Optimal Contracts for Central Bankers: Calls on Inflation

Christian-Oliver Ewald[a,*], Johannes Geißler[b]

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[a] Adam Smith Business School - Economics, University of Glasgow, UK
[b] Bayerische Landesbank - Credit Risk, Munich, Germany

Abstract

We consider a framework featuring a central bank, private and financial agents as well as a financial market. The central bank's objective is to maximize a functional, which measures the classical trade-off between output and inflation plus income from the sales of inflation linked calls minus payments for the liabilities that the inflation linked calls produce at maturity. Private agents have rational expectations and financial agents are averse against inflation risk. Following this route, we explain demand for inflation linked calls on the financial market from a no-arbitrage assumption and derive pricing formulas for inflation linked calls, which lead to a supply-demand equilibrium. We then study the consequences that the sales of inflation linked calls have on the observed inflation rate and price level. Similar as in Walsh (1995) we find that the inflationary bias is significantly reduced, and hence that markets for inflation linked calls provide a mechanism to implement inflation contracts as discussed in the classical literature.

Keywords: Monetary policy, inflation contracts, inflationary bias, mechanisms, inflation indexed securities
JEL Subject Classification: E52; E61; C73; E44;

*Corresponding author at: Adam Smith Business School, University of Glasgow, Glasgow, G12 8QQ, United Kingdom, phone: +44(0)141 330 5354, fax: +44 (0) 141 330 2000
E-mail addresses: christian.ewald@glasgow.ac.uk (C.-O. Ewald)

1 Introduction

Inflation contracts have been widely discussed in the macro-economic and monetary policy literature. In principal agent manner, the central banker is offered a compensation, which depends on the realized inflation level. This approach was developed by Persson and Tabellini (1993) as well as Walsh (1995) and shown to be a useful device in order to remove inflationary bias, and in this way raise general welfare. The contracts approach was further discussed in Svensson (1997) and Muscatelli (1999), and put into relation with the concept of inflation targeting. Muscatelli highlighted some advantages of the contracts approach as compared to the inflation targeting approach, in the context of uncertainties in preferences and output targets.

The contracts proposed by Persson and Tabellini (1993) and Walsh (1995) are between the government acting as a principal and the central banker as an agent. In this note we show that this principal agent relationship is not required, and that financial markets for inflation indexed securities can reduce inflationary bias in the same way as the inflationary contracts discussed in the classical literature. Rather than forcing the central banker to enter an inflation contract with the government, we allow the central banker to sell inflation contracts in form of inflation linked calls to financial agents. The central banker can choose the quantity of contracts placed on the market and in this way determines the supply. Demand is mainly determined by the financial agents' level of risk aversion toward inflationary risk and their expectations about the central banks inflation policy. The type of contract proposed in this article is realistic, in fact, according to Deacon et al. (2004) no less than 27% of UK government debt is inflation indexed and treasury inflation protected securities (TIPS) issued by the US Treasury contain as their final payment an inflation linked call.

The model that we present is a full general equilibrium model, in which quantity and price of the inflation linked calls is determined by equating supply and demand, and private as well as financial agents reflect rational expectations in their inflation forecast. In this latter aspect, the current article differs structurally form Ewald and Geissler (2013), who assumed adaptive expectations (as well as a different contract structure). The supply side is modeled in analogy with Walsh (1995) and most of the other inflation contract literature, with the difference that the inflation contract is replaced by a quantity of inflation linked calls, which the central bank can choose. The demand side is modeled in analogy to Black-Scholes, assuming that the price level follows a geometric Brownian motion, whose drift rate is the inflation level chosen by the central bank.

While the identification of markets for inflation linked securities as a Walsh (1995) like mechanism contributes to the economic literature, our results also contribute to the financial literature. Almost all of the classical financial literature on pricing of inflation indexed securities takes inflation as exogenously given, and applies standard Black-Scholes type theory or the Heath-Jarrow-Morton approach to term structure modeling in order to develop pricing formulas, see for example Deacon et al. (2004) and Jarrow and Yilderim (2003). These models ignore the central bank's role in issuing inflation indexed securities and the feedback effects on monetary policy this has. To the best of our knowledge, this article presents the first pricing formulas for inflation linked calls in which monetary policy is integrated within a full general equilibrium framework.

2 Central Bank's Supply of Inflation Linked Calls

An inflation linked call issued at time s with strike $\tilde{K}$ and maturity T is a financial derivative that pays off nominal

$$\left(\frac{P_T}{P_s} - \tilde{K}\right)^+ = \left(e^{\pi(T-s)} - e^{K(T-s)}\right)^+ := \max\left(e^{\pi(T-s)} - e^{K(T-s)}, 0\right) \tag{1}$$

at maturity time T. Here we have set $K = \frac{1}{T-s} \log(\tilde{K})$. Deacon, Derry and Mirfedereski (2004) present an excellent overview about all types of traded inflation indexed securities. Here P_t denotes the price level at time t and π denotes the average instantaneous inflation rate over time $T - s$. We consider a simplistic setup, in which the central bank can issue inflation linked calls at time $s = 0$ which mature at time $T = 1$. Denoting log-output with y, natural log-output with y_n and a target value with k, the classic quadratic loss function of the central bank has to be modified in the following way

$$Z^{CB} := \frac{1}{2} \left[\lambda \left(y - y_n - k \right)^2 + \pi^2 \right] + N \cdot d \left[\left(e^{\pi} - e^{K} \right)^{+} - p \right], \tag{2}$$

in order to take account of the central bank's profits from the sales of inflation linked calls. Here p denotes the price of one inflation linked call and N the number of inflation linked calls issued by the central bank. The factor d is a weight and measures the contribution of the financial position of the central bank in relations to its output and inflation objectives. Expression (2) is of similar type as expressions (6) and (7) in Muscatelli (1999), which include a penalty function and an exogenously defined inflation contract instead of the profits from inflation linked call sales. However, in contrast to Muscatelli and all other literature, in our case the price p and the quantity N, and as such the inflation contract itself will be determined endogenously within the model, via the financial market modeled in the next section.

As in the classical literature we assume that output is given by a Lucas-type aggregate supply function of the form

$$y = y_n + a(\pi - \pi^e) + \varepsilon, \tag{3}$$

with a being the slope of the Phillips curve, π^e expected inflation and ε a stochastic shock to the economy with zero mean under the central bank's measure. We assume in the following that $\epsilon \sim \mathcal{N}(0,\sigma)$.[1]

We assume that the central bank is able to observe the economy shock ε, but private agents are not.[2] This means that the central bank's inflation policy can depend on ε, i.e. $\pi = \pi(\epsilon)$, while private agents' expectations must not. Substitution of (3) into (2) and taking expectations gives

$$V := \mathbb{E}\left\{\frac{1}{2}\lambda\left[a\left(\pi-\pi^e\right)+\epsilon-k\right]^2+\frac{1}{2}\pi^2+N\cdot d\left[\left(e^{\pi}-e^{K}\right)^+-p\right]\right\}. \tag{4}$$

The quantity N and price p of inflation linked calls will be determined in full equilibrium in section 4. For given N and p we will now compute the central bank's optimal inflation policy. To stress that this policy in general depends on N we write $\pi(N)$ in the following. We will later look for a rational expectation equilibrium, where $\pi^e = \mathbb{E}(\pi)$, and as π depends on N, so will π^e. Hence we will include N in the notation for the expected inflation rate as well, and assume for the moment that $\pi^e = \pi^e(N)$ is given.

Proposition 1. *The central bank's optimal choice for the inflation rate $\pi(N)$ as a function of the shock ε is given by*

$$\pi^*(N,\varepsilon) = \begin{cases} \xi(N,\varepsilon) & \textit{if} \quad \varepsilon \geq \eta(N) \\ K \quad \textit{if} & \theta(N) \leq \varepsilon \leq \eta(N) \\ \psi(N,\varepsilon) & \textit{if} \quad \varepsilon \leq \theta(N), \end{cases}$$

[1]For a more general discussion on noise, please compare Sun et al. (2010a,b) as well as Li and Jin (2012)

[2]In a more dynamic version of the model private agents would observe the economy shock with a time delay. How to deal with time delay conceptually is discussed in Sun et al. (2014) and Sun et al. (2015)

where

$$\begin{aligned}
\xi(N,\varepsilon) &:= \frac{a^2\lambda\pi^e(N)+a\lambda k}{1+a^2\lambda} - \frac{a\lambda\varepsilon}{1+a^2\lambda} \\
\psi(N,\varepsilon) &:= \xi(N,\varepsilon) - \frac{1}{T-s}\mathbb{W}\left(\frac{dN(T-s)^2 e^{\xi(N,\varepsilon)(T-s)}}{1+a^2\lambda}\right)
\end{aligned}$$

and

$$\begin{aligned}
\eta(N) &:= a\pi^e(N)+k-\frac{(1+a^2\lambda)K}{a\lambda} \\
\theta(N) &:= \eta(N)-\frac{dN(T-s)e^{K(T-s)}}{a\lambda}.
\end{aligned}$$

Here $\mathbb{W}$ *denotes the Lambert W-function, compare Abramowitz and Stegun (1965).*

Proof. To find the optimal $\pi^*(N,\epsilon)$, we can carry out the minimization of (4) pointwise for each ϵ inside the expectation. Also note that p in (4) can be treated as an additive constant and as such can be ignored in the minimization. We omit the arguments N and ϵ from π, π^e, ξ and ψ and denote

$$\begin{aligned}
V_1 &= \frac{1}{2}\lambda(a(\pi-\pi^e)+\varepsilon-k)^2+\frac{1}{2}\pi^2+dN\left(e^{\pi(T-s)}-e^{K(T-s)}\right)^+, \\
V_2 &= \frac{1}{2}\lambda(a(\pi-\pi^e)+\varepsilon-k)^2+\frac{1}{2}\pi^2, \\
V_3 &= \frac{1}{2}\lambda(a(\pi-\pi^e)+\varepsilon-k)^2+\frac{1}{2}\pi^2+dN\left(e^{\pi(T-s)}-e^{K(T-s)}\right).
\end{aligned}$$

Note that V_1 corresponds to V, while V_2 respectively V_3 correspond to the two cases where the inflation linked call is not exercised respectively exercised. Verifying the first order conditions for V_2 and V_3 it can easily be seen that ξ minimizes V_2, while ψ minimizes V_3. Therefore, by Lemma 1 following below, the optimal choice π^* for V_1 is ξ if $\xi \leq K$ and ψ if $\psi \geq K$. If the shock is such that $\psi \leq K \leq \xi$ (or alternatively $\theta(N) \leq \epsilon \leq \eta(N)$) the first statement in Lemma 1 lets us conclude that $\pi^* \geq K$ while the second statement lets us conclude that $\pi^* \leq K$, which together implies

$\pi^* = K$. □

Lemma 1. *Let V_1, V_2 and V_3 be the functions defined in the proof of Proposition 1.*

1. *V_1 attains its minimum for some $\pi < K$ if and only if V_2 attains its minimum for that particular π.*

2. *V_1 attains its minimum for some $\pi > K$ if and only if V_3 attains its minimum for that particular π.*

Proof. First note that all three functions V_1, V_2 and V_3 are convex in π for $\lambda > -\frac{1}{a^2}$. This can be easily verified from the second order derivatives of the corresponding functions. We assume $\lambda > -\frac{1}{a^2}$ in the following. Therefore each function has exactly one global minimum and no local maximum. As it is easy to check ξ minimizes V_2, while ψ minimizes V_3. In particular we see that V_2 is always minimized for larger values of π than V_3 and therefore there is no inconsistency in the conditions above.

For the first statement note that $V_2(\pi) \leq V_1(\pi)$ with equality for all $\pi \leq K$. Hence if V_2 is minimal for some $\pi^* \leq K$ then V_1 is also minimal for π^*. For the inverse implication suppose there is some $\pi^* < K$ minimizing V_1. Then there exists an $\varepsilon > 0$ such that $V_2(\pi) = V_1(\pi)$ for all $\pi \leq \pi^* + \varepsilon$. Hence π^* minimizes V_2 over the interval $(-\infty, \pi^* + \varepsilon]$ and by the above convexity argument we know that it is the unique global minimum of V_2.

The second statement can be shown analogously using $V_3(\pi) \leq V_1(\pi)$ with equality for all $\pi \geq K$, whilst for the inverse implication we use the convexity of $V_3(\pi)$. □

So far we have established how the central bank would optimally choose the inflation rate given a specific number of inflation linked calls it has issued, the announced private agent's expected inflation rate π^e and the economy shock ε. Let us now consider how private agents build their expectations in this modified setup. While private agents do not know the outcome of the economy shock, we do assume that they know its distribution. This assumption differs from Walsh (1995), where the linear nature of the contract permits, that private agents only have to know that the shock is neutral, i.e. does have expectation zero. As indicated earlier, we assume that private agents have rational expectations, i.e. $\pi^e(N) = \mathbb{E}(\pi(N))$. Using Proposition 1 we obtain

$$\begin{aligned} \pi^e(N) &= \mathbb{P}(\varepsilon \geq \eta(N))\mathbb{E}(\xi(N)|\varepsilon \geq \eta(N)) + \mathbb{P}(\theta(N) \leq \varepsilon \leq \eta(N))K \\ &+ \mathbb{P}(\varepsilon \leq \theta(N))\mathbb{E}(\psi(N)|\varepsilon \leq \theta(N)). \end{aligned} \tag{5}$$

Note that the right hand side of (5) depends on $\pi^e(N)$ through $\xi(N), \psi(N), \theta(N)$ and $\eta(N)$ and in fact represents a fixed point equation. The fact that ε is normally distributed allows in principle to write down and compute the expectations on the right hand side, however it remains impossible to solve for π^e. For the general case this has to be done by an iterative procedure, which we carried out in order to obtain our numerical results.

If $N = 0$, i.e. no inflation linked calls are issued by the central bank, the objective function (4) is identical with the classical quadratic loss function, and we should obtain the classical results $\pi = \pi(0, \epsilon) = a\lambda k - \left(\frac{a\lambda}{1+a^2\lambda}\right)\epsilon$ and $\pi^e = \pi^e(0) = a\lambda k$. This is confirmed in the following remark

Remark 1. *For the case $N = 0$, in which no inflation linked calls are issued, we find that $\xi(0)$ and $\psi(0)$ coincide and so do $\theta(0)$ and $\eta(0)$. Hence the middle summand in*

(5) vanishes and therefore, omitting all arguments $N = 0$, we find that

$$
\begin{aligned}
\pi^e =& \mathbb{P}(\varepsilon \geq \eta)\mathbb{E}(\xi|\varepsilon \geq \eta) + \mathbb{P}(\varepsilon \leq \eta)\mathbb{E}(\xi|\varepsilon \leq \eta) \\
=& \int_\eta^\infty \xi(\varepsilon) d\mathbb{P}(\varepsilon) + \int_{-\infty}^\eta \xi(\varepsilon) d\mathbb{P}(\varepsilon) \\
=& \frac{a^2\lambda\pi^e + a\lambda k}{1+a^2\lambda} - \frac{a\lambda\mathbb{E}\varepsilon}{1+a^2\lambda}. \\
=& \frac{a^2\lambda\pi^e + a\lambda k}{1+a^2\lambda}.
\end{aligned}
$$

The last equality results from the fact that the shock has zero mean. Hence as in the classical case without inflation linked calls we obtain that $\pi^e = a\lambda k$. Substitution into the expressions for ξ and ψ in Proposition 1 then gives $\pi(\epsilon) = \xi(\epsilon) = \psi(\epsilon) = a\lambda k - \left(\frac{a\lambda}{1+a^2\lambda}\right)\epsilon$, which is the classical result.

We will see though in section 5, that $N = 0$ is in general far from optimal, and that there will be positive demand $N > 0$ for inflation linked calls on the financial market.

3 Demand for Inflation Linked Calls on Financial Markets

The financial market in our model features a safe asset paying a nominal interest rate r_i as well as inflation linked calls as defined in the previous section. Demand for inflation linked calls arises from our assumption that financial agents are risk averse towards inflation. This aversion is expressed in an inflation risk premium, which manifests itself in the use of an appropriate risk neutral measure. Financial agents buy inflation linked calls in order to reduce/eliminate their exposure to inflationary risk. As indicated earlier, we assume that $\varepsilon \sim \mathcal{N}(0, \sigma)$ under the central bank's measure $\mathbb{P}$. For the market to be arbitrage free, the price $p = p(N)$ of one inflation linked call has to satisfy

$$p(N) = e^{-r_i} \tilde{\mathbb{E}} \left(e^{\pi(N)} - e^{K} \right)^{+}, \tag{6}$$

where $\tilde{\mathbb{E}}$ denotes the expectation under a risk-neutral pricing measure $\tilde{\mathbb{P}}$. In difference to say a call option on a stock, the price level as such is not tradeable, as the consumption good is assumed to be perishable. This has the consequence that a risk neutral measure is not unique. We assume that financial agents are averse toward inflation risk and attach a specific risk premium to the economy shock in the following way. We write $\epsilon = \sigma \cdot w$ with $w \sim \mathcal{N}(0,1)$ under $\mathbb{P}$. The measure transformation leading to the risk neutral measure is then as follows: $\tilde{w} = w + \chi$ with $\tilde{w} \sim \mathcal{N}(0,1)$ under $\tilde{\mathbb{P}}$ and

$$\epsilon = \sigma \left(\tilde{w} - \chi \right), \tag{7}$$

where χ represents the market price of economy risk. We then have that $\varepsilon \sim \mathcal{N}\left(-\sigma \cdot \chi, \sigma^2\right)$ under $\tilde{\mathbb{P}}$. Identifying w with the realization of a Brownian motion at time $t = 1$, this transformation can be identified as Girsanov transformation, which is used in classical Black-Scholes theory to determine the risk-neutral measure.

4 General Equilibrium with Inflation Linked Calls

Our framework involves three players, the central bank, private agents and financial agents. These have been discussed separately within the previous sections. Obviously their decisions are linked with each other. Private agents can infer on the actual number N of inflation linked calls sold by the central bank as well as financial agent's objectives to build their expectations π^e, while the central bank depends on π^e and N in order to determine $\pi(N)$ and p and financial agents depend on π^e in order to determine their demand for inflation linked calls $p(N)$.

Proposition 2. *A general equilibrium* $(p^*, N^*, \pi^*, \pi^{*e})$ *of our model consisting of price* p^* *for one inflation linked call, a quantity* N^* *of inflation linked calls issued, actual inflation* π^* *and expected inflation* π^{*e} *is determined by the following conditions*

$$\begin{aligned}
p^* &= p(N^*) \text{ where the right hand side is given by (6)} \\
\pi^* &= \pi^*(N^*) \text{ with } \pi^*(N) \text{ given by Proposition 1} \\
\pi^{*e} &= \pi^e(N^*) \text{ with } \pi^e(N) \text{ satisfying equation (5)} \\
N^* &= \arg\min_N \left\{ \frac{1}{2}\lambda\mathbb{E}(a(\pi(N) - \pi^e(N)) + \varepsilon - k)^2 + \frac{1}{2}\mathbb{E}\pi(N)^2 + dN(M(N) - p^*) \right\} \\
&\quad \text{with } \pi(N) \text{ given by Proposition 1 and } \pi^e(N) \text{ satisfying equation (5).}
\end{aligned}$$

Proof. This follows directly from the results obtained in sections 2 and 3. □

In Proposition 2 $\arg\min_N$ denotes the minimizer of the expression in the pointy brackets. In order to compute the equilibrium in Proposition 2, we need to solve equation (5) numerically for a discrete grid of values of N and work from there through the other conditions.

It can be concluded from (5) as well as the definition of $\xi(N), \psi(N), \eta(N)$ and $\theta(N)$ and the properties of the Lambert $\mathbb{W}$-function that the solution $\pi^e(N)$ of (5) is decreasing in N. In particular, if inflation linked calls are issued π^{*e} will be lower than $a\lambda k$, the expected inflation without a market for inflation linked calls. If the strike price K of the inflation linked calls is not chosen to high, $p(N)$ in equation (6) will be strictly positive and hence demand for inflation linked calls from financial agents will exist. It then becomes evident from (4) that the central bank, by issuing a positive number of inflation linked calls can improve its objective function V as compared to the case without inflation linked calls. Hence our analysis shows that all three players, central bank, private agents and financial agents improve their position as compared to the case when $N = 0$. Therefore the existence of a market for inflation linked calls leads to a Pareto improvement as compared to the classical case, and inflationary bias is significantly reduced. Table 1 summarizes these conclusions.

	$N > 0$
private agents	inflation decreasing
central bank	value V increasing
financial agents	reducing inflationary risk

Table 1: Benefits from inflation linked calls.

5 Numerical Results

In this section we provide a short numerical experiment. We choose the following set of parameters as introduced in sections 2 and 3: $K = 0.02$, $\sigma = 0.09$, $r_i = 0.05$, $\chi = \frac{1}{3}$, $a = 3$, $k = 0.15$, $\lambda = \frac{1}{9}$ and $d = 10^{-11}$. The expected inflation rate in the classical model without inflation linked calls is then given by $\pi^e = a\lambda k = 0.05$.

As seen in Figure 1, the expected inflation rate with inflation linked calls is a decreasing function in N and turns out to be less than the strike $K = 0.02$ for $N \approx 6 \times 10^{11}$.

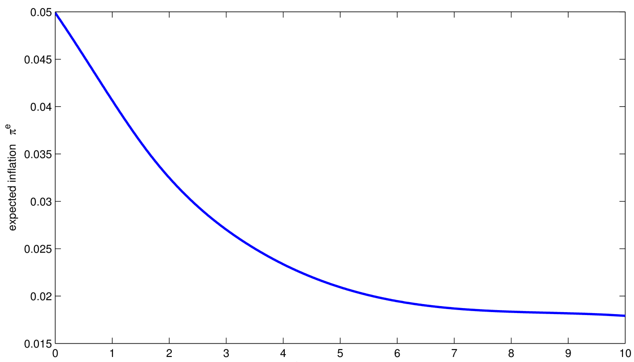

Figure 1: The expected inflation rate is decreasing in the number N of inflation linked calls that the central bank sells.

Knowing $\pi^e(N)$ it is possible to compute the corresponding expected payoff for the inflation linked calls. The inverse demand function $p(N)$ for inflation linked calls is given by (6). This equation determines the number N the central bank can issue on the market, when choosing a particular price. A numerical computation, as indicated in section 3, then shows that the number of inflation linked calls issued in equilibrium by the central bank is $N \approx 5.25 \times 10^{11}$ for an equilibrium price of $p \approx 0.001925$ per inflation linked call. Private agents' rationale expectation for the inflation rate in this example is then reduced from 5% without inflation linked calls to $\pi^e \approx 0.02067 = 2.067\%$ with inflation linked calls, which can also be observed from Figure 1.

6 Conclusions

We have studied the effects of markets for inflation linked calls on the central bank's monetary policy. We presented a model featuring a central bank, private and financial agents as well as a financial market, in which the central bank can adjust inflation and in addition can issue inflation linked calls, which it sells on the financial market. Within this models we have derived equilibrium prices and quantity for the inflation linked calls issued. Our model features rational expectations for the private agents. We have shown, that the introduction of inflation linked calls can reduce the central bank's inflationary bias, and that central bank, private agents as well as financial agents in our model are better off with inflation linked calls than without. In this way inflation linked calls should be seen as effective and powerful monetary policy instruments, which can implement the sort of inflation contracts discussed in the classical literature, without the requirement of a classical principal agent relationship between government and central bank.

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