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ELECTROMAGNETIC FORMATION FLYING WITH ECCENTRIC REFERENCE ORBITS

Leonel Palacios*, Matteo Ceriotti†, and Gianmarco Radice‡

Over the last decade, a considerable amount of research work has been done in the area of spacecraft formation flight, with particular emphasis on control techniques using thruster-based systems. Nevertheless, thrusters require propellant to work and this limit the lifetime of the mission. Electromagnetic Formation Flight (EMFF) is presented in this paper as a fuel-less strategy to control spacecraft formations by means of electromagnets. In EMFF, spacecraft can be equipped with one or more coils and reactions wheels which could be arranged in several combinations according to mission requirements. An electric current flows through the coils in order to produce a magnetic dipole in a specific direction. The magnetic field of a spacecraft reacts against the magnetic dipoles of the others, generating forces and torques which in turn could be used as control inputs. The main objective of this paper is to provide a formulation for EMFF when a formation is moving in eccentric reference orbits and for this purpose, the Tschauner and Hempel model will be used. Results are presented after analysing different formation scenarios providing the necessary magnetic requirements for station keeping and resolving which cases are suitable to be controlled by this technology. High-Temperature Semiconductor (HTS) plays an important role in EMFF and for that reason the paper also investigates the correlation of the magnetic force and the coil mass, which in turn affects the total mass of the spacecraft.

INTRODUCTION

In recent years, a significant amount of work has been done in spacecraft formation flight, especially in the area of control by means of conventional thruster-based systems. This concept has some drawbacks, including the need of propellant mass on-board, optical contamination, plume impingement, thermal emission and vibration excitation[1]. Electromagnetic Formation Flight (EMFF) is presented in this paper as a propellant-less strategy to control the relative movement and attitude of spacecraft with the aid of coils in order to create forces and torques needed to maintain a desired formation and orientation of a spacecraft. In EMFF, spacecraft can be equipped with one or more coils and reactions wheels which could be arranged in several combinations according to mission requirements. An electric current, which could be generated by solar panels attached to each element in the array, flows through the coils in order to produce a

* PhD. Student, School of Engineering, University of Glasgow, University Avenue, G12 8QQ, UK
† Lecturer, School of Engineering, University of Glasgow, University Avenue, G12 8QQ, UK
‡ Lecturer, School of Engineering, University of Glasgow, University Avenue, G12 8QQ, UK

magnetic dipole in a specific direction. The magnetic field of a spacecraft reacts against the magnetic dipoles of the others, generating forces and torques which in turn could be used for element deployment, formation keeping or reconfiguration and rendezvous operations. The material used to make the coils is High-Temperature Semiconductor (HTS), due to its interesting property of providing large electric currents with almost zero resistance[2]. Although HTS plays an important role in EMFF, one of the drawbacks of its use is that the magnetic force generated depends, among other factors, on the mass of the material: the greater the mass, the greater the force[Y]. This condition imposes an important cost in spacecraft weight.

Many aspects regarding magnetic theory and applicability of EMFF have been covered in previous research such as magnetic field models through dipole approximations[2] and effectiveness in arrays of two and three spacecraft[3] [4]. With regards to dynamics and control of electromagnetic formations, different topics have been investigated including control algorithms for Near-Earth Orbits[5], formation control with collision avoidance in deep space[6], full three-dimensional models for a two-vehicle array[1] and docking control algorithms for a 6 degree-of-freedom model[7]. However, these papers related to control only consider arrays which move in circular reference trajectories leaving aside eccentric reference orbits. Therefore, the main objective of this paper is to provide a formulation for EMFF when a formation is in station-keeping while moving in eccentric reference orbits and for this purpose, the Tschauner and Hempel model[8] will be used. Results are presented after analysing different formation scenarios providing the necessary magnetic requirements for station keeping and resolving which cases are suitable to be controlled by this technology.

To accomplish these tasks, equations of motion were derived and solved numerically for three different cases considering the effects of two-body gravitational forces and the magnetic control inputs. These formations were selected with more than two spacecraft and taking into account design parameters such as separation distance between elements, array configuration and eccentricity and period of the reference orbit. Additionally, the paper also presents the formulation of the correlation of the magnitude of the strength of the magnetic dipole and results of the necessary coil mass to produce it for every case considered. The rest of the paper is organized as follows: first, the derivation of the equations of relative dynamics and magnetic forces will be derived and explained together with a brief description of HTS characteristics. Next, the formation cases will be studied and the necessary magnetic and HTS mass requirements for controllability for each spacecraft. Finally, conclusions on results and future work will be stated.

EQUATIONS OF RELATIVE MOTIONS AND MAGNETIC FORCES

The Tschauner and Hempel Model for Relative Dynamics

In this section the formation flying relative dynamics model is derived. First, consider two spacecraft orbiting around Earth and assign them the names leader and deputy. We will derive next the equations of relative motion for the deputy with respect to the leader. Quantities related to the leader will be written without subscript while those designated to the deputy will have the subscript *D*. In order to develop the equations of relative motion between these two spacecraft, consider an *Earth-Centred inertial* frame (ECI), denoted by *I*, with an orthogonal basis $\{\hat{\boldsymbol{X}}\,\hat{\boldsymbol{Y}}\,\hat{\boldsymbol{Z}}\}$. The unit vectors $\hat{\boldsymbol{X}}$ and $\hat{\boldsymbol{Y}}$ lie in the equatorial plane with $\hat{\boldsymbol{X}}$ aligned with the line of equinoxes. $\hat{\boldsymbol{Z}}$ is pointing in the direction of the North Pole to complete the triad. The relative dynamics between spacecraft is better described in a *Local-Vertical-Local-Horizontal* (LVLH) reference frame (also known as the Euler-Hill Frame), denoted by *L*, with centre at the leader satellite or at a central point of the formation with orthogonal basis $\{\hat{\boldsymbol{x}}\,\hat{\boldsymbol{y}}\,\hat{\boldsymbol{z}}\}$ and angular velocity $\boldsymbol{\omega}$ normal to the orbital plane, with $\hat{\boldsymbol{x}}$ aligned to the radius vector $\vec{\boldsymbol{r}}$ of the leader from the ECI, $\hat{\boldsymbol{z}}$ is normal to $\hat{\boldsymbol{x}}$ and the orbit plane and $\hat{\boldsymbol{y}} = \hat{\boldsymbol{z}} \times \hat{\boldsymbol{x}}$ to complete the triad, as illustrated in Figure 1.

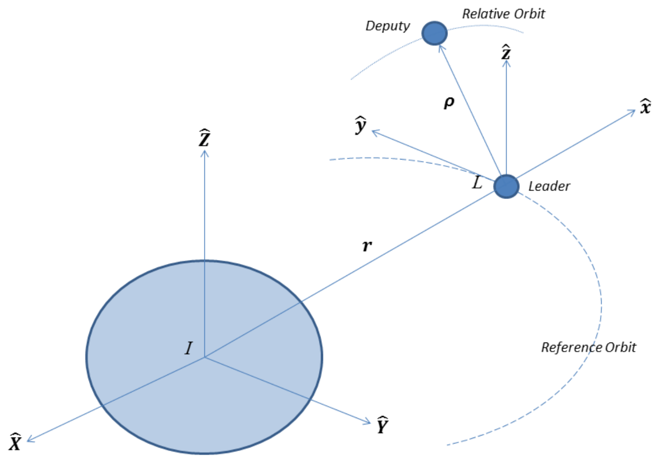

Figure 1. Earth-Centred (ECI) and Local-Vertical-Local-Horizontal (LVLH) Reference Frames.

The inertial equations of orbital motion of the leader and deputy are given by:

$$\ddot{\boldsymbol{r}} = -\frac{\mu}{r^3}\boldsymbol{r} \tag{1}$$

$$\ddot{\boldsymbol{r}}_{\boldsymbol{D}} = -\frac{\mu}{r_D^3}\boldsymbol{r}_{\boldsymbol{D}} \tag{2}$$

Then, the relative movement between the two spacecraft is the difference between Equation (2) and (1):

$$\boldsymbol{\rho} = \boldsymbol{r}_{\boldsymbol{D}} - \boldsymbol{r} \tag{3}$$

Expressing the relative acceleration in the L frame as[9]:

$$\ddot{\boldsymbol{\rho}} = [\ddot{\boldsymbol{\rho}}]_{\boldsymbol{L}} + \boldsymbol{2\omega} \times [\dot{\boldsymbol{\rho}}]_{\boldsymbol{L}} + \dot{\boldsymbol{\omega}} \times \boldsymbol{\rho} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{\rho}) \tag{4}$$

and taking into account that:

$$\boldsymbol{\omega} = \left[0, 0, \dot{\theta}\right]^{\boldsymbol{T}} \tag{5}$$

$$\boldsymbol{\rho} = [x, y\ , z]^{\boldsymbol{T}} \tag{6}$$

we arrive to the following system of nonlinear equations of relative movement for an arbitrary orbit of the leader in component form[10]:

$$\ddot{x} - 2\dot{\theta}\dot{y} - \ddot{\theta}y - \dot{\theta}^2 x = -\frac{\mu(r+x)}{[(r+x)^2 + y^2 + z^2]^{3/2}} + \frac{\mu}{r^2}$$
$$\ddot{y} - 2\dot{\theta}\dot{x} - \ddot{\theta}x - \dot{\theta}^2 y = -\frac{\mu y}{[(r+x)^2 + y^2 + z^2]^{3/2}} \quad (7)$$
$$\ddot{z} = -\frac{\mu z}{[(r+x)^2 + y^2 + z^2]^{3/2}}$$

This system can be simplified assuming the leader is in circular orbit. In this case $\dot{\theta} = n = constant, \ddot{\theta} = 0$, and $r = a = constant$, where a is the semi-major axis and n is the mean motion of the leader orbit and the resulting motion can be solved analytically. Substitution of these conditions and expansion of the right-hand side of equations (7) into Taylor series about the origin provides the system of equations called the *Hill-Clohessy-Wiltshire* (HCW) model[11]:

$$\ddot{x} - 2n\dot{y} - 3n^2 x = f_x$$
$$\ddot{y} + 2n\dot{x} = f_y \quad (8)$$
$$\ddot{z} + n^2 z = f_z$$

Nevertheless, the work developed in this paper is based on the relative movement between spacecraft when the leader is following elliptic orbits. Therefore, equations (8) are not useful for this approach and a more suitable model must be used. It can be observed that Equations (7) depend implicitly on the independent variable *time* but this system is easily analysed when a substitution of the independent variable by the true anomaly is performed and after scaling the relative position by the radius of the leader. Next, by expanding in series of Legendre polynomials and neglecting second and higher order terms a set of equations known as the *Tschauner-Hempel* (TH) model is obtained:

$$\ddot{x} - \ddot{\theta}y - 2\dot{\theta}\dot{y} - \dot{\theta}^2 x + \frac{\mu}{r^3}x = f_x$$
$$\ddot{y} + \ddot{\theta}y + 2\dot{\theta}\dot{x} - \dot{\theta}^2 x - \frac{2\mu}{r^3}y = f_y \quad (9)$$
$$\ddot{z} + \frac{\mu}{r^3}z = f_z$$

The TH model is valid for any value of the eccentricity of the leader's orbit and allows analytical solutions in terms of a special integral known as Lawden's integral[12]. Furthermore, the TH model contains the HCW model when the eccentricity is set to a value of zero.

Steerable Dipole

In this paper, each spacecraft is assumed to have attached a single circular loop of current mathematically approximated to a magnetic dipole $\tilde{\mu}$ with *N* number of turns as in Equation (10). This moment reacts against the magnetic fields of other spacecraft in the formation producing the required control force which in turn depends on spatial orientation of each spacecraft, separation distance, intensity of the current and mass of the conductor. As one spacecraft rotates in any direction to satisfy the magnetic requirements of the control – by the aid of reaction wheels – so does the magnetic dipole and this is represented schematically in Figure 2. In order to present a simpler insight of the dynamical response of each element in the formation the spatial rotation of the magnetic dipole – and therefore, of each spacecraft – is represented in a spherical coordinate system with the coordinates r, θ and φ as observed in Figure 3.

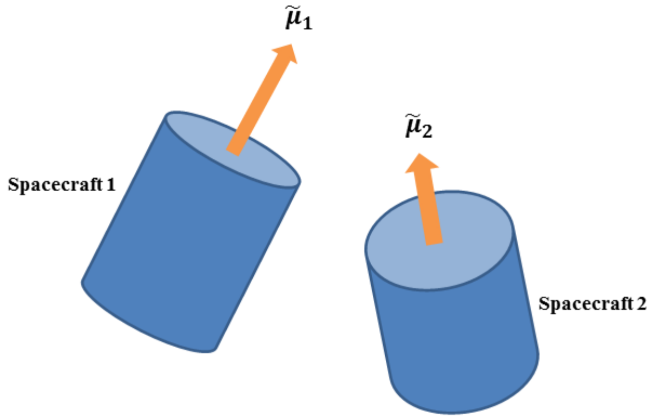

Figure 2. Representation of the Magnetic Moment of a Spacecraft in a Spherical Reference Frame.

$$\tilde{\mu} = NIA \tag{10}$$

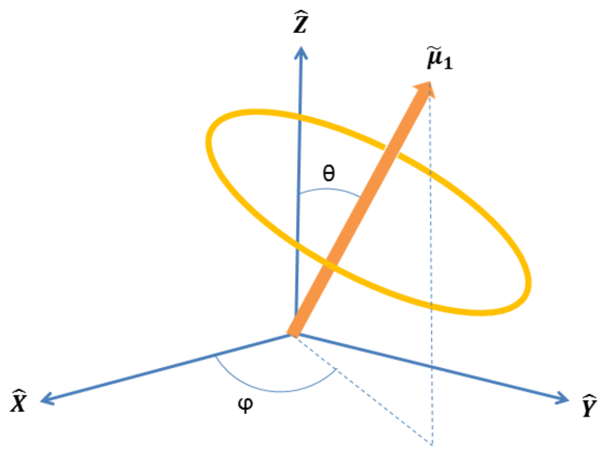

Figure 3. Representation of the Magnetic Moment of a Spacecraft in a Spherical Reference Frame.

Magnetic Force Model

The magnetic fields and forces models in this paper do not include electric fields and are based on the classical theory of electromagnetism for the case of magnetostatics, as the magnetic fields and currents are considered to vary very slowly with respect to time. Then, a magnetic field $\boldsymbol{B}$ can be determined by Maxwell's equations[13] as stated next:

$$\nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} \quad \text{(Ampère Law)} \tag{11}$$

where $\mu_0 = 4\pi 10^{-07}\, N/A^2$ is the permeability of free space.

Maxwell's equations together with the Lorentz force for a line charge distribution $\boldsymbol{I} = \lambda \boldsymbol{v}$ in Equation (12) define a complete formulation for the magnetostatic problem when electric fields are not present.

$$\boldsymbol{F} = \int (\boldsymbol{v} \times \boldsymbol{B}) \lambda dl = \int (\boldsymbol{I} \times \boldsymbol{B}) dl \tag{12}$$

According to Helmoltz theorem, the static magnetic field $\boldsymbol{B}$ is completely defined by its curl and its divergence and since the divergence is equal to zero, $\boldsymbol{B}$ can be defined only by means of its curl. Thus – mathematically speaking – a vector field can be defined as the curl of another arbitrary vector field $\boldsymbol{A}$. Therefore, the magnetic field $\boldsymbol{B}$ can de stated as:

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} \tag{13}$$

Substitution of Equation (13) in *Ampère Law* of Equations (11) provides the result:

$$\nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2 \boldsymbol{A} = \mu_0 \boldsymbol{J} \tag{14}$$

Until now there is no mention about the divergence of the vector $\boldsymbol{A}$ and also we must consider that this vector is not a unique solution of $\boldsymbol{B}$ as it can be defined with the addition of a term that vanishes with the curl, for example the gradient of any scalar. Recalling Equation (13) we have:

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} = \boldsymbol{\nabla} \times [\boldsymbol{A} + \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{A})] = \boldsymbol{\nabla} \times \boldsymbol{A} + \boldsymbol{\nabla} \times \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{A}) \tag{15}$$

and:

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{A}) = \boldsymbol{0} \tag{16}$$

As the curl of a gradient and the gradient of a divergence is always zero we could assume that:

$$\boldsymbol{\nabla} \cdot \boldsymbol{A} = \boldsymbol{0} \tag{17}$$

Therefore, the *Ampère Law* can be written as:

$$\nabla^2 \boldsymbol{A} = -\mu_0 \boldsymbol{J} \tag{18}$$

This is the Poisson equation and assuming $\boldsymbol{J}$ tends to zero at infinity the solution is expressed as:

$$\boldsymbol{A}(\boldsymbol{s}) = \frac{\mu_0}{4\pi} \iiint \frac{\boldsymbol{J}(\boldsymbol{s})}{|\boldsymbol{s} - \boldsymbol{\varrho}|} d\boldsymbol{\varrho} \tag{19}$$

where $\boldsymbol{s}$ is the distance to the point of study and $\boldsymbol{\rho}$ is the distance to the moving charge.

These exact models contain integrals that cannot be solved analytically. After using Equation (19) to determining the vector potential of a circular loop and expanding the resulting denominator in Taylor series we can obtain the next expression for $\boldsymbol{A}$:

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\frac{\widetilde{\boldsymbol{\mu}} \times \boldsymbol{s}}{s^3} \tag{20}$$

Substitution of Equation (20) in (13) provides us a simple expression for the magnetic field:

$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\left[\frac{3s(\widetilde{\boldsymbol{\mu}} \cdot \boldsymbol{s})}{s^5} - \frac{\widetilde{\boldsymbol{\mu}}}{s^3}\right] \tag{21}$$

With no further derivation, the force over a dipole *j* generated by the magnetic field of a dipole *i* is represented by the equation[13]:

$$\boldsymbol{F_{ji}} = -\boldsymbol{\nabla}(-\widetilde{\boldsymbol{\mu}}_{\boldsymbol{j}} \cdot \boldsymbol{B_i}) \tag{22}$$

By substitution of Equation (21) into (22) we arrive to the equation for the magnetic force between two dipoles:

$$\boldsymbol{F_{ji}} = -\frac{3\mu_0}{4\pi}\left[-\frac{\widetilde{\boldsymbol{\mu}}_{\boldsymbol{i}} \cdot \widetilde{\boldsymbol{\mu}}_{\boldsymbol{j}}}{s^5}\boldsymbol{s} - \frac{\widetilde{\boldsymbol{\mu}}_{\boldsymbol{i}} \cdot \boldsymbol{s}}{s^5}\widetilde{\boldsymbol{\mu}}_{\boldsymbol{j}} - \frac{\widetilde{\boldsymbol{\mu}}_{\boldsymbol{j}} \cdot \boldsymbol{s}}{s^5}\widetilde{\boldsymbol{\mu}}_{\boldsymbol{i}} + \frac{5(\widetilde{\boldsymbol{\mu}}_{\boldsymbol{i}} \cdot \boldsymbol{s})(\widetilde{\boldsymbol{\mu}}_{\boldsymbol{j}} \cdot \boldsymbol{s})}{s^7}\boldsymbol{s}\right] \tag{23}$$

HTS Requirements

A good coil design would be one with an adequate balance between the terms of Equation (10) that is the number of turns *N*, the intensity of the current *I* and the area of the loop *A*. In order to achieve a large value of the strength of the magnetic dipole a high current must be created and/or a coil with a large mass must be attached to the spacecraft. Common conductors such as copper, aluminium and gold are not suitable options due to the fact that they present a considerable resistance values limiting the amount of electric current employed to generate the magnetic dipole. To overcome this issue, the use of *High Temperature Semiconductors* (HTS) is presented in EMFF. This material has the special characteristic of allowing the electrical current to flow with almost zero resistance when they are cooled below a critical temperature value. If we use large amounts of HTS we would be able to produce large magnetic dipoles, although with a cost on spacecraft mass. To obtain the HTS requirements for the results to be presented in the next section we will use the HTS model presented by Elias[1]:

$$I_{max} = I_C A_C \tag{24}$$

$$M_C = 2N\pi R_C A_C \rho_C \tag{25}$$

$$I_C/\rho_C \approx 16{,}250\ A \cdot m/kg\ \ at\ 77K \tag{26}$$

where I_{max} is the maximum electrical current, I_C is the critical current density, A_C is the cross-sectional area of the HTS wire, M_C is the mass of the coil, R_C is the coil radius, ρ_C is the density of the HTS wire and I_C/ρ_C is the value of the technology parameter for the current state-of-the-art HTS. Substitution of equations (24), (25) and (26) in (10) provide an equation for the necessary coil mass for a given value of the magnitude of the magnetic dipole, that is:

$$M_C = \frac{\mu}{8125 R_C} \tag{27}$$

CONTROLABILITY REQUIREMENTS FOR STATION-KEEPING

As stated in the introduction, the main objective of this paper is to provide a formulation for EMFF when a formation is in station-keeping while moving in eccentric reference orbits. The formation control inputs for each spacecraft will be provided by the magnetic forces generated by magnetic dipoles interaction among the different magnetic fields. To obtain such formulation we will follow the procedure used by Schweighart[2] to obtain a set of equations of motion that includes two-body gravitational and magnetic forces. Thus, the system to be solved is one composed by *q* vector equations:

$$\begin{gathered}
\boldsymbol{F}_1^M(\boldsymbol{r}_1, \boldsymbol{r}_2, \cdots, \boldsymbol{r}_q, \tilde{\boldsymbol{\mu}}_1, \tilde{\boldsymbol{\mu}}_2, \cdots, \tilde{\boldsymbol{\mu}}_q) + \boldsymbol{F}_1^G = 0 \\
\boldsymbol{F}_2^M(\boldsymbol{r}_1, \boldsymbol{r}_2, \cdots, \boldsymbol{r}_q, \tilde{\boldsymbol{\mu}}_1, \tilde{\boldsymbol{\mu}}_2, \cdots, \tilde{\boldsymbol{\mu}}_q) + \boldsymbol{F}_2^G = 0 \\
\vdots \\
\boldsymbol{F}_q^M(\boldsymbol{r}_1, \boldsymbol{r}_2, \cdots, \boldsymbol{r}_q, \tilde{\boldsymbol{\mu}}_1, \tilde{\boldsymbol{\mu}}_2, \cdots, \tilde{\boldsymbol{\mu}}_q) + \boldsymbol{F}_q^G = 0
\end{gathered} \tag{28}$$

This system can be arbitrarily reduced to a scalar system with *(3q-3)* equations by eliminating one of the equations without loose of generality and providing a Cartesian representation of each of the dipoles as observed in Equations (29) where F_q^M corresponds to magnetic forces and F_q^G to gravitational forces . This operation also opens the possibility of having a non-zero free dipole whose coordinates can be chosen at will.

$$\begin{gathered}
F_1^M(\boldsymbol{r}_1, \boldsymbol{r}_2, \cdots, \boldsymbol{r}_q, \tilde{\mu}_{11}, \tilde{\mu}_{21}, \cdots, \tilde{\mu}_{3q}) + F_1^G = 0 \\
F_2^M(\boldsymbol{r}_1, \boldsymbol{r}_2, \cdots, \boldsymbol{r}_q, \tilde{\mu}_{11}, \tilde{\mu}_{21}, \cdots, \tilde{\mu}_{3q}) + F_2^G = 0 \\
\vdots \\
F_{(3q-3)}^M(\boldsymbol{r}_1, \boldsymbol{r}_2, \cdots, \boldsymbol{r}_q, \tilde{\mu}_{11}, \tilde{\mu}_{21}, \cdots, \tilde{\mu}_{3q}) + F_{(3q-3)}^G = 0 \\
+ \textit{Free Dipole}
\end{gathered} \tag{29}$$

The components of the free magnetic dipole are known in advance and the results to be obtained with Equations (29) are each of the three magnetic dipole components for every spacecraft in the formation. The terms corresponding to the relative dynamics of the formation are composed by the TH acceleration requirements for station keeping making the results also dependable on true anomaly of the reference orbit. Finally, these results are presented as plots of the strength of the magnetic dipole (SMD) and the angles describing the orientation of every magnetic moment in response to the control requirements requested by the station-keeping process during the movement of the formation around the reference orbit. In the analysis performed in the next cases a special notation will be followed: each spacecraft will be named as *SPK* where *K* corresponds to the number of spacecraft (e.g. Spacecraft 5 would be named *SP5*). Additionally, all spacecraft will have the same mass of 50 kg and all cases will follow the same reference orbit with an eccentricity of 0.25 and a period of 8267 sec. Frequent analysis can be found in literature about formations consisting of only two spacecraft hence in this paper three different scenarios where chosen with arrays having three, four and six spacecraft.

Three Spacecraft in Collinear Formation

A formation composed by three spacecraft in collinear formation is presented in the first case and its schematic representation can be observed in Figure 4. Specific parameters like the position of each spacecraft in the Hill frame and the components of every magnetic dipole to be determined are summarized in Table 1.

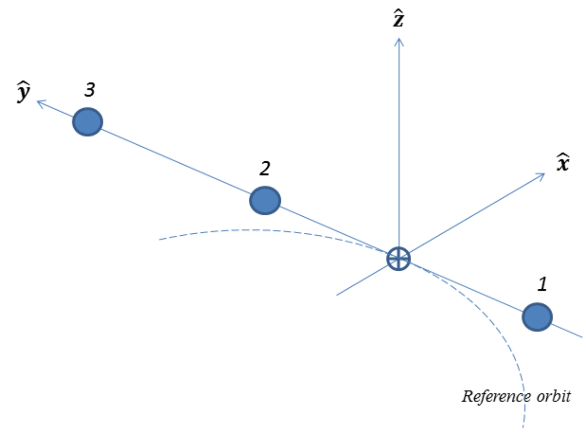

Figure 4. Three Spacecraft in Formation.

Table 1. Parameters for the Formation of Three Spacecraft.

Spacecraft 1	Spacecraft 2	Spacecraft 3
Total mass 50 (kg)	Total mass 50 (kg)	Total mass 50 (kg)
$\boldsymbol{\rho_1} = [0, y_1, 0]^T$ (m)	$\boldsymbol{\rho_2} = [0, y_2, 0]^T$ (m)	$\boldsymbol{\rho_3} = [0, y_3, 0]^T$ (m)
$\boldsymbol{\tilde{\mu}_1} = [\tilde{\mu}_{11}, \tilde{\mu}_{21}, \tilde{\mu}_{31}]$	$\boldsymbol{\tilde{\mu}_2} = [m_{12}, m_{22}, m_{32}]$	$\boldsymbol{\tilde{\mu}_3} = [m_{13}, m_{23}, m_{33}]$

Results were obtained first by selecting *SP1* as the free dipole with constant magnetic dipole components and then, forming the necessary equations of motion with the aid of Equations (29). These set of equations were solved numerically in order to calculate the rest of the magnetic dipole components necessary to maintain the specific formation. By inspecting the resulting plots of the *SMD* in Figure 5 we can observe that for *SP1* and *SP3* constant values for the components of the magnetic dipole are required during the movement while for *SP2* there is an oscillatory variation from values between 200 and 1500 Am^2. Regarding the rotational response of the spacecraft this can be observed in Figures 6, 7 and 8 where for SP1 and SP3 a fixed orientation is required while SP2 is oscillating according to the gravitational requirements for station-keeping.

The three largest magnetic moments strengths required for this analysis are 1732.1, 1514.12 and 1000 Am^2 for which the HTS mass requirements are expressed in Table 2 when a radius of 0.3m is considered for the coils.

Table 2. HTS Mass Requirements for the Formation with Three Spacecraft.

Magnetic moment (Am^2)	Mass of HTS coil (k)
1732.1	0.71
1514.12	0.62
1000	0.41

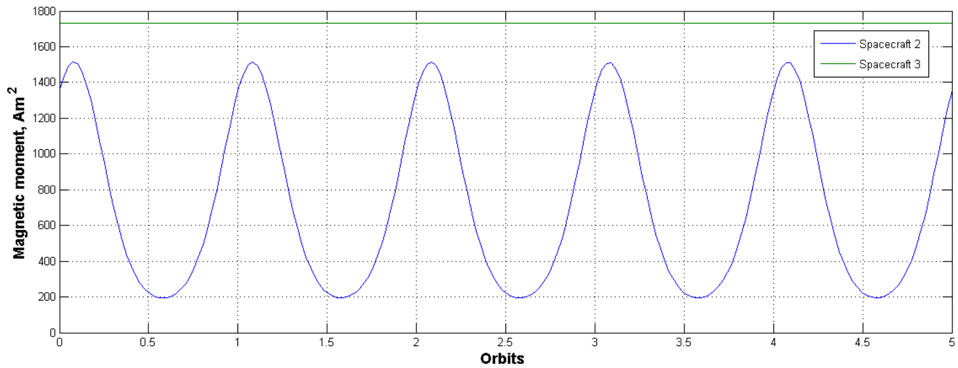

Figure 5. Strength of magnetic moment for SP2 and SP3.

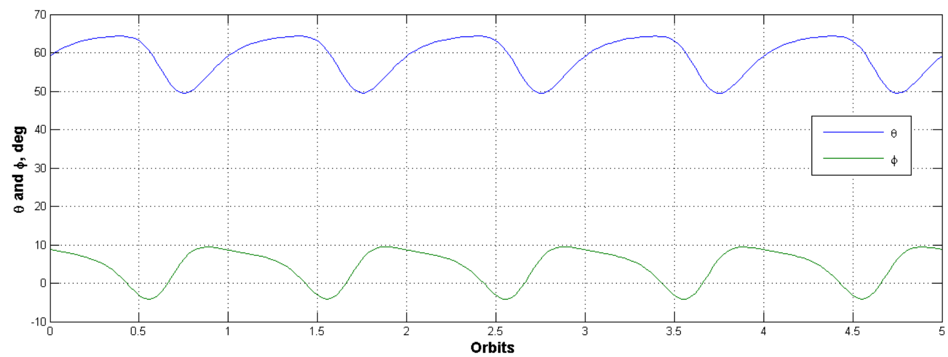

Figure 6. Rotational Behaviour for SP2

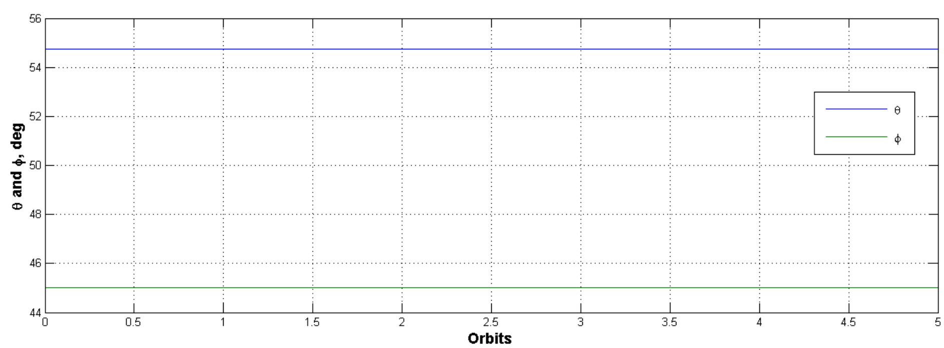

Figure 7. Rotational Behaviour for SP3

The higher mass in Table 2 represents the 1.42% of the total mass of the spacecraft. The necessary hardware to produce the magnetic dipoles could be considered as 85% of the sum of the mass of the propulsion system and the propellant of the spacecraft. That is:

$$Magnetic\ System = 0.85(Propellant + Propulsion\ System) \qquad (30)$$

which should not be higher than 5.1% of the total mass of the spacecraft[14]. The 15% remaining is left for an emergency propulsion system. As the necessary mass does not exceed these design parameters proposed it is assumed that the current results are adequate for this formation.

Four Spacecraft in Tetrahedral Formation

The next arrangement to be considered is a formation consisting of four spacecraft arranged as a tetrahedron and the reader can observe its schematic representation in Figure 8. Specific parameters like the position of each spacecraft in the Hill frame and the components of every magnetic dipole to be determined are summarized in Table 3.

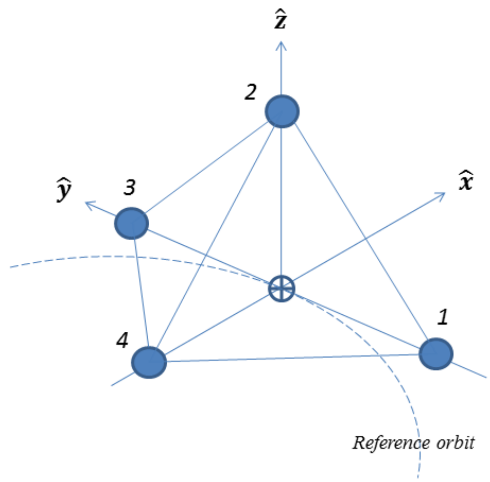

Figure 8. Four Spacecraft in Formation.

As before, two types of plots are presented: response of the SMD and rotation response for every spacecraft. In Figure 9 it is observed that the SMD of all spacecraft are oscillating in response to the control requirements to counteract the gravitational forces and in this case SP2 have the largest variation from values between 500 and 6300 Am^2. Regarding the orientation of the elements of the formation, it can be observed in Figures 10, 11 and 12 an oscillatory movement to correspond the required control values with SP3 having the largest range from 30 to 95 deg.

Table 3. Parameters for the Tetrahedral Formation.

Spacecraft 1	Spacecraft 2	Spacecraft 3	Spacecraft 4
Total mass 50 (kg)	Total mass 50 (kg)	Total mass 50 (kg)	Total mass 50 (kg)
$\boldsymbol{\rho_1} = [0, y_1, 0]^T$ (m)	$\boldsymbol{\rho_2} = [0, y_2, 0]^T$ (m)	$\boldsymbol{\rho_3} = [0, 0, z_3]^T$ (m)	$\boldsymbol{\rho_4} = [x_4, 0, 0]^T$ (m)
$\tilde{\boldsymbol{\mu}}_1 = [\tilde{\mu}_{11}, \tilde{\mu}_{21}, \tilde{\mu}_{31}]$	$\tilde{\boldsymbol{\mu}}_2 = [\tilde{\mu}_{12}, \tilde{\mu}_{22}, \tilde{\mu}_{32}]$	$\tilde{\boldsymbol{\mu}}_3 = [\tilde{\mu}_{13}, \tilde{\mu}_{23}, \tilde{\mu}_{33}]$	$\tilde{\boldsymbol{\mu}}_4 = [\tilde{\mu}_{14}, \tilde{\mu}_{24}, \tilde{\mu}_{34}]$

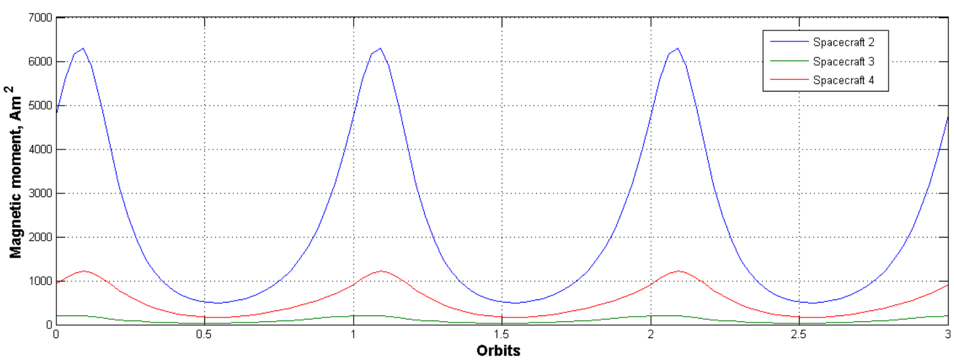

Figure 9. Strength of Magnetic Moment for SP2, SP3 and SP4.

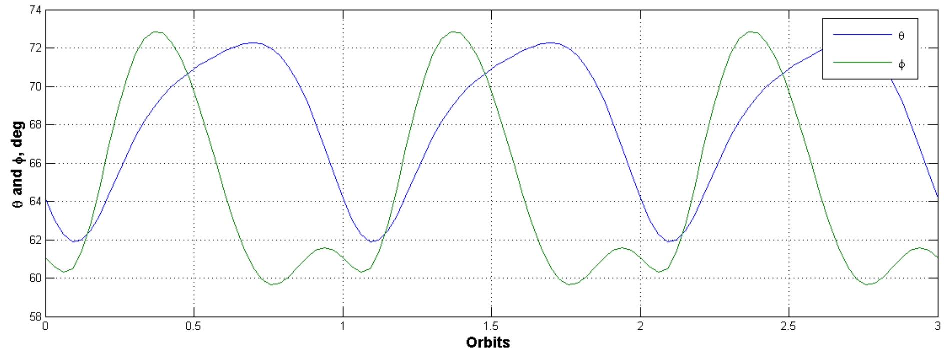

Figure 10. Rotational Behaviour for SP2.

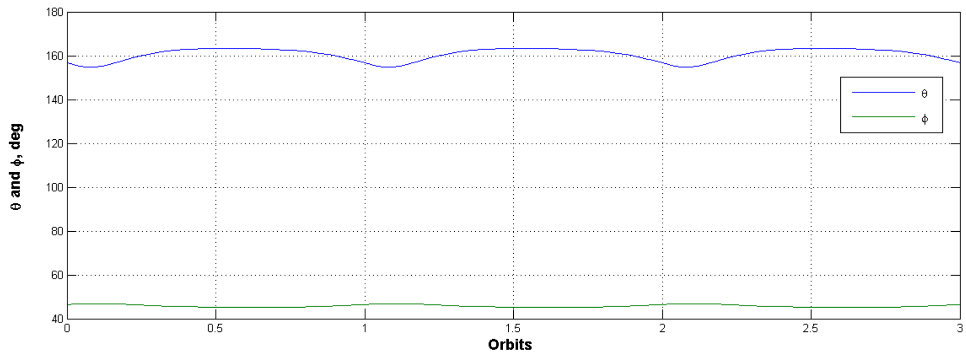

Figure 11. Rotational Behaviour for SP3.

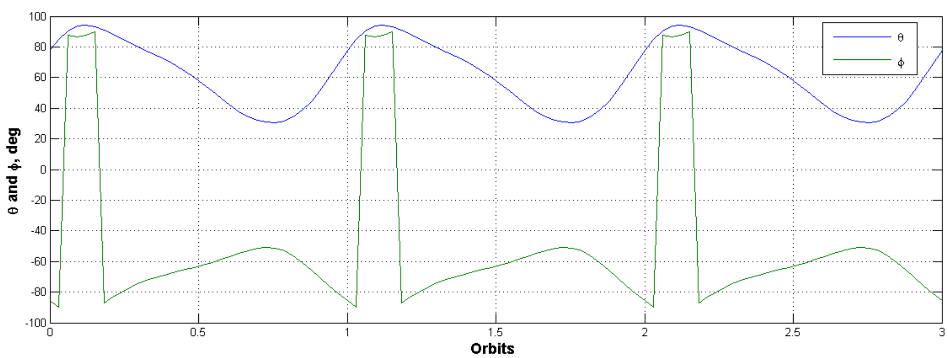

Figure 12. Rotational Behaviour for SP4

The three largest magnetic moments in this case were 6295.8, 6000 and 1219.54 Am^2 for which the HTS mass requirements can be observed in Table 4 when a radius of 0.3 m is considered for all the coils.

In this scenario, the value of the higher mass represents the 5.1% of the total mass of the spacecraft which – according to Equation (30) – is an acceptable design value capable of providing the necessary controllability requirements for this formation.

Table 4. HTS Mass Requirements for the Tetrahedral Formation.

Magnetic moment (Am^2)	Mass of HTS coil (kg)
6295.8	2.58
6000	2.46
1219.54	0.5

Six Spacecraft in Formation

Finally, a formation composed by six spacecraft equally spaced in a circle is analysed next and its schematic view can be observed in Figure 13. Specific parameters like the position of each spacecraft in the Hill frame and the components of every magnetic dipole to be determined are summarized in Table 5 and 6.

Table 5. Parameters for the Six spacecraft Formation.

Spacecraft 1	Spacecraft 2	Spacecraft 3	Spacecraft 4
Total mass 50 kg	Total mass 50 kg	Total mass 50 kg	Total mass 50 kg
$\boldsymbol{\rho_1} = [0, y_1, z_1]^T$, m	$\boldsymbol{\rho_2} = [0, y_2, z_2]^T$, m	$\boldsymbol{\rho_3} = [0, y_3, z_3]^T$, m	$\boldsymbol{\rho_4} = [0, y_4, z_4]^T$, m
$\boldsymbol{m_1} = [m_1, m_2, m_3]$	$\boldsymbol{m_2} = [m_4, m_5, m_6]$	$\boldsymbol{m_3} = [m_7, m_8, m_9]$	$\boldsymbol{m_4} = [m_{10}, m_{11}, m_{12}]$

Table 4. Parameters for the Six Spacecraft Formation (Continuation).

Spacecraft 5	Spacecraft 6
Total mass 50 kg	Total mass 50 kg
$\boldsymbol{\rho_5} = [0, y_5, z_5]^T$, m	$\boldsymbol{\rho_6} = [0, y_6, z_6]^T$, m
$\boldsymbol{m_5} = [m_{13}, m_{14}, m_{15}]$	$\boldsymbol{m_6} = [m_{16}, m_{17}, m_{18}]$

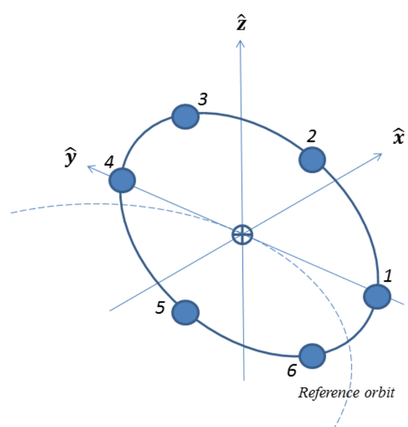

Figure 13. Six Spacecraft in Formation.

Plots regarding the SMD and the rotational response for every spacecraft in the formation where obtained in order to inspect the behaviour of each element due to the control requirements to maintain the aforementioned configuration. Referring to Figure 14 it is noticed that SP2, SP3 and SP5 present the largest SMD with SP5 values ranging between 5500 and 6125 Am^2. In

Figure 15 the SMD of SP4 and SP6 are observed with an oscillatory behaviour and with the lowest values in the formation. Regarding the rotational response, this can be observed in Figures 16 to 20 and it is viewed that all spacecraft are rotating to correspond the controllability requirements of the gravitational forces.

The three largest magnetic moments generated by this formation are 6126.7, 4500 and 1363.3 Am^2 for which the HTS mass requirements can be observed in Table 5 when a radius of 0.3 m is considered for all the coils.

Table 5. HTS Mass Requirements.

Magnetic moment (Am^2)	Mass of HTS coil (kg)
6126.7	2.5
4500	1.64
1363.3	0.55

The larger mass represents the 5% of the total mass of the spacecraft and this value is considered an acceptable design parameter according to Equation (30).

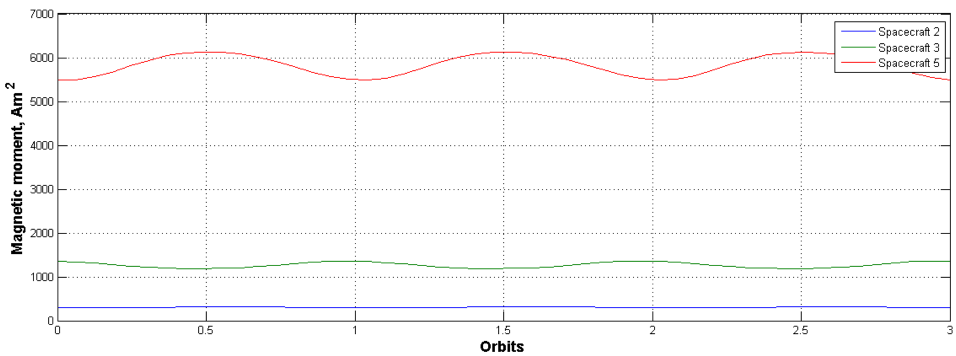

Figure 14. Magnetic Strength Response for SP2, SP3 and SP5.

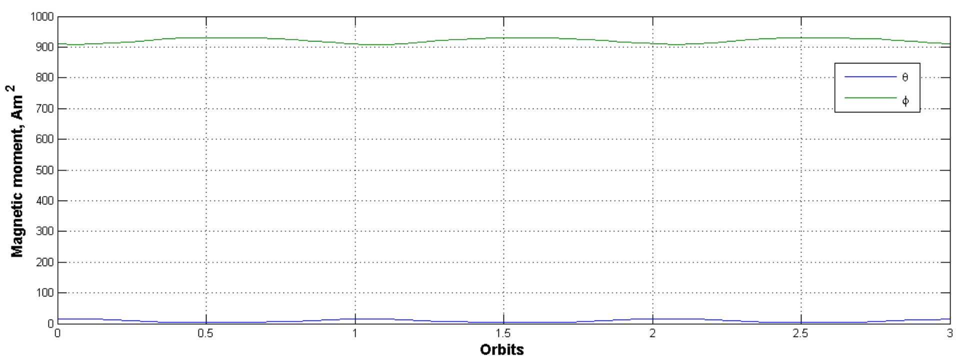

Figure 15. Magnetic Strength Response for SP4 and SP6.

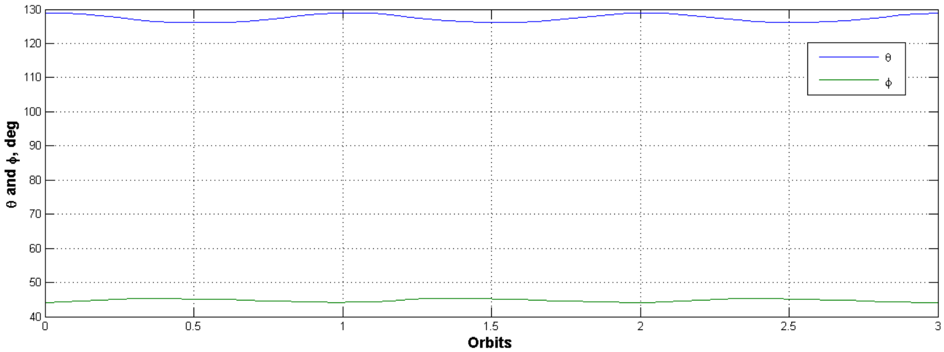

Figure 16. Rotational Behaviour for SP2.

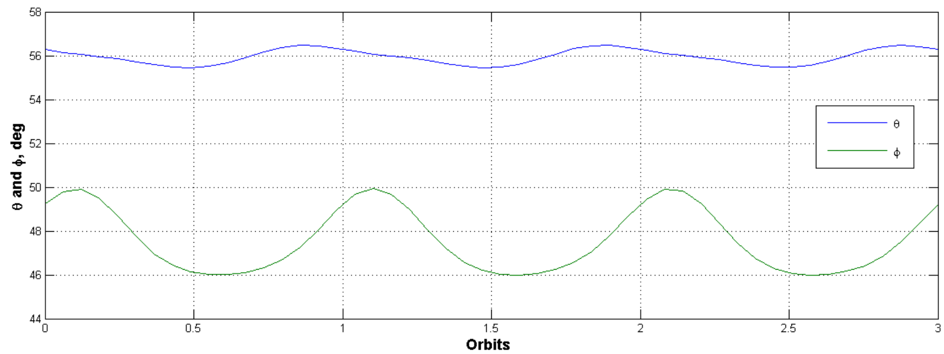

Figure 17. Rotational Behaviour for SP3.

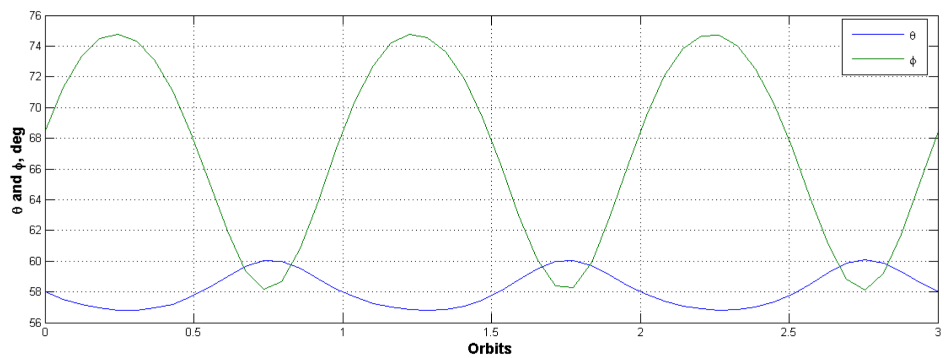

Figure 18. Rotational Behaviour for SP4.

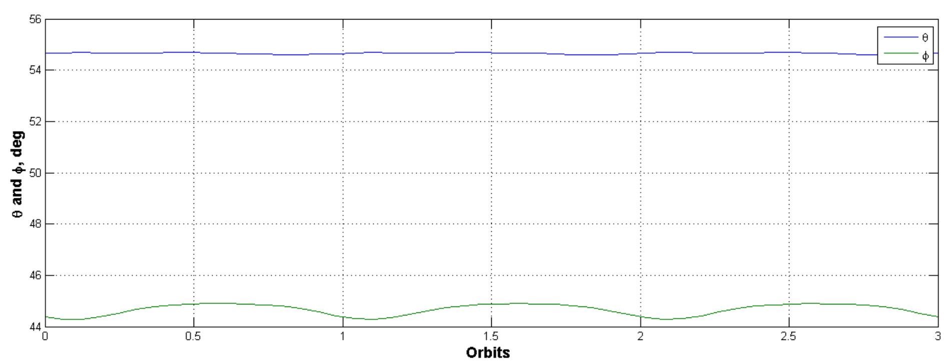

Figure 19. Rotational Behaviour for SP5.

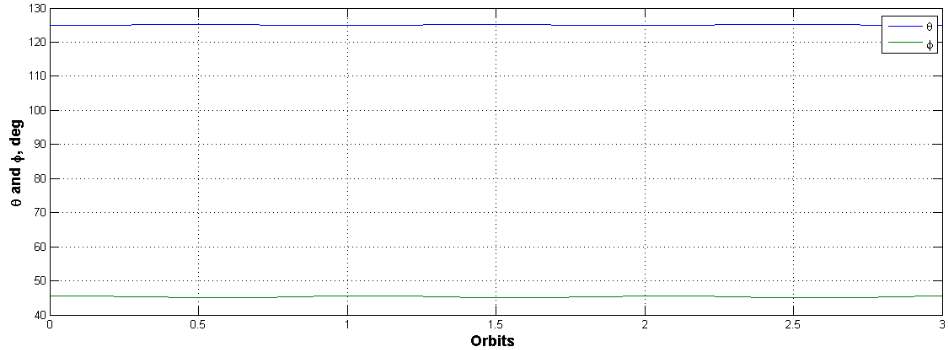

Figure 20. Rotational Behaviour for SP6.

CONCLUSIONS

The main contribution of this paper is to provide an EMFF formulation for spacecraft in formation following elliptical orbits by means of the Tschauner and Hempel model for relative dynamics. Different scenarios were modelled, simulated and analysed in order to provide results to satisfy the controllability requirements for station-keeping such as magnetic dipoles components and the rotational response of every spacecraft. Additionally, the HTS mass requirements for every spacecraft in a formation were obtained in order to evaluate if such controllability conditions could be satisfied with realistic coil designs. Results show that EMFF is adequate for formations following eccentric orbits when composed by small satellites arranged in configurations with short separation distances. Therefore, these features make this technology feasible for missions such as rendezvous, tight formations and collision avoidance especially when the spacecraft does not require a fast reaction. HTS future advances regarding characteristics such as volumetric and current density could also be beneficial for EMFF as better semiconductor materials would increase the strength of the magnetic dipole to adequate levels while maintaining a low mass, even for larger separation distances between spacecraft.

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