UNCONVENTIONAL LAMINATE DESIGN USING THIN-PLY TECHNOLOGIES

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Background

The work describe here is part of an on-going study addressing **Mechanically Coupled** (Composite) **Laminates**.

24 distinct classes of coupled laminate have previously been identified, containing all possible interactions between *Extension*, *Shearing*, *Bending* and *Twisting*.

These laminate classes were derived for UD material using (but not restricted to) combinations of standard fibre angle orientations, i.e. 0, 90 and/or ±45° (= ±θ°).

The derivation of these laminate classes involves the added restrictions that *each layer in the laminate*:

- *has identical material properties;*
- *has identical thickness;*
- *differs only by its orientation.*

This presentation focuses on important aspect of laminate designs, including **taper** and **ply contiguity**, firstly for **UD** material and then for thin-ply **Non-Crimp Fabric** and **Woven cloth** materials, in which:

- *stacking sequence symmetries are unconstrained;*
- *the coupling matrix, **B** = **0**!*

Laminate Characterisation

The thermo-mechanical behaviour of coupled laminates may be determined from the specific form of the **ABD** stiffness matrix:

$$\begin{Bmatrix} N_x + N_x^{Thermal} \\ N_y + N_y^{Thermal} \\ N_{xy} + N_{xy}^{Thermal} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} M_x + M_x^{Thermal} \\ M_y + M_y^{Thermal} \\ M_{xy} + M_{xy}^{Thermal} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

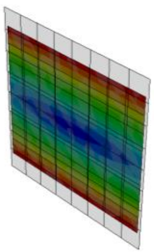
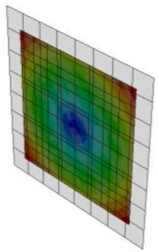
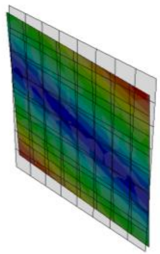
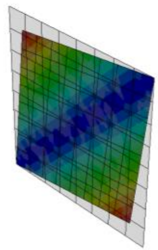
Couplings exist between:

- in-plane and out-of-plane actions, when $B_{ij} \neq 0$ (5 independent forms of the **B** matrix!),
- extension and shearing, when A_{16}, $A_{26} \neq 0$, and
- bending and twisting, when D_{16}, $D_{26} \neq 0$.

A given laminate can be described in terms of its physical response, due an applied set of force and/or moment resultants …..

Table 1 – Response based labelling for the 4 laminate classes of interest.

Illustrations represent exaggerated thermal contraction responses (in-plane only, since coupling matrix **B** = **0**!) following a typical high temperature curing process.

Uncoupled in Extension		*Extension-Shearing*	
Uncoupled in Bending	***Bending-Twisting***	**Uncoupled in Bending**	***Bending-Twisting***
			
$[+/-_2/\bigcirc/+_2/-]_T$	$[+/-/-/+]_T$	$[\pm/\bigcirc/-/\bigcirc/-_3/\bigcirc/-_3/\bigcirc/+]_T$	$[+/+]_T$
Simple laminate	***B-T coupled laminate***	***E-S coupled laminate***	***E-S;B-T coupled laminate***

1. Simple laminates (balanced and symmetric?)

2. *B-T* coupled laminates (balanced and symmetric!?)

Tapered designs are certified for symmetric laminate construction, but have severe design constraints, e.g., 1 angle-ply termination requires a further 3 angle-ply terminations to maintain balanced and symmetric construction!

B-T coupled laminates are known to be weaker in compression buckling than the equivalent *Simple* laminate (with matching stiffness properties), but are potentially stronger in shear buckling (direction dependent!).

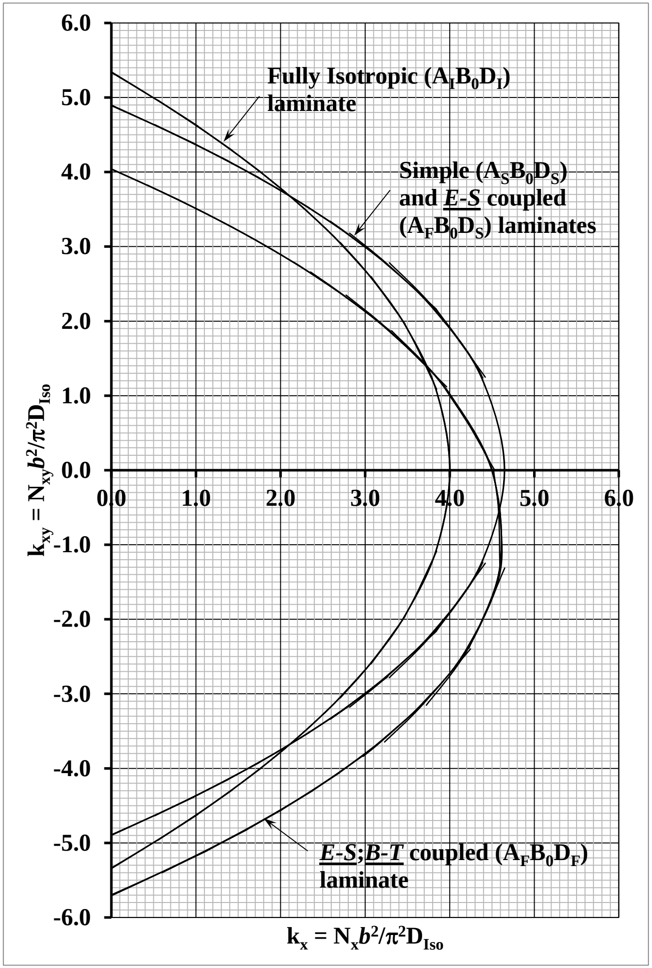


Figure 1 – Buckling interaction envelopes for an infinitely long plate with simply supported edges, highlighting the effect of isolated mechanical coupling properties.

Table 2 – Number (%) of *B-T* coupled laminate stacking sequences for each ply number grouping, *n*, arranged by sub-sequence symmetry.

	←Grid-stiffened Fuselage ↔					Traditional Fuselage →			
n	8	9	10	11	12	13	14	15	16
NC									
NN		16.7		35.8	20.0	52.1	32.0	68.0	54.0
NS				7.5	6.2	10.8	11.2	10.5	11.8
SC				3.8	2.8	0.9		0.9	1.1
SN						4.8	4.8	4.3	4.0
SS	100	83.3	100	52.8	71.0	31.4	52.0	16.3	29.1
Σ	15	36	56	212	290	1,336	1,500	9,666	10,210

C – Cross-symmetric; *N* – Non-symmetric; *S* – Symmetric

NC:
$[+/-/+/○/-/●/●/+/●/-/○/○/-/●/-/+/+]_T$

NN:
$[+/●/-/●/-/+/-/+/●]_T$

NS:
$[+/-/-/-/+/●/+/+/+/-/-]_T$

SC:
$[+/●/○/-/○/●/●/-/●/○/+]_T$

SN:
$[+/●/●/○/○/-/●/-/●/●/●/○/+]_T$

SS:
$[+/-/-/+]_T$

Table 3 – Number of *Bending-Twisting* coupled laminates from (*n* =) 16 plies down to (*n* =) 8 plies, subject to contiguity constraints.

	Ply Contiguity			
n	1	2	>2	Σ
16	210	5,717	4,283	10,210
15	602	5,452	3,612	9.666
14	40	940	520	1,500
13	156	722	458	1,336
12	6	197	87	290
11	40	108	64	212
10	-	42	14	56
9	14	14	8	36
8	-	12	3	15

Table 4 – Single ply termination algorithm applied to <u>*B-T*</u> coupled laminates between (*n* =) 16 plies and (*n* =) 8 plies; with ply contiguity ≤ 2.

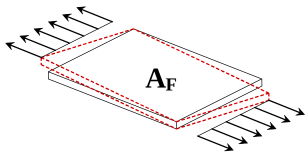
(1) *n*	(2) No. Seq. from *n* (Compatible with *n*+1.)	(3) No. Seq. from *n* (Compatible with *n*-1.)	(4) No. Solutions (○ or ●/+ or –)	(5) No. Seq. from *n* (Compatible with *n*-1.)	(6) No. Solutions (○ or ●/+ or –)
16	2,844 (↘3,066)	(18↖)	36 (18/0)	(286↖)	752 (286/0)
15	1,496 (↘2,976)	(18↖) 18	36 (18/0)	(286↖) 286	286 (143/0)
14	484 (↘954)	(18↖) 18	36 (18/0)	(294↖) 286	588 (294/0)
13	332 (↘492)	(18↖) 18	36 (18/0)	(294↖) 294	294 (147/0)
12	96 (↘**203**)	(18↖) 18	36 (18/0)	**198**	
11	62 (↘98)	(18↖) 18	36 (18/0)		
10	22 (↘42)	(18↖) 18	36 (18/0)		
9	18 (↘22)	(18↖) 18	18 (9/0)		
8	- (↘**12**)	**12**			

Table 5 – Sub-sequence symmetries in compatible *Bending-Twisting* coupled laminate stacking sequences for single ply terminations, corresponding to the results of: (a) Column (2) and; (b) Column (5) of Table 4.

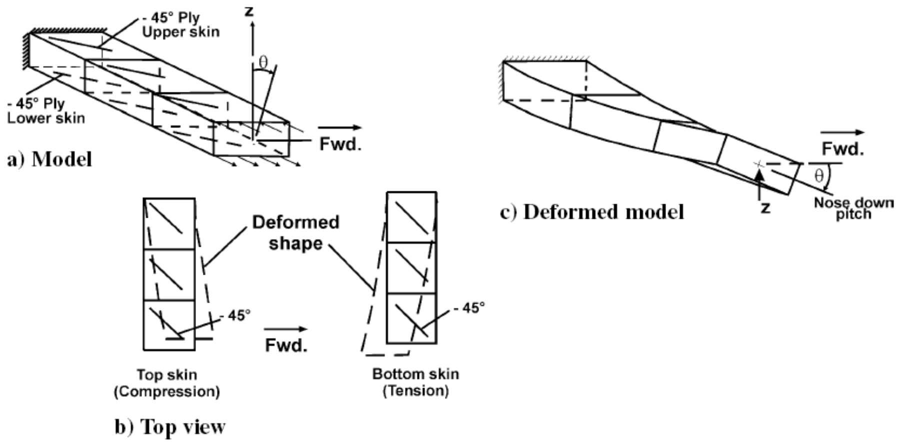
	(a)									(b)				
	Ply contiguity = 1/Ply contiguity = 2									Ply contiguity ≤ 2				
n	8	9	10	11	12	13	14	15	16	12	13	14	15	16
NN					4/24	8/84	34/170	74/490	81/1,654	(44)	58	58	58	58
NS						-/16	-/16	-/152	-/152	(6)	12	12	12	12
SC					2/2	-/12	-/-	-/-	8/25	(2)	4	4	4	4
SN							-/24	-/76	-/96					
SS	(-/6)	12/10	-/22	32/44	-/72	102/170	-/274	286/676	-/968	(146)	220	212	212	212

3. ***E-S*** coupled laminates (unbalanced!)

Laminate tailoring strategies for ***E-S*** or ***E-S***;***B-T*** coupled laminates



(a)



(b)

Figure 1 – Illustration of (a) Extension-Shearing (*E-S*) coupling as a result of fully populated **A** matrix [$\mathbf{A}_F$], producing (b) Bending-Twisting deformation in aircraft wing-box structures when top and bottom skins have identical (n_+) fibre alignment (eliminated with opposing alignment!), essential for avoiding divergence in forward-swept wings or for reducing drag in aft-swept wings.

4. *E-S*;*B-T* coupled laminates

Table 6 – Number (%) of *E-S*;*B-T* coupled laminate stacking sequences for each ply number grouping, *n*, arranged by sub-sequence. Symmetric laminates of the form $[+/\ldots/+]_T$ have been disregarded in all of the results presented.

n	8	9	10	11	12	13	14	15	16
AC								0.03	
AN								0.03	
AS		1.9		1.3		0.7		0.3	
NC								0.1	
NN	4.0	20.6	13.4	37.3	23.5	53.1	40.1	67.9	56.2
NS		5.0	3.3	9.3	6.2	10.1	8.5	9.5	9.4
SC					0.7	1.4	1.1	0.7	0.4
SN		2.5		1.8	1.3	3.2	2.1	3.7	3.5
SS	96.0	70.1	83.3	50.3	68.3	31.5	48.2	17.7	30.5
Σ	50	321	241	1,843	1,191	11,651	6,847	83,573	43,830

A – Anti-symmetric; *C* – Cross-symmetric; *N* – Non-symmetric; *S* – Symmetric

AC:
[+/○/●/–/–/●/○/–/●/○/+/+/○/●/–]$_T$

AN:
[+/○/●/–/–/●/●/–/○/●/+/+/○/●/–]$_T$

AS:
[+/–/–/–/+/+/–]$_T$

NC:
[+/●/+/○/+/○/+/+/+/●/+/●/+/○/+]$_T$

NN:
[+/●/●/●/+/+/●]$_T$

NS:
[+/–/–/–/●/+/–/+/–]$_T$

SC:
[+/●/○/○/●/○/●/●/○/+]$_T$

SN:
[+/●/○/○/●/●/●/○/+]$_T$

SS:
[+/–/+]$_T$

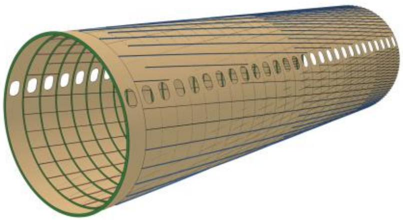


Table 7 – Number of *Extension-Shearing and Bending-Twisting* coupled laminates from (*n* =) 16 plies down to (*n* =) 8 plies, subject to contiguity constraints.

	Ply Contiguity			
n	1	2	>2	Σ
16	414	19,949	23,467	43,830
15	3,413	39,622	40,538	83,573
14	88	3,463	3,296	6,847
13	925	5,382	5,344	11,651
12	10	665	516	1,191
11	243	845	755	1,843
10	4	145	92	241
9	75	122	124	321
8	-	35	15	50

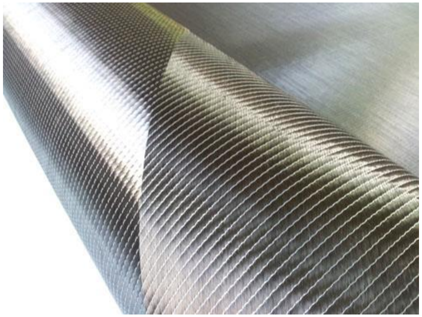
Table 8 – Single ply termination algorithm applied to <u>*E-S*</u>-<u>*B-T*</u> coupled laminates between (*n* =) 16 plies and (*n* =) 8 plies; with ply contiguity ≤ 2.

(1) *n*	(2) No. Seq. from *n* (Compatible with *n*+1.)	(3) No. Seq. from *n* (Compatible with *n*-1.)	(4) No. Solutions (○ or ●/+/–)	(5) No. Seq. from *n* (Compatible with *n*-1.)	(6) No. Solutions (○ or ●/+/–)
16	20,329 (↘20,355)	(1,791↖)	3,582 (934/930/784)	(4,373↖)	7,911 (2,086/2,020/1,719)
15	11,273 (↘21,243)	(1,791↖) 1,791	1,791 (467/465/392)	(4,391↖) 4,391	4,391 (1,156/1,118/961)
14	3,167 (↘3,551)	(637↖) 637	1,274 (340/324/270)	(1,637↖) 1,637	3,274 (948/714/664)
13	2,111 (↘3,645)	(637↖) 637	637 (170/162/135)	(1,637↖) 1,637	1,637 (474/357/332)
12	623 (↘**675**)	(231↖) 231	462 (122/126/92)	**675**	-
11	463 (↘673)	(231↖) 231	231 (61/63/46)		
10	137 (↘149)	(87↖) 87	174 (52/36/34)		
9	107 (↘141)	(87↖) 87	87 (26/18/17)		
8	- (↘**35**)	**35**	-		

Table 9 – Sub-sequence symmetries in compatible *Extension-Shearing* and *Bending-Twisting* coupled laminate stacking sequences for single ply terminations, corresponding to the results of: (a) Column (2) and; (b) Column (5) of Table 8.

	(a)									(b)				
	Ply contiguity = 1/Ply contiguity = 2									Ply contiguity ≤ 2				
n	8	9	10	11	12	13	14	15	16	12	13	14	15	16
AC								4/20	-/-					
AN								8/8	-/-					
AS		-/6	-/-	-/24	-/-	24/74	-/-	40/186	-/-				16	
NC								-/32	-/-					
NN	(-/2)	6/28	4/12	24/364	8/128	218/3,056	64/1,240	1,184/28,212	59/10,454	(136)	280	280	1,165	1,147
NS		-/4	-/2	-/68	-/32	32/442	16/212	120/3,074	8/1,394	(32)	60	60	216	232
SC			-/2	8/20	2/4	24/108	8/50	100/336	6/118	(6)	14	10	85	57
SN		-/4	-/-	-/8	-/4	-/148	-/48	64/1,316	16/616	(4)	4	8	85	113
SS	(-/33)	69/80	-/129	211/361	-/497	627/1,554	-/1,913	1,893/6,438	-/7,367	(497)	1,279	1,279	2,824	2,824

C-Ply (bi-angle) non-crimp fabric material



… two plies of carbon fibre, at 0° and a shallow angle (shown here at 45°), stitched together.

The new design solutions, reported here, follow the repeating bi-angle philosophy, $[\underline{\theta/0}]_{rT}$, which possesses *Extension-Shearing* and *Bending-Twisting* coupling, but now with immunity to thermal warping distortions; warping is eliminated in $[\underline{\theta/0}]_{rT}$ laminates only when the number (r) of repeats becomes very large.

n	Uncoupled in Extension		*Extension-Shearing*	
	Uncoupled in Bending	***Bending-Twisting***	**Uncoupled in Bending**	***Bending-Twisting***
4 (8)	-	4	-	5
5 (10)	-	-	-	-
6 (12)	-	-	-	88
7 (14)	-	-	-	-
8 (16)	35	419	-	683

(*n*) for UD laminate equivalent.

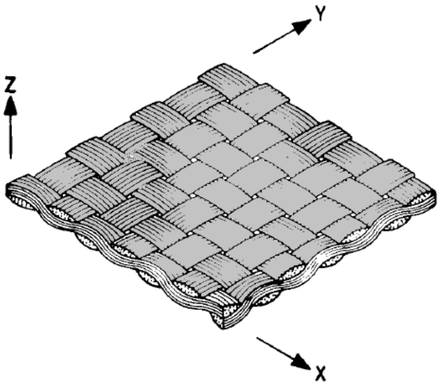
Note that a 24-ply fully isotropic (π/4) laminate can also be constructed from 0/45 and 0/-45 bi-angle NCF:

[-45/90/0/45/0/45/90/45/-45/0/-45/90/-45/90/45/90/0/-45/0/45/0/45/-45/90]$_T$

by are either flipping/reversing (-45/0), rotating (90/45) or both (45/90 and -45/90).

Balanced plain weave material

What lessons can be brought forward from UD to woven cloth architectures?



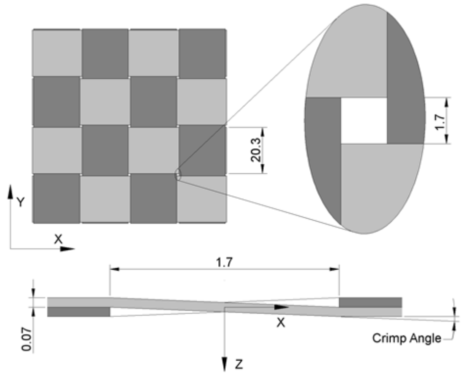


Figure 2 – Balanced plain weave architecture ≡ TeXtreme™

Due to the balanced nature of a single layer of plain weave, i.e. equal reinforcement (fibre volume fraction) in the 0 and 90º directions, the warp and weft directions are indistinguishable from each other.

Hence standard ply angle orientations, 0, 90 and ±45º, simplify to 0 and 45º if the equal modulus ($E_1 = E_2$) condition is assumed; orthogonal counterparts, 90 and -45°, have identical properties, respectively.

A single layer of balanced plain weave material also possesses equal thermal expansion coefficients ($\alpha_1 = \alpha_2$; $\alpha_{12} = 0$ is implied) and is known to be immune to thermal warping distortions.

Stiffness matrices and associated lamination parameters

The elements of the extensional, [**A**], coupling, [**B**] and bending [**D**] stiffness matrices can be calculated from laminate invariants, U_i, and lamination parameters ξ_i:

$A_{11}=\{U_1+\xi_1 U_2+\xi_2 U_3\}H$	$B_{11}=\{\xi_5 U_2+\xi_6 U_3\}H^2/4$	$D_{11}=\{U_1+\xi_9 U_2+\xi_{10} U_3\}H^3/12$
$A_{12}=A_{21}=\{-\xi_2 U_3+U_4\}H$	$B_{12}=B_{21}=\{-\xi_6 U_3\}H^2/4$	$D_{12}=D_{21}=\{U_4-\xi_{10} U_3\}H^3/12$
$A_{16}=A_{61}=\{\xi_3 U_2/2+\xi_4 U_3\}H$	$B_{16}=B_{61}=\{\xi_7 U_2/2+\xi_8 U_3\}H^2/4$	$D_{16}=D_{61}=\{\xi_{11} U_2/2+\xi_{12} U_3\}H^3/12$
$A_{22}=\{U_1-\xi_1 U_2+\xi_2 U_3\}H$	$B_{22}=\{-\xi_5 U_2+\xi_6 U_3\}H^2/4$	$D_{22}=\{U_1-\xi_9 U_2+\xi_{10} U_3\}H^3/12$
$A_{26}=A_{62}=\{\xi_3 U_2/2-\xi_4 U_3\}H$	$B_{26}=B_{62}=\{\xi_7 U_2/2-\xi_8 U_3\}H^2/4$	$D_{26}=D_{62}=\{\xi_{11} U_2/2-\xi_{12} U_3\}H^3/12$
$A_{66}=\{-\xi_2 U_3+U_5\}H$	$B_{66}=\{-\xi_6 U_3\}H^2/4$	$D_{66}=\{-\xi_{10} U_3+U_5\}H^3/12$

$H = n \times t$.

Note that laminate invariant $U_2 = (Q_{11} - Q_{22})/2$

where $Q_{11} = E_1/(1 - \nu_{12}\nu_{21})$

and $Q_{22} = E_2/(1 - \nu_{12}\nu_{21})$

However, for balanced plain weave material with $E_1 = E_2$

$$Q_{11} = Q_{22}$$

hence $U_2 = 0$

Stiffness matrices and associated lamination parameter simplifications

For balanced plain weave material, the elements of the **ABD** matrix can be calculated from a reduced set of laminate invariants, U_i, and lamination parameters ξ_i:

$A_{11}=\{U_1+\xi_2 U_3\}H$	$B_{11}=\{\xi_6 U_3\}H^2/4$	$D_{11}=\{U_1+\xi_{10} U_3\}H^3/12$
$A_{12}=A_{21}=\{-\xi_2 U_3+U_4\}H$	$B_{12}=B_{21}=\{-\xi_6 U_3\}H^2/4$	$D_{12}=D_{21}=\{U_4-\xi_{10} U_3\}H^3/12$
$A_{16}=A_{61}=\{\xi_4 U_3\}H$	$B_{16}=B_{61}=\{\xi_8 U_3\}H^2/4$	$D_{16}=D_{61}=\{\xi_{12} U_3\}H^3/12$
$A_{22}=\{U_1+\xi_2 U_3\}H$	$B_{22}=\{\xi_6 U_3\}H^2/4$	$D_{22}=\{U_1+\xi_{10} U_3\}H^3/12$
$A_{26}=A_{62}=\{-\xi_4 U_3\}H$	$B_{26}=B_{62}=\{-\xi_8 U_3\}H^2/4$	$D_{26}=D_{62}=\{-\xi_{12} U_3\}H^3/12$
$A_{66}=\{-\xi_2 U_3+U_5\}H$	$B_{66}=\{-\xi_6 U_3\}H^2/4$	$D_{66}=\{-\xi_{10} U_3+U_5\}H^3/12$

$H = n \times t$.

Lamination parameters are defined by:

$$\xi_2=\sum_{k=1}^{n} \cos 4\theta_k (z_k - z_{k-1}) \qquad \xi_6=\sum_{k=1}^{n} \cos 4\theta_k (z_k^2 - z_{k-1}^2)/2 \qquad \xi_{10}=\sum_{k=1}^{n} \cos 4\theta_k (z_k^3 - z_{k-1}^3)/3$$

$$\xi_4=\sum_{k=1}^{n} \sin 4\theta_k (z_k - z_{k-1}) \qquad \xi_8=\sum_{k=1}^{n} \sin 4\theta_k (z_k^2 - z_{k-1}^2)/2 \qquad \xi_{12}=\sum_{k=1}^{n} \sin 4\theta_k (z_k^3 - z_{k-1}^3)/3$$

z_k = k^{th} layer interface distance with respect to the laminate mid-plane.

Similarly, the thermal force and moment vectors:

$$\begin{Bmatrix} N_x^{Thermal} \\ N_y^{Thermal} \\ N_{xy}^{Thermal} \end{Bmatrix} = \frac{H}{2} \begin{Bmatrix} (U_1+U_4)(\alpha_1+\alpha_2)+U_2(\alpha_1-\alpha_2)+\xi_1[U_2(\alpha_1+\alpha_2)+(U_1+2U_3-U_4)(\alpha_1-\alpha_2)] \\ (U_1+U_4)(\alpha_1+\alpha_2)+U_2(\alpha_1-\alpha_2)-\xi_1[U_2(\alpha_1+\alpha_2)+(U_1+2U_3-U_4)(\alpha_1-\alpha_2)] \\ \xi_3[U_2(\alpha_1+\alpha_2)+(U_1+2U_3-U_4)(\alpha_1-\alpha_2)] \end{Bmatrix} \Delta T$$

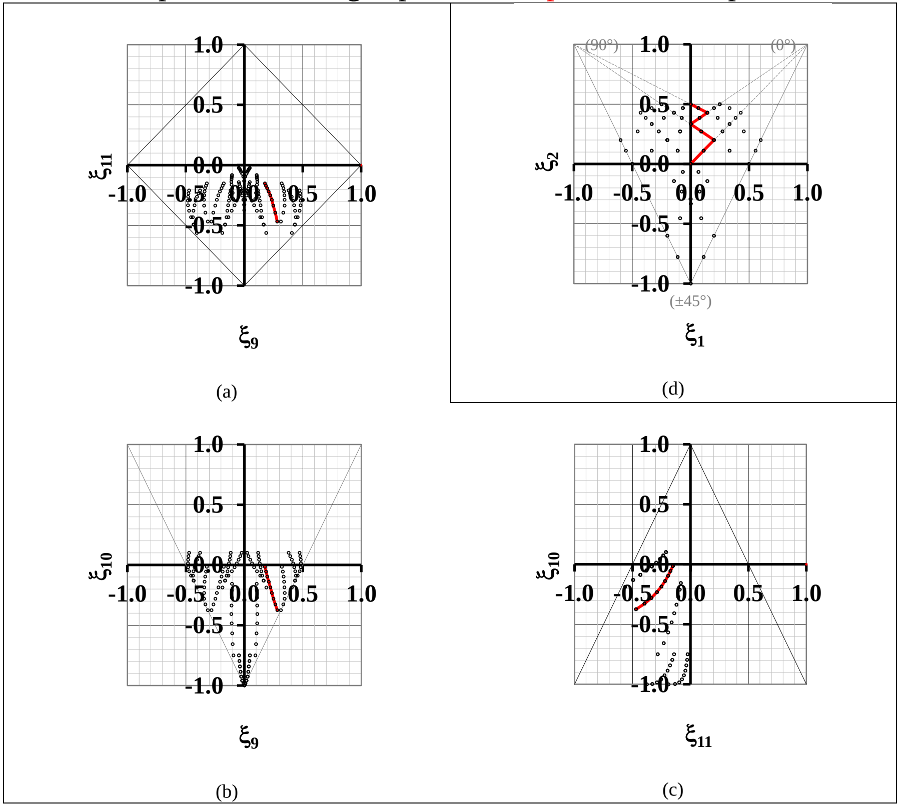
$$\begin{Bmatrix} M_x^{Thermal} \\ M_y^{Thermal} \\ M_{xy}^{Thermal} \end{Bmatrix} = \frac{H^2}{8} \begin{Bmatrix} \xi_5[U_2(\alpha_1+\alpha_2)+(U_1+2U_2-U_4)(\alpha_1-\alpha_2)] \\ -\xi_5[U_2(\alpha_1+\alpha_2)+(U_1+2U_3-U_4)(\alpha_1-\alpha_2)] \\ \xi_7[U_2(\alpha_1+\alpha_2)+(U_1+2U_2-U_4)(\alpha_1-\alpha_2)] \end{Bmatrix} \Delta T$$

simplify substantially due to the assumption of equal moduli ($E_1 = E_2$, hence $U_2 = 0$) and equal thermal coefficients ($\alpha_1 = \alpha_2 = \alpha_{Iso}$; $\alpha_{12} = 0$ is implied):

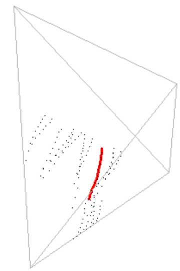
$$\begin{Bmatrix} N_x^{Thermal} \\ N_y^{Thermal} \\ N_{xy}^{Thermal} \end{Bmatrix} = H \begin{Bmatrix} (U_1+U_4)\alpha_{iso} \\ (U_1+U_4)\alpha_{iso} \\ 0 \end{Bmatrix} \Delta T \qquad \text{(Thermal isotropy!)}$$

$$\begin{Bmatrix} M_x^{Thermal} \\ M_y^{Thermal} \\ M_{xy}^{Thermal} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Lamination parameter design spaces for tapered *B-T* coupled UD laminates with 8-16 plies:



Isometric view of $(\xi_9,\xi_{10},\xi_{11})$ design space:

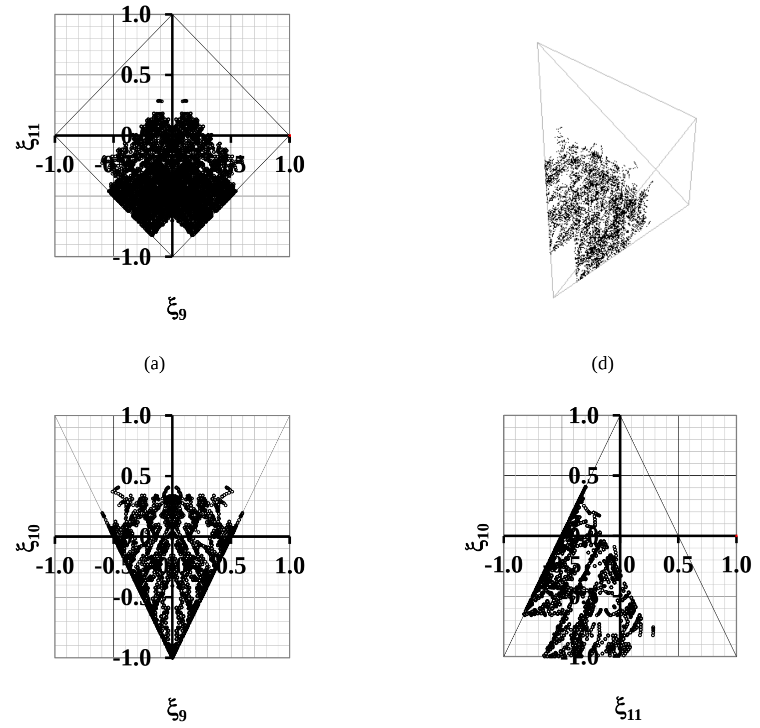


Corresponding tapered laminate:

[+/○/–/●/○/●/○/●/●/○/●/○/●/–/○/+]$_{T(SS)}$
[+/○/–/●/○/●/○/●/○/●/○/●/–/○/+]$_{T(SS)}$
[+/○/–/●/○/●/○/○/●/○/●/–/○/+]$_{T(SS)}$
[+/○/–/●/○/●/○/●/○/●/–/○/+]$_{T(SS)}$
[+/○/–/●/○/●/●/○/●/–/○/+]$_{T(SS)}$
[+/○/–/●/○/●/○/●/–/○/+]$_{T(SS)}$
[+/○/–/●/○/○/●/–/○/+]$_{T(SS)}$
[+/○/–/●/○/●/–/○/+]$_{T(SS)}$
[+/○/–/●/●/–/○/+]$_{T(SS)}$

Strings of points are clearly discernible in these design spaces, and in fact represent a series of 9 compatible stacking sequences forming, collectively, the tapered solutions identified.

Lamination parameter design spaces for tapered *E-S*;*B-T* coupled UD laminates with 12-16 plies:



(a) (d)

(b) (c)

[+/+/○/●/●/+/●/+/+/●/+/●/●/○/+/+]$_{T(SS)}$
[+/+/○/●/●/+/●/+/●/+/●/●/○/+/+]$_{T(SS)}$
[+/+/○/●/●/+/●/●/+/●/●/○/+/+]$_{T(SS)}$
[+/+/○/●/●/+/●/+/●/●/○/+/+]$_{T(SS)}$
[+/+/○/●/●/+/+/●/●/○/+/+]$_{T(SS)}$

[+/○/○/●/●/○/+/–/–/+/●/○/○/●/○/+]$_{T(SN)}$
[+/○/○/●/●/○/+/–/+/●/○/○/●/○/+]$_{T(SN)}$
[+/○/○/●/●/○/+/+/●/○/○/●/○/+]$_{T(SN)}$
[+/○/○/●/●/○/+/●/○/○/●/○/+]$_{T(SN)}$
[+/○/○/●/●/○/●/○/○/●/○/+]$_{T(SN)}$

[+/●/○/+/○/●/+/●/●/+/○/●/+/●/○/+]$_{T(SN)}$
[+/●/○/+/○/●/+/●/+/○/●/+/●/○/+]$_{T(SC)}$
[+/●/○/+/○/●/+/+/○/●/+/●/○/+]$_{T(SC)}$
[+/●/○/+/○/●/+/○/●/+/●/○/+]$_{T(SC)}$
[+/●/○/+/○/●/○/●/+/●/○/+]$_{T(SC)}$

[+/–/+/●/+/–/●/+/+/●/+/–/●/–/+/+]$_{T(NS)}$
[+/–/+/●/+/–/●/+/●/+/–/●/–/+/+]$_{T(NS)}$
[+/–/+/●/+/–/●/●/+/–/●/–/+/+]$_{T(NS)}$
[+/–/+/●/+/–/●/+/–/●/–/+/+]$_{T(NS)}$
[+/–/+/●/+/–/+/–/●/–/+/+]$_{T(NS)}$

[+/○/●/–/○/+/●/+/+/●/○/+/–/●/+/○]$_{T(NN)}$
[+/○/●/–/○/+/●/+/●/○/+/–/●/+/○]$_{T(NN)}$
[+/○/●/–/○/+/●/●/○/+/–/●/+/○]$_{T(NN)}$
[+/○/●/–/○/+/●/○/+/–/●/+/○]$_{T(NN)}$
[+/○/●/–/○/+/○/+/–/●/+/○]$_{T(NN)}$

Single angle-ply terminations!

Balanced plain weave material, *continued*….

Simplifications with respect to UD material

Only two parent classes are possible for laminates with balanced plain weave and standard ply angle orientations, in comparison to the 24 parent classes for UD material (or unbalanced weave):

Simple *E*-*B*-*S*-*T*

Figure 3 – Parent laminate classes with balance plain weave. Illustrations represent free thermal contraction responses; the response of the *E*-*B*-*S*-*T* coupled laminate ($B_{ij} \neq 0$) represents un-balanced plain weave since balanced plain weave is hygro-thermally curvatures stable, i.e. warp-free.

Mechanical Coupling in Balanced Plain Weave Laminates.

Coupling characteristics can be obtained from the parent laminate classes by applying off-axis material alignment (β)… the parent class with non-zero coupling [**B**] stiffness matrix is omitted here!

The form for the extensional [**A**] and bending [**D**] matrices may be either *Simple* or possess *Extension-Shearing* and *Bending-Twisting*, respectively. All matrices are Square symmetric!

Table 10 – Square symmetric forms of the Extensional [**A**] and Bending [**D**] stiffness matrices for Simple (β = *m*π/2) and coupled (β ≠ *m*π/2) behaviour (*m* = 0, 1, 2, 3).

Extensional [**A**]		Bending [**D**]	
Simple	*E-S*	Simple	*B-T*
$\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{11} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}$	$\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{11} & -A_{16} \\ A_{16} & -A_{16} & A_{66} \end{bmatrix}$	$\begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{11} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}$	$\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{11} & -D_{16} \\ D_{16} & -D_{16} & D_{66} \end{bmatrix}$

Table 11 – Summary on the number of ***Simple*** **laminates** for each ply number grouping, *n*, and the number of **quasi-homogeneous** or **fully isotropic laminates**, where $\alpha = \beta + \pi/4$.

n	Simple $\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$	Quasi-homogeneous $\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$	Fully Isotropic $\mathbf{A}_I\mathbf{B}_0\mathbf{D}_I$	Fully Isotropic stacking sequences
8	9	1	1	$[\alpha/\beta_2/\alpha/\beta/\alpha_2/\beta]_T = [45/0_2/45/0/45_2/0]_T$
9	26	1		
10	24	1		
11	76	5		
12	69	1	1	$[\alpha/\beta/\alpha/\beta_3/\alpha_3/\beta/\alpha/\beta]_T$
13	236	12		
14	214	7		
15	760	12		
16	696	7	7	$[\alpha/\beta_3/\alpha_4/\beta_2/\alpha/\beta_2/\alpha/\beta/\alpha]_T$

Quasi-homogeneity signifies that $\mathbf{A}^*$ (= A_{ij}/H) = $\mathbf{D}^*$ (= $12D_{ij}/H^3$)

Concluding Remarks

UD laminates:

Thin laminates must exploit non-symmetric and potentially unbalanced stacking sequence configurations to fully exploit the available design space.

Tapered laminate solutions have been demonstrated in non-symmetric laminates, whereby immunity to thermal warping and consistent mechanical coupling (or uncoupled!) properties are maintained.

Balanced plain weave laminate architecture ≡ TeXtreme™

Benchmark stacking sequences have been derived for *uncoupled* balanced plain weave laminates including those with either extensionally isotropic or fully isotropic properties.

All solutions (including those with non-zero coupling [**B**] stiffness matrices) possess immunity to thermal warping and therefore provide a robust manufacturing solution for integrating (complex) mechanical coupling response, as an enabling technology, in future smart materials and structures.

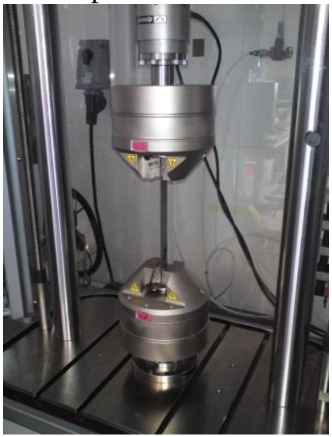
Concluding Remarks continued

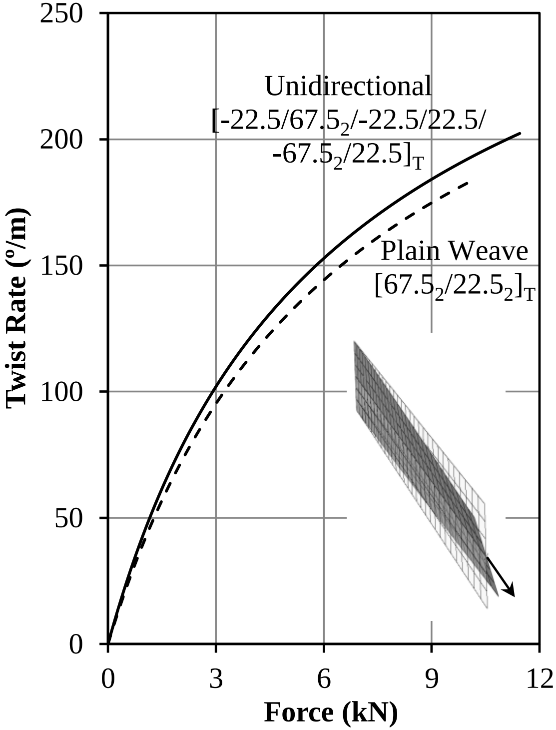
Prospects for exploitation of thin-ply technologies:

Whilst not explicitly stated in the foregoing, it is clear that thin-ply technologies will facilitate a significant reduction in overall laminate thickness and therefore allow an exponential increase in tailoring opportunities; this will bring design flexibilities found only in traditionally thick laminate construction into the thin laminate domain.

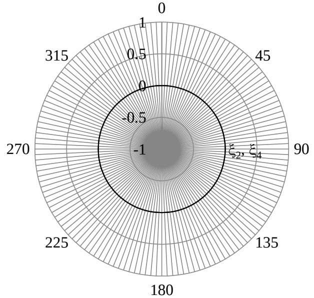
This statement is evident from details of the exploitable design spaces for the 4 Hygro-Thermally Curvature-Stable (or warp-free) laminate classes, e.g. where solutions exist only with 7 plies and above for Simple laminates and 14 plies and above for Extension-Shearing coupled laminates. Additionally, the relative increase in design flexibility for thickness tapering, as the minimum number of plies is increased, has been demonstrated for laminates with Extension-Shearing and/or Bending-Twisting coupling.

Instron E10000 Tension-Torsion test for Extension-Twisting coupled laminates.

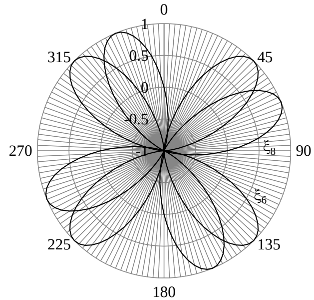




Lamination parameters:



A and **D** matrices



B matrix

Figure 4 – Twist Rate vs Axial Force simulations for *Extension-Twisting* coupled laminates with unidirectional and balanced plain weave of equal thickness. Maximum force applied corresponds to Tsai-Wu (first ply) failure prediction.

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