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Electron acceleration by magnetic collapse during decoupling

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ABSTRACT
This paper identifies the non-equilibrium evolution of magnetic field structures at the onset of large-scale recombination of an inhomogeneously ionized plasma. The context for this is the Universe during the epoch of recombination. The electromagnetic treatment of this phase transition can produce energetic electrons scattered throughout the Universe, localized near the edges of magnetic domains. This is confirmed by a numerical simulation in which a magnetic domain is modelled as a uniform field region produced by a thin surrounding current sheet. Conduction currents sustaining the magnetic structure are removed as the charges comprising them combine into neutrals. The induced electric field accompanying the magnetic collapse is able to accelerate ambient stationary electrons (that is, electrons not participating in the current sheet) to energies of up to order 10keV. This is consistent with theoretical predictions. The localized electron acceleration leads to local imbalances of charge which has implications for charge separation in the early Universe.

Key words: magnetic fields – cosmology: theory.

1 INTRODUCTION

In this paper, a theoretical model first developed by Diver & Teodoro (2008) will be expanded. Here we will develop a numerical model used to simulate the motion of charged particles accelerated by the transient electric and magnetic fields induced in a rapidly evolving magnetized plasma.

For some years now, it has been proposed that the early Universe contained large-scale coherent magnetic fields before the recombination era (Kandus, Kunze & Tsagas 2011; Ryu et al. 2012; Widrow et al. 2012). If this was indeed the case, then the field must have been pseudo-stochastically arranged over very large length-scales, such that the total field summed to zero (Moffatt 1978; Ichiki et al. 2006; Durrer 2007). This assumption preserves the cosmological principles of isotropy and homogeneity. Therefore, it is reasonable to assume that if large-scale magnetic fields did exist at this epoch, then the Universe was divided into magnetic domains, each of which contains a spatially coherent magnetic field not necessarily aligned with neighbouring domains.

Here we define a domain as a cylindrical region of radius 10 Mpc. Each domain contains a uniform magnetic field that is coherent over a length-scale far longer than the radius of the domain.

The boundary between domains must have been provided by conduction currents in the high-conductivity plasma, which would be perturbed after the start of global recombination as the current densities vanishes. The model presented in this paper will consider the electromagnetic behaviour within the current-carrying boundary of one such domain, and examine the consequences that magnetic collapse holds for the remaining free charges inside that domain.

During recombination, the Universe made the transition from fully ionized to just 1 part in 10^5 ionized (Padmanabhan 1993). This large-scale neutralization must have removed charges from the entire plasma, including the current-carrying parts. During such a transition, the displacement current would assert itself to restore the balance and sustain the magnetic field structure. The time-varying electric field of the displacement current would accelerate any remaining free charges in the plasma within a certain locus of the original conduction current density. This is a direct physical consequence of the large-scale neutralization of an inhomogeneously magnetized plasma. In the early Universe, the resulting small population of energetic charges has a very small cross-section for recombination due to their high kinetic energy. This means that many of the charges will persist as a non-equilibrium population.

This idea was developed by Diver & Teodoro (2008) and also by Teodoro, Diver & Hendry (2008) where they posed the question: What is the correct physical model that encompasses the critical physics? The paper constructed the model by identifying the appropriate time-scales that have to be considered. In order to accurately model the physics of this epoch, the crucial, shortest time-scale is that of electromagnetic changes communicated at the speed of light. In other words, the time is right for standard magnetohydrodynamic (MHD) plasma cosmological models (Durrer, Kahniashvili & Yates 1998; Subramanian & Barrow 1998a,b; Jedamzik,

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Katalinić & Olinto 2000) to be extended. It was concluded that the only model that takes account of the appropriate physics time-scales is a fully electromagnetic one.

2 THEORETICAL MODEL

Magnetic domains as defined in Section 1 are assumed to be sustained by an azimuthal current density caused by the differential drift of the plasma species in the domain boundary; the resulting magnetic field is assumed coherent over a scalelength greater than the radius of the domain. This must be true since in order to refract the coherent magnetic field, i.e. to change the direction of it at the boundary between two domains, there must either be a material change or a current density located at the boundary. In the cosmological context, the latter case must be true. Furthermore, the current density located at the boundary of a domain sustains the coherent magnetic field within it.

Consider one such single magnetic domain, the field in which we shall approximate simply as that arising from an infinite solenoid. Diver & Teodoro (2008) predicted that electrons in such a domain could reach energies of ∼7 keV when accelerated by the magnetic collapse induced by the local domain recombining. Here we will test their predictions, and the model will be expanded and generalized such that more complex systems and arrangements of magnetic field can be studied.

3 ELECTROMAGNETIC EVOLUTION

Consider an infinite solenoid (Fig. 1) with the axis pointing in the axial (z-)direction of a cylindrical polar coordinate system. The current density $\boldsymbol{J}$ is entirely confined to a narrow band in the azimuthal (ϕ-)direction. Accordingly, the magnetic field sustained by the conduction current is uniform in the axial direction inside the solenoid, and zero everywhere outside it. This geometry means that when the current density decreases with time, the resultant-induced electric field is azimuthal only, and the induced magnetic perturbation is confined to the z-direction.

To derive the equation describing the electric field evolution, begin with Faraday's law and the complete form of Ampere's law:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{1}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}, \tag{2}$$

where $\boldsymbol{E} = \hat{\phi} E_\phi$ is the electric field, $\boldsymbol{B} = \hat{z} B_z$ is the total magnetic field, $\boldsymbol{J} = \hat{\phi} J_\phi$ is the current density, μ_0 is the permeability of free space and c is the speed of light in vacuum. In a normal cosmological model, the scale factor $a(t)$ would be present in equations (1) and (2): however, since our model describes only the relatively short epoch of recombination, it is reasonable to consider $a(t)$ to be constant, as a first approximation.

Recasting in cylindrical polar coordinates $(\hat{r}, \hat{\phi}, \hat{z})$, taking the curl of equation (1) and the time-derivative of equation (2), and combining to eliminate B_z, a second-order hyperbolic differential equation in E_ϕ emerges:

$$\frac{1}{c^2}\frac{\partial^2 E_\phi}{\partial t^2} - \frac{\partial^2 E_\phi}{\partial r^2} - \frac{1}{r}\frac{\partial E_\phi}{\partial r} + \frac{1}{r^2} E_\phi = -\mu_0 \frac{\partial J_\phi}{\partial t}. \tag{3}$$

The main variables and derivatives can be substituted as follows. In each case the quantity with subscript 0 is the dimensional part, and the Greek letter is the dimensionless variable to cast the equations in:

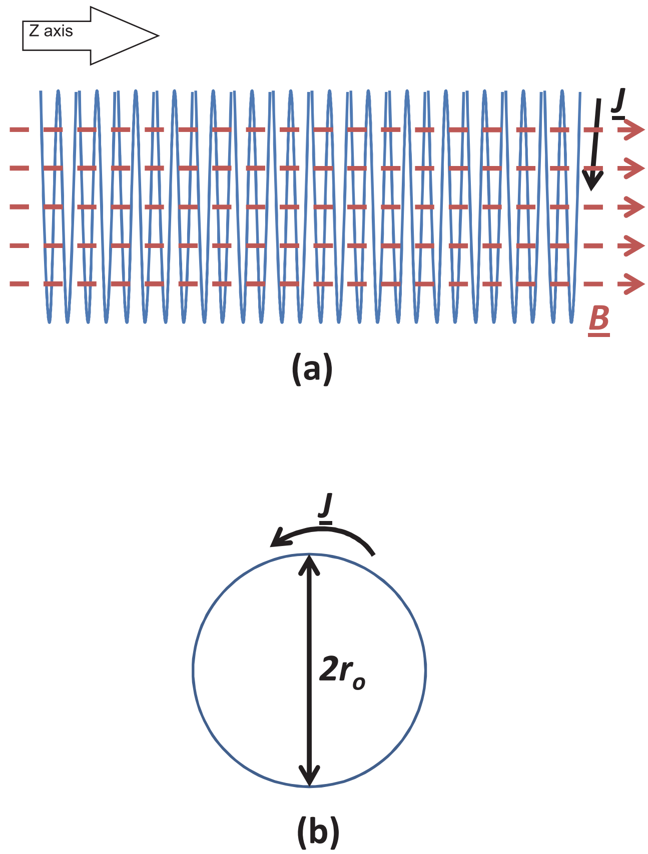

Figure 1. Diagram of an infinite solenoid. The top picture (a) shows azimuthal current loops of radius $r_{\rm o}$ carrying current density $\boldsymbol{J}$ in the ϕ-direction only, resulting in a uniform magnetic field $\boldsymbol{B}$ aligned along the z-direction inside the coil. For clarity, picture (b) shows the same solenoid from the perspective along the z-axis in the antiparallel direction to the magnetic field.

$$r = r_0 \rho \tag{4}$$

$$t = t_0 \tau \tag{5}$$

$$\frac{\partial}{\partial r} = \frac{1}{r_0}\frac{\partial}{\partial \rho} \tag{6}$$

$$\frac{\partial}{\partial t} = \frac{1}{t_0}\frac{\partial}{\partial \tau}. \tag{7}$$

Now, recognizing that if the current density is constant then there will be no electric field, the variables are separable as follows:

$$J_\phi(\rho, \tau) = J_0 f(\rho) T(\tau) \tag{8}$$

$$E_\phi(\rho, \tau) = E_0 g(\rho) \partial_\tau T(\tau) \tag{9}$$

$$B_z(\rho, \tau) = B_0 h(\rho) T(\tau), \tag{10}$$

where $f(\rho)$ and $T(\tau)$ are dimensionless functions representing the spatial form and temporal form of the current density, respectively. These are specified according to the physical problem studied. The spatial forms of the electric and magnetic fields are $g(\rho)$ and $h(\rho)$, respectively, and are to be solved for.

The simplest form of $f(\rho)$ that can be chosen is $f(\rho) = \delta(\rho - \rho_j)$, i.e. a Dirac delta function centred on $\rho = \rho_j$, where ρ_j is the radius of the solenoid. The simplest form of $T(\tau)$ is an exponential decay. J_0, E_0 and B_0 are the dimensional parts of the current density and fields.

Substituting equations (4)–(10) into equation (3) and simplifying:

$$\partial_{\rho\rho} g(\rho) + \partial_\tau T(\tau) + \frac{1}{\rho}\partial_\rho g(\rho)\partial_\tau T(\tau) - \frac{1}{\rho^2} g(\rho)\partial_\tau T(\tau)$$

$$-\frac{{r_0}^2}{{t_0}^2}\frac{1}{c^2} g(\rho)\partial_{\tau\tau\tau} T(\tau) = \frac{\mu_0 r_0^2 J_0}{t_0 E_0} f(\rho)\partial_\tau T(\tau), \tag{11}$$

where in the usual fashion a ∂ symbol denotes a derivative, with the subscript indicating the independent variable. Now assume that

$$\frac{\partial\tau\tau\tau T(\tau)}{\partial_\tau T} = \kappa^2, \tag{12}$$

where κ is a constant. This gives the form of $T(\tau)$ as

$$T(\tau) = \exp[(-\kappa\tau)], \tag{13}$$

and if β is defined as

$$\beta = \frac{r_0\kappa}{ct_0}, \tag{14}$$

then equation (11) now becomes

$$\partial_{\rho\rho} g(\rho) + \frac{\partial_\rho g(\rho)}{\rho} - \left(\frac{1}{\rho^2} + \beta^2\right) g(\rho) = \frac{\mu_0 {r_0}^2 J_0}{t_0 E_0} f(\rho). \tag{15}$$

Applying the boundary conditions of the solenoid, the electric field can be expressed fully as

$$E_\phi(\rho, \tau) = \frac{\mu_0 {r_0}^2 J_0}{ct_0 E_0}\kappa\rho_j K_1(\beta\rho_j) I_1(\beta\rho)\exp(-\kappa\tau). \tag{16}$$

Combining equations (1) and (16), the magnetic field evolution can be shown to be

$$B_z(\rho, \tau) = B_0 \exp(-\kappa\tau)\left(\frac{r_0\mu_0 J_0}{B_0} + \frac{\beta^2 r_0\mu_0 J_0}{B_0}\rho_j K_1(\beta\rho_j) I_0(\beta\rho)\right). \tag{17}$$

Each of the six plots in Fig. 2 show a snapshot in time evolution, beginning with the instant an exponential current density decay was first imposed. The blue axes and lines show the magnitude of the z-magnetic field, while the red axes and lines display the total magnitude (i.e. in the ϕ-direction) of the electric field.

4 RESULTS

If the initial value of the background magnetic field, sustained by the current density of the azimuthally drifting plasma, is chosen to be the upper limit of $B_0 \sim 10^{-12}$ T (Barrow, Ferreira & Silk 1997; Jedamzik et al. 2000; Chen et al. 2004; Yamazaki, Ichiki, Kajino & Mathews 2010; Yamazaki et al. 2012), then the electron cyclotron period informs the characteristic time-scale $t_0 \sim 5$ s. The typical length-scale of the magnetic domain radius can be identified as the Larmor radius for the electrons, here we consider a characteristic length-scale of 5×10^8 m. These length and time-scales combine to give a characteristic speed scale of the order of 10^8 m s^{-1}.

Fig. 3 shows results for simulations which can be interpreted as a system with these scalelengths. The simulation was carried out for 100 electrons, evenly spaced throughout the domain from the centre of the solenoid to just short of the location of the current density. In the plot, the points show the mean final electron velocity (i.e. the gyro motion is averaged out over 10 or so cycles leaving only the drift component of motion) while the lines denote the deviation from the mean. The usual standard deviation formula was used to calculate this. Large lines indicate that the gyro motion is still significant, while small lines show that the drift motion completely dominates.

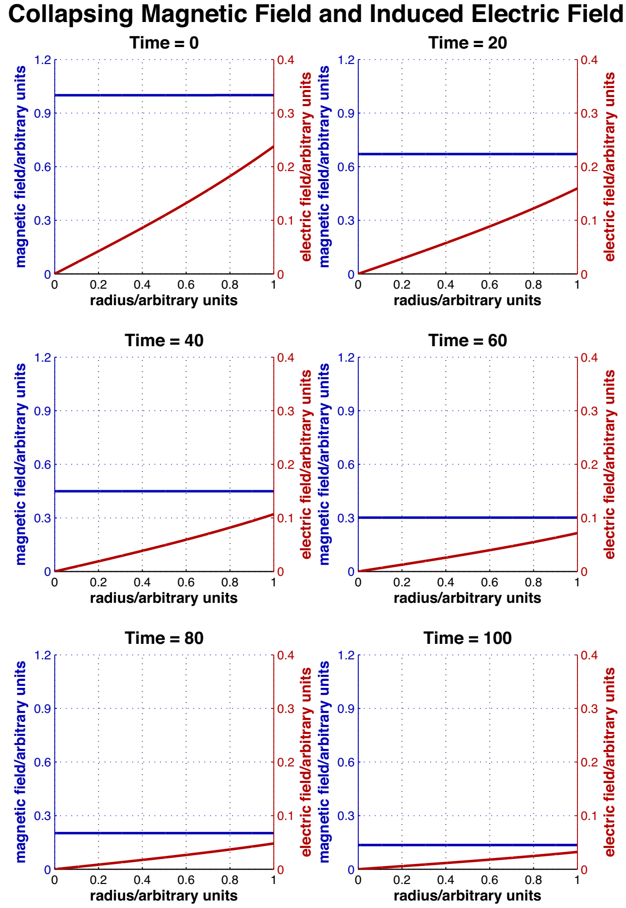

Figure 2. Snapshots of the magnitude of magnetic field (blue lines and axes) and electric field (red lines and axes) as a function of radius inside an infinite solenoid representing a single magnetic domain. Radius = 1 corresponds to the conduction current-carrying part of the solenoid. The conduction current density in the solenoid decreases exponentially with time, with the decrease beginning at the instant time = 0 as shown in the top-left plot. Subsequent plots show snapshots as time advances. The magnetic field remains approximately uniform as it decays along with the current that sustained it.

The final mean kinetic energy of electrons spaced throughout the domain ranges up to ∼9 keV for those nearest the domain edge. In the context that the 'thermal' energy is around 0.3 eV, a possible increase in kinetic energy of four orders of magnitude above the background means that even a tiny proportion of these energetic electrons could have an effect in the post-decoupling Universe.

Figs 4 and 5 show the full position and velocity space data for two individual electrons after they have been accelerated, at opposite extremes of the domain. Note that the magnitudes of velocity are different by nearly two orders of magnitude. The cyclotron frequency is almost identical for both because the large-scale magnetic field is approximately uniform for both over the timesteps shown. However, the electric field is very different because it falls in magnitude with time and distance as it sweeps into the domain from the edge (i.e. from radius = 1 in the figures), hence the large difference in drift speed.

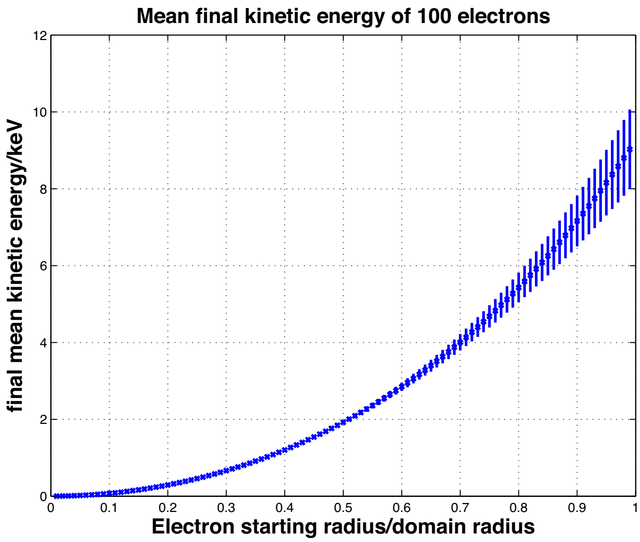

Figure 3. Each point denotes the final kinetic energy in keV of a typical electron after acceleration by the transient electric field caused by the current collapse. The motion has been averaged over 10 cyclotron periods. The electrons were initially stationary, and evenly spaced along the radius of the cylindrical magnetic domain. The x-axis denotes radial distance from the centre of the magnetic domain, with $x = 0$ the centre axis and $x = 1$ as the location of the current-carrying region. The transient electric field caused by magnetic collapse propagates into the domain from $x = 1$. The error bars accompanying each point essentially show the gyro motion of each electron, calculated by standard deviation from the mean kinetic energy. Large error bars indicate that the gyro motion is still significant, while smaller ones show that the drift motion completely dominates.

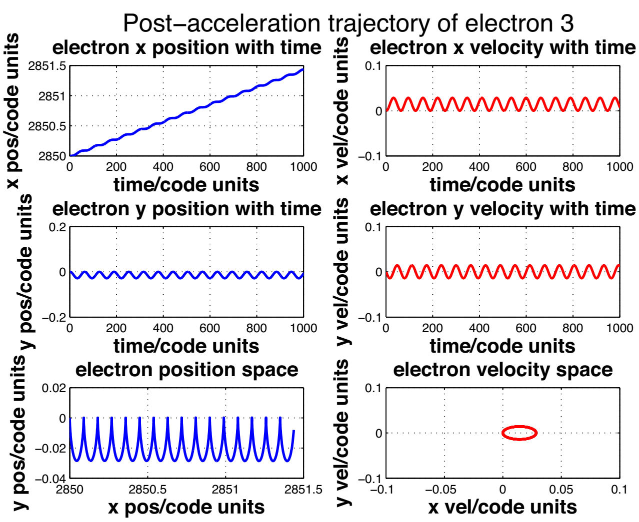

Figure 4. The post-acceleration behaviour of a typical single free electron at starting radius 0.03 in Fig. 3. This plot shows the full position and velocity space. The position and velocity coordinates are in code units, which are related to the characteristic scalelengths described in the text of Section 4. Note that there are 10^5 code units in one magnetic domain radius as depicted in Figs 1–3.

5 DISCUSSION

If one assumes that the Universe was magnetized pre-decoupling, then regardless of how the magnetic fields were first induced there are electromagnetic consequences *during* the epoch of decoupling. One of these consequences is the presence of an enduring population

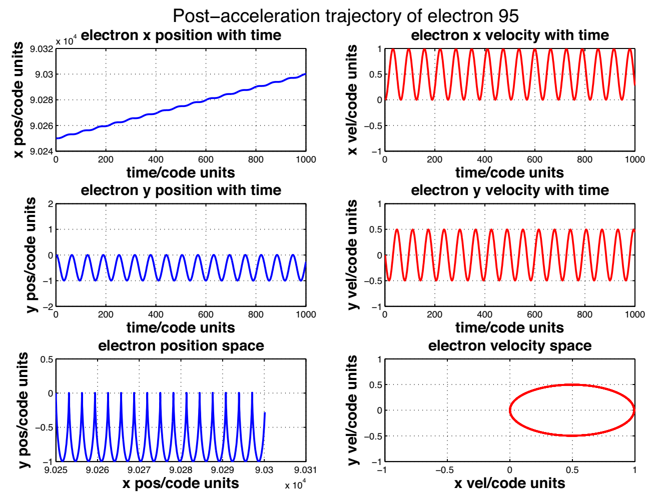

Figure 5. The post-acceleration behaviour of a typical single free electron at starting radius 0.95 in Fig. 3. This plot shows the full position and velocity space of the electron. The position and velocity coordinates are in code units, which are related to the characteristic scalelengths described in the text of Section 4. Note that there are 10^5 code units in one magnetic domain radius as depicted in Figs 1–3. Note also the difference in velocity magnitude between this plot and Fig. 4: this is because the transient electric field falls in magnitude with time and distance as it sweeps across the domain, leading to less acceleration further from the domain edge.

of energetic electrons (and associated energetic ions) in the post-recombination Universe. The implications of this include charge separation and therefore localized non-zero electrostatic potential which could contribute to structure formation.

Any large-scale magnetic structure in the early Universe must have been stochastically arranged in domains, in order to preserve the cosmological principle of anisotropy. The magnetic field within each domain must itself have been sustained by a conduction current around the outer edges.

In this paper, we have shown that within one such domain, the following sequence of events must occur during the epoch of recombination.

(i) Electrons and ions recombine into neutrals throughout the domain, including in the narrow current-carrying region.

(ii) Recombination causes the current density to collapse as charge carriers are removed.

(iii) The changing conduction current forces changes in the magnetic field.

(iv) By Maxwell's equations, the magnetic fluctuation produces a radiating electromagnetic impulse.

(v) Free charges in the domain encountering that electromagnetic pulse can be accelerated to energies of up to 9 keV.

(vi) The difference in mobility between accelerated electrons and ions may result in charge separation if the energy gained is not just from moving at the drift speed.

For physical scalelengths consistent with much of the literature, the orders of magnitude of kinetic energy gained by the electrons in these simulations are consistent with those predicted by the simple drift speed calculation in Diver & Teodoro (2008). The simulations show electrons could be accelerated up to ∼10 keV, compared with a thermal energy of ∼0.3 eV. Once the electrons are accelerated to such energies, their collisional cross-section is tiny and so generally the accelerated electrons would carry on uninterrupted into the otherwise cool, dark Universe. The agreement of the results from this simple computational model with the earlier prediction motivates the development of a more detailed model, for example one that models the current-carrying region to higher resolution. One might also consider the signal emitted from the accelerated electrons – which would be redshifted into the UV today.

The modelling approximations allow scope to try out alternative models for the progress of global neutralization, for example a propagating neutralization front travelling at the sound speed, or a spotty domain with localized hotspots of neutralization that spread outwards until the domain is fully neutral.

Since the magnetic energy density varies with the square of the field intensity, lower magnetic field magnitudes in the domains result in lower energies gained by accelerated electrons; in an extreme lower limit (such as the 10^{-22} T in Ichiki et al. 2006), the magnetic energy contribution may be lost in the thermal noise.

5.1 The impact on structure formation

Diver & Teodoro (2008) constructed their model with the simplifying assumption that the $E \times B$ drift was uniform. This cannot by itself lead to charge separation. We have shown in this article that the electric field is transient in both space and time, meaning electrons are non-uniformly accelerated to higher energies than simply acquiring the drift speed. This more sophisticated model permits the possibility of charge separation. We can estimate the magnitude of this as follows.

Assume a fraction of unbalanced charge Ψ develops as a result of this charge separation. Comparing the self-electrostatic potential of the domain with the self-gravitational potential, we see that

$$\frac{\Phi_E}{\Phi_G} = \frac{\Psi^2 n^2 e^2}{4\pi\epsilon_0 R}\frac{R}{G n^2 m_{\rm p}^2} = \frac{\Psi^2 e^2}{4\pi\epsilon_0\, G m_{\rm p}^2}. \tag{18}$$

Note that $\frac{\Phi_E}{\Phi_G}$ is independent of separation length-scale R. Substituting all of the physical constants, we find $\frac{\Phi_E}{\Phi_G} = 1.24 \times 10^{36}\,\Psi^2$. The ratio will be of order unity if

$$\Psi \geq 10^{-18}. \tag{19}$$

To put this in context, consider the fraction of electrons in a single magnetic domain that are most affected by the magnetic collapse: those electrons participating in the current density sustaining the magnetic field. Recall that in Section 1, the magnetic domains were of the order of 10 Mpc across. The current-carrying region at a boundary between two adjacent domains, by contrast, is a cylinder of the order of 10 pc in diameter. Since the current-carrying region forms the boundary between two domains it would have length of the order of 10 Mpc.

Therefore post-recombination, electrons that were originally located in the current-carrying region represent a fraction $f_{\rm e} = 10^{-12}$ of all electrons in the magnetic domain. Recall also that the ionization fraction after recombination was $f_{\rm i} = 10^{-5}$. The final factor of Ψ is the fraction of electrons $f_{\rm d}$ within the current-carrying region which gain a velocity over and above the drift speed. We now have

$$\Psi = f_{\rm e}\, f_{\rm i}\, f_{\rm d} = 10^{-17}\, f_{\rm d}. \tag{20}$$

The electrostatic potential from charge separation will be of similar magnitude to the self-gravitational potential of the magnetic domain if $f_{\rm d} \geq 0.1$, i.e. if just 10 per cent of electrons in the current-carrying region are accelerated to speed faster than the drift speed.

In a region with no dark matter, this electrostatic residual from the decay of pre-existing magnetic field could nudge the matter towards gravitational collapse. Observed today however, in the absence of magnetic fields, the gravitational collapse would be assumed to have been caused by a density perturbation but in fact would trace out primordial magnetic field lines.

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