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Shaft inflation

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A B S T R A C T

A new family of inflation models is introduced and studied. The models are characterised by a scalar potential which, far from the origin, approximates an inflationary plateau, while near the origin becomes monomial, as in chaotic inflation. The models are obtained in the context of global supersymmetry starting with a superpotential, which interpolates from a generalised monomial to an O'Raifearteagh form for small to large values of the inflaton field respectively. It is demonstrated that the observables obtained, such as the scalar spectral index and the tensor to scalar ratio, are in excellent agreement with the latest observations. Some discussion of initial conditions and eternal inflation is included.

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The latest CMB observations from the Planck satellite have confirmed the broad predictions of the inflationary paradigm, in that the Universe is found to be spatially flat with a predominantly Gaussian curvature perturbation that is almost (but not quite) scale invariant [1]. However, the precision of these observations is so high that they put tension to (or even exclude) entire classes of inflationary models, e.g. chaotic inflation.[1]

The Planck observations seem to support an inflationary scalar potential which asymptotes to a constant, i.e. an inflationary plateau is favoured [3]. In view of this fact, in this letter we present a new class of inflationary potentials, which we call shaft inflation. The idea is that the inflationary plateau is pierced by shafts such that, when the inflaton field finds itself close to one of them it slow-rolls inside the shaft, until inflation ends and gives away to the hot big bang cosmology. Assuming a shaft at the origin, the scalar potential approximates a constant at large values of the inflaton field, but at small values the potential becomes similar to monomial chaotic inflation. In that respect, shaft inflation is similar to the so-called T-model inflation [4] but the scalar potential in our case features a power-law (in contrast to exponential) dependence on the inflaton field. Although we attempt to design the model in the context of global supersymmetry, this is by no means restrictive since the phenomenology really stems out from the form of the scalar potential, which can be obtained via a different, possibly more realistic (and complicated) setup. Indeed, as we discuss, one of the realisations of shaft inflation can be iden-

tified with S-dual superstring inflation [5] or with radion assisted gauge inflation [6].

After the first draft of this letter was produced, the first data of the BICEP2 experiment were released, which show that inflation may produce substantial gravitational waves. According to the findings of BICEP2, the tensor to scalar ratio is $r \simeq 0.20 \pm 0.07$ [7]. We show that shaft inflation can accommodate such a large value of r.

We use natural units, where $c = \hbar = 1$ and Newton's gravitational constant is $8\pi G = m_P^{-2}$, with $m_P = 2.43 \times 10^{18}$ GeV being the reduced Planck mass.

Let us begin with the following superpotential:

$$W = M^2 \frac{|\phi|^{nq+1}}{(|\phi|^n + m^n)^q} \tag{1}$$

where M, m are mass-scales, n, q are real parameters and ϕ is a real scalar field (corresponding to a superfield made real by suitable field redefinitions). Without loss of generality, we assume that $\phi > 0$ so we can write $|\phi| = \phi$ and assume that there is a Z_2 symmetry $\phi \to -\phi$. In the limit $\phi \gg m$ the above superpotential reduces to an O'Raifearteagh form $W \simeq M^2\phi$, which leads to de-Sitter expansion. However, in the limit $\phi \ll m$ the superpotential becomes $W \simeq M^2\phi(\phi/m)^{nq}$, which leads to monomial chaotic inflation. To simplify the potential we may assume $q = -1/n$, in which case the superpotential becomes

$$W = M^2(\phi^n + m^n)^{1/n}. \tag{2}$$

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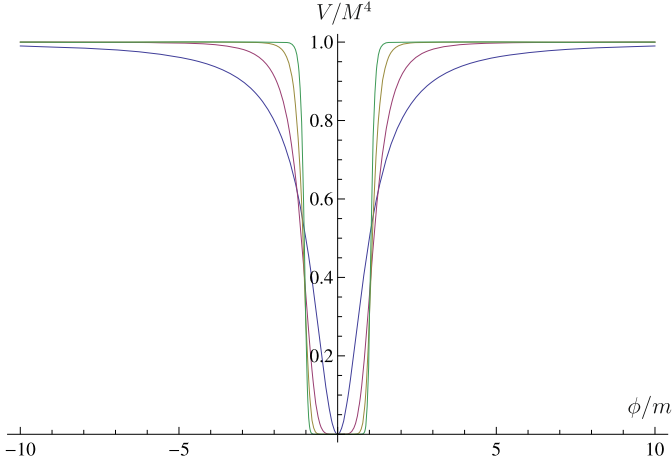[1] Unless excited states are assumed instead of the usual Bunch–Davis vacuum [2].

Fig. 1. The scalar potential in shaft inflation for $n = 2, 4, 8$ and 16. The shaft becomes sharper as n grows. Far from the origin the potential approximates the inflationary plateau with $V \approx M^4$. Near the origin the potential becomes monomial, as in chaotic inflation.

Thus, in the limit $\phi \ll m$ the above becomes $W \simeq M^2 m + \frac{1}{n} M^2 m(\phi/m)^n$, which leads to a monomial F-term potential. [2]

For the superpotential in Eq. (2), the corresponding F-term scalar potential is:

$$V(\phi) = M^4 \phi^{2n-2} \left(\phi^n + m^n\right)^{\frac{2}{n}-2}. \tag{3}$$

From the above we see that the scalar potential has the desired behaviour for $n > 1$, i.e. it approaches a constant for $\phi \gg m$, while for $\phi \ll m$ the potential becomes monomial, with $V \propto \phi^{2(n-1)}$, see Fig. 1. When $n = 1$ the potential is exactly flat and the shaft disappears.

For the slow-roll parameters we find

$$\epsilon \equiv \frac{1}{2} m_P^2 \left(\frac{V'}{V}\right)^2 = 2(n-1)^2 \left(\frac{m_P}{\phi}\right)^2 \left(\frac{m^n}{\phi^2 + m^n}\right)^2 \tag{4}$$

$$\eta \equiv m_P^2 \frac{V''}{V} = 2(n-1)\left(\frac{m_P}{\phi}\right)^2 \left(\frac{m^n}{\phi^2 + m^n}\right)$$
$$\times \frac{(2n-3)m^n - (n+1)\phi^n}{\phi^n + m^m}, \tag{5}$$

where the prime denotes derivative with respect to the inflaton field. Hence, the spectral index of the curvature perturbation is

$$n_s = 1 + 2\eta - 6\epsilon = 1 - 4(n-1)\left(\frac{m_P}{\phi}\right)^2 \frac{m^n[(n+1)\phi^n + nm^n]}{(\phi^n + m^n)^2}. \tag{6}$$

It is straightforward to see that inflation is terminated when $|\eta| \simeq 1$ so that, for the end of inflation, we find

$$\phi_{\text{end}} \simeq m_P \left[2(n^2 - 1)\alpha^n\right]^{1/(n+2)}, \tag{7}$$

where we assumed that $\phi > m$ (so that the potential deviates from a chaotic monomial) and we defined

$$\alpha \equiv \frac{m}{m_P}. \tag{8}$$

Using this, we obtain $\phi(N)$

$$N = \frac{1}{m_P^2} \int\limits_{\phi_{\text{end}}}^{\phi} \frac{V}{V'} d\phi$$

$$\simeq \frac{1}{2(n-1)(n+2)\alpha^n} \left[\left(\frac{\phi}{m_P}\right)^{n+2} - \left(\frac{\phi_{\text{end}}}{m_P}\right)^{n+2}\right] \tag{9}$$

$$\Rightarrow \quad \phi(N) \simeq m_P \left[2(n-1)(n+2)\alpha^n \left(N + \frac{n+1}{n+2}\right)\right]^{1/(n+2)}, \tag{10}$$

where N is the remaining e-folds of inflation and we considered $\phi > m$ again. Inserting the above into Eqs. (4) and (6) respectively we obtain the tensor to scalar ratio r and the spectral index n_s as functions of N:

$$r = 16\epsilon = 32(n-1)^2 \alpha^{\frac{2n}{n+2}}$$
$$\times \left[2(n-1)(n+2)\left(N + \frac{n+1}{n+2}\right)\right]^{-2\left(\frac{n+1}{n+2}\right)} \tag{11}$$

$$n_s = 1 - 2\frac{n+1}{n+2}\left(N + \frac{n+1}{n+2}\right)^{-1} \tag{12}$$

An interesting choice is $n = 2$, in which case the scalar potential becomes

$$V(\phi) = M^4 \frac{\phi^2}{\phi^2 + m^2}. \tag{13}$$

We see that the above can be thought of as a modification of quadratic chaotic inflation, because after the end of inflation, the inflaton field oscillates in a quadratic potential. However, for large values of the inflaton the potential approaches a constant. This potential has been obtained also in S-dual superstring inflation [5] with $\alpha = 1/4$ and also in radion assisted gauge inflation [6] with $\alpha \sim 10^{-3/2}$. In this case, Eqs. (11) and (12) become

$$r = \frac{32\alpha}{[8(N + \frac{3}{4})]^{3/2}} \quad \text{and} \quad n_s = 1 - \frac{3}{2}\left(N + \frac{3}{4}\right)^{-1} \tag{14}$$

For the moment, let us ignore the BICEP2 results and try to satisfy the Planck observations only. Assuming $\alpha \simeq 1$, for $N \simeq 60$ {$N \simeq 50$} we readily obtain $n_s = 0.975$ and $r = 2.99 \times 10^{-3}$ {$n_s = 0.970$ and $r = 3.91 \times 10^{-3}$}. As shown in Fig. 2, these values fall within the 95% {68%} c.l. contour of the Planck observations. Things improve further if we enlarge n.

Indeed, in the limit $n \gg 1$ Eqs. (11) and (12) become

$$r = \frac{8\alpha^2}{n^2(N+1)^2} \to 0 \quad \text{and} \quad n_s = 1 - \frac{2}{N+1}. \tag{15}$$

The spectral index is now the same as in the original R^2 inflation model [8] (also in Higgs inflation [9]), which is not surprising since we expect power-law behaviour to approach the exponential when $n \to \infty$. Now, for $N \simeq 60$ {$N \simeq 50$} we obtain $n_s = 0.967$ {$n_s = 0.961$}, which is very close to the best fit point for the Planck data, as shown in Fig. 2.

Now, let us incorporate in our thinking the BICEP2 results, which suggest that $r = 0.20 \pm 0.07$ [7]. From Eq. (11) it is readily seen that $r \propto \alpha^{2n/(n+2)}$. This means that the tensor production can be enhanced if the shaft is appropriately widened, i.e. if m is somewhat larger than m_P without affecting the scalar spectral index, as seen in Eq. (12). Indeed, it is easy to show that $\alpha \simeq 50$ is enough to boost the tensor signal up to BICEP2 values. For example, assuming $N \simeq 50$ and $\alpha = 50$, Eqs. (11) and (12) give Table 1.
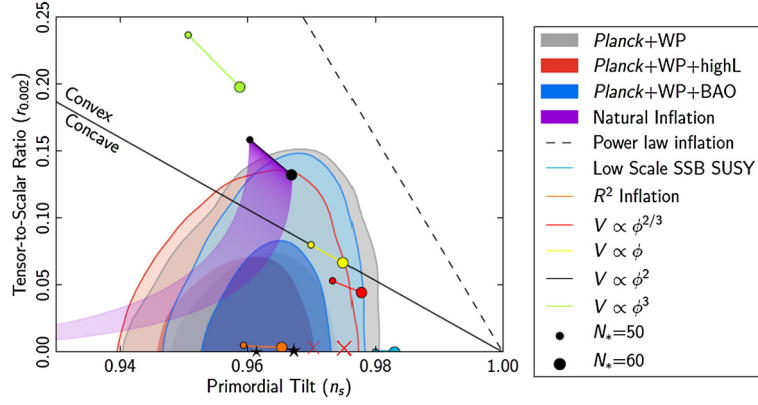
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[2] The form of the superpotential in Eq. (1) is dictated by the requirement that it gives rise to the envisaged scenario. The simplifying relation $q = -1/n$ reduces the parameters and can have physical interpretation for specific values of n, e.g. $n = 2$ (see below). However, the physical meaning of Eq. (2) falls beyond the scope of this letter; while in a sense, can be thought as the definition of shaft inflation.

Fig. 2. Shaft inflation for $n = 2$ is depicted with the large {small} cross for $N \simeq 60$ {$N \simeq 50$}. Shaft inflation for $n \gg 1$ is depicted with the large {small} star for $N \simeq 60$ {$N \simeq 50$}. The difference with R^2-inflation (orange dots) stems from taking into account the contribution of ϕ_{end} so that $N \to N + 1$ in Eq. (15). Intermediate values of n lie in-between the depicted points. As evident, there is excellent agreement with the Planck observations. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 1
Values of r and n_s for shaft inflation with $N = \alpha = 50$ and $n = 2, 4, 6$ and 8.

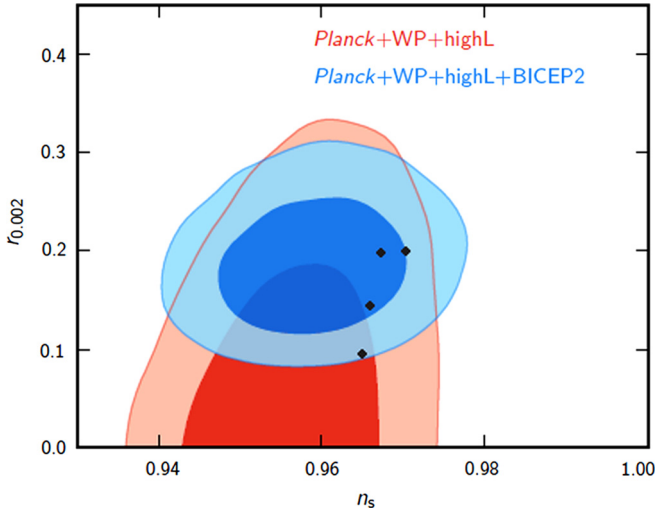| n	r	n_s
2	0.200	0.970
4	0.199	0.967
6	0.141	0.966
8	0.098	0.965

Fig. 3. Allowing for a running spectral index, shaft inflation can accommodate the BICEP2 results. The bullets depict the predictions of shaft inflation for $n = 2, 4, 6$ and 8 from right to left respectively. As evident, there is excellent agreement with the observations. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

Allowing for a running spectral index the BICEP2 results suggest the blue contours shown in Fig. 3. However, it is easy to show that

$$\frac{dn_s}{d\ln k} = -\frac{2(\frac{n+1}{n+2})}{(N + \frac{n+1}{n+2})^2} \sim -\frac{2}{N^2}, \tag{16}$$

which gives $\frac{dn_s}{d\ln k} \sim 10^{-3}$ for $N \simeq 50$, so the running is not substantial.

An estimate for the required value of M is obtained enforcing the COBE bound onto the curvature perturbation. Using Eqs. (3) and (10) we find

$$\sqrt{\mathcal{P}_\zeta} = \frac{1}{2\sqrt{3}\pi} \frac{V^{3/2}}{m_P^3 |V'|}$$

$$\Rightarrow \quad \left(\frac{M}{m_P}\right)^2 = 4\sqrt{3}(n-1)\alpha^{-\frac{n}{n+2}}\pi\sqrt{\mathcal{P}_\zeta}$$

$$\times \left[2(n-1)(n+2)\left(N + \frac{n+1}{n+2}\right)\right]^{-\frac{n+1}{n+2}}. \tag{17}$$

For $n = 2$, $N = 60$ and $\alpha = 1$ {$\alpha = 50$} and taking $\sqrt{\mathcal{P}_\zeta} = 4.706 \times 10^{-5}$ we get $M = 7.7 \times 10^{15}$ GeV {$M = 1.6 \times 10^{15}$ GeV}, which is close to the scale of grand unification, as expected.

Provided ϕ can be arbitrarily large [the vacuum density for large ϕ is constant and remains sub-Planckian, since $M \ll m_P$ and $V(\phi \gg m) \simeq M^4$] one can show that slow-roll inflation can last for a huge number of e-folds. However, far away from the shaft, the potential becomes so flat that the inflaton finds itself in the so-called quantum diffusion zone, leading to eternal inflation [10]. The criterion is as follows.

For eternal inflation to occur, the classical variation of the inflaton field $|\dot{\phi}|$ needs to become subdominant to the quantum variation of ϕ, which is given by the Hawking temperature $\delta\phi = H/2\pi$ per Hubble time $\delta t = H^{-1}$. Comparing the two it is easy to show that

$$|\dot{\phi}| \gtrsim \frac{\delta\phi}{\delta t} \quad \Leftrightarrow \quad |V'| \gtrsim \frac{3}{2\pi} H^3, \tag{18}$$

where we used the slow-roll equation of motion $3H\dot{\phi} \simeq -V'$. In view of Eq. (3) and using the Friedman equation $V(\phi) \simeq 3(Hm_P)^2$, after some algebra, one can show

$$|\dot{\phi}| \gtrsim \frac{\delta\phi}{\delta t} \quad \Leftrightarrow \quad \frac{N}{N_*} \simeq \frac{N + \frac{n+1}{n+2}}{N_* + \frac{n+1}{n+2}} \lesssim (\sqrt{\mathcal{P}_\zeta})^{-\frac{n+2}{n+1}} \sim 10^{4\text{-}6}, \tag{19}$$

where we also used Eq. (17), we considered that $1 < \frac{n+2}{n+1} < \frac{3}{2}$ and with N_* we have denoted the remaining e-folds, when the cosmological scales leave the horizon, i.e. $N_* \simeq 60$.

Thus, we see that, even though the multiverse may be undergoing eternal inflation, our region finds itself relatively close to the potential shaft such that slow-roll takes over and the inflaton gradually moves into the shaft. The inflaton slow-rolls for a few millions of e-folds before the cosmological scales exit the horizon and $N_* \simeq 60$ after this. Eventually, inflation, in our region, ends and the inflaton oscillates at the bottom of the shaft, leading to (p)reheating and the hot big bang. Meanwhile, elsewhere in the multiverse, eternal inflation continues. One can imagine that there may be a

large number of shafts puncturing the inflationary plateau (leading to different vacua, possibly with different values of n). Some regions of the multiverse are close to the shafts in such a way that eternal inflation is superseded by classical slow-roll which attracts the system into the shaft in question. Our observable universe is such a case.

In summary, we introduced and studied a new family of inflation models, which we called shaft inflation. The models correspond to the scalar potential given in Eq. (3) and are parametrised by $n > 1$. We obtained the models in the context of global supersymmetry starting with a superpotential, which interpolates from a generic monomial to an O'Raifearteagh form for small to large values of the inflaton field respectively. However, shaft inflation can be obtained in different setups, as mentioned, for example, in the case $n = 2$. We showed that we obtain values for the spectral index n_s or the tensor to scalar ratio r that are in excellent agreement with the latest observations of the Planck satellite and the BICEP2.

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