Are Estimates of Asymmetric First-Price Auction Models Credible?

Semi & Nonparametric Scrutinizations

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Abstract

Structural first-price auction estimation methods, built upon Bayesian Nash Equilibrium (BNE), have provided prolific empirical findings. However, due to the latent nature of underlying valuations, the assumption of BNE is not feasibly testable with field data, a fact that evokes harsh criticism on the literature. To respond to skepticism regarding credibility, we provide a focused answer by scrutinizing estimates derived from experimental asymmetric auction data in which researchers observe valuations. We test the statistical equivalence between the estimated and true value distributions. The Kolmogorov-Smirnov test fails to reject the distributional equivalence, strongly supporting the credibility of structural asymmetric auction estimates.

Keywords and Phrases:

Empirical Auction, Asymmetric Auction, Risk Aversion, and Semi/Noneparametric Estimation

JLE Classifications:

C13 - Estimation: General and D44 - Auctions

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1 Introduction

By scrutinizing the estimates derived from the experimental auction data in which researchers observe laboratory-assigned valuations, and by statistically testing the equivalence between estimated and observed valuations, this research establishes the credibility of broadly used asymmetric first-price auction estimates, both structural and semi/nonparametric, which has not been reported in the literature.[1]

In empirical first-price auction literature, researchers are interested in describing strategic interactions among bidders for understanding and designing auction markets based on underlying economic incentives. Traditionally, reduced-from regressions had been used, despite the restriction on linearity had been the hindrance to describe strategic behaviors and to obtain profound insights. In order to overcome the linearity restriction, structural[2] and nonparametric estimation methods arose and have been widely used over the last twenty years for investigating testable implications and drawing policy recommendations supported by counterfactual experiments.[3] In addition, due to the ubiquity of asymmetry among bidders, the estimation methods are extended to asymmetric auctions. In such empirical auction research, estimated valuations are particularly important to both researchers and industry practitioners as market designs, such as setting reserve prices or detecting collusions, essentially depend on empirical estimates. As a result, asymmetric auction estimates now come to serve as the vital foundations of many auction market research for addressing numerous positive and normative questions.

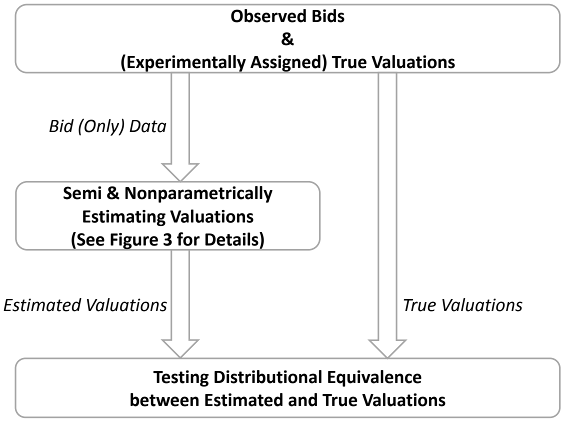
However, while more and more asymmetric first-price auction estimates are reported in the literature, there is a fundamental difficulty in evaluating the performance of these estimates. Structural methods, in which researchers strictly assume that observed bids are derived from the Bayesian Nash equilibrium, estimate the bidders' valuations. However, despite the fact that the comparison between estimated and true valuations is essential in accurately measuring the performance of estimates, the truth is *bidders' valuations are latent* in empirical first-price auctions. This latent nature of bidders' valuations in empirics lead to an infeasible

[1]In this paper, we repeatedly use the terminology of "performance" and "accuracy" to mean the results of statistical tests that compare the distributions of estimated and true valuations. Specifically, we use the two-sample Modified Kolmogorov-Smirnov test.

[2]Given the assumption of Bayesian Nash Equilibrium (BNE), the structural elements are potentially heterogeneous von-Neumann-Morgenstern (vNM) payoff functions and potentially affiliated bidders' value distributions.

[3]The cornerstone work in the dawn of empirical and structural first-auction literature should be credited. To the best of our knowledge, the literature was initiated by the contribution made by the Ph.D. thesis of Paarsch (1992) [65] with parametric models. Donald and Paarsch (1993, 1996) [22] [23], Elyakime, Laffont, Loisel, and Vuong (1994) [24], Laffont, Ossard, and Vuong (1995) [46] established statistically rigorous yet flexible parametric estimation methods. The survey paper of Hendricks and Paarsch (1995) [30] and Perrigne and Vuong (1999) [67] concisely illustrate the early contributions in the literature.

Figure 1: Research Outline



comparison between estimated and true valuations.[4] Such an infeasible comparison, along with the rigid Bayesian Nash equilibrium assumptions for describing bidders' behavior, then becomes the target of harsh criticism and skepticism on empirical auction estimates.[5]

Against such criticism and skepticism, Bajari and Hortaçsu (2005) [12] provide a concrete and focused[6] response by using symmetric first-price auction data from laboratory study in which researchers observe

[4]This difficulty has been widely recognized from the beginning of the literature. The survey of Hendricks and Paarsch (1995) [30] concisely summarizes the difficulty as "*The difficulty with field data.....is that neither the valuations of potential buyers nor the probability law determining these valuations is observed by the researcher.*" In addition, McAfee and Vincent (1992) [56] commented the difficulty of unobserved valuation (signal) as "*The most obvious roadblock to test auction theory is the heavy use made of unobservables in the theory. Bidders choose optimal bids based on signals that are not observed by econometrician studying auction behavior.*"

[5]There are at least two sorts of reported criticism. The first is made by the group of robust mechanism design researchers who have the skeptical view on the bidders' abilities to find Beyesian Nash Equilibrium, especially in asymmetric auctions, that is the solutions of intricate best-response functions. These theoretical researchers claim bidders are not able to find BNE and suggest to use weaker equilibrium concepts, such as the prior-distribution-free iterations of dominated-strategy eliminations in second price auctions, to design auction markets. See Wilson (1987) [76] for the critique on BNE. The second criticism comes from the group of labor economics researchers who claim assumptions made for structural analyses are implausibly strong. For example, Angrist and Pischke (2010) [4] describe structural elements as "*superstructure of assumptions*" and "*industrial disorganization*" and suggest to use reduced-from analyses of field-experiment data to obtain policy implications. As Bajari and Hortaçsu (2005) [12] mention, these skepticism are not without merit.

[6]Here, we use the word "focused" as Bajari and Hortaçsu (2005) [12] use laboratory experimental data, which has a more simplified framework compared to field auctions.

experimentally-assigned true valuations, as depicted in Figure 1.[7] By using the valuations observed in the experiment as a benchmark, they compare the estimates generated by various structural models with nonparametric estimation methods. Their analyses shows that estimates based on the Constant Relative Risk Averse (CRRA) Bayes-Nash model can recover the distributions of latent valuations in symmetric auctions with a statistically acceptable degree of accuracy,[8] at least under but not limited to a laboratory environment, and demonstrates the great potential of structural and nonparametric auction estimation methods.

This research contributes to the empirical auction literature by extending the seminal symmetric first-price auction study of Bajari and Hortaçsu (2005) [12] to an asymmetric auction framework. Given the ubiquity of asymmetry among bidders in real-world auctions, prevalence of structural asymmetric first-price auction studies over the last 10 years,[9] and difficulties reported in theoretical asymmetric auction literature,[10] it is our belief that establishing the credibility of asymmetric auction estimates by testing the performance and pointing out the potential improvements are invaluable, as they crucially matter to auction research and auction market design policies. Following the precedent set by Bajari and Hortaçsu (2005) [12], we likewise use laboratory data as it is able to capture insightful views on the performance of auction estimates. Both strengths and shortcomings of estimates are explicitly detected with laboratory data, and such findings are essential for improving the performance of asymmetric first-price auction estimates. Also, despite the fact that empirical asymmetric auctions have been and continue to be actively investigated, to the best of our knowledge, direct evaluations of asymmetric auction estimates have not previously been investigated in the

[7] In a typical auction laboratory experiment, valuations of an object is exogenously and randomly assigned to bidders, often by using computer-generated random numbers, and researchers are able to observe valuations.

[8] In Bajari and Hortaçsu (2005) [12], the accuracy of estimates is statistically supported by the two-sample Modified Kolmogorov-Smirnov test.

[9] The nonparametric and structural auction estimation methods are extended to an asymmetric auction framework by Campo, Perrigne, and Vuong (2003) [17]. Numerous empirical asymmetric auction studies follow. The incomplete list of such empirical asymmetric auction studies includes Flambard and Perrigne (2006) [25] for Canadian snow removal contracts, Marion (2007) [57] for bid preference in Californian highway procurement, Lu and Perrigne (2008) [55] for US Forest Service timbers, Krasnokutskaya (2011) [43] for Californian highway procurement, Krasnokutskaya and Seim (2011) [44] for bid preference in Californian highway procurement, Campo (2012) [16] for Californian construction procurement, Nakabahashi (2013) [63] for small businesses set-aside in Japanese construction procurement, and Balat (2013, working paper) [14] for dynamic analysis of Californian construction procurement. All of the empirical asymmetric auction studies use estimated latent valuations to investigate the policy implications in which performances (accuracy) of estimated valuables are crucial for answering important policy questions. By establishing the credibility of asymmetric auction estimates, our research indirectly supports these empirical studies and findings.

[10] Krishna (2009) [45], one of the most popular auction-theory textbooks, describes the theoretical difficulties in extending symmetric auction results to asymmetric environments as "*Much of the theory developed in the symmetric case is fragile and does not extend to situations in which bidders are asymmetric.*"

literature, and it is these estimated valuations that market designs heavily reply upon.

Specifically, we investigate the performance of de-facto standardly used asymmetric first-price auction estimation methods introduced by Isabelle Perrigne, Quang Vuong, and their coauthors.[11] We choose these methods since, due to the versatility in allowing asymmetry in value distributions and computational tractablity, they are now the standard used by numerous empirical works for investigating auction markets.

For this investigation, we use the unique dataset from the asymmetric private value first-price auctions collected in the laboratory experiment conducted by Chernomaz (2012) [20]. The data contains submitted bids and laboratory-assigned valuations for each bidder in a repeatedly-conducted experiment. Additionally, Chernomaz (2012) [20] investigates the effects caused by asymmetry among bidders under exogenously changing auction environments, while the majority of bidder valuations remain fixed before and after such exogenous changes. Figure 2 visually illustrates the details of such exogenous changes and repetition in the laboratory environment.

Given such exogeneity and repetition in the laboratory experiment, this is the bottom line of our estimations strategy: exploiting exogenous changes in auction environments to identify both bidders' payoff functions and underlying value distributions, as bid distributions vary before and after the exogenous change while the valuations that bidders hold remain unchanged. We construct, then estimate, the *compatibility conditions* based on such exogenous changes. For empirical investigation, this bottom line ideas allows us to scrutinize the accuracy of derived estimates.

The primary analytic methodology employed in our research straightforwardly follows those implemented in Bajari and Hortaçsu (2005) [12], as depicted in Figure 1, yet we newly extend their analyses to three empirically important dimensions.[12] The first extension is that we investigate asymmetric value distributions

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[11] Specifically, we investigate the accuracy of structural and semi/nonparametric estimation methods for asymmetric auctions with exogenous variations proposed by Campo, Perrigne, and Vuong (2003) [17], Guerre, Perrigne, and Vuong (2009) [28], and Campo, Guerre, Perrigne, and Vuong (2011) [18]. In the literature, these methods are proposed and evolve as follows: based on the cornerstone work of Guerre, Perrigne, and Vuong (2000) [27] that proposes a method for symmetric first-price auctions, Campo, Perrigne, and Vuong (2003) [17] extend the nonparametric estimation method to asymmetric auctions in which bidders draw their valuations from asymmetric distributions. In addition, Guerre, Perrigne, and Vuong (2009) [28], and Campo, Guerre, Perrigne, and Vuong (2011) [18] broaden the estimation method to allow both homogeneous and heterogeneous risk-averse preferences among bidders.

[12] Bajari and Hortaçsu (2005) [12] also conduct the analyses of Adaptive Learning and Quantal Response Models. We exclude these models from this research as they are not popularly used in empirical auction literature, although we recognize these models have intriguing aspects for understanding bidding behavior.

among bidders. Second, we allow and test potential affiliations among underlying value distributions as empirical researchers seldom have priori knowledge of the independence of underlying valuations. The last, but not least, extension is that, in addition to the semi-parametric models, we investigate the nonparametric von-Neumann-Morgenstern (vNM) functions that allow flexibility of bidders' risk preferences.

Based on the comparisons between estimated and true private valuations, we report these main conclusions: (1) the risk-neutral model assumption, which is often assumed for simplicity and tractability in the literature, tends to inflate estimated valuations; (2) the assumption of risk-averse bidders is indispensable as it enables nonnegligible improvements in the accuracy of estimates; (3) among semi and nonparametric risk-averse models, the nonparametric model with conventional-wisdom-based shape restrictions provides the most accurate results;[13] (4) when advanced risk-averse models are employed, the two-sample Modified Kolmogoro-Smirnov test fails to reject the statistical equivalence between the estimated and tue value distributions, positively supporting the empirical findings reported in the empirical asymmetric auction literature; (5) estimated value distributions of stochastically-dominated bidders are relatively more accurate compared to those of stochastically-dominating bidders;[14] and (6) if the true data generating process is independent private value, the assumption of affiliated or independent private value only create a negligible difference in accuracy, strongly encouraging the usage of affiliated private value models originally proposed by Li, Perrigne, and Vuong (2002) [52] in any empirical auction research. While degrees of accuracy and improvements differ by applications, the facts we find in this research are widely extendable to any empirical asymmetric auction research. Additionally, these findings are achieved through relatively small sample size, keeping them in line with most empirical auction research that also have restrictions in terms of sample size available to researchers.

The paper is organized as follows: Section 2 illustrates the experimental data that contains both valuation and bid information; Section 3 explains the theoretical auction models that are the basis for structural estimations; Section 4 describes the semi and nonparametric asymmetric auction estimation methods that are used to generate estimates; Section 5 visually reports the estimation results then statistically tests the performance of asymmetric auction estimates; and lastly, Section 6 provides the external validity and conclusions.

[13]We employ the nonparametric sieve estimation method with shape restrictions in which restrictions are based commonly accepted economic theory. Specifically, we restrict the lower bound of slope on nonparametrically estimating functions in the regions where identifications are challenging.

[14]As Bajari and Hortaçsu (2005) [12] mention, and as auctions in the laboratory could differ from those in the real-world, the strong caveat to the external validity (validity outside the reach of the experimental laboratory) should be explicitly noted. We will discuss the external validity in the conclusion section.

2 Data Descriptions

In this section, we illustrate the laboratory auction data used to obtain results described in the empirical section. Key to this section is that bidders in experiments participated in two exogenously varying formats of auctions while the majority (two out of three, as illustrated in Figure 2) of their valuations remained unchanged. Such exogeneity provides us with the exclusion restrictions and enables us to identify both risk-averse vNM payoff functions and value distributions in the empirical section. With the emphasis on such exogenous change, we first describe the laboratory auction procedures, then explicate the summary statistics for illustrating the differences in bidding behavior before and after the change.

The data is from Chernomaz (2012) [20], which investigates the results of joint bids in independent private value first-price auctions.[15] The participants were recruited from undergraduate students at Ohio State University and paid a $6 show-up fee. There were three experiment runs (denoted as Experiment Run I, II, and III), conducted at different times, and participants were not allowed to join more than one experiment run. A computer-based laboratory was used for this experiment, and participants interacted only through computer screens. Table 1 summarizes the number of participating bidders and observed bids in each experiment run. At the beginning of each experiment run, bidder types (joint and solo) were randomly assigned, and every participant remained the same type throughout the experiment run. Thus, a participating bidder kept playing the same type throughout an entire experiment run. In each experiment run, bidders initially experienced two practice rounds, then they participated in twenty-four rounds involving monetary incentives. Figure 2 depicts the stages within each round.[16] At the beginning of each round, participating bidders were randomly matched to form a three-bidder group. Out of three within a matched group, two were from the pool of joint-type bidders, and the remaining one was from the pool of solo-type bidders. Then, valuations were drawn from i.i.d. uniform distribution $U[\$0, \$18.75]$, denoted by v_1 to a joint-type bidder, v_2 to another joint-type bidder, and v_3 to a solo-type bidder, as depicted in Figure 2. Within each round, there were symmetric- and asymmetric-auction stages. In a symmetric-auction stage, three bidders submitted one bid

[15]A joint bid (also known as a consortium bid) is defined as two or more bidders who form a group and submit one joint (consortium) bid in an auction. Joint bids were allowed in Mexico and Louisiana Gulf Outer Continental Shelf (OSC) wildcat auctions, and as a result the implications of joint bids are now intensively scrutinized in empirical auction literature. Hendricks and Porter (1998) [35], Campo, Perrigne, and Vuong (2003) [17], and Hendricks, Pinkse, and Porter (2003) [37] investigate joint bids in wildcat auctions and the associated asymmetry in available economic resources. Note that our laboratory procedures in Figure 2 can be viewed as a miniature of hypothetical wildcat auctions in which both (non-collusively) individual and joint bids are allowed to submit to an auction where an auctioneer randomly determines whether or not to allow a joint bid.

[16]In summary, there were three experiment runs (I, II, and III). In each run, there were twenty-four rounds, excluding the two practice rounds. In each round, there were several stages, including symmetric- and asymmetric-auction stages, as depicted in Figure 2.

each (denoted as b_1, b_2, and b_3), yet the outcome of a symmetric-stage auction was not announced until the result-announcement stage. Next, at the beginning of an asymmetric-auction stage, the two joint-type bidders aggregated their valuations as $\max\{v_1, v_2\}$.[17] In an asymmetric-auction stage, a solo-type bidders submitted a bid b_{Solo}, based on her valuation of v_3. On the other hand, each joint-type bidder submitted a respective bid, based on the aggregated valuation of $\max\{v_1, v_2\}$.[18] At an asymmetric-auction stage, these two joint-type bids (denoted as $b_{1,\text{Joint}}$ and $b_{2,\text{Joint}}$) were separately submitted by each joint-type bidder; then the experiment organizer (i.e. the auctioneer) randomly chose one of them with equal probability (described as 50% and 50% in Figure 2) to be the chosen joint-type bid.[19] At the result-announcement stage, within-a-matched-group results, including assigned valuations (v_1, v_2, and v_3), aggregated valuation ($\max\{v_1, v_2\}$), bids in each stage (b_1, b_2, b_3, chosen b_{Joint}, and b_{Solo}), and winning/losing statuses in each stage were announced to the matched group members, yet the identities of bidders were kept hidden. Therefore, the participants in experiment played against anonymous opponent bidders. In addition, monetary payoffs were calculated and added to each participating bidder's account.[20,21] Lastly, at the end of each round, a matched group was dissolved,

[17] This way of value aggregation, adopting a maximum valuation among joint-type bidders, corresponds to the empirical observations that joint (consortium) bidders share their economic resources, such as the most available cost-saving technology, the closest geographical locations, and the information of best-available resale opportunities. In our experiment, this means that one of the joint-type bidders kept having the same valuation before and after the exogenous value aggregation, and this invariant nature is exploited in the estimation section.

[18] In an asymmetric-auction stage, the two within-a-matched-group joint-type bidders are informed of their aggregated valuation (i.e. $\max\{v_1, v_2\}$) through their respective computer screens. However, verbal or textual communication between joint-type bidders was strictly forbidden. Therefore, a bid made by a joint-type bidder in an asymmetric stage (i.e. $b_{1,\text{Joint}}$ or $b_{2,\text{Joint}}$ in Figure 2) was derived from a single-agent payoff-maximization problem. This single-agent decision nature is advantageous for estimating preferences among bidders in the estimation section.

[19] This randomized choice among joint-type bids (i.e. among $b_{1,\text{Joint}}$ and $b_{2,\text{Joint}}$) with equal probability was designed for investigating behavioral difference in bidding behavior between a consortium-leader-firm-like joint-type bidder whose valuation draw was originally $\max\{v_1, v_2\}$ and a consortium-follower-firm-like joint-type bidder whose valuation draw was originally $\min\{v_1, v_2\}$. We encourage interested researchers to see Chernomaz (2012) [20] for further details on the experiment design and behavioral differences.

[20] Monetary payoffs were calculated as follows. After a result-announcement stage, the experiment organizer (i.e. the auctioneer) randomly selected with equal probability an auction stage in which an outcome was actually paid. Note that as far as bidders' vNM functions are additively separable, which is usually assumed and accepted in auction literature, this random selection does not affect bidders' payoff-maximization problems in each auction stage. This randomized selection process was empirically motivated by the factual observation of timber auctions in which the U.S. Forest Service randomized different auction rules. Lu and Perrigne (2008) [55] and Athey, Levin, and Seira (2011) [10] exploit such randomization of timber auctions for detailed investigations of identifications and bidding behavior.

[21] In addition, if an asymmetric-stage auction was selected (by the experimental organizer) and if a chosen (by the experimental organizer) joint-type bid exceeded a solo-type bid, the payoff of the joint-type bidders was equally split. This means that, conditional

and participants returned to the pool of bidders.

Results were announced only to matched-group members at the end of each round, and the results of a specific matched group were NOT available to members of any other groups. As a natural consequence, we observe sizable learning and adjusting behavior in the first half of the rounds in each experiment run. For investigating the strategic interactions and estimates of valuations without concern for the learning effect, this research excludes the data from the first half of the rounds, and only the data from the second half of the rounds is used in the empirical investigations. Table 1 summarizes the sample size in each auction stages.

Table 2 lists the summary statistics of observed bids in both symmetric- and asymmetric-auction stages. In theory, bidders are predicted to bid less in asymmetric-auction stages, as symmetric-auction stages consist of three bidders while asymmetric-auction stages consist of only two bidders (i.e. a chosen joint-type bidder and a solo-type bidder). In most of the experiment runs, both joint- and solo-type bidders decreased their bids in asymmetric-auction stages, yet we observe a small increase in bids in Experiment Run II that could negatively affect the performance of structural estimations. Figure 3 depicts the pairs of symmetric-auction stage bids and chosen (and also announced announced) asymmetric-auction stage bids in each experiment run for which we later apply apply kernel density estimations. Although we observe the slight rounding-up/down effect (eg. bidders tend to bid in increments of five -\$0, \$5, \$10, and \$15), this does not seem to severely affect the estimates of distributional functions as, overall, bids appear widely spread out.[22]

on being selected, chosen, and winning, each joint-type bidder's payoff was $\frac{1}{2}(\max\{v_1, v_2\} - b_{\text{Joint}}^{\text{chosen}})$ as illustrated in Figure 2, where $b_{\text{Joint}}^{\text{chosen}}$ is a chosen joint-type bid. Although this splitting rule does not affect the risk neutral and CRRA models, it affects the estimation methods of joint-type bidders' risk preferences for CARA and nonparametric models. See Appendix for details.

[22]As this research emphasizes the empirical perspective of asymmetric auction data, we refrain from examining the data of laboratory-assigned valuations until Estimation Result section. Nonetheless, the plots of laboratory-assigned valuations and bids are reported in Appendix, and we observe large overbidding (bidding more than risk neutral BNE) in our laboratory auction data.

Figure 2: Stages within a Round

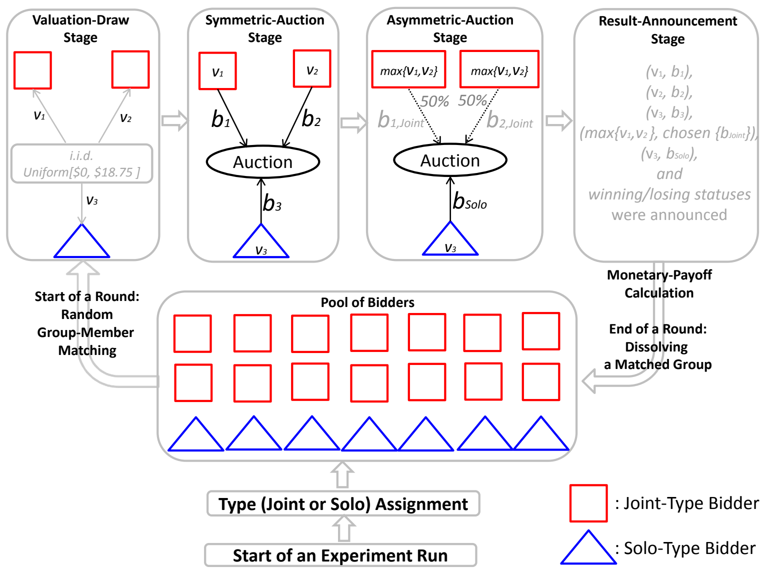


Table 1: Sample Size

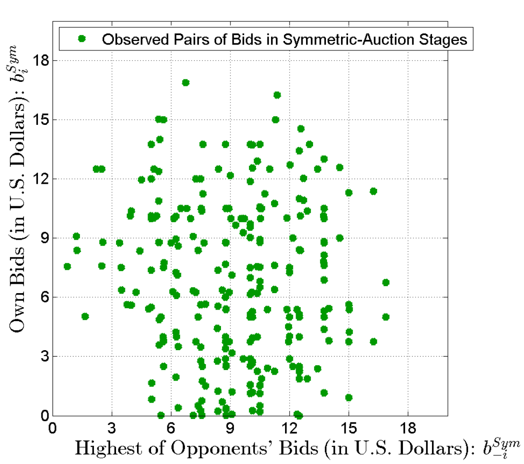
		Number of Participants	Symmetric-Auction Stage Bids Observed	Asymmetric-Auction Stage Bids Observed
Experiment Run I	Joint Type	14 Bidders	168 Bids (84 Chosen Bids)	84 Bids
	Solo Type	7 Bidders	84 Bids	84 Bids
Experiment Run II	Joint Type	16 Bidders	192 Bids (96 Chosen Bids)	96 Bids
	Solo Type	8 Bidders	96 Bids	96 Bids
Experiment Run III	Joint Type	12 Bidders	142 Bids (72 Chosen Bids)	72 Bids
	Solo Type	6 Bidders	72 Bids	72 Bids

Table 2: Summary Statistics of Bid Data (in U.S. Dollars)

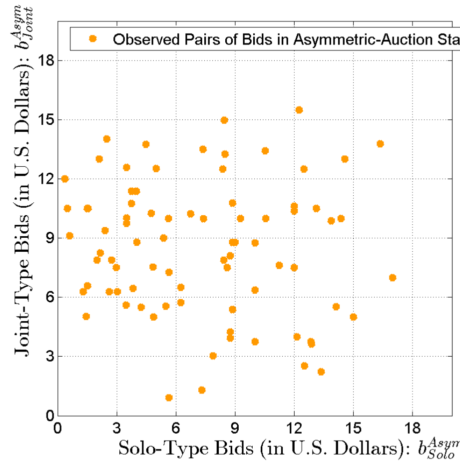
			Mean	Standard Deviation	Quantile 10th	25th	50th	75th	90th
Experiment Run I	Joint Type	Symmetric	8.77	3.13	5.00	6.38	8.88	10.50	12.62
		Asymmetric	8.17	3.22	4.22	5.53	7.84	10.38	12.55
		Difference	0.60		0.78	0.85	1.04	00.13	00.07
	Solo Type	Symmetric	7.09	4.38	1.57	3.44	6.94	10.69	12.63
		Asymmetric	7.01	4.47	1.50	3.49	6.50	10.28	13.15
		Difference	0.08		0.08	-0.05	0.44	0.41	-0.52
Experiment Run II	Joint Type	Symmetric	9.54	3.75	5.00	6.28	9.99	12.40	14.98
		Asymmetric	9.58	3.95	4.75	6.76	9.28	12.34	15.00
		Difference	-0.04		0.25	-0.49	0.71	0.06	-0.02
	Solo Type	Symmetric	6.79	4.60	1.02	2.58	6.69	11.06	12.96
		Asymmetric	6.92	4.64	0.75	2.56	7.22	10.93	13.75
		Difference	-0.13		0.27	0.01	-0.54	0.14	-0.79
Experiment Run III	Joint Type	Symmetric	10.60	4.30	3.16	7.55	11.49	14.18	15.79
		Asymmetric	10.22	4.39	3.16	7.38	11.25	13.64	15.63
		Difference	0.38		0.00	0.17	0.24	0.54	0.17
	Solo Type	Symmetric	8.52	4.88	2.03	4.53	8.61	12.78	15.27
		Asymmetric	8.43	4.85	2.03	4.53	8.31	12.75	15.08
		Difference	0.09		0.00	0.00	0.30	0.03	0.19

Figure 3: Observed Bids: LEFT - Symmetric Auction Stages; RIGHT - Asymmetric-Auction Stages

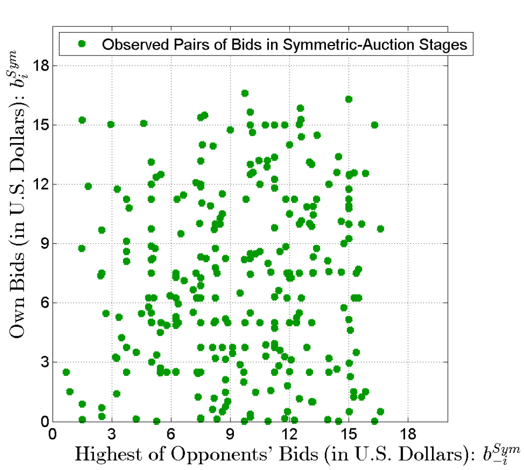
(a) Experiment Run I: b_{-i}^{Sym} and b_i^{Sym}



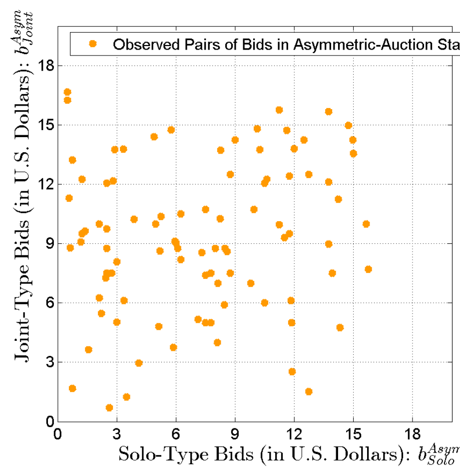
(b) Experiment Run I: $b_{\text{Solo}}^{\text{Asym}}$ and $b_{\text{Joint}}^{\text{Asym}}$



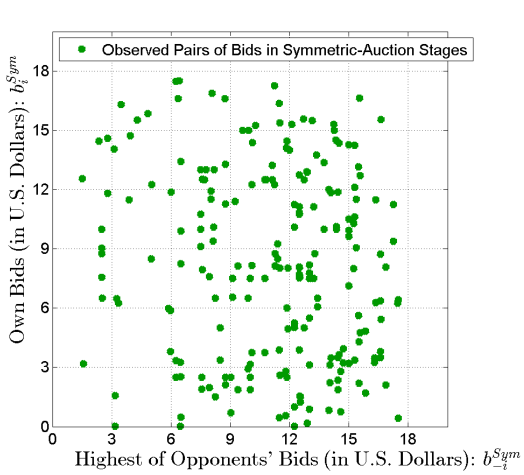
(c) Experiment Run II: b_{-i}^{Sym} and b_i^{Sym}



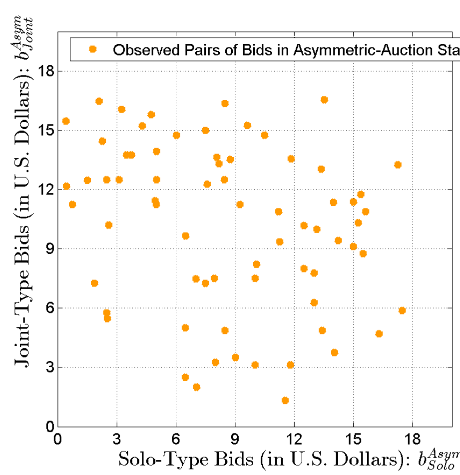
(d) Experiment Run II: $b_{\text{Solo}}^{\text{Asym}}$ and $b_{\text{Joint}}^{\text{Asym}}$



(e) Experiment Run III: b_{-i}^{Sym} and b_i^{Sym}



(f) Experiment Run III: $b_{\text{Solo}}^{\text{Asym}}$ and $b_{\text{Joint}}^{\text{Asym}}$



3 Auction Models

This section describes the theoretical models of an affiliated private value (APV) auction that include an independent private value (IPV) auction as a special case. Although the bid data used in this research is generated from the experiments of IPV auctions, APV models are initially employed for the following empirically pragmatic considerations.[23] In many empirical investigations, researchers seldom have enough prior evidence to determine the independence of underlying value distributions. Given the pervasiveness of insufficient initial information, therefore, it is empirically prudent for researchers to first estimate latent valuations with a model that can allow potential affiliations, then test independence. Formal and statistical tests for distributional independence are demonstrated in the Estimation Result section.[24] These cautious model-specification procedures are also based on the robustness requisition proposed by the well-known and influential critique by Leamer (1983) [47], which is largely concerned with the credibility of estimates in general empirical research due to the lack of robustness in changes to model assumptions.[25] With the emphasis on generality in modeling assumptions, we first explain the symmetric auction models, then illustrate the asymmetric auction models.

3.1 Symmetric Auction Models

A single and indivisible object is sold in an auction to N bidders who have the von-Neumann-Morgenstern (vNM) function $U(\cdot)$ that is twice differentiable with $U'(\cdot) > 0$ and $U''(\cdot) \leqslant 0$ to allow potential risk aversion. As a vNM function is unique up to the positive-affine transformation, the normalization of $U(0) = 0$ and $U(1) = 1$ is imposed without loss of generality. For the same of clear notations, we use a capital letter for describing a random variable and a lower-case letter for describing the realization of a random variable. Assuming that $N = 3$ bidders in an auction with index $i \in \{1, 2, 3\}$, bidders draw private valuations $\{v_1, v_2, v_3\}$ from the potentially affiliated joint distribution $F_{V_1,V_2,V_3}(v_1, v_2, v_3)$. The arguments of the joint distribution are exchangeable in its $N = 3$ elements, meaning that the model is distributionally symmetric.[26] Given that

[23] Surprisingly, APV models provide slightly more precise estimates compared to those derived from IPV models, and we will investigate the reasons behind this in the Estimation Result section.

[24] Given the existence of behavioral bidders who may not strictly play BNE, the independence of observed bids does not imply the independence of valuations (vice versa).

[25] See the recent development on the Leamer Critique argued in Angrist and Pischke (2010) [4] and Sims (2010) [74] for a detailed description of model robustness.

[26] This setting is called the symmetric affiliated private value (APV) model, and its econometric identification and rationalizability are intensively investigated in Li, Perrigne, and Vuong (2002) [52]. By the Monte Carlo simulations, they investigate the problem of misspecification, estimating distributions of valuations under the IPV assumption when the true data-generating process is APV. They report that such misspecification tends to result in overestimation of private values, specifically in the upper domain of private values. This overestimation is caused by the neglect of modeling a strategic behavior in an APV environment; if a bidder

other bidders employ a symmetric equilibrium strategy, bidder i's expected payoff maximization problem is

$$\max_{b_i} U(v_i - b_i) \cdot F_{Y_{-i}|V_i}(\phi(b_i)|v_i),$$

where Y_{-i} is a random variable of the highest valuations among opponent bidders with its realization $y_{-i} = \max_{j \neq i} v_j$, and $\phi(\cdot)$ is the inverse of a symmetric equilibrium bidding function. In addition, $F_{Y_{-i}|V_i}(y_{-i}|v_i)$ denotes the conditional distribution function of Y_{-i} given v_i. The first-order necessary condition[27] of the above maximization problem, where a bidder equates marginal cost and benefit of changing her bid, can be written in the form of

$$v_i = b_i + \underbrace{\lambda^{-1}\left(\frac{F_{Y_{-i}|V_i}(\phi(b_i)|v_i)}{\frac{F_{Y_{-i}|V_i}(\phi(b_i)|v_i)}{dy_{-i}} \cdot \phi'(b_i)}\right)}_{\text{Shading Function}},$$

where, according to the tradition of empirical auction literature, we define $\lambda(\cdot) \equiv U(\cdot)/U'(\cdot)$ and $\lambda^{-1}(\cdot)$ as a corresponding inverse function. We refer to a function $\lambda^{-1}(\cdot)$ as a *shading function* after its role of describing the difference between a valuation and a bid. We also refer to the argument of the shading function as a *reciprocal factor*.[28] Since in field-auction data we cannot empirically observe latent valuations that appear in the argument of the shading function, we now need to replace unobserved valuations with observed bids. We denote that B_{-i} is a random variable of a highest bid among opponent bidders with its realization $b_{-i} = \max_{j \neq i} b_j$. In addition, we denote the conditional distribution of B_{-i} given b_i as $G_{B_{-i}|B_i}(b_{-i}|b_i)$ and its derivative as $g_{B_{-i}|B_i}(b_{-i}|b_i)$. By assuming bidder i also employs a symmetric equilibrium bidding strategy, the probabilistic relation between observable bids and latent valuations is $G_{B_{-i}|B_i}(x|b_i) = F_{Y_{-i}|V_i}(\phi(x)|v_i)$. Moreover, by the fundamental theorem of calculus, the conditional density is obtained as $g_{B_{-i}|B_i}(x|b_i) = \big(dF_{Y_{-i}|V_i}(\phi(x)|\phi(b_i))/dy_{-i}\big) \cdot \phi'(x)$. Then, by using these probabilistic relations that bridge the unobservable

has a high valuation, other bidders are also likely to have high valuations, and she needs to bid aggressively. Conversely, we in this research investigate the empirically prudent misspecification with the laboratory data, estimating value distributions under the APV assumption when the true data-generating process is IPV. We are happy to report that empirical asymmetric auction estimates in our research are robust (and even more accurate) against such empirically prudent misspecification, as we explain in the Estimation Result section.

[27]Throughout this research, we assume that second-order conditions are satisfied.

[28]This slightly awkward naming comes after the fact that Guerre, Perrigne, and Vuong (2009) [28] use the notation R to express this argument (see p.1202 of their paper for details), yet they do not provide a memorable name for this object. *"R"eciprocal factor* represents a quotient of "probability of winning" divided by "marginal probability of winning." It is actually the reciprocal of semi-elasticity $\equiv [(dh(x)/dx)/h(x)]$ where $h(x)$ is a winning probability and dx is a change in bid.

to the observable, the first-order necessary condition can be equivalently written as

$$v_i = b_i + \lambda^{-1}\left(\frac{G_{B_{-i}|B_i}(b_i|b_i)}{g_{B_{-i}|B_i}(b_i|b_i)}\right), \tag{1}$$

where components of the right-hand side of the equation are observable or estimatable. Furthermore, as the distributional functions in the right hand side of equation (1) share the same conditional variable, we exploit the definitions of conditional density and distribution functions, as suggested by Li, Perrigne, and Vuong (2002) [52], $g_{B_{-i}|B_i}(z|b_i) = g_{B_{-i},B_i}(z,b_i)/g_{B_i}(b_i)$ and $G_{B_{-i}|B_i}(x|b_i) = \left(\int_{\underline{b}}^{x} g_{B_{-i},B_i}(z,b_i)dz\right)/g_{B_i}(b_i)$ where $\underline{b}$ is the lower bound of bid distribution. Thus, the equation (1) can be re-written in an unconditional fashion as

$$v_i = b_i + \lambda^{-1}\left(\frac{\int_{\underline{b}}^{b_i} g_{B_{-i},B_i}(z,b_i)dz}{g_{B_{-i},B_i}(b_i,b_i)}\right). \tag{2}$$

For the sake of organized notations for later empirical use, we introduce the convenient terms of $\Gamma(x,b_i|g_{B_{-i},B_i}) = \int_{\underline{b}}^{x} g_{B_{-i},B_i}(z,b_i)dz$ and of reciprocal factor as $R[x,y|\Gamma_{B_{-i},B_i}, g_{B_{-i},B_i}]$ $= \Gamma(x,y|g_{B_{-i},B_i})/g_{B_{-i},B_i}(x,y)$. Given these simplified notations, we can denote the first-order necessary condition of symmetric auction as

$$v_i^{\text{Sym}} = b_i^{\text{Sym}} + \lambda^{-1}\left(\underbrace{R^{\text{Sym,Affi}}\left[b_i^{\text{Sym}}, b_i^{\text{Sym}}\middle|\Gamma_{B_{-i},B_i}^{\text{Sym,Affi}}, g_{B_{-i},B_i}^{\text{Sym,Affi}}\right]}_{\text{(i)}}\right), \tag{3}$$

where the upper indices of "Sym" and "Sym,Affi" emphasize that the auction model is symmetric and of affiliated value. In addition, if a researcher further assumes the independence of private valuations, the bivariate functions are simplified as $g_{B_{-i},B_i}(z,b_i) = g_{B_{-i}}(z)$ and $\int_{\underline{b}}^{x} g_{B_{-i},B_i}(z,b_i)dz = G_{B_{-i}}(x)$. Accordingly, first-order necessary condition equation (3) becomes the well-known equation in empirical auction literature,

$$v_i = b_i + \lambda^{-1}\left(\frac{G_{B_{-i}}(b_i)}{g_{B_{-i}}(b_i)}\right). \tag{4}$$

Lastly, by denoting a reciprocal factor as $R[x|G_{B_{-i}}, g_{b_{-i}}] = G_{B_{-i}}(x)/g_{B_{-i}}(x)$, we can write the first-order necessary condition as

$$v_i^{\text{Sym}} = b_i^{\text{Sym}} + \lambda^{-1}\left(\underbrace{R^{\text{Sym,Inde}}\left[b_i^{\text{Sym}}\middle| G_{B_{-i}}^{\text{Sym,Inde}}, g_{B_{-i}}^{\text{Sym,Inde}}\right]}_{\text{(ii)}}\right), \tag{5}$$

where the upper indices of "Sym" and "Sym,Inde" emphasize that the auction model is symmetric and of independent value.

3.2 Asymmetric Auction Models

We next introduce asymmetric auction models. In order to be notationally minimalistic, we henceforward focus on the simplest environment, a two-type and two-bidder asymmetric auction on which our experimental asymmetric auction data is based. We define the index of the bidder type as $t \in \{\text{Joint}, \text{Solo}\}$. By exploiting the two-type-two-bidder nature, we use a convenient notation of $-t$ for representing an opponent bidder's type. We denote $U_t(\cdot)$ as a vNM function of type t bidder, as we allow for the possibility of joint- and solo-type bidders having different payoff functions. Bidders draw their private valuations from joint distribution $F_{V_{-t},V_t}(v_{-t}, v_t)$ in which arguments are not exchangeable and valuations are potentially affiliated.[29,30] Assuming a type t bidder draws her valuation of v_t and assuming an opponent bidder employs an equilibrium strategy $\phi_{-t}(\cdot)$, her expected payoff maximization problem is[31]

$$\max_{b_t} \; U_t(v_t - b_t) \cdot F_{V_{-t}|V_t}(\phi_{-t}(b_t)|v_t), \tag{6}$$

where $F_{V_{-t}|V_t}(V_{-t}|v_t)$ denotes the conditional distribution function of V_{-t} given v_t. By differentiating the above expected payoff function with respect to b_t, a type t bidder equates the marginal cost and benefit of changing her bid, and we have the first-order necessary condition, written as

$$v_t = b_t + \underbrace{\lambda_t^{-1}\left(\frac{F_{V_{-t}|V_t}(\phi_{-t}(b_t)|v_t)}{\frac{dF_{V_{-t}|V_t}(\phi_{-t}(b_t)|v_t)}{dv_{-t}} \cdot \phi'_{-t}(b_t)} \right)}_{\text{Shading Function}},$$

where $\lambda_t(\cdot) \equiv U_t(\cdot)/U_t'(\cdot)$ for each type of $t \in \{\text{Joint}, \text{Solo}\}$ and $\lambda_t^{-1}(\cdot)$ is its inverse function (called as a shading function in this research). Next, similar to the symmetric case, we derive the relations between empirically unobservable valuations and observable bids. We denote the conditional distribution of an opponent type's bid B_{-t} given b_t as $G_{B_{-t}|B_t}(b_{-t}|b_t)$ and its derivative as $g_{B_{-t}|B_t}(b_{-t}|b_t)$. By further assuming a type t bidder employs an equilibrium strategy $\phi_t(\cdot)$, the probabilistic relation between latent valuations and observable bids

[29] The model explored in this subsection is the simplest version of an asymmetric and affiliated private value auction model, and its econometric identifications and rationalizability are established by Campo, Perrigne, Vuong (2003) [17].

[30] According to theoretical asymmetric auction literature, for example, Maskin and Riley (2000a, 200b) [59] [60], we assume all types share the common support of private values. We follow Milgrom and Weber (1982) [61] for the definition of affiliation among private values.

[31] Technically speaking, as joint-type bidders equally split a monetary payment, the maximization problem for a joint-type bidder is $\max_{b_{\text{Joint}}} \{U_{\text{Joint}}((v_{\text{Joint}} - b_{\text{Joint}})/2) \cdot F_{V_{\text{Solo}}|V_{\text{Joint}}}(\phi_{\text{Solo}}(b_{\text{Joint}})|v_{\text{Joint}})\}$. Here, for simplicity's sake, we abbreviate explanation as seen in (6) and continue using it for maximization problems of both joint- and solo-type bidders. The detailed explanations are found in Appendix. Note that this equal-split-payment rule does not affect the discussion of theoretical asymmetric auction models (as we can re-define a vNM function as $\widetilde{U}_{\text{Joint}}(x) \equiv U_{\text{Joint}}(x/2)$), yet it slightly affects the structural estimations.

is $G_{B_{-t}|B_t}(x|b_t) = F_{V_{-t}|V_t}(\phi_{-t}(x)|v_t)$. In addition, by the fundamental theorem of calculus, the conditional density is obtained as $g_{B_{-t}|B_t}(x|b_t) = \big(dF_{V_{-t}|V_t}(\phi_{-t}(x)|v_t)/dv_{-t}\big) \cdot \phi'_{-t}(x)$. Therefore, using these probabilistic relations, the first-order necessary condition can be equivalently written as the function of observable bids,

$$v_t = b_t + \lambda_t^{-1}\left(\frac{G_{B_{-t}|B_t}(b_t|b_t)}{g_{B_{-t}|B_t}(b_t|b_t)}\right). \tag{7}$$

Furthermore, similar to the symmetric case, by exploiting the definitions of conditional density and distribution functions, $g_{B_{-t}|B_t}(z|b_t) = g_{B_{-t},B_t}(z,b_t)/g_{B_t}(b_t)$ and $G_{B_{-t}|B_t}(x|b_t)$ $= \left(\int_{\underline{b}}^{x} g_{B_{-t},B_t}(z,b_t)dz\right)/g_{B_t}(b_t)$ where $\underline{b}$ is the lower bound of bid distributions, the first order necessary condition equation (7) can be written in an unconditional fashion as

$$v_t = b_t + \lambda_t^{-1}\left(\frac{\int_{\underline{b}}^{b_t} g_{B_{-t},B_t}(z,b_t)dz}{g_{B_{-t},B_t}(b_t,b_t)}\right). \tag{8}$$

To simplify notations, we again introduce the convenient terms of $\Gamma(x, b_t|g_{B_{-t},B_t}) = \int_{\underline{b}}^{x} g_{B_{-t},B_t}(z,b_t)dz$ and of reciprocal factor as $R[x,y|\Gamma_{B_{-t},B_t}, g_{B_{-t},B_t}] = \Gamma(x,y|g_{B_{-t},B_t})/g_{B_{-t},B_t}(x,y)$. Given these simplified notations, we can denote the first-order necessary condition as

$$v_t^{\text{Asym}} = b_t^{\text{Asym}} + \lambda_t^{-1}\left(\underbrace{R^{\text{Asym,Affi}}\left[b_t^{\text{Asym}}, b_t^{\text{Asym}}\,\middle|\,\Gamma_{B_{-t},B_t}^{\text{Asym,Affi}}, g_{B_{-t},B_t}^{\text{Asym,Affi}}\right]}_{\text{(iii)}}\right), \tag{9}$$

where upper indices of “Asym” and “Asym,Affi”emphasize that the auction model is asymmetric and of affiliated value. In addition, if a researcher further assumes the independence of private valuations, the bivariate functions are simplified as $g_{B_{-t},B_t}(z,b_t) = g_{B_{-t}}(z)$ and $\int_{\underline{b}}^{x} g_{B_{-t},B_t}(z,b_t)dz = G_{B_{-t}}(x)$. Accordingly, equation (8) becomes

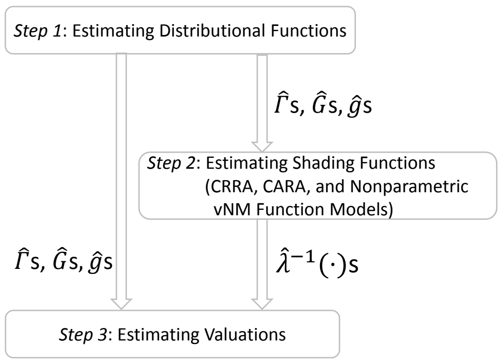
$$v_t = b_t + \lambda_t^{-1}\left(\frac{G_{B_{-t}}(b_t)}{g_{B_{-t}}(b_t)}\right). \tag{10}$$

Lastly, by denoting a reciprocal factor as $R_t[x|G_{B_{-t}}, g_{B_{-t}}] = G_{B_{-t}}(x)/g_{B_{-t}}(x)$, we can write the first-order necessary condition as

$$v_t^{\text{Asym}} = b_t^{\text{Asym}} + \lambda_t^{-1}\left(\underbrace{R^{\text{Asym,Inde}}\left[b_t^{\text{Asym}}\,\middle|\,G_{B_{-t}}^{\text{Asym,Inde}}, g_{B_{-t}}^{\text{Asym,Inde}}\right]}_{\text{(iv)}}\right). \tag{11}$$

where the upper indices “Asym” and “Asym,Inde” emphasize that the auction model is asymmetric and of independent value.

Figure 4: Estimation Steps and Methods



Step	Object	Method
Step 1	$\widehat{\Gamma}$s, $\widehat{g}$s $\widehat{G}$s	Nonparametric Kernel Density Empirical CDF
Step 2	CRRA $\widehat{\lambda}^{-1}(\cdot)$s CARA $\widehat{\lambda}^{-1}(\cdot)$s Nonparametric $\widehat{\lambda}^{-1}(\cdot)$s	OLS NLLS Sieve
Step 3	$\widehat{v}$s	Substituting the objects estimated in *Step 1* and *Step 2* in f.o.c.s

4 Structural Estimation Methods

In this section, we illustrate the estimation methods for recovering valuations. Estimation procedures are summarized into the three steps as depicted in Figure 4: *Step 1* – nonparametrically estimating distributional functions; *Step 2* – by applying semi or nonparametric methods, estimating shading functions (i.e. $\lambda_t^{-1}(\cdot)$s); *Step 3* – estimating valuations based on estimated distributional functions and shading functions. As the main purpose of this research is to investigate the accuracy of asymmetric auctions estimates, we primarily recover valuations from bids observed in asymmetric auctions, and bid data from symmetric auctions is subsidiarily used solely for the purpose of estimating the shading functions. For the sake of clear notations, we introduce the following indices: $r \in \{1, \ldots, R\}$ as an auction round index; $m \in \{1, \ldots, M\}$ as a (within-a-round) matched group index; and $i \in \{1, \ldots, N\}$ where $N = 3$ as a bidder index in a symmetric-auction stage. In addition, as estimates are separately calculated for each experiment run, we omit an index for experiment runs.[32]

Given the goal of obtaining the estimates of valuations, this section is organized as follows. The first subsection illustrates *Step 1* with descriptions of nonparametric estimation method for distributional functions. The following subsections illustrate *Step 2* and *Step 3* in the order of risk neutral, general estimation framework for risk-averse models, constant relative risk averse (CRRA), constant absolute risk averse (CARA), nonparametric vNM function, and heterogeneous risk averse attitude models, as the latter models require incrementally advanced estimation methods.[33]

[32] Any estimation results reported in the rest of this research are separately calculated for each experiment run.

[33] As the rest of this research focus on private-value auctions, implications of risk averse preferences in a common-value auction should be concisely mentioned here. In a common-value framework, risk-aversions does not play a major role in explaining bidding behavior as there are two opposing and canceling effects. In a common-value first-price auction, a risk averse bidder have incentives to (1) raise her bid to insure a higher winning probability (with the cost of additional payment) and (2) reduce her bid to avoid the payoff fluctuations caused by stochastic common valuation (with the cost of lower probability of winning). As (1) and (2) are canceled out, the assumption of risk averse or neutral bidders does not create empirically meaningful differences. See Paarsch (1992)

4.1 Nonparametric Estimations of Distributional Functions

Here, as *Step 1*, we nonparametrically estimate the distributional functions that are the basis for further estimations of shading functions and valuations. Nonparametric kernel estimation methods are employed throughout this research, and we use the *Gaussian kernel* with Silverman's rule of thumb bandwidths. For bi-variate distributional function estimations, we use a product kernel. In addition, we exploit the anonymous-identity nature of our experiment; a bidder did not know identities of opponents in each auction game. This anonymous-identity nature enables us to aggregate the distributional functions against which a bidder is best responding.[34]

4.1.1 Affiliated Value Assumption

The distributional functions under affiliated private value assumption are estimated as follows. By using symmetric-auction stage bid data, we estimate (remind that b_{-i} is the highest of opponents' bids)[35,36]

$$\begin{aligned}\widehat{\Gamma}^{\text{Sym,Affi}}_{B_{-i},B_i}(x,y) &= \int_{\underline{b}}^{x} \widehat{g^{\text{Sym,Affi}}_{B_{-i},B_i}(z,y)}dz \\ &= \frac{1}{h_{\Gamma}^{\text{Sym,Affi}}}\frac{1}{RMN}\sum_{r=1}^{R}\sum_{m=1}^{M}\sum_{i=1}^{N}\mathbb{1}(b^{\text{Sym}}_{r,m,-i} \leqslant x)\cdot K\left(\frac{b^{\text{Sym}}_{r,m,i}-y}{h_{\Gamma}^{\text{Sym,Affi}}}\right),\end{aligned} \tag{12}$$

[65], Li, Perrigne, and Vuong (2000) [51], Hendricks, Pinkse, and Porter (2003) [37], and Philip, Hong, and Shum (2003, working paper) [29] for empirical investigations of common-value auctions with risk-neutral bidders.

[34] We can interpret a bidder (say, bidder i) in equilibrium is best responding to: (i) the average bidding strategy of all bidders (including bidder i herself in other auction rounds) in symmetric-auction stages [described by Equation (12), (14) (18), and (20)]; and (ii) the average bidding strategy of all opponent-type bidders in asymmetric-auction stages [described by Equation (13), (15) (19), and (21)] .

[35] In this subsection (and only in this subsection), we generically use x for a first functional argument, y for a second functional argument, and z for an integrating variable. Note that a first function argument is a realized random variable of an opponent who has the highest valuation in a symmetric-auction stage or of an opponent-type bidder in an asymmetric-auction stage. A second functional argument is a realized random variable of a bidder i in a symmetric-auction stage or of a type t bidder in an asymmetric-auction stage.

[36] Note that, in round $r \in \{1,\dots,R\}$ within matched group $m \in \{1,\dots,M\}$, we use $\{b_{r,m1}, b_{r,m,2}, b_{r,m,3}\}$ in a symmetric auction stage and $\{b_{r,m,\text{Joint}}, b_{m,r,\text{Solo}}\}$ in an asymmetric-auction stage for estimating distributional functions. Although there are two observed joint-type bids in an asymmetric-auction stage (as depicted in Figure 2), only a chosen (and announced) joint-type bid in an asymmetric-auction stage is used for estimations of distributional functions. In other words, a non-chosen (and unannounced) joint-type bid in an asymmetric-auction stage is not used for estimations of distributional functions.

and by using asymmetric-auction stage bid data, for each type of $t \in \{\text{Joint}, \text{Solo}\}$, we estimate[37]

$$
\begin{aligned}
\widehat{\Gamma}^{\text{Asym,Affi}}_{B_{-t},B_t}(x,y) &= \int_{\underline{b}}^{x} \widehat{g^{\text{Asym,Affi}}_{B_{-t},B_t}(z,y)}dz \\
&= \frac{1}{h_{\Gamma}^{\text{Asym,Affi}}} \frac{1}{RM} \sum_{r=1}^{R} \sum_{m=1}^{M} \mathbb{1}(b^{\text{Asym}}_{r,m,-t} \leqslant x) \cdot K\left(\frac{b^{\text{Asym}}_{r,m,t} - y}{h_{\Gamma}^{\text{Asym,Affi}}}\right),
\end{aligned}
\tag{13}
$$

where $h_{\Gamma}^{\text{Sym,Affi}} = c_{\Gamma} \cdot (RMN)^{-1/5}$ and $h_{\Gamma}^{\text{Asym,Affi}} = c_{\Gamma} \cdot (RM)^{-1/5}$ with $c_{\Gamma} = 1.06 \cdot \hat{\sigma}_b$, and $\hat{\sigma}_b$ is the empirical standard deviation of corresponding observed bids.[38] For the bi-variate density functions, we estimate

$$
\widehat{g}^{\text{Sym,Affi}}_{B_{-i},B_i}(x,y) = \frac{1}{h_{g_x}^{\text{Sym,Affi}} \cdot h_{g_y}^{\text{Sym,Affi}}} \frac{1}{RMN} \sum_{r=1}^{R} \sum_{m=1}^{M} \sum_{i=1}^{N} K\left(\frac{b^{\text{Sym}}_{r,m,-i} - x}{h_{g_x}^{\text{Sym,Affi}}}\right) \cdot K\left(\frac{b^{\text{Sym}}_{r,m,i} - y}{h_{g_y}^{\text{Sym,Affi}}}\right),
\tag{14}
$$

and for each type of $t \in \{\text{Joint}, \text{Solo}\}$,

$$
\widehat{g}^{\text{Asym,Affi}}_{B_{-t},B_t}(x,y) = \frac{1}{h_{g_x}^{\text{Asym,Affi}} \cdot h_{g_y}^{\text{Asym,Affi}}} \frac{1}{RM} \sum_{r=1}^{R} \sum_{m=1}^{M} K\left(\frac{b^{\text{Asym}}_{r,m,-t} - x}{h_{g_x}^{\text{Asym,Affi}}}\right) \cdot K\left(\frac{b^{\text{Asym}}_{r,m,t} - y}{h_{g_y}^{\text{Asym,Affi}}}\right),
\tag{15}
$$

where $h_{g_x}^{\text{Sym,Affi}} = c_g \cdot (RMN)^{-1/6}$, and $h_{g_x}^{\text{Asym,Affi}} = c_g \cdot (RM)^{-1/6}$ with $c_g = 1.06 \cdot \hat{\sigma}_b$ and $\hat{\sigma}_b$ is the empirical standard deviation of corresponding observed bids. In addition, $h_{g_y}^{\text{Sym,Affi}}$ and $h_{g_y}^{\text{Asym,Affi}}$ are determined in the same manner. Given these estimated distributional functions with affiliated value assumption, we can calculate reciprocal factors as

$$
\underbrace{R^{\text{Sym,Affi}}\left[b^{\text{Sym}}_{r,m,i}, b^{\text{Sym}}_{r,m,i} \,\middle|\, \widehat{\Gamma}^{\text{Sym,Affi}}_{B_{-i},B_i}, \widehat{g}^{\text{Sym,Affi}}_{B_{-i},B_i}\right]}_{\widehat{\text{(i)}}} = \underbrace{\widehat{R}^{\text{Sym,Affi}}_{B_{-i},B_i}\left[b^{\text{Sym}}_{r,m,i}, b^{\text{Sym}}_{r,m,i}\right]}_{\text{Shorthand Notation}} = \frac{\widehat{\Gamma}^{\text{Sym,Affi}}_{B_{-i},B_i}(b^{\text{Sym}}_{r,m,i}, b^{\text{Sym}}_{r,m,i})}{\widehat{g}^{\text{Sym,Affi}}_{B_{-i},B_i}(b^{\text{Sym}}_{r,m,i}, b^{\text{Sym}}_{r,m,i})},
\tag{16}
$$

and for each type of $t \in \{\text{Joint}, \text{Solo}\}$,

$$
\underbrace{R^{\text{Asym,Affi}}\left[b^{\text{Asym}}_{r,m,t}, b^{\text{Asym}}_{r,m,t} \,\middle|\, \widehat{\Gamma}^{\text{Asym,Affi}}_{B_{-t},B_t}, \widehat{g}^{\text{Asym,Affi}}_{B_{-t},B_t}\right]}_{\widehat{\text{(iii)}}} = \underbrace{\widehat{R}^{\text{Asym,Affi}}_{B_{-t},B_t}\left[b^{\text{Asym}}_{r,m,t}, b^{\text{Asym}}_{r,m,t}\right]}_{\text{Shorthand Notation}} = \frac{\widehat{\Gamma}^{\text{Asym,Affi}}_{B_{-t},B_t}(b^{\text{Asym}}_{r,m,t}, b^{\text{Asym}}_{r,m,t})}{\widehat{g}^{\text{Asym,Affi}}_{B_{-t},B_t}(b^{\text{Asym}}_{r,m,t}, b^{\text{Asym}}_{r,m,t})}.
\tag{17}
$$

The plots of these affiliated value distributional function estimates based on observed bids are found in Online Appendix.

[37] As there are only two bids in an asymmetric-auction stage used for estimations, (one is submitted by a solo-type bidder, and the other is submitted by a chosen joint-type bidder) and as we estimate asymmetric-auction stage distributional functions for each type of bidder, we drop summations over bidder types in the following equations.

[38] Following Bajari and Hortaçsu (2005) [12], in this research we use the unbounded-support *Gaussian kernel* with rule of thumb, $c = 1.06 \cdot \hat{\sigma}_b$ (see Li and Racine (2007) [53] page 26, for example). Note that, some of preceding and influential works (eg. Li, Perrigne, and Vuong (2002) [52] and Campo, Perrigne, and Vuong (2002) [17]) use the bounded-support *triweight kernel* with the rule of thumb, $c = 2.978 \times 1.06 \cdot \hat{\sigma}_b$. See Härdle (1990, 1991) [31] [32] for the detailed description of bandwidth choices.

4.1.2 Independent Value Assumption

Next, the distributional functions under independent private value assumption are estimated as follows. By using symmetric-auction stage bid data, we derive the empirical CDF as

$$\widehat{G}_{B_{-i}}^{\text{Sym,Inde}}(x) \quad = \quad \frac{1}{RMN}\sum_{r=1}^{R}\sum_{m=1}^{M}\sum_{i=1}^{N}\mathbb{1}(b_{r,m,-i}^{\text{Sym}} \leqslant x), \tag{18}$$

and by using symmetric-auction stage bid data, for each type of $t \in \{\text{Joint}, \text{Solo}\}$, we derive

$$\widehat{G}_{B_{-t}}^{\text{Asym,Inde}}(x) \quad = \quad \frac{1}{RM}\sum_{r=1}^{R}\sum_{m=1}^{M}\mathbb{1}(b_{r,m,-t}^{\text{Asym}} \leqslant x). \tag{19}$$

For uni-variate density functions, we estimate

$$\widehat{g}_{B_{-i}}^{\text{Sym,Inde}}(x) \quad = \quad \frac{1}{h_{g_x}^{\text{Sym,Inde}}}\frac{1}{RMN}\sum_{r=1}^{R}\sum_{m=1}^{M}\sum_{i=1}^{N}K\left(\frac{b_{r,m,-i}^{\text{Sym}} - x}{h_{g_x}^{\text{Sym,Inde}}}\right), \tag{20}$$

and, for each type of $t \in \{\text{Joint}, \text{Solo}\}$,

$$\widehat{g}_{B_{-t}}^{\text{Asym,Inde}}(x) \quad = \quad \frac{1}{h_{g_x}^{\text{Asym,Inde}}}\frac{1}{RM}\sum_{r=1}^{R}\sum_{m=1}^{M}K\left(\frac{b_{r,m,-t}^{\text{Asym}} - x}{h_{g_x}^{\text{Asym,Inde}}}\right), \tag{21}$$

where $h_{g_x}^{\text{Sym,Inde}} = c_g \cdot (RMN)^{-1/5}$, and $h_{g_x}^{\text{Asym,Inde}} = c_g \cdot (RM)^{-1/5}$ with $c_g = 1.06 \cdot \hat{\sigma}_b$ and $\hat{\sigma}_b$ is the empirical standard deviation of corresponding observed bids. Accordingly, we can calculate reciprocal factors as

$$\underbrace{R^{\text{Sym,Inde}}\left[b_{r,m,i}^{\text{Sym}} \middle| \widehat{G}_{B_{-i}}^{\text{Sym,Inde}}, \widehat{g}_{B_{-i}}^{\text{Sym,Inde}}\right]}_{\widehat{\text{(ii)}}} = \underbrace{\widehat{R}_{B_{-i}}^{\text{Sym,Inde}}\left[b_{r,m,i}^{\text{Sym}}\right]}_{\text{Shorthand Notation}} = \frac{\widehat{G}_{B_{-i}}^{\text{Sym,Inde}}(b_{r,m,i}^{\text{Sym}})}{\widehat{g}_{B_{-i}}^{\text{Sym,Inde}}(b_{r,m,i}^{\text{Sym}})}, \tag{22}$$

and for each type of $t \in \{\text{Joint}, \text{Solo}\}$,

$$\underbrace{R^{\text{Asym,Inde}}\left[b_{r,m,t}^{\text{Asym}} \middle| \widehat{G}_{B_{-t}}^{\text{Asym,Inde}}, \widehat{g}_{B_{-t}}^{\text{Asym,Inde}}\right]}_{\widehat{\text{(iv)}}} = \underbrace{\widehat{R}_{B_{-t}}^{\text{Asym,Inde}}\left[b_{r,m,t}^{\text{Asym}}\right]}_{\text{Shorthand Notation}} = \frac{\widehat{G}_{B_{-t}}^{\text{Asym,Inde}}(b_{r,m,t}^{\text{Asym}})}{\widehat{g}_{B_{-t}}^{\text{Asym,Inde}}(b_{r,m,t}^{\text{Asym}})}. \tag{23}$$

The plots of these independent value distributional function estimates based on observed bids are found in Online Appendix.

4.2 Estimation Method for Risk Neutral Model

We begin by assuming bidders behave according to the simplest model, risk neutral (RN) vNM function with $U_t(x) = x$, $\lambda_t(x) = U_t(x)/U_t'(x) = x$, and the shading function $\lambda_t^{-1}(y) = y$. Under the risk neutral model, by substituting estimated distributional functions, equilibrium first order condition equations (9) under the

APV assumption and (11) under the IPV assumption become

$$\hat{v}^{\text{Asym,Affi}}_{\text{RN},r,m,t} = b^{\text{Asym}}_{r,m,t} + \underbrace{R^{\text{Asym,Affi}}\left[b^{\text{Asym}}_{r,m,t}, b^{\text{Asym}}_{r,m,t}\middle| \hat{\Gamma}^{\text{Asym,Affi}}_{B_{-t},B_t}, \hat{g}^{\text{Asym,Affi}}_{B_{-t},B_t}\right]}_{\widehat{\text{(iii)}}} \tag{24a}$$

$$\hat{v}^{\text{Asym,Inde}}_{\text{RN},r,m,t} = b^{\text{Asym}}_{r,m,t} + \underbrace{R^{\text{Asym,Inde}}\left[b^{\text{Asym}}_{r,m,t}\middle| \hat{G}^{\text{Asym,Inde}}_{B_{-t}}, \hat{g}^{\text{Asym,Inde}}_{B_{-t}}\right]}_{\widehat{\text{(iv)}}} \tag{24b}$$

for each type of $t \in \{\text{Joint}, \text{Solo}\}$. Accordingly, by substituting observed bids with estimated distributional functions in the right hand side of the above equations, we obtain the estimates of valuations $\{\hat{v}^{\text{Asym,Affi}}_{\text{RN},r,m,t}\}^{m=1,\cdots,M}_{r=1,\cdots,R}$ under the APV assumption and $\{\hat{v}^{\text{Asym,Inde}}_{\text{RN},r,m,t}\}^{m=1,\cdots,M}_{r=1,\cdots,R}$ under the IPV assumption for each type of $t \in \{\text{Joint}, \text{Solo}\}$.

4.3 Semi & Nonparametric Estimations of Risk Averse Models

In empirical auctions, the assumption of risk neutrality is justified when a bidder can be seen as a large firm whose wealth is relatively large compared to the to the value of an object under auction. However, in reality, a bidder is likely to be a representative of a firm whose personal incentives (eg. individual bonus, promotion, or opportunity costs) depend on the result of an auction. Under such situation, bidders are seen to have risk averse preferences, and derived model implications could be largely different from those derived from the risk neutral model.[39] Accordingly, for investigating bidders' risk averse preferences, we now assume that bidders have preferences with type-homogeneous (i.e. bidders share the same risk attitude within a same type) risk averse vNM functions,[40] $U_t(\cdot)$. In *Step 2*, we use quantile restrictions to derive *compatibility conditions*, which in turn are used for semi and nonparametrically estimating the shapes of type-homogeneous shading functions, $\lambda_t^{-1}(\cdot)$s.[41] We introduce the notation for $b^{\text{Sym}}_{i,\alpha}$ to denote the αth quantile for distribution of observed symmetric-auction stage bids submitted by all types of bidders. Similarly, we denote $b^{\text{Asym}}_{t,\alpha}$ for the αth quantile for distribution of observed asymmetric-auction stage bids submitted by type t bidders. In addition, we denote $v_{i,\alpha}$ as αth quantile of value distribution among all types of bidders and $v_{t,\alpha}$ as αth quantile of value distribution among type t bidders. Then, the quantile notations of equilibrium first order

[39]The empirical evidences and estimates of risk averse preferences are reported by Athey and Levin (2001) [9], Lu and Perrigne (2008) [55], and Campo, Guerre, Perrigne, and Vuong (2011) [18] in their investigations of U.S. Forest Service timber auctions and Campo (2012) [16] in her scrutiny of Los Angeles construction contract auctions. Specifically, these empirical investigations emphasize the risk-averse preferences among small-size firms.

[40]To avoid the verbal confusions, in this research, the terminologies of symmetric and asymmetric are used to express the diversity in value distributions, while the terminologies of homogeneous (type-homogeneous) and heterogeneous are used to express the diversity in bidders' risk preferences.

[41]Quantile restrictions with the resulting *compatibility conditions* are proposed by Guerre, Perrigne, and Vuong (2009) [28], Campo, Guerre, Perrigne, and Vuong (2011) [18], Campo (2012) [16] and also used by Bajari and Hortaçsu (2005) [12].

conditions (3) and (5) for the symmetric-auction models are

$$v_{i,\alpha}^{\text{Sym,Affi}} = b_{i,\alpha}^{\text{Sym}} + \lambda^{-1}\left(R^{\text{Sym,Affi}}\left[b_{i,\alpha}^{\text{Sym}}, b_{i,\alpha}^{\text{Sym}}\middle| \Gamma_{B_{-i},B_i}^{\text{Sym,Affi}}, g_{B_{-i},B_i}^{\text{Sym,Affi}}\right]\right) \tag{25a}$$

$$v_{i,\alpha}^{\text{Sym,Inde}} = b_{i,\alpha}^{\text{Sym}} + \lambda^{-1}\left(R^{\text{Sym,Inde}}\left[b_{i,\alpha}^{\text{Sym}}\middle| G_{B_{-i}}^{\text{Sym,Inde}}, g_{B_{-i}}^{\text{Sym,Inde}}\right]\right), \tag{25b}$$

and of equilibrium first order conditions (9) and (11) for the asymmetric-auction models are

$$v_{t,\alpha}^{\text{Asym,Affi}} = b_{t,\alpha}^{\text{Asym}} + \lambda_t^{-1}\left(R^{\text{Asym,Affi}}\left[b_{t,\alpha}^{\text{Asym}}, b_{t,\alpha}^{\text{Asym}}\middle| \Gamma_{B_{-t},B_t}^{\text{Asym,Affi}}, g_{B_{-t},B_t}^{\text{Asym,Affi}}\right]\right) \tag{26a}$$

$$v_{t,\alpha}^{\text{Asym,Inde}} = b_{t,\alpha}^{\text{Asym}} + \lambda_t^{-1}\left(R^{\text{Asym,Inde}}\left[b_{t,\alpha}^{\text{Asym}}\middle| G_{B_{-t}}^{\text{Asym,Inde}}, g_{B_{-t}}^{\text{Asym,Inde}}\right]\right). \tag{26b}$$

Next, by exploiting the fact that the majority of bidders in experiments did not change their valuations within a round while they submitted distinct bids in symmetric- and asymmetric-auction stages, we take advantage of the observed differences of bids between auction stages. Note that, as the valuations of a solo-type bidder and of one of the joint-type bidders were unchanged in both symmetric- and asymmetric-auction stages, we have the equivalence of valuations, $v_{r,m,i=t}^{\text{Sym}} = v_{r,m,t}^{\text{Asym}}$ with the slight abuse of notation $i = t$ for each type of $t \in \{\text{Joint}, \text{Solo}\}$, meaning that a type t bidder did not change her valuation across auction stages. Accordingly, by using this unchanged nature of valuations, we match the quantiles of bidders' private value distribution $v_{i=t,\alpha}^{\text{Sym}} = v_{t,\alpha}^{\text{Asym}}$, where $v_{i=t,\alpha}^{\text{Sym}}$ and $v_{t,\alpha}^{\text{Asym}}$ denote the αth quantiles of type t bidders' value distribution.[42] Thus, for each type $t \in \{\text{Joint}, \text{Solo}\}$, we can equate the equilibrium first order condition equations (25a) and (26a) under the APV assumption and (25b) and (26b) under the IPV assumption. Then, by assuming that a type t bidder reveals the same preference $U_t(\cdot)$ and $\lambda_t^{-1}(\cdot)$ in different and exogenously changing auction stages, we have the following *compatibility condition* equations for each type of $t \in \{\text{Joint}, \text{Solo}\}$:

$$\begin{aligned} &b_{i=t,\alpha}^{\text{Sym}} - b_{t,\alpha}^{\text{Asym}} = \\ &\lambda_t^{-1}\left(R^{\text{Asym,Affi}}\left[b_{t,\alpha}^{\text{Asym}}, b_{t,\alpha}^{\text{Asym}}\middle| \Gamma_{B_{-t},B_t}^{\text{Asym,Affi}}, g_{B_{-t},B_t}^{\text{Asym,Affi}}\right]\right) - \lambda_t^{-1}\left(R^{\text{Sym,Affi}}\left[b_{i=t,\alpha}^{\text{Sym}}, b_{i=t,\alpha}^{\text{Sym}}\middle| \Gamma_{B_{-i},B_i}^{\text{Sym,Affi}}, g_{B_{-i},B_i}^{\text{Sym,Affi}}\right]\right) \end{aligned} \tag{27a}$$

$$\begin{aligned} &b_{i=t,\alpha}^{\text{Sym}} - b_{t,\alpha}^{\text{Asym}} = \\ &\lambda_t^{-1}\left(R^{\text{Asym,Inde}}\left[b_{t,\alpha}^{\text{Asym}}\middle| G_{B_{-t}}^{\text{Asym,Inde}}, g_{B_{-t}}^{\text{Asym,Inde}}\right]\right) - \lambda_t^{-1}\left(R^{\text{Sym,Inde}}\left[b_{i=t,\alpha}^{\text{Sym}}\middle| G_{B_{-i}}^{\text{Sym,Inde}}, g_{B_{-i}}^{\text{Sym,Inde}}\right]\right), \end{aligned} \tag{27b}$$

[42]Note that, as we exploiting the invariance of laboratory assigned valuations, our quantile restrictions are unconditional. Guerre, Perrigne, and Vuong (2009) [28] Campo, Guerre, Perrigne, and Vuong (2011) [18] and Campo (2012) [16] establish identifications based on conditional quantile restrictions (that allow, observed characteristics of auction objects, endogenous participation, and unobserved heterogeneity) that accommodate a broad class of auction models and data generating processes. Our unconditional restrictions in this subsection are the special case of their conditional restrictions.

where we use the notation of $b^{\text{Sym}}_{i=t,\alpha}$ for the αth quantile of observed bids made by type t bidders in symmetric-auction stages.[43] See Appendix for a detailed description on the constructions of quantile points.[44] Given the quantiles of bid distributions, in *Step 2*, the equation (27a) and (27b) can be estimated by the semi and nonparametric methods explained later in this subsection. Once we obtain the estimates of shading functions, $\widehat{\lambda}^{-1,\text{Affi}}_t(\cdot)$ from the equation (27a) and $\widehat{\lambda}^{-1,\text{Inde}}_t(\cdot)$ from the equation (27b), in *Step 3*, we can obtain the estimates of valuations by substituting observed the bids and estimated objects in *Step 1* and *Step 2* in first order necessary conditions as

$$\widehat{v}^{\text{Asym,Affi}}_{r,m,t} = b^{\text{Asym}}_{r,m,t} + \widehat{\lambda}^{-1,\text{Affi}}_t \left(\underbrace{R^{\text{Asym,Affi}} \left[b^{\text{Asym}}_{r,m,t}, b^{\text{Asym}}_{r,m,t} \middle| \widehat{\Gamma}^{\text{Asym,Affi}}_{B_{-t},B_t}, \widehat{g}^{\text{Asym,Affi}}_{B_{-t},B_t} \right]}_{\widehat{\text{(iii)}}} \right) \tag{28a}$$

$$\widehat{v}^{\text{Asym,Inde}}_{r,m,t} = b^{\text{Asym}}_{r,m,t} + \widehat{\lambda}^{-1,\text{Inde}}_t \left(\underbrace{R^{\text{Asym,Inde}} \left[b^{\text{Asym}}_{r,m,t} \middle| \widehat{G}^{\text{Asym,Inde}}_{B_{-t}}, \widehat{g}^{\text{Asym,Inde}}_{B_{-t}} \right]}_{\widehat{\text{(iv)}}} \right). \tag{28b}$$

We now introduce the semi and nonparametric specifications of shading functions.[45]

4.3.1 Semiparametric Estimation for CRRA Model

We assume that bidders have the preference of constant relative risk averse (CRRA) vNM functions $U_t(x) = x^{\theta_t}$ where $0 < \theta_t \leqslant 1$ for $t \in \{\text{Joint}, \text{Solo}\}$. The CARA model has the advantage, as it nests the risk neutral model as the the special case of $\theta_t = 1$ that is empirically testable. Under the CARA model, we have $\lambda_t(x) = U_t(x)/U_t'(x) = x/\theta_t$ and the shading function, $\lambda_t^{-1}(y) = \theta_t \cdot y$. Then, in *Step 2*, the *compatibility condition* equations (27a) and (27b) with estimated distributional functions ($\widehat{\Gamma}$s, $\widehat{G}$s, and $\widehat{g}$s) for each type of

[43] Precisely describing, within a matched group (of three bidders), a solo-type bidder and one of the joint-type bidders did not change valuations as depicted in Figure 2 . For the semi and nonparametric estimations of risk-averse vNM functions, we abandon the bid data of joint-type bidders whose valuations were exogenously changed within a round.

[44] To the best of our knowledge, the literature has not settled a method to choose quantile points αs (i.e. how and how many αs a researcher should use). Guerre, Perrigne, and Vuong (2009) [28] suggest a recursive construction of quantile points, yet they also indicate the potential problem of serial correlations and accumulated errors. The recent work of Zincenko (2014, working paper) [77] establishes the identification and uniform consistency of nonparametric $\lambda^{-1}(\cdot)$ function by using the minmax absolute distance estimator in which a maximum distance is chosen over quantile points on bid space, and a minimum distance is chosen over a sieve space.

[45] Once we estimate $\widehat{\lambda}^{-1}_t(\cdot)$, we can analytically or numerically recover a payoff function $\widehat{U}_t(x)$ by solving the differential equation of $\widehat{\lambda}_5(x) = \widehat{U}_t(x)/\widehat{U}_t'(x)$ with the normalized initial condition of $\widehat{U}(1) = 1$; leading the solution of $\widehat{U}(x) = \exp\left[\int_1^x 1/\widehat{\lambda}(z)dz\right]$.

$t \in \{\text{Joint}, \text{Solo}\}$ become

$$\widehat{b}^{\text{Sym}}_{i=t,\alpha} - \widehat{b}^{\text{Asym}}_{t,\alpha} = \theta_t \cdot \left\{ \widehat{R}^{\text{Asym,Affi}}_{B_{-t},B_t} \left[\widehat{b}^{\text{Asym}}_{t,\alpha}, \widehat{b}^{\text{Asym}}_{t,\alpha} \right] - \widehat{R}^{\text{Sym,Affi}}_{B_{-i},B_i} \left[\widehat{b}^{\text{Sym}}_{i=t,\alpha}, \widehat{b}^{\text{Sym}}_{i=t,\alpha} \right] \right\} + \varepsilon_{t,\alpha} \tag{29a}$$

$$\widehat{b}^{\text{Sym}}_{i=t,\alpha} - \widehat{b}^{\text{Asym}}_{t,\alpha} = \theta_t \cdot \left\{ \widehat{R}^{\text{Asym,Inde}}_{B_{-t}} \left[\widehat{b}^{\text{Asym}}_{t,\alpha} \right] - \widehat{R}^{\text{Sym,Inde}}_{B_{-i}} \left[\widehat{b}^{\text{Sym}}_{i=t,\alpha} \right] \right\} + \varepsilon_{t,\alpha}, \tag{29b}$$

where we use the shorthand notations for simplicity. With the bid quantile data of $\left\{b^{\text{Sym}}_{t,\alpha_q}\right\}_{q=0,\cdots,Q}$ and $\left\{b^{\text{Asym}}_{t,\alpha_q}\right\}_{q=0,\cdots,Q}$, we can apply the OLS estimation to the equations (29a) and (29b) to obtain $\widehat{\theta}^{\text{Affi}}_t$ and $\widehat{\theta}^{\text{Inde}}_t$ for each type of $t \in \{\text{Joint}, \text{Solo}\}$. Consequently, in *Step 3*, we obtain the estimates of valuations $\{\widehat{v}^{\text{Asym,Affi}}_{\text{CRRA},r,m,t}\}^{m=1,\cdots,M}_{r=1,\cdots,R}$ under the APV assumption and $\{\widehat{v}^{\text{Asym,Inde}}_{\text{CRRA},r,m,t}\}^{m=1,\cdots,M}_{r=1,\cdots,R}$ under the IPV assumption by substituting $\widehat{\lambda}^{-1,\text{Affi}}_t(\cdot)$ and $\widehat{\lambda}^{-1,\text{Inde}}_t(\cdot)$ into equations (28a) and (28b).[46]

4.3.2 Semiparametric Estimation for CARA Model

Next, we assume that bidders have the preference of constant relative risk averse (CARA) vNM functions by modeling bidders payoff functions as $U_t(x) = \frac{1-\exp(-\zeta_t \cdot x)}{1-\exp(-\zeta_t)}$. As decisions made by CARA-preference bidders are not affected by the level of their wealth (i.e. no wealth/income effect), the CARA model has the advantage to control bidders' heterogeneity in their wealth levels that are rarely observed in empirical auction research. Given the CARA model, we have $\lambda_t(x) = \frac{1}{\zeta_t} \cdot [\exp(\zeta_t \cdot x) - 1]$, and the shading function has the form of $\lambda_t^{-1}(y) = \frac{1}{\zeta_t} \cdot \ln(1 + \zeta_t \cdot y)$. Then, in *Step 2*, the *compatibility condition* equations (27a) and (27b) with estimated distributional functions ($\widehat{\Gamma}$s, $\widehat{G}$s, and $\widehat{g}$s) for each type of $t \in \{\text{Joint}, \text{Solo}\}$ become

$$\widehat{b}^{\text{Sym}}_{i=t,\alpha} - \widehat{b}^{\text{Asym}}_{t,\alpha} = \frac{1}{\zeta_t} \left\{ \ln\left(1 + \zeta_t \cdot \widehat{R}^{\text{Asym,Affi}}_{B_{-t},B_t} \left[\widehat{b}^{\text{Asym}}_{t,\alpha}, \widehat{b}^{\text{Asym}}_{t,\alpha} \right]\right) - \ln\left(1 + \zeta_t \cdot \widehat{R}^{\text{Sym,Affi}}_{B_{-i},B_i} \left[\widehat{b}^{\text{Sym}}_{i=t,\alpha}, \widehat{b}^{\text{Sym}}_{i=t,\alpha} \right]\right) \right\} + \varepsilon_{t,\alpha} \tag{30a}$$

$$\widehat{b}^{\text{Sym}}_{i=t,\alpha} - \widehat{b}^{\text{Asym}}_{t,\alpha} = \frac{1}{\zeta_t} \left\{ \ln\left(1 + \zeta_t \cdot \widehat{R}^{\text{Asym,Inde}}_{B_{-t}} \left[\widehat{b}^{\text{Asym}}_{t,\alpha} \right]\right) - \ln\left(1 + \zeta_t \cdot \widehat{R}^{\text{Sym,Inde}}_{B_{-i}} \left[\widehat{b}^{\text{Sym}}_{i=t,\alpha} \right]\right) \right\} + \varepsilon_{t,\alpha}, \tag{30b}$$

where we use the shorthand notations for simplicity. With the bid quantile data of $\left\{b^{\text{Sym}}_{t,\alpha_q}\right\}_{q=0,\cdots,Q}$ and $\left\{b^{\text{Asym}}_{t,\alpha_q}\right\}_{q=0,\cdots,Q}$, we can apply the non-linear least square (NLLS) estimation to the equations (30a) and (30b) to obtain $\widehat{\zeta}^{\text{Affi}}_t$ and $\widehat{\zeta}^{\text{Inde}}_t$ for each type of $t \in \{\text{Joint}, \text{Solo}\}$. Consequently, in *Step 3*, we obtain the estimates of valuations $\{\widehat{v}^{\text{Asym,Affi}}_{\text{CARA},r,m,t}\}^{m=1,\cdots,M}_{r=1,\cdots,R}$ under the APV assumption and $\{\widehat{v}^{\text{Asym,Inde}}_{\text{CARA},r,m,t}\}^{m=1,\cdots,M}_{r=1,\cdots,R}$ under the IPV assumption by substituting $\widehat{\lambda}^{-1,\text{Affi}}_t(\cdot)$ and $\widehat{\lambda}^{-1,\text{Inde}}_t(\cdot)$ into equations (28a) and (28b).[47]

[46] The empirical drawback of the CRRA estimation model under the specification of $U(x) = x^\theta$ is that the estimated θ is not guaranteed to be in $(0, 1]$. $\widehat{\theta}$ could be negative (which is not compatible with economic theory) or larger than 1 (which indicates bidders are risk loving). Note that, if we affine transform the CRRA vNM function into the form of $U(x) = \frac{x^{1-\nu}-1}{1-\nu}$, the negative estimated coefficients could be interpreted as extreme risk aversion.

[47] As joint-type bidders' monetary payments are equally split, we need slight modifications to equations (28a), (28b), (30a), and (30b), for joint-type bidders. See Appendix for details of such slight modifications.

4.3.3 Nonparametric Estimation for vNM Function Model

Finally, we estimate the nonparametric (NP) vNM function model that is proposed by Guerre, Perrigne, and Vuong (2009) [28] as it allows the most flexibility to the shapes of shading functions, $\lambda_t^{-1}(\cdot)$.[48] Based on the method indicated in their seminal research, we use the sieve method to estimate the shading functions $\lambda_t^{-1}(\cdot) \in \Lambda^{-1}$, where Λ^{-1} is a set of differentiable and (strict) monotonically increasing functions.[49,50,51] In practice, we choose Λ^{-1} as the set of polynomial functions $\mathrm{Pol}(y;\eta_{t,n}) = \sum_{k=1}^{\overline{K}} \eta_{t,k} \cdot y^k$ without intercept terms, where $\eta_{t,n}$ stands for the coefficient vector of nth order polynomial. In addition, a polynomial order $\overline{K}$ flexibly changes. Then, in *Step 2*, as polynomials are linear in their coefficients, the *compatibility condition* equations (27a) and (27a) with estimated distributional functions ($\widehat{\Gamma}$s, $\widehat{G}$s, and $\widehat{g}$s) for each type of $t \in \{\mathrm{Joint}, \mathrm{Solo}\}$ become

$$\widehat{b}_{i=t,\alpha}^{\mathrm{Sym}} - \widehat{b}_{t,\alpha}^{\mathrm{Asym}} = \sum_{k=1}^{\overline{K}} \eta_{t,k}^{\mathrm{Affi}} \left\{ \left(\widehat{R}_{B_{-t},B_t}^{\mathrm{Asym,Affi}} \left[\widehat{b}_{t,\alpha}^{\mathrm{Asym}}, \widehat{b}_{t,\alpha}^{\mathrm{Asym}} \right] \right)^k - \left(\widehat{R}_{B_{-i},B_i}^{\mathrm{Sym,Affi}} \left[\widehat{b}_{i=t,\alpha}^{\mathrm{Sym}}, \widehat{b}_{i=t,\alpha}^{\mathrm{Sym}} \right] \right)^k \right\} + \varepsilon_{t,\alpha} \tag{31a}$$

$$\widehat{b}_{i=t,\alpha}^{\mathrm{Sym}} - \widehat{b}_{t,\alpha}^{\mathrm{Asym}} = \sum_{k=1}^{\overline{K}} \eta_{t,k}^{\mathrm{Inde}} \left\{ \left(\widehat{R}_{B_{-t}}^{\mathrm{Asym,Inde}} \left[\widehat{b}_{t,\alpha}^{\mathrm{Asym}} \right] \right)^k - \left(\widehat{R}_{B_{-i}}^{\mathrm{Sym,Inde}} \left[\widehat{b}_{i=t,\alpha}^{\mathrm{Sym}} \right] \right)^k \right\} + \varepsilon_{t,\alpha}, \tag{31b}$$

where we use the shorthand notations for simplicity. Under this polynomial specification, Λ^{-1} becomes the linear space, and we can solve the minimization problems by the least square method with the bid quantile data of $\left\{b_{t,\alpha_q}^{\mathrm{Sym}}\right\}_{q=0,\cdots,Q}$ and $\left\{b_{t,\alpha_q}^{\mathrm{Asym}}\right\}_{q=0,\cdots,Q}$. In practice, we have to select a number of polynomial terms.[52] For selecting $\widehat{\eta}_t^{\mathrm{Affi}}$ among $\left\{\widehat{\eta}_{t,1}^{\mathrm{Affi}}, \widehat{\eta}_{t,2}^{\mathrm{Affi}}, \widehat{\eta}_{t,3}^{\mathrm{Affi}}, \cdots, \widehat{\eta}_{t,n}^{\mathrm{Affi}}, \cdots\right\}$ and $\widehat{\eta}_t^{\mathrm{Inde}}$ among $\left\{\widehat{\eta}_{t,1}^{\mathrm{Inde}}, \widehat{\eta}_{t,2}^{\mathrm{Inde}}, \widehat{\eta}_{t,3}^{\mathrm{Inde}}, \cdots, \widehat{\eta}_{t,n}^{\mathrm{Inde}}, \cdots\right\}$, we adopt the Akaike Information Criterion (AIC) [Akaike (1973) [2]].[53] Consequently, in *Step 3*, we obtain the estimates of valuations $\{\widehat{v}_{\mathrm{CARA},r,m,t}^{\mathrm{Asym,Affi}}\}_{r=1,\cdots,R}^{m=1,\cdots,M}$ under the APV assumption and $\{\widehat{v}_{\mathrm{CARA},r,m,t}^{\mathrm{Asym,Inde}}\}_{r=1,\cdots,R}^{m=1,\cdots,M}$ under the

[48] To the best of our knowledge, this is the first applied auction work of their nonparametric method for identifying and estimating risk-averse preferences in auction research.

[49] Chen (2007) [19] extensively surveys the recent developments of sieve estimations.

[50] As we normalize a vNM function, for all sieve estimations, we impose the theoretical restrictions of Restriction1: $\lambda_t^{-1}(0) = 0$ and Restriction2: $0 < \frac{d}{dR}\lambda_t^{-1}(R) \leqslant 1$. In programming, for Restriction1, we remove the intercept terms in polynomials. For Restriction2, we impose the restrictions $\varepsilon \leqslant \frac{d}{dR}\lambda_t^{-1}(R) \leqslant 1$ where $\varepsilon = 10^{-8}$ on the MATLAB `fmincon` subroutine.

[51] This research takes a view that sieve is a nonparametric estimation method. Some researchers use the terminology "semi-nonparametric" to describe sieve.

[52] In programming, we choose the maximum number of polynomial terms as $\overline{K}_{\max} = 14$.

[53] We use the AIC, which assumes error terms are normally and homoskedastically distributed, for the computational tractability. The cross validation criterion, which is shown to be asymptotically optimal with heteroskedastic error terms by Andrews (1991) [3], is computationally expensive and is not used in this research.

IPV assumption by substituting $\widehat{\lambda}_t^{-1,\text{Affi}}(\cdot)$ and $\widehat{\lambda}_t^{-1,\text{Inde}}(\cdot)$ into equations (28a) and (28b).[54]

However, the fundamental limitation of this sieve estimation method based on differenced variables is that researchers identify a $\lambda^{-1}(\cdot)$ only on limited domains witch are away from boundaries. This limitation comes from two facts: (1) we estimate nonlinear shading function $\lambda^{-1}(\cdot)$ not by applying nonlinear-recursive-projection estimator (as originally proposed by Guerre Perrigne Vuong (2009) [28]) but by applying a linear (in coefficient) difference estimator in which researchers cannot obtain data points in near-upper-boundary domains; and (2) researchers practically need to trim the quantile points to avoid the well-know boundary problem in nonparametric density estimations. See Appendix for details of local identification. To overcome these difficulties, we apply conventional-wisdom-based shape restrictions to extrapolate sieve polynomial estimations to near-upper-boundary domains while exploiting the additive-coefficient-preserving nature in domains where data variations are available. Appendix provides the details of these shape restrictions with figures. In the next section, we report the estimation results based on the following shape restrictions: (1) minimalistic slope restrictions based only on economic theory;[55] (2) shape restrictions based on homogeneously-treated (across bidder types) slope restrictions; and (3) shape restrictions based on heterogeneously-treated slopes, and showing empirical usefulness of such restrictions.

5 Estimation and Test Results

This section reports the results of estimations and statistical tests under various modeling assumptions. Here, we analyze estimates of valuations derived from asymmetric auction models.[56] We first visually compare laboratory-assigned true valuations and estimated variations. Then, we statistically test their distributional equivalence, independence, and asymmetry. Regarding the model restrictions on vNM payoff functions, we start with the risk neutral model. Then, we discuss risk-averse models in the order of CRRA, CARA, and finally nonparametric models, since latter models in general enjoy higher distributional equivalence.

5.1 Estimation Results

The estimation results are plotted in Figure 5, 6, 7, 8 9, and 10, which depict laboratory-assigned true valuations on the horizontal axis and estimated valuations on the vertical axis. For measuring the deviations

[54] Similar to the CARA model case, as joint-type bidders' monetary payments are equally split, we need slight modification to equations (28a), (28b), (31a), and (31b) for joint-type bidders. See Appendix for details of such slight modifications.

[55] The minimalistic restrictions are $\lambda^{-1}(0) = 0$ and $\varepsilon < \frac{d}{dR}\lambda^{-1}(R) \leqslant 1$ on $R \in [0, R_{\max}]$ where $\varepsilon = 10^{-8}$.

[56] In this research, symmetric-auction stage bid data is subsidiarily used solely for the purpose of recovering bidders' risk-averse preferences.

from true valuations, a 45-degree line is added.[57] In addition, for providing crude yet comparable measures of estimates' accuracy we also calculate the L^1 norm for measuring an averaged distance from true valuation and L^2 norm for measuring a dispersion, defined by

$$L_t^1 = \frac{1}{RM}\sum_{r=1}^{R}\sum_{m=1}^{M}\left|\hat{v}_{r,m,t}^{\text{Asym}} - v_{r,m,t}\right| \qquad L_t^2 = \left(\frac{1}{RM}\sum_{r=1}^{R}\sum_{m=1}^{M}\left|\hat{v}_{r,m,t}^{\text{Asym}} - v_{r,m,t}\right|^2\right)^{\frac{1}{2}}$$

for each experiment run and for each bidder type.

There are four empirical findings across these figures. First, in general, more advanced estimation methods with risk averse vNM functions provide better model fits, accessed by both L^1 and L^2 measures. Second, within each estimation method, the estimates of solo-type bidders (i.e. valuations of stochastically dominated bidders) have smaller norms compared to those of joint-type bidders, indicating that estimated valuations are relatively more accurate for solo-type bidders. Third, as true valuations become larger, the distance from true to estimated valuations, on average, also gets larger. Fourth, the assumptions of independent and affiliated private valuations create minor difference in estimates.[58] We now discuss the findings and details of each model.

5.1.1 Risk Neutral Model Estimates

The estimates of valuations under the assumption of risk neutral bidders are plotted in Figure 5. We discover a severe over-estimation of asymmetric risk neutral model estimates, and this re-confirms the same finding detected in the symmetric risk neutral model reported by Bajari and Hortaçsu (2005) [12]. This over-estimation result suggests that, beyond the standardly-used risk neutral model, empirical auction researchers are recommended to investigate more advanced alternative models in order to achieve higher accuracy in estimates. We now discuss the gains in accuracy from such alternative models.

[57] Also, for creating equally-scaled figures (so that a 45-degree line is exactly tilted at 45 degrees), the estimated valuations are censored from above at \$30. Note that a few of the estimated valuations, especially ones derived from the risk neutral model, exceed \$100. Regarding the order, the plots are ordered from left to right as (a) joint-type bidders under APV, (b) joint-type bidders under IPV, (c) solo-type bidders under APV, and (d) solo-type bidders under IPV.

[58] The surprising finding in the estimates of valuations in this research is that affiliated and independent private value models do not have distinctive differences in estimates, although the true data generating process in our experiment is of independent private value. This finding strongly encourages the usage of the affiliated private value assumption in empirical auction research, as originally suggested by Campo, Perrigne, and Vuong (2003) [17].

5.1.2 Semiparametric CRRA and CARA Model Estimates

The estimated valuations derived from CRRA model plotted in Figure 6. The problem of over-estimation is largely resolved, yet we now have the systematic under-estimation problem, especially under-estimation among joint-type bidders is severe. The ordinal least square estimates of risk averse parameters, θ_ts, derived from CRRA model are reported in Table 3. All CRRA parameters are in the range of $(0, 1]$ as economic theory predicts, although the degree of Arrow-Pratt relative risk aversion $(1 - \theta_t)$ is remarkably large, and the null hypotheses of risk neutral bidders (H_0: $\theta_t = 1$) are easily rejected.

Next, the estimated valuations derived from CARA model plotted in Figure 7. The problem of under-estimation is slightly worsen among the high valuation domains, as shading function of CARA model is downwardly suppressing compared to that of CRRA model (See Figure 12 and 13 for the differences in functional forms). Table 4 lists the nonlinear least square (NLLS) estimates of risk averse parameters, ζ_ts. While the estimates for solo-type bidders reveal a slightly large yet predicted signs of risk attitudes, those of joint-type bidders are remarkably large. We find that these large numbers in estimates of joint-type bidders are cause by two reasons: (1) the half splitting payoff rule of joint-type bidders and (2) the scale variant nature of CARA payoff functions.[59] Regarding the performance of estimates, we again find the underestimation of valuations, and such underestimation is severe among joint-type bidders.

Note that these systematic underestimation problems are consistent with the preceding finding of semi-parametric estimates of symmetric auctions in Bajari and Hortaçsu (2005) [12].[60] From the practical point of view, the systematic underestimations among joint-type (stochastically dominating) bidders are required to be improved, since they directly cause systematic biases in market design policies, such as setting reservation prices or calculating expected revenues.

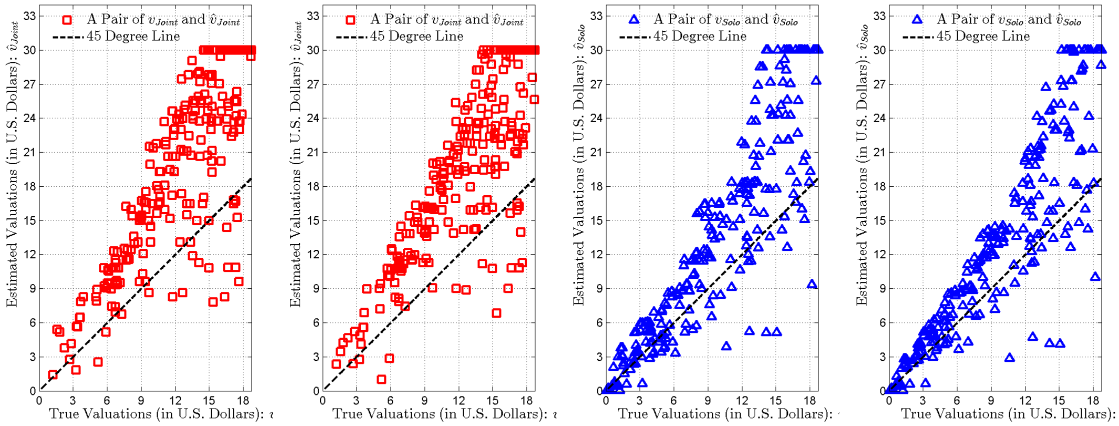
5.1.3 Nonparametric vNM Function Model Estimates with Shape Restrictions

For solving the systematic under-estimation problem in semiparametric estimates, we now advance the non-parametric sieve estimation results. The estimate are plotted in Figure 8 (with minimalistic restrictions based on auction theory), 9 (with soft slope restrictions, Shape Restriction 1), and 10 (with slope restrictions, Shape Restriction 2). See Appendix for the validity and details for the these shape restrictions. We observe that, as tighter slope restrictions are applied the systematic under-estimation problem is gradually alleviated, proving the empirical usefulness of nonparametric estimation method.

[59] Although CARA preferences are invariant to wealth/income levels, it is variant to scale changes. See the detail in Online Appendix for the details.

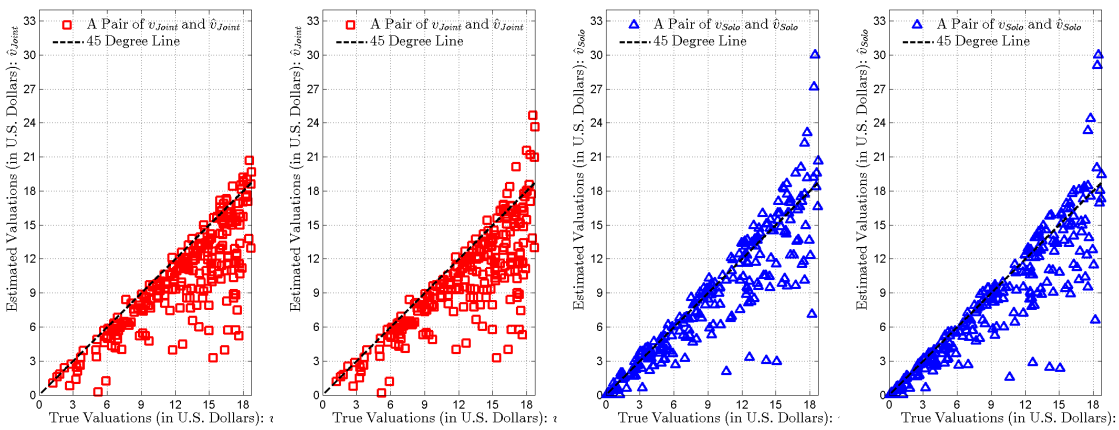
[60] See the histograms on pp. 723 of their work for the under-estimation problem of semiparametric estimates.

Figure 5: Risk-Neutral Model:
True (horizontal-axis) vs Estimated (vertical-axis) Valuations in Asymmetric Auctions



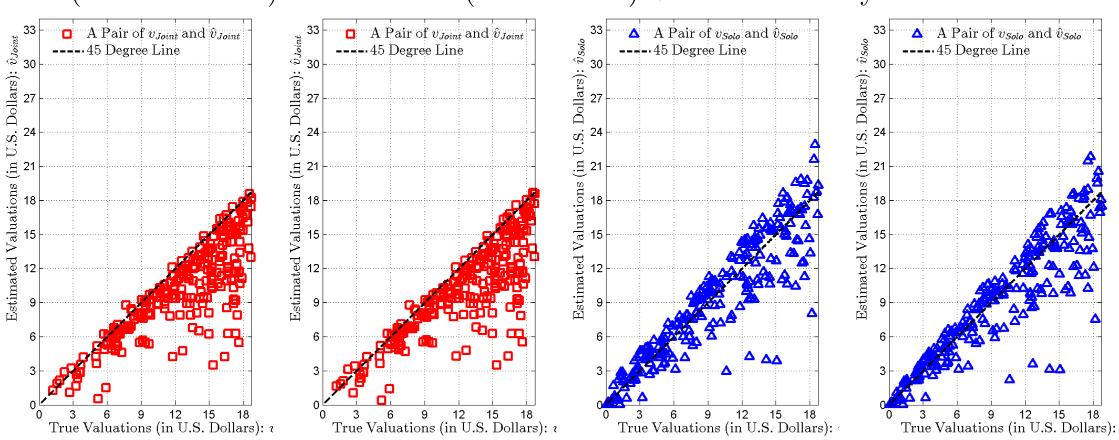
(a) Joint Type: APV (b) Joint Type: IPV (c) Solo Type: APV (d) Solo Type: IPV

Figure 6: CRRA Model:
True (horizontal-axis) vs Estimated (vertical-axis) Valuations in Asymmetric Auctions



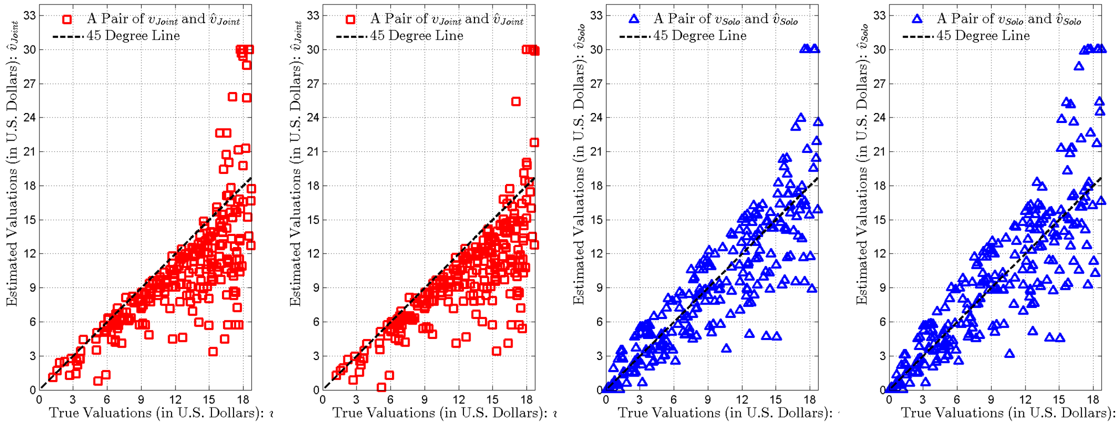
(a) Joint Type: APV (b) Joint Type: IPV (c) Solo Type: APV (d) Solo Type: IPV

Figure 7: CARA Model:
True (horizontal-axis) vs Estimated (vertical-axis) Valuations in Asymmetric Auctions



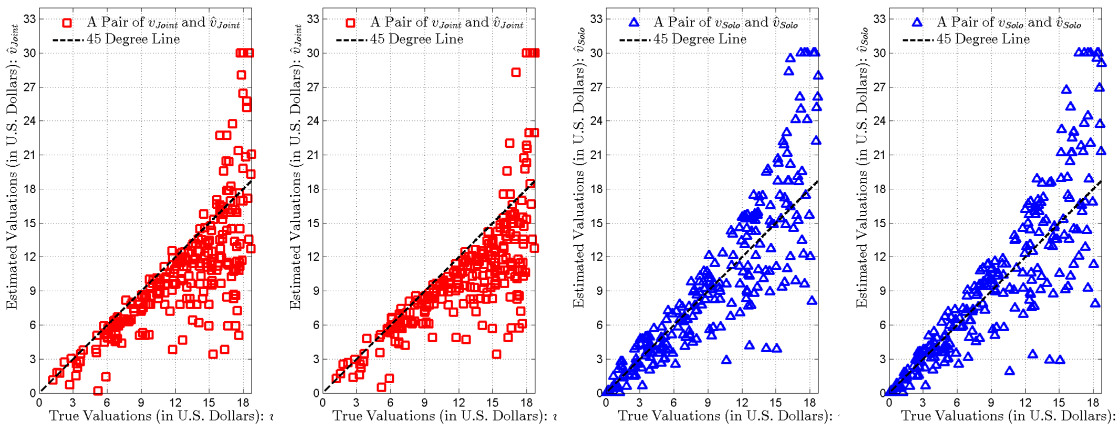
(a) Joint Type: APV (b) Joint Type: IPV (c) Solo Type: APV (d) Solo Type: IPV

Figure 8: Nonparametric Model:
True (horizontal-axis) vs Estimated (vertical-axis) Valuations in Asymmetric Auctions



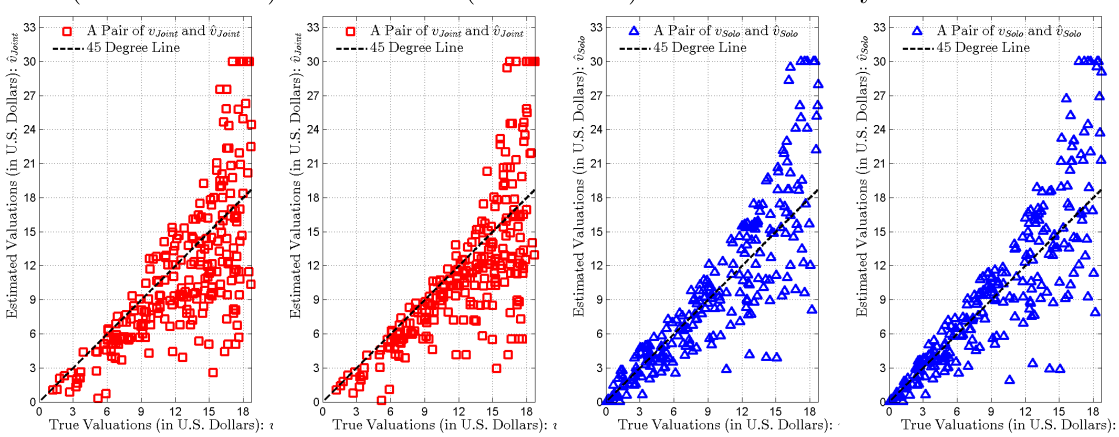
(a) Joint Type: APV (b) Joint Type: IPV (c) Solo Type: APV (d) Solo Type: IPV

Figure 9: Nonparametric Model with Shape Restriction 1:
True (horizontal-axis) vs Estimated (vertical-axis) Valuations in Asymmetric Auctions



(a) Joint Type: APV (b) Joint Type: IPV (c) Solo Type: APV (d) Solo Type: IPV

Figure 10: Nonparametric Model with Shape Restriction 2:
True (horizontal-axis) vs Estimated (vertical-axis) Valuations in Asymmetric Auctions



(a) Joint Type: APV (b) Joint Type: IPV (c) Solo Type: APV (d) Solo Type: IPV

Table 3:
OLS Estimates of CRRA Risk-Averse Parameters: $U_t(x) = x^{\theta_t}$

	Bidder Type	CRRA: $\hat{\theta}_t$ under Affiliated PV	CRRA: $\hat{\theta}_t$ under Independent PV
Experiment Run II	Joint Type	0.050 (0.056, 0.044)	0.051 (0.059, 0.043)
	Solo Type	0.271 (0.302, 0.239)	0.090 (0.185, -0.004)
Experiment Run III	Joint Type	0.026 (0.028, 0.024)	0.039 (0.042, 0.036)
	Solo Type	0.056 (0.064, 0.049)	0.169 (0.196, 0.143)

*Inside of parentheses are 95 % confidence intervals with heteroskedasticity-robust standard errors

Table 4:
NLLS Estimates of CARA Risk Averse Parameters: $U_t(x) = \frac{1-\exp(-\zeta_t \cdot x)}{1-\exp(-\zeta_t)}$

	Bidder Type	CARA: $\hat{\zeta}_t$ under Affiliated PV	CARA: $\hat{\zeta}_t$ under Independent PV
Experiment Run II	Joint Type	14.002 (16.048, 11.956)	13.888 (16.079, 11.698)
	Solo Type	00.568 (00.653, 00.483)	01.519 (02.793, 00.246)
Experiment Run III	Joint Type	20.134 (21.901, 18.367)	18.413 (20.006, 16.820)
	Solo Type	02.433 (02.763, 02.103)	00.804 (00.939, 00.669)

*Inside of parentheses are 95 % confidence intervals with heteroskedasticity-robust standard errors

5.2 Tests of Distributional Equivalence, Independence, and Asymmetry

In this subsection, we examine the testable hypotheses that measures the performance of asymmetric auction estimates. We first provide the answer to the nitty-gritty of the dispute on empirical auctions, the accuracy of asymmetric auction estimates. We then address the results from independency and asymmetry tests.

5.2.1 Test of Equivalence between True and Estimated Value Distributions

The core question in the empirical auction literature is the accuracy of estimated valuations. If estimated valuations are statistically different from true ones, we should not expect any policy insights, such as optimal reserve prices, derived are not credible as such insights are based on in accurate estimates. For examining the equivalence between true and estimated value distributions, we here investigate the two-sample Kolmogorov-Smirnov statistics that is the difference between two empirical distribution functions.

5.2.2 Test of Independence

In many empirical auction research investigations, the independency of valuations are assumed beforehand, yet it is empirically diligent to test the independency to avoid potential model-misspecification problem. In this subsection, we test the null hypothesis of independent valuations against the alternative of non-independent valuations. Formally, we employ the nonparametric test of independence proposed by Blum, Kiefer, and Rosenblatt (1961, henceforth BKR) [15] with the test statistic of $\frac{1}{2}\pi^4 \cdot N \cdot B_N$ where $B_N = N^{-4}\sum_{l=1}^{N}\{N_1(l)\cdot N_4(l) - N_2(l)\cdot N_3(l)\}^2$ with the index $l \in \{1, \cdots, RM\}$ where R is the number of rounds in each experiment run and M is the number of (within-a-round) matched groups.[61] Here, $N_1(l)$, $N_2(l)$, $N_3(l)$, and $N_4(l)$ are defined as the number of data points that fall in the region of $\{(x, y) : x \leqslant X_l, y \leqslant Y_l\}$, $\{(x, y) : x \geqslant X_l, y \leqslant Y_l\}$, $\{(x, y) : x \leqslant X_l, y \geqslant Y_l\}$, and $\{(x, y) : x \geqslant X_l, y \geqslant Y_l\}$ respectively.[62] Table 5 reports the results of the BKR tests.[63] Independencies are not rejected in all Experiment Runs.

5.2.3 Test of Asymmetry

In empirical asymmetric auction studies, researchers often assume the asymmetry among bidders (or firms) based on the priori information, such as differences in firm size, capacity size, or observed bid distributions.

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[61] For instance, RM is equal to 84 (= 12 rounds × 7 matched groups) in Experiment Run I.

[62] Campo, Perrigne, and Vuong (2003) [17] use the same test statistics to investigate the affiliation of estimated valuations Outer Continental Shelf (OCS) wildcat lease auction data and reject the null of independent valuations.

[63] The p-value of the BKR statistic with 5% significance level(= 2.844) is listed in Table II on p.497 of Blum, Kiefer, and Rosenblatt (1961) [15].

Figure 11: Comparison between Experimental and Field Auctions

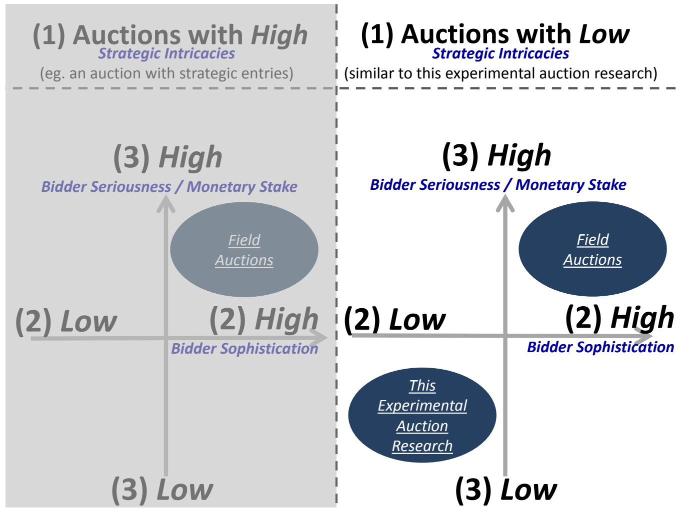


Table 5: :Testing Bivariate Independence: Nonparametric Blum, Kiefer, and Rosenblatt Statistics (nB_n)

	BKR Statistic
Experiment Run I: $RM = 84$	0.0236 (0.5056) Not Rejected
Experiment Run II: $RM = 96$	0.0350 (0.2281) Not Rejected
Experiment Run III: $RM = 72$	0.0254 (0.4496) Not Rejected

6 Conclusion

To provide an answer to the criticism and skepticism on empirical asymmetric auction studies, this research provides laboratory evidence to support the credibility of asymmetric first-price auction estimates. We have investigated the blind spot of empirical asymmetric auction research, that being latent valuations, and have provided new statistical and visual evidence of estimate accuracy by manifesting how the estimates of valuations fit the true valuations. We have also shown that incorporating the risk-aversion has nonnegligible improvements in estimates. Although the *degree* of such improvements will differ by applications, the *fact* of improvement is translatable to other empirical research.

Finally, the external validity (i.e. generalizability and translatablity) of our results to other auctions, especially empirical auction research with field auction data, must be addressed. We recognize that getting one good estimate result in one specific auction environment does not guarantee that researchers will get a similar result in other situations. However, we can deductively bring conservative yet practical insights by contrasting our experimental auctions to field auctions in the following ways. The experimental auctions differ from field ones in, at least, three dimensions: (1) strategic intricacies of auctions; (2) bidder sophistication; and (3) bidder seriousness caused by associated monetary stakes. The discussion below, as depicted in Figure 11, breaks down (1) into two parts (i.e. high/advanced and low/similar strategic intricacies compared to this research), then examines the generalizability of our results regarding (2) and (3).

Asymmetric auctions with high/advanced strategic intricacies, as compared to this research (left hand side of Figure 11): If an environment of undertaking empirical asymmetric auction research is more intricate than the one we have discussed in this research, such as endogenous and strategic participation in auctions or binding reserve prices, our research has the limited external validity on the accuracy of estimates. Bidders who face such a high degree of strategic intricacies may behave differently than what we observe in this research. Further investigation on the accuracy of estimates derived from experimental data, or any field data that directly or indirectly contains the information of underlying valuations, will extend the results of our research for such intricate auctions.

Asymmetric auctions with low/similar strategic intricacies, as compared to this research (right hand side of Figure 11): In our experimental asymmetric auctions, the participants were recruited undergraduate students. Thus, given the lack of their real-world business experience, their degree of strategic sophistication is reasonably expected to be lower than the ones observed in real-world competitive business industry (i.e. low degree of (2)). In addition, as the monetary stake in our experiment is relatively low compared to the stakes observed in real-world auctions, the associated seriousness among bidders in the experiment laboratory is also expected to be low (i.e. low degree of (3)).

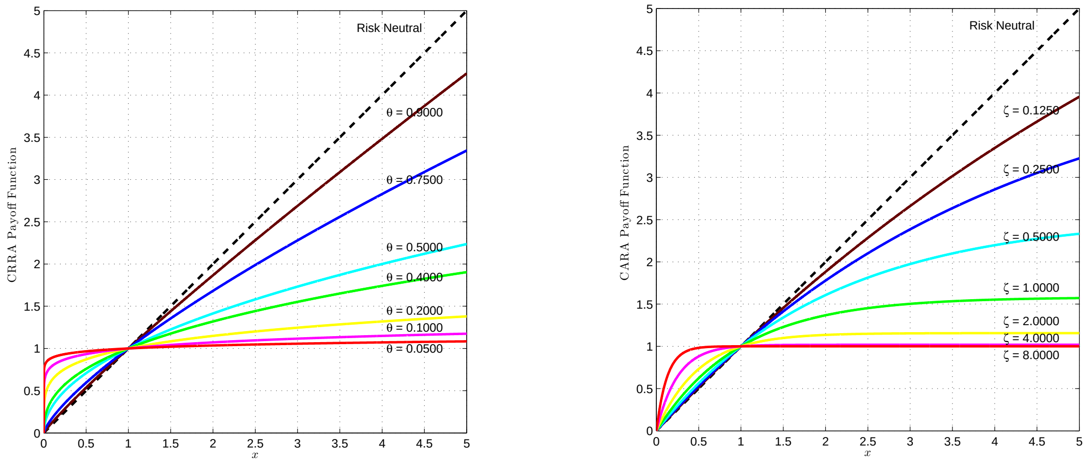
However, the positive finding of this research is that structural estimates derived from bids submitted by such strategically unsophisticated and less seriousness bidders are statistically shown to be accurate. Therefore, we can deductively translate the positive finding in the accuracy of estimates reported in this research into the estimates generated from bids submitted by professional industry bidders in field auctions by the following reasons: first, professional industry bidders must have a high degree of strategic sophistication in order to survive harsh industry competition (i.e. high degree of (2)), and secondly, as the monetary stakes in real-world auctions are high, the associated seriousness among bidder is also high (i.e. high degree of (3)). It stands to reason then, compared to our experiment participants, such professional industry bidders are more likely to profoundly recognize underlying strategic interactions in auctions as prescribed by BNE and less likely to make optimization errors. Accordingly, because structural estimations are rigidly based on BNE, the estimates derived from such sophisticated and seriously considered bids are likely to be more accurate than those reported in this research. Thus, we deductively conclude that, as far as the strategic intricacy of underlying asymmetric auction market is not vastly different from the one discussed in this research and as far as industry bidders are maximizing expected payoffs, what holds accurate in our laboratory auctions also holds accurate in a real-world industry setting.

For these reasons, this research not only contributes to providing support for estimates previously reported in the literature but also pushes the credibility of present and future empirical asymmetric auction research further.

7 Appendix

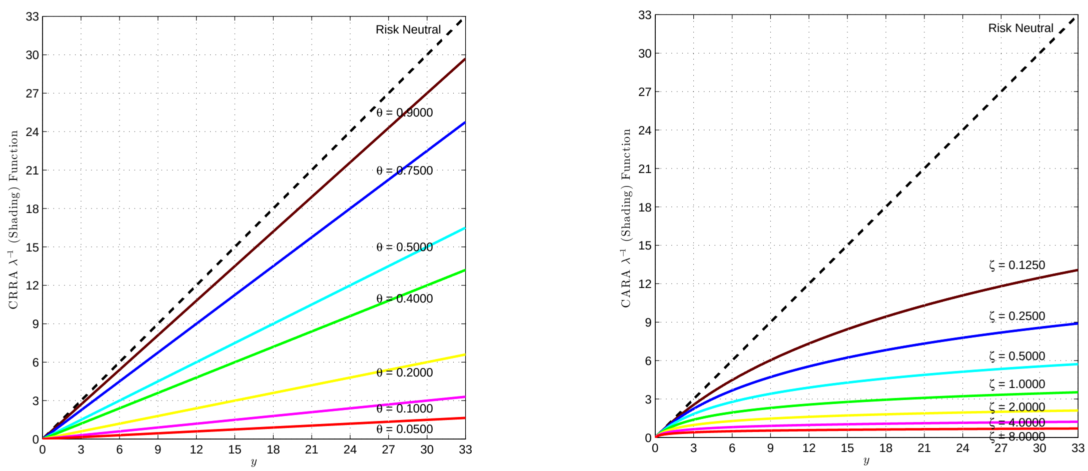
7.1 Shapes of CRRA and CARA Payoff and Shading Functions

Figure 12: Visual Comparison of CRRA and CARA Payoff Functions



(a) CRRA Payoffs: $U(x) = x^{\theta}$ (b) CARA Payoffs: $U(x) = \frac{1-\exp(-\zeta \cdot x)}{1-\exp(-\zeta)}$

Figure 13: Visual Comparison of CRRA and CARA Shading Functions



(a) CRRA Shading Functions: $\lambda^{-1}(y) = \theta \cdot y$ (b) CARA Shading Functions: $\lambda^{-1}(y) = \frac{\ln(1+\zeta \cdot y)}{\zeta}$

7.2 Construction of Quantile Points

In the semi and nonparametric estimations of vNM functions, this research uses a large number of point αs. In practice, we choose the sequence of quantile points, $\{\alpha_q\}_{q=0,\cdots,Q}$ where $\alpha_q \in [0.50, 0.75]$ and $Q = 250$, and calculate the quantiles of bid distributions at equally-spaced quantile points in $[0.50, 0.75]$ and denote them as $\left\{b^{\text{Sym}}_{t,\alpha_q}\right\}_{q=0,\cdots,Q}$ and $\left\{b^{\text{Asym}}_{t,\alpha_q}\right\}_{q=0,\cdots,Q}$.[64] We use the notation of $\alpha_{\text{lower}} = 0.50$ and $\alpha_{\text{upper}} = 0.75$. As suggested by Bajari and Hortaçsu (2005) [12] quantile points less than the 25th and larger than the 75th quantiles are not used in order to avoid the well-known boundary problems in nonparmetric kernel density estimations. In addition, we do not use $\alpha \in (0.25, 0.50)$, as R function plots (posted earlier in Appendix) suggest that R functions in this domain suffer from non-monotonicities and discontinuous jumps, potentially due to the small sample size or bidders' non-expected-payoff maximizing behavior.[65]

7.3 Sieve Polynomial Estimation: Details

This subsection explains the sieve polynomial estimation method that provides a local identification and the conventional-wisdom-based shape restrictions. We first explain the recursively-constructed nonparametric identification method proposed by Guerre, Perrigne, and Vuong (2009) [28]. As noted in their paper, the method faces computational and empirical tractability challenges. Second, we explain the sieve polynomial estimation method that provides not full-range but local identification. Lastly, we illustrate the conventional-wisdom-based shape restrictions used in this research that supplementally fill the gap between full-range and local identifications.

7.3.1 Guerre, Perrigne, and Vuong's (2009) Recursive Identification Method

In their seminal paper, Guerre, Perrigne, and Vuong (2009) [28] propose the following recursive identification method, which is illustrated in Figure 14,[66]

PROPOSITION 3 [from Guerre, Perrigne, and Vuong (2009)] *: Under the previous assumptions* [proposed in their research], $\lambda^{-1}(\cdot)$ *is identified nonparametrically on* $\mathcal{R}_1$*. Specifically,* $\lambda^{-1}(0) = 0$ *and for any* $u_0 \in \mathcal{R}_1 \backslash \{0\}$, $\lambda^{-1}(\cdot)$ *is given by*

$$\lambda^{-1}(u_0) = \sum_{t=0}^{+\infty} \Delta b(\alpha_t),$$

where $\Delta b(\alpha_t) = b_2(\alpha) - b_1(\alpha)$ *and the sequence* $\{\alpha_t\}$ *is strictly decreasing with* $0 < \alpha_t \leqslant 1$ *satisfying the nonlinear recursive relation* $R_1(\alpha_t) = R_2(\alpha_{t-1})$ *with initial condition* $R_1(\alpha_0) = u_0$*. Moreover,* $F(\cdot)$ *is identified nonparametrically on* $p1\underline{v}, \overline{v}]$ *with* $F(\cdot) = G_j(\xi_j^{-1}(\cdot))$ *for* $j = 1, 2$*.*

This identification method is computationally and empirically challenging, as researchers have to guess (then

[64]To enable a high-order nonparametric sieve polynomial estimation, we use this relatively large number of quantile points in this research.

[65]In general, non-monotonicity and discontinuous jumps of R functions are especially severe under the affiliated private value assumption in which the convergence rates of distributional functions are slower than those under the independent private value assumption.

[66]Figure 14 is the replication of FIGURE 1 (on pp. 1204) in Guerre, Perrigne, and Vuong (2009) [28].

match with observed bids) equilibrium bidding functions.[67] Also, as the authors mention, the recursive constructions of $\{\alpha_t\}$ leads to an accumulated error problem. Additionally, given the small sample size in most empirical auction research, the identification of near-boundary domain areas is practically challenging, as estimated bid distributions suffer from the well-known boundary problem.

7.3.2 Sieve Polynomial Estimation

Alternatively, this research pursues the second estimation method that Guerre, Perrigne, and Vuong (2009) [28] suggest, sieve polynomial estimations. Specifically, in a sieve polynomial estimation, we restrict a shading function $\lambda^{-1}(R)$ on the domain of $R \in [0, R_{\alpha=1.00}]$ to be in the linear space of (without-intercept[68]) polynomial functions with the basis of $PB(R) = \left\{R, R^2, R^3, \cdots, R^{\overline{K}}\right\}$, where PB stands for "polynomial basis" and the order of polynomial $\overline{K}$ flexibly changes. We henceafter omit the bidder type subscript $t \in \{\text{Joint}, \text{Solo}\}$ for simplicity. Then, we exploit the additively-coefficient-preserving nature of polynomial linear space. The (mathematically imprecise yet) intuitive illustration of the estimation strategy is depicted in Figure 15.[69] With the polynomial linear space restriction, a compatibility condition becomes[70]

$$\begin{aligned} b_\alpha^{\text{Sym}} - b_\alpha^{\text{Asym}} &= \overbrace{\sum_{k=1}^{\overline{K}} \eta_k \cdot (R_\alpha^{\text{Asym}})^k}^{=\lambda^{-1}(PB(R^{\text{Asym}}))} - \overbrace{\sum_{k=1}^{\overline{K}} \eta_k \cdot (R_\alpha^{\text{Sym}})^k}^{=\lambda^{-1}(PB(R^{\text{Sym}}))} \\ &= \underbrace{\sum_{k=1}^{\overline{K}} \eta_k \cdot \left\{(R_\alpha^{\text{Asym}})^k - (R_\alpha^{\text{Sym}})^k\right\}}_{=\lambda^{-1}(PB(R^{\text{Asym}}) - PB(R^{\text{Sym}}))}. \end{aligned}$$

Ideally, empirical researchers should obtain an estimate of $\lambda^{-1}(R)$ on the entire domain, $[0, R_{\alpha=1.00}]$. However, this estimation method proides only local identification. Specifically, as we construct quantile points between α_{lower} and α_{upper} (and as depicted in Figure 15), it is not possible to estimate $\lambda^{-1}(\cdot)$ on near-lower-boundary and near-upper-boundary domains of $[0, R_{\alpha=1.00}]$.[71] This is due to two facts: (1) we use not a

[67] We would like to leave this computational challenge as a future research question by encouraging computationally-skilled researchers to overcome the numerical difficulties.

[68] An intercept term is eliminated to enable the theoretical restriction of $\lambda^{-1}(0) = 0$.

[69] The easiest way to understand the right hand side of Figure 15 is by restricting the polynomial basis to order 1, $PB(B) = \{R\}$. In this special case, as a shading function becomes $\lambda^{-1}(R) = \eta_1 \cdot R$, a polynomial estimation is equivalent to the semiparametric CRRA estimation.

[70] For joint-type bidders, as they equally split their monetary payoffs, we need a slight modification to a compatibility condition equations. See Appendix for details.

[71] In a sieve polynomial estimation, we do not have the data points in the near-lower-boundary domain of $[0, R_{\alpha=1.00}]$, yet this is practically less problematic as we can exploit the theoretical restriction of $\lambda^{-1}(0) = 0$ in estimations.

recursive but a linear difference estimator,[72] that is, after taking a difference we do not have data points in the near-upper-boundary area of domain;[73] and (2) to overcome the boundary value problem we shrink the quantiles of data variations by setting α_{lower} and α_{upper}.

In addition, as reported by Bajari and Hortaçsu (2005) [12], who estimate the special case of this seive polynomial estimation method, (i.e. polynomial order $\overline{K} = 1$ CRRA specification), it is known that this method overestimates the risk preferences that, in turn, leads to underestimation of valuations among bidders.[74] From a practical point of view, this underestimation of valuations among the upper quantile of valuations is problematic as many policy implications, such as expected revenue calculations, largely depend on estimated valuations. Thus, an extrapolation method that has a basis on and extends local identification results is inevitably required to obtain empirically reasonable estimates of valuations.

7.3.3 Conventional-Wisdom-Based Shape Restrictions

To overcome the local identification problem, we now restrict the shapes of shading function $\lambda^{-1}(R)$ on the domain in which researchers cannot obtain data points after variables are differenced. Specifically, we restrict slopes of $\lambda^{-1}(\cdot)$ on a near-upper-boundary domain, $\left[\left(PB(R^{\text{Asym}}_{\alpha_{\text{upper}}})-PB(R^{\text{Sym}}_{\alpha_{\text{upper}}})\right), PB(R^{\text{Asym}}_{\alpha=1.00})\right]$ so that we can smoothly extrapolate the estimate of a shading function, while exploiting the additively-coefficient-preserving nature of polynomial functions in the domain where data variations are available. One may consider this as a minimalistic usage of a calibration method for a part of an auction model that is challenging to estimate.[75] Specifically, we apply the slope restrictions based on a DRRA (decreasing relative risk aversion) preference

[72] Notably, by applying a minmax estimator (in which min is chosen over sieve linear space and max is chosen over quantile pionts with a normed distance), the recent working paper of Zincenko (2014, working paper) [77] establishes the identification of $\lambda^{-1}(\cdot)$ on $[0, \overline{\alpha}]$ where $\overline{\alpha} \to 1$ as a sample size goes to infinity. We tried this minmax estimator. Although our trial suffered from the small sample size problem and the results are not reported here, we believe this minmax estimator is an empirically powerful tool when researchers have a large sample size. Specifically, the property of $\overline{\alpha} \to 1$ is attractive.

[73] As Figure 14 depicts, the recursively-constructed identification method projects the shading function at a given quantile value to lower quantile points. From a practical point of view, as researchers often suffer from boundary-value problems, we believe that, even in the recursive identification method, some sort of shape restrictions are required for obtaining a reasonable estimate of $\lambda^{-1}(\cdot)$ on the near-boundary domains.

[74] See the histogram figures on pp. 723 of Bajari and Hortaçsu (2005) [12] for this problem. They apply the order $\overline{K} = 1$ polynomial, CRRA specification, to symmetric auctions in which they use the observed difference of bids between three-bidder (less competitive) and six-bidder (more competitive) markets.

[75] Note that this type of minimilistic usage of calibration is common in empirical structural estimations. For example, empirical structural estimation researchers commonly do not estimate but calibrate a discount factor (eg. Rust (1994) [71]) or an extra cost of exceeding capacity constraint (eg. Snider (2009, working paper)), as they are practically infeasible to estimate.

that has established empirical evidence and a clear interpretation.[76] These restrictions behaviorally mean that as an expected payoff increases, a bidder becomes relatively less risk averse (or becomes relatively closer to risk neutral) in the measure of the Allow-Pratt relative risk aversion (Pratt (1964)][69]). Furthermore, the measure of relative risk aversion (RRA), as seen in Figure 13, is linearly associated with the slope of $\lambda^{-1}(\cdot)$. Thus, DRRA is straightforwardly interpreted as gradually increasing slopes of $\lambda^{-1}(\cdot)$. In addition, as Rabin (2000) [70] illustrates, when a researcher exports any reasonable risk preferences on gambling at the lower wealth level to the higher wealth level, it is well-known in the theoretical literature that an agent becomes implausibly too risk averse in the higher wealth levels (i.e. her vNM function becomes too curvy in the higher wealth levels), supporting the usage of DRRA. Specifically, we apply the following slope restrictions (note: polynomial basis notations are omitted):

Shape Restriction - Minimalistic :

For both joint- and solo-type bidders,

$$\left\{ 10^{-8} \leqslant \frac{d}{dR}\widehat{\lambda}^{-1}(R) \leqslant 1.00 \quad \text{for} \quad 0 \leqslant R < \widehat{R}_{\alpha=1.00} \right. ,$$

Shape Restriction - Homogeneous (Same Restriction across Bidder Types):
For both joint- and solo-type bidders,

$$\left\{ \begin{array}{lllllllll} 10^{-8} & \leqslant & \frac{d}{dR}\widehat{\lambda}^{-1}(R) & \leqslant & 1.00 & \text{for} & 0 & \leqslant R < & (\widehat{R}^{\text{Asym}}_{\alpha\text{upper}} - \widehat{R}^{\text{Sym}}_{\alpha\text{upper}}) \\ 0.32 & \leqslant & \frac{d}{dR}\widehat{\lambda}^{-1}(R) & \leqslant & 0.59 & \text{for} & (\widehat{R}^{\text{Asym}}_{\alpha\text{upper}} - \widehat{R}^{\text{Sym}}_{\alpha\text{upper}}) & \leqslant R < & \frac{7}{8}(\widehat{R}^{\text{Asym}}_{\alpha\text{upper}} - \widehat{R}^{\text{Sym}}_{\alpha\text{upper}}) + \frac{1}{8}\widehat{R}^{\text{Asym}}_{\alpha=1.00} \\ 0.59 & \leqslant & \frac{d}{dR}\widehat{\lambda}^{-1}(R) & \leqslant & 0.85 & \text{for} & \frac{7}{8}(\widehat{R}^{\text{Asym}}_{\alpha\text{upper}} - \widehat{R}^{\text{Sym}}_{\alpha\text{upper}}) + \frac{1}{8}\widehat{R}^{\text{Asym}}_{\alpha=1.00} & \leqslant R \leqslant & \widehat{R}^{\text{Asym}}_{\alpha=1.00} \end{array} \right. ,$$

Shape Restriction - Heterogeneous (Different Restrictions across Bidder Types):[77]
For joint-type bidders,

$$\left\{ \begin{array}{lllllllll} 10^{-8} & \leqslant & \frac{d}{dR}\widehat{\lambda}^{-1}(R) & \leqslant & 1.00 & \text{for} & 0 & \leqslant R < & (\widehat{R}^{\text{Asym}}_{\alpha\text{upper}} - \widehat{R}^{\text{Sym}}_{\alpha\text{upper}}) \\ 0.85 & \leqslant & \frac{d}{dR}\widehat{\lambda}^{-1}(R) & \leqslant & 0.90 & \text{for} & (\widehat{R}^{\text{Asym}}_{\alpha\text{upper}} - \widehat{R}^{\text{Sym}}_{\alpha\text{upper}}) & \leqslant R < & \frac{7}{8}(\widehat{R}^{\text{Asym}}_{\alpha\text{upper}} - \widehat{R}^{\text{Sym}}_{\alpha\text{upper}}) + \frac{1}{8}\widehat{R}^{\text{Asym}}_{\alpha=1.00} \\ 0.90 & \leqslant & \frac{d}{dR}\widehat{\lambda}^{-1}(R) & \leqslant & 0.95 & \text{for} & \frac{7}{8}(\widehat{R}^{\text{Asym}}_{\alpha\text{upper}} - \widehat{R}^{\text{Sym}}_{\alpha\text{upper}}) + \frac{1}{8}\widehat{R}^{\text{Asym}}_{\alpha=1.00} & \leqslant R \leqslant & \widehat{R}^{\text{Asym}}_{\alpha=1.00} \end{array} \right. ,$$

For solo-type bidders,

$$\left\{ \begin{array}{lllllllll} 10^{-8} & \leqslant & \frac{d}{dR}\widehat{\lambda}^{-1}(R) & \leqslant & 1.00 & \text{for} & 0 & \leqslant R < & (\widehat{R}^{\text{Asym}}_{\alpha\text{upper}} - \widehat{R}^{\text{Sym}}_{\alpha\text{upper}}) \\ 0.15 & \leqslant & \frac{d}{dR}\widehat{\lambda}^{-1}(R) & \leqslant & 0.32 & \text{for} & (\widehat{R}^{\text{Asym}}_{\alpha\text{upper}} - \widehat{R}^{\text{Sym}}_{\alpha\text{upper}}) & \leqslant R < & \frac{7}{8}(\widehat{R}^{\text{Asym}}_{\alpha\text{upper}} - \widehat{R}^{\text{Sym}}_{\alpha\text{upper}}) + \frac{1}{8}\widehat{R}^{\text{Asym}}_{\alpha=1.00} \\ 0.32 & \leqslant & \frac{d}{dR}\widehat{\lambda}^{-1}(R) & \leqslant & 0.59 & \text{for} & \frac{7}{8}(\widehat{R}^{\text{Asym}}_{\alpha\text{upper}} - \widehat{R}^{\text{Sym}}_{\alpha\text{upper}}) + \frac{1}{8}\widehat{R}^{\text{Asym}}_{\alpha=1.00} & \leqslant R \leqslant & \widehat{R}^{\text{Asym}}_{\alpha=1.00} \end{array} \right. ,$$

[76] The empirical evidence of the DRRA model with household-level panel data is provided by Morin and Suarez (1983) [62] and Ogaki and Zhang (2001) [64].

[77] We apply this heterogeneous treatment across bidder types for the following reasons: (1) joint-type bidders equally split their payoffs; (2) as listed in Table 4, the estimates of CARA risk averse preference (which is not invariant to payoff scales) parameters for joint-type bidders provide implausibly high numbers; and (3) a sieve polynomial estimation is also not invariant to payoff scales.

These RRA numbers are taken from the highly cited work of Holt and Laury (2002) [39], in which the majority of bidders' relative risk preferences fall into these bounds. Also, the steeper-slope restrictions in Restriction 2 are motivated by Harrison (1989) [33] work in which expected profits in his experiment are close to the joint-type expected profits in our experimental setting. As these results are widely accepted in the literature, we treat their RRA numbers and the bounds listed above as conventional wisdom.[78] Intuitively, these DRRA restrictions assume that, as an expected payoff increases in the domain where researchers cannot obtain data points (due to the limitations of polynomial difference estimator), the measure of bidders' risk attitudes gradually becomes close to the RRA preferences conventionally reported in literature. It is important to note that under these slope restrictions, although the slopes gradually become closer to $\frac{d}{dR}\lambda^{-1}(R) = 1$ (which is the slope of shading function in the risk neutral model), this does not mean that the shading function is getting closer to $\lambda^{-1}(R) = R$ (the shading function of risk-neutral model), as depicted in the bottom-right plot of Figure 15. While the validity of applying these shape restrictions to our experimental setting is debatable and should be scrutinized more in future research, our investigation results plotted in Figure 9 and 10 depict that there are sizable improvements in the accuracy of value estimates, showing empirical usefulness of these shape restrictions.[79]

[78] As Holt and Laury (2002) [39] report, researchers typically observe a wide range of risk averse attitudes in experiments, from highly risk averse to slightly risk loving participants. Yet, the majority of bidders' risk preferences in their research fall into the bounds specified here. Regarding bidders' preference heterogeneity in our research, although we tried the heterogeneous risk preference models with CRRA and CARA specifications for estimating individual-bidder-specific vVM functions, our results suffer from the small sample size problem and are not reported here. MATLAB code for individual bidder specific risk preference models is available upon request.

[79] To the extent of minimalistic usage of calibration, in general, the CRRA coefficient estimates reported in Lu and Perrigne (2008) [55] and Campo, Guerre, Perrigne, and Vuong (2011) [18] are invaluable for many empirical auction investigations, as researchers may export these CRRA estimates to their own research when exogenous variations are not available. Note that Lu and Perrigne (2008) [55] estimate the risk attitude by using the difference between first-price and ascending auctions, while Campo, Guerre, Perrigne, and Vuong (2011) [18] establish then apply the conditional quantile restrictions with observed characteristics of auctioned objects.

Figure 14: Nonparametric and Recursive Identification Method in Guerre, Perrigne and Vuong (2009)

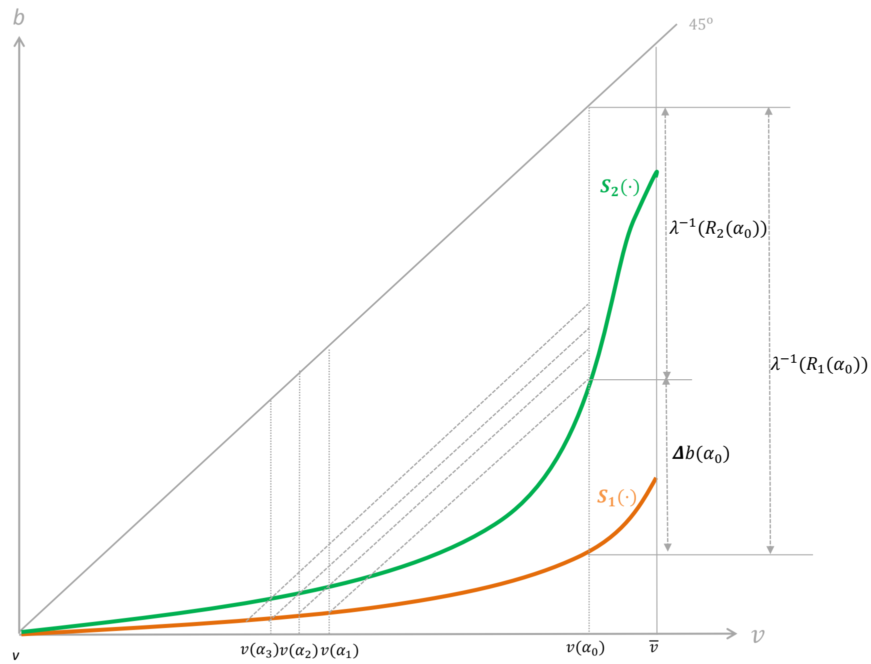
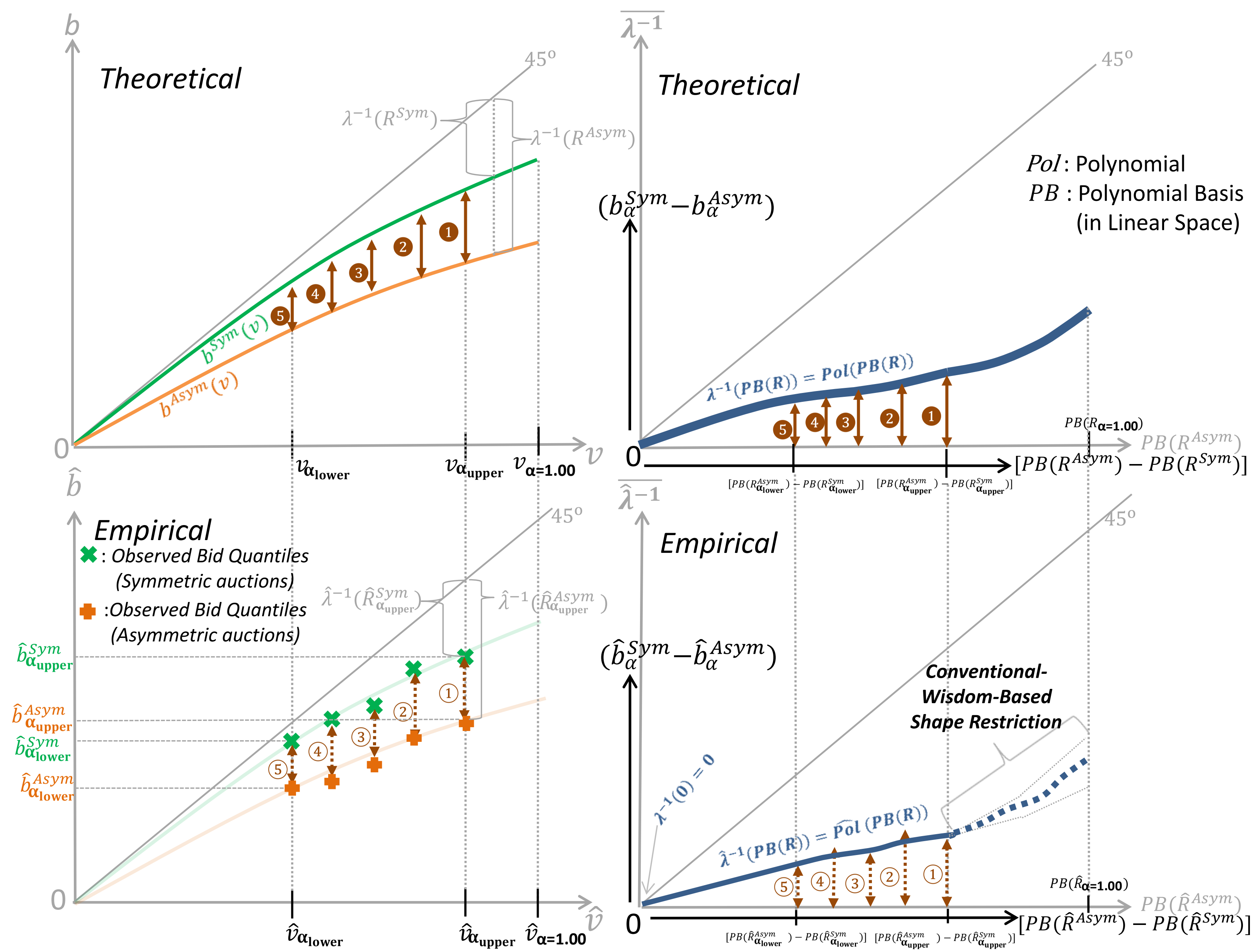


Figure 15: Sieve Polynomial Estimation in This Research: Outline



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