INFLUENCE OF BENDING–TWISTING COUPLING ON COMPRESSION BUCKLING STRENGTH.

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PRESENTATION CONTENTS

- Stacking sequence listings are derived for *Bending-Twisting* coupled laminates with up to 21 plies.
- The common design rule of balanced and symmetric stacking sequences will be shown to predominantly give rise to *Bending-Twisting* coupling; all exceptions are presented in *Journal of Aircraft*, 2009, 46 (4) pp. 1114-25.
- The symmetry rule will be shown to be a constraint that serves only to restrict the number of possible configurations to a very small proportion of the total design space.
- Dimensionless parameters will be presented from which the laminate properties are readily calculated.
- Expressions relating the dimensionless parameters to the well-known lamination parameters are also given, together with graphical representations of the design space.
- Finally, bounds on buckling performance under compression will be presented with specific reference to the lamination parameter design space.

CHARACTERIZATION

Composite laminate materials are typically characterized in terms of their response to mechanical or thermal loading, which is generally associated with a description of the coupling behavior, described by the **ABD** relation:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ & A_{22} & A_{26} \\ \text{Sym.} & & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ & B_{22} & B_{26} \\ \text{Sym.} & & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ & B_{22} & B_{26} \\ \text{Sym.} & & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ & D_{22} & D_{26} \\ \text{Sym.} & & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Couplings generally exist between:

- in-plane (extension or membrane) and out-of-plane (bending or flexure) actions, when $B_{ij} \neq 0$,
- shear and extension, when $A_{16} = A_{26} \neq 0$, and
- bending and twisting, when $D_{16} = D_{26} \neq 0$.

Balanced and symmetric stacking sequences, may or may not possess *Bending-Twisting* coupling, and are therefore characterized, respectively, by the designations:

$$\mathbf{A}_S\mathbf{B}_0\mathbf{D}_F \text{ or } \mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$$

signifying, in both cases, that the elements of the extensional stiffness matrix (**A**) are ***Simple*** or orthotropic in nature, i.e.:

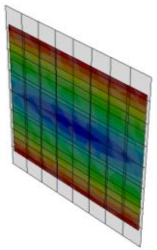
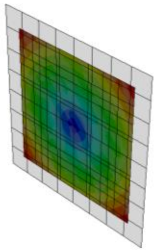
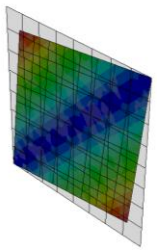
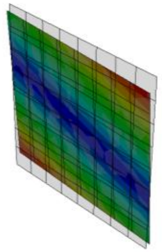
$$\begin{bmatrix} A_{11} & A_{12} & 0 \\ & A_{22} & 0 \\ \text{Sym.} & & A_{66} \end{bmatrix}$$

the (bending-extension) coupling matrix (**B**) is null, whilst all elements of the bending stiffness matrix (**D**) are either **Finite**, i.e.:

$$\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ & D_{22} & D_{26} \\ \text{Sym.} & & D_{66} \end{bmatrix}$$

or of the same *Simple* or orthotropic form as the extensional stiffness matrix.

Table 1 – Unrestrained thermal (contraction) response of square, initially flat, composite laminates. Stacking sequence configurations containing angle- and cross-ply sub-sequences are a representative sample from the minimum ply number grouping of each class of laminate. Note that cross-ply laminates with stacking sequences $[\bigcirc/\bigcirc]_T$ also represent the minimum ply number grouping for designation $\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$, but such configurations are not considered in this study.

Uncoupled in Extension ($\mathbf{A}_S$)		**Extension-Shearing ($\mathbf{A}_F$)**		
Uncoupled in Bending ($\mathbf{D}_S$)	**Bending-Twisting ($\mathbf{D}_F$)**	**Bending-Twisting ($\mathbf{D}_F$)**	**Uncoupled in Bending ($\mathbf{D}_S$)**	
$\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$ $[+/-_2/\bigcirc/+_2/-]_T$  *Simple* laminate	$\mathbf{A}_S\mathbf{B}_0\mathbf{D}_F$ $[+/-/-/+]_T$  <u>*B-T*</u>	$\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F$ $[+/+]_T$  <u>*E-S*</u>;<u>*B-T*</u>	$\mathbf{A}_F\mathbf{B}_0\mathbf{D}_S$ $[\pm/\bigcirc/-/\bigcirc/-_3/\bigcirc/-_3/\bigcirc/+]_T$  <u>*E-S*</u>	**Uncoupled ($\mathbf{B}_0$)**

ARRANGEMENT AND FORM OF STACKING SEQUENCE DATA

Stacking sequences are characterized by sub-sequence symmetries using a **double prefix** notation:

the **first character** relates to the form of the **angle-ply** sub-sequence and

the **second character** to the **cross-ply** sub-sequence.

The double prefix contains combinations of the following characters:

A to indicate **Anti-symmetric** form;

N for **Non-symmetric**; and

S for **Symmetric**.

Additionally, for cross-ply sub-sequence only,

C is used to indicate **Cross-symmetric** form.

To avoid the trivial solution of a stacking sequences with cross plies only, the first ply in every sequence is an angle-ply (+).

Fully uncoupled laminates have the following forms of sub-sequence symmetries:

Form (Number of stacking sequences):	Example stacking sequence:
AC (210)	[+/○/●/–/–/●/○/●/●/○/+/+/○/●/–]$_{T}$
AN (14,532)	[+/○/●/–/–/●/●/●/○/●/+/+/○/●/–]$_{T}$
AS (21,609)	[+/–/●/–/●/+/●/+/–]$_{T}$
${}_{+}\boldsymbol{NN}_{+}$ (5,498)	[+/–/–/○/+/○/–/+/+/+/–/–/–/○/+]$_{T}$
${}_{+}\boldsymbol{NN}_{-}$ (15,188)	[+/–/●/+/–/●/●/●/–/–/+/+/–/+/+/●/●/–]$_{T}$
${}_{+}\boldsymbol{NN}_{\circ}$ = ${}_{+}\boldsymbol{NN}_{\bullet}$ (10,041)	[+/–/●/○/○/●/–/+/○/–/●/+/+/●/–/○]$_{T}$
${}_{+}\boldsymbol{NS}_{+}$ (220)	[+/–/–/–/+/+/+/+/–/–/+/–/–/+/–/+]$_{T}$
${}_{+}\boldsymbol{NS}_{-}$ (296)	[+/●/–/+/–/–/–/+/+/–/+/+/–/–/+/+/●/–]$_{T}$
SC (12)	[+/–/○/–/●/+/●/●/○/○/+/○/–/●/–/+]$_{T}$
SN (192)	[+/–/●/○/–/●/●/+/●/+/○/○/–/●/●/–/+]$_{T}$
SS (1,029)	[+/–/–/●/+/●/●/+/●/–/–/+]$_{T}$

Design space (%) comparisons for each sub-symmetric grouping:

Fully uncoupled ($\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$) or *Simple* laminates

n	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**Σ%**
AC	-	-	-	-	-	-	-	-	-	-	-	6.7	3.3	4.6	-	2.7
AN	-	-	-	-	-	-	-	-	-	-	-	6.7	-	12.7	8.2	8.8
AS	-	-	-	100	100	100	100	100	84.0	100	86.4	80.0	74.4	54.7	58.3	61.8
NN	-	-	-	-	-	-	-	-	-	-	-	5.6	11.9	24.0	24.6	20.5
NS	-	-	-	-	-	-	-	-	-	-	-	-	0.6	0.7	1.5	0.9
SC	-	-	-	-	-	-	-	-	-	-	-	-	0.6	0.2	-	0.1
SN	-	-	-	-	-	-	-	-	-	-	-	-	-	0.4	0.5	0.4
SS	-	-	-	-	-	-	-	-	16.0	-	13.6	1.1	9.2	2.7	6.9	4.9
Σ*n*	**0**	**0**	**0**	**2**	**1**	**6**	**6**	**24**	**25**	**84**	**88**	**360**	**360**	**1,832**	**1,603**	**4,391**

Bending-Twisting coupled ($\mathbf{A}_S\mathbf{B}_0\mathbf{D}_F$) laminates

n	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**Σ%**
NC	-	-	-	-	-	-	-	-	-	-	-	-	-	0.1	-	0.1
NN	-	-	-	-	-	16.7	-	35.8	20.0	52.1	32.0	68.0	54.0	79.9	69.5	72.4
NS	-	-	-	-	-	-	-	7.5	6.2	10.8	11.2	10.5	11.8	8.0	10.3	9.4
SC	-	-	-	-	-	-	-	3.8	2.8	0.9	-	0.9	1.1	0.3	-	0.3
SN	-	-	-	-	-	-	-	-	-	4.8	4.8	4.3	4.0	3.8	4.7	4.2
SS	100	100	100	100	100	83.3	100	52.8	71.0	31.4	52.0	16.3	29.1	7.9	15.5	13.7
Σ*n*	**1**	**2**	**4**	**8**	**15**	**36**	**56**	**212**	**290**	**1,336**	**1,500**	**9,666**	**10,210**	**75,540**	**73,068**	**171,944**

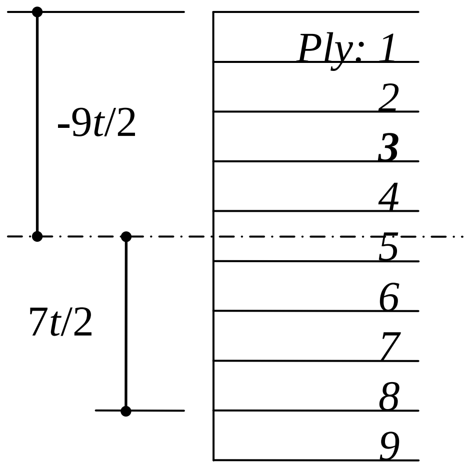
DEVELOPMENT OF NON-DIMENSIONAL PARAMETERS

The derivation of non-dimensional bending stiffness parameters is readily demonstrated for the 9-ply *NN* 5 laminate, with stacking sequence $[+/-/\bigcirc_2/+/-_2/\bigcirc/+]_T$, where the bending stiffness terms,

$$D_{ij} = \sum_{k=1}^{n} Q'_{ij}(z_k^{\,3} - z_{k-1}^{\,3})/3$$

may be written in sequence order as:

$$D_{ij} = \{Q'_{ij+}((-7t/2)^3 - (-9t/2)^3) + Q'_{ij-}((-5t/2)^3 - (-7t/2)^3) + Q'_{ij\bigcirc}((-3t/2)^3 - (-5t/2)^3) + Q'_{ij\bigcirc}((-t/2)^3 - (-3t/2)^3) + Q'_{ij+}((t/2)^3 - (-t/2)^3) + Q'_{ij-}((3t/2)^3 - (t/2)^3) + Q'_{ij-}((5t/2)^3 - (3t/2)^3) + Q'_{ij\bigcirc}((7t/2)^3 - (5t/2)^3) + Q'_{ij+}((9t/2)^3 - (7t/2)^3)\}/3$$



where subscripts i,j = 1, 2, 6.

The bending stiffness contributions from the different ply orientations are:

$$D_{ij+} = 96.75t^3/3 \times Q'_{ij+} = \zeta_+ t^3/12 \times Q'_{ij+} \qquad \zeta_+ = \mathbf{387}$$

$$D_{ij-} = 42.75t^3/3 \times Q'_{ij-} = \zeta_- t^3/12 \times Q'_{ij-} \qquad \zeta_- = \mathbf{171}$$

$$D_{ij\bigcirc} = 42.75t^3/3 \times Q'_{ij\bigcirc} = \zeta_\bigcirc t^3/12 \times Q'_{ij\bigcirc} \qquad \zeta_\bigcirc = \mathbf{171}$$

$$\zeta_+ + \zeta_- + \zeta_\bigcirc = n^3 = 729$$

CALCULATION OF THE LAMINATE BENDING STIFFNESS

Calculation of the bending (**D**) stiffness matrix, follows from the dimensionless parameters using:

$$D_{ij} = \{\zeta_+ Q'_{ij+} + \zeta_- Q'_{ij-} + \zeta_{\circ} Q'_{ij\circ} + \zeta_{\bullet} Q'_{ij\bullet}\} \times t^3/12 \qquad (1)$$

The transformed reduced stiffness terms in Eqs. (1) are given by:

$$Q'_{11} = Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\cos^2\theta\sin^2\theta + Q_{22}\sin^4\theta$$

$$Q'_{12} = Q'_{21} = (Q_{11} + Q_{22} - 4Q_{66})\cos^2\theta\sin^2\theta + Q_{12}(\cos^4\theta + \sin^4\theta)$$

$$Q'_{16} = Q'_{61} = \{(Q_{11} - Q_{12} - 2Q_{66})\cos^2\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin^2\theta\}\cos\theta\sin\theta$$

$$Q'_{22} = Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\cos^2\theta\sin^2\theta + Q_{22}\cos^4\theta$$

$$Q'_{26} = Q'_{62} = \{(Q_{11} - Q_{12} - 2Q_{66})\sin^2\theta + (Q_{12} - Q_{22} + 2Q_{66})\cos^2\theta\}\cos\theta\sin\theta$$

$$Q'_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\cos^2\theta\sin^2\theta + Q_{66}(\cos^4\theta + \sin^4\theta) \qquad (2)$$

and the reduced stiffness terms by:

$$Q_{11} = E_1/(1 - \nu_{12}\nu_{21}) \qquad Q_{12} = \nu_{12}E_2/(1 - \nu_{12}\nu_{21}) \qquad Q_{22} = E_2/(1 - \nu_{12}\nu_{21}) \qquad Q_{66} = G_{12}$$

$$(3)$$

LAMINATION PARAMETERS

For optimum design of laminates with angle- and cross-ply orientations, lamination parameters are often preferred, since these allow the stiffness terms to be expressed as linear design variables.

$$\xi_9 = \xi_1^D = \{\zeta_+\cos(2\theta_+) + \zeta_-\cos(2\theta_-) + \zeta_\circ\cos(2\theta_\circ) + \zeta_\bullet\cos(2\theta_\bullet)\}/n^3$$

$$\xi_{10} = \xi_2^D = \{\zeta_+\cos(4\theta_+) + \zeta_-\cos(4\theta_-) + \zeta_\circ\cos(4\theta_\circ) + \zeta_\bullet\cos(4\theta_\bullet)\}/n^3$$

$$\xi_{11} = \xi_3^D = \{\zeta_+\sin(2\theta_+) + \zeta_-\sin(2\theta_-) + \zeta_\circ\sin(2\theta_\circ) + \zeta_\bullet\sin(2\theta_\bullet)\}/n^3$$

$$\xi_{12} = \xi_4^D = \{\zeta_+\sin(4\theta_+) + \zeta_-\sin(4\theta_-) + \zeta_\circ\sin(4\theta_\circ) + \zeta_\bullet\sin(4\theta_\bullet)\}/n^3 \qquad (4)$$

Calculation of the bending (**D**) stiffness matrix, follows from the dimensionless parameters using:

$D_{11} = \{U_1 + \xi_9 U_2 + \xi_{10} U_3\} \times H^3/12$	$U_1 = \{3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}\}/8$
$D_{12} = D_{21} = \{U_4 - \xi_{10} U_3\} \times H^3/12$	$U_2 = \{Q_{11} - Q_{22}\}/2$
$D_{16} = D_{61} = \{\xi_{11} U_2/2 + \xi_{12} U_3\} \times H^3/12$	$U_3 = \{Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}\}/8$
$D_{22} = \{U_1 - \xi_9 U_2 + \xi_{10} U_3\} \times H^3/12$	$U_4 = \{Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}\}/8$
$D_{26} = D_{62} = \{\xi_{11} U_2/2 - \xi_{12} U_3\} \times H^3/12$	$U_5 = \{Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}\}/8$
$D_{66} = \{-\xi_{10} U_3 + U_5\} \times H^3/12$ (5)	(6)

H ($= n \times t$) is the laminate thickness.

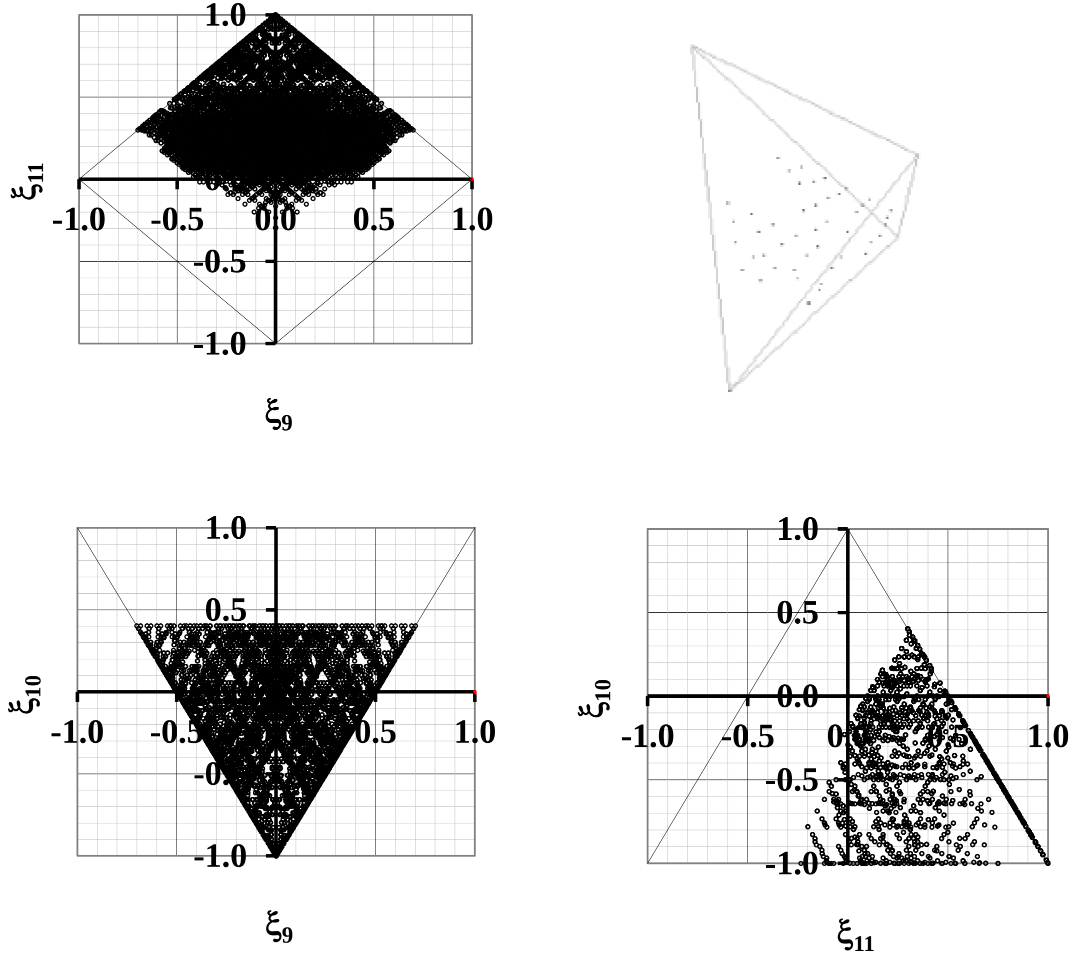


Figure 7 – Bending lamination parameter (ξ_9, ξ_{10}, ξ_{11}) design space for symmetric (***SS***) laminates with up to 18 plies, representing 23,470 configurations. Note that $\xi_{12} = 0$ for $\theta = \pm 45°$.

BUCKLING STRENGTH ASSESSMENTS

Numerous closed form solutions have been proposed in the literature.
The most recent being:

$$k_{x,\infty} = 2(1 - 4\delta\gamma - 3\delta^4 + 2\delta^2\beta)^{1/2} + 2(\beta - 3\delta^2)$$

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involving non-dimensional parameters, consisting of an orthotropic parameter, β, and two anisotropic parameters, γ and δ.

$$\beta = (D_{12} + 2D_{66})/(D_{11}D_{22})^{1/2}$$

$$\gamma = D_{16}/(D_{11}^{3}D_{22})^{1/4}$$

$$\delta = D_{26}/(D_{11}D_{22}^{3})^{1/4}$$

However, results for the fully uncoupled laminate ($\gamma = \delta = 0$) do not match the relative buckling load, $N_{x,\infty}$ (with half-wavelength λ), across the lamination parameter design space due to normalization with respect to $(D_{11}D_{22})^{1/2}$, i.e.:

$$k_{x,\infty} = N_x b^2/\pi^2(D_{11}D_{22})^{1/2} \text{ with } N_{x,\infty} = \pi^2\left[D_{11}\left[\frac{1}{\lambda}\right]^2 + 2(D_{12} + 2D_{66})\frac{1}{b^2} + D_{22}\left[\frac{1}{b^4}\right]\lambda^2\right]$$

An alternative assessment of the buckling strength between fully uncoupled (or *Simple*) laminates and *Bending-Twisting* coupled laminates is now considered. Here, a mapping to the lamination parameter design space is developed.

An 18-ply quasi-homogeneous laminate configuration ($\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$) was chosen to check for convergence of the buckling strength predictions of an exact infinite strip method, VICONOPT[1], since it represents the simplest form of laminate, i.e.:

$$D_{ij} = A_{ij}H^2/12 \qquad (7)$$

Additionally, this laminate becomes a fully isotropic (π/3) laminate when angle plies (+/–) are changed from +45/–45 to +60/–60. This is used as a datum case.

$[+/-/-/\bigcirc/\bigcirc/\bigcirc/+/+/\bigcirc/-/+/+/-/-/-/\bigcirc/\bigcirc/+]_T$ ($\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$), ($\mathbf{A}_I\mathbf{B}_0\mathbf{D}_I$)

Buckling factor results have been normalised against D_{Iso}, representing the equivalent isotopic composite bending stiffness, where

$$D_{Iso} = E_{Iso}H^3/(1 - \nu_{Iso}^2) = U_1H^3/12 \qquad (8)$$

$$E_{Iso} = 2(1 + \nu_{Iso})G_{Iso} = U_1(1 - \nu_{Iso}^2) \qquad (9)$$

$$\nu_{Iso} = U_4/U_1 \qquad (10)$$

[1] Williams FW, Kennedy D, Butler R, Anderson MS. VICONOPT: program for exact vibration and buckling analysis or design of prismatic plate assemblies. AIAA J 1991;29:1927–8.

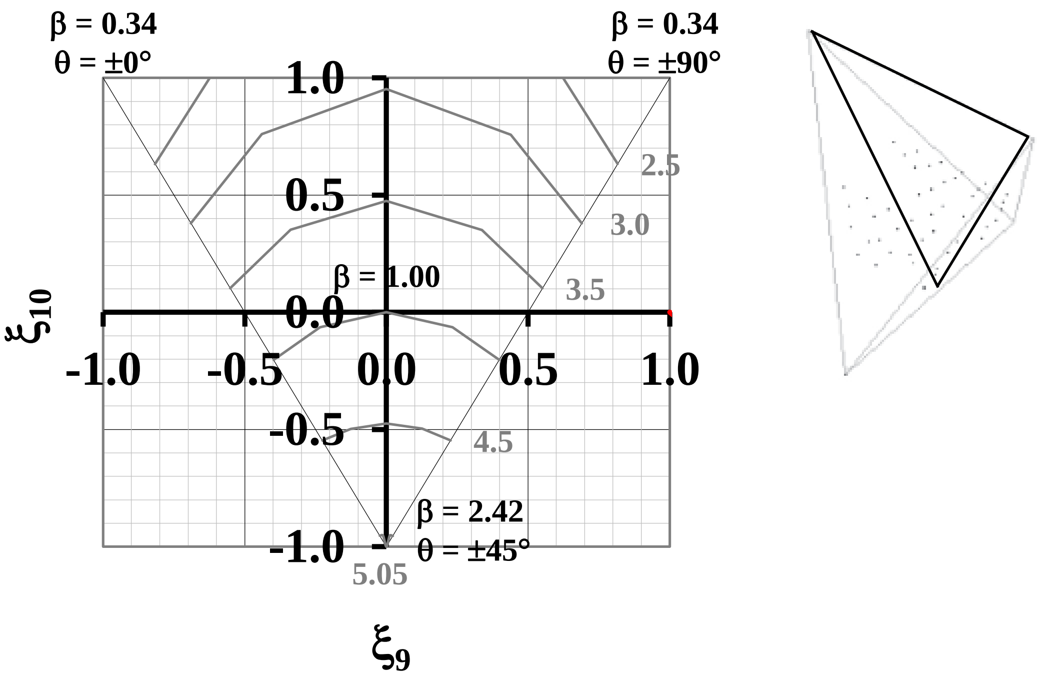


Figure 1 – Compression buckling contours, k_x (= $N_x b^2/\pi^2 D_{Iso}$), for fully uncoupled laminates, i.e. $\boldsymbol{\xi_{11} = 0}$.

The closed form buckling solution, representing an infinitely long, simply supported plate, and from which the contours are subsequently plotted, can be derived from 15 lamination parameter points, giving:

$$k_{x,\infty} = 4.000 - 1.049\xi_{10} - 1.217\xi_9^{\ 2} + 0.340\xi_{10}\xi_9^{\ 2} - 0.360\xi_9^{\ 4} - 0.340\xi_{10}^{\ 2}\xi_9^{\ 2}$$

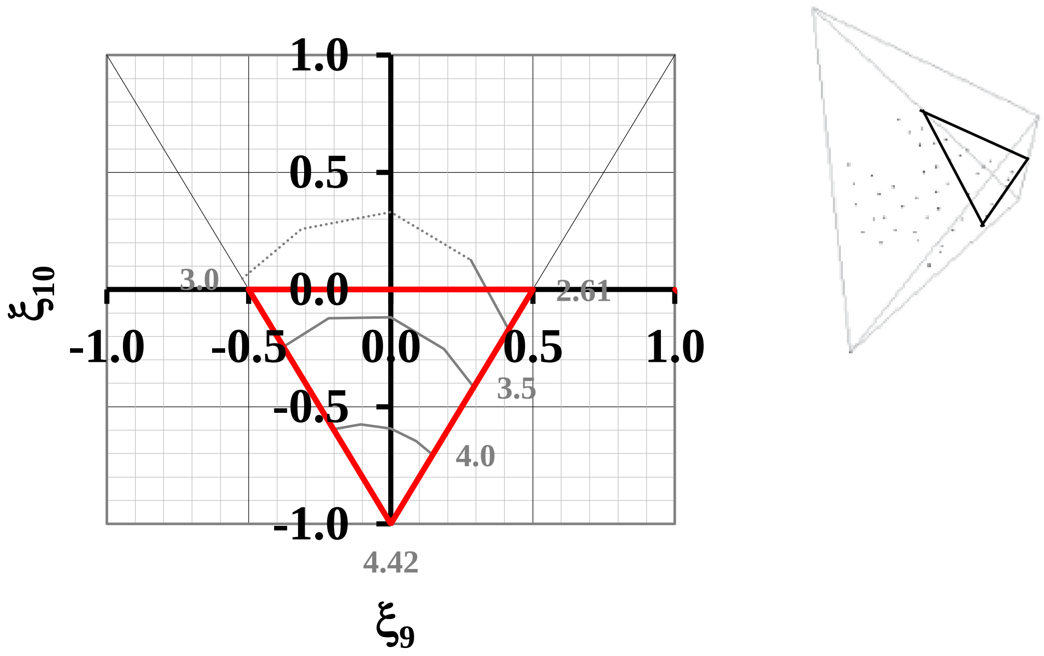
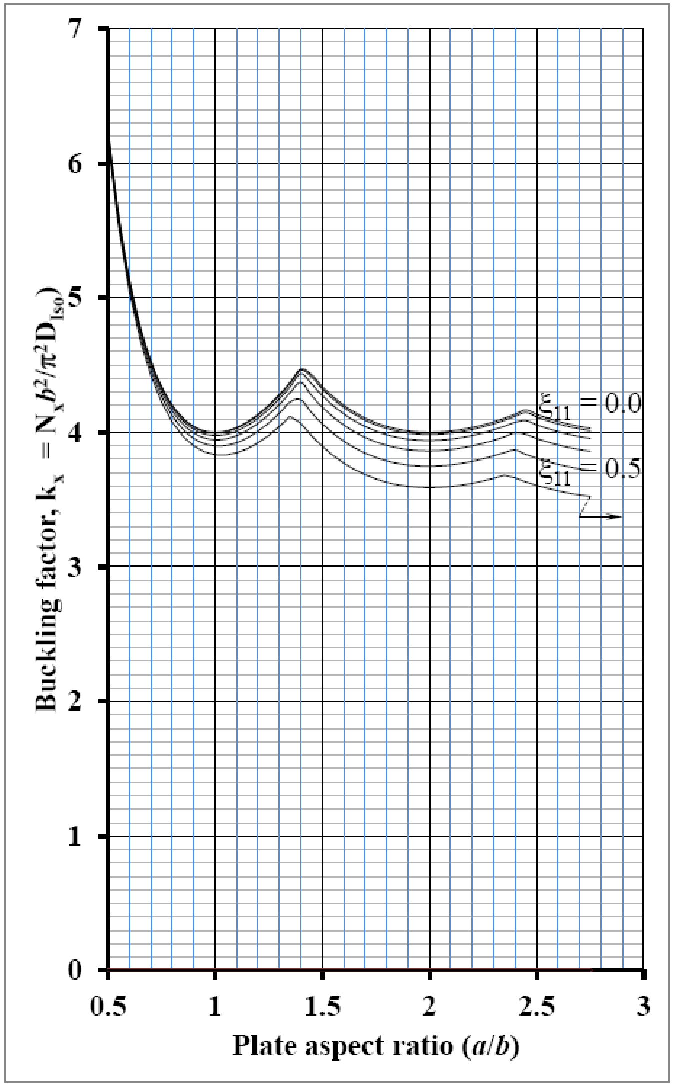


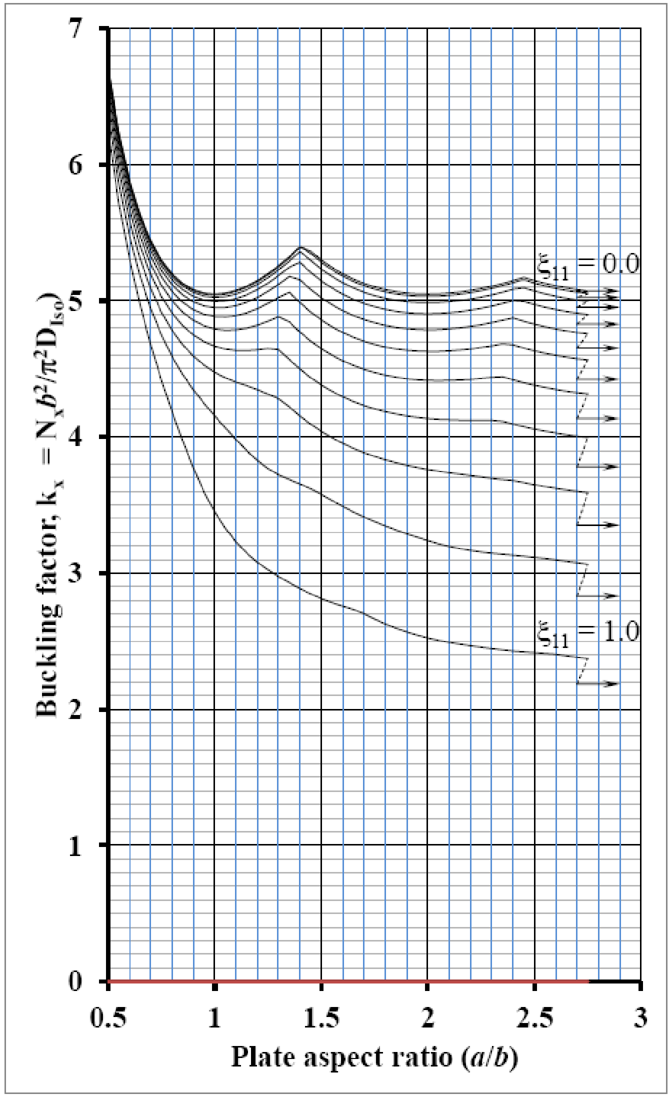
Figure 2 – Compression buckling factor contours, k_x (= $N_x b^2/\pi^2 D_{Iso}$), for $\boldsymbol{\xi_{11}}$ **= 0.5**, representing *Bending-Twisting* coupled laminates, demonstrating *increasing* asymmetry. When ξ_{11} = 1.0, the design space degenerates to a single point with k_x = 2.19.

For ξ_{11} = 0.5, the new closed form buckling solution for the infinitely long plate can be stated as:

$$k_{x,ortho,\infty} = 3.374 - 0.329\xi_9 - 1.06\xi_{10} - 1.742\xi_9^2 - 0.012\xi_{10}^2 + 0.145\xi_{10}\xi_9 - 0.598\xi_9^3 - 0.001\xi_{10}^3 - 0.014\xi_{10}^2\xi_9 + 0.671\xi_{10}\xi_9^2 - 1.456\xi_9^4 - 0.003\xi_{10}^3\xi_9 - 0.083\xi_{10}^2\xi_9^2 - 0.008\xi_{10}\xi_9^3$$



(a)



(b)

Figure 3 –Compression buckling factor curves for:

(a) quasi-homogeneous, quasi-isotropic laminates with $(\xi_9, \xi_{10}) = (0,0)$ and $0 \leq \xi_{11} \leq 0.5$ and;

(b) angle-ply laminates with $(\xi_9, \xi_{10}) = (0,-1)$ and $0.0 \leq \xi_{11} \leq 1.0$.

Asymptotes represent $k_{x,\infty}$ for the infinitely long plate, and reveal bounds on buckling strength reductions of 16% for the quasi-homogeneous, quasi-isotropic laminates and 57% for angle-ply laminates.

Results for isotropic skew plates – mode shape analogy.

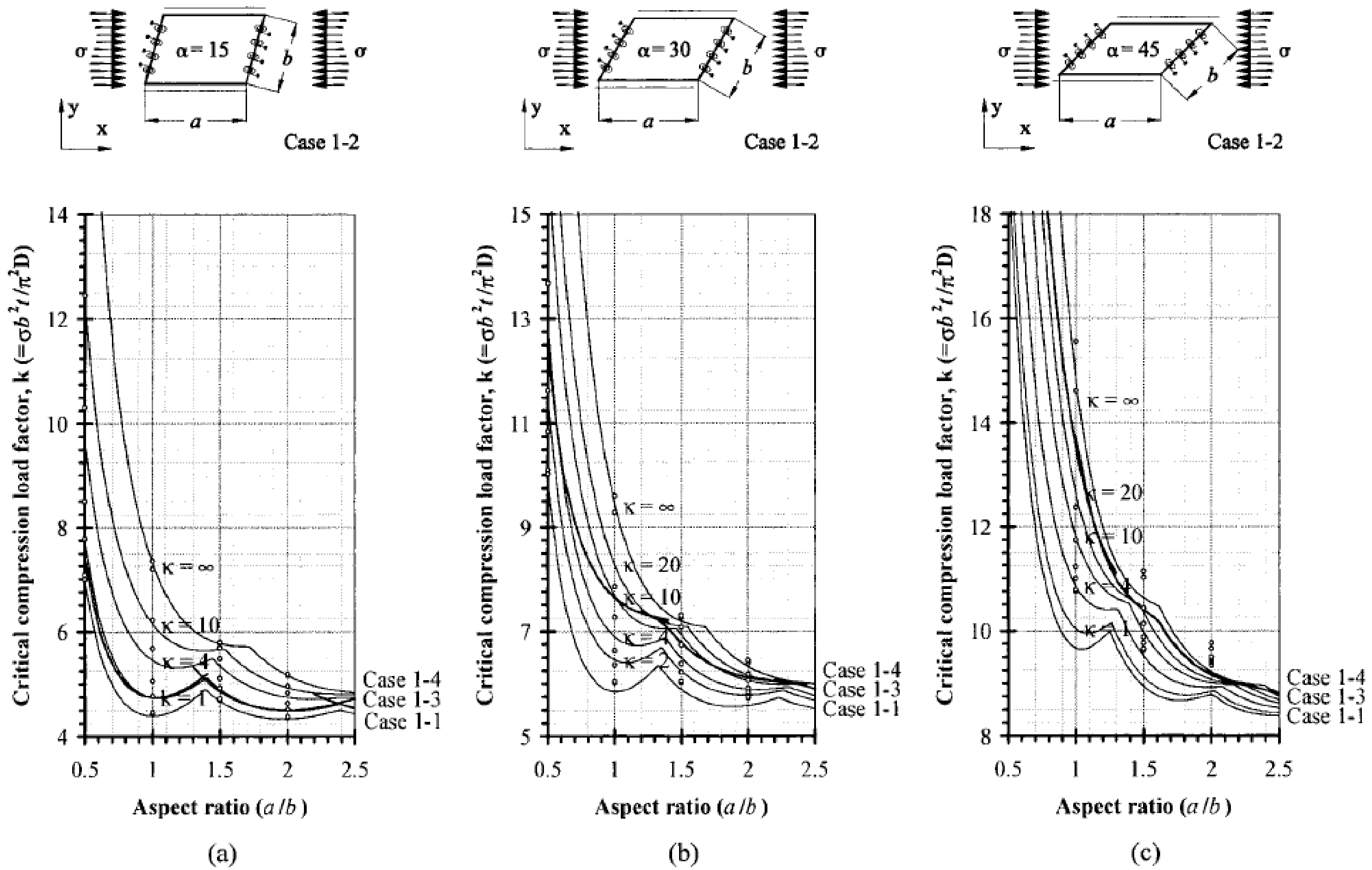


FIG. 6. Case 1-2 Design Curves for α =: (a) 15°; (b) 30°; (c) 45° with Skew-Transverse Edges Elastically Restrained against Rotation and Longitudinal Edges Simply Supported. Discrete Results [Mizusawa and Kajita (1986)] shown (κ = 0, 0.1, 1, 2, 5, 10, 100 and ∞) for Comparison

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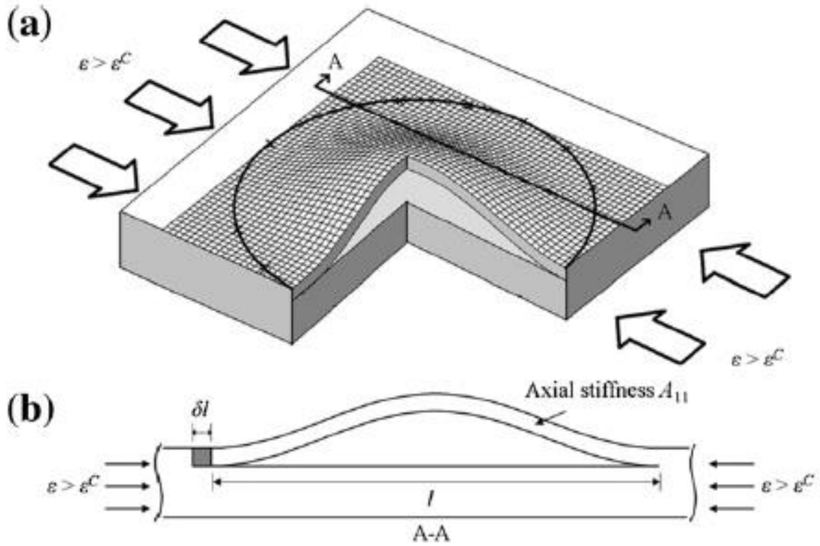
CONSIDERATIONS FOR FML

Initial buckling strength.

Material properties of FML are essentially isotropic.
$[Al/\underline{+45/-45}/Al/\underline{-45/+45}/Al]_T$ gives the highest magnitude of *Bending-Twisting* coupling, but the buckling curves have no diminishing cusps, as seen previously in the Uni-Directional (UD) CFRP designs.
The buckling strength of this design increased in comparison to equivalent design with 'isotropic' fibre layers, since $\xi_{10} < 0$; see trends in buckling factor contours in Figs. (1) and (2).

Delamination buckling

Favourable designs were found to be dominated by anti-symmetric UD laminate designs.
The use of woven cloth or Non-Crimp Fabric (NCF) designs using thin ply technologies may improve isotropic characteristics and potentially improve damage tolerance.



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Woven Cloth Mateial.

Application to FML using Aluminium and Boron- or Carbon-Fibre epoxy woven cloth (TeXtreme™) layers, involving thin ply laminate technology with areal weights of 50g/m^2, compared to standard materials with 250g/m^2, allows the possibility of designing isotropic layers, e.g.

$$[\alpha/\beta/\alpha/\beta_3/\alpha_3/\beta/\alpha/\beta]_T$$

with $\alpha = \beta + \pi/4$, possessing similar moduli to Aluminium, and within the thickness constraints found in standard FML, such as Glare, e.g.:

$$[\mathrm{Al.}/\alpha/\beta/\alpha/\beta_3/\alpha_3/\beta/\alpha/\beta/\mathrm{Al.}/\alpha/\beta/\alpha/\beta_3/\alpha_3/\beta/\alpha/\beta/\mathrm{Al.}]_T.$$

Titanium may be required to avoid galvanic corrosion between Aluminium and Carbon-Fibre material.

Non-Crimp Fabric (C-Ply)

The four design freedoms associated with the stacking sequences for standard UD laminate manufacture, with ply orientations 0, 90 and ±45°, are increased to eight using 0/45 and 0/-45 bi-angle NCF: by flipping (-45/0 and 45/0), rotating (90/-45 and 90/45) or both (45/90 and -45/90). The underlining helps to differentiate between 0/45 and 0/-45 plies.
A comparable isotropic sub-laminate to the TeXtreme design is given by:

[135/90/0/45/0/45/90/45/-45/0/135/90/135/90/45/90/0/-45/0/45/0/45/135/90]$_T$

Addendum: Matching of Stiffness and Thermal properties in FML

Table 2 - Engineering constants, thermal expansion coefficients and specific gravity of typical unidirectional composites together with equivalent Isotropic laminate properties.

Type [Material]	E_1 (E_{Iso}) (GPa)	E_2 (E_{Iso}) (GPa)	ν_{12} (ν_{Iso})	G_{12} (G_{Iso}) (GPa)	α_1, α_2 (α_{Iso}) (μm/m)/K	ρ (g/cm^3)
T300/5208	181	10.3	0.28	7.17	0.02, 22.5	1.6
[Graphite/Epoxy]	(69.7)	(69.7)	(0.30)	(26.9)	(11.3)	
B(4)/5505	204	18.5	0.23	5.59	6.1, 30.3	2.0
[Boron/Epoxy]	(78.5)	(78.5)	(0.32)	(29.7)	(18.2)	
AS/3501	138	8.96	0.30	7.1	-0.3, 28.1	1.6
[Graphite/Epoxy]	(54.8)	(54.8)	(0.28)	(21.4)	(13.9)	
Scotchply 1002	38.6	8.27	0.26	4.14	8.6, 22.1	1.8
[Glass/Epoxy]	(18.97)	(18.97)	(0.27)	(7.47)	(15.3)	
Kevlar 49/Epoxy	76	5.5	0.34	2.3	-4.0, 79.0	1.46
[Aramid/Epoxy]	(29)	(29)	(0.32)	(10.95)	(37.5)	
Aluminium 2014-T4	73		0.33	28	23.0	2.7
Titanium	114		0.33	43	9.5	4.4

CONCLUSIONS

- Definitive listings of *Bending-Twisting* coupled laminates demonstrate that the vast majority of the stacking sequences are non-symmetric.
- Symmetric laminates with up to 18 plies occupy less than 7% of the total design space for *Bending-Twisting* coupled laminates.
- Interrogation of these feasible design spaces has facilitated the calculation of bounds on the buckling strength of infinitely long simply supported plates.
- Buckling strength comparisons for infinitely long laminated plates have revealed bounds on buckling strength reductions between fully uncoupled and *Bending-Twisting* coupled laminates of 16% for the quasi-homogeneous, quasi-isotropic laminates and 57% for angle-ply laminates.
- FML designs possess buckling behaviour consistent with the equivalent isotropic laminate, despite the presence of *Bending-Twisting* coupling in the fibre reinforcement.
    - The potential for improvements in delamination buckling strength (i.e., compression strength after impact) remains to be explored….