model has not worked well, it is worth asking whether the reason is the spatial generalization procedure, or an underlying inability of the model to simulate flows at the particular catchment, even when calibrated locally using gauged data. The two examples help to illustrate the issues.

[45] In the case of catchment 30004, the generalized simulations and confidence intervals underestimate the gauged peak flows. Using the as-ungauged parameter estimates θ^* as a starting point, manual recalibration was carried out for this catchment, by varying the routing store constants k_1 and k_b. The results are plotted in Figure 7, along with the empirical frequency curve, spatially generalized 90% confidence intervals, and the simulation corresponding to setting parameters equal to the estimates, θ^*. It can be seen that the recalibration (even for only two of the PDM parameters) considerably improves upon the generalized estimates. However, there remains a problem in that the recalibration overestimates the larger flood peaks and tends to underestimate the smaller peaks. The mismatch in fitted distribution shape between the empirical curve and the modeled data remains, and further attempts to improve the calibration by varying c_{max} and f_c were not able to resolve it. Although it is difficult to "prove a negative" in model calibration, it appears that the PDM formulation used here may not be able to model the distribution of peak flows in this catchment effectively. The station is known to be bypassed at high flows, and it may be that the larger peaks visible in Figure 7d are therefore underestimated in the gauged data.

[46] In the case of catchment 54034 recalibration successfully corrected the simulated peak flows (Figure 8). The calibrated values and as-ungauged estimates of the model parameters for this catchment were, respectively, $f_c = 0.8$, $f_c^* = 0.8$, $c_{max} = 7$ mm, $c_{max}^* = 61$ mm, $k_1 = 25$ h, $k_1^* = 25$ h and $k_b = 24$ h(mm)^2, $k_b^* = 107$ h(mm)^2. Setting
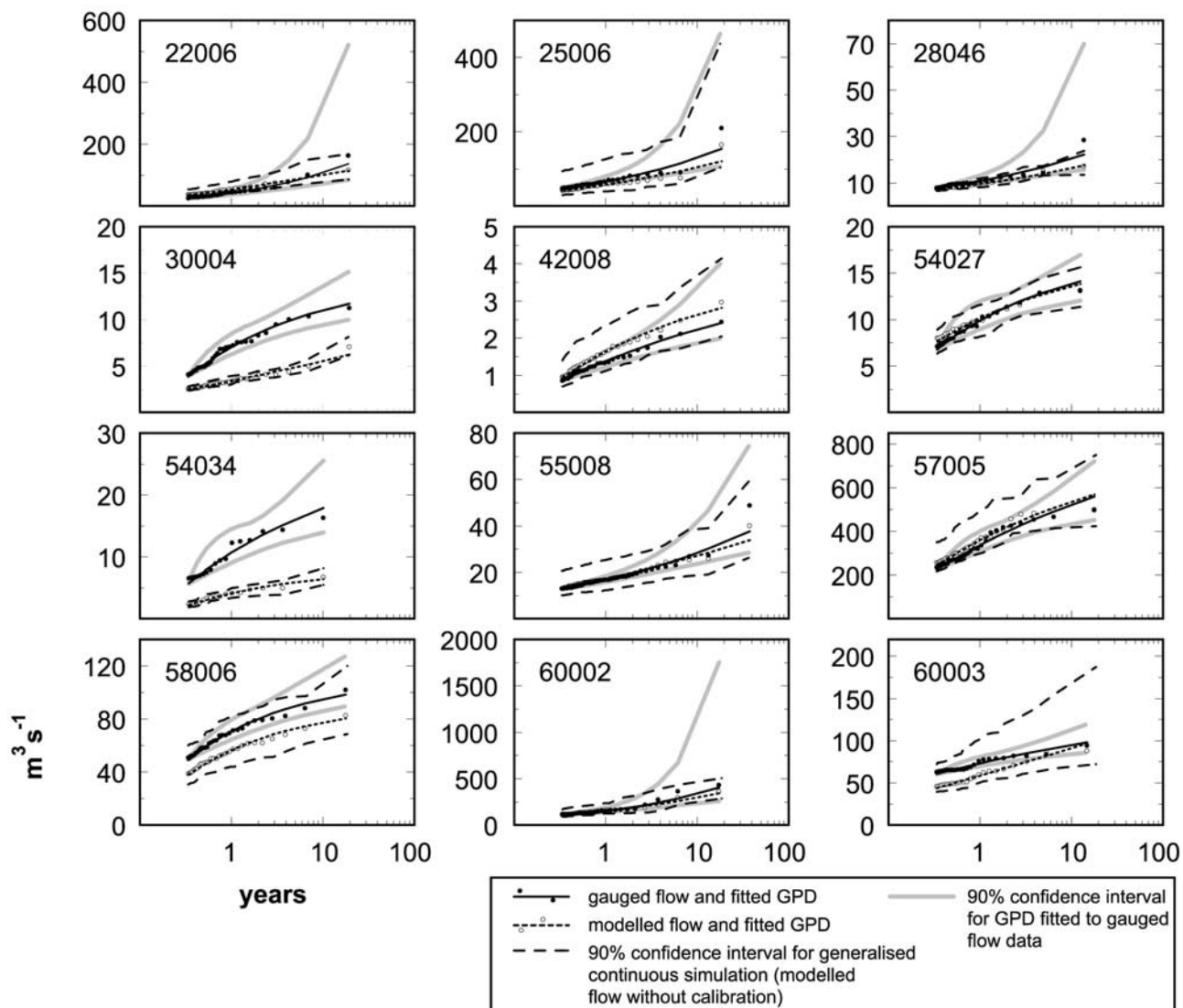
Figure 6. Flood frequency curves showing peak flow on vertical axes, plotted against average return period in years. Five digit numbers are U.K. National Water Archive catchment index numbers. "Modeled flow" and "90% confidence interval for generalized continuous simulation" were simulated using spatially generalized parameters to represent an application to an ungauged catchment.

c_{max} and k_b to their calibrated values would obviously be expected to improve the simulation, but, in fact, all that was required to obtain the improved fit shown in Figure 8 was to reset k_b to the calibrated value. In this case, it therefore appears that the reason for the failure of the generalized simulation is the calibration and generalization process itself, rather than any particular difficulty in simulating catchment responses with the PDM.

[47] It is recognized that the curves we refer to as confidence intervals are only approximate, and depend on the fitting of a combined hydrological and regression model that, as is generally the case for such models, has to be accepted as an imperfect model of the physical system. It is however of interest to compare the uncertainty about the generalized PDM simulations with the uncertainty associated with the generalized Pareto distribution (GPD) that has been fitted to the gauged flow data using the method of probability weighted moments. To construct confidence intervals for the GPD we used a likelihood-based approach described by *Clarke* [1994]. The log likelihood is

$$l = -N \log s - (1 - \kappa) \sum_{i=1}^{N} - \frac{1}{\kappa} \log\left(1 - \frac{\kappa y_i}{s}\right), \quad (3)$$

where N is the number of extracted peak flow data, $y_i = q_i - u$ is the ith exceedance above threshold u, and s and κ are scale and shape parameters of the GPD,

$$F(y) = 1 - \left(1 - \frac{\kappa y}{s}\right)^{1/\kappa}. \quad (4)$$

The log likelihood was maximized using the Nelder-Mead simplex algorithm within the MATLAB5.3 package to perform a local search of the (s, κ) parameter space. The search was initialized from the probability weighted
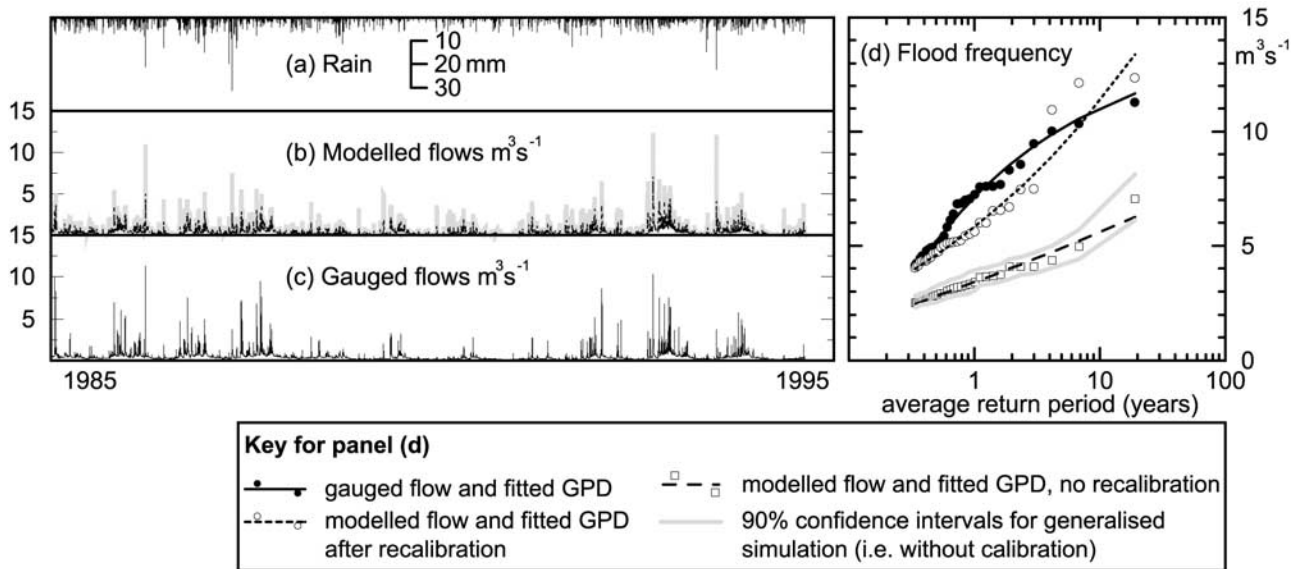
Figure 7. Recalibration for catchment 30004 (Partney Lymn at Partney Mill, 60 km²) showing (a) hourly rainfall (mm), (b) modeled flow time series (solid line is as-ungauged estimate using parameters derived from catchment properties, and shaded line is recalibrated model), and (c) gauged flows. (d) Flood peaks and fitted distributions, including 90% confidence intervals for the generalized model, representing an application to an ungauged catchment.

moment estimates (which, in practice, were found to be very close to the resulting maximum likelihood estimates). The (s, κ) space was then searched to define the confidence region

$$2[\max\{l(s, \kappa; y)\} - l(s, \kappa; y)] \leq \chi^2_{d,\alpha} , \quad (5)$$

where $\chi^2_{d,\alpha}$ is the χ^2 distribution for $d = 2$ degrees of freedom with significance level α. Now $100(1 - \alpha)\%$ confidence intervals correspond to the largest and smallest quantile values of the GPD obtained for all (s, κ) on the boundary of the confidence region defined by the inequality (5). These quantile intervals were calculated for each of the plotting positions defined by the N exceedances in the peaks-over-threshold series.

[48] The GPD confidence intervals consistently enclose the observed flood peak data. This is to be expected, given

that the GPD was fitted directly to these data. However, the uncertainty about the GPD as a "model" for the flood frequency data was not found to be markedly less than the uncertainty in the generalized hydrological model simulations. For many of the study catchments, the 90% GPD confidence intervals were found to be qualitatively comparable in width to the spatially generalized hydrological model intervals, as shown in Figure 6. We have interpreted this tentatively to suggest that although the continuous simulation model can fail for some specific catchments, the uncertainty in the spatially generalized hydrological model is not necessarily greater than the sample uncertainty present when fitting a distribution directly to a relatively short, single-site, gauged flow record.

[49] In many cases, the GPD confidence intervals are much narrower for lower return periods, but then expand rapidly for longer return periods toward the tail of the
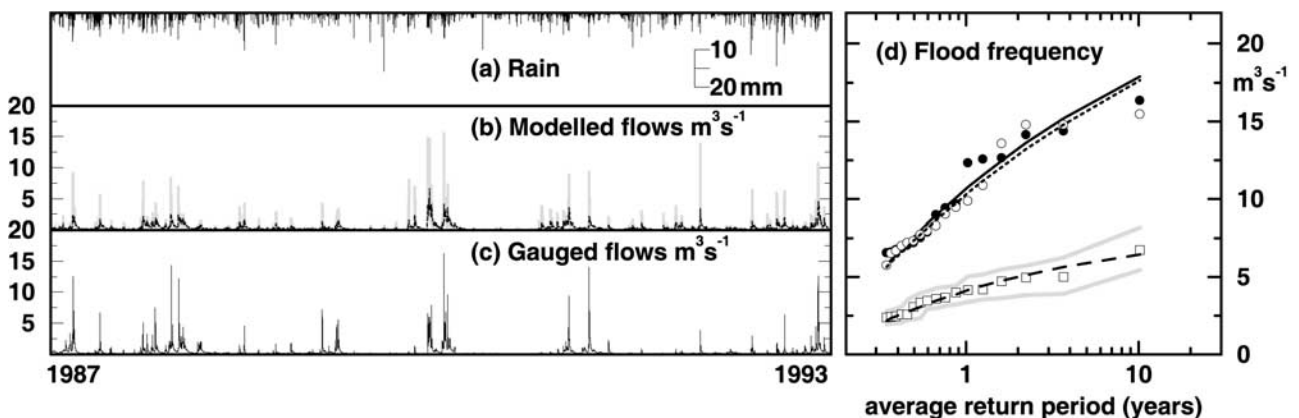

Figure 8. Recalibration for catchment 54034 (Dowles Brook at Oak Cottage, 42 km²); explanation same as for Figure 7.

distribution. The joint confidence region of the GPD parameters was found to extend into $\kappa < 0$ for many of the study catchments, in which case the GPD confidence intervals would always tend to expand steeply toward longer return periods, owing to the very nonlinear nature of equation (4). This is not, however, inevitable for the generalized CS intervals, reflecting perhaps the greater physical/conceptual structure of the PDM (or any other CS model).

8. Discussion

[50] The generalized continuous simulation confidence intervals are often similar in width to comparable intervals plotted around a flood frequency distribution fitted to gauged flows. The relatively large impact of sampling uncertainty for the GPD, especially for the tail of the distribution, is one of the motivations for the use of pooled or regional analyses in statistical frequency estimation. The pooled analysis can be viewed as "borrowing" information from other sources to constrain uncertainty in fitting a statistical model at a specific location (which may be either a gauged or an ungauged site). An important interpretation of the results presented in this paper may be that the combination of a conceptual hydrological model (which implicitly includes some structural constraints on runoff responses) with parameterization based on catchment properties can lead to similar constraints on uncertainty.

[51] For reasons stated earlier, we have only considered flood return periods up to the limits imposed by available gauged flow records, whereas, for most practical purposes, there is an interest in longer return periods. At longer return periods, the slope of the rainfall growth curve increasingly controls the flood frequency curve [Sivapalan et al., 1990], presumably making continuous simulation of rare floods sensitive to uncertainty in modeled rainfall. Future work should test whether continuous simulation driven by rainfall modeling provides flood estimates with comparable confidence to a pooled statistical analysis at an ungauged site.

[52] A fundamental problem remains the multivariate nature of the calibration/generalization problem. The sequential procedure described in this paper, in which catchment properties are used to help in constraining the model calibration process, is suggested as one practical way of addressing this issue. Even for a highly simplified hydrological model, we have still found that interactions between parameters can produce some unexpected results. Consider, for example, the generalized simulations for catchments 28046 and 57005, for which the as-ungauged parameter estimates θ^* were plotted in Figure 5. In both cases the 90% intervals enclose the empirical flood frequency curve (Figure 6). Inspection of the parameter estimates in Figure 5 reveals that for catchment 57005, the calibrated estimates are within the distributions surrounding the spatially generalized regression estimate, as might be expected. In contrast, however, the calibrated values for 28046 differ significantly from the generalized estimates based on catchment properties, even allowing for uncertainty around the regression relationships. Despite this, the generalized estimate of flood flows at catchment 28046 is a relatively good one and the simulated 90% intervals largely contain the empirical data.

[53] A limitation in judging how well the spatially generalized model has worked is that there is not a truly

independent group of sites available for testing. Long continuous hourly records, as used in this work, are costly and time consuming to collate. Given a limited number of catchments, it was decided to make use of all available sites in the generalization procedure, rather than split the sites into separate groups for fitting the generalized parameters and subsequent testing.

[54] A procedure based on the jackknife could, in principle, be used to provide a more objective test. In brief, this would involve excluding one site when fitting the relationships between PDM parameters and catchment properties, with the process being repeated n times, excluding each site in turn. However, the sequential approach tested here is a new development that has not so far been implemented in a completely automated form, and so each iteration of the above procedure would be a nontrivial task. It is largely for this pragmatic reason that we have assumed, for present purposes, that the regression relationships fitted across the available sites are reasonably independent of any one site.

[55] A basic test of the assumption was carried out by assessing whether particular catchments exert strong leverage on the fitted regression relationships. For each of the regression models for the four PDM parameters, the leverages (given by the diagonal elements of the matrix $\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$) were calculated and examined. If p is the number of explanatory terms in the regression, then sites that have a leverage of greater than $2p/n$ exert a strong control on the regression relationship. For c_{max}, k_1 and k_b, only one catchment (in each case) was found to have a high degree of leverage. For f_c there were three catchments where the leverage was greater than $2p/n$. We tested the influence of these sites by removing them from the fitted relationships and examining the resulting changes in "predicted" PDM parameter values. The changes were found to be very small (on average less than a factor of 0.02), with the exception of one catchment for which the value of c_{max} predicted from catchment properties was much improved in the refitted regression model.

[56] The simple test, described above, does not account for the effects that excluding a site might have through successive steps in the sequential model generalization procedure. However, it does at least suggest that the model parameters calculated from catchment properties are not dominated by any single site in the current analysis. Given that the sites were chosen to be qualitatively representative of a broad range of catchment types, we can therefore speculate that the results obtained (i.e., the simulated flood frequency confidence intervals) would be similar for an independent group of test catchments.

[57] The methods used in this paper do not explicitly account for uncertainty in the calibration of the PDM against gauged flow data. This is not an oversight, rather a deliberate decision to simplify the development of a procedure for estimating uncertainty in the spatially generalized model. The issue is discussed here because of the implications for generalized parameterization. Having different but similarly good estimates of model parameters for a single site may arguably be tolerated in some circumstances (for example if a model is used to in-fill short periods of data, and is not used for extrapolation), but for

spatial generalization it raises the question of which estimate should be used?

[58] An alternative approach would be to account explicitly in some way for the uncertainty about the calibration estimates of parameters. In this paper, we have used point estimates of parameter values inferred from gauged flows, which, despite the many recent advances in methods for handling calibration uncertainty, remains a common approach in practice. Calibration uncertainty enters into this analysis in the sense that there is assumed to be a random error component implicit in fitting a regression equation using each "sample" of calibrated parameter values; parameter interactions leading to uncertainty are mitigated by use of a parsimonious hydrological model. A different approach will be needed to allow for this calibration uncertainty more explicitly, albeit at the cost of greater complexity.

[59] To account more completely for calibration uncertainty, an approach should be taken that would allow the variance of the regression estimates to reflect uncertainty about the calibrated values in a more general way. One approach that we have experimented with is based on importance sampling, where the calibration uncertainty at gauged sites was expressed by assigning weights within the parameter space according to how well each set of model parameters could simulate gauged flows. We hope to report the results in a later paper. The methodology presented here is, however, a first attempt to make a quantitative assessment of uncertainty in generalized flood estimation by continuous simulation.

Notation

α confidence level (tail probability).
c_{max} PDM maximum soil moisture storage capacity.
d degrees of freedom.
k_1 PDM fast flow routing store constant.
k_b PDM slow flow (base flow) routing store constant.
v_c constant proportion of PDM runoff entering fast flow routing pathway.
κ shape parameter of the generalized Pareto distribution (GPD).
l log likelihood for GPD.
m number of model parameters.
M regression model.
n number of catchments.
O objective function.
p number of catchment properties.
q river flow.
θ hydrological model parameter.
f_c constant PDM rainfall/volume adjustment factor.
s scale parameter of the GPD.
σ standard deviation.
t quantile of the t distribution.
u threshold of the GPD.
x catchment property.
$\mathbf{X}$ matrix of catchment properties.
$\mathbf{x}_k$ properties for catchment k.
y flow exceedance series.

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