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Scale-lengths and instabilities in magnetized classical and relativistic plasma fluid models

D A Diver and E W Laing

SUPA, School of Physics and Astronomy, Kelvin Building, University of Glasgow, Glasgow G12 8QQ, Scotland UK

E-mail: declan.a.diver@glasgow.ac.uk

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Abstract

The validity of the traditional plasma continuum is predicated on a hierarchy of scale-lengths, with the Debye length being considered to be effectively unresolvable in the continuum limit. In this article, we revisit the strong magnetic field case in which the Larmor radius is comparable or smaller than the Debye length in the classical plasma, and also for a relativistic plasma. Fresh insight into the validity of the continuum assumption in each case is offered, including a fluid limit on the Alfvén speed that may impose restrictions on the validity of magnetohydrodynamics (MHD) in some solar and fusion contexts. Additional implications concerning the role of the firehose instability are also explored.

Keywords: fluid plasmas, Debye length, Larmor radius, particle transport, scale hierarchy, instability

1. Introduction and motivation

Conventionally, plasmas are described by continuum descriptions such as MHD or kinetic theory, each of which assumes that the density of discrete charges in the medium is sufficiently large that very small-scale effects associated with random fluctuations are negligible in the continuum limit, and therefore can be safely neglected. This concept is discussed in most of the classical textbooks (for example, [1–5]) where the Debye length is defined as the smallest considered scale-length for plasmas, at which any discontinuity is averaged or smoothed when constructing the macroscopic equations.

In this article, a fundamental question is addressed: what are the consequences for the microscopic and macroscopic models if the Debye length is not shorter than the Larmor radius?

Such plasmas are discussed in general terms, with an emphasis on the implications for the validity of fluid models, rather than a detailed analysis of collision and transport parameters in the kinetic limit.

The discussion is then extended to relativistic plasmas, where the significance of the approximations can be judged and interpreted.

The authors' original motivation was to inform wave analysis in the relativistic limit. However, the transition from the classical plasma continuum model into one in which the scale-lengths over which charge imbalance persists are significant has long been a matter of routine concern in plasma physics generally, and in particular for low-temperature plasma modelling, and hence the conclusions in this article may have wider impact.

2. Classical plasma parameters

Conventionally, for a non-relativistic electron-ion plasma at temperature T (and therefore possessing a Maxwellian distribution), the Debye length λ_{D} is given by [1–5]

$$\lambda_{\mathrm{D}} = \left(\frac{\epsilon_0 k_B T}{ne^2}\right)^{1/2} = \frac{c_s}{\omega_{pe}}, \tag{1}$$

where n is the number density of electrons. The Debye length can be considered as the distance travelled by a pressure disturbance (that is, at the sound speed $c_s \simeq (k_b T/m_e)^{1/2}$), synonymous with the thermal speed) in a plasma period (the reciprocal of the plasma frequency $\omega_{pe} = [ne^2/(\epsilon_0 m_e)]^{1/2}$). The physical significance of λ_{D} is that it represents the minimum scale-length over which the plasma may be considered electrically neutral—a key assumption for continuum (macroscopic) models such as MHD. Moreover, λ_{D} may be considered as the scale-length over which stochastic electrostatic and thermodynamic fluctuation energies are equilibrated.

These three parameters: $\lambda_{\rm D}$, c_s and ω_{pe} are fluid quantities that express the collective nature of the plasma in its electromagnetic interactions.

The magnetic field enters the scales hierarchy via the electron cyclotron frequency $\omega_{ce} = eB/m_e$, which is a single-particle concept, not dependent on plasma collective effects.

Charged particles moving in a uniform magnetic field execute circular orbits in the plane perpendicular to the magnetic field; for a plasma at temperature T, the representative equilibrium transverse speed of plasma particles can be taken to be the average thermal speed, allowing the corresponding (average) Larmor radius $R_{\rm L}$ for the electron to be defined as follows:

$$R_{\rm L} = \frac{c_s}{\omega_{ce}}. \tag{2}$$

Note that in the kinetic description of a plasma (on the mesoscopic scale), the electrons move with a distribution of speeds about the mean; since the Larmor radius is a particle concept, as opposed to a plasma collective phenomenon, there must be an equivalent distribution of electron Larmor radii in this description.

Analogous to the thermal speed is the Alfvén speed, $c_a = B/(\mu_0\rho)^{1/2}$, where $\rho \approx nm_i$ is the plasma mass density, dominated by the positive ion mass m_i, and where the thermodynamic pressure is replaced by magnetic pressure. Hence to the three critical collective parameters can be added a further three magnetically defined ones: $R_{\rm L}$, c_a and ω_{ce}. In macroscopic fluid models such as MHD, only the fluid speeds c_s and c_a have physical significance, since the fluid-scale lengths and times are too large to admit consideration of the other parameters ($\lambda_{\rm D}$, $R_{\rm L}$, ω_{pe} and ω_{ce}). However, there are fundamental inter-relationships between all these parameters which restrict the range of physically relevant macroscopic parameters, even in MHD.

It is therefore natural to ask the fundamental question: what are the consequences for the microscopic and macroscopic models if the Debye length is not shorter than the Larmor radius? Since the former is the equilibration scale between thermal and electrostatic energies, does the presence of an additional smaller length constraint have appreciable consequences?

3. Critical ratios

As a first step to answering these questions, consider the ratio of the Debye length to the Larmor radius:

$$\left(\frac{\lambda_{\rm D}}{R_{\rm L}}\right) = \frac{c_s}{\omega_{pe}}\frac{\omega_{ce}}{c_s} = \left(\frac{\epsilon_0 B^2}{nm_e}\right)^{1/2} = \frac{c_a}{\kappa c}, \tag{3}$$

where $c_a = B/(\mu_0\, nm_i)^{1/2}$ is the Alfvén speed, with m_i being the ion mass, $m_i \gg m_e$, and $c = (\mu_0\epsilon_0)^{-1/2}$ is the speed of light. The mass ratio $\kappa = (m_e/m_i)^{1/2} \ll 1$ is the factor required to allow the identification of c_a in the ratio. Notice that there is no temperature dependence, since T featured in both $\lambda_{\rm D}$ and $R_{\rm L}$ in the same way, and cancelled in the ratio.

It can therefore be stated that in a classical magnetized fluid plasma, the requirement that the Debye length is smaller than the Larmor radius constrains the Alfvén speed to be significantly less than the vacuum speed of light:

$$\lambda_{\rm D} < R_{\rm L} \Leftrightarrow c_a < \kappa c. \tag{4}$$

This is a restriction on the fluid model that arises from the physical validity of the microscopic parameters: these constraints do not arise self-consistently from within the MHD framework, but are essential to ensure $\lambda_{\rm D} < R_{\rm L}$. The consequence of not ensuring this hierarchy will be explored shortly. Note that $\lambda_{\rm D} > R_{\rm L}$ does not necessarily force a superluminal Alfvén speed. In fact, the condition for the Larmor radius to be smaller than the Debye length in the plasma can be quantified [6, 7]:

$$\lambda_{\rm D} > R_{\rm L} \Leftrightarrow c_a > \kappa c \Leftrightarrow Bn^{-1/2} > (\mu_0 m_e)^{1/2} c \approx 3.2 \times 10^{-10}\,{\rm Tm}^{3/2}. \tag{5}$$

In a tokamak, where $B \approx 3$ T, and $n \approx 10^{20}$ m^{-3}, $Bn^{-1/2} \approx 3 \times 10^{-10}$ Tm$^{3/2}$, implying that λ and $R_{\rm L}$ are very similar in value; in the quiescent (ie non-flaring) solar corona, however, where a typical value of the magnetic field is around 10^{-3} T, and the number density in the range 10^{10}–10^{16} m^{-3}, $Bn^{-1/2}$ lies in the range 10^{-11}–10^{-8} Tm$^{3/2}$, a range which encompasses the critical value.

Moreover, since

$$\frac{\omega_{ce}^2}{\omega_{pe}^2} = \frac{c_a^2}{\kappa^2 c^2} \tag{6}$$

we have the additional insight that in order to ensure that the Debye length is smaller than the (average) Larmor radius, the cyclotron frequency must be less than the plasma frequency:

$$\lambda_{\rm D} < R_{\rm L} \Leftrightarrow c_a < \kappa c \Leftrightarrow \omega_{ce} < \omega_{pe}. \tag{7}$$

There is an important clarification required here: given that in any kinetic description of a plasma there is a distribution of speeds about the mean, then when calculating the Larmor radius for any given particle, the particle's actual random speed v should be used, rather than the plasma's mean thermal speed, c_s:

$$\frac{R_{\rm L}}{\lambda_{\rm D}} = \frac{v}{\omega_c}\frac{\omega_p}{c_s} = \frac{v}{c_s}\frac{\omega_p}{\omega_c}. \tag{8}$$

Thus to ensure $\lambda_{\rm D} < R_{\rm L}$ requires $v > c_s(\omega_c/\omega_p)$, which is possible for all but a minority of particles if $\omega_p \gg \omega_c$, but practically impossible if $\omega_c \gg \omega_p$. This is clear from considering the classical Maxwellian distribution as a function of speed, $f_{\rm M}(v)$:

$$f_{\rm M}(v) = \frac{4}{\sqrt{\pi}\, v_p^3} v^2 \exp\left(-v^2/v_p^2\right) \tag{9}$$

in which $v_p = \sqrt{2k_B T/m} \approx c_s$ is the most probable speed. By integrating with respect to v, and setting $v_p = 1$ for simplicity, the fraction $\phi(b)$ of the total distribution in the speed interval $[0,\ b]$ is given by

$$\phi(b) = \int_0^b \frac{4}{\sqrt{\pi}} v^2 \exp\left(-v^2\right) \mathrm{d}v = \mathrm{erf}\,(b) - 2b \exp\left(-b^2\right) \Big/ \sqrt{\pi}. \quad (10)$$

If $\omega_c/\omega_p = 0.1$, then the fraction of the distribution for which particles have Larmor radii less than the Debye length is $<10^{-3}$; however, if the two frequencies are comparable, then $\sim$40% of the distribution has $\lambda_D > R_L$.

3.1. Physical significance for fluid models

The physical significance for thermodynamic macroscopic fluid models if $R_L < \lambda_D$ can be addressed as follows.

In general, for a plasma electron, there can be localized, random electric fields present within a Debye length of the electron's position, but which endure for only a plasma period (on average) before changing in amplitude and direction. If $R_L \gg \lambda_D$, then, averaged over a cyclotron period, there will be a negligible perturbation superimposed on the electron's Larmor orbit.

For the sake of illustration, consider a random, spontaneously arising electric field E associated with thermal fluctuations of electrons, taken to be a constant in the x-direction for simplicity of analysis, with the homogeneous magnetic field B in the z-direction. Consider the dynamics of an electron in the x, y-plane.

Solving the equations of motion for an electron initially at rest yields

$$v_x = -v_D \sin(\omega_c t), \qquad v_y = -v_D\left[1 - \cos(\omega_c t)\right], \quad (11)$$

where $v_D = E/B_0$ is the drift velocity. For the case $\omega_c t \ll 1$, $|v_x| \sim eEt/m$ and $v_y \sim 0$, recovering the case of simple acceleration under an electric field when $B = 0$. The distance travelled by the particle in this case is $\sim\lambda_D$ in $t \sim 1/\omega_p$ (assuming that $E \neq 0$ for a plasma period) and the energy gained over that period is $(eE/\omega_p)^2/(2\,m) \approx k_B T/2$: the particle is thermalized.

Now consider the calculation in which the electric field is still present for the same period and in addition there is a constant magnetic field present, the influence of which is not negligible: the motion is now two-dimensional, and the speed v gained by the particle is given by $v = \mu v_D$, where $\mu^2 = 2[1 - \cos(\omega_c/\omega_p)]$. Treating the electric field as resulting from random thermal noise, then it must endure on average for only $t \sim 1/\omega_p$. Hence if $\omega_c/\omega_p \ll 1$ then the electron gains $\approx k_B T/2$ energy from interacting with the electric field, as before. However, if the magnetic field is not negligible, so that $\omega_c/\omega_p \not\ll 1$, then the energy gained by the electron is significantly different from the thermal case. Since

$$v_D = \frac{E}{B} = \frac{eE/m}{eB/m} = \frac{c_s \omega_p}{\omega_c} \quad \text{since } eE/(m\omega_p) = c_s \quad (12)$$

then $v_D/c_s < 1$ in this situation, and consequently the energy ε gained by the particle is

$$\varepsilon = \frac{1}{2}\mu^2\, m v_D^2 < m c_s^2 = k_{BT}. \quad (13)$$

Hence it is possible to contrast the energy gain from a spontaneously arising electric field in an unmagnetized plasma with one in which there is a non-negligible perpendicular magnetic field present. In the latter case, thermalization over the plasma period from the stochastic electric impulse is hampered if the cyclotron period is similar to the plasma period.

Of course, electric fields that spontaneously arise parallel to the magnetic field produced thermalization as before, and hence it is clear that for $R_L < \lambda_D$, cross-field transport is inhibited. One macroscopic interpretation of this is the evolution of an anisotropic temperature distribution, where the temperature (and consequently pressure) in the plane perpendicular to the magnetic field is suppressed relative to the field direction itself. Denoting the direction perpendicular and parallel to the magnetic field by the subscripts $\perp$ and $\|$, the corresponding pressure p and temperature T profiles can be identified as follows:

$$\frac{p_\perp}{p_\|} = \frac{T_\perp}{T_\|} \leqslant \frac{v_D^2}{c_s^2} < 1, \quad (14)$$

where the random energy in the plane perpendicular to the magnetic field is identified as $(1/2)\mu^2 m v_D^2 \sim k_B T_\perp$. Given that $v_d = c_s \omega_p/\omega_c$, we can recast the classical firehose instability (associated with $T_\| > T_\perp$), given by [8]

$$p_\| > \frac{B_0^2}{\mu_0} + p_\perp \quad (15)$$

to the form

$$1 - \frac{\omega_p^2}{\omega_c^2} > \frac{2}{\beta}, \quad (16)$$

where $\beta = 2\mu_0 p_\| \big/ B_0^2$ is the usual plasma pressure parameter. Note that equation (16) already contains the requirement that $\lambda_D > R_L$, via equation (7), since it is only valid if $\omega_c > \omega_p$. Hence we can conclude that if $R_L \leqslant \lambda_D$ then the plasma is vulnerable to the firehose instability, which can destabilize Alfvén waves, producing mass motion and charge streaming.

The effect of non-standard transport has been addressed in the classical kinetic limit: conventional scattering theory is altered, leading for example to anomalous acceleration of low-energy electrons, [6, 9–11], and free-streaming of electrons and anomalous instabilities and damping [12, 13]. A recent detailed analysis of the relaxation rates derived from a consideration of interparticle collisions in strongly magnetized kinetic plasmas [14] also concludes that anisotropic temperature evolution is significantly influenced by strong magnetic fields characterized by the Larmor radius being much smaller than the Debye length, and effectively replacing the latter as the collision cut-off distance. There are also consequences for beam–plasma interactions in terms of altered ion stopping distances [7] of significance to astrophysical plasmas, as well as surface–plasma interactions such as near tokamak divertors [15, 16] where the effect of the magnetic field measurably perturbs the angular distribution of particles arriving at the surface, a result that can be attributed to the magnetically altered sheath scaling [17, 18].

4. Relativistic plasma

In the classical analysis of the preceding sections, it was shown that the validity of continuum solutions may be open to question if the plasma is strongly magnetized; it is reasonable to examine how the relativistic plasma might react to such conditions. In this context, the plasma is relativistic because it is intrinsically very energetic, not because it is moving relative to the observer. Hence for comparison with the classical case, the only plasma parameter that characterizes the energetic nature of the plasma is the normalized temperature a, defined below; the number density and magnetic field are unchanged from the classical case, to facilitate direct comparisons.

The relativistic kinetic theory of plasmas uses the generalized distribution function f^R, where

$$f^R = \left(4\pi m_e^3 c^3\right)^{-1} \frac{a}{K_2(a)} e^{-a\gamma} \tag{17}$$

is the Maxwell–Boltzmann–Jüttner distribution [19] in which K_2 is the modified Bessel function of order 2, $\gamma = (1 - v^2/c^2)^{-1/2}$ is the usual relativistic factor, and

$$a = \frac{m_e c^2}{k_B T} \tag{18}$$

is the reciprocal temperature parameter, which plays a central role in the phenomenological descriptions. The distribution can also be written in the form

$$f^R = \frac{a\gamma^2\beta}{K_2(a)} e^{-a\gamma}, \tag{19}$$

where $\beta = v/c$. This latter form for f^R is often encountered in hard gamma-ray spectra from blazar jets [20, 21].

In the classical case, $a \gg 1$, whereas for ultra-relativistic plasmas, $a \ll 1$. Strictly, for electron-ion plasmas there should be separate reciprocal temperature parameters for each species: we shall assume here that the ultrarelativistic limit will mean that all such parameters are small (see [19, 22] for more detailed discussions of multi-species distribution functions).

The relativistic sound speed c_s^R is defined by [19]:

$$\left(\frac{c_s^R}{c}\right)^2 = \frac{1}{G}\frac{\mathrm{d}G}{\mathrm{d}a}\left(a\frac{\mathrm{d}G}{\mathrm{d}a} + \frac{1}{a}\right)^{-1}, \tag{20}$$

where $G = K_3(a)/K_2(a)$. For $a \gg 1$, $c_s^R/c \approx [5/(3a)]^{1/2}$, consistent with the adiabatic result in the earlier section. However, in the limit $a \ll 1$, $c_s^R/c \approx 1/\sqrt{3}$.

The relativistic plasma frequency ω_{pe}^R changes via its mass dependence:

$$\omega_{pe}^R = \left(\frac{ne^2}{\gamma\epsilon_0 m_e}\right)^{1/2}. \tag{21}$$

Hence the relativistic Debye length λ_{D}^R can be written in the form

$$\left(\lambda_{\mathrm{D}}^R\right)^2 = \frac{c^2}{G}\frac{\mathrm{d}G}{\mathrm{d}a}\left(a\frac{\mathrm{d}G}{\mathrm{d}a} + \frac{1}{a}\right)^{-1}\frac{\gamma\epsilon_0 m_e}{ne^2} \tag{22}$$

which, in the ultra-relativistic limit, reduces to

$$\left(\lambda_{\mathrm{D}}^R\right)^2 \approx \frac{\epsilon_0\gamma m_e c^2}{3ne^2}. \tag{23}$$

In the relativistic plasma, the most probable speed u_p^R is given by [19]:

$$u_p^R = c\left(\frac{5\gamma^2 - 3}{a\gamma^3}\right)^{1/2}, \tag{24}$$

which, in the ultra-relativistic limit $\gamma \to \infty$, yields

$$u_p^R \approx c\left(\frac{5}{a\gamma}\right)^{1/2} \approx c \tag{25}$$

since $a \approx 5/\gamma$ as $\gamma \to \infty$. The relativistic Larmor radius R_{L}^R is given by

$$\begin{aligned} R_{\mathrm{L}}^R &= \frac{\gamma m_e c}{eB}\left(\frac{5\gamma^2 - 3}{a\gamma^3}\right)^{1/2} \\ &\approx \frac{\gamma m_e c}{eB} \quad \text{if } a \ll 1. \end{aligned} \tag{26}$$

The ratio of the Debye length to the Larmor radius can be written in the following form, for the general case:

$$\left(\frac{\lambda_{\mathrm{D}}^R}{R_{\mathrm{L}}^R}\right)^2 = \left(\frac{c_s^R}{\omega_{pe}^R}\right)^2\left(\frac{\omega_{ce}^R}{u_p^R}\right)^2 \tag{27}$$

$$= \frac{1}{\gamma}\left(\frac{c_s^R}{u_p^R}\right)^2\left(\frac{\lambda_{\mathrm{D}}}{R_{\mathrm{L}}}\right)^2. \tag{28}$$

In the classical limit, $\gamma \to 1$, $c_s^R \to u_p^R$ and the result of (3) is recovered. In the ultra-relativistic case, $c_s^R \to u_p^R/\sqrt{3}$ and $\gamma \approx 5/a$, so that the ratio is now

$$\left(\frac{\lambda_{\mathrm{D}}^R}{R_{\mathrm{L}}^R}\right)^2 \approx \frac{1}{3\gamma}\left(\frac{\lambda_{\mathrm{D}}}{R_{\mathrm{L}}}\right)^2 \tag{29}$$

in the ultra-relativistic limit. Taking $\gamma \approx 5/a$ yields

$$\left(\frac{\lambda_{\mathrm{D}}^R}{R_{\mathrm{L}}^R}\right)^2 \approx \frac{a}{15}\left(\frac{\lambda_{\mathrm{D}}}{R_{\mathrm{L}}}\right)^2. \tag{30}$$

The implication of (28) is clear: there is a relativistic factor that changes the ratio of Debye length to Larmor radius as the plasma becomes more energetic, simply because the Larmor radius increases faster ($\propto\gamma$) than the Debye length

($\propto\gamma^{1/2}$). For classical, non-relativistic plasmas, this factor is approximately unity, but as γ becomes larger, this factor produces a much more significant correction to the classical case. In fact, moving to a relativistic plasma description restores the temperature to the calculation of the ratio of the Debye length to the Larmor radius, through the ratio's dependence on γ.

5. Discussion

This article shows that there are restrictions on fundamental plasma length scales that are directly encoded in the macroscopic plasma parameters, even if the plasma description does not refer explicitly to the microscopic scale hierarchy.

In order to ensure that the Larmor radius for electrons is not smaller than the Debye length (and thereby avoid the transport implications), then in the classical case, the Alfvén speed must be less than the speed of light times the square root of the electron to ion (rest) mass ratio:

$$c_a < (m_e/m_i)^{1/2}c. \tag{31}$$

Plasmas that do not satisfy this criterion are vulnerable to firehose instability.

In the relativistic case, this is modified to yield (using (28) combined with (3), neglecting any numerical prefactors)

$$c_a < \left(\frac{m_e}{am_i}\right)^{1/2} c, \tag{32}$$

and given that $a \to 0$ as the plasma becomes more relativistic, this makes it easier to ensure the Larmor radius exceeds the Debye length. Note that (32) is in agreement with the modified relativistic Alfvén speed given in equation (51) of [22], derived from considerations of low-frequency relativistic dispersion relations; in both cases, the appropriate relativistic Alfvén speed is significantly less than the classical one.

In fluid models such as MHD, microscopic concepts such as the Debye length and Larmor radius do not appear, since the model has assumed that the continuum physics is consistent with fluid dynamics at these scales. However, the macroscopic Alfvén speed does appear, and so offers a route by which the fluid approximation can be assessed for microscopic consistency, even though such considerations are beyond the remit of the continuum model. This is a key point: whilst much sophisticated analysis of transport processes in the mesoscopic limit has highlighted the instabilities that may arise from anisotropic processes influenced by a non-classical cut-off distance (e.g. [6, 9, 12, 14]) it has tacitly been assumed that in the fluid limit, the models are immune from such considerations. It is our contention that these kinetic restrictions are encoded into the physics of the continuum, and may not be ignored.

For plasmas in which the parameters suggest a contradiction in the conventional scales hierarchy if the *classical* continuum model is chosen, there is always the possibility of moving to a relativistic model via a choice of a that restores the Debye length to be appropriately subdominant, so long as that choice of a does not contradict the desired plasma electron temperature. If a resolution cannot be found via this transformation, then either the physical model must move to a non-continuum one, or anomalous transport must be accommodated.

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