Detecting Wash Trade in Financial Market Using Digraphs and Dynamic Programming

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Abstract— **Wash trade refers to the illegal activities of traders who utilise carefully designed limit orders to manually increase the trading volumes for creating a false impression of an active market. As one of the primary formats of market abuse, wash trade can be extremely damaging to the proper functioning and integrity of capital markets. Existing work focuses on collusive clique detections based on certain assumptions of trading behaviours. Effective approaches for analysing and detecting wash trade in a real-life market have yet to be developed. This paper analyses and conceptualises the basic structures of the trading collusion in a wash trade by using a directed graph of traders. A novel method is then proposed to detect the potential wash trade activities involved in a financial instrument by first recognizing the suspiciously matched orders and then further identifying the collusions among the traders who submit such orders. Both steps are formulated as a simplified form of the Knapsack problem, which can be solved by dynamic programming approaches. The proposed approach is evaluated on seven stock datasets from NASDAQ and the London Stock Exchange. Experimental results show that the proposed approach can effectively detect all primary wash trade scenarios across the selected datasets.**

***Index Terms*—Market Abuse, Directed Graph, Dynamic Programming, Wash Trade.**

I. Introduction

Surveillance of a financial exchange market for preventing market abuse activities has been attracting significant academic and industrial attention after the financial crisis in 2008 and especially since the flash crash in 2010. The abuse of financial markets can occur in a variety of ways, all of which can be extremely damaging to the proper functioning and integrity of the market. Trade-based manipulation, where the manipulation tactic is carried out only by simply buying and selling (Franklin & Douglas, Stock Price Manipulation, 1992), is one of the primary forms. Price and volume are usually two major objects to be manipulated, and the former format, price manipulation, is thoroughly studied in a previous work by the authors (Cao, Li, Coleman, Belatreche, & T.M.McGinnity, Adaptive Hidden Markov Model with Anomaly States for Price Manipulation Detection, 2015) (Cao, Li, Coleman, Belatreche, & McGinnity, A Hidden Markov Model with Abnormal States for Detecting Stock Price Manipulation, Oct. 2013) (Cao, Li, Coleman, Belatreche, & McGinnity, Detecting price manipulation in the financial market, Mar. 2014) (Cao, Li, Coleman, Belatreche, & McGinnity, Detecting wash trade in the financial market, Mar. 2014) (Zhai & Cao, Mar. 2014). Another format of trade-based abuse is volume manipulation, the manipulation actions intending to increase the transaction volume for the purpose of giving a false impression of high trading volume on the market (Franklin & Douglas, Stock Price Manipulation, 1992) (Franklin, Litov, & Mei, Large investors, price manipulation, and limits to arbitrage: An anatomy of market corners, 2006). The major form of volume manipulation is wash trade, which occurs when the same individuals or a group of collusive clients are on both sell and buy sides of a financial instrument (i.e. stock) trading. While there is no beneficial change in ownership, wash trading has the effect of creating a misleading appearance of an active interest in the stock (Douglas, Feng, & Aitken, 2012).

Wash trade usually does not contain any illegal actions such as financial rumour spreading and market resource squeezing but is carried out only by legitimate trading activities. With carefully designed buy and sell order sequences, manipulators can make the transaction follow their expectation. In the wash trade tactics, a series of orders is often submitted as a number of order pairs. The monitoring of any single leg of one pair or part of a pair would not be concluded as collusive trading. Most of the existing related literature studies the collusive cliques according to the “activity similarity”, which is defined under certain assumptions. Very few address quantitative analysis of the features of different wash trade scenarios and the corresponding detection approaches. This paper follows on from our previous work on trade-based manipulation (Cao, Li, Coleman, Belatreche, & T.M.McGinnity, Adaptive Hidden Markov Model with Anomaly States for Price Manipulation Detection, 2015) and proposes a detection approach that considers a complete spectrum of the wash trade detection. The main contributions of the work are as follows: the problem of wash trade is thoroughly discussed including the analysis of all possible scenarios, from which the key features are extracted and quantified. This provides a clear problem formulation and explains the significance of exploring the conceptual models. To the best of our knowledge, this is the first theoretical study of wash trade market manipulation. A two-step algorithm is proposed to detect wash trade activities. The proposed two

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steps, which consist of discovering the matching orders and further recognizing the collusions, are both formulated as a combinatorial optimisation problem and solved by one unified algorithm. Extensive experiments have been conducted on real data from both USA and UK markets for testing the practicability of the proposed wash trade detection method in real-life.

The remainder of the paper is organised as follows. Section II provides a review of wash trade manipulation and the corresponding detection methods. The features of all types of wash trade scenarios as well as the proposed detection approach are analysed, formulated and characterised in Section III. Performance evaluation of the proposed approach is provided in Section IV. Finally, Section V concludes the paper and discusses potential improvements and future work.

II. Wash Trade and its Detection

A. Wash Trade

In capital markets, limit orders, indicate the trading intention of the trader to buy or sell volumes of a specific equity at a specific price or better (SEC, 2011) ("Better price" refers to higher selling prices or lower buying prices). The transaction occurs when eligible orders meet order-matching rules. The outstanding unmatched limit orders are recorded in order books of the exchange market, in which the highest buying price decides the best bid price while the lowest selling price is the best ask price. The gap between the best bid and ask price is defined as bid-ask spread (Kojo & Paudyal, 2000). In most of the exchange markets, the matching rule selects the earliest order with the matched price for execution. In the following examples in Table 1, three limit orders, #01, #02 and #03 are submitted in sequence to the exchange market. According to the matching rule, order #03 is firstly executed by 300 shares with #01, which has the same price but is earlier than order #02, and then the remaining 100 shares are executed with #02.

Table 1 Limit order sequences

Order #	Trader	Time	Buy/Sell	Price	Volume
01	***A***	09:00:000	Buy	125	300
02	***A***	09:05:000	Buy	125	300
03	***B***	09:06:100	Sell	125	400

Wash trades follow the same matching rules as legitimate transactions with the special feature defined by the Financial Conduct Authority (FCA) as "no change in beneficial interest or market risk", or "the transfer of beneficial interest or market risk only between parties acting in concert or collision, other than for legitimate reasons" (FSA, 2006). The Committee of European Securities Regulators (CESR) further indicates that a wash trade is the "deliberate arrangement in concert or collusion" (EU, Market Abuse Directive, 2014). On August 28 2014, Chicago Mercantile Exchange (CME) released a new rule (adopted by U.S. Commodity Futures Trading Commission CFTC), termed as "Rule 575" (CME, 2014). Rule 575 clearly states that "no person shall enter messages to the market as pre-arranged collusion (wash trade) with intent to mislead other participants". The definition in Rule 575 in the U.S. shows the consistent regulation to CESR in Europe that the pre-arranged collusive trading is wash trade and shall be strictly prohibited. Although clearly defined the wash trade activity, the regulators (FCA, CESR, and CFTC) do not provide any quantitative approach on detecting such activities.

As illustrated by the example in Table 2, the simplest format of wash trade is the simultaneous submission of two opposite limit orders with identical price (125 in Table 2) and similar volume (495 in Table 2) from one trader ***A***. By the matching rules, order #01 and #02 match and 495 shares are executed immediately after the submission. Additionally, the wash trade actions can be also carried out by multiple orders and traders as the example formats shown in Table 3 and Table 4. In Table 3, order #03 is matched and executed with #01 and #02 sequentially so that a transaction of 490 shares can be artificially created by trader ***A***. In Table 4, two transactions are created by four matched orders between traders ***A*** and ***B***. After the transactions (450 matched volumes), there is almost no effective transfer of beneficial interest among the two traders.

Table 2 Basic format of wash trade

Order #	Trader	Time	Buy/Sell	Price	Volume
01	***A***	09:00:000	Buy	125	500
02	***A***	09:00:001	Sell	125	495

Summarising the typical formats in Table 2 and Table 4 as well as the definitions from the regulators, we obtain three features of a successful execution of a wash trade manipulation:

1. Tight submission intervals between the matched buy and sell orders (to minimize the risk of the orders being unintentionally picked up by other traders);
2. Executable prices (to make the orders an immediate execution);
3. Mostly matched volumes (to minimize the risk of loss from the unmatched volumes executed with other traders).

Table 3 Wash trade with multiple orders

Order #	Trader	Time	Buy/Sell	Price	Volume
01	***A***	09:00:000	Buy	125	250
02	***A***	09:00:001	Buy	125	250
03	***A***	09:00:002	Sell	125	490

Table 4 Wash trade with multiple traders

Order #	Trader	Time	Buy/Sell	Price	Volume
01	***A***	09:00:000	Buy	125	500
02	***B***	09:00:001	Sell	124.2	490
03	***B***	09:10:000	Buy	125.5	490
04	***A***	09:10:001	Sell	125	500

Perfect matching orders, which have the same price, volume and submission time according to the summarised features, guarantee the execution but are obviously easy to be suspected as market abuse trade by the regulators. Therefore, to avoid being easily detected, "smart manipulators" design the wash trade orders to be "mostly matched" such as the examples in Table 2 and Table 4, where around 99% volumes are executed, respectively. Similarly, due to the matching rules in most exchange markets (Bowen, 2013), that is buy (sell) limit order matching sell (buy) limit orders with the same price or lower (higher), the limit prices in the examples in Table 4, which are different but executable, are also deliberately designed to avoid inspection. In Table 4, order #02 can be executed with order #01 at price 125 and order #04 can be executed with order #03 at price 125.5. The 125 and 125.5 are the execution prices of the two possible transactions; we refer to such prices as transaction prices.

B. *Wash Trade Detection*

To the best of our knowledge, there is no related work on the detection of wash trade activities in capital markets. The only analogous research is work on the detection of collusive cliques based on certain similar trading behaviours, which are defined as the buy/sell activities of equities in a similar way. A spectral clustering based approach was developed (Markus., Hoser, & Schröder, 2007), where a trading-behavioural network is generated and any behaviour that deviates from the network is reported as an irregularity. The assumption of this work is the strong consistency between a trader's current behaviours and his/her previous trading network. A graph clustering algorithm for detecting a set of collusive traders has been proposed in (Palshikar & Apte, 2008). The relationship between traders is constructed as a stock flow graph, and those with "heavy trading" within their network are clustered as a collusion set.

A new trading collusion detection approach, the correlation matrix of one trading day, was presented in recent work (Wang, Zhou, & Guan, 2012), where trader behaviour was represented by an aggregated time series of signed volumes of submitted orders. The similarities of behaviours among multiple traders are measured by Pearson's product-moment coefficient and the cliques with a coefficient higher than a user-specified threshold were considered as suspicious collusions. The experiments of this study evaluated the real order data of futures traded in the Shanghai Futures Exchange. The "signed order volume" is constructed by volumes and directions (buy/sell) of the order. The order price information is ignored according to the assumption that order prices are not related to the trader's behaviours (Wang, Zhou, & Guan, 2012). However, the market impact measure shows that order price significantly impacts the market (Hautsch & Huang, The market impact of a limit order, 2012) so that the market moves caused by the traders' own actions (orders) become the principal part of the transaction costs (ITG, 2013). It is therefore unacceptable to ignore the order price information, which not only distinguishes traders' intention, but is a key feature of wash trade manipulation tactics.

A technique developed by the Chicago Mercantile Exchange (CME) to prevent wash trades at the "engine level" was rolled out in the middle of 2011 (Patterson, Strasburg, & Trindle, 2013) and updated in the summer of 2013 (Bowen, 2013). However, it only monitored the same-priced buy/sell orders from trading accounts with the same beneficial ownership (Bowen, 2013) (example in Table 2). The lack of the surveillance mechanisms for wash trades with multiple orders or traders (example illustrations in Table 3 and

Table 4) left it possible for collusive parties to create a number of transactions that give a false appearance of large trading volumes.

In December 2012, a wash trade case was manually inspected and documented by the Securities and Exchange Commission of Pakistan (Jamal, 2012). In March 2013, the US regulators started to investigate traders acting as both buyer and seller in the same transactions and reported that several hundred potential wash trades occur each day on CME and Intercontinental Exchange (ICE) (Patterson, Strasburg, & Trindle, 2013). In June 2012, the Hong Kong financial regulator claimed that the attempts of entering wash trade or matched trade were financial manipulation crimes whether or not the wash trade or matched trade in fact has, or is likely to have, the effect of misleading appearance (Loh & Cumming, 2012). This ruling was also accepted by "Rule 575" (CME, 2014) and the Market Abuse Directive II (EU, European Commission, 2015). This rule provided an aggressive restriction: any attempts of wash trade or matched trade are financial crimes.

To date, academic research has mainly focused on detecting the overall trading collusions according to defined analogous behaviours. The detection of mass market behaviours can hardly reach a precise and determinable manipulation detection result but can show a collective correlation of trading activities among different trader clusters. Industry techniques merely covered the simple format of wash trade scenarios. A slightly improved manipulation tactic can bypass the wash trade monitoring. However, no efforts appear to have been made in the analysis of wash trade strategic behaviour or the design of a detection approach identifying any tactics of attempts of wash trade. Given the gap in the field, it is this aspect of market manipulation that this paper seeks to address. This paper proposes a wash trade detection algorithm that monitors all incoming limit orders that can possibly attempt to compose a wash trade. Recognising such attempts helps the regulators to prevent market abuse by a strict regulation.

III. Wash Trade Detection Methodology

A. *Analysis Terminologies*

To analyse the wash trade strategic behaviours, the definitions and terminologies in (Tsang, Olsen, & Masry, 2013) are adopted and revised to formalise the trading properties and market changes. The effect of wash trade can be represented by the position of the whole trading collusion, where "position" is the amount of equities held by a trader. As the wash trade is merely fraudulent activities rather than true trading actions, each participated trader tends to maintain his own positions unchanged for minimising the unnecessary financial loss, and therefore the position of the whole wash trade collusive group is also not changed. During the wash trade process, the position change is caused by a number of orders from the trader in the collusive group and can be defined as:

$$\text{Position} + \text{Orders} \rightarrow \text{Position},$$

Position is comprised of a sequence of orders:

$$\text{Position} = \{(\mathrm{Order}_1), (\mathrm{Order}_2) \ldots (\mathrm{Order}_n)\},$$

where each order is defined as:

$$\text{Order} = (\text{Trader\_ID, Type, Price, Volume}),$$

where Type = buy | sell. Representing the order Type buy and sell by positive and negative signs respectively and affixing the sign to the Trader_ID and Volume, a sell order can be represented as:

$$\text{Order} = (-\text{Trader\_ID}, \text{Price}, -\text{Volume}). \qquad (1)$$

By this, the orders in Table 4 can be illustrated as:

Position={(A, 125, 500), (-B, 124.2, -450),
(B, 125.5, 450), (-A, 125, -500) }.

The buy/sell orders having matched prices can be merged as:

Position={ (A-B, 125, 500-450=50),
(B-A, 125.5, 450-500=-50) },

As discussed in Section II.A, prices 125 and 125.5 are represented as transaction prices. The difference between the executable limit prices is calculated as the margins of the transaction prices. In this case, the transaction price 125 has the margin 125-124.2=0.8 and the transaction price 125.5 has the margin 125.5-125=0.5. We merge the potential transactions who price margins are overlapped, i.e. 125+0.8 and 125.5+0.5 are overlapped. After the merge, we re-represent the positions, i.e. the margin between 124.2 and 125.5 is represented as: 124.85±0.65.

Position={ A-B+B-A, 124.85±0.65, 50-50 },
={ “0”, 124.85±0.65, 0 },

where the Trader_ID calculation is carried out as a symbolic operation and 0.65 is represented as the transaction margin δ^T and 124.85 is the transaction price P^T. The zero-valued “signed trader ID” implies that each collusive trader transact at both sides (buy and sell) of the market and the zero “signed volume” indicate the total amounts of the transactions in both sides are zero: no equity is really bought or sold. Therefore, the unchanged position, represented through zero-valued “signed trader ID” and “signed volume”, indicate the wash trade activities in certain collusion.

B. Wash Trade among multiple traders

As the FCA and CESR pointed out in their consultation reports (FSA, 2006) (EU, Market Abuse Directive, 2014), it is difficult to distinguish a wash trade because the format of trading collusions varies and the collusive transactions can be buried in mass numbers of normal trading activities, such as the complex network reported by Nanex on 31 May 2013 (NANEX, Chicago PMI, 2013), where vertices illustrate traders and directional connections among vertices represent the transaction between traders. We utilise this idea in (NANEX, Chicago PMI, 2013) and represent submitted limit orders (from a number of traders) by a graph, where vertices represent traders, the short arrows affixed to the vertex represent the orders submitted by the trader (buying and selling orders are represented by arrows pointing inward and outward, respectively) and the dotted arrow lines represent the possible executed orders according to the matching rule discussed in Section II.A. An example of wash trade action mixed up with legitimate trading orders is shown in Table 5 and illustrated by the graph in Fig. 1. Among the 14 orders submitted by six traders in this example, four pairs (#1-#4 in Table 5) of wash trade orders are deliberately submitted by four traders with tight submission intervals, executable prices, and mostly matched volumes so that orders in each pair are suspiciously easy to match and execute. In Fig. 1, the possible executions of the orders are illustrated by four dotted arrow lines: each dotted arrow line connecting one pair of matched orders and the arrowhead indicating the transaction direction of the financial equity, i.e. ***A*** pointing to ***B*** means trader ***A*** sells shares of equity to trader ***B***. From the illustration in Fig. 1, when participating wash trade activities, traders (***A***, ***B***, ***C***, and ***D***) connect as a closed simple cycle (dotted arrow lines) and continuous transactions among the traders flow throughout the cycle in one single direction (either “clockwise” or “counter clockwise”) with each trader along the pathway “passing the parcel” (Aitken, Harris, & Ji, Nov. 2009). After a complete transaction loop, the beneficial interest has been transferred across the collusive group and no traders in the group have an actual position change.

Table 5 Example of wash trade in a sequence of limit orders from a number of traders

#	Trader	Time	Buy/Sell	Price	Volume	Pairs
01	***A***	9:00:000	Sell	125.00	1450	# 1
02	***B***	9:00:001	Buy	125.01	1500	
03	***B***	9:05:000	Sell	124.95	1500	# 2
04	***C***	9:05:001	Buy	125.01	1450	
05	***E***	9:16:000	Sell	124.90	200	
06	***C***	9:20:000	Buy	124.90	235	
07	***C***	9:30:001	Sell	125.00	1450	# 3
08	***D***	9:30:002	Buy	125.01	1500	
09	***C***	9:45:000	Sell	124.80	250	
10	***F***	10:05:000	Buy	124.70	350	
11	***D***	10:50:000	Sell	125.01	1450	# 4
12	***A***	10:50:001	Buy	125.01	1450	
13	***F***	11:35:000	Sell	124.80	200	
14	***E***	11:50:000	Buy	124.50	550	

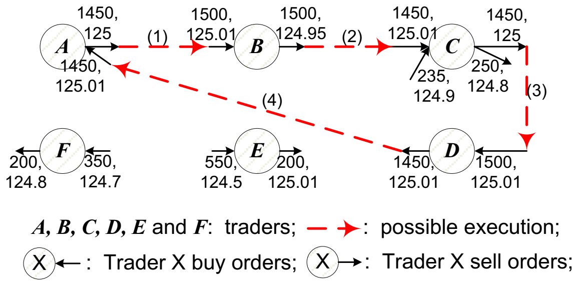


Fig. 1 Closed connection cycle of traders and the possible execution flow along the cycle in wash trade action (14 orders in Table 5 are mapped to the graph)

The “no beneficial interest change” of all collusive traders in wash trade activities can also be calculated by the terminologies defined in Section III.A as the equation (2).

Equation (2) shows the possible execution (the dotted arrow (1) in Fig. 1) of two orders in pair #1 in Table 5 due to the matching rule, execution occurring on earliest orders with matched prices, discussed in Section II.A. Similarly, the executions of matched pairs #2-4 in Table 5 (and the dotted arrows (2)-(4) in Fig. 1) are represented by equation (2). The aggregated results of those executions are calculated in equation (2), where 50 shares of volumes are remained due to the "mostly matched volumes" tactic between any two "smart manipulator neighbours" to avoid regulatory inspections (Aitken, Harris, & Ji, Nov. 2009). The unmatched volumes (for example 2%) can then be defined as the *matching margin*(δ_v). Similarly, the differences between the limit order prices and transaction prices can be defined as limit price margin $(\delta_\mathrm{p})$and the transaction margin(δ_p^T) respectively. In the following case, $\delta_\mathrm{p}^T = 0.005$.

$$\begin{array}{ll}
\text{Position}=\{ & (-\boldsymbol{A}, 125.00, -1450), (\boldsymbol{B}, 125.01, 1500), \\
 & (-\boldsymbol{B}, 124.95, -1500), (\boldsymbol{C}, 125.01, 1450), \\
 & (-\boldsymbol{C}, 125.00, -1450), (\boldsymbol{D}, 125.01, 1500), \\
 & (-\boldsymbol{D}, 125.01, -1450), (\boldsymbol{A}, 125.01, 1450) \quad \} \\
=\{ & (-\boldsymbol{A}+\boldsymbol{B}, 125.00+0.01, +50), \\
 & (-\boldsymbol{B}+\boldsymbol{C}, 124.95+0.06, -50), \\
 & (-\boldsymbol{C}+\boldsymbol{D}, 125.00+0.01, +50), \\
 & (-\boldsymbol{D}+\boldsymbol{A}, 125.01+0, 0) \quad \} \\
=\{ & \text{`}-\boldsymbol{A}+\boldsymbol{B}-\boldsymbol{B}+\boldsymbol{C}-\boldsymbol{C}+\boldsymbol{D}-\boldsymbol{D}+\boldsymbol{A}\text{'}, \\
 & 124.95+0.06, 50-50+50+0 \quad \} \\
=\{ & \text{`}0\text{'}, 125.005\pm0.005, +50 \quad \}
\end{array} \tag{2}$$

Furthermore, as shown in Table 5, the time intervals between different pairs can vary as random events occurred in one single trading day. To avoid being detected as suspiciously trading action, in practice, "smart manipulators" tactically place the pairs at separated time points as the examples in Table 5, where the time differences among any two pairs are completely different and random. To achieve this, manipulators carefully design each pair of matched orders to minimise the possible financial loss from price changes in the time period (i.e. from 9:00 to 10:50 in Table 5) and to maintain the positions of their whole collusive group at zero. The separated arrangement of the matched pairs increases the complexity of detecting a wash trade under a mixture environment of both "normal" and "manipulative" trades.

Additional to the example in Table 5 and Fig. 1, the matched pairs among any two manipulators can also be constructed by a number of limit orders as illustrated in Table 3, rather than simply matched one-to-one sell and buy orders (as the pairs in Table 5). For example, the matched pair #1 in Table 5 can be constituted by four selling orders and one buying orders as shown in Table 6 and illustrated in Fig. 2.

Table 6 Example of matched pair composed of multiple orders in wash trade activity

#	Trader	Time	Buy/Sell	Price	Volume	Pairs
01	***A***	9:00:000	Sell	124.99	450	# 1
02	***A***	9:00:000	Sell	124.98	450	
03	***A***	9:00:000	Sell	124.97	350	
04	***A***	9:00:000	Sell	124.96	200	
05	***B***	9:00:001	Buy	125.01	1500	

In the examples, the submission of four sell orders is followed tightly by one large buy order, which matches, potentially executes and removes all (or most) volumes of previous four sell orders. The graph of the traders and the transaction flow are revised in Fig. 2, where the #1 matched pair between ***A*** and ***B*** is illustrated by four short outward arrows affixed to ***A*** connecting with one short inward arrows affixed to ***B*** through the dotted arrow and other parts of the structure of the whole closed cycle of the traders is remained. In the example in Table 6, since the buy order #05 is submitted later than the sell orders, it will be executed at the prices of four sell orders, i.e., order #05 will be firstly executed as 450 shares at 124.99 with order #01 and then another 450 shares executed at 124.98 with order #02 and so on.

C. *Wash trade features*

From the discussion in Section III.A and III.B, the strategy that constructs a wash trade activity has the following two key features:

Feature 1: Matched orders - as the first step of wash trade manipulation, traders deliberately submit the matched orders to the market in tiny time intervals to guarantee the execution; those orders can be one-to-one (examples in Table 5) or one-to-many matched (example in Table 6); this feature refers to dotted arrow lines in Fig. 1 and Fig. 2;

Feature 2: Closed transaction cycle - any single execution of the matched orders does not refer to wash trade manipulation unless those executions constitute a closed cycle as illustrated in the examples shown in Fig. 1 and Fig. 2; this feature refers to closed cycle of dotted arrows among the traders in Fig. 1 and Fig. 2.

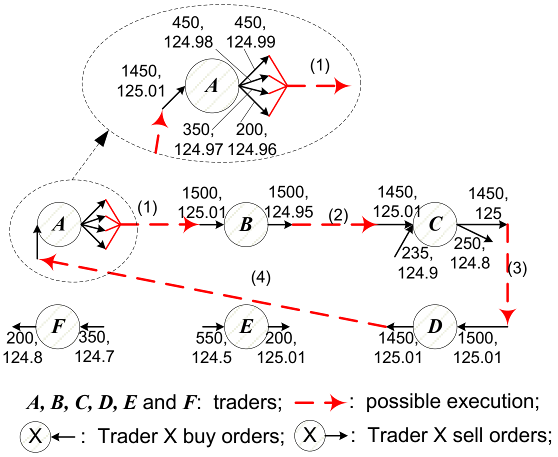


Fig. 2 Multiple matched orders between two manipulators in wash trade action (14 orders in Table 5 and five orders in Table 6 are mapped into the graph)

Considering the example in Table 5, the manipulators set up the matched orders from the #01 order at time 9:00:000, but the wash trade is not completely constructed until the submission of the #12 order at time 10:50:001, which closes the transaction cycle. Therefore, wash trade can be detected through detecting the matched orders and closed cycle in two steps:

Step 1: Detect the suspiciously matched order pairs S according to the matching rule and wash trade features, tight submission intervals, executable prices and mostly matched volumes:

$$\text{Order Pair} = \{\sum Orders\} = \{+T_m - T_n, \mathrm{P}^\mathrm{T} \pm \delta_p^T, \pm\delta_v\},$$

where $+T_m - T_n$ represents trader T_n selling shares of equity to T_m and δ_v and δ_p^T represent the *matching margin* of volume and transaction price P^T;

Step 2: Among S, find the order pairs whose transaction price margins are overlapped, in those pairs, if some pairs fulfil the condition:

$$\text{Position} = \{\textstyle\sum_{k\in S} \text{Order Pair}_{k}\} = \{\text{"0"}, \mathrm{P}^{\mathrm{T}} \pm \delta_p^T, \pm\delta_v\},$$

a wash trade alert is triggered.

To further formulate those features, we define the *#k* order *L* submitted by trader T_n at time t_k as:

$$L_k = (t_k, \pm T_n, P_k, \pm V_k)$$

where P_k and V_k are *#k* order price and volume respectively and the positive and negative sign $\pm$ represent buy and sell operation. The *matching margin* δ is defined as a vector $\delta = [\delta_p, \delta_t, \delta_v]$ with three small positive values for price, time and volume respectively. If buy order #***K*** is matched with ***K****-1* sell orders from *#1* to #***K****-1*, their features have 1) tiny time interval:

$$|t_1 - t_{\boldsymbol{K}}| < \delta_t, \tag{3}$$

2) executable tiny price difference:

$$P_{\boldsymbol{K}} - \min(P_1, \dots, P_{\boldsymbol{K}-1}) < \delta_p, \tag{4}$$

3) and mostly matched volume:

$$|\textstyle\sum_{k=1}^{K-1} V_k - V_{\boldsymbol{K}}| < \delta_v. \tag{5}$$

If ***K*** orders among *N* traders construct wash trade action, their features meet the following condition, where $r_{nk}$is the indicator that if order *#k* from trader $T_n$is a sell order, then $r_{nk} = -1$, and $r_{nk} = +1$ for buy order.

$$\begin{aligned} \text{Position} &= \{\textstyle\sum_{n=1}^{N} r_{nk}T_n, \mathrm{P}^{\mathrm{T}} \pm \delta_p^T, \pm\delta_v\} \\ &= \{\text{"0"}, \mathrm{P}^{\mathrm{T}} \pm \delta_p^T, \pm\delta_v\} \end{aligned} \tag{6}$$

The features in equations (2)-(5) are detected in Step 1 and the feature in equation (6) is detected in Step 2.

D. Problem formulation

To discover the wash trade before it completely occurs (fulfilling the recent regulations on preventing the attempts of wash trade), the detection approach is applied to the limit order streams instead of the trade records. The order stream is the sequence of limit orders received by the trading platform from numerous traders. The stream is updated by the "order event", which could be submission, modification, cancellation or execution. As shown in Table 1, an order includes ID, trader ID, time, buy/sell sign, price and volume. In this study, we assume that the orders in the stream are on one specific stock. Thus the stock information in the stream can be ignored once the specific stock is determined. This assumption, on one hand, narrows the scope of this study specifically on the underlying problem, on the other hand conforms the practical trading platform environment, where the algorithm can be easily applied to selected equity.

The Step 1, detecting the suspiciously matched order pairs according to equations (3)-(5), is termed as coarse detection while Step 2, recognising the closed cycle based on equation (6), is termed as fine detection. The limit order stream is then required to be pre-organised to commence with those two tasks. A physical time sliding window sized θ_T is specified and the trading order stream can be split into two queues of consecutive orders: *buy order queue,* Q_b and *sell order queue,* Q_s each of which maintains a size θ_T. That is, if a new orderL_k is a buy order, push it into Q_b; otherwise push it into Q_s. If the length of the updated queue is larger than θ_T, pop the earliest orders to maintain the length of the sliding window. The algorithm is described in Algorithm 1. Since the order stream is measured in "order event time", θ_T is maintained by calculating the difference between the physical time stamps of the first and the last orders in the queue. Hence the number of orders in each queue ultimately depends on the underlying frequency of order activities and differs across time. (Algorithm 1 is named as WASH_TRADE_DETECT because it will involves all detection sub-functions, which are discussed in follow up sections.)

The intention of the wash trade, increasing transaction volume, indicates that wash trades are usually associated with large-sized orders. Consequently, the orders with volumes smaller than a predefined threshold θ_V are ignored, where the threshold can be setup according to the requirements of the detection solidness. Given the limit order queues Q_b and Q_s, the *coarse detection* can then be formulated as follows: for a large incoming order, examine in the opposite order queue for one or multiple potential matching orders which are characterised by equation (3)-(5). The result of the *coarse detection* comprises all order combinations matched with the incoming order. Collusions may exist among those combinations.

Algorithm 1. Wash Trade Detection – Pre-organisation

```
WASH_TRADE_DETECT($L_k$ )
   $Q_s = \emptyset$; $Q_b = \emptyset$
      while $L_k$ is a valid limit order
         if $L_k$ is buy
            Push $L_k$ into $Q_b$
            while $Q_b$ length > $\theta_T$
               Pop $Q_{b,1}$ to maintain $\theta_T$
         else
            Push $L_k$ into $Q_s$
            while $Q_s$ length > $\theta_T$
               Pop $Q_{s,1}$ to maintain $\theta_T$
```

Similarly, the *fine detection* can be formulated as follows: given the matched order pairs, find certain sets of pairs in which the sum of "signed trader ID" and "signed volume" have zero values as the illustrations in equation (6). Defining *coarse detection* and *fine detection* as the function **COARSE_DETECT** and **FINE_DETECT** respectively, the wash trade detection is further designed in following section.

E. Coarse Detection - Matching Search

The matching relationship of wash trade order pairs is summarised in equations (3)-(5). In the *Coarse Detection* process, three conditions are sequentially checked to identify the potential matching.

The time matching margin δ_t in equation (3) shows the tiny interval between the orders in a pair. Setting the length of the order queue θ_T in Algorithm 2 and Algorithm 1 equivalent to δ_t, the *coarse detection* is designed as the illustration in Fig. 3: given the incoming order $\mathrm{L_k}$, examining the opposite orders in previous δ_t (θ_T) period for potential matched orders, which are determined by price and volume margin, δ_p and δ_v. Algorithm 1 is then revised as Algorithm 2 which includes both the **COARSE_DETECT** and **FINE_DETECT** functions, where the $\{MP\}$ is the detected matched pairs of **COARSE_DETECT**.

In financial markets, only the orders following executable price rules (Bowen, 2013) match and execute. Therefore the price margin δ_p in equation (4) is constrained by the following rules:

Rule 1. Sell order matches buy orders with equal or higher prices;

Rule 2. Buy order matches sell orders with equal or lower prices.

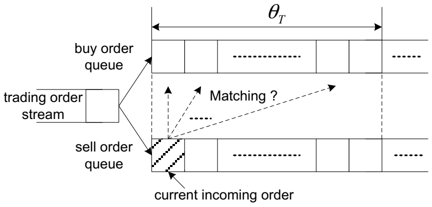


Fig. 3 Coarse Detection Scheme

The example in Table 6, where the #5 buy order price is slightly higher than all previous sell orders, shows the Rule 2 of price margin δ_p. Considering the price margin δ_p, the *coarse detection* is designed as follows: given the incoming buy (sell) order $\mathrm{L_k}$, among all executable orders (in terms of the executable limit prices) in the previous δ_t (θ_T) period, find the order pairs having the best matching volumes.

Algorithm 2. Wash Trade Detection Algorithm

```
WASH_TRADE_DETECT($\mathrm{L_k}$)
$Q_s = \emptyset$; $Q_b = \emptyset$; {Matched Pairs}$= \emptyset$;
while $\mathrm{L_k}$ is a valid limit order
    if $\mathrm{L_k}$ is buy
        Push $\mathrm{L_k}$ into $Q_b$
        while $Q_b$ length > $\theta_T$
            Pop $Q_{b,1}$;
        {MP}= COARSE_DETECT($Q_s$, $\mathrm{L_k}$);
    else
        Push $\mathrm{L_k}$ into $Q_s$
        while $Q_s$ length > $\theta_T$
            Pop $Q_{s,1}$;
        { MP }= COARSE_DETECT($Q_b$, $\mathrm{L_k}$);
    if { MP } $\neq \emptyset$
        FINE_DETECT({MP});
```

The volume matching can be defined as a function **VOL_MATCH**$(Q^{t,p}, L_k)$, where $Q^{t,p}$ is a set of orders after being filtered by δ_t and δ_p. Given this, **COARSE_DETECT**(Q, L_k) is defined in Algorithm 3, where Q contains all opposite orders in the previous δ_t periods and $\mathrm{L_k}$ is the incoming order. Based on the above discussions and the constraints in equation (5), the function **VOL_MATCH**$(Q^{t,p}, L_k)$ is defined as follows: given incoming order L_k and a set of matched orders $Q^{t,p}$, find subsets S of the order pairs from $Q^{t,p}$ such that

$$|(\sum_{i\in S} V_i) - V_k| \leq \delta_v.$$

The number of limit orders in subset S is n_s (n_s is smaller than the size of $Q^{t,p}$). In essence, the problem of **VOL_MATCH** is a practical case of a more general problem called the *Knapsack Problem* (Rumen, Vincent, & Sanjay, 2000) (Vincent, Yanev, & Andonov, 2009) (Zukerman, Jia, Neame, & Woeginger, 2001). The name *Knapsack* refers to the problem of filling a knapsack of capacity W using a subset of m items $\{1, \ldots, m\}$, each of which has a mass and a value, such as the total weight of the selected items is less than or equal W and their total value is maximised. The volume matching problem can be viewed as a simplified form of the *Knapsack Problem*: given a capacity V_k (the knapsack size) and a set $Q^{t,p}$ of items, each having non-negative size V_i, find all possible subsets S of items to eventually make

$$|(\sum_{i\in S} V_i) - V_k| \leq \delta_v.$$

Due to the similarity of the two problems, the widely used approach solving the *Knapsack Problem,* dynamic programming, is employed in **VOL_MATCH**$(Q^{t,p}, L_k)$. The main principles of dynamic programming are that we have to come up with a number of sub-problems so that each sub-problem can be solved easily from "smaller" sub-problems, and the solution of the original problem can be obtained easily once we know the solutions to all the sub-problems (Kleinberg & Tardos, 2005). Dynamic programming has been studied thoroughly in optimization problems in (Zhen, He, Wen, & Xu, 2013) (Jiang & Jiang, 2014).

Algorithm 3. Coarse Detection

```
COARSE_DETECT ($Q$, $\mathrm{L_k}$)
    $Q^{t,p} = \emptyset$;
    if $\mathrm{L_k}$ is a valid buy order
        for each order $L_i$ in Q
            if $\mathrm{P_i}$ <= $\mathrm{P_k}$
                push $L_i$ into $Q^{t,p}$
    else if $\mathrm{L_k}$ is a sell order
        for each order $L_i$ in Q
            if $\mathrm{P_i}$ >= $\mathrm{P_k}$
                push $L_i$ into $Q^{t,p}$
    if $Q^{t,p} \neq \emptyset$
        $S$ = VOL_MATCH($Q^{t,p}$, $\mathrm{L_k}$)
```

To solve the special form of the *Knapsack Problem* under N limit orders and volume V_k, denoting the final subset of orders in an optimum solution for the original problem as S_N, we then use the notation $\mathbf{OPT}(N, V_k)$ to denote the sum of the order volumes of the first N orders in the subset S under the constraint $|\mathbf{OPT}(N, V_k) - V_k| \leq \delta_v$. The sum in the first *N-1, N-2,...,1* orders can then be represented as $\mathbf{OPT}(N-1, V_k)$, $\mathbf{OPT}(N-2, V_k)$, …, $\mathbf{OPT}(1, V_k)$. To determine $\mathbf{OPT}(N, V_k)$, we not only need the solution of $\mathbf{OPT}(N-1, V_k)$, but also need to know $\mathbf{OPT}(N-1, V_k - V_N)$, the best solution for the first *N-1* orders with the remaining capacity $V_k - V_N$, which constructs the constraint as $|\mathbf{OPT}(N-1, V_k) - (V_k - V_N)| \leq \delta_v$. The recursion can then be summarised as follows: if L_N is not one of the orders in the final subset S_N, we can ignore the order *N* and determine $\mathbf{OPT}(N-1, V_k)$; however if L_N is one of the orders, we need to seek an optimal solution for the remaining orders, *1,.., N-1*, which is $\mathbf{OPT}(N-1, V_k - V_N)$. Using this set of sub-problems, we are able to express the $\mathbf{OPT}(N, V_k)$ as a simple expression in terms of values from "smaller" problems. Therefore, the recursion is summarised as two conditions:

1. If $L_i \notin S_N$, then $\mathrm{OPT}(N, V_k)$= $\mathrm{OPT}(N-1, V_k)$;

2. If $L_i \in S_N$, then $\mathrm{OPT}(N, V_k)$=V_N+ $\mathrm{OPT}(N-1, V_k - V_k)$;

This recursive process is re-organised based on the above two conditions to give Algorithm 4. This recursive algorithm can be used by invoking OPT (N, V_k) for N limit orders and the capacity V_k.

F. Fine detection – collusion search

$\mathrm{S_N}$, orders from Q matched with the incoming order L_k, is the result of the *coarse detection*. To further detect the potential closed cycle of transactions, the orders in $\mathrm{S_N}$ are represented by equation (1), where the trader ID and volumes are affixed with trading direction signs. After the conversion, $\mathrm{S_N}$ is defined asS_N^c, the input of the fine detection algorithm FINE_DETECT. As discussed in Section III.A and equation (2), the order pairs with potential transaction prices with overlapped price margins are grouped together for potential collusion detection.

Detecting trader collusion is treated as discovering the combinations C from S_N such that the sum of the signed trader equals zero as illustrated in equation (6). This process can be considered equivalent to a special case of the previously defined volume matching problem: given a capacity W=0 (the knapsack size) and a set of signed trader pairs, each having a value (e.g. $\{+\mathrm{A}\}$ and $\{-\mathrm{A}\}$), select all possible subsets C of signed trader pairs to make $\sum_{n\in\mathrm{C}} r_{nk} T_n = \mathrm{W}$, where the symbolic computation of the signed trader pairs are defined in Section A and can be implemented by operator overloading. The subset C is considered as trading collusion in a wash trade.

Algorithm 5, derived from Algorithm 4, provides the recursive solution for FINE_DETECT($\mathrm{S_N^c}$).

Algorithm 4 Volume Matching Detection by recursion

```
VOL_MATCH($Q^{t,p}, L_k$)              // original limit order set $Q^{t,p}$;
   $S_N = \emptyset$;                          // solution subset, initialized to empty;
   $N$ = length($Q^{t,p}$);                 // $N$: size of $Q^{t,p}$;
OPT($N$ , $L_k$)                         // $N$ decreases on each recursion step;
   if $N < 1$ or $|L_k| \le \delta_v$         // if $N$ reaches the last one or $\delta_v$
      return;                          condition is satisfied;
   if $|V_N - L_k| \le \delta_v$              // if condition is satisfied, then orders
      output $S_N$;                     in $S_N$ is one solution;
   push $L_N$ into $S_N$                   // assume $L_N \in S_N$;
   OPT$(N-1, V_N - L_{N,v})$;           // recursively find solution by
                                       condition 2;
   Discard $L_N$ from $S_N$                // assume $L_N \notin S_N$;
   OPT($N-1, L_v^n$);                  // recursively find solution by
                                       condition 1;
end of OPT
return  $S_N$;
```

Algorithm 5 Collusion Search by recursion

```
FINE_DETECT ($S_N^c$);                 // original signed trader set $S_N^c$
$\vec{C} = \emptyset$;                               // solution subset, initialized to empty;
$N$ = length($S_N^c$); sum = 0 ;         // N: size of $S_N^c$;
OPT($N$ , sum)                          // N decreases on each recursion step;
   if $N < 1$                            // if N reaches the last one, done
         return;
   if rT + sum = 0                      // if the sum of signed trader
                                         including current one is zero
         output $\vec{C}$;                      // the signed trader in $S_N^c$ is a solution;
   push rT into $\vec{C}$;                     // assume $\overrightarrow{t_N} \in \vec{C}$;
   OPT($N-1$ , sum + rT);             // recursively find solution by
                                        condition 2;
   Discard rT from $\vec{C}$;                  // assume $\overrightarrow{t_N} \notin \vec{C}$;
   OPT($N-1$ , sum);                   // recursively find solution by condition 1;
end of OPT
return  $\vec{C}$;
```

IV. Experiments and Evaluation

Evaluating a detection model usually relies on real data of both "normal" and "abuse" cases. However, due to the limited reports on wash trade manipulation and regulatory rules prohibiting the disclosure of illegitimate market data, the availability of the examples of wash trade behaviours in capital markets is far less than the availability of routine normal trading records. Therefore to evaluate the proposed detection model, it is acceptable to the financial industry that all the characteristic patterns of wash trade examples are reproduced then injected into original trading records to generate a mixed dataset of normal and abuse cases (NANEX, Exploratory Trading in the eMini, 2013). Randomly synthesised exploratory manipulation cases can mimic any possibility of wash trade scenarios, i.e. we can generate the matched order at any time with any volume size as well as matching margins. Synthetic exploratory financial data are also accepted in academia for evaluating the proposed model when real market data are hard to collect (Palshikar & Apte, 2008) (Ou, Cao, Luo, & Zhang, Dec. 2008) (Markus., Hoser, & Schröder, 2007). In this paper, the experimental evaluation is composed of two parts:

Part 1: experimental evaluation using original trading datasets from the market;

Part 2: experimental evaluation using original trading datasets injected with synthetically generated wash trade scenarios following the analysis in Section II.A.

A. Experiment Setup

The experimental data used in this work involve real market data (trading orders) of seven stocks Google (GOOG), Microsoft (MSFT) and Apple (AAPL) from NASDAQ and First Quantum Minerals (FQM), Yamana Gold (YAU), Gazprom (OGZD), and Vodafone (VOD) from London Stock Exchange (LSE). The selection of these datasets is due to their active trading activities, relatively high trading volumes and more volatile price fluctuation, the factors that might increase the likelihood of market abuse across the exchanges (Douglas, Feng, & Aitken, 2012) (Lee, Eom, & Park, 2013). The datasets from NASDAQ cover messages over five trading days from 11th – 15th June 2012, and consist of more than 400,000 trading orders in total for each stock. The datasets from LSE cover 23rd – 27th May 2011, and consist of more than 100,000 orders in total for each stock. Table 1 shows an excerpt of the trading records used in this study. The wash trade detection algorithms are evaluated on the original seven datasets for detecting any transactions which are suspiciously similar to wash trade manipulation. Additionally, typical wash trade activities are reproduced according to the discussions and examples in Table 5Table 6 and injected into those seven datasets for further experimental evaluations.

B. Determining the marginal parameters

As discussed in Section II.A, the submissions of the matched orders in a wash trade are usually within tiny time intervals δ_t so that the manipulated execution can compete against the action of normal traders who may pick the orders unintentionally (FSA, 2006) (Bowen, 2013). Consequently, the normal execution time shows a reasonable reference to the time interval δ_t, which otherwise is not available because of the lack of the statistical studies of the real wash trade cases.

Usually, the execution time of a limit order is strongly associated with its volume (Douglas, Feng, & Aitken, 2012) (Hautsch & Huang, The market impact of a limit order, 2012) (Hautsch & Huang, Limit Order Flow, Market Impact and Optimal Order Sizes: Evidence from NASDAQ TotalView-ITCH Data, 2011). Therefore, a more reasonable measure of the average execution time of normal limit orders can be given by volume-weighted average execution time (VWAT), defined as

$$T_{VWAT} = \frac{\sum_j (T_j * v_j)}{\sum_j v_j}, \quad (7)$$

where T_{VWAT} is volume-weighted average execution time; T_j is the execution time of order j; v_j is the volume of order j; and j is each individual order (Hautsch & Huang, Limit Order Flow, Market Impact and Optimal Order Sizes: Evidence from NASDAQ TotalView-ITCH Data, 2011). In practice, if the wash trade orders are submitted with time intervals larger than T_{VWAT}, they are apparently easy to pick by other legitimate traders. Accordingly, by setting $\delta_t = T_{VWAT}$, this approach covers a time period for all possible wash trade activities. The order execution time T_j and the T_{VWAT} across the seven stocks in the test dataset are calculated and summarized in Table 7.

Theoretically, the wash trade can be carried out by a large number of small orders. However, in practice, the wash trade orders are usually larger than the average volume of the normal trading orders because a large number of orders can significantly increase the uncertainty of the order executions, which may bring a risk of loss if it does not follow the expected arrangements. Therefore, the average order volume of each stock is selected as the threshold θ_V for the order volume filtering discussed in Section D. The average volume across seven stocks is also calculated and summarized in Table 7.

Table 7 Volume weighted average execution Time and average volume

	T_j (sec.)	T_{VWAT}(sec.)	Avg. Vol (share)
GOOG	2.77	118.79	635.57
MSFT	3.07	107.68	530.70
AAPL	5.92	87.04	900.04
FQM	10.19	83.87	163.20
YAU	14.35	104.25	878.46
OGZD	6.04	52.35	796.30
VOD	12.97	71.15	661.16

In addition, the volume matching margin δ_v is selected as percentages: 0%, 1%, 2%, 3%, 4% and 5% indicating the ratio of not matching (1% refers identifying orders with 99% matching volumes). In the example in Table 6, the #5 buy order volume (1500 shares) is around 96.7% matched with all previous sell orders (1450 shares). The price margin δ_p is unconstrained in the detection so that any orders following the price matching rules Rule 1 and Rule 2 are scanned for possible matching pairs under the condition in equation (5).

Under the configurations of δ_t, θ_V,δ_v, and δ_p, Algorithm 4 reflects the fact that given an order L_k, among all executable priced orders (unconstrained δ_p but following Rule 1 and 2) with volume not smaller than θ_V in a previous δ_t time period, find the matched orders that executed at least (1-δ_v)% volumes of L_k.

C. Part 1: Experiments on original datasets

In Part 1 experiment, the wash trade detection algorithm is evaluated on the original seven datasets using the parameters in Section IV.B. The evaluation shows the applicability of the proposed algorithms to real transaction data and also examines the legitimacy of the transactions in original dataset. Since the original datasets do not contain any reported wash trade manipulation activities, it is assumed to only contain legitimate transactions. Thus the evaluation measure is based on *false negative rate*, $FNR = \frac{FN}{FN+TP}$, which is based on *false negative*, *FN*, defined as normal cases detected as a wash trade, and *true positive*, *TP*, defined as normal cases detected as normal.

The results of the experiments (max FNR values on each stock dataset are highlighted) are shown in Table 8. It is clear that in each dataset, some transactions are detected as suspicious wash trade actions and the numbers of the detected actions increase across the increases of volume margins. Most of the datasets do not contain any suspicious actions when the volume margin is set to 0% and the Apple stock shows the highest FNR rate (1.263%) at the 5% volume margin.

With careful inspection and consultation with the financial industry experts, we determined that the detected false negative cases show very similar features to the wash trade actions although not reported by the regulators. The detected false negative cases fall into two formats as shown in Table 9.

Table 8 Experiment results (FNR) across original datasets of seven stocks

stock	Volume Margins					
	0%	1%	2%	3%	4%	5%
GOOG	0.000%	0.046%	0.059%	0.073%	0.093%	**0.096%**
MSFT	0.000%	0.030%	0.176%	0.275%	0.519%	**0.530%**
AAPL	0.000%	0.000%	0.166%	0.576%	1.153%	**1.263%**
FQM	0.389%	0.499%	0.526%	0.553%	0.673%	**0.926%**
YAU	0.000%	0.669%	0.761%	0.780%	0.853%	**1.186%**
OGZD	0.000%	0.346%	0.519%	0.680%	0.693%	**0.853%**
VOD	0.000%	0.953%	1.048%	1.066%	1.143%	**1.219%**

In case #1, the trader Client12 sold 6600 shares to Client3 at price 58.0 and bought 6606 back 2 seconds later at the same price. The 99.9% matched transacted volumes, 100% matched prices and the closed cycle of the transaction directions between Client12 and Client3 make this case extremely suspicious and potentially be a wash trade action according to the regulation (SEC, 2011) although not reported yet. Detecting such suspicious activities shows effectiveness of the proposed algorithms, although recognising the real intention behind such cases requires more inspections from the regulators, which is out of the scope of our work. In case #2, Client1 sold 15000 shares to Client5 at price 58.56 at market closing time and bought them all back at slightly higher price at market opening time on the next day. Those transactions also fulfill the conditions of a wash trade action except the trading dates. According to the suggestions from financial experts, case #2 refers to pre-arranged trading, which is defined as “a sell is coupled with a buy back at the same or pre-arranged price that limits the risks” (SEC, 2011). The only difference between pre-arranged trading and wash trade is that the former is usually among merely two parties and may occur in different days and the latter can involve a number of collusive traders and usually occurs as intra-day trading. When only targeting wash trade, the proposed algorithms can be applied on intra-day transactions to avoid picking up the pre-arranged trading as case #2, although the pre-arranged trading is also illegal (SEC, 2011) and needs to be monitored and banned from the capital markets.

Table 9 False Negative cases of stock AAPL

case#	time	volume	price	seller	buyer
1	21/06/2012 15:18:48.768	6600	58.00	Client12	Client3
	21/06/2012 15:20:11.811	6606	58.00	Client3	Client12
2	21/06/2012 16:28:40.629	15000	58.56	Client1	Client5
	22/06/2012 10:00:45.187	15000	58.60	Client5	Client1

D. *Part 2: Experiments on datasets with injected wash trade*

Testing with synthetic data can mimic any possible wash trade cases and also can evaluate the robustness of the proposed algorithms under any wash trade scenarios, i.e. random combinations of one or multiple traders wash trade activities.

1) *Wash Trade Case Generation*

The typical wash trade activities are reproduced and injected in each stock dataset. The activities are reproduced in two format groups:

Group 1: one order matched with single opposite order, termed as “single-matching”;

Group 2: one order matched with multiple opposite orders, termed as “multi-matching”.

Each group contains three different sets according to trader numbers in the wash trade collusion: set #1 has examples with one trader in a trading collusion; set #2 and #3 has two and four traders in a trading collusion respectively. To ensure a comprehensive assessment of the approach, in each set, volume matching margin δ_v is selected as a percentage 0%, 1%, 2%, 3%, 4% and 5% indicating the ratio of not matching (1% indicating the orders from two sides are 99% matching). There are 10 examples for each combination of the above parameters.

Table 10 Generated single matched Wash Trade cases (δ_v=5%)

Case	Trader	Time	Buy/Sell	Price	Volume	pairs
#1	***A***	9:00:000	Sell	58.00	5000	1
	B	9:00:001	Buy	58.01	4750	
	A	9:15:000	Buy	58.01	5000	2
	B	9:15:001	Sell	58.00	4750	
#2	***A***	12:16:000	Sell	58.00	5000	1
	B	12:16:100	Buy	58.05	4750	
	C	13:00:001	Buy	58.05	4750	2
	B	13:00:002	Sell	58.00	5000	
	C	13:20:001	Sell	58.00	5000	3
	D	13:20:002	Buy	58.05	4750	
	A	14:20:001	Buy	58.05	4750	4
	D	14:20:002	Sell	58.00	5000	

Table 11 Generated multiple matched Wash Trade cases (δ_v=5%)

Case	Trader	Time	Buy/Sell	Price	Volume	pairs
#3	***A***	9:00:100	Sell	58.00	1100	1
	A	9:00:100	Sell	58.01	1200	
	A	9:00:100	Sell	58.02	1000	
	A	9:00:100	Sell	58.03	1400	
	B	9:00:101	Buy	58.05	5000	
	B	10:10:000	Sell	58.00	1000	2
	B	10:10:000	Sell	58.01	1300	
	B	10:10:000	Sell	58.02	1200	
	B	10:10:000	Sell	58.03	1250	
	A	10:10:100	Buy	58.05	5000	

The examples in Table 10 and Table 11 show an excerpt of the generated wash trade cases: case #1: two traders with 5% single matched volumes; case #2: four traders with 5% single matched volumes; case #3: two traders with 5% multiple matched volumes. The volume, time and matching margin of the synthetic orders are all randomly generated. For example, in case 3 in Table 11, buy order volume v_b in pair 1 is randomly generated (under condition: $v_b \geq \theta_V$) and all sell orders in pair 1 are also randomly generated under the condition that volume sum V_s of all sell orders satisfies: $v_b * (1 - \delta_v) \leq V_s \leq v_b$. The time of orders in pair 2 are also randomly generated as long as they are much later than the time of pair 1. Similarly, the prices of order in each pairs are randomly generated following the price matching rules discussed in Section III.E. Similar to the examples in Table 6, two order pairs in Table 11 have different transaction prices. The buy order in pair #1 in Table 11 will be executed with the previous four sell orders at 58, 58.01, 58.02 and 58.03 respectively. Therefore, the generated examples have different transaction prices within transaction margins.

Such random generation of synthetic cases provides the possibility of thorough evaluation of the proposed algorithms using any possible wash trade cases.

As discussed before, the models are tested on seven real stocks, each of which contains two groups of injected examples. Each group has three sets (1, 2 and 4 traders) and each set contains six margin configurations. Under each configuration, there are 10 examples. There are overall $7\times2\times3\times6\times10 = 2520$ different experiments carried out as a robust evaluation plan for the proposed detection model.

The generated wash trade orders are then injected into the data of corresponding stocks making the test data a mixture of both “normal” and “abuse” patterns. The time intervals between different pairs are selected randomly as examples in Table 10 and Table 11. For example, in case #1 in Table 10, time of pair 2 is randomly selected after the pair 1 occurs. In addition, the generated orders in each pair are separated by several normal orders in original datasets to mimic the practical case in the markets. This is a practical approach to simulate how these wash trade scenarios occur in the real world (Cao, Ou, & Yu, 2012).

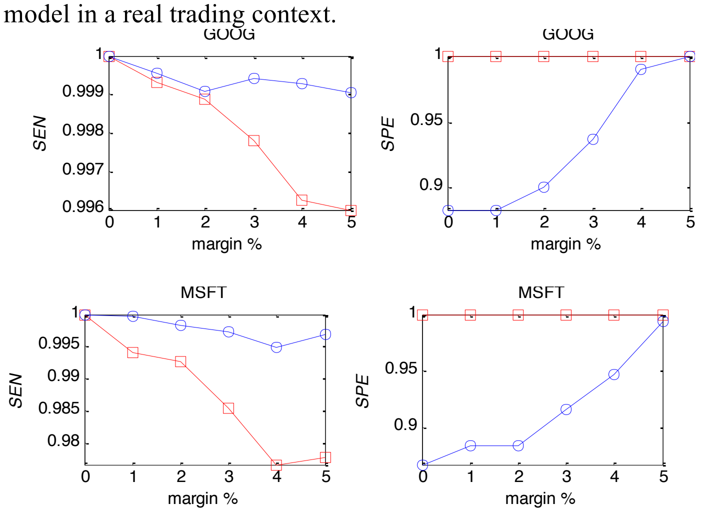
2) Performance evaluation metrics

The performance evaluation of the proposed model is based on two popular statistical measures: sensitivity (SEN) and specificity (SPE). Both of them are based on the confusion matrix, where a false positive (FP) is defined as a wash trade case detected as normal; a true negative (TN) is defined as a wash trade case detected as wash trade, and a false negative (FN) and a true positive (TP) which are defined in Section IV.C. The sensitivity, defined as $\mathrm{SEN} = \mathrm{TP}/(\mathrm{TP} + \mathrm{FN})$, represents the rate of correctly detecting normal trading orders (a.k.a. the true positive rate) while the specificity, defined as $\mathrm{SPE} = \mathrm{TN}/(\mathrm{FP} + \mathrm{TN})$, refers to the rate of correctly detecting wash trade cases (a.k.a the true negative rate).

3) Experimental Results

The experimental evaluations across seven stocks are summarized in Fig. 4, where the average SEN and SPE values across different numbers of traders are illustrated against the margin values.

From Fig. 4, the SPE values for single-matching show that the algorithm completely detects the single matching cases, which is the simplest wash trade format and is apparently easy to detect. The SPE values for multi-matching vary across the margins and the different stocks as the illustrations in Fig. 4. The SPE values increase with the increase of the margins and approach 100% when the margin is higher than 5%. The result conforms to the design expectation of the detection approach: more possible collusions will be detected under bigger matching margins. As discussed in Section II.A, "mostly matched" (for example 98%) orders might be built by "smart manipulators" for standing aside from the inspections. A big marginal value compensates this "smart tactic" and the configurability of the margin increases the practicability of the model in a real trading context.



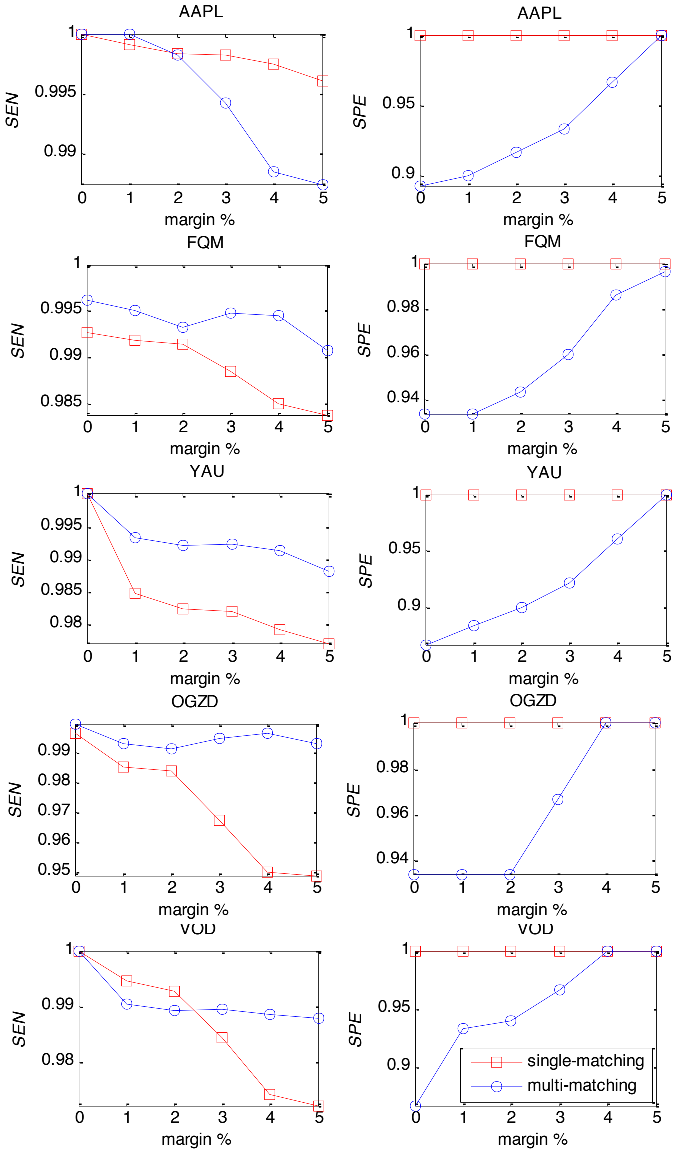


Fig. 4 Experiment results across seven stock dataset.

The SEN values show more volatile results across the margins. In most experiments, the sensitivity values reduce as the margin increases indicating more normal activities incorrectly detected as wash trade cases. On the contrary, the highest SEN value appears at the zero margin value.

From the experimental results, it can be concluded that the proposed approach detects the primary wash trade scenarios effectively and consistently across the selected stocks with SEN values in a range of 97% to 100%.

V. Conclusion and Future Work

A wash trade activity detection approach is proposed after thoroughly studying the various scenarios of wash trade behaviours. The analysis of the collusive activities in wash trades through a graph of traders with transactions represented by the directed connections among the vertexes shows the basic structure of the collusion among multiple traders following a closed cycle of the transactions among certain traders. Further studies also show that the limit orders in wash trades are usually submitted fast with mutually executed prices and matched volumes. According to the analysed features, the proposed method is then split into steps defined separately in Algorithm 4 and Algorithm 5.

There are two major innovations in the proposed method:

1. Graph theory has been used to represent and model the collusive relationships of the traders in wash trade activities. The concluded fundamental structure of the closed cycle structure within a trader graph simplifies the detection from the complexity of the collusive networks;
2. The wash trade order detection has been approached as a Knapsack problem which can be solved in two steps by traditional dynamic programming approaches.

Instead of only detecting the same-priced buy/sell orders in the "engine level" detection mechanism in CME, the proposed method determines the wash trade activities by considering the suspicious matched orders as well as the collusive groups, which are according to the trading activities in a certain time period rather than a tiny time interval in real-time detection. Therefore, the proposed approach best suits over-night detection in real financial world. However, the rapidly growing trading frequency challenges detection mechanisms and hence implementing the proposed approach in real-time in a computationally efficient way will be the focus of future work.

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