An Informational Model for Cellular Automata Aesthetic Measure

Mohammad Ali Javaheri Javid ✉ [1] and **Mohammad Majid al-Rifaie** [2] and **Robert Zimmer** [3]

Abstract. This paper addresses aesthetic problem in cellular automata, taking a quantitative approach for aesthetic evaluation. Although the Shannon's entropy is dominant in computational methods of aesthetics, it fails to discriminate accurately structurally different patterns in two-dimensions. We have adapted an informational measure to overcome shortcomings of entropic measure by using information gain measure. This measure is customised to robustly quantify the complexity of multi-state cellarer automata patterns. Experiments are set up with different initial configurations in a two-dimensional multi-state cellular whose corresponding structural measures at global level are analysed. Preliminary outcomes on the resulting automata are promising, as they suggest the possibility of predicting the structural characteristics, symmetry and orientation of cellular automata generated patterns.

1 INTRODUCTION

Cellular Automata (CA) initially invented by von Neumann in the late 1940's as a material independent system to address self-reproduction. A cellular automaton consists of a lattice of uniformly arranged finite state automata each of which taking input from the neighbouring automata; they in turn compute their next states by utilising a state transition function. A synchronous interactive application of state transition function (also known as a *rule*) over states of automata (also referred to as *cells*) generates the global behaviour of a cellular automaton.

The formation of complex patterns from simple rules sometimes with high aesthetic quality has been contributed to the creation of many digital art works since the 1960's. The most notable ones are "*Pixillation*", one of the early computer generated animations [32], the digital art works of Peter Struycken [31, 36], Paul Brown [4, 11] and evolutionary architecture of John Frazer [17]. Although classical one-dimensional CA with binary states can generate complex behaviours, experiments with two-dimensional multi-state CA have shown that adding more states significantly increases the complexly of behaviour, therefore, generating very complex symmetrical patterns with high aesthetic qualities [20, 21]. These observations have led to the quest of developing a quantitative model to evaluate the aesthetic quality of multi-state CA patterns. This work follows Birkhoff's tradition in studying mathematical bases of aesthetics, especially the association of aesthetic judgement with the degree of complexity of a stimulus. Shannon's information theory provided an objective measure of complexity. It led to emergence of various informational theories of aesthetics. However due to its nature, the entropic measure fails to take into account spacial characteristics of two-dimensional patterns which is fundamental in addressing aesthetic problem for CA generated patterns.

[1] Goldsmiths, University of London, email: m.javaheri@gold.ac.uk
[2] Goldsmiths, University of London, email: m.majid@gold.ac.uk
[3] Goldsmiths, University of London, email: r.zimmer@gold.ac.uk

2 CELLULAR AUTOMATA ART

The property of CA that makes them particularly interesting to digital artists is their ability to produce interesting and logically deep patterns on the basis of very simply stated preconditions. Iterating the steps of a CA computation can produce fabulously rich output. The significance of CA approach in producing digital art was outlined by Wolfram in his classical studies on CA behaviours collected in [39]. Traditional scientific intuition, and early computer art, might lead one to assume that simple programs would always produce pictures too simple and rigid to be of artistic interest. But extrapolating from Wolfram's work on CA, it becomes clear that even a program that may have extremely simple rules will often be able to generate pictures that have striking aesthetic qualities-sometimes reminiscent of nature, but often unlike anything ever seen before [39, p.11].

Knowlton developed "*Explor*" system for generating two-dimensional patterns, designs and pictures from explicitly provided 2D patterns, local operations and randomness. It aimed not only to provide the computer novice with graphic output; but also a vehicle for depicting results of simulations in natural (i.e., crystal growth) and hypothetical (e.g. cellular automata) situations, and for the production of a wide variety of designs [22]. Together with Schwartz and using *Explor*'s CA models, they generated "*Pixillation*", one of the early computer generated animations [32]. They contested in the *Eighth Annual Computer Art Contest* in 1970 with two entries, "*Tapestry I*" and "*Tapestry II*" (two frames from *Pixillation*). The "*Tapestry I*" won the first prize for "*new, creative use of the computer as an artist's tool*" as noted by selecting committee and covering the front page of *Computers & Automation* on Aug. 1970.

Meertens and Geurts also submitted an entry to the *Eighth Annual Computer Art Contest* with "*Crystalization*" as an experimental computer graphics generated by a asynchronous cellular automaton. Their entries were four drawings intended to generated patterns that combine regularity and irregularity in a natural way [19]. Peter Struycken, the Dutch contemporary digital artist has created many of his works "*Computer Structures*" (1969), "*Four Random Drawings for Lien and Ad*" (1972), *Fields* (1979-1980) with binary and multi-state CA [31, 36]. Brown, the British contemporary digital artists also applied various CA rules in his static and kinematic computer arts. "*Neighbourhood Count*" (1991), "*Infinite Permutations V1*" (1993-94), "*Infinite Permutations V2*" (1994-95), "*Sand Lines*" (1998), "*My Gasket*"(1998) "*Chromos*" (199-2000) [4, 11] are some of his CA generated works.

John F. Simon Jr Generated some of his art projects (art appliances) using CA based software and LCD panels to exhibit CA pattern formations. Every Icon (1996), ComplexCity (2000), Automata Studies (2002) are examples of his CA art works. Driessen and Verstappen have produced "*Ima Traveler*" (1996) and "*Breed*"(1995-2007) digital arts in a three-dimensional CA space. Dorin's *Meniscus* [12] and McCormack's *Eden* [26] are further examples of interactive artworks built on bases of CA rules. In addition, a combination of CA with other Alife techniques (e.g. evolutionary computing or L-systems) has been used to explore a set of rules generating patterns with aesthetic qualities [8, 34]. Fig. 1 shows some experimental patterns generated by the authors to demonstrate the generative capabilities of CA in creating appealing complex patterns.

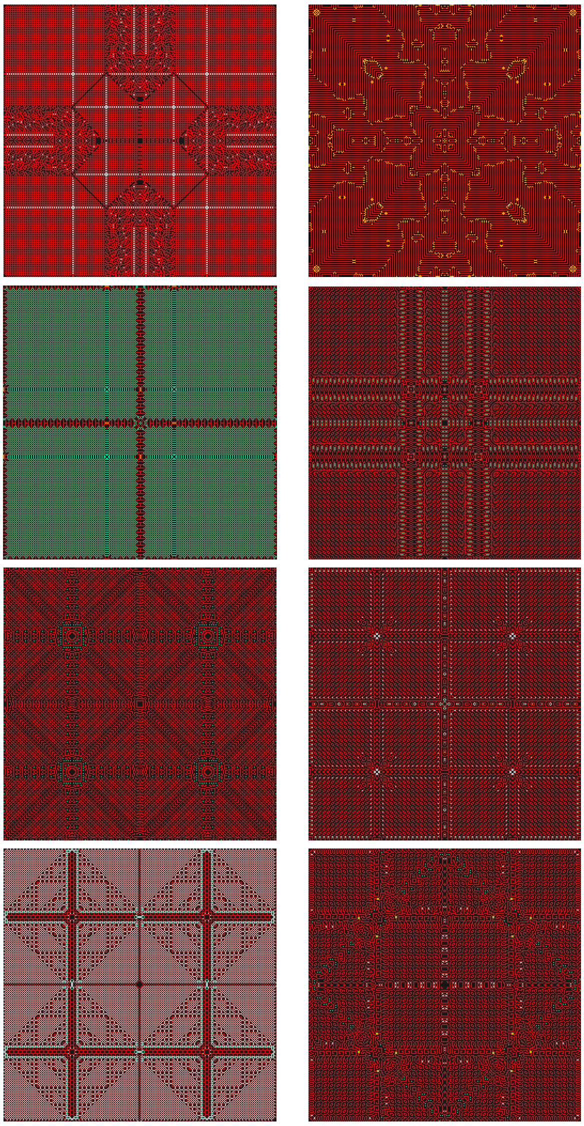


Figure 1: Sample CA generated complex symmetrical patters

3 DEFINITION OF CELLULAR AUTOMATA

In this section, formal notions of CA are explained and later referred to in the rest of the paper.

Definition 1 A cellular automaton is a regular tilling of a lattice with uniform deterministic finite state automata as a quadruple of $\mathcal{A} = \langle L, S, N, f \rangle$ such that:

1. L is an infinite regular *lattice* in $\mathbb{Z}$,
2. $S \subseteq \mathbb{N}^0$ is a finite set of integers as *states* $S = \{s_0, .., s_n\}$,
3. $N \subseteq \mathbb{N}^+$ is a finite set of integers as *neighbourhood* $N = \{n_1, .., n_n\}$,
4. $f : S^{|N|} \mapsto S$ is the *state transition function*.

The transition function f maps from the set of neighbourhood states $S^{|N|}$ where $|N|$ is the cardinality of neighbourhood set, to the set of states $\{s_0, .., s_n\}$ synchronously in *discrete time* intervals of $t = \{0, 1, 2, 3, ..., n\}$ where t_0 is the *initial time* of a CA with *initial configuration* (IC). In a two-dimensional square lattice ($\mathbb{Z}^2$) if the opposite sides of the lattice (up and down with left and right) are connected, the resulting *finite lattice* forms a torus shape (Fig.2) which is referred as a lattice with *periodic boundary condition*.

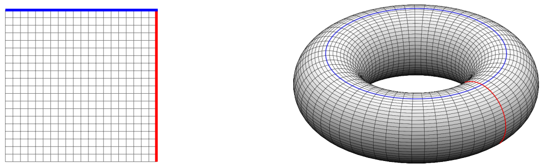


Figure 2: Connecting the opposite sides of a lattice forms a torus

The state of each cell (automaton) at time $(t+1)$ is determined by the states of immediate surrounding neighbourhood cells at time (t) given their neighbourhood template. There are two commonly used neighbourhood templates considered for two-dimensional CA. A five-cell *von Neumann neighbourhood* (Eq. 1) and a nine-cell *Moor neighbourhood* (Eq. 2). A mapping that satisfies $f(s_0, ..., s_0) = s_0$ where $(s_0 \in S)$ is called a *quiescent state*.

$$s_{i,j}^{t+1} = f\begin{pmatrix} & s_{(i,j+1)}^t & \\ s_{(i-1,j)}^t & s_{(i,j)}^t & s_{(i+1,j)}^t \\ & s_{(i,j-1)}^t & \end{pmatrix} \tag{1}$$

$$s_{i,j}^{t+1} = f\begin{pmatrix} s_{(i-1,j+1)}^t & s_{(i,j+1)}^t & s_{(i+1,j+1)}^t \\ s_{(i-1,j)}^t & s_{(i,j)}^t & s_{(i+1,j)}^t \\ s_{(i-1,j-1)}^t & s_{(i,j-1)}^t & s_{(i+1,j-1)}^t \end{pmatrix} \tag{2}$$

Since the elements of the S are non-negative integers and discrete instances of time are considered, the resulting cellular automaton is a *discrete time-space* cellular automaton. These type of CA can be considered as *discrete dynamical systems*.

4 INFORMATIONAL AESTHETICS

The topic of determining aesthetics or aesthetic measures have been a heated debate for centuries. There is a great variety of computational approaches to aesthetics in visual and auditory forms including mathematical, communicative, structural, psychological and neuroscience. A thorough examination of these methodologies from different perspective has been provided in [18]. In this section, some informational aesthetic measures are presented. Our review is focused on informational theories of aesthetics as these are the ones that conform with this work directly.

Birkhoff suggested an early aesthetic measure by arguing that the measure of aesthetic (M) is in direct relation with the degree of *order*

(O) and in reverse relation with the *complexity* (C) of an object [10]. Given that order and complexity are measurable parameters the aesthetic measure of (M) is:

$$M = \frac{O}{C} \quad (3)$$

Even though the validity of Birkhoff's approach to the relationship and definition of order and complexity has been challenged [14, 15, 13, 38], the notion of *complexity* and objective methods to quantify it remains a prominent parameter in aesthetic evaluation functions. Shannon's introduction of *information theory* provided a mathematical model to measure the degree of uncertainty (entropy) associated with a random variable [33]. The entropy H of a discrete random variable X is a measure of the average amount of uncertainty associated with the value of X. So $H(X)$ as the entropy of X is:

$$H(X) = -\sum_{x\in\mathcal{X}} P(x)\log_2 P(x) \quad (4)$$

The definition of entropy for X has a logarithm in the base of 2 so the unit of measure of entropy is in $bits$. Moles [27], Bense [6, 7, 5] and Arnheim [1, 2, 3] were pioneers of the application of Shannon's entropy to quantify order and complexity in Birkhoff's formula by adapting statistical measure of information in aesthetic objects. Berlyne used informational approach in his psychological experiments to determine humans perceptual curiosity of visual figures [9]. Bense argued that aesthetic objects are "vehicles of aesthetical information" where statistical information can quantify the aesthetical information of objects [6]. For Bense order is a process of artistic selection of elements from a determined repertoire of elements. The aesthetic measure (M_B) is a the relative redundancy (R) of the reduction of uncertainty because of selecting elements from a repertoire ($H_{max} - H$) to the absolute redundancy (H_{Max}).

$$M_B = \frac{R}{H_{max}} = \frac{H_{max} - H}{H_{max}} \quad (5)$$

where H quantifies entropy of the selection process from a determined repertoire of elements in $bits$ and H_{max} is maximum entropy of predefined repertoire of elements [5]. His informational aesthetics has three basic assumptions. (1) Objects are material carriers of aesthetic state, and such aesthetic states are independent of subjective observers. (2) A particular kind of information is conveyed by the aesthetic state of the object (or process) as *aesthetic information* and (3) objective measure of aesthetic objects is in relation with degree of order and complexity in an object [28].

Herbert Franke put forward an *aesthetic perception* theory on the ground of *cybernetic aesthetics*. He made a distinction between the amount of information being stored and the rate of information flowing through a channel as *information flow* measured in $bits/sec$ [16]. His theory is based on psychological experiments which suggested that conscious working memory can not take more than 16 $bits/sec$ of visual information. Then he argued that artists should provide a flow of information of about 16 $bits/sec$ for works of art to be perceived as beautiful and harmonious.

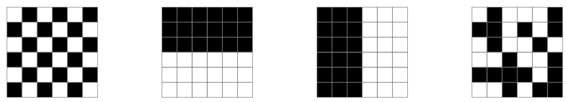
Staudek in his multi criteria approach (informational and structural) as *exact aesthetics* to Birkhoff's measure applied information flow I' by defining it as a measure assessing principal information transmission qualities in time. He used 16 $bits/sec$ reference as channel capacity $C_r = 16\ bits/sec$ and a time reference of 8 seconds ($t_r = 8s$) to argue that artefacts with $I > 128\ bits$ will not fit into the conscious working memory for absorbing the whole aesthetic message [35].

Adapting Bense's informational aesthetics to different approaches of the concepts of order and complexity in an image in [29, 30], three measures based on Kolmogorov complexity [24], Shannon entropy (for RGB channels) and Zurek's physical entropy [40] were introduced. Then the measures were are applied to analyse aesthetic values of several paintings (Mondrian, Pollock, and van Gogh). Machado and Cardoso [25] proposed a model based on relation between *image complexity* (IC) and *processing complexity* (PC) by arguing that images with high visual complexity, are processed easily so they have highest aesthetic value.

$$M(I) = \frac{IC(I)}{PC(I)} \quad (6)$$

5 INFORMATION GAIN MODEL

Despite the domination of entropic measures to aesthetic evaluation functions, it has a major shortcoming in terms of reflecting structural characteristics of two-dimensional patterns. Examples in Fig.3 illustrate this shortcoming by showing the calculations of entropy for two-dimensional patterns with the same density but different structural regularities and complexities. Fig.3a is a uniformly distributed patterns (a highly ordered pattern), Fig.3b and Fig.3c are patterns with identical structures but in vertical and horizontal orientations. Fig.3d is randomly arranged pattern (a random pattern). As it is evident from the comparison of the patterns and their corresponding entropy value, all of the patterns have the same entropy value. This clearly demonstrates that Shannon's entropy fails to differentiate structural differences among these patterns. In case of measuring complexity of CA generated patterns especially with multi-state structures, it would be problematic if only entropy used as a measure of complexity for the purpose of aesthetic evaluation.



(a) $H = 1$ (b) $H = 1$ (c) $H = 1$ (d) $H = 1$

Figure 3: The measure of entropy H for structurally different patterns with the same density of 50%

In order to overcome this problem we have adapt a mean information gain model as a measure reflecting structural characteristics of two-dimensional patterns. The information gain IG (also known as Kullback-Leibler divergence [23]) is the amount of information required to select a random variable X with state j if prior information about of variable X is known at the state of i.

$$IG_{x_{ij}} = -\log P_{(x_i|x_j)} \quad (7)$$

where $P_{(x_i|x_j)}$ the conditional probability of the variable x at state i given its state i. From Eq.7 we can define a mean information gain $\overline{IG}$ as the average information gain from possible states $(i|j)$ [37]:

$$\overline{IG} = \sum_{i,j} P(i,j) IG_{ij} = -\sum_{i,j} P_{i,j}\log P(i|j) \quad (8)$$

where $P_{(i,j)}$ is the joint probability of the variable x at state i and variable x at state j. Taking Eq.8 we can now define a structural complexity measure for a multi-state cellular automaton as follows:

Definition 2 *A structural complexity measure is the mean information gain of a cell having a heterogeneous neighbouring cell in a two-dimensional multi-state CA pattern.*

$$\overline{IG} = -\sum_{i,j} P_{(i,j)} \log_2 P_{(i|j)} \quad (9)$$

where $P_{(i,j)}$ is the joint probability of a cell having the i state (colour) and the neighbouring cell (in one of the four directions of up, low, left or right) has the state (colour) j. And $P_{(i|j)}$ is the conditional probability of the state (colour) i given that its neighbouring cell has state (colour) j. Since the logarithm is in the base of 2 so unit of $\overline{IG}$ measures information gain in $bits$.The mean information gain $\overline{IG}$ defined by equation (9) measures the lack of information about other elements of the structure (e.g. the state of the neighbouring cell in one of the four directions), when some properties of the structure are known (e.g. the state of a cell). It can be noted that the combined probabilities of $P_{i,j}$ and $P_{i|j}$ describe spatial correlations in the a pattern so that the $\overline{IG}$ complexity can detect inherent correlations of patterns. Considering neighbourhood structure of CA cells following variations of $\overline{IG}$ can be defined where for each cell in i state a neighbouring cell in j state in one of four directions defined as followings:

$$\overline{IG}_u = -\sum_{i,j_{(x,y+1)}} P_{(i,j_{(x,y+1)})} \log_2 P_{(i|j_{(x,y+1)})} \quad (10)$$

$$\overline{IG}_d = -\sum_{i,j_{(x,y-1)}} P_{(i,j_{(x,y-1)})} \log_2 P_{(i|j_{(x,y-1)})} \quad (11)$$

$$\overline{IG}_l = -\sum_{i,j_{(x-1,y)}} P_{(i,j_{(x-1,y)})} \log_2 P_{(i|j_{(x-1,y)})} \quad (12)$$

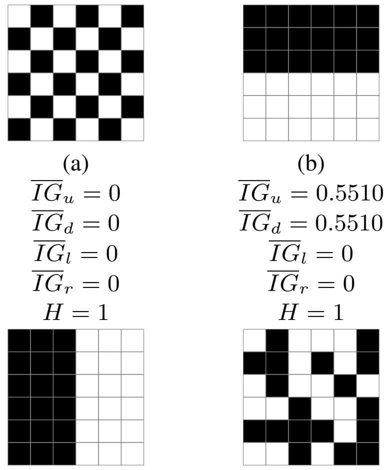
$$\overline{IG}_r = -\sum_{i,j_{(x+1,y)}} P_{(i,j_{(x+1,y)})} \log_2 P_{(i|j_{(x+1,y)})} \quad (13)$$

6 EXPERIMENTS AND RESULTS

A set of experiments were designed to examine how effectively $\overline{IG}$ is able to discriminate structurally different patterns generated by a multi-state CA rule.

The size of the CA lattice is set to 129×129 cells. The black background cells are quiescent state cells (s_0). The experimental rule chosen for cellular automaton maps three states represented with *green*, *red* and *white* colour cells. The experiments are conducted with a single cell and a randomly seeded initial configurations with 50% destiny of three states. All the experiments are conducted for 200 successive time steps, however a sample of time steps ($\{0, 10, 20, 40, 50, 60, 80, 100, 200\}$) are presented in the figures. The $\overline{IG}$ for all four directions along with their corresponding entropy H are calculated for all of the time steps.

Figs. 5 and 6 illustrate the formation of CA patterns starting from two different initial configurations and their corresponding $\overline{IG}$ and H for sample time steps. The $\overline{IG}$ measures in Fig. 5 which shows the formation of CA patterns from a single cell are conforming in directional calculations; it means that each cell in the patterns have exactly the same amount of information regarding their neighbouring cell in one of four directions. Therefore it indicates that the development of the patterns are symmetrical in four directions. In other words, the CA rule with a single cell has created pattens with 4-fold rotational symmetry.



(a) $\overline{IG}_u = 0$ $\overline{IG}_d = 0$ $\overline{IG}_l = 0$ $\overline{IG}_r = 0$ $H = 1$

(b) $\overline{IG}_u = 0.5510$ $\overline{IG}_d = 0.5510$ $\overline{IG}_l = 0$ $\overline{IG}_r = 0$ $H = 1$

(c) $\overline{IG}_u = 0$ $\overline{IG}_d = 0$ $\overline{IG}_l = 0.5510$ $\overline{IG}_r = 0.5510$ $H = 1$

(d) $\overline{IG}_u = 0.9839$ $\overline{IG}_d = 0.9871$ $\overline{IG}_l = 0.9377$ $\overline{IG}_r = 0.9473$ $H = 1$

Figure 4: The measure of entropy H and $\overline{IG}$ for structurally different patterns but the same density of 50%

The measures in Fig. 6 starts with $\overline{IG} \approx 1.7$ for a random initial configuration and with $H \approx 1.5$ (maximum entropy for a three-state patterns since $\log_2 3 = 1.5848$). The formation of patterns with local structures reduced the value of $\overline{IG}$. The value of $\overline{IG}$ are not conforming in any directional calculations which is an indicator of the development of less ordered ("chaotic") patterns. From the comparison of H with $\overline{IG}$ in the set of experiments, it is clear that it would be very unlikely to discriminate the structural differences of patterns with a single measure of H given the diversity of patterns that can be generated by various CA models.

Computing directional measures of $\overline{IG}$ and comparing their values provide a more subtle measure of structural order or complexity. The conformity or non-conformity of $\overline{IG}$ measure in up, down, left and right neighbouring cells clearly gives us not only an accurate measure of structural characteristics of a two-dimensional pattern but they also provide us with information about the orientation of the patterns as well.

7 CONCLUSION

CA, which are to fundamental to the study of self-replicating systems, are powerful tools in generating computer art. The multi-state CA rule space is a vast set of possible rules which might generate interesting patterns with high aesthetic qualities. The application of CA in digital art has been reviewed; and the concepts of order and complexity form Shannon's information entropy perspective in the CA framework has been analysed. Based on an informational approach to aesthetics, mean information gain model were adapted to measure the aesthetic values of generated patterns in the global level of multi-state CA environment. The results of experiments show that not only the mean information gain model can distinguish the structural complexity of patterns compared to entropic approaches but also it can distinguish the symmetrical orientation of the patterns as well. In this paper we only focused on the complexity of patterns in discrete instances of CA time evolution. Having a model to evaluate the aes-

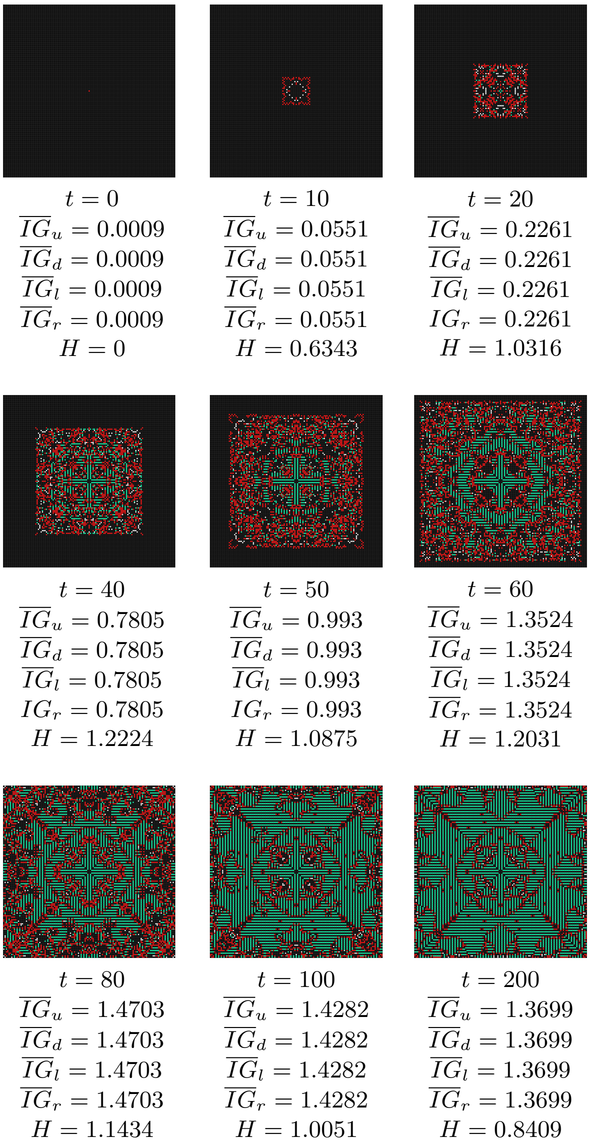


Figure 5: Patterns generated from a single cell as initial configuration and their corresponding $\overline{IG}$ and H values

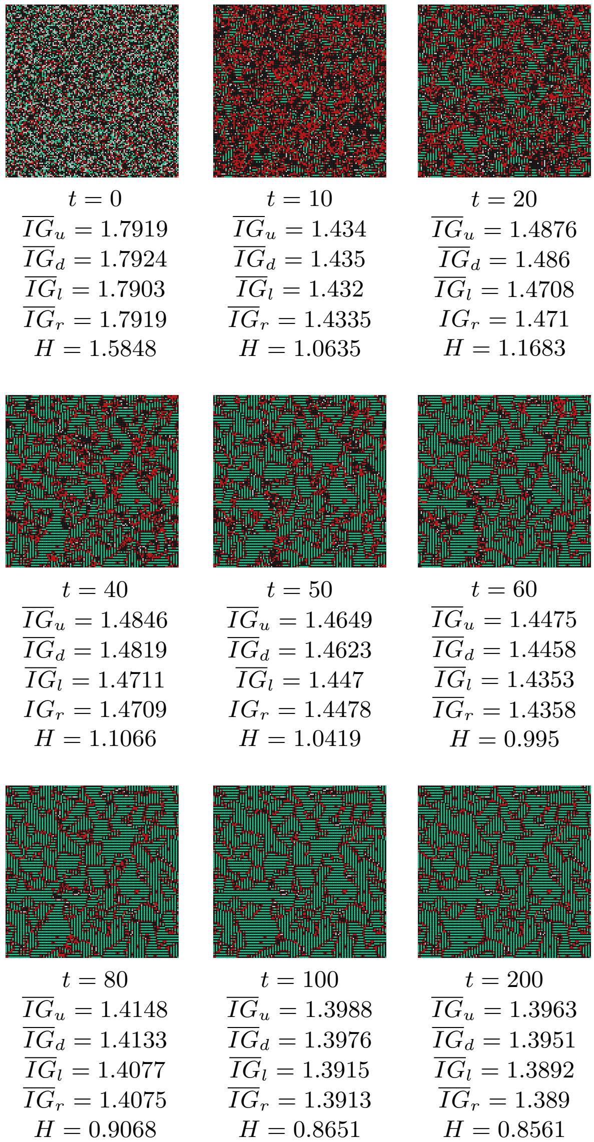


Figure 6: Patterns generated from a 50% seeded density as initial configuration and their corresponding $\overline{IG}$ and H values

thetic qualities of CA generated patterns could potentially enable us to have an integrated process of generation-evaluation which is a subject of on going research.

REFERENCES

[1] Rudolf Arnheim, *Art and visual perception: A psychology of the creative eye*, Univ of California Press, 1954.
[2] Rudolf Arnheim, 'Towards a psychology of art/entropy and artan essay on disorder and order', *The Regents of the University of California*, (1966).
[3] Rudolf Arnheim, *Visual thinking*, Univ of California Press, 1969.
[4] Honor Beddard and Dodds Dodds, *Digital Pioneers*, V&A Pattern, V&A Publishing, 2009.
[5] M. Bense, *Kleine abstrakte ästhetik*, E. Walther, 1969.
[6] Max Bense, *Aestetica: Programmierung des Schönen, allgemeine Texttheorie und Textästhetik*, Agis-Verlag, 1960.
[7] Max Bense, *Aesthetica, Einfuehrung in die neue Aesthetik*, Agis-Verlag, 1965.
[8] Katie A Bentley, 'Exploring aesthetic pattern formation', in *Generative Art 2002 conference proceedings*, (2002).
[9] Daniel Ellis Berlyne, 'Conflict and information-theory variables as determinants of human perceptual curiosity.', *Journal of experimental psychology*, **53**(6), 399, (1957).
[10] G.D. Birkhoff, *Aesthetic Measure*, Harvard University Press, 1933.
[11] Paul Brown, 'Stepping stones in the mist', in *Creative evolutionary systems*, 387–407, Morgan Kaufmann Publishers Inc., (2001).
[12] Alan Dorin, 'The virtual ecosystem as generative electronic art', in *Applications of Evolutionary Computing*, 467–476, Springer, (2004).
[13] Hans J Eysenck, 'An experimental study of aesthetic preference for polygonal figures', *The Journal of General Psychology*, **79**(1), 3–17, (1968).
[14] Hans Jürgen Eysenck, 'The empirical determination of an aesthetic formula.', *Psychological Review*, **48**(1), 83, (1941).
[15] Hans Jurgen Eysenck, 'The experimental study of the 'good gestalt' –a new approach.', *Psychological Review*, **49**(4), 344, (1942).
[16] Herbert W Franke, 'A cybernetic approach to aesthetics', *Leonardo*, 203–206, (1977).
[17] John Frazer, *An evolutionary architecture*, Architectural Association Publications, Themes VII, 1995.

[18] Philip Galanter, 'Computational aesthetic evaluation: past and future', in *Computers and Creativity*, eds., Jon McCormack and Mark d'IInverno, 255–293, Springer, (2012).

[19] Leo Geurts and Lambert Meertens, 'Crystallization', *Computers and Automation*, **19**(8), 22, (1970).

[20] Mohammad Ali Javaheri Javid, Mohammad Majid al Rifaie, and Robert Zimmer, 'Detecting symmetry in cellular automata generated patterns using swarm intelligence', in *Theory and Practice of Natural Computing*, eds., Adrian-Horia Dediu, Manuel Lozano, and Carlos Martn-Vide, volume 8890 of *Lecture Notes in Computer Science*, pp. 83–94. Springer International Publishing, (2014).

[21] Mohammad Ali Javaheri Javid and Rene te Boekhorst, 'Cell dormancy in cellular automata', in *International Conference on Computational Science (3)*, eds., Vassil N. Alexandrov, G. Dick van Albada, Peter M. A. Sloot, and Jack Dongarra, volume 3993 of *Lecture Notes in Computer Science*, pp. 367–374. Springer, (2006).

[22] Kenneth C Knowlton, 'Explor-a generator of images from explicit patterns, local operations, and randomness', in *Proceedings of 9th Meeting of UAIDE*, pp. 544–583, (1970).

[23] S. Kullback and R. A. Leibler, 'On information and sufficiency', *The Annals of Mathematical Statistics*, **22**(1), pp. 79–86, (1951).

[24] Ming Li, *An introduction to Kolmogorov complexity and its applications*, Springer, 1997.

[25] Penousal Machado and Amílcar Cardoso, 'Computing aesthetics', in *Advances in Artificial Intelligence*, 219–228, Springer, (1998).

[26] Jon McCormack, 'Evolving sonic ecosystems', *Kybernetes*, **32**(1/2), 184–202, (2003).

[27] Abraham Moles, *Information theory and esthetic perception. Trans. JE Cohen.*, U. Illinois Press, 1968.

[28] Frieder Nake, 'Information aesthetics: An heroic experiment', *Journal of Mathematics and the Arts*, **6**(2-3), 65–75, (2012).

[29] Jaume Rigau, Miguel Feixas, and Mateu Sbert, 'Informational aesthetics measures', *Computer Graphics and Applications, IEEE*, **28**(2), 24–34, (2008).

[30] Jaume Rigau, Miquel Feixas, and Mateu Sbert, 'Conceptualizing birkhoff's aesthetic measure using shannon entropy and kolmogorov complexity', in *Proceedings of the Third Eurographics conference on Computational Aesthetics in Graphics, Visualization and Imaging*, pp. 105–112. Eurographics Association, (2007).

[31] Ir Remko Scha, 'Kunstmatige kunst', *De Commectie*, **2**(1), 4–7, (2006).

[32] L.F. Schwartz and L.R. Schwartz, *The Computer Artist's Handbook: Concepts, Techniques, and Applications*, W W Norton & Company Incorporated, 1992.

[33] Claude Shannon, 'A mathematical theory of communication', *The Bell System Technical Journal*, **27**, 379–423 & 623–656, (October 1948).

[34] Karl Sims, 'Interactive evolution of dynamical systems', in *Toward a practice of autonomous systems: Proceedings of the first European conference on artificial life*, pp. 171–178, (1992).

[35] Tomáš Staudek, *Exact Aesthetics. Object and Scene to Message*, Ph.D. dissertation, Faculty of Informatics, Masaryk University of Brno, 2002.

[36] Peter Struycken, 'Splash 1972/1974', in *Artist and computer*, ed., Ruth Leavitt, 30–31, Harmony Books, (1976).

[37] Renate Wackerbauer, Annette Witt, Harald Atmanspacher, Jürgen Kurths, and Herbert Scheingraber, 'A comparative classification of complexity measures', *Chaos, Solitons & Fractals*, **4**(1), 133–173, (1994).

[38] D. J. Wilson, 'An experimental investigation of birkhoff's aesthetic measure', *The Journal of Abnormal and Social Psychology*, **34**(3), 390, (July 1939).

[39] Stephen Wolfram, *A New Kind of Science*, Wolfram Media Inc., 2002.

[40] Wojciech H Zurek, 'Algorithmic randomness and physical entropy', *Physical Review A*, **40**(8), 4731, (1989).