VIDEO MEASUREMENT OF LINEAR DISPLACEMENT ALONG AN OBLIQUE LINE USING THE CROSS-RATIO

Mark Colpus [1] and Mark Goss-Sampson [1]

Centre for Sports Science and Human Performance, University of Greenwich, United Kingdom[1]

The purpose of this study was to rediscover the cross-ratio and assess its effectiveness for measuring linear displacement when the image plane is not parallel to the object plane. In the laboratory a fixed, 4m object length was reconstructed with a mean absolute error of 2.6mm (s.d. = 1.6mm, maximum = 4.9mm). In the field, two cameras filmed a fixed, 8m object length with a mean absolute error of 13mm (s.d. = 5mm, maximum = 20mm). The method is very accessible to non-specialists in projective geometry and the results are both valid and reliable.

KEY WORDS: cross-ratio, camera, calibration, perspective.

INTRODUCTION: Ongoing technological development and increased use of video has meant that human movement is being filmed in more environments than ever before. Whilst qualitative feedback can be made available immediately and viewed globally, quantitative feedback still has to be processed and confirmed to be reliable and valid. Ease of use and affordability have secured the dominance of non-metric digital cameras and their employment by a wider, non-specialist user group, exacerbating the need to evidence quality of data. Accessible techniques have not evolved accordingly.
Video data recorded on an oblique plane, such as position on a running track or football pitch, has to be processed through the use of advanced, linear transformations within a Euclidean space. Analysing the data in its original, projective space enables the exploitation of projective invariant properties such as the cross-ratio. To date there appears to be no published data on the use of cross-ratio in sports biomechanics.
This paper investigates the cross-ratio and its suitability as an accessible technique for the digital age. Milne (1911) stated that Lazare Carnot wrote about the cross-ratio in 1803 but its concepts can be traced back to Pappus of Alexandria (c.290 – c.350 AD). The cross-ratio measures the projective invariance of four collinear points in Euclidean Space and directly links the object and image space, Figure 1.

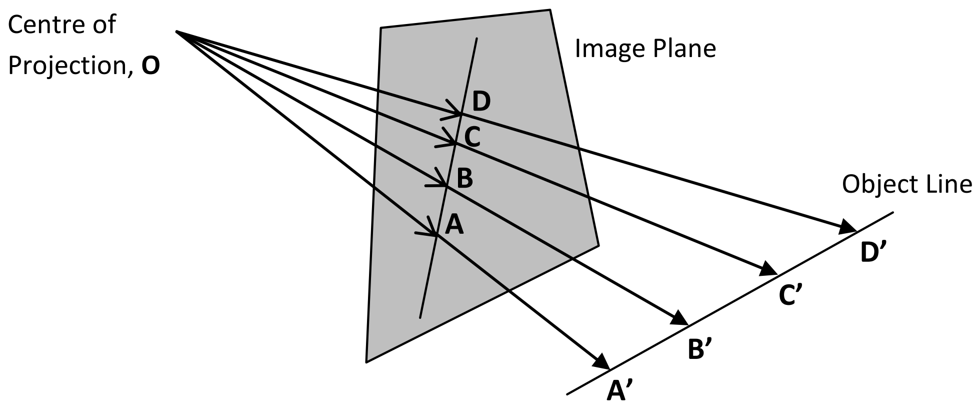


Figure 1: Projective invariance of the cross-ratio.

The cross-ratio can be defined by the relationships below:

$$\text{Cross-ratio (A'B'C'D')} = \frac{A'C'}{C'B'} \div \frac{A'D'}{D'B'} = \frac{A'C'}{C'B'} \times \frac{D'B'}{A'D'} = \frac{AC}{CB} \times \frac{DB}{AD} = \text{Cross-ratio (ABCD)}$$

If the points A, B and C are fixed then the location of D at any other point on the line returns a unique value for the cross-ratio (ABCD), Figure 2. Measurement of an image cross-ratio (ABCD) and knowledge of the object points A', B', and C' enables the direct calculation of the object point D'. The points A, B, C, and D can be any order.

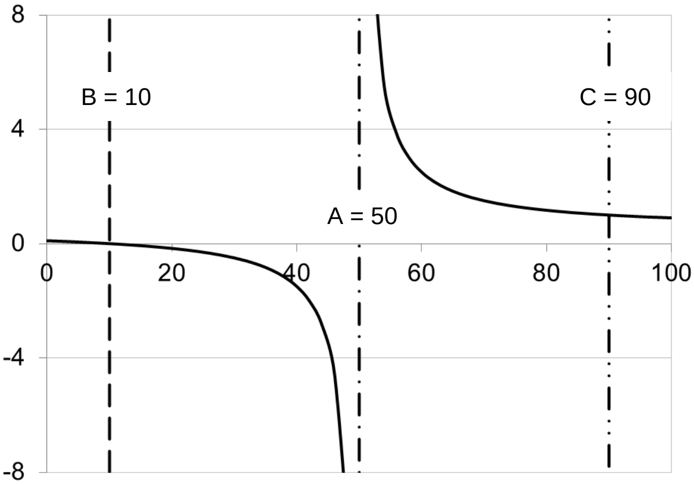
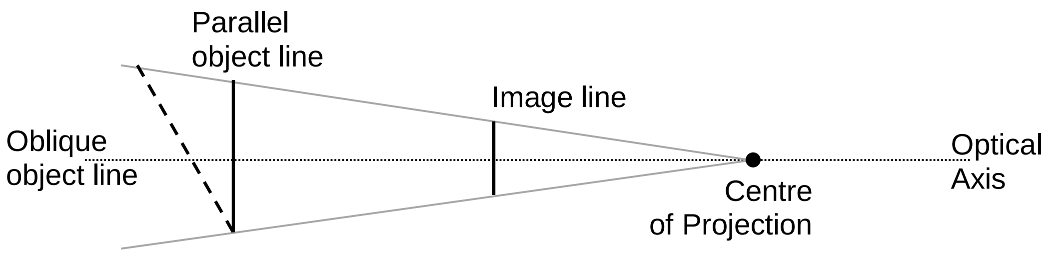


Figure 2: Unique variation of cross-ratio (ABCD) for fixed point A (centre, x = 50), point B (left, x = 10), point C (right, x = 90) and variable point D (range, 0 to 100).

For planar movement the method of similar triangles means that the image:object scale is constant, but only if the image plane is parallel to the object plane, Figure 3. For non-parallel planes, scaling can be achieved through a 3 × 3 linear transformation but there is no accepted methodology for the comparatively simple task of linear tracking, a task for which the cross-ratio would appear perfectly suited.

Figure 3: Method of similar triangles ensures constant scaling only when the image and object lines are parallel.



This study aims to introduce and validate the cross-ratio to sports biomechanics as a simple measurement tool valid for modern, digital media. After the viability of the cross-ratio was confirmed in the laboratory by reconstructing a 5m calibrated object line, its wider, practical application was then investigated using two camera positions to film a 30m line outside.

METHODS: In the laboratory a 5m line was secured to the floor and calibrated at 0.65m, 2.65m and 4.65m. Markers were placed at 0.1m intervals and filmed at an oblique angle using a Panasonic HX-WA30 digital camcorder, recording in 1080-30p mode.
Outside, a 30m line was calibrated at 1.5m, 10.5m, 19.5m and 28.5m with markers placed at 1.0m intervals. The line was part filmed at either end using the same HX-WA30 camcorder from distances of about 50m.
Using a Windows 7 computer the videos were replayed and digitised through a html5 canvas object displayed on a modern web browser. Image resolution was 1920 × 1080 pixels. The

image co-ordinates were saved in an Excel spreadsheet and a separate cross-ratio calculated for each marker. Combining the object calibration points with the image cross-ratios enabled the recalculation of the original marker positions.

RESULTS: In the 5m reconstruction mean absolute error was 2.6mm (s.d. = 1.6mm, maximum = 4.9mm). Mean error was slightly higher for the far half of the line (mean = 3.3mm, s.d. = 1.4mm) compared to the near half (mean = 1.9mm, s.d. = 1.5mm). It can be observed from Figure 4 that the error in the far half tended to be negative whilst the error in the near half tended to positive.

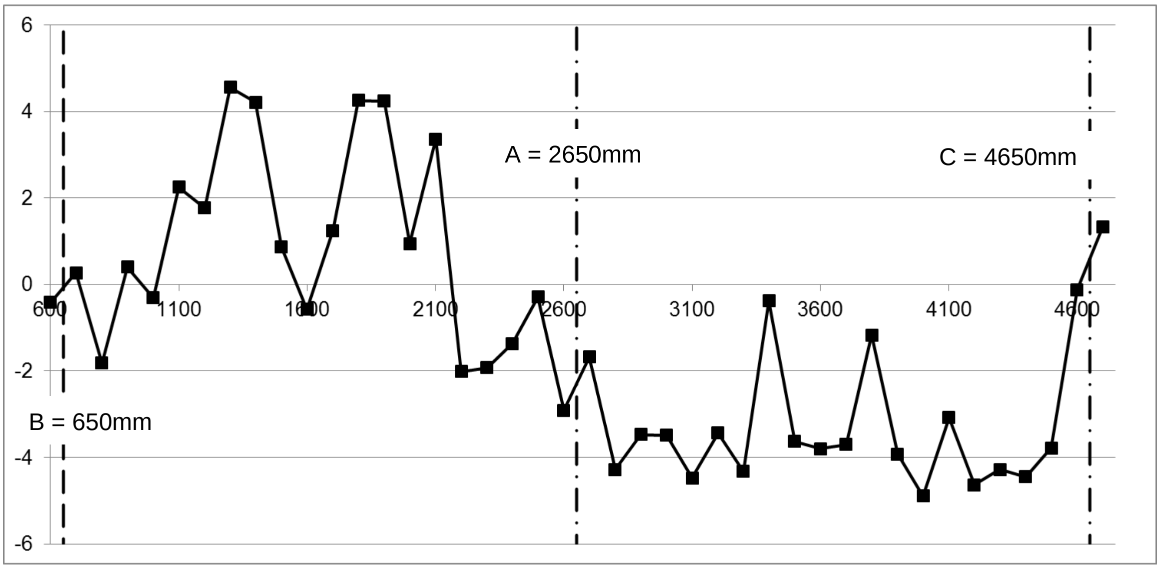


Figure 4: Variation of calculation error along the 5m calibration line (all measurements in mm).

The 30m reconstruction consisted of a near camera (2m to 19m, mean = 73mm, s.d. = 31mm) and a far camera (11m to 28m, mean = 83mm, s.d. = 38mm). The calculation errors in Figure 5 show the same positive/negative trends about the mid-point as Figure 4. The mid-section (11m to 19m) was recorded by both cameras and the systemic errors oppose each other, giving a mean, absolute error of 13mm (s.d. = 5mm, maximum = 20mm).

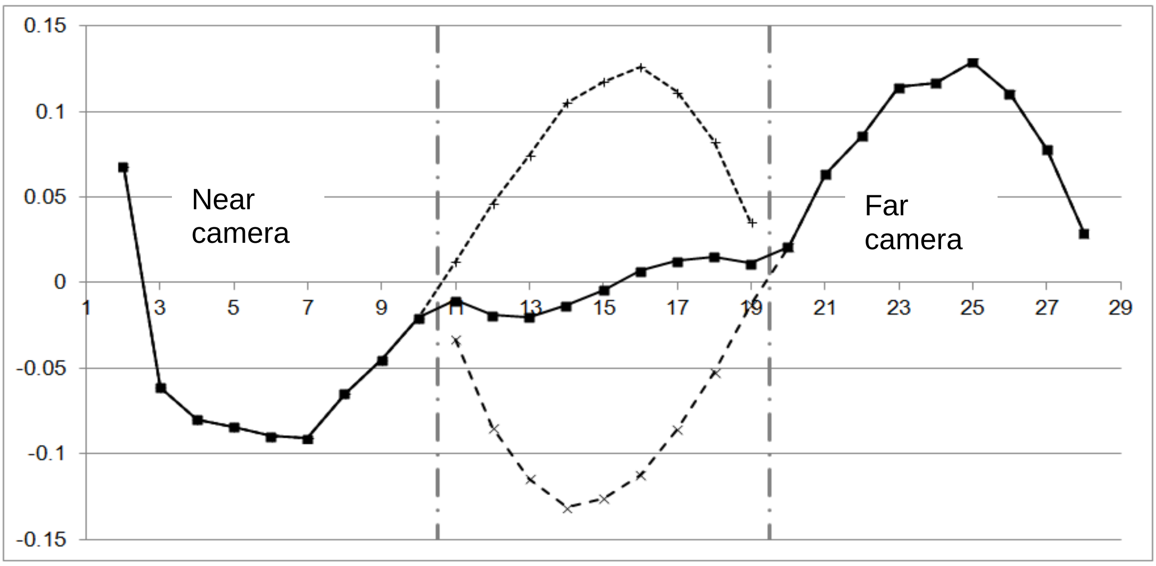


Figure 5: Variation of calculation error along the 30m calibration line (all measurements in m).

DISCUSSION: The 5m cross-ratio was accurate between the calibration points from 0.65m to 4.65m. The generally positive errors in the near half and generally negative errors in the far half would be consistent with the displacements from the midpoint being measured in opposite directions.

Validity was limited by the resolution of the image, causing digitised points to be quantised to the nearest pixel, paradoxically forcing high reliability. The local maxima and minima observed on the graph are a direct result of the quantised data. A systemic quantisation error measured in one direction only would manifest itself as a shortened displacement in the near half and extended displacement in the far half. This quantisation error could be reduced by using a higher definition image and/or using a digitiser with sub-pixel resolution.

As the object points move further from the image then the image:object scale decreases and the relative size of the quantisation errors would increase. This was observed with the slightly higher mean error observed in the far half. Unlike many linear techniques in two-dimensional analysis the cross-ratio clearly demonstrates its sensitivity to the changing distance between the object line and image plane.

Though the individual results for the 30m calibration had the same characteristics as the 5m line the errors were larger even though the curves were smoother because of better picture resolution. Single camera results appear to be good enough for small, lab-based displacements, e.g. single steps and jumps. For longer distances it appears the cross-ratio is not accurate enough to be of practical value but a second, shifted cross-ratio can cancel systemic error to give results dependent on image quality only. This would be the preferred method for displacement calculations within large, external environments such as running tracks and football pitches.

CONCLUSION: This study shows that the cross-ratio enables a valid object line calculation from an oblique image plane. Unsurprisingly, the quality of the results is dependent upon the variability of the image:object scale, resolution of the image and digitising process.

Two cameras filming the same oblique line and generating opposing cross-ratios can produce valid and reliable results. Simple displacement measures such as step length can be measured with non-metric, digital camcorders. Mathematically the calculation is simple, accessible and requires little specialist knowledge or software.

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