Reasoning Mechanism for Cardinal Direction Relations

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Abstract. In the classical Projection-based Model for cardinal directions [6], a two-dimensional Euclidean space relative to an arbitrary single-piece region, a, is partitioned into the following nine tiles: North-West, NW(a); North, N(a); North-East, NE(a); West, W(a); Neutral Zone, O(a);East, E(a); South-West, SW(a); South, S(a); and South-East,SE(a). In our Horizontal and Vertical Constraints Model [9], [10] these cardinal directions are decomposed into sets corresponding to horizontal and vertical constraints. Composition is computed for these sets instead of the typical individual cardinal directions. In this paper, we define several whole and part direction relations followed by showing how to compose such relations using a formula introduced in our previous paper [10]. In order to develop a more versatile reasoning system for direction relations, we shall integrate mereology, topology, cardinal directions and include their negations as well.

Keywords: Cardinal directions, composition table, mereology, topology, qualitative spatial reasoning, vertical and horizontal constraints model.

1 Introduction

Cardinal directions are generally used to describe relative positions of objects in large-scale spaces. The two classical models for reasoning about cardinal direction relations are the cone-shaped and projection-based models [6] where the latter forms the basis of our Horizontal and Vertical Constraints Model.

Composition tables are typically used to make inferences about spatial relations between objects. Work has been done on the composition of cardinal direction relations of points [6], [7], [13] which is more suitable for describing positions of point-like objects in a map. Goyal et. al [8] used the direction-relation matrix to compose cardinal direction relations for points, lines as well as extended objects. Skiadopoulos et. al [15] highlighted some of the flaws in their reasoning system and thus developed a method for correctly computing cardinal direction relations. However, the set of basic cardinal relations in their model consists of 218 elements which is the set of all disjunctions of the nine cardinal directions. In our Horizontal and Vertical Constraints Model, the nine cardinal directions are partitioned into sets based on horizontal and vertical constraints. Composition is computed for these sets instead of the individual cardinal directions, thus helping collapse the typical disjunctive relations into smaller sets. We employed the constraint network of binary direction relations to evaluate the consistency of the composed set relations. Ligozat

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[11] has worked on constraint networks for the individual tiles but not on their corresponding vertical and horizontal sets. Some work relating to hybrid cardinal direction models has been done. Escrig et.al [5] and Clementini et.al [2] combined qualitative orientation combined with distance, while Sharma et. al [14] integrated topological and cardinal direction relations. In order to come up with a more expressive model for direction relations, have extended existing spatial language for directions by integrating mereology, topology, and cardinal direction relations. Additionally, to develop a more versatile reasoning system for such relations, we have included their negations as well.

2 Cardinal Directions Reasoning Model

2.1 Projection-Based Model

In the Projection-based Model for cardinal directions [6], a two-dimensional Euclidean space of an arbitrary single-piece region, a, is partitioned into nine tiles. They are North-West, NW(a); North, N(a); North-East, NE(a); West, W(a); Neutral Zone, O(a); East, E(a); South-West, SW(a); South, S(a); and South-East, SE(a). In this paper, we only address finite regions which are bounded. Thus every region will have a minimal bounding box with specific minimum and maximum x (and y) values (in Table 1). The boundaries of the minimal bounding box of a region *a* is illustrated in Figure 1. The definition of the nine tiles in terms of the boundaries of the minimal bounding box is listed below. Note that all the tiles are regarded as closed regions. Thus neighboring tiles share common boundaries but their interior will remain disjoint.

Table 1. Definition of Tiles

Definition of tiles	
$N(a) \equiv \{\langle x,y\rangle \mid Xmin(a) \le x \le Xmax(a) \wedge y \ge Ymax(a)\}$ $NE(a) \equiv \{\langle x,y\rangle \mid x \ge Xmax(a) \wedge y \ge Ymax(a)\}$ $NW(a) \equiv \{\langle x,y\rangle \mid x \le Xmin(a) \wedge y \ge Ymax(a)\}$ $S(a) \equiv \{\langle x,y\rangle \mid Xmin(a) \le x \le Xmax(a) \wedge y \le Ymin(a)\}$ $SE(a) \equiv \{\langle x,y\rangle \mid x \ge Xmax(a) \wedge y \le Ymin(a)\}$	$SW(a) \equiv \{\langle x,y\rangle \mid x \le Xmin(a) \wedge y \le Ymin(a)\}$ $E(a) \equiv \{\langle x,y\rangle \mid x \ge Xmax(a) \wedge Ymin(a) \le y \le Ymax(a)\}$ $W(a) \equiv \{\langle x,y\rangle \mid x \le Xmin(a) \wedge Ymin(a) \le y \le Ymax(a)\}$ $O(a) \equiv \{\langle x,y\rangle \mid Xmin(a) \le x \le Xmax(a) \wedge Ymin(a) \le y \le Ymax(a)\}$

Table 2. Definitions for the Horizontal and Vertical Constraints Model

Definitions for the Horizontal and Vertical Constraints Model	
WeakNorth(*a*) is the region that covers the *tiles* NW(*a*), N(*a*), and NE(*a*); WeakNorth(*a*) ≡ NW(*a*) ∪ N(*a*) ∪ NE(*a*). Horizontal(*a*) is the region that covers the *tiles* W(*a*), O(*a*), and E(*a*); Horizontal(*a*) ≡ W(*a*), O(*a*), and E(*a*). WeakSouth(*a*) is the region that covers the *tiles* SW(*a*), S(*a*), and SE(*a*); WeakSouth(*a*) ≡ SW(*a*) ∪ S(*a*) ∪ SE(*a*).	WeakWest(*a*) is the region that covers the *tiles* SW(*a*), W(*a*), and NW(*a*); WeakWest(*a*) ≡ SW(*a*) ∪ W(*a*) ∪ NW(*a*). Vertical(*a*) is the region that covers the *tiles* S(*a*), O(*a*), and N(*a*); Vertical(*a*) ≡ S(*a*) ∪ O(*a*) ∪ N(*a*). WeakEast(*a*) is the region that covers the *tiles* *NE*(*a*), E(*a*), and SE(*a*); WeakEast(*a*) ≡ NE(*a*) ∪ E(*a*) ∪ SE(*a*).

2.2 Horizontal and Vertical Constraints Model

In the Horizontal and Vertical Constraints Model [9, 10], the nine tiles are collapsed into six sets based on horizontal and vertical constraints as shown in Figure 1. The definitions of the partitioned regions are shown in Table 2 and the nine cardinal direction *tiles* can be defined in terms of horizontal and vertical sets (see Table 3).

Table 3. Definition of the tiles in terms of Horizontal and Vertical Constraints Sets

NW(*a*)≡ WeakNorth(*a*) ∩ WeakWest(*a*) N(*a*)≡ WeakNorth(*a*) ∩ Vertical(*a*) NE(*a*)≡ WeakNorth(*a*) ∩ WeakEast(*a*)	W(*a*)≡ Horizontal(*a*) ∩ WeakWest(*a*) O(*a*)≡ Horizontal(*a*) ∩ Vertical(*a*) E(*a*)≡ Horizontal(*a*) ∩ WeakEast(*a*)	SW(*a*)≡ WeakSouth(*a*) ∩ WeakWest(*a*) S(*a*)≡ WeakSouth(*a*) ∩ Vertical(*a*) SE(*a*)≡ WeakSouth(*a*) ∩ WeakEast(*a*)

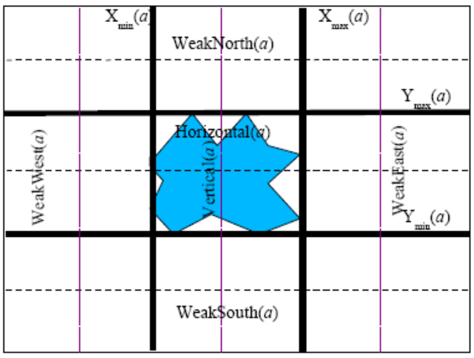

Fig.1. Horizontal and Vertical Sets of *Tiles*

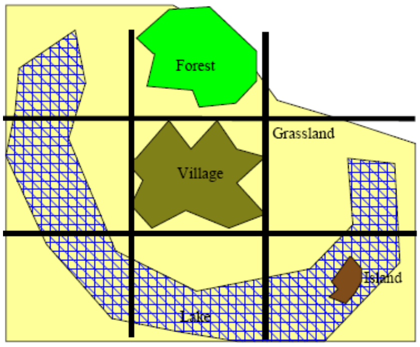

Fig. 2. Spatial Relationships between regions

2.3 RCC Binary Relations

In this paper, we shall use the RCC-5 [3] JPED binary topological relations for regions. They are: PP(*x*, *y*) which means '*x* is a proper part of *y*'; PPi(*x*, *y*) which means '*y* is a proper part of *x*'; EQ(*x*, *y*) which means '*x* is identical with *y*'; PO(*x*, *y*) which means '*x* partially overlaps *y*'; DR(*x*,*y*) which means '*x* is discrete from *y*'. The relations EQ, PO, and DR are symmetric while the rest are not. PPi is also regarded as the inverse of PP. However, in this paper, the relationship PPi will not be considered because all tiles (except for tile O) are unbounded.

2.4 *Whole* or *Part* Cardinal Direction Relations

In our previous paper [8], we created an expressive hybrid mereological, topological and cardinal direction relation model. Here we shall improve the definitions of $A_{\mathbf{R}}(b, a)$ which means that the *whole* destination region, b, is in the tile **R**(*a*) while $P_{\mathbf{R}}(b, a)$ means that *part* of *b* is in tile **R**(*a*).

Cardinal direction relations defined in terms of *tiles*

In this section, we shall introduce several terms to extend the existing spatial language for cardinal directions to facilitate a more versatile reasoning about their relations. We shall use RCC-5 relations to define three categories of direction relations: *whole*, *par*t, and *no part.* $A_N(b, a)$ means *whole* of *b* is in the North *tile* of *a:* $A_N(b, a) \equiv \mathrm{PP}(b, \mathrm{N}(a)) \vee \mathrm{EQ}(b, \mathrm{N}(a))$

Here we adopt the natural language meaning for the word *part* which is '*some but not all*'. $P_N(b, a)$ represents *part* of *b* is in the North tile of *a*. When *part* of *b* is in the North *tile* of *a*, this means that *part* of *b* covers the North *tile* and possibly one or more of the complementary *tiles* of North.

$$P_N(b, a) \equiv \mathrm{PO}(b, \mathrm{N}(a))$$

We shall use the Skiadopoulos et. al [2004] definition of multi-tile cardinal direction relations. As an example, if *part* of *b* is in the North *tile* and the remaining *part* of *b* is in the NorthWest *tile* of *a* (or in other words, *part* of *b* is only in the North and NorthWest *tiles* of *a*) and vice versa, then its representation is

$$P_{N:NW}(b, a) \equiv \mathrm{PO}(b, \mathrm{N}(a)) \wedge \mathrm{PO}(b, \mathrm{NW}(a)) \wedge \mathrm{DR}(b, \mathrm{NE}(a)) \wedge \mathrm{DR}(b, \mathrm{W}(a)) \wedge \mathrm{DR}(b, \mathrm{O}(a)) \wedge \mathrm{DR}(b, \mathrm{E}(a)) \wedge \mathrm{DR}(b, \mathrm{SE}(a)) \wedge \mathrm{DR}(b, \mathrm{S}(a)) \wedge \mathrm{DR}(b, \mathrm{SW}(a))$$

or $P_{N:NW}(b, a) \equiv A_N(b1, a) \wedge A_{NW}(b2, a)$ where $b = b1 \cup b2.$

$\Phi_N(b, a)$ means *no part* of *b* is in the North tile of *a*. When b has no part in the North tile of a, this means that b could be in one or more the complementary tiles of North so

$$\Phi_N(b, a) \equiv \mathrm{DR}(b, \mathrm{N}(a))$$

If *no part* of *b* is in North and Northwest tiles (or in other words, *b* could only be in one or more of the complementary *tiles* of North and Northwest), then the representation is

$$\Phi_{N:NW}(b, a) \equiv \mathrm{DR}(b, \mathrm{N}(a)) \wedge \mathrm{DR}(b, \mathrm{NW}(a))$$

Assume U = {N, NW, NE, O, W, E, S, SW, SE}. The general definition of the following direction relations are in Table 4:

Table 4. Definition of direction relations

D1. $A_R(b, a) \equiv \mathrm{PP}(b, \mathrm{R}(a)) \vee \mathrm{EQ}(b, \mathrm{R}(a))$ where $\mathrm{R} \in \mathrm{U}$ **D2**. $P_R(b, a) \equiv \mathrm{PO}(b, \mathrm{R}(a))$ where $\mathrm{R} \in \mathrm{U}$ **D3.1**. $P_{R1:...:Rn}(b,a) \equiv \mathrm{PO}(b,\mathrm{R1}(a)) \wedge ... \wedge \mathrm{PO}(b,\mathrm{Rn}(a)) \wedge \mathrm{DR}(b,\mathrm{R}'(a))$ where $\mathrm{R1},...,\mathrm{Rn} \in \mathrm{U}$, $1 \le n \le 9$ and $\mathrm{R}' \in \mathrm{U} - \{\mathrm{R1},...,\mathrm{Rn}\}$ **D3.1.** $P_{R1:...:Rn}(b,a) \equiv A_{R1}(b1, a) \wedge ... \wedge A_{Rn}(bn, a)$ where $b=b1 \cup ... \cup bn$, where $\mathrm{R1},...,\mathrm{Rn} \in \mathrm{U}$ and $1 \le n \le 9$	**D4**. $\Phi_R(b, a) \equiv \mathrm{DR}(b, \mathrm{R}(a))$ where $\mathrm{R} \in \mathrm{U}$ **D5**. $\Phi_{R1:...:Rn}(b, a) \equiv \mathrm{DR}(b, \mathrm{R1}(a)) \wedge ... \wedge \mathrm{DR}(b,\mathrm{Rn}(a))$ where $\mathrm{R},...,\mathrm{Rn} \in \mathrm{U}$ and $1 \le n \le 9$. **D6**. $\neg A_R(b, a) \equiv \Phi_R(b, a) \vee P_R(b, a)$ where $\mathrm{R} \in \mathrm{U}$. **D7**. $\neg P_R(b,a) \equiv A_R(b,a) \vee \Phi_R(b,a)$ where $\mathrm{R} \in \mathrm{U}$. **D8.** $\neg\Phi_R(b,a) \equiv A_R(b,a) \vee P_R(b,a)$ where $\mathrm{R} \in \mathrm{U}$.

Negated cardinal direction relations defined in terms of *tiles*

In this section, we shall define three categories of negated cardinal direction relations: not whole, not part, and not no part. Negated direction relations could be used when reasoning with incomplete knowledge. Assume B is a set of the relations, {PP, EQ, PO, DR}. $\neg A_N(b, a)$ means that b is not wholly in North tile of a. It is represented by:

$$\neg A_N(b, a) \equiv \neg[\mathrm{PP}(b, \mathrm{N}(a)) \vee \mathrm{EQ}(b, \mathrm{N}(a))]$$

Use De Morgan's Law and we have $\neg A_N(b, a) \equiv \neg\mathrm{PP}(b, \mathrm{N}(a)) \wedge \neg\mathrm{EQ}(b, \mathrm{N}(a))$

The complement of PP and EQ is {PO, DR} so the following holds:

$$\neg A_N(b, a) \equiv [\mathrm{PO}(b, \mathrm{N}(a))] \vee \mathrm{DR}(b, \mathrm{N}(a))$$

Use **D2** and **D4** and we have *part* of *b* is **not** or *no part* of *b* is in North *tile* of *a* so

$$\neg A_N(b, a) \equiv \Phi_N(b, a) \vee P_N(b, a)$$

$\neg P_N(b, a)$ means b is not partly in North tile of a so $\neg P_N(b, a) \equiv \neg\mathrm{PO}(b, \mathrm{N}(a))$

The complement of PO is {PP, EQ, DR} so the following holds:

$$\neg P_N(b, a) \equiv [\mathrm{PP}(b, \mathrm{N}(a)) \vee \mathrm{EQ}(b, \mathrm{N}(a))] \vee \mathrm{DR}(b, \mathrm{N}(a))$$

Use **D1**, **D4**, we have $\neg P_N(b, a) \equiv A_N(b, a) \vee \Phi_N(b, a)$

$\neg\Phi_N(b, a)$ means not no part of b is in the North tile of a. Thus

$\neg\Phi_N(b, a) \equiv \neg\mathrm{DR}(b, \mathrm{N}(a))$ or $\neg\Phi_N(b, a) \equiv [\mathrm{PP}(b, \mathrm{N}(a)) \vee \mathrm{EQ}(b, \mathrm{N}(a))] \vee \mathrm{PO}(b, \mathrm{N}(a))$

Use **D1**, **D2** and **D4**, we have the following: $\neg\Phi_N(b, a) \equiv A_N(b,a) \vee P_N(b, a)$

Assume U = {N, NW, NE, O, W, E, S, SW, SE}. The general definition of the *negated* direction relations are in Table 4. Here we shall give an example to show how some of the aforementioned *whole-part* relations could be employed to describe the spatial relationships between regions. In Figure 2, we shall take the village as the referent region while the rest will be destination regions. The following is a list of possible direction relations between the village and the other regions in the scene:

- A_N(forest,village): The whole forest is in the North tile of the village and A_{SE}(island,village): the whole island is in the SouthEast tile of the village.
- $P_{NW:W:SW:S:SE:E}$(lake,village): Part of the lake is in the NorthWest, West, SouthWest, South, SouthEast and East tiles of the village.
- $\Phi_{O:N:NE}$(lake,village): This is another way to represent the direction relationship between the lake and village. t means no part of the lake is in the Neutral, North and NorthEast tiles of the village.
- $P_{O:N:NE:NW:W:SW:S:SE:E}$(grassland,village): Part of the grassland is in all the tiles of the village.

Next we shall show how negated direction relations could be used to represent incomplete knowledge about the direction relations between two regions. Assume that we have a situation where the hills are not wholly in the North tile of the village. We can interpret such incomplete knowledge using D6, part or no part direction relations: P_N(hills, village)$\vee\Phi_N$(hills,village). In other words, either there is no hilly region is in the North tile of the village or part of the hilly region covers the North tile of the village. If we are given this piece of information 'it is not true that no part of the lake lies in the North tile of the village', we shall use D8 to interpret it. Thus we have the following possible relations: A_N(lake,village)$\vee P_N$(lake,village). This means that the whole or only part of the lake is in the North tile of the village.

2.5 Cardinal Direction Relations Defined in Terms of Horizontal or Vertical Constraints

The definitions of cardinal direction relations expressed in terms of horizontal and vertical constraints are similar to those shown in the previous section (**D1** to **D8**). The only difference is that the universal set, U is {WeakNorth (WN), Horizontal (H), WeakSouth (WS), WeakEast (WE), Vertical (V), WeakWest (WW)}.

Whole and *part* cardinal direction relations defined in terms of horizontal and vertical constraints

In this section, we use examples to show how *whole* and *part* cardinal direction relations could be represented in terms of horizontal and vertical constraints. We shall exclude the inverse and negated relations for reasons that will be given in the later part of this paper. We shall use abbreviations {*WN, H, WS*} for {*WeakNorth, Horizontal, WeakSouth*} and {*WE, V, WW*} for {*WeakEast, Vertical, WeakWest*} respectively.

D9. $A_N(b, a) \equiv A_{WN}(b, a) \wedge A_V(b, a)$
D10. $P_N(b, a) \equiv P_{WN}(b, a) \wedge P_V(b, a)$
D11. $P_{N:NW}(b, a) \equiv A_N(b_1, a) \wedge A_{NW}(b_2, a) \equiv [A_{WN}(b_1, a) \wedge A_V(b_1, a)] \wedge [A_{WN}(b_2, a) \wedge A_{WW}(b_2, a)]$ where $b = b_1 \cup b_2$
D12. $\Phi_N(b, a) \equiv \Phi_{WN}(b, a) \wedge \Phi_V(b, a)$
D13. $\Phi_{N:NW}(b,a) \equiv \Phi_N(b,a) \wedge \Phi_N(b,a) \equiv [\Phi_{WN}(b,a) \wedge \Phi_V(b,a)] \wedge [\Phi_{WN}(b, a) \wedge \Phi_{WW}(b, a)]$

Next we shall use the *part* relation as a primitive for the definitions of the *whole* and *no part* relations. Once again assume U = {N, NW, NE, O, W, E, S, SW, SE}.
D14.1. $A_R(b, a) \equiv P_R(b, a) \wedge [\neg P_{R1}(b, a) \wedge \neg P_{R2}(b, a) \wedge \ldots \wedge \neg P_{Rm}(b, a)]$ where R$\in$ U, Rm $\in$ U – {R} (which is the complement of R), and $1 \le m \le 8$. As an example,
$A_N(b,a) \equiv P_N(b,a) \wedge [\neg P_{NE}(b,a) \wedge \neg P_{NW}(b,a) \wedge \neg P_w(b,a) \wedge \neg P_O(b,a) \wedge \neg P_E(b,a) \wedge \neg P_{SW}(b,a) \wedge \neg P_S(b,a) \wedge \neg P_{SE}(b,a)]$
D14.2. $A_{HR}(b, a) \equiv P_{HR}(b, a) \wedge [\neg P_{HR1}(b, a) \wedge \ldots \wedge \neg P_{HRn}(b, a)]$ where HR $\in$ {WN, H, WS}, HRm is the complement of HR, and $1 \le n \le 3$ .As an example, $A_{WN}(b, a) \equiv P_{WN}(b, a) \wedge [\neg P_H(b, a) \wedge \neg P_{WS}(b, a)]$
D14.3. $A_{VR}(b, a) \equiv P_{VR}(b, a) \wedge [\neg P_{VR1}(b, a) \wedge \ldots \wedge \neg P_{VRn}(b, a)]$ where VR$\in$ {WW, V, WE}, VRm is the complement of VR), and $1 \le n \le 3$ .As an example, $A_{WW}(b, a) \equiv P_{WW}(b, a) \wedge [\neg P_V(b, a) \wedge \neg P_{WE}(b, a)]$
D15.1.$\Phi_R(b, a) \equiv \neg P_R(b, a) \wedge [P_{R1}(b, a) \vee P_{R2}(b, a) \vee \ldots \vee P_{Rm}(b, a)]$ where R$\in$ U, Rm $\in$ U – {R}, and $1 \le m \le 8$ .As an example,
$\Phi_N(b,a) \equiv \neg P_N(b,a) \wedge [P_{NE}(b, a) \vee P_{NW}(b,a) \vee P_w(b,a) \vee P_O(b,a) \vee P_E(b,a) \vee P_{SW}(b,a) \vee P_S(b,a) \vee P_{SE}(b,a)]$
D15.2.$\Phi_{HR}(b, a) \equiv \neg P_{HR}(b, a) \wedge [P_{HR1}(b, a) \vee P_{HR2}(b, a)]$ where HR$\in$ {WN,H, WS}, while HR1 and HR2 constitute its complement. As an example, $\Phi_{WN}(b, a) \equiv \neg P_{WN}(b, a) \wedge [P_H(b, a) \vee P_{WS}(b, a)]$.
D15.3.$\Phi_{VR}(b, a) \equiv \neg P_{VR}(b, a) \wedge [P_{VR1}(b, a) \vee P_{VR2}(b, a)]$ where VR$\in$ {WW,V, WE}, while VR1 and VR2 constitute its complement. As an example, $\Phi_{WW}(b, a) \equiv \neg P_{WW}(b, a) \wedge [P_V(b, a) \vee P_{WE}(b, a)]$.

3 Composition Table for Cardinal Directions

Ligozat (1988) obtained the outcome of the composition of all the nine tiles in a *Projection Based Model* for point objects by composing the constraints {<, =, >}. However, our composition tables (Tables 5 and 6) are computed using the vertical and horizontal constraints of the sets of direction relations. We shall abstract several composition rules in Table 5. Similar rules apply to Table 6. Assume U is { A_{WE}, A_V, A_{WW} }. *WeakEast*(*WE*) is considered the converse of *WeakWest* (*WW*) and vice versa.
Rule 1 (Identity Rule): $R \wedge R = R$ where $R \in U$.
Rule 2 (Converse Rule): $S \wedge S' = U$, $A_V \wedge S = P_V \vee P_S$ where $S \in \{A_{WE}, A_{WW}\}$ and S' is its converse.

Here we shall introduce several axioms that are necessary for the direction reasoning mechanism. In the next section we shall show how to apply these axioms and some logic rules for making inferences about direction relations.

Axiom 1. $A_R(b_1,a) \wedge A_R(b_2,a) \wedge ... \wedge A_R(b_k,a) \rightarrow A_R(b,a)$ where R $\in$ U, $1 \leq k \leq 9$ and $b_1 \cup b_2 \cup ... \cup b_k = b$

Axiom 2. $A_{R1}(b_1,a) \wedge A_{R2}(b_2,a) \wedge ... \wedge A_{Rn}(b_k,a) \rightarrow P_{R1:R2:...:Rn}(b,a)$ where Rn $\in$ U, $1 \leq k \leq 9$ and $b_1 \cup b_2 \cup ... \cup b_k = b$

Axiom 3. $P_R(c_k,a) \wedge PP(c_k,c) \rightarrow P_R(c,a)$ where R $\in$ U, and $1 \leq k \leq 9$

Axiom 4. $[P_{R1}(c_1,a) \wedge PP(c_1,c)] \wedge [P_{R2}(c_2,a) \wedge PP(c_2,c)] \wedge ... \wedge [P_{Rk}(c_k,a) \wedge PP(c_k,c)] \rightarrow P_{R1:R2:...:Rk}(c,a)$ where $1 \leq k \leq 9$, and Rk $\in$U.

Axiom 5. $A_R(c_k,a) \wedge PP(c_k,c) \rightarrow P_R(c,a)$ where R $\in$ U, and $1 \leq k \leq 9$

Axiom 6. $\neg\{[P_{WW}(c_1,a) \wedge PP(c_1,c)] \wedge [P_{WE}(c_2,a) \wedge PP(c_2,c)]\}$ where $c_1 \cup c_2 = c$ (because c is a single connected piece)

Axiom 7. $\neg\{[P_{WN}(c_1,a) \wedge PP(c_1,c)] \wedge [P_{WS}(c_2,a) \wedge PP(c_2,c)]\}$ where $c_1 \cup c_2 = c$ (because c is a single connected piece)

3.1 Formula for Computation of Composition

In our previous paper [10], we introduced a formula (obtained through case analyses) for computing the composition of cardinal direction relations. Here we shall modify the notations used for easy comprehension. Skiadopoulos et. al [15] introduced additional concepts such as rectangular versus nonrectangular direction relations, bounding rectangle, westernmost (etc...) to facilitate the composition of relations. They have separate formulae for the composition of rectangular and non-rectangular regions. However, in this paper we shall apply one formula for the composition of all types of direction relations. The basis of the formula is to first consider the direction relation between a and each individual part of b followed by the direction relation between each individual part of b and c. Assume that the region b covers one or more tiles of region a while region c covers one or more tiles of b. The direction relation between a and b is $R(b,a)$ while the direction relation between b and c is $S(c,b)$. The composition of direction relations could be written as follows:

$$R(b,a) \wedge S(c,b)$$

Firstly, establish the direction relation between a and each individual part of b.

$R(b,a) \wedge S(c,b) \equiv [R_1(b_1,a) \wedge R_2(b_2,a) ... \wedge R_k(b_k,a)] \wedge [S(c,b)]$……….where $1 \leq k \leq 9$…………………....(1)

Consider the direction relation of each individual part of b and c. Equation (1) becomes: $[R_1(b_1,a) \wedge S_1(c,b_1)] \wedge [R_2(b_2,a) \wedge S_2(c,b_2)] \wedge ... \wedge [R_k(b_k,a) \wedge S_k(c,b_k)]$…where $1 \leq k \leq 9$……(2)

3.2 Composition of Cardinal Direction Relations

Previously we have grouped the direction relations into three categories namely: whole, part, and no part. If we include their respective inverses and negations, there will be a total of 9 types of direction relations. However, we do not intend to delve into the composition of inverse and negated relations due to the high level of uncertainty involved. Typically, the inferences drawn would consist of the universal set of tiles, which is not beneficial. In this paper, we shall demonstrate several types of composition. The type of composition shown in this part of the paper involves the composition of vertical and horizontal sets which is different from Skiadopoulos et. al's work [15] involving the composition of individual tiles. Use Tables 5 and 6 to obtain the outcome of each composition. The meaning of the two following notations $U_V(c,a)$ and $U_H(c,a)$ are in Tables 5 and 6.

Table 5. Composition of Vertical Set Relations

		WeakEast	Vertical	WeakWest
		$A_{WE}(c,b)$	$A_{V}(c,b)$	$A_{WW}(c,b)$
WeakEast	$A_{WE}(b,a)$	$A_{WE}(c,a)$	$A_{WE}(c,a)$	$U_{V}(c,a)$
Vertical	$A_{V}(b,a)$	$P_{WE} \vee P_{V}(c,a)$	$A_{V}(c,a)$	$P_{WW} \vee P_{V}(c,a)$
WeakWest	$A_{WW}(b,a)$	$U_{V}(c,a)$	$A_{WW}(c,a)$	$A_{WW}(c,a)$

Note: $U_{V}(c,a)$=[$P_{WE}(c,a) \vee P_{V}(c,a) \vee P_{WW}(c,a)$].Therefore the possible set of relations is {[$A_{WE}(c,a)$, $A_{V}(c,a)$, $A_{WW}(c,a)$, $P_{WE:V:WW}(c,a)$, $P_{WE:V}(c,a)$, $P_{WW:V}(c,a)$}.

Table 6. Composition of Horizontal Set Relations

		WeakNorth	Horizontal	WeakSouth
		$A_{WN}(c,b)$	$A_{H}(c,b)$	$A_{WS}(c,b)$
WeakNorth	$A_{WN}(b,a)$	$A_{WN}(c,a)$	$A_{WN}(c,a)$	$U_{H}(c,a)$
Horizontal	$A_{H}(b,a)$	$P_{WN} \vee P_{H}(c,a)$	$A_{H}(c,a)$	$P_{WS} \vee P_{H}(c,a)$
WeakSouth	$A_{WS}(b,a)$	$U_{H}(c,a)$	$A_{WS}(c,a)$	$A_{WS}(c,a)$

Note: $U_{H}(c,a)$=[$P_{WN}(c,a) \vee P_{H}(c,a) \vee P_{WS}(c,a)$]. Therefore the possible set of relations is {$A_{WN}(c,a)$, $A_{H}(c,a)$, $A_{WS}(c,a)$, $P_{WN:H:WS}(c,a)$, $P_{WN:H}(c,a)$, $P_{WS:H}(c,a)$]}.

Example 1

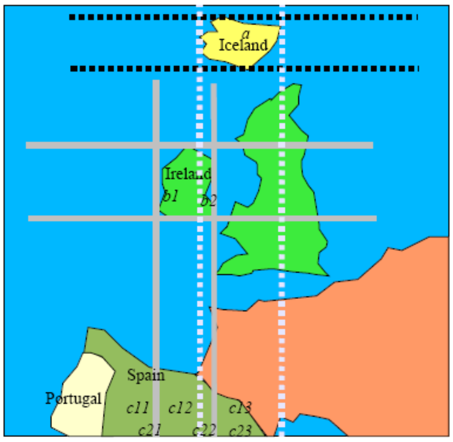

Fig. 3. Spatial relationships among regions in Europe

In Figure 3, *part* of Ireland (b) is only in the South and SouthWest *tiles* of Iceland (a) while the *part* of Spain (c) is in the SouthWest, South and SouthEast *tiles* of Ireland. We have to make an inference about the direction relation between Iceland and Spain. We shall represent the information as:

$P_{SW:S}$(Ireland,Iceland)$\wedge$ $P_{SW:S:SE}$(Spain,Ireland)

Use the abbreviations a, b, c to represent Iceland, Ireland, and Spain respectively. The above expression is written as:

$P_{SW:S}(b, a) \wedge P_{SW:S:SE}(c, b)$................(3a)

Firstly, establish the direction relation between a and each individual part of b. Use **D3** and expression in (3a) becomes

$[A_{SW}(b_1,a) \wedge A_{S}(b_2,a)] \wedge [P_{SW:S:SE}(c, b)]$.............(3b)

Use the extended boundaries of part region b_1 to partition c. As depicted in Figure 3, c is divided into 3 subregions (c_{11}, c_{12}, and c_{13}). Establish direction relations between these regions and b_1. We have $A_{SW}(c_{11},b_1)$,$A_{S}(c_{12},b_1)$, and $A_{SE}(c_{13},b_1)$.Repeat the same procedure for b_2 and we have the following direction relations between b_2 and its corresponding subregions:

$A_{SW}(c_{21},b_2)$, $A_{S}(c_{22},b_2)$ and $A_{SE}(c_{23},b_2)$

Expression (3b) becomes:

$[A_{SW}(b_1,a)\wedge A_S(b_2,a)]\wedge\{[A_{SW}(c_{11},b_1)\wedge A_S(c_{12},b_1)\wedge A_{SE}(c_{13},b_1)]\wedge [A_{SW}(c_{21},b_2)\wedge A_S(c_{22},b_2)\wedge A_{SE}(c_{23},b_2)]\}$...(3c)

Apply formula (2) into expression (3c) and we have

$$\{[A_{SW}(b_1,a)\wedge A_{SW}(c_{11},b_1)] \wedge [A_{SW}(b_1,a)\wedge A_S(c_{12},b_1)]\wedge[A_{SW}(b_1,a) \wedge A_{SE}(c_{13},b_1)]\}\wedge$$
$$\{[A_S(b_2,a)\wedge A_{SW}(c_{21},b_2)]\wedge [A_S(b_2,a) \wedge A_S(c_{22},b_2)]\wedge [A_S(b_2,a)\wedge A_{SE}(c_{23},b_2)]\}\text{............(3d)}$$

We shall compute the vertical and horizontal constraints separately and apply formulae similar to **D9.**

Composition of Horizontal Constraints

$$[[A_{WS}(b_1,a)\wedge A_{WS}(c_{11},b_1)]\wedge [A_{WS}(b_1,a)\wedge A_{WS}(c_{12},b_1)]\wedge[A_{WS}(b_1,a)\ A_{WS}(c_{13},b_1)]] \wedge$$
$$[[A_{WS}(b_2,a)\wedge A_{WS}(c_{21},b_2)]\wedge [A_{WS}(b_2,a) \wedge A_{WS}(c_{22},b_2)]\wedge[A_{WS}(b_2,a) \wedge A_{WS}(c_{23},b_2)]]$$

Use Table 6 and we have

$$[A_{WS}(c_{11},a)\wedge A_{WS}(c_{12},a)\wedge A_{WS}(c_{13},a)] \wedge [A_{WS}(c_{21},a)\wedge A_{WS}(c_{22},a)\wedge A_{WS}(c_{23},a]$$

However, as shown earlier, $c_{11}\cup c_{12}\cup c_{13}$ = d $c_{21}\cup c_{22}\cup c_{23} = c$. Use Axiom 1 and the modus ponens inference rule (P □Q; P, □Q) and the above expression becomes $A_{WS}(c,a)\wedge A_{WS}(c,a)$ which equals $A_{WS}(c,a)$.

Composition of Vertical Constraints

$$[[A_{WW}(b_1,a)\wedge A_{WW}(c_{11},b_1)]\wedge[A_{WW}(b_1,a)\wedge A_V(c_{12},b_1)]\wedge[A_{WW}(b_1,a)\wedge A_{WE}(c_{13},b_1)]] \wedge$$
$$[[A_V(b_2,a)\wedge A_{WW}(c_{21},b_2)]\wedge [A_V(b_2,a) \wedge A_V(c_{22},b_2)]\wedge [A_V(b_2,a) \wedge A_{WE}(c_{23},b_2)]]$$

Use Table 5 and we have

$$[A_{WW}(c_{11},a)\wedge A_{WW}(c_{12},a)\wedge U_V(c_{13},a)]\wedge[(P_{WW}\vee P_V)(c_{21},a)\wedge A_V(c_{22},a)\wedge (P_{WE}\vee P_V)(c_{23},a)]$$

Use **Axiom 5**, **D15.3**, and the expression becomes:

$\{P_{WW}(c, a) \wedge P_{WW}(c, a) \wedge [(P_{WW} \vee P_V\vee P_{WE})(c, a)]\}\wedge \{(P_{WW}\vee P_V)(c, a)] \wedge P_V(c, a) \wedge [(P_{WE}\vee P_V)(c, a)]\}$

Use **Axiom 6**, distributivity, idempotent, and absorption rules to compute the first part of the expression

$$\{P_{WW}(c, a) \wedge P_{WW}(c, a) \wedge [(P_{WW} \vee P_V\vee P_{WE})(c, a)]\}$$
$$=\{P_{WW}(c, a) \wedge [(P_{WW} \vee P_V\vee P_{WE})(c, a)]\}$$
$$=[P_{WW}(c, a) \wedge P_{WW}(c, a)]\vee [P_{WW}(c, a) \wedge P_V(c, a)]\vee[P_{WW}(c, a) \wedge P_{WE}(c, a)]$$
$$=[P_{WW}(c, a)]\vee [P_{WW}(c, a) \wedge P_V(c, a)]$$
$$=P_{WW}(c, a)\text{...(4a)}$$

Use absorption rule to compute the second part of the expression

$$\{[(P_{WW}\vee P_V)(c, a)] \wedge P_V(c, a) \wedge [(P_{WE}\vee P_V)(c, a)]\}$$
$$= P_V(c, a) \wedge [(P_{WE}(c, a) \vee P_V(c, a)]\text{..................(4b)}$$

Combine the computed expressions in (4a) and (4b) and apply distributivity rule:

$$P_{WW}(c, a) \wedge P_V(c, a) \wedge [(P_{WE}(c, a) \vee P_V(c, a)]$$
$$=[P_{WW}(c, a) \wedge P_V(c, a) \wedge (P_{WE}(c, a)] \vee [P_{WW}(c, a) \wedge P_V(c, a)]$$

The outcome of the composition could be written as

$$A_{WS}(c,a) \wedge[P_{WW:V:WE}(c, a) \vee P_{WW:V}(c, a)]$$

which means c covers the SouthWest, South and SouthEast or SouthWest and South *tiles* of a. And this is confirmed by the direction relation between Iceland and Spain depicted in Figure 3.

4 Conclusion

In this paper, we have developed and formalised *whole part* cardinal direction relations to facilitate more expressive scene descriptions. We have also introduced a refined formula for computing the composition of such type of binary direction relations. Additionally, we have shown how to represent constraint networks in terms of weak cardinal direction relations. We demonstrated how to employ them for evaluating the consistency of composed weak direction relations.

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