Mutual calculations in creating accounting models: a demonstration of the power of matrix mathematics in accounting education.

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Keywords: Accounting Education, Matrix Accounting, Accounting Theory Course, Accounting Teaching Technique.

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History

Received: September, 2014
Revised: May, 2015
Revised: September, 2015
Revised: November 2015
Accepted: May, 2016

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Abstract

The aim of this paper is to describe the innovative teaching approach used in the Southern Federal University (SFedU), Russia, to teach accounting via a form of matrix mathematics. It thereby contributes to disseminating the technique of teaching to solve accounting cases using mutual calculations to a worldwide audience. The approach taken in this course provides useful insights for both accounting educators and accounting students by providing an alternative explanation and understanding of the main principles of accounting, and does so using a simple game simulation.

This paper provides readers with an awareness of the broader international environment of accounting education and of the techniques that can be adopted in the teaching of accounting. The paper also demonstrates that any accounting operation can be represented by matrix equations, and, as a result, adds to the understanding of alternative processes of recording transactions and the mathematical generation of essential accounting reports. The paper thereby reveals the potential value of matrix based approaches to accounting within the accounting educational environment.

Keywords: Accounting Education, Matrix Accounting, Accounting Theory, Problem-Solving

Introduction

This paper describes a specific teaching practice that has delivered positive results via the utilisation of a problem-oriented case study that introduces a matrix mathematics approach in the teaching of accounting. The course “Accounting theory” (or “Accounting and Analysis”, as it is now called) was formed by a process of trial and error in response to social, political and economic changes in Russia. In this context it is important that accounting is seen to be a discipline in which theory has been developed to be coherent with practice and that accounting in the University sector is seen as a legitimate ‘science’. In addition modern practice requires that accounting be suitable for operation using information technologies (IT). In this context the significance of the adoption of a mathematical approach in this field has increased, as this provides a logical and sequential solution of the problem which is consistent with the scientific approach of other disciplines. The concept of using matrices in accounting is not new, as it was developed for macro accounting by Leontief (1930 and 1940), by Mattessich in the 1950s (see Mattessich, 2008) and further development in the 1980s by Mepham, 1988 and Leech, 1986. However, developed as an alternative language in accounting and due to its application in IT based systems, situational-matrix modelling has become an ‘in-demand’ approach in accounting (Danilenko *et al.*, 2015; Kolvakh, 1996, 1999, 2000; Mepham , 1988; Sbitneva, 2013; Vysotskaya, 2011).

This study is dedicated to the detailed description of the teaching practice used in-class in the compulsory second year (first semester) course “Accounting and Analysis” and a discussion of some of the implications of a comparison of this method with more conventional teaching approaches. The course was developed and introduced by the first two authors in 2000 at SFedU, and has been used successfully since. This course is delivered to accounting and audit majors and the section of prime interest here is dedicated to “Accounting and situational modelling”.

Contribution

The first contribution of this paper is to increase the awareness of accounting educators to the potential for adopting a matrix algebra approach by illustrating how matrix transaction records can be adopted in the process of teaching accounting, with a fairly minimal level of matrix algebra knowledge. The paper thereby provides accounting academics with a more detailed understanding of the broader international field of accounting education. Additionally the paper contributes to the field by arguing that the described approach is coherent and has potential benefits in accounting theory and other accounting courses (e.g. financial accounting, IFRS, and the facilitation of the transformation of reports to and from IFRS formats, etc.). The paper does this by illustrating; how the approach makes problem-solving integral to student learning; how the foundations of mathematics can help in the teaching process; and, how to introduce students to the main notions and concepts used in accounting practice via a game based example.

This article is organized as follows. The next section provides an overall view of the course and its context, this is followed by sections on the approach taken, and the detailed case problem used. The paper concludes, after a brief note on the success of the approach, by discussing the solution suggested and the implications of the approach adopted.

The context

The course “Accounting and Analysis” is substantially based on the Russian national accounting system, which includes learning the details of the standard chart of accounts that applies to all Russian organizations, the nature of the allowable types of accounting entries and the standard report formats used within Russia (Stoner and Vysotskaya, 2012). The course is the first substantive course on accounting that students encounter and is comprised of 6 Sections over 72 class hours during the first semester of the second year (Stoner and Vysotskaya, 2012). This paper is concerned with only part of the course, Section 5, which is dedicated to “Accounting and Situational Modelling”, the technique at the core of this paper. To put this section in context, the full schedule of Sections and Topics is detailed in Table 1. Broadly, the first two sections of the course aim to explain accounting principles and, in the context of accounting, the third introduces the concepts of modelling and the forth revises and builds on students’ prior knowledge of matrix algebra, a compulsory course in the previous academic year. The concluding section of the course further develops the situational modelling introduced in section 5. Throughout the course the class meets twice a week; Section 5 is covered in 3 Lectures and 3 tutorials (12 contact hours).

Table 1: Schedule of Sections and Topics for the Accounting Theory Course	
№ Section/ Topic	**Name of the section/topic**
Section 1	**Accounting and its place in the economic system**
T 1	*Aims, content and structure of the course*
T 2	*Classification of assets*
T 3	*Financial processes and transactions*
Section 2	**The subject and aims of accounting**
T 1	*Accounting: basic principles*
T 2	*Organization of accounting in the enterprise*
T 3	*Balance sheet and its meaning*
T 4	*The Double-entry system*
T 5	*Accounts and the chart of accounts*
Section 3	**Modelling as a means of theory development and improving methodology in accounting**
T 1	*Standards and principles in accounting*
T 2	*Accounting as an object for mathematic modelling: status and prospects*
T 3	*Accounting and symmetry as a demonstration of financial transaction's double nature*
Section 4	**Concept of the accounting information equivalence and its transformation algorithms**
T 1	*Theoretical bases of equivalence concept and its transformation algorithms*
T 2	*Equivalence of accounting transactions syntactic forms and their logical-mathematical form*
T 3	*Classification and equivalence of accounting methods*
Section 5	**Accounting and situational modelling**
T 1	*Concept of the situational modelling as accounting language*
T 2	*Balance equations in accounting situational modelling systems*
T 3	*Methodological problems of situational modelling algorithmization*
T 4	*Some examples of situational modelling algorithmization in the field of accounting transactions*
Section 6	**Methodology and methods of constructing the system matrix and situation-matrix models in accounting**
T 1	*Matrix model of chess-balance formation*
T 2	*Matrix model of accounting: from transactions to balance-sheet.*
T 3	*Pacioli's and Pisani's postulates: their proof in the proposed situation-matrix modelling system*
T 4	*Matrix models of dynamic ACL-equation (Active-Capital-Liabilities)*
This table is adapted from Stoner and Vysotskaya, 2012, Appendix C.	

The accounting and situational modelling section (5) is concerned with learning how mutual calculations are used to create accounting models, partly via the utilisation of a set of problems to be solved by using this accounting tool alongside a standard method. A variety of presentations and resources are used for the class, including a mix of PowerPoint presentations, lecture material, handouts and spreadsheet (MS Excel) tables[1], which are available to students in advance.

The teaching practice that is employed in this part of the course, including its structure and examples, are described and explained in the following sections, starting with a statement of the principles of the class case, a statement of the case example used and a description and explanation of the class content and process of student learning.

[1] These spreadsheets are available from the first author on request.

The principles of the class case

The starting point of this section of the course is that in order to properly understand accounting it is necessary, as for example Smirnova, Sokolov and Emmanuel (1995) and Sangster (2010) indicate, to teach students how to use, operate and understand basic forms of recording (registration) of business transactions. It is also clear that despite the near universal adoption of the concepts of the basic double entry bookkeeping (DEB) model (Debits and Credits) that at a detailed level accounting technologies provide multiple ways of recording transactions and account balances. Further, given the predominance of computerisation of bookkeeping systems, it is likely to be useful for students to understand, or at least be aware of, mathematics based algorithmic methods that are well suited to the digital environment. In this context the classes help students to learn how to formulate and operate a set of mathematical processes and algorithms as an interrelated system for the recording and processing of accounting records, and the mathematical creation of accounting reports.

The case statement: example used.

The case problem used for the examples in this section is based on the case statements:

1. Assume that there are three main players who play a game: A, B, and C. The result of each play is a money reward (or loss). The game is played in pairs and can be played between any pairs of the players: A and B, A and C, B and C.
2. Also assume that as a result of the game, the players owe each other the sums of money they gain or lose, and that there is somebody (e.g. the accountant) who records (registers) the emerging assets and liabilities over a set period of time (e.g., a month), and does so in chronological order.

The case process

Illustration proceeds by providing students the opportunity to learn to record the example case using two different methods, traditional "T accounts" (which are not detailed in this paper on the grounds that this process is well understood) and situational (matrix) modelling, and to appreciate the differences between them in terms of, for example, effectiveness and information value. By doing so, we clearly demonstrate the benefits of chronological registers, in accounting. The demonstration deals with this process in the following sequence:

a) The liabilities that will occur can be registered, for instance, in the form:

1. September, 4 - A won the sum of money of 10 c.u. (currency units) from B.
2. On the same day - C won 4 c.u. from B.
3. On September, 7 - B won 3 c.u. from A, etc.

b) The events can be registered chronologically in a transaction log, waste book or list of journal entries (typically the Journal of Business

Operation in Russian). The full set of (example) events are shown in Table 2.

Table 2: Journal of Operations, September

№	Date	Liabilities		Sum, c.u.
		To Receive	To Pay	
1	4.09	A	B	10
2	4.09	C	A	4
3	7.09	B	A	3
4	15.09	A	C	7
5	17.09	B	C	8
6	24.09	C	B	6
7	28.09	A	B	9
8	30.09	C	A	3
Total:				**50**

Rows 1 to 3 represent the examples detailed in a) above, the rest of the table represent the additional transactions (games) in the class example.

The basic notions to be learned in this part of the course are given as follows [authors' translation from the teaching materials]:

1. The transaction log refers to the group of chronological accounting registers, as it contains information on the entries in chronological order.
2. The date of the event can't be used to uniquely identify a transaction as on any day several events can take place. Events are therefore identified by serial or record number.
3. Players are indicated (identified) by notional names (in effect personal account names) in the journal of operations, (e.g. A – to receive, B – to pay). In accounting terms the relationship shown in a row (e.g. A to receive from B) is termed the 'correspondence of accounts'.
4. The record of the operation is called an accounting entry, or an accounting transaction.

Educational goals here are to encourage students to consider the following issues:

1. What is the link between the examples and accounting practice?
2. How might the examples be represented in accounting records using T-accounts?
3. How and to what extent can we use matrix algebra devices to represent and record these transactions?

Students complete the exercise in writing in class time followed by approximately half of the lesson used to discuss the results.

The solution

A reasonable start to the solution of the case follows these steps:

1. Calculate the total sums of liabilities ("rewards – losses").
2. Record the entries using the following notation and equation:

$$S(X,Y) = S_{x,y} \quad ,$$

Where the liabilities [S(X,Y)] mean: X – to receive, Y – to pay, with the amount of the equation on the left [$S_{x,y}$]in currency units (c.u.).

In our example the sum of all liabilities "A – to receive, B – to pay" (transactions 1 and 7) will be equal to:

$$S\ (A, B) = S_1\ (A, B) + S_7\ (A, B) = 10 + 9 = 19 \text{ c.u.}$$

Accordingly (transactions 2 and 8),

$$S\ (C, A) = S_2\ (C, A) + S_8\ (C, A) = 4 + 3 = 7 \text{ c.u.}$$

The other transacting pairs have only one transaction, thus, for example;

$$S\ (B, A) = S_3\ (B, A) = 3 \text{ c.u.}$$

and so on.

3. Record the resulting sums into the Ledger (Table 3).
In our example it has the form of a systematic register of all of the gross liabilities.

Table 3: The Ledger of players A, B, C with the entries (September).

№	Liabilities		Sum, in c.u.
	To receive	To pay	
1	A	B	19
2	A	C	7
3	B	A	3
4	B	C	8
5	C	A	7
6	C	B	6
Total			50

Such a Ledger refers to the group of systematic accounting registers as it systematizes the data that transpired during the whole period in a convenient order: an order which is suitable from the point of view of solving the example question about the resulting balances, regardless of the dates of events within the period.

Although the transaction log may include any number of entries, because they can be repeated in relation to the same pairs of players, the number of entries in the Ledger (in this form) is limited to the number of possible pairs of players. In our example the number is equal to six pairs: AB, AC, BA, BC, CA, CB. In general, the number of players is determined according to the formula $m(m$-1), where m - is the number of settlement players.

The data in this type of Ledger is sufficient to complete the balance sheet balances as well as to solve the problem of determining balances between participants A, B, and C.

The final operation of settlement (payment) can be processed in two ways:

1. Using the gross settlement method which is broadly used for calculations between bank clients.
This is possible by fulfilling the following operation for the calculations, for example, between A and B:
A receives 19 c.u. from B, B receives 3 c.u. from A.

The sum of funds necessary for calculations between all of the players is equal to the total sum of the Ledger (that is 50 c.u.).

2. By means of netting or clearing debts. In this case it is necessary to calculate the difference using the next formula:

$$\Delta S (X, Y) = S (X, Y) - S (Y, X),$$

where X, Y are taken as A, B and C, accordingly.

In our example:

$$\Delta S (A, B) = S (A, B) - S (B, A) = 19 – 3 = +16 \text{ c.u.},$$

$$\Delta S (A, B) = S (A, B) - S (B, A) = 19 – 3 = +16 \text{ c.u.} > 0,$$

$$\Delta S (B, A) = S (B, A) - S (A, B) = 3 - 19 = -16 \text{ c.u.} < 0$$

Where the symbol «+» means «To receive», and the symbol «-», correspondingly, means «To pay».

Such calculations can be systematized by re-writing the data of the Ledger in the form of a 'chess balance'. Where 'chess balance' refers to the method of recording counterparty transactions as a balanced matrix (with row sums shown equal to column sums).

In order to represent these we follow the next processes:

1. Construct a Matrix of Debit Turnovers (MDT) to represent the liabilities to receive funds, using the values of S (X,Y) as cell entries. In the example it can be represented as follows[2] (noting the zeros in the diagonal):

MDT =

To receive	To pay			Total to receive
	A	B	C	
A	*0*	*19*	*7*	*26*
B	3	0	8	11
C	7	6	0	13
Total to pay	10	25	15	50

By transposing this matrix, by replacing its columns and lines, we will create a matrix of liabilities to pay, which is called the Matrix of Credit Turnovers (**MCT**).

MCT =

To receive	To pay			Total to receive
	A	B	C	
A	*0*	3	7	10
B	*19*	0	6	25
C	*7*	8	0	15
Total to pay	*26*	11	13	50

[2] As a rule, accounting uses tables (matrices) with totaling sections (rows and columns). Matrices in this form are referred to as framed matrices.

2. The next step is to subtract the matrix of liabilities to pay funds (MCT) from the matrix of liabilities to receive funds (MDT) and, as a result, we will obtain a matrix of balance (MB):

$$\mathbf{MDT} - \mathbf{MCT} = \mathbf{MB} \qquad \text{(F-1).}$$

The difference obtained as a result of this subtraction is the balance matrix which is represented below.

MB =

To receive	To pay			Total to receive
	A	B	C	
A	0	+16	0	+16
B	-16	0	+2	-14
C	0	-2	0	-2
Total to pay	-16	+14	+2	0

3. By using the given formula (F-1), we can represent mathematically how the balance matrix is obtained, as follows:

$$\underbrace{\left(\begin{array}{c|ccc|c} to & to & pay & & \\ receive & A & B & C & \Sigma \\ \hline A & 0 & 19 & 7 & 26 \\ B & 3 & 0 & 8 & 11 \\ C & 7 & 6 & 0 & 13 \\ \hline \Sigma & 10 & 25 & 15 & 50 \end{array}\right)}_{\mathbf{MDT}} - \underbrace{\left(\begin{array}{c|ccc|c} to & to & receive & & \\ pay & A & B & C & \Sigma \\ \hline A & 0 & 3 & 7 & 10 \\ B & 19 & 0 & 6 & 25 \\ C & 7 & 8 & 0 & 15 \\ \hline \Sigma & 26 & 11 & 13 & 50 \end{array}\right)}_{\mathbf{MCT}} = \underbrace{\left(\begin{array}{c|ccc|c} to & to & pay & & \\ receive & A & B & C & \Sigma \\ \hline A & 0 & +16 & 0 & +16 \\ B & -16 & 0 & +2 & -14 \\ C & 0 & -2 & 0 & -2 \\ \hline \Sigma & -16 & +14 & +2 & 50 \end{array}\right)}_{\mathbf{MB}}$$

In this formula a matrix of credit turnovers (MCT) is obtained from a matrix of debit turnovers (MDT) via transposition: MCT = (MDT)′ (where the subscript “′” indicates the inverse, transposed, matrix):

$$\left(\begin{array}{c|ccc|c} to & to & receive & & \\ pay & A & B & C & \Sigma \\ \hline A & 0 & 3 & 7 & 10 \\ B & 19 & 0 & 6 & 25 \\ C & 7 & 8 & 0 & 15 \\ \hline \Sigma & 26 & 11 & 13 & 50 \end{array}\right) = \left\langle\left(\begin{array}{c|ccc|c} to & to & pay & & \\ receive & A & B & C & \Sigma \\ \hline A & 0 & 19 & 7 & 26 \\ B & 3 & 0 & 8 & 11 \\ C & 7 & 6 & 0 & 13 \\ \hline \Sigma & 10 & 25 & 15 & 50 \end{array}\right)\right\rangle'$$

A balance matrix (MB) is an algebraic matrix which represents the balances of accounting correspondences with the help of symbols: the sums for receiving are shown with the symbol plus (+), the sums for paying with the symbol minus (–).

Such matrix possesses two qualities:

a) The elements $\Delta S_{X,Y} = S_{X,Y} - S_{Y,X}$ are symmetrical reflections around the main diagonal, i.e. $\Delta S_{X,Y} = -\Delta S_{Y,X}$ and $\Delta S_{Y,X} = -\Delta S_{X,Y}$, which follows from the direct comparison of the formulas from which the balances are calculated.

b) The sum of the elements of a balance matrix is always equal to zero:

$$\sum_{X,Y} \Delta S_{X,Y} = 0 .$$

In fact, the first quality of the mentioned matrix leads to the situation when the sum of every pair of symmetric elements is equal to zero: $\Delta S_{X,Y} + \Delta S_{Y,X} = 0$. Therefore, the sum of all non-diagonal elements of the matrix of balance is equal to zero. The sum of diagonal elements is also equal to zero as every diagonal element is equal to zero: $\Delta S_{X,Y} = -\Delta S_{Y,X}$, as a result, $\Delta S_{X,Y} + \Delta S_{Y,X} = 0$. Hence, it follows that the sum of all elements of a matrix of balance is equal to zero.

4. By understanding the formula (F-1) in step 2, and its use in recording the operations in this example, in step 3, we have made an important step in the direction of understanding a new, qualitatively different non-operational notion of accounting reporting technologies.
   Formula (F-1) is by definition true for any pair of debit and credit matrices of turnover, all of which will be square matrices and can be of any size. This means that it is valid for any finite number of settlement participants, for instance, 1000, 100,000 and so on. Accordingly, the conclusions about the general properties of these matrices and the balance matrix are also valid for any finite number of participants and monies involved in the calculations.

It should further be noted that in general the equation (F-1) includes a matrix of balance, which can be rewritten in the following way:

$$\mathbf{MB_0 + MDT - MCT = MB_1} \qquad \text{(F-2)}.$$

Here MB_0 – is a matrix of opening balances, MDT – is a matrix of debit turnovers for a certain period, MCT = MDT′ - is a matrix of credit turnovers for the same period which is transposed to MDT, and MB_1 is a matrix of balance for the end of the period derived from the equation.

If, as before, we suppose that the players did not have any liabilities at the beginning of the period, the matrix of opening balances will be equal to zero (therefore all the matrix elements are zerox). The general equation, using the data of our example will there look as follows:

MB_0

to receive	to pay A	B	C	Σ
A	0	0	0	0
B	0	0	0	0
C	0	0	0	0
Σ	0	0	0	0

+

MDT

to receive	to pay A	B	C	Σ
A	0	19	7	26
B	3	0	8	11
C	7	6	0	13
Σ	10	25	15	50

−

MCT

to receive	to pay A	B	C	Σ
A	0	3	7	10
B	19	0	6	25
C	7	8	0	15
Σ	26	11	13	50

=

MB_1

to receive	to pay A	B	C	Σ
A	0	+16	0	+16
B	−16	0	+2	−14
C	0	−2	0	−2
Σ	−16	+14	+2	50

Now we can turn from the matrix equation to the total, or vector, equation model of mutual calculations. In this example the vectors represent the total columns of corresponding matrices, which for the example is shown as:

$$\begin{array}{c} \underline{\textit{Accounts}} \\ A: \\ B: \\ C: \\ \hline \Sigma \end{array} \begin{pmatrix} \underline{\mathbf{VB_0}} \\ 0 \\ 0 \\ \underline{0} \\ 0 \end{pmatrix} + \begin{pmatrix} \underline{\mathbf{VDT}} \\ 26 \\ 11 \\ \underline{13} \\ 50 \end{pmatrix} - \begin{pmatrix} \underline{\mathbf{VCT}} \\ 10 \\ 25 \\ \underline{15} \\ 50 \end{pmatrix} = \begin{pmatrix} \underline{\mathbf{VB_1}} \\ +16 \\ -14 \\ \underline{-2} \\ 0 \end{pmatrix} \qquad \text{(F-3).}$$

In the balance equations presented, the *identifications* of settlement players A, B, C are refered to as the 'accounts', which correspond to the notion of *personified accounts* in accounting records (which have been used since the early evolutions of double entry bookkeeping and accounting development, even back to the times of Luca Pacioli.

Here VB_0 denotes a vector of opening balances, VDT is a vector of debit turnover, VCT is a vector of credit turnovers, and VB_1 is a vector of period end (closing) balances which can be derived from the equation. Here the first vector (VB_0) is a zero vector, a vector of zero opening balances, since there are no opening balances, as is indicated by the absence of an opening matrix of opening balances in the initial equation (F-1), or it is a zero matrix in the equation (F-2). Thus, its total column – a vector of opening balances (VB_0) will also be a zero vector.

In this way, every vector equation can be written in its general form which is valid for any number of calculations and players.

$$\mathbf{VB}_0 + \mathbf{VDT} - \mathbf{VCT} = \mathbf{VB_1} \qquad (\mathbf{F}\text{-}3').$$

With the help of the equation (F-3′) we can establish the connection between incoming (opening) and outgoing (closing) balances through account turnovers. In this example turnovers are understood as the totals of corresponding entries of liabilities to receive and to pay. In this example turnover debits represents the total of monies receivable, and credit turnover the total monies payable.

The equation in this form (F-3′) can be referred to as (called) an 'algebraic equation of the turnover balance sheet', since the balance of payment and the balance of receiving are presented in it with the help of symbols, namely, plus and minus. The total of incoming and outgoing balances is always equal to zero, which follows from the property of reflection symmetry of the beginning and final matrices of balances.

In many ways, the term debit in accounting refers to the left side of the 'T-account' table – to receive and credit refers to the right side of the table – to pay. It all corresponds to an original meaning of these terms. Debit meaning "someone owes us money" – debt liability; credit, correspondingly, meaning "we owe money to someone" – credit liability.

The algebraic equation in the form (F-3) can be substituted with the table of Algebraic Turnover Balances (Table 4), which can be a basis for transition to another form of data presentation – a table of Accounting Turnover Balances (Table 5).

Table 4 – Algebraic Turnover Balance:

$$VB_0 + VDT - VCT = VB_1$$

Accounts	Balance (+,-)	Turnovers		Balance (+,-)
		Debit	Credit	
A	0	26	10	+16
B	0	11	25	-14
C	0	13	15	-2
Total:	0	50	50	0

⇧ ⇩

Table 5 – Accounting Turnover Balances:

$$(VDB\text{-}VCB)_0 + VDT - VCT = (VDB\text{-}VCB)_1$$

Accounts	Balance		Turnovers		Balance	
	Debit	Credit	Debit	Credit	Debit	Credit
A	0	0	26	10	16	0
B	0	0	11	25	0	14
C	0	0	13	15	0	2
Total:	0	0	50	50	16	16

Here, the column of ending balances (Table 4) shows what needs to be done in order to settle the accounts between the players A, B and C. In this example players need to fund payments of 14 c.u. and 2 c.u. respectively, and, in turn, participant A must receive his money reward of 16 c.u. The explicit netting operations in the example require just 16 c.u. compared to the 50 c.u. needed (the total accounts turnovers) for gross settlement operations.

The transition shown between Tables 4 and 5 illustrates the positional notation of the Accounting Turnover Balances, compared to the balances in the Algebraic form: surpluses recorded in the left position - in the debit, negative balances in the right position - in credit. Theses tables of turnover balances contain balance equations in each corresponding line, which establish the connections of opening and closing balances with incoming and outgoing turnovers for each account. The total line of the table of Accounting Turnover Balance (Table 5) contains balance equations identities known as “Pacioli's and Pisani's postulates”:

Postulate 1. The totals of turnovers on debit and credit are always equal.
(In our example it is: 50 = 50).

Postulate 2. The totals of balances on debit and credit are always equal.
(In our example as follows: 0=0 and 16=16).

The interpretation of the data of the algebraic balance in Table 4, including its line of totals, is relatively straightforward, being the tabular presentation of the vector equation model (F-3). Hence, in Table 4 the connection of incoming and outgoing balances is established with the help of equations of the following type:

$$\mathbf{B_0 + DT - CT = B_1} \qquad \text{(F-4).}$$

Where B_0 – denotes an algebraic value (+,-) of opening balances;
DT – is a debit turnover,
CT – is credit turnover; and
B_1 – is an algebraic value (+,-) of the balances for the end of period derived from the equation.

But how should we interpret balance equations which are presented in Table 5: the accounting balance? In particular, how should we interpret the data of its totals line which contain the balance identities considered above?

The problem can be easily solved by the considering the record of balance equations in a position (differences) or accounting form:

$$\mathbf{(DB - CB)_0 + DT - CT = (DB - CB)_1} \qquad \text{(F-5),}$$

Where $B_0 = (DB - CB)_0$ that denotes the difference between opening debit and credit balances;
DT – debit turnover,

CT – credit turnover,
$B_1 = (DB - CB)_1$ is a difference between opening debit and credit balances derived from the equation.

In Exhibit 1 we show the illustration of the connections derived from balance equations in algebraic and accounting forms using the data of the considered example:

Exhibit 1: Algebraic and Accounting Equivalence

Algebraic equations:	⇨ ⇦	Accounting record:
A: 0 + 26 – 10 = +16		A: (0 -0)+ 26 – 10 = (16-0)
B: 0 + 11 – 25 = -14		B: (0 -0) + 11 – 25 = (0-14)
C: 0 + 13 – 15 = -2		C: (0 -0) + 13 – 15 = (0 -2)
∑: 0 + 50 – 50 = 0		∑: (0 -0) + 50 – 50 = (16-16)

The record of balance equations in accounting form is equivalent to their record in algebraic form. This means that it is always possible to pass from the accounting form of record to the algebraic form, and vice versa. In other words, it is always possible to pass from the record in columns to the position based accounting record, and, from the position based accounting record of balances to their record in columns.

I conventional bookkeeping terms the data in the balances are probably more easily understood, at least by accountants, if they are recorded as debits and credits positions on T-accounts, rather than in the columnar form of Tables 4 or 5. To achieve the equivalent records in traditional double entry T-accounts each transaction is recorded twice, in the corresponding accounts (the double entry). In our example there are 3 such T-accounts: A, B and C. The entries in the 'journal' (assuming no summarising day books) for the first transaction, $S_1(A,B) = 10$, would be journalised as in Exhibit 2:

Exhibit 2: Sample Journal (S1)

			Dr	**Cr**
J1		X September		
A		Game 1 - B	10	
	B	Game 1 - A		10
			10	10

In these terms the mathematics based notation can be interpreted as:

S_1	(A	, B)	= 10
Transaction 1	Dr account A	, Cr account B	with amount 10

The transfer of this entry to the 'ledger' (T-account) being achieved by taking the 10 c.u. Dr to the left side of account A and the 10 c.u. Cr to the right side of the corresponding account, account B, as in Exhibit 3:

Exhibit 3: Sample T-Accounts (S1)

A		B		C	
Dr	CR	Dr	CR	Dr	CR
1) 10			1) 10		

In the same way, the data from the other example entries in the journal of operations can be transferred into the three relevant T-accounts, as shown in Exhibit 4.

Exhibit 4: Sample T-Accounts – Turnover form (all transactions)

A		B		C	
1) 10	2) 4	3) 3	1) 10	2) 4	4) 7
4) 7	3) 3	5) 8	6) 6	6) 6	5) 8
7) 9	8) 3		7) 9	8) 3	
DT= 26	CT=10	DT =11	CT =25	DT =13	CT =15

In this form the debit turnovers (DT), the amounts to be received by the account holder, are provided by the (pre balanced) Dr totals, and credit turnovers (CT), the amounts to pay by each account, by the Cr totals. The above T-accounts can be called turnover T-accounts, since they show only debit and credit turnovers of an account (no opening of closing balances).

In accounting texts and in practice the T-accounts more usually include opening balances and are periodically balanced rather than showing just turnover. In effect in a form similar to that shown in Exhibit 5, where the upper part of an account, above turnovers, shows the incoming / opening balances for the beginning of the period, and in the bottom part of an account the ending/closing balances are calculated, after casting the turnover figures, according to the balance equations.

Exhibit 5: Sample T-Accounts – Balance form (all transactions)

A		B		C	
DB_0 = 0	CB_0 = 0	DB_0 = 0	CB_0 = 0	DB_0 = 0	CB_0 = 0
1) 10	2) 4	3) 3	1) 10	2) 4	4) 7
4) 7	3) 3	5) 8	6) 6	6) 6	5) 8
7) 9	8) 3		7) 9	8) 3	
DT = 26	CT = 10	DT = 11	CT = 25	DT = 13	CT = 15
DB_1 = 16			CB_1 = 14		CB_1 = 2

Exhibit 6: Balanve equations

A: (0 -0) + **26 – 10** = (16-0), where **26** = 10+7+9, **10** = 4+ 3 + 3

B: (0 -0) + **11 – 25** = (0-14), where **11** = 3 + 8, **25** = 10 + 6 + 9

C: (0 -0) + **13 – 15** = (0 -2), where **13** = 4 + 6+ 3, **15** = 7 + 8

∑: (0 -0) + 50 – 50 = (16-16)

The balance equations in Exhibit 6 correspond to the entries in turnover balance T-accounts (Exhibit 4), using the total figures: These records of the balance equations shows not only the turnovers, but also their breakdown (the details in "where"), showing how the sums are calculated. In this way the equations presented are shown to be full equivalents of the turnover balance T-accounts (Exhibit 5). Thus, it is always possible to pass from T-account presentation of accounting information to corresponding balance equations presentation, and vice versa, provided the opening account balances are known, or the transactions the transactions that underlie those balances. Further, provided the current and past transaction data is available in matrix or vector form it is possible to calculate the balances following additional transactions mathematically, and to provide the data for common accounting reports.

The equivalence is important as it demonstrates (albeit here for a simple example) that the matrix modelling approach is capable of achieving all that is obtained via the more common DEB methodology, without loss of information. There are, however, significant differences. Principally, these arise from the nature and processes of classic DEB, which are also reflected in many business recording processes. These differences and their implications are discussed in the penultimate section of this paper.

Evaluation

The situational modelling approach described in this paper was introduced at SFedU, a major regional University within Russia, in 2000, and has been implemented, refined and developed ever since. The course has been, and is, successful in engaging student interest, however, given this long period of implementation formal experimental evaluation would be problematic (not least ethically), as would be quasi experiments, and comparisons with other institutions. The course has, however, been evaluated, mainly in terms of student satisfaction, and the results of that evaluation are reported in Stoner and Vysotskaya (2012). In summary the survey results in that study indicated that the course, in general, was positively rated (scores the positive side of the neutral score, 3, with more positive than negative ratings) and the course was not considered more difficult than other course in the curriculum.

Student destinations and employer opinion is a potentially expedient measure of degree programme success, however using such data to assess the usefulness of individual interventions in a course is highly problematic. That notwithstanding the good employment record of students is a positive indicator: almost all graduates achieve good graduate relevant employment or places in further education. Although official destination statistics are not collected Department staff are aware of the demand for and destinations of students, in large part due to the employer relationships resulting from the work placement schemes that are part of the degree (see Stoner and Vysotskaya, 2012), and this is supported by *ad hoc* local destination data[3]. Further, the bare destination data is supplemented by data from employers and student placement contacts that indicate that the skills developed via the problem-oriented matrix approach, that is adopted in several elements of the programme, is seen to help develop valued student skills. Further evidence of the perceived quality of the programme, in which this approach is a significant core element of multiple courses, is visible in the high demand for places on the course. The programme attracts a significant number of students beyond the state funded quota: these additional students are willing to pay fees because of the positive record of the degree and its graduates. Additionally the programme, of which this course is a core element, is well respected within the accounting academy within Russia.

Discussion: situational modelling compared with DEB and implications

Classical DEB solutions for the handling of transaction and accounting data requires the data be processed through a range of standard procedures, and or the creation of several files or documents. For example, real world accounting systems utilise a range of accounting and control practices, each of which frequently require different systems of accounting records and registers and potentially different algorithms of data transformation, both in the data processing and the production of accounting reports. For example, in order to create the data for the turnover balance report considered above it would be necessary to disaggregate the aggregated T-account balances, or to differently aggregate the core accounting data in parallel or additional processes.

In stark contrast, in principal the matrix modelling approach can solve the balances and produce the data for reports mathematically. The situational modelling approach is capable of doing this because it models the accounting process as a generalised mathematical model. As such the process can be completed, and manipulated, mathematically, which means, *inter alia*, that accounting effects and transactions can be conveniently modelled using analytic mathematics. One natural consequence of this is that modelling of future results, for example, as well as records of the past, can

[3] However, the last available, locally collected, data (for 2011 graduates) indicates that only 1 of the 44 graduates (2.2%) was not employed in a relevant graduate occupation, in further education (7) or in the Army(2) c.15 months after graduation.

be mathematically created driven by mathematically expressed parameters and assumptions.

The mathematical form of the situational modelling approach, additionally, potentially makes the model amenable to operation in mathematically controlled environments, including digital environments that are often optimised (at least at their core) for mathematical operations. This being one of the reasons that such methods are often utilised at the core of modern information systems, such as many Enterprise Resource Planning (ERP) systems (Stoner and Vysotskaya, 2012) that are an integral element of many accounting systems in practice. An understanding of the mathematical method therefore has potential to improve students' understanding of the operations, limitations and management of many business information system environments, and the information derived from them.

In the context of financial reporting, the facility to mathematically derive, or simulate, accounting results creates opportunities to, for example, automatically derive, via the use of different processing transformation matrices, primary financial statements (balance sheets, income statements and cash flows) under different accounting treatments. This is a useful function in practice, or in class, to produce comparative accounts under different versions of GAAP (e.g. IFRS versus local GAAP, or existing and proposed GAAP) from the underlying transaction and other data, or to provide a more automated process for the production of different versions of the accounts. Further, classroom use of this approach to the task can allow an analytic approach to the comparison to aid student understanding: an approach that is used on higher level classes on the accounting and audit programme to enhance students' use of accounting in decision making, problem solving and audit risk assessment, via analytic review.

At an advanced level an understanding of the matrix approach can also be used, in an adaptation of the approach, to help compare and analyse the differences between the accounts of an entity over time, for example between the opening and closing balance sheets of a company. In addition to possibilities in financial analysis this approach has application in audit, where it can be used as part of the analytic review of an entity's financial statements, and to help students analyse and understand how an entity's accounts change over time. This is another application that is developed in more advanced courses in the accounting and audit programme at SFedU.

In the context of planning and decision making the described matrix accounting and modelling approach has advantages beyond the choice between accounting methods, as the approach can be used in the modelling of forecast financial statements, as well as (in adapted form) additional and more detailed reports, such as those used in investment appraisal. This is an arena of financial work that in practice is dominated by the use of spreadsheet modelling (to the extent it has not become the domain of ERP, or similar type systems). Spreadsheets are a powerful tool in this role, but their use can be problematic in practice, not least as their convenience tends to be paired with a lack of control and the incidence of errors. The mathematical approach can help to control for at least some of the problems in this sphere, as well as helping users and students to analytically investigate the effects of decisions, rather than relying simply on simulations of effects, such as "what if" analysis.

Conclusion

By demonstrating the situational-matrix modelling approach that is introduced in the introductory ‘Accounting and Analysis’ course and employed elsewhere within the accounting and audit programme, we have contributed to our primary aim of increasing awareness of this alternative approach to recording accounting transactions and its use in accounting education. We have also illustrated how this can be done via the use of a game based example. Further, by discussing the comparison with DEB and the implications of this we have introduced accounting educators to the potential of adopting a matrix based approach, argued the validity of using this approach in classes on accounting theory, for example with respect to alternative accounting choices, and introduced how this approach can make problem-solving more integral to student learning.

In the process of doing these things we have also provided accounting academics with a more detailed understanding of the broader international field of accounting education, in particular showing how the foundations of mathematics, which is an integral part of accounting as science in some cultures, can help in the teaching process. The overall mathematical modelling approach to the course shows that the basic model of accounting, the accounting equation, introduced in the course proceeds to a model of transactions as mathematical transformations. It also provides the mathematical proofs that the processing maintains the duality of double entry bookkeeping and accounting. It is also important that the processes at the core of this course are also reflected in many computer based systems, such as ERP, that are important in accounting in practice.

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