ON MECHANICALLY COUPLED TAPERED LAMINATES WITH BALANCED PLAIN WEAVE AND NON-CRIMP FABRICS.

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Background

A recent study on single-ply terminations[1] investigated the extent of the design space for tapered laminates when the ubiquitous balanced and/or symmetric design rules are relaxed.

Consideration of design heuristics, involving ply contiguity (≤ 2), representing the most severe design constraints for tapered laminates, were found to significantly influenced the form of the designs.

Several key observations were concluded from the study:

- Tapered laminate solutions arise in non-symmetric, as well as symmetric laminates, with consistent mechanical behaviour and immunity to thermal warping

- Single cross-ply terminations are permissible in *Simple* or *Bending-Twisting* coupled laminates, but only within a mid-plane ply block, which can be non-symmetric.

- Single angle-ply terminations are permissible only in laminate configurations possessing *Extension-Shearing* and *Bending-Twisting* coupling.

- Consistent mechanical *Extension-Shearing* and *Bending-Twisting* coupling can be preserved by modifying a mid-plane symmetric ply block, within an otherwise non-symmetric laminate.

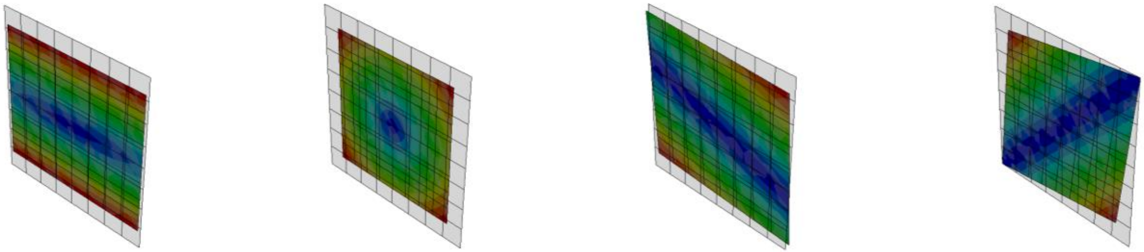
[1] York, C.B. (2015). On tapered wap-free laminates with single-ply terminations, *Composites Part A – Applied Science and Manufacturing*, **72**, 127-138.

Similar tailoring strategies are now applied to balanced plain weave (BPW) and bi-angle non-crimp fabric (NCF) laminate designs.

The strategy focuses on **single-ply** or, where necessary, **multiple-ply terminations**, <u>to investigate the extent to which individual layers can be terminated without introducing *unwanted mechanical* coupling, or to maintain *Extension-Shearing* and/or *Bending-Twisting* throughout the tapered laminate</u>.

The results presented here are based on the four laminate classes, illustrated under free thermal contractions in Fig. 1.

Uncoupled in Extension		***Extension-Shearing***	
Uncoupled in Bending	***Bending-Twisting***	**Uncoupled in Bending**	***Bending-Twisting***



[+/●/○/–/–/○/●/+]$_A$	[+/●/–/○/○/–/●/+]$_T$	[No NCF Solutions]	[+/●/+/○/○/+/●/+]$_T$
Simple laminate	***<u>B-T</u> coupled laminate***	***<u>E-S</u> coupled laminate***	***<u>E-S</u>;<u>B-T</u> coupled laminate***

Figure 1 – In-plane thermal contraction responses (not to scale) resulting from a typical high temperature curing process. All examples shown are square, initially flat, NCF composite laminates.

Derivation of Stacking Sequences

The common feature relating the ***Simple***, ***Extension-Shearing*** and/or ***Bending-Twisting*** coupled laminate classes is that all are decoupled, i.e. $\mathbf{B}_{\mathrm{ij}} = 0$.

The constitutive relations therefore simplify as follows:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} \mathrm{A}_{11} & \mathrm{A}_{12} & \mathrm{A}_{16} \\ & \mathrm{A}_{22} & \mathrm{A}_{26} \\ \mathrm{Sym.} & & \mathrm{A}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} \mathrm{D}_{11} & \mathrm{D}_{12} & \mathrm{D}_{16} \\ & \mathrm{D}_{22} & \mathrm{D}_{26} \\ \mathrm{Sym.} & & \mathrm{D}_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

where the elements of the stiffness matrices are derived from the well-known relationships:

$$\mathrm{A}_{\mathrm{ij}} = \Sigma \mathrm{Q}'_{\mathrm{ij}}(z_k - z_{k\text{-}1}) \qquad \mathrm{B}_{\mathrm{ij}} = \Sigma \mathrm{Q}'_{\mathrm{ij}}(z_k^2 - z_{k\text{-}1}^2)/2 = 0 \qquad \mathrm{D}_{\mathrm{ij}} = \Sigma \mathrm{Q}'_{\mathrm{ij}}(z_k^3 - z_{k\text{-}1}^3)/3$$

In the derivation of the definitive listings of stacking sequences, which assumes (but is not restricted to) combinations of standard fibre angle orientations, i.e. 0, 90 and/or ±θ° (= ±45°), the <u>general rule of symmetry is relaxed</u>.

The derivation of the bi-angle **NCF laminates** involves the added restrictions that: each layer in the two-ply pairing has identical orthotropic material properties and; identical thicknesses, *t*.

The four design freedoms associated with the stacking sequences for standard UD laminate manufacture, with ply orientations 0, 90 and ± 45°, are now increased to eight using 0/45 and 0/-45 bi-angle NCF by:

- Flipping (-45/0 and 45/0), or
- Rotating (90/-45 and 90/45) or
- Both (45/90 and -45/90)

For **BPW material**, only two pairing constraints need to be imposed, i.e., 45/-45 and 0/90, since -45/45 and 90/0 give identical stiffness properties, respectively.

For compatibility with the previously published data, similar symbols have been adopted for defining all stacking sequences, i.e., ○, ●, + and – are used in place of standard ply angle orientations 0, 90, +45 and –45°, respectively.

Non-dimensional parameters

The development of non-dimensional parameters involves the summations of $(z_k - z_{k-1})$, $(z_k^2 - z_{k-1}^2)$ and $(z_k^3 - z_{k-1}^3)$, relating to the **A**, **B** and **D** matrices, respectively, for each ply orientation.

Together with the transformed reduced stiffness, Q'_{ij}, for each ply orientation of constant thickness, *t*, facilitate simple calculation of the elements of the extensional, coupling and bending stiffness:

$$A_{ij} = \{n_+Q'_{ij+} + n_-Q'_{ij-} + n_{\circ}Q'_{ij\circ} + n_{\bullet}Q'_{ij\bullet}\}t$$
$$B_{ij} = 0$$
$$D_{ij} = \{\zeta_+Q'_{ij+} + \zeta_-Q'_{ij-} + \zeta_{\circ}Q'_{ij\circ} + \zeta_{\bullet}Q'_{ij\bullet}\}t^3/12$$

The Extension-Shearing and Bending-Twisting coupled ($\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F$) laminate satisfies the following non-dimensional parameter criteria:

$$n_+ \neq n_-$$
$$\zeta_+ \neq \zeta_-$$

whilst $n_+ = n_-$ and/or $\zeta_+ = \zeta_-$ are the conditions giving rise to the *Bending-Twisting* coupling and *Simple* or *Extension-Shearing* coupled laminate classes in Fig. 1, respectively.

Definitive listing results

The definitive listings of Balanced Plain Weave (BPW) laminates are summarized in Table 1; revealing that tapering can potentially be achieved with single ply terminations.

Table 1: Number of BPW laminate solutions for *Simple* ($A_SB_0D_S$), *Bending-Twisting* coupled ($A_SB_0D_F$) and *Extension-Shearing* and *Bending-Twisting* coupled ($A_FB_0D_F$) warp-free laminate classes corresponding to, n_{BPW}, with BTW layers and equivalent number, n_{UD}, with UD layers.

(1) $n_{BPW}(n_{UD})$	(2) $A_SB_0D_S$	(3) $A_IB_0D_F$	(4) $A_FB_0D_F$
4(8)	2	1	1
5(10)	4	–	4
6(12)	4	–	4
7(14)	10	–	10
8(16)	9	2 (+ 1 $A_IB_0D_I$)	6
9(18)	26	–	26
10(20)	24	–	24
11(22)	76	–	76
12(24)	69	27 (+ 1 $A_IB_0D_I$)	41

The coupled laminate classes, presented in columns (3) and (4) of Table 1, are achieved by off-axis alignment of the *Simple* laminate designs.

The definitive listings for bi-angle Non Crimp Fabric (NCF) laminates are summarized in Table 2; revealing that at least two NCF ply terminations will be required for laminates with *Extension-Shearing Bending-Twisting* coupling and four NCF ply terminations for *Simple* and *Bending-Twisting* coupled laminates.

Table 2: Number of NCF laminate solutions for *Simple* ($A_SB_0D_S$), *Bending-Twisting* coupled ($A_SB_0D_F$) and *Extension-Shearing* and *Bending-Twisting* coupled ($A_FB_0D_F$) warp-free laminate classes corresponding to, n_{NCF}, with bi-angle NCF layers and equivalent number, n_{UD}, with UD layers.

(1) $n_{NCF}(n_{UD})$	(2) $A_SB_0D_S$	(3) $A_SB_0D_F$	(4) $A_FB_0D_F$
4(8)	–	4	5
5(10)	–	–	–
6(12)	–	–	88
7(14)	–	–	–
8(16)	35	419	683
9(18)	–	–	–
10(20)	–	–	22,568
11(22)	–	–	–
12(24)	2,413 (+ 1 $A_IB_0D_I$)	243,498	356,242

Lamination parameters permit an interrogation of the extent of resulting design space for these definitive listings, e.g., for *Extension-Shearing* and *Bending-Twisting* coupled laminates…

Lamination parameters

Lamination parameter offer an alternative set of non-dimensional expressions when ply angles are a design constraint.

These ply orientation dependent lamination parameters are related to the non-dimensional parameters, used in the development of the definitive listings, by the following expressions:

$$\xi_1 = \{n_+\cos(2\theta_+) + n_-\cos(2\theta_-) + n_\circ\cos(2\theta_\circ) + n_\bullet\cos(2\theta_\bullet)\}/n$$
$$\xi_2 = \{n_+\cos(4\theta_+) + n_-\cos(4\theta_-) + n_\circ\cos(4\theta_\circ) + n_\bullet\cos(4\theta_\bullet)\}/n$$
$$\xi_3 = \{n_+\sin(2\theta_+) + n_-\sin(2\theta_-) + n_\circ\sin(2\theta_\circ) + n_\bullet\sin(2\theta_\bullet)\}/n$$

$$\xi_9 = \{\zeta_+\cos(2\theta_+) + \zeta_-\cos(2\theta_-) + \zeta_\circ\cos(2\theta_\circ) + \zeta_\bullet\cos(2\theta_\bullet)\}/n^3$$
$$\xi_{10} = \{\zeta_+\cos(4\theta_+) + \zeta_-\cos(4\theta_-) + \zeta_\circ\cos(4\theta_\circ) + \zeta_\bullet\cos(4\theta_\bullet)\}/n^3$$
$$\xi_{11} = \{\zeta_+\sin(2\theta_+) + \zeta_-\sin(2\theta_-) + \zeta_\circ\sin(2\theta_\circ) + \zeta_\bullet\sin(2\theta_\bullet)\}/n^3$$

Noting that for standard ply orientations 0, 90 and ±45°, assumed here, $\xi_4 = \xi_{12} = 0$.

Elements of the extensional (A) and bending (D) stiffness matrices are each related to lamination parameters, ξ_i, and laminate invariants, U_i, respectively, by:

$$A_{11} = \{U_1 + \xi_1 U_2 + \xi_2 U_3\} \times H \qquad D_{11} = \{U_1 + \xi_9 U_2 + \xi_{10} U_3\} \times H^3/12$$
$$A_{12} = \{-\xi_2 U_3 + U_4\} \times H \qquad D_{12} = \{U_4 - \xi_{10} U_3\} \times H^3/12$$
$$A_{16} = \{\xi_3 U_2/2 + \xi_4 U_3\} \times H \qquad D_{16} = \{\xi_{11} U_2/2 + \xi_{12} U_3\} \times H^3/12$$
$$A_{22} = \{U_1 - \xi_1 U_2 + \xi_2 U_3\} \times H \qquad D_{22} = \{U_1 - \xi_9 U_2 + \xi_{10} U_3\} \times H^3/12$$
$$A_{26} = \{\xi_3 U_2/2 - \xi_4 U_3\} \times H \qquad D_{26} = \{\xi_{11} U_2/2 - \xi_{12} U_3\} \times H^3/12$$
$$A_{66} = \{-\xi_2 U_3 + U_5\} \times H \qquad D_{66} = \{-\xi_{10} U_3 + U_5\} \times H^3/12$$

where the laminate thickness H (= number of plies, n, × constant ply thickness, t).

The laminate invariants, U_i, are calculated from the reduced stiffness terms, Q_{ij}:

$$U_1 = \{3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}\}/8 \qquad Q_{11} = E_1/(1 - \nu_{12}\nu_{21})$$
$$U_2 = \{Q_{11} - Q_{22}\}/2 \qquad Q_{12} = \nu_{12}E_2/(1 - \nu_{12}\nu_{21})$$
$$U_3 = \{Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}\}/8 \qquad Q_{22} = E_2/(1 - \nu_{12}\nu_{21})$$
$$U_4 = \{Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}\}/8 \qquad Q_{66} = G_{12}$$
$$U_5 = \{Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}\}/8$$

There are further simplifications for BPW material i.e., $E_1 = E_2$, hence, $U_2 = 0$.

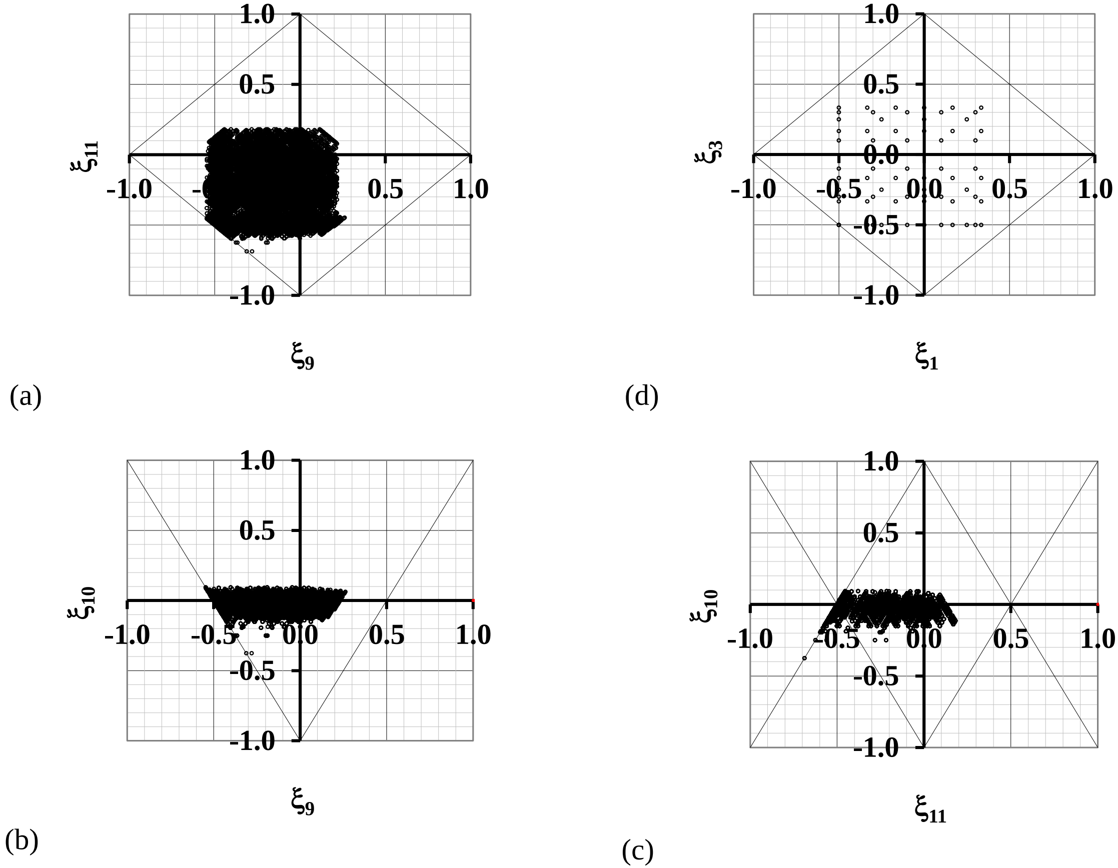


Figure 2 – Lamination parameter design spaces for $\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F$ bi-angle NCF laminates with up to n_{NCF} = 12 (n_{UD} = 24), for: (a) – (c) bending stiffness and; (d) extensional stiffness (ξ_2 = 0).

Laminate Tapering Algorithm

Tapered laminates are certified for symmetric laminate construction, but the vast majority possess *Bending-Twisting* coupling.

Such designs have a severe design constraint, i.e., a single angle-ply termination requires a further three angle-ply terminations to maintain balanced and symmetric construction.

This work investigates the extent to which this restriction can be overcome by employing NCF or BPW material without changing the mechanical behaviour or introducing undesirable warping distortions.

Tapered laminate designs have been developed in a two stage process…

The first stage of the termination scheme involves:

- *m* ply terminations, applied in turn to specific ply combinations in every stacking sequence with *n*-ply layers of either NCF or balanced plain weave material;
- comparison with all stacking sequences with *n*-*m* plies and;
- recording exact matches.

Notes:

- The first (or upper surface) ply is assumed to be continuous throughout the tapering process; this represents a practical design constraint to prevent surface ply delamination.
- This constraint also applies to the last (or lower surface) ply for practical design, but results have been also reported in the paper to reveal the propensity for such terminations.
- Repeated sequences are removed from the data when multiple matches arise from different ply terminations within a single stacking sequence.

The first stage serves to produce a reduced design space and forms a starting point for the second stage of the tapering algorithm.

The second stage of the tapering algorithm can be described as a bottom up process.

- It begins with compatible stacking sequences representing the minimum ply number grouping (*n*) of interest.
- These sequences are then algorithmically filtered through higher ply number groupings, in turn, but now only sequences compatible with the minimum ply number grouping are retained.
- This procedure ensures that the solutions will be compatible with higher ply number groupings beyond those reported here.

The results, following this second stage process for are given in a tabulated addendum and Table 1 from the published article….

Table (Addendum) – First stage, Top down termination algorithm results for *Simple* ($A_SB_0D_S$) *tapered* laminate designs corresponding to number, n_{BPW}, of BPW layers.

n_{BPW}	No. of Stacking sequences		Compatible tapered sequences (n_{BPW} + 1)	
	Symmetric	Non-symmetric	Symmetric	Non-symmetric
12	32	37	32 (↘32)	28 (↘37)
11	32	44	32 (↘32)	16 (↘44)
10	16	8	16 (↘16)	6 (↘8)
9	16	10	16 (↘16)	2 (↘10)
8	8	1	8 (↘8)	1 (↘1)
7	8	2	8 (↘8)	– (↘2)
6	4	–	4 (↘4)	–
5	4	–	4 (↘4)	–
4	2	–	2 (↘2)	–

Table 1 – Second stage, Bottom up termination algorithm results for *tapered* bi-angle NCF laminate solutions for *Simple* ($A_SB_0D_S$), *Bending-Twisting* coupled ($A_SB_0D_F$) and *Extension-Shearing* and *Bending-Twisting* coupled ($A_FB_0D_F$) warp-free laminate classes corresponding to number, n_{NCF}, of NCF layers and equivalent number, n_{UD}, of UD layers.

(1) $n_{NCF}(n_{UD})$	(2) $A_SB_0D_S$	(3) $A_SB_0D_F$	(4) $A_FB_0D_F$
4(8)	–	4	5
5(10)	–	–	–
6(12)	–	–	68
7(14)	–	–	–
8(16)	35	370	503
9(18)	–	–	–
10(20)	–	–	6,732
11(22)	–	–	–
12(24)	309	97,103	58,058

Lamination parameters once again permit interrogation of the extent of the resulting design space …

…. for the tapered NCF laminate designs of column (4) with *Extension-Shearing Bending Twisting* coupled laminates …

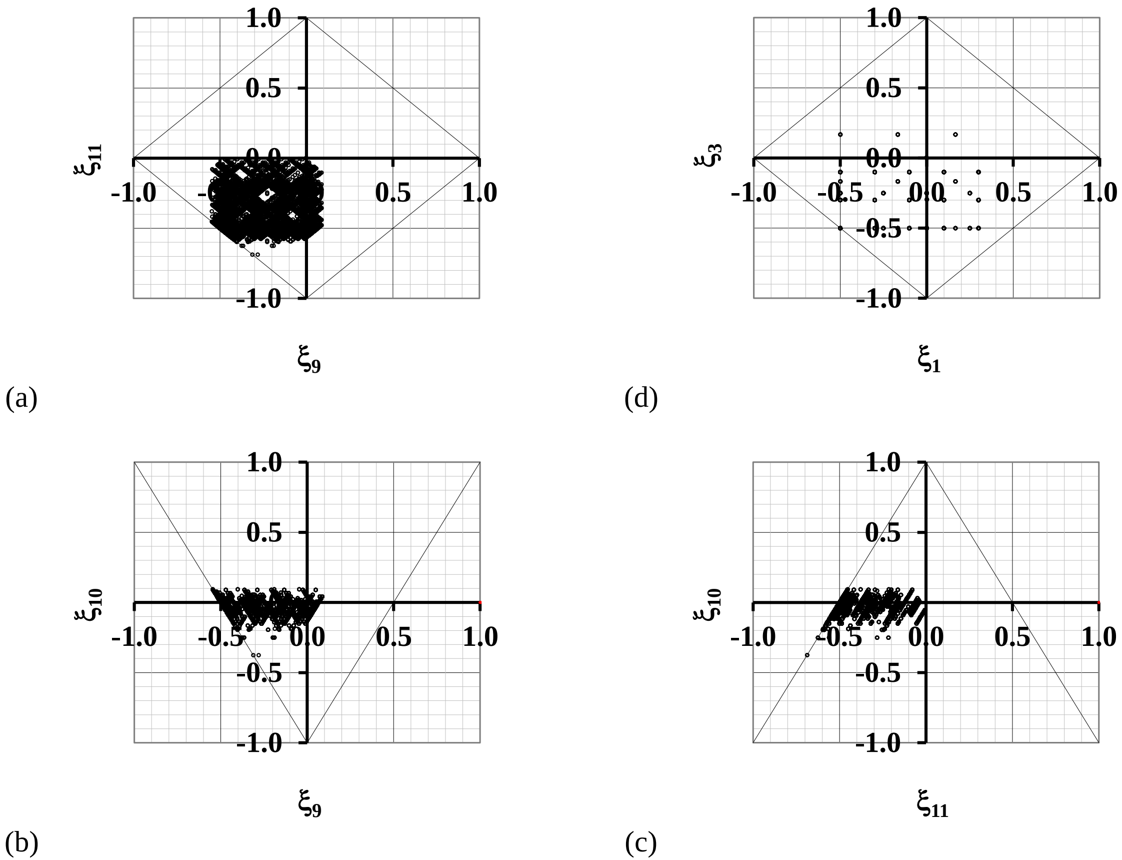
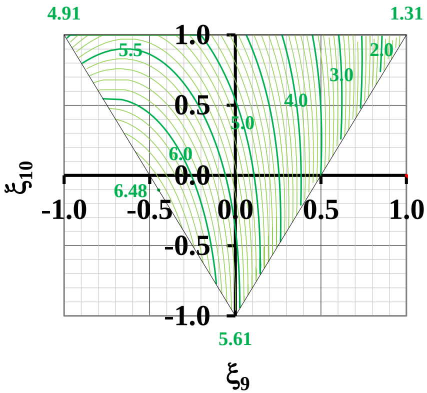
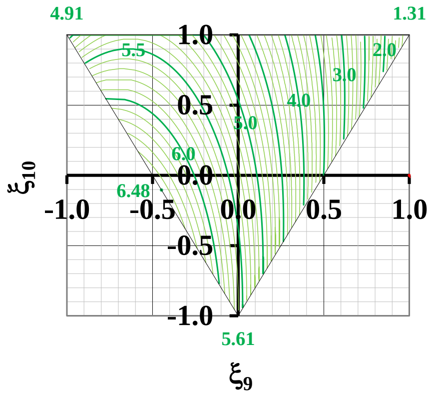


Figure 3 – Lamination parameter design spaces for *tapered* $\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F$ bi-angle NCF laminates with up to n_{NCF} = 12 (n_{UD} = 24), for: (a) – (c) bending stiffness and; (d) extensional stiffness.

(Addendum) Interpretation of lamination parameter design spaces: cross section at $\xi_{11} = 0$



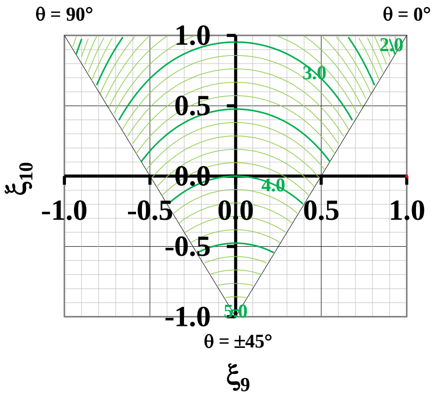
(a) $+k_{xy,\infty}$



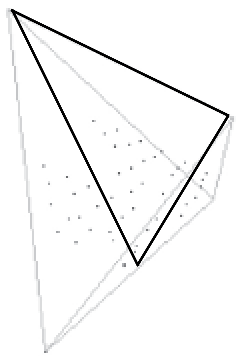
(b) $-k_{xy,\infty}$

Buckling factor mapping for infinitely long simply supported plates.

Compression buckling contours, $k_{x,\infty}$ ($k_{xy,\infty}$) = 4.00 (5.34), for fully isotropic laminates, i.e. $\xi_9 = \xi_{10} = \xi_{11} = 0$

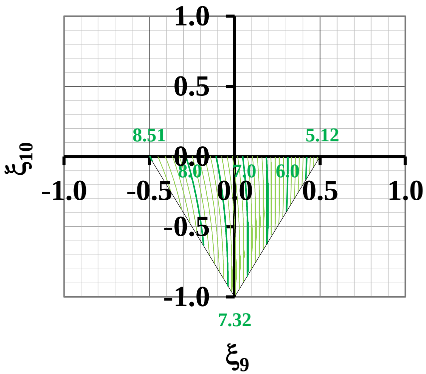


(c) $k_{x,\infty}$

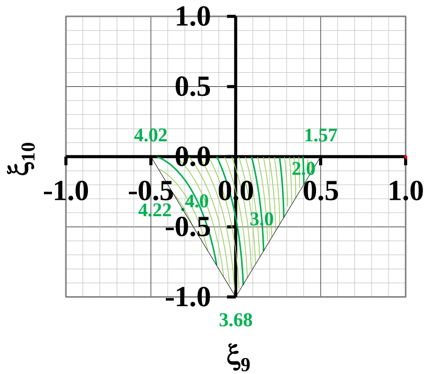


$\xi_{11} = 0$

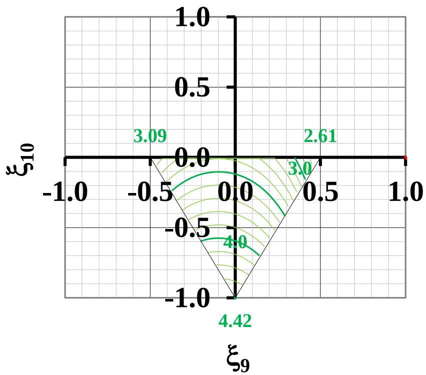
(Addendum) Interpretation of lamination parameter design spaces: cross section at ξ_{11} = 0.5



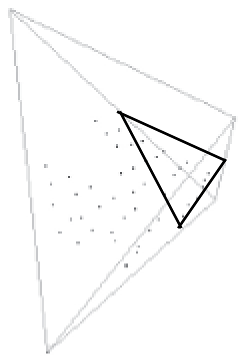
(a) $+k_{xy,\infty}$



(b) $-k_{xy,\infty}$



(c) $k_{x,\infty}$



ξ_{11} = 0.5

Examples of *Simple* tapered BPW laminate designs with *single*-ply terminations:

Non-symmetric design

+/○/○/+/○/+/○/+/○/+/○/○/+/+/○
+/○/○/+/○/+/○/○/+/○/○/+/+/○
+/○/○/+/○/+/○/+/○/○/+/+/○
+/○/○/+/○/+/+/○/○/+/+/○
+/○/○/+/○/+/○/○/+/+/○

Symmetric design:

+/+/○/+/+/○/+/+
+/+/○/+/○/+/+
+/+/○/○/+/+
+/+/○/+/+
+/+/+/+

These demonstrate a similar termination pattern to that found for single ply terminations in UD material[2], i.e., by the **termination of consecutive pairs of cross ply or angle ply orientations at the laminate mid-plane** to maintain symmetry within the central ply block.

[2] C.B. York, On tapered warp-free laminates with single-ply terminations, *Composites Part A - Applied Science and Manufacturing*, **72,** 2015, pp. 127-138.

By contrast, this example of an *Extension-Shearing* and *Bending-Twisting* coupled tapered bi-angle NCF stacking sequence design with *two*-ply (highlighted) terminations (n_{NCF} = 12 – 4) demonstrates symmetric terminations throughout a non-symmetric laminate.

+/○/○/+/+/●/○/–/○/–/○/+/○/+/–/○/–/○/●/+/+/○/+/○
+/○/○/+/+/●/○/–/○/+/○/+/–/○/●/+/+/○/+/○
+/○/○/+/+/●/○/+/○/+/●/+/+/○/+/○
+/○/○/+/○/+/○/+/+/○/+/○
+/○/○/+/○/+/+/○

Conclusions

- A multiple-ply termination algorithm has necessarily been employed to develop permissible tapered designs for Balanced Plain Weave (BPW) and Non-Crimp Fabric (NCF) laminates in which consistent mechanical coupling characteristics and immunity to thermal warping distortion are preserved.

- *Simple*, or uncoupled, BPW designs can be achieved with single ply terminations, but designs for bi-angle NCF laminates require a minimum of 2 or 4 ply terminations to maintain these consistent characteristics.

- Lamination parameter design spaces help to indicate the extent to which the available NCF stacking sequences solutions are reduced when practical design constraints for laminate taper are applied to the newly derived definitive listings of stacking sequences.

- The constraint of employing bi-angle NCF (in comparison to UD) material on the magnitude of *Extension-Shearing* and *Bending-Twisting* coupling response, can also be readily assessed by inserting the lamination parameter co-ordinates into the stiffness equations provided.
  - These constraints have also been demonstrated in the context of compression and shear buckling strength, via a mapping of the lamination design space.

Acknowledgement

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