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Modelling of Dynamic Crack Propagation in 3D Elastic Continuum Using Configurational Mechanics

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ABSTRACT

This paper presents the theoretical basis and numerical implementation for simulating dynamic crack propagation in 3D hyperelastic contina within the context of configurational mechanics. The approach taken is based on the principle of global maximum energy dissipation for elastic solids, with configurational forces determining the direction of crack propagation. The work builds on the developments made by the authors for static analysis [1], incorporating the influence of the kinetic energy. The nonlinear system of equations are solved in a monolithic manner using Newton-Raphson scheme. Initial numerical results are presented.

Keywords: *Dynamic fracture; 3D crack propagation; Configurational mechanics*

1. Introduction

Numerical simulation of dynamic crack propagation in three-dimensional hyperelastic materials is studied within the context of linear elastic fracture mechanics (LEFM) and configurational mechanics. Although dynamic fracture has been widely investigated in continuum mechanics, this is still a challenging topic. We require a physical and mathematical description to determine (a) when a crack will propagate, (b) the direction of propagation and (c) how far/fast the crack will propagate. Furthermore, we require a numerical setting to accurately resolve the evolving displacement discontinuity.

In this study, we present a mathematical derivation and numerical implementation that can achieve these goals, solving for conservation of momentum in both the spatial and material domains. The spatial (or physical) domain is what we observe and the material domain is the evolving reference domain due to crack evolution. The theory is an interpretation of linear elastic fracture mechanics and consistent with Griffith's fracture criterion. This paper represents a generalisation of the authors' previous work on static crack propagation [1]. The approach taken is based on the principle of global maximum energy dissipation for elastic solids, with configurational forces determining the direction of crack propagation. This approach has been successfully adopted by a number of other authors in the context of quasi-static analysis, e.g. [1,2]. At present we restrict ourselves to the consideration of elastic bodies with energy dissipation limited to the creation of new crack surfaces.

In the context of the numerical setting, we have adopted the Arbitrary Lagrangian-Eulerian (ALE) method, which is a kinematic framework to describe movement of the nodes of the finite element mesh independently of the material. Thus, we are able to resolve the propagating crack without influence from the original finite element mesh, and maintain mesh quality. We are primarily concerned with solving crack propagation in large 3D problems. The efficient solution of such problems, with a large numbers of degrees of freedom, requires the use of an iterative solver for solving the system of algebraic equations. In such cases, controlling element quality enables us to optimise matrix conditioning, thereby increasing the computational efficiency of the solver.

The resulting systems of equations are highly nonlinear and requires a solution strategy that enables us to trace the entire transient response.

2. Body with propagating crack

Differentiable mappings relate the reference material domain to both the current spatial and the current material domains. These mappings are utilized to independently observe the deformation of material in the physical space Ω_t and the evolution of the crack surface
in the material space $\mathcal{B}_t$, see Figure 1.

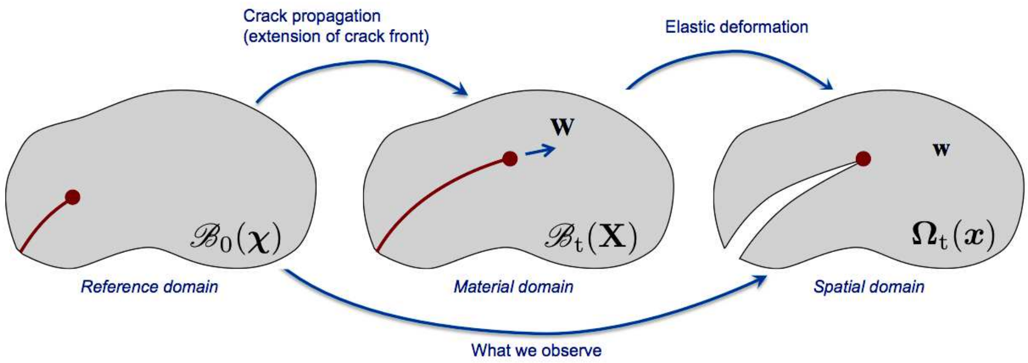

Figure 1: Deformation and structural change of a body with a propagating crack

The spatial domain defines a set of coordinates $\mathbf{x}$ corresponding to the position of all mesh nodes in the body, while the material domain has set of coordinates $\mathbf{X}$ corresponding to the position of all material points. These sets are used to calculate spatial $\mathbf{w}$ and material $\mathbf{W}$ displacement fields relative to the reference domain, $\boldsymbol{\chi}$:

$$\mathbf{w} = \mathbf{x} - \chi \tag{1}$$

$$\mathbf{W} = \mathbf{X} - \chi \tag{2}$$

The physical displacement $\mathbf{u}$ is given by:

$$\mathbf{u} = \mathbf{w} - \mathbf{W} \tag{3}$$

3. Numerical implementation

The finite element approximation is applied for both the physical and material space. Three dimensional domains are discertised with tetrahedral elements with hierarchical basis functions of arbitrary polynomial order, following the work of Ainsworth and Coyle [3]. The higher-order approximations are only applied to displacements in the spatial configuration, whereas a linear approximation is used for displacements in the material space.

The resulting residuals in the spatial and material domain, that represent the two primary, nonlinear equations that need to be solved, are expressed as:

$$\mathbf{r}_{\mathrm{s}} = \lambda\left(t\right)\mathbf{f}_{\mathrm{s,ext}} - \mathbf{f}_{\mathrm{s,int}} \tag{4}$$

$$\mathbf{r}_{\mathrm{m}} = \mathbf{f}_{\mathrm{m,res}} - \mathbf{f}_{\mathrm{m,driv}} \tag{5}$$

where λ is the load factor that scales the external reference load, $\mathbf{f}_{\mathrm{s,ext}}$, $\mathbf{f}_{\mathrm{s,int}}$ is the internal force vector, $\mathbf{f}_{\mathrm{m,res}}$ is the material resistance and $\mathbf{f}_{\mathrm{m,driv}}$ is the driving force for crack propagation. These equations are linearised and solved using a Newton-Raphson procedure.

The spatial internal force vector is expressed as

$$\mathbf{f}_{\mathrm{s,int}} = \int \mathbf{B}_{\mathrm{X}}^{\mathrm{T}}\mathbf{P}\,\mathrm{dV} + \int \rho\mathbf{N}^{\mathrm{T}}\mathbf{a}\,\mathrm{dV} \tag{6}$$

and the material driving force is expressed as

$$\mathbf{f}_{\mathrm{m,driv}} = \int \mathbf{B}_{\mathrm{X}}^{\mathrm{T}} \boldsymbol{\Sigma}\, \mathrm{dV} - \int \rho_0 \left(\mathbf{F}^{\mathrm{T}} \mathbf{a} + \dot{\mathbf{F}}^{\mathrm{T}} \mathbf{v} \right) \mathrm{dV} = \mathbf{G} - \int \rho_0 \left(\mathbf{F}^{\mathrm{T}} \mathbf{a} + \dot{\mathbf{F}}^{\mathrm{T}} \mathbf{v} \right) \mathrm{dV} \tag{7}$$

where $\mathbf{P}$ is the first Piola Kirchoff stress tensor, $\boldsymbol{\Sigma}$ is the Eshelby stress, $\mathbf{a}$ is the spatial acceleration, $\mathbf{v} = \dot{\mathbf{u}} = \dot{\mathbf{w}} - \mathbf{F}\dot{\mathbf{W}}$ is the spatial velocity, $\mathbf{F}$ is the deformation gradient and $\mathbf{G}$ is the configurational force.

In the restricted case of quasi-static analysis, the inertia and velocity terms in (6) and (7) vanish and the formulation reverts to that presented in [1]. Thus, equations (6) and (7) represent an important development in the modelling of dynamic crack propagation, generalising the configurational mechanics formulation.

4. Mesh quality

The continuous adaptation of the finite element mesh to resolve the propagating crack will result in a degeneration of the mesh quality. For large problems, where it is necessary to use iterative solvers, we need to control the quality of the elements to ensure good matrix conditioning. The key challenge is to enforce constraints to preserve element quality for each Newton-Raphson iteration, without influencing the physical response. Thus, we introduce a measure of element quality for tetrahedral elements in terms of their shape and use this to drive mesh improvement. Here we include both node movement and changes in element topology (face flipping).

Thus, equations (4) and (5) are augmented by a third residual, $\mathbf{f}_{\mathrm{q}}$, defined as

$$\mathbf{f}_{\mathrm{q}} = \int \mathbf{B}_{\mathrm{X}}^{\mathrm{T}} \mathbf{Q}\, \mathrm{dV} \tag{8}$$

where $\mathbf{Q}$ is a pseudo "stress" at the element level, as a counterpart to the first Piola Kirchhoff stress. Since the conditioning of the finite element stiffness matrix is controlled by the quality of the worst elements, we advocate that $\mathbf{Q}$ is a function of a log-barrier objective function as a means of evaluating the quality of an entire mesh (whilst punishing harshly the worst quality element), and the volume-length quality measure [4]. The latter does not directly measure dihedral angles, but it has been shown to be very effective at eliminating poor angles, thus improving stiffness matrix conditioning and interpolation errors. As the volume-length mesh quality measure is a smooth function of vertex positions, its gradient is straightforward and computationally cheap to calculate.

5. Time integration

The dynamic problem is integrated in time using a Newmark scheme. The system of equations for conservation of the spatial (4) and material (5) momentum, and mesh quality (8) are solved using a Newton-Raphson iterative process within each time step. The challenge is to ensure that the time integration algorithm adapts the size of the time step. When the problem is behaving in a purely elastic manner, the time step can be relatively large. However, as we approach the time when a crack starts to propagate, potentially in an unstable manner, it is necessary to reduce the time step. Detecting this transition point in an effective manner is the current focus of this work.

6. Numerical example

A three-point bend test with a corner notch is presented. The experiments include Figure 2(a) shows the crack development at an early stage. Figure 2(b) shows the nonlinear load-displacement path for two different meshes.

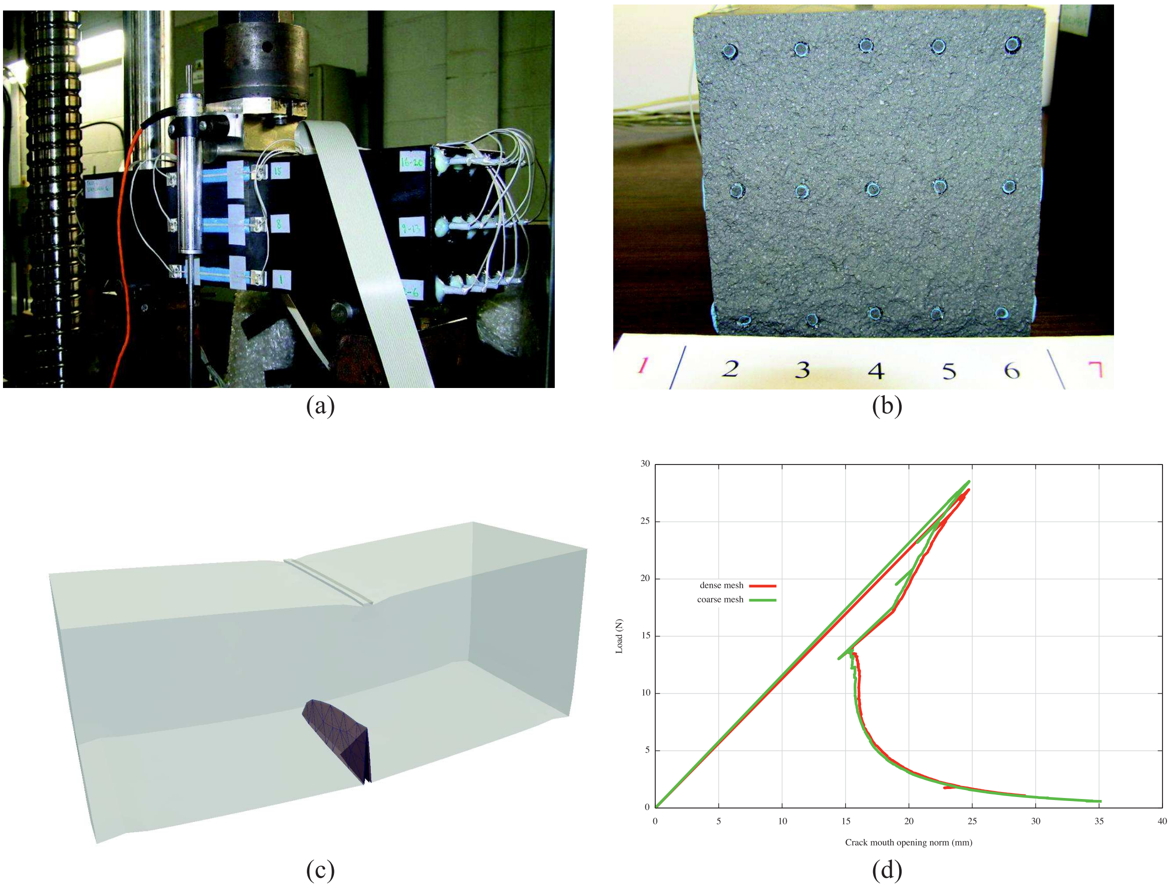

Figure 2: Three point bend test with corner notch. (a) Experimental set-up; (b) cracked beam, showing electrically isolated longitudinal rods for measuring crack front speed; (c) numerically predicted crack path, (d) load-displacement respons

7. Conclusions

In this study, the basis for dynamic crack propagation using configurational mechanics has been presented. The highly nonlinear system of equations are implemented in and solved using MoFEM, a finite element code for multi-physics problems which is developed at the University of Glasgow.

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