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ORBITAL DYNAMICS OF LARGE SOLAR POWER SATELLITES

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Designs for geostationary SPS are extremely large in scale, more than an order of magnitude larger than the International Space Station. The problem of how to control the orbital motion of such large structures, accounting for various perturbing forces, is therefore a topic worthy of further study. The primary objective of the proposed research is to perform a detailed study of SPS orbit dynamics, obtaining a comprehensive understanding of the effect of perturbations on orbits of large SPS structures over a time-frame commensurate with proposed SPS lifetimes (30-40 years). Analytical equations derived by the process of averaging of the SPS equations of motion shall be used in determining the long-term orbital behaviour. Previous studies have simply assumed a geostationary orbit (GEO) then designed control systems for maintaining it thus. It is found that an alternative SPS orbital location known as the geosynchronous Laplace plane (GLP) is superior to GEO. An SPS in GLP requires virtually no fuel to maintain its orbit, avoids the main orbital debris population originating from GEO satellites and is extremely robust, i.e. loss of control is inconsequential. The GLP SPS saves of order 10^4 to 10^5 kg per year in fuel compared to a GEO SPS for equivalent power delivery compared to GEO.

I INTRODUCTION

I.I Aims, Objectives and Outcomes

The main aim of this research is to understand the long term orbital dynamics of large space structures, in particular futuristic solar power satellite (SPS) systems.

Firstly, a comprehensive understanding of the effect of perturbations on orbits of large SPS structures over a time-frame commensurate with proposed SPS lifetimes (30-40 years) is sought. The results of the orbital dynamics study shall be used to assess the performance of a SPS over mission lifetime.

The following three cases shall be studied and compared: an SPS in a controlled geostationary orbit; an SPS initially in geostationary but left uncontrolled; and finally, an uncontrolled SPS placed in a geosynchronous Laplace plane (GLP) orbit.

The GLP SPS provides comparable performance in terms of power delivered to the controlled GEO SPS while requiring nominal fuel to maintain its orbit. Additional benefits are the avoidance of the main orbital debris population at GEO altitude, improved robustness and avoidance of conflict with GEO communication satellites.

I.II Concept of the Solar Power Satellite

The solar power satellite (SPS) is conceptually simple: a large satellite designed to act as an electric power plant in orbit. It consists of three main segments: a solar energy collector to convert the solar energy into direct current (DC) electricity, a DC-to-microwave converter, and a large antenna which beams the microwave power to the ground. Designs for GEO SPS are extremely large in scale, more than an order of magnitude larger than the International Space Station. Understanding the long-term orbital motion of these structures and addressing the problem of how to control the orbital motion, accounting for various perturbing forces, is therefore a topic worthy of further study.

I.III Solar Power Satellite Designs

Various concepts for how to realize the solar power satellite (SPS) have been formulated since the idea was first proposed by Peter Glaser.[1] In this paper we consider three designs, shown in Figure 1, chosen for the relative simplicity of their orbital dynamics and as being representative of a range of area-to-mass (A/m) ratios. All SPS designs have a high-area-to-mass ratio (HAMR) as compared with con-

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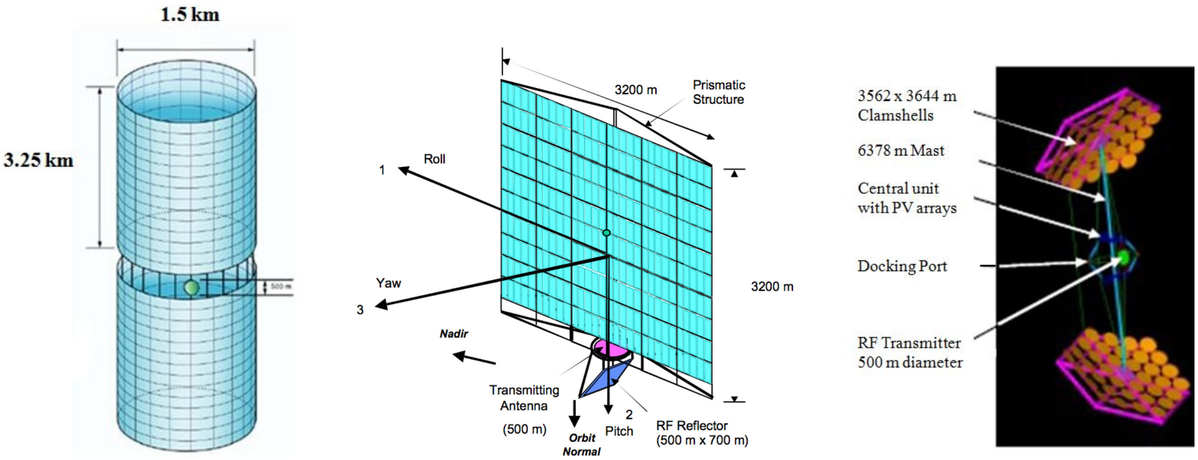


(a) Cylindrical SPS. (b) Abacus (c) Integrated Symmetrical Concentrator (ISC).

Figure 1. SPS designs.[4]

ventional satellites, this leads to an increased effect on the orbit due to solar radiation pressure (SRP). Consequently, this is the defining parameter which distinguishes their orbital dynamics. For a given satellite semi-major axis, a, reflectivity, ρ, and A/m value, an angle known as the SRP perturbation angle can be defined as:[2]

$$\tan \Lambda = \frac{3\beta}{2}\sqrt{\frac{a}{\mu\mu_s a_e(1-e_e^2)}} \tag{1}$$

where μ and μ_s are the gravitational parameters of the Earth and Sun, respectively, and a_e and e_e are the Earth's semi-major axis and eccentricity. SRP perturbation becomes strong as $\Lambda \to \pi/2$ and weak as $\Lambda \to 0$. Therefore the angle Λ characterizes the strength of the SRP perturbation. Values of Λ for the three SPS designs are calculated in Table 1.

Table 1. Area-to-mass ratios and the corresponding values for SRP perturbation angle for different SPS designs.

SPS	$A/m(m^2/kg)$	ρ	$\Lambda(^\circ)$
Cylindrical	0.15	0.3	0.12
Abacus	0.40	0.3	0.33
ISC	0.87	1.0	1.09

I.IV Retro-Directive Phased Array Antennas

One of the primary reasons GEO was first suggested for SPS was due to the simple geometry between transmitting antenna and receiving antenna (rectenna) on the ground. Minimal repointing of the power beam is required, therefore removing the need to mechanically reorient the transmitter and rectenna throughout an orbital period. However, there exists a method of wireless power transmission using a so-called retro-directive phased array that allows for the beam to be electronically steered with no major mechanical repointing necessary, i.e. off-axis power beaming is possible. Therefore orbits other than the conventional geostationary become more feasible. This paper shall focus on the orbit dynamics of SPS, and as such it is sufficient to select a reference system retro-directive phased array antenna, as proposed by Frank Little et al.[3] the details of which are given in Table 2. The parameters given in Table 2 are illustrated in Figure 2.

Table 2. Retro-directive phased array antenna reference system[3]

Property	Symbol	Value
Antenna Diameter	D_T	0.5 km
Rectenna Diameter	D_R	8.85 km
Power Transmitted	P_t	1.78 GW
Frequency	f	5.8 GHz
Wavelength	λ	5.17 cm
Separation	x	$35,786km + \Delta x$
Beam Steering	$\Delta\beta$	$\pm 3^\circ$

I.V Orbital Location

The vast majority of previous SPS studies have mainly concentrated on SPS located in GEO. However, this may not be the best option in terms of orbital dynamics of the system. An alternative system with an SPS located in a geosynchronous Laplace plane orbit is considered.

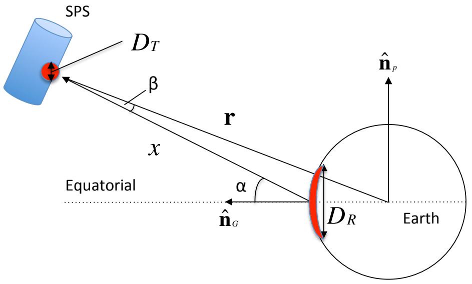


Figure 2. SPS-ground rectenna geometry. $\hat{n}_p$ is the unit vector in the direction of Earth's rotation pole, α is the incident angle of the beamed radiation. The remaining symbols are defined in Table 2.

Geostationary

A satellite in geostationary earth orbit (GEO) is stationary with respect to a point on the Earth's surface. It's altitude is such that it's orbital rate is equal to the rotational rate of the Earth. This occurs for an altitude of 35,786 km. It has approximately zero inclination and eccentricity. As well as providing near 24 hour access (with only short outages around the equinoxes), GEO minimizes scanning losses as minimal slewing of the power beam is necessary. The main disadvantages are the high cost of launch to GEO and the divergence of the power beam over the large distance from GEO to the ground-based receiver drives the overall system size up.

The Laplace Plane

While studying Jupiter's satellites in 1805, Laplace[5] recognized that the combined effect of a planet's oblateness and the solar tide induced a so-called 'proper' inclination in satellite orbits with respect to the planetary equator. He found that the proper inclination depended upon the distance of the satellite from the planet, increasing with increasing distance. This proper inclination defines a plane between the orbital plane of the planet around the Sun and the planet's equatorial plane, as illustrated in Figure 3. This is what is now known as the Laplace plane. Recent contributions to the understanding of the Laplace plane and the effect of solar radiation pressure on the Laplace plane have been made by Rosengren and Scheeres.[6] The Laplace plan is essentially a region of space where the secular evolution of the combined effects of the lunisolar gravitational and Earth planetary oblateness perturbations cancel each other out. Consequently, the orbits lying within this plane are effectively frozen. The approximate inclination of the Laplace plane with respect to the equatorial plane can be calculated according to the theory of Allan and Cook,[7] with the results for different semi-major axis shown in Figure 4. For SPS, we are interested in maintaining the geosynchronous nature of the orbit to allow for 24 hour power beaming. From Figure 4, the Laplace plane inclination, Φ, at the altitude required for geosynchronous is approximately 7.5°.

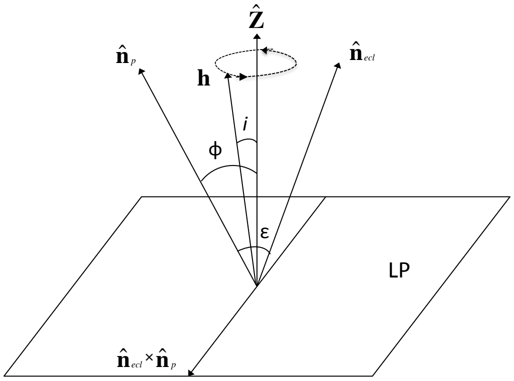


Figure 3. The normal to the local Laplace plane, $\hat{Z}$, lies between, and is coplanar with, the planets spin pole, $\hat{n}_p$, and the normal to the ecliptic, $\hat{n}_{ecl}$. The angular momentum vector, h, or the normal to an arbitrary objects orbit plane, will precess around $\hat{Z}$, at approximately constant inclination, i, sweeping out a cone. The Earth's obliquity, ϵ, is simply the angle between the vectors $\hat{n}_p$ and $\hat{n}_{ecl}$. The Laplace plane angle, Φ, represents the angle between $\hat{n}_p$ and the $\hat{Z}$ axis. As the semi-major axis, a, changes, the relative strengths of the Suns and planets perturbations vary and hence, the Laplace plane will shift. Based on figure from Tamayo et al.[8]

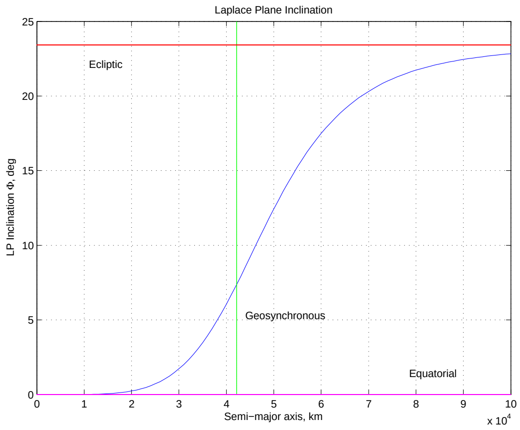


Figure 4. Laplace plane inclination with respect to the equatorial plane for various semi-major axis.[7]

Previous Investigation of Laplace Plane SPS

The possibility of locating SPS in the so-called Laplace plane has been investigated before by Graf,[9] however, to the author's knowledge, this is not widely known. Graf studied the long-term evolution of the eccentricity and inclination of a geosynchronous Laplace plane orbits using analytical methods. The ground-tracks of these orbits for varying argument of perigee were found. Graf also considered the possibility of an orbit with non-zero initial eccentricity, for which it appears the amplitude of the yearly oscillations in eccentricity are decreased for the first decade or so. However, no analysis was made of the consequences for the operation of an SPS in such an orbit, compared to the conventional GEO. Using a semi-analytical orbit propagation technique, we obtain more accurate and longer-term predictions of SPS orbits than Graf and assess various parameters related to the performance of the SPS system.

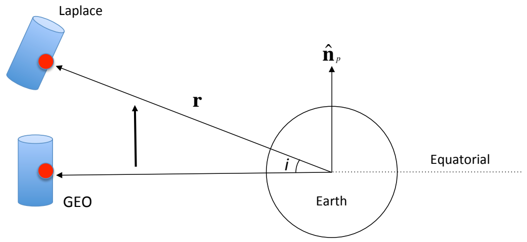


Figure 5. Geosynchronous laplace plane orbit relative to geostationary. Both have a semi-major axis of 42,164 km, but GLP has an inclination of 7.5° to the equatorial plane, while GEO has $i = 0$ and $e = 0$.

II ORBITAL MODELING

In order to justify the large initial investment, a large scale SPS should have a operating lifetime of at least 30 years. Perishable items such as the solar arrays may be replenished periodically but the main structure could be in orbit for even longer. Therefore when considering the orbital dynamics it is desirable to understand the evolution of the orbit over timescales of this order. Hence, an averaged formulation developed by Rosengren and Scheeres[10] for the propagation of high-area-to-mass ratio (HAMR) objects in Earth orbit which accounts for solar radiation pressure, Earth oblateness and luni-solar gravitational perturbations is utilized. This is a first-order averaged model, for further detail consult "Long-term dynamics of high area-to-mass ratio objects in high-Earth orbit",[10] in which the advantages of this method are summarized:

> The averaged equations can be numerically integrated, with significantly reduced computational requirements, and often reveal the essential characteristics of the exact solution in a more satisfactory way than a numerical solution of the non-averaged equations of motion.

This approach allows one to easily capture both the qualitative and quantitative effects of perturbations on the orbits of over long-time spans commensurate with proposed SPS lifetimes. In order to enhance confidence in the results obtained from the averaged equations of motion, numerical integration of the full equations of motion over short time spans was also performed using Runge-Kutta integration.

III EVALUATION OF SPS PERFORMANCE

The results of the long-term orbit propagation may be used to evaluate how an SPS in such an orbit would perform. In order to achieve this, we define SPS performance metrics. Firstly the distance between the transmitting antenna and rectenna, x is evaluated. Variation in this distance causes a fluctuation in the beam coupling efficiency, η_t:[11]

$$\eta_t \backsim 1 - \exp(-\tau^2) \tag{2}$$

where

$$\tau = \frac{\pi D_T D_R}{4\lambda x} \tag{3}$$

and D_T and D_R are the diameters of the transmitting antenna and receiving antenna (rectenna) respectively (illustrated in Figure 2), and λ is the wavelength of beamed radiation.

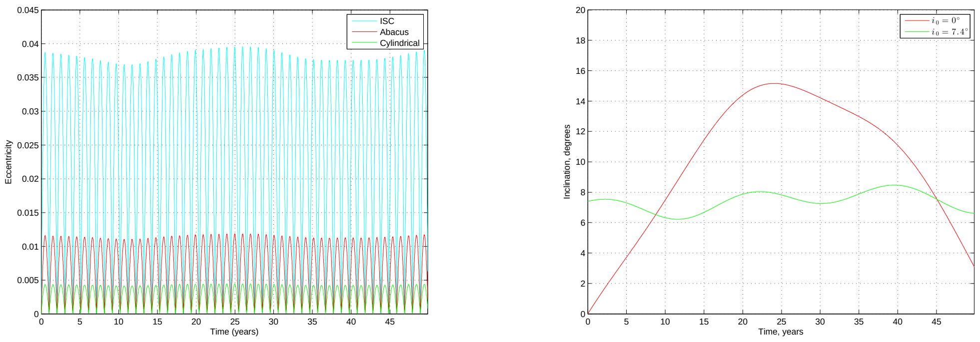
The power received by the ground station can then be calculated according to:

$$P_r = P_t \eta_t \cos^2 \alpha \tag{4}$$

where P_t is the power transmitted, as given in Table 2, η_t is given by Equation 2, and α is the incident angle of the beamed radiation, which can be evaluated knowing the ground station position vector and SPS position vector, $\boldsymbol{r}$. The off-axis beaming angle, β required to aim the beam at the rectenna is also evaluated. The limit for the reference antenna chosen was $\beta \leq \pm 3°$.[3]

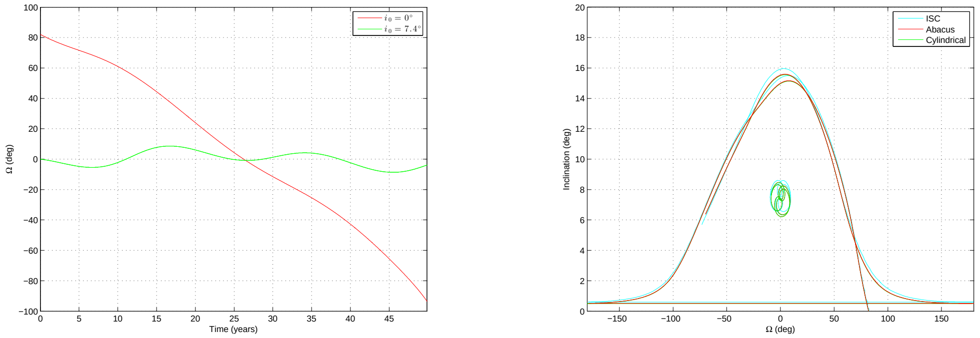
IV RESULTS

The results of the long-term SPS orbit propagation for a geosynchronous laplace plane **(GLP)** orbit, $i_0 = 7.4°$, and an uncontrolled initially geostationary orbit **(U-GEO)**, $i_0 = 0°$, are presented in Figure 6. **U-GEO** is considered in order to understand the long-term natural evolution of an SPS orbit starting in GEO. The implications of the perturbation effects on the orbits for the delivery of power to a single equatorial ground receiving antenna (rectenna) are also assessed. The daily, yearly, and long-term evolution of SPS performance parameters for **GLP** are given by Figures 8, 9, 10 respectively. The long-term evolution of SPS performance parameters for **U-GEO** are given in Figure 11.



(a) Eccentricity vs Time.

(b) Inclination vs Time



(c) RAAN vs Time

(d) Inclination vs RAAN

Figure 6. Long-term orbital element variation. (a) Eccentricity variation for 3 SPS designs with $i_0 = 7.4°$. Similar results for $i_0 = 0°$ omitted to maintain clarity. (b) $i_0 = 0$ Long-period variation, with maximum value of $15°$. Stable inclination evolution for $i_0 = 7.4$, oscillation caused by Moon's nodal motion, Saros period 18.61 years.

IV.I Long-Term Orbit Propagation

Eccentricity, e

GLP: The three SPS designs, Figure 1, share similar orbital behavior, the main difference is in their eccentricity evolution due to their different Λ values, defined in Equation 1 and shown in Table 1. The effect of solar radiation pressure depends upon Λ. The larger the value of Λ, the greater the amplitude of yearly eccentricity variation, Figure 6(a). The other perturbation effects cause minor variation in the amplitude of the yearly eccentricity variation.

U-GEO: No significant difference in this behavior is observed between the U-GEO or GLP cases (i.e. for $i_0 = 0°$ or $i_0 = 7.4°$).

Inclination, i

GLP: The long-term inclination evolution is shown for both $i_0 = 0°$ and $i_0 = 7.4°$ in Figure 6(b). When the initial inclination is chosen to be close to the Laplace Plane inclination for the chosen altitude, it is stable and oscillates with a period of 18.6 years due to the moon's nodal motion, i.e. the lunar orbital plane is not fixed in the ecliptic plane, but is itself regressing around the pole of the ecliptic with the so-called Saros period of 18.6 years. The amplitude of the oscillation is small for $i_0 = 7.4°$.The results obtained agree well with those of Graf.[9] In order to obtain a solution without the oscillations due to the Moon, one would have to average over the moon's nodal motion. This is planned in future work and should allow an extremely stable solution for the GLP to be found.

U-GEO: The inclination of the initially GEO ($i_0 = 0°$) satellite immediately begins to increase and shows long-term periodic motion with period 52.86 years, reaching a maximum after approximately 26 years then decreasing again.

This oscillation is due to the luni-solar gravitational perturbations and agrees well with the results of Allan and Cook,[7] who found the period for inclination variation for a geosynchronous orbiter to be 52.9 years. When started in GEO, the plane of the satellite's orbit is at the obliquity angle (23.4°) to the ecliptic plane. The moon and the sun orbits are not in the same plane, hence their gravity pulls the satellite out of its initial orbital plane. The stable Laplace plane inclination at geosynchronous altitude is approximately 7.5°, therefore, when not started at or near to this inclination, the inclination will oscillate around 7.5° which explains the maximum value of 15°. The rate of inclination growth for the first 20 years is approximately $\Delta i = 0.7°$/year.

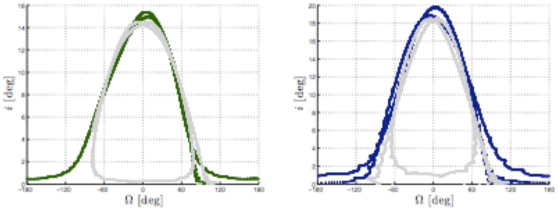
Right Ascension of the Ascending Node (RAAN), Ω

GLP: For the GLP orbit, the node oscillates around $\Omega = 0°$ with small amplitude. Again, the period is equal to the Saros period (18.6 years), as this is caused by the nodal motion of the moon. The stable Laplace Plane solution is at $\Omega = 0°$ for all altitudes as this is where the ecliptic, Laplace and equatorial planes intersect. The stability of the solution in RAAN-inclination space can be seen in Figure 6(d). This shows results for all three SPS, and illustrates that solar radiation pressure has an effect on the Laplace inclination, i.e. the greater the effect of SRP (due to higher Λ) the higher the inclination of the Laplace plane.

U-GEO: The node of the spacecraft regresses due to the combined effects of J_2 and luni-solar perturbations for $i_0 = 0°$, see Figure 6(c).

Orbital Debris Avoidance

Another major advantage of the GLP orbit which has been observed is that it avoids the main orbital debris population originating in GEO.[10] This is clear when the inclination-ascending node space graph of the GLP orbit in Figure 6(d) is compared with the work of Rosengren and Scheeres[10] shown in Figure 7. While the $i_0 = 0°$ case clearly lies within the path of the debris, the $i_0 = 7.4°$ is comfortably removed from the path of this orbital debris population.



(a) $\Lambda = 0.85°$ (b) $\Lambda = 4.26°$

Figure 7. Predicted HAMR debris originating in GEO.[10]

IV.II SPS Performance

Incident Angle of Beamed Radiation, α

GLP: The motion of the satellite relative to the ground station causes a variation in the absolute value of the incident angle of the beamed power, α, as defined in Figure 2. Initially, α varies between 0° and 8° twice daily as shown in Figure 8. The long-term variation in α for the GLP orbit is shown in Figure 10. The value of α reaches a maximum of $\alpha = 15°$ during the lifetime of the SPS due a combination of the long-term small oscillations in inclination and RAAN.

U-GEO: The incident angle increases in a linear fashion, see Figure 11, mainly due to the regression of the RAAN. It takes only 7 years for α to reach the maximum value of the GLP solution. After 23 years α reaches 90° and power transmission is no longer possible. It will become unrealistic and uneconomical to beam power earlier than this however, and other studies have suggested $\alpha = 60°$ as the realistic upper limit.[12] This limit is reached after approximately 18 years.

Off-Axis Beaming Angle, β

GLP: In order to continuously beam power, the off-axis beaming angle, β, must not exceed 3°. The GLP orbit satisfies this constraint over a long-period of time, see Figure 10.

U-GEO: The initially GEO SPS exceeds this limit after approximately 9 years, see Figure 11, after which it can no longer beam power to the ground station for this particular retro-directive phased array antenna.

Inter-Antenna Distance, x

GLP: The small amplitude oscillation in x due to the inclination of the orbit, i.e. at the ascending and descending node the value is 35,786 km and this increases slightly when the satellite is either at the 'bottom' or 'top' of it's orbit with respect to the equatorial plane, Figure 8. The variation in eccentricity caused by SRP causes x to vary, with peak amplitude, Figure 9, coinciding with peak eccentricity. The maximum amplitude is dependent upon the maximum value of eccentricity reached, and therefore the Λ value of the SPS.

U-GEO: The effect of the yearly eccentricity variation on x can be clearly seen in Figure 11. The growth in x is mainly caused by the regression of the RAAN, with the inclination growth and eccentricity change also having a small effect.

Power Received, P_r

GLP: The power received drops by approximately 20MW twice per day, as shown in Figure 8, mainly due to the variation in the incident angle, α. The long-term variation in α, Figure 10, causes a maximum power loss compared to a *controlled* GEO SPS of 3% which corresponds to $\alpha = 15°$ and a resultant minimum power, P_r(min)=1.67GW. However, the power required for the electric ion thrusters in the controlled GEO case has not been accounted for in this comparison. This, along with the fact that the SPS solar arrays in GLP will be closer to perpendicular with respect to the Sun's rays throughout the Earth's orbit about the Sun (due to the GLP orbit being closer to the ecliptic plane and assuming no solar beta angle tracking) means that this small power loss will actually be approximately cancelled out. The variation in the beam coupling efficiency also has a small effect on the overall power received.

U-GEO: The power received drops below the GLP P_r(min) after approximately 7 years. P_r continues to decrease until it reaches zero after approximately 24 years.

Satellite Ground Track

GLP: The satellite ground track is a figure of eight centered on the equatorial ground station, see Figure 8. The centre of the figure of eight slowly moves along the longitude axis according to the change in the RAAN. For the 1 year results, the ground track moves west, Figure 9. This is due to the RAAN initially regressing, Figure 6(c). The figure of eight ground track moves East and West along the longitude axis over a longer period of time between ±10°, see Figure 10. Growth in eccentricity causes a distortion in the figure of eight.

U-GEO: As the inclination of the satellite grows, the oscillations in latitude grow. The longitude of the satellite regresses due to the regression of the RAAN. This is the main reason the satellite becomes out of range of the ground station.

Beam Coupling Efficiency, η_t

GLP: The beam coupling efficiency, η_t, is inversely related to the inter-antenna distance, x, through Equation (2). The variation in this parameter for GLP is reasonably small (±0.1%) for cylindrical SPS and indicates that eccentricity control is not required to maintain good efficiency of transmission. The variation of η_t is relatively small for the values of Λ considered.

U-GEO: The beam coupling efficiency shows considerable decrease from 97 to 92% in 25 years. However, this is insignificant compared to the effect of the increase in α on the power received over this timescale.

IV.III GEO Controlled - Fuel Requirements

In order to compare the uncontrolled GLP orbit with GEO, the fuel required to maintain a GEO SPS is calculated. The orbit correction due to SRP and lunisolar gravitation are considered separately. The SRP force acts largely in the orbital plane, therefore we can hold the position vector $\boldsymbol{r}$ fixed and integrate the acceleration due to SRP to obtain the Δv_{ecc}. The Δv_{NS} needed to correct the inclination growth is calculate according to:

$$\Delta v_{NS} = 2v \sin \Delta i/2 \tag{5}$$

where v is the magnitude of the orbital velocity, and Δi is the change in inclination desired, which is obtained from the graph of inclination vs time in Figure 6(b) for $i_0 = 0°$. The Rocket equation is used to calculate the annual fuel requirements for the three different SPS designs, assuming an $I_{sp} = 3000s$, are given in Table 3. In reality a GEO is controlled by periodically correcting the orbit within a certain tolerance. However, this method gives an approximate mass of fuel required for orbit maintenance. As discussed earlier, the variation in eccentricity causes a minimal change in the beam coupling efficiency and hence, the power received. Therefore it is not necessary to control the eccentricity either for the GLP orbit or the GEO case. Hence, the GLP offers savings only in the correction of the orbit inclination. The mass of fuel saved depends upon the overall mass of the satellite. This indicates that the larger the mass of the satellite, the more beneficial the GLP orbit is in terms of fuel saved.

Table 3. Fuel estimates for GEO controlled SPS.

SPS	$\Lambda(°)$	Ecc. Fuel/yr (kg)	Mass SPS (kg)	Inc. Fuel/yr (kg)	Total Fuel/yr (kg)
Cylindrical	0.12	54,000	6.6×10^7	87,600	141,600
Abacus	0.33	60,000	2.5×10^7	33,200	93,200
ISC	1.09	139,200	1.7×10^7	22,600	161,800

V DISCUSSION

The main advantages of the Laplace plane orbit are: reduction in fuel requirements due to not having to correct for inclination drift; robustness, if the control system were to go offline for any reason it does not matter as it will stay in that orbit; the major advantages of the geostationary SPS (24 hour access, low transmitter/receiver relative velocity) are maintained; the main population of orbital debris originating in geostationary earth orbit is avoided; and finally the congestion of geostationary earth orbit is avoided along with possible contention with the communication satellite industry.

Although the fuel saved in GLP over the SPS lifetime is relatively small when compared to the mass of the satellite, approximately 5% of the overall mass for 40 year lifetime, the problem may be supplying the quantity of xenon gas required to maintain a network of large SPSs.

If instead of using the GLP orbit the SPS is initially placed in a geostationary orbit and left uncontrolled, within approximately 9 years it will be unable to beam power to the ground station due to exceeding the off-axis beaming limit of the chosen reference system. For a system capable of greater beam pointing, then this option is still inferior due to the increase in the incident angle of the beamed radiation caused by the combined effect of regression of the orbital node and increase in the inclination. The power received will be less than the minimum received by the GLP option after only 7 years. It will eventually reach zero after approximately 24 years. This indicates if the SPS is initially placed in GEO then it must be use some active control thrusting in order to remain a useful resource for the planned 30-40 year lifetime. As an option that requires minimal control thrusting, the GLP looks attractive, as it only suffers small losses in power delivered and maintains a stable power supply for the full 30-40 year lifetime.

The "exact" Laplace plane has not been found in this analysis, as oscillation in inclination and ascending node is observed. In order to find the Laplace plane more precisely and to obtain minimal variation in the inclination and ascending node, it is necessary to average over the nodal period of the moon, as the precession of the moon's node is responsible for the periodic motion observed in inclination and ascending node.

The Earth tesseral harmonic term, J_{22}, has not been included in this analysis. It is important to consider due to the resonance it causes at geosynchronous altitude. This will cause an East-West drift in the longitude of the satellite unless the SPS is located in one of the stable 'sinks' at either 75°E or 105°W. Otherwise continuous thrusting would be required to offset the East-West drift. For this study it has been assumed that the SPS are located in a stable sink. The averaging of this effect is more difficult, but will be pursued. This may a complex effect for an orbit with non-zero inclination and eccentricity, as considered here. A previous study by Ely[13] indicates that chaotic motion is possible for synchronous, inclined and eccentric orbits. Adoption of the non-classical stationkeeping algorithm proposed by Ely[14] may be necessary to maintain the ground-track of the SPS within acceptable limits. The period of the orbit is very important because if it changes from geosynchronous period, even slightly, it will become out of synch with the ground station and will consequently be out of beaming range for long periods. The effect of shadowing should be considered, as shadowing will lead to a small secular change in semi-major axis, which would throw the period off. Small thrusting may be needed to maintain the semi-major axis.

Due to the use of an extraordinarily large transmitting antenna for beaming of the microwave energy from orbit to the Earth's surface, there is also a reactive force acting opposite the satellite to ground station beaming direction. This is significant enough to warrant consideration and to be included as a perturbing effect in future studies (6N for 500 m 1.78 GW antenna). For a geostationary SPS, this could be incorporated simply by tweaking the earth's mass as the reaction force will be radially outwards. However, the situation is more complicated for the Laplace plane SPS as the reaction force angle relative to the radial direction will be continually changing.

A higher fidelity model of solar radiation pressure should be incorporated to take into account the non-spherical nature of SPS designs. Although the cannonball model used for solar radiation pressure in this study is able to capture the general nature of the SRP perturbation, it does not provide a precise prediction of the orbital evolution of an individual object. The long-term effects of solar eclipses on the orbit shall also be incorporated into the model in future work.

The somewhat overlooked option of locating an SPS in the geosynchronous Laplace plane orbital should be investigated further. A trade-off study between the power lost through use of an inclined orbit with continual beam redirecting versus the fuel saved from low station-keeping requirements should be performed. Although minimal orbit control may be necessary for this type of orbit, active control of the SPS attitude may be required in order to maintain sun-pointing of the large solar arrays/concentrators. In order to first gain an understanding of the orbital dynamics, the attitude of the SPS has not been considered in detail here, however, it shall be considered in future studies.

VI CONCLUSIONS

The geosynchronous Laplace plane orbit has been confirmed as a viable alternative to the conventional geostationary nominally proposed for the large solar power satellite. The geosynchronous Laplace plane offers significant advantages for an SPS. By locating an SPS in this type of orbit the orbital control fuel requirements are reduced by the order of 10^4 to 10^5kg/year. The congestion of satellites in geostationary is avoided and greater robustness is obtained, as if active control were to be lost, this would not be a problem. The main population of orbital debris originating in geostationary orbit shall be avoided. It has been shown that an SPS located in a geosynchronous Laplace plane orbit will stay within range of the receiving antenna on the Earth's surface with minimal active control.

An understanding of the orbital evolution of an uncontrolled initially geostationary solar power satellite has also been gained. Placing an solar power satellite in an initially geostationary orbit and leaving it uncontrolled is not a feasible option for providing a long-term source of energy to a single ground station due to the natural evolution of the orbit.

The geosynchronous Laplace plane orbit should now be considered the ideal location for a large solar power satellite rather than geostationary.

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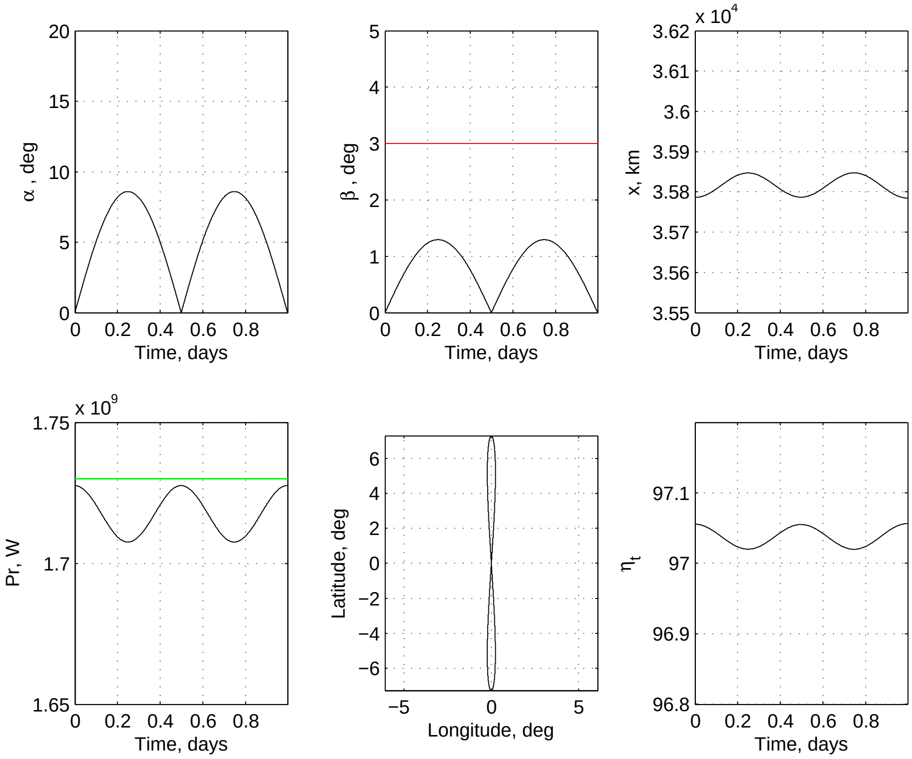

Figure 8. Variation of SPS performance related parameters over 1 day. The red line in middle-top figure represents the limit of off-axis beaming for the chosen retro-directive beaming system.[3] The green line in the bottom-left figure represents the steady power delivery offered by a controlled GEO SPS.

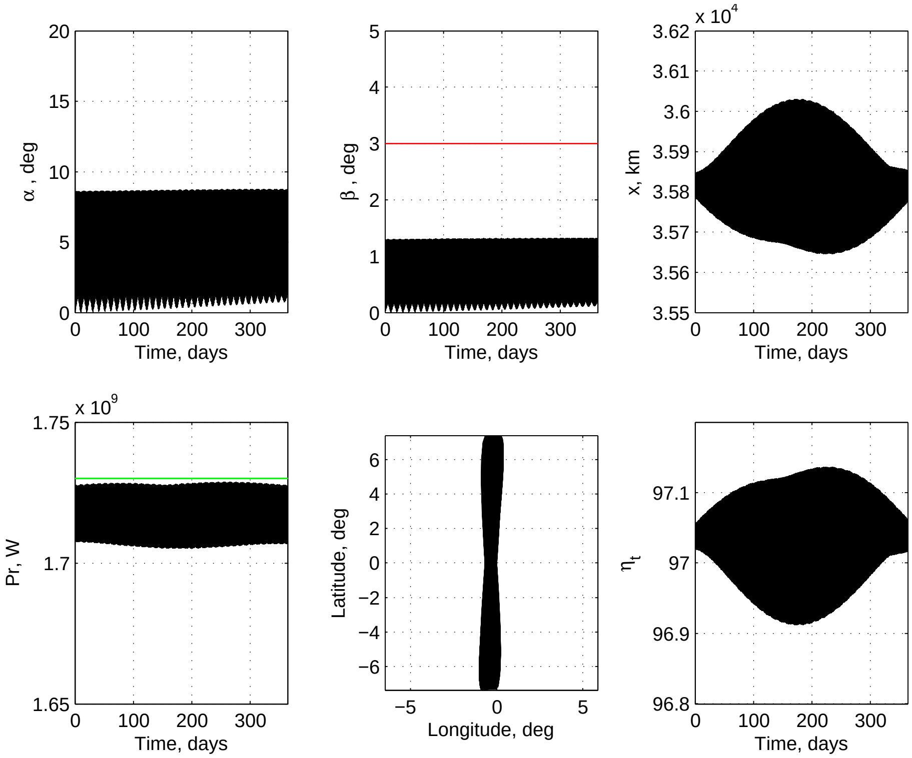

Figure 9. Variation of cylindrical SPS performance related parameters over 1 year for GLP.

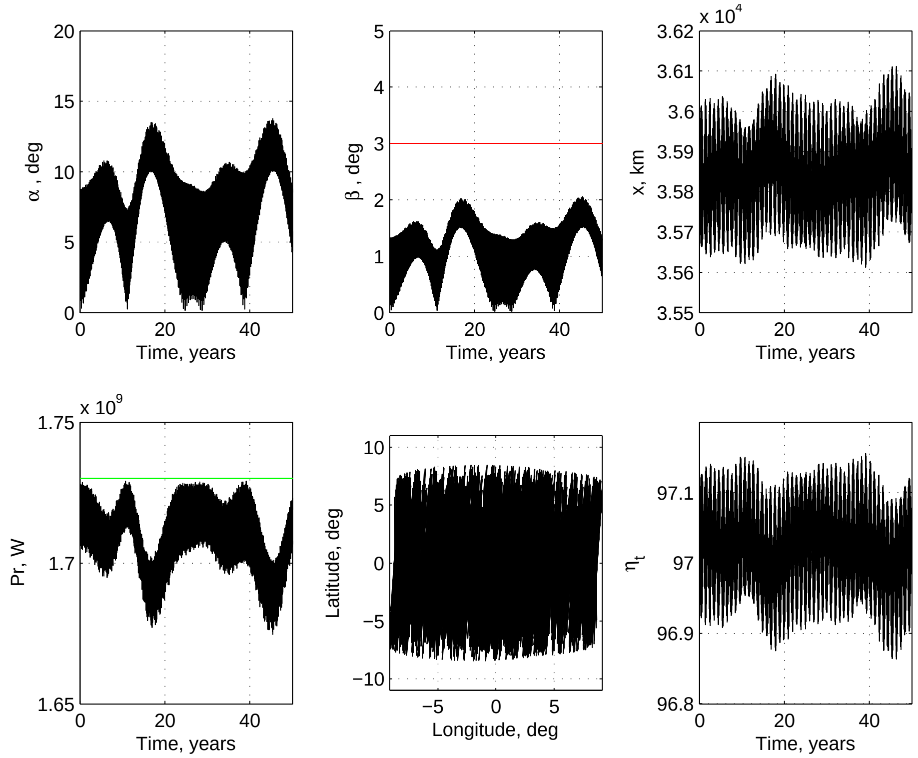

Figure 10. Long-term variation in cylindrical SPS performance parameters for GLP.

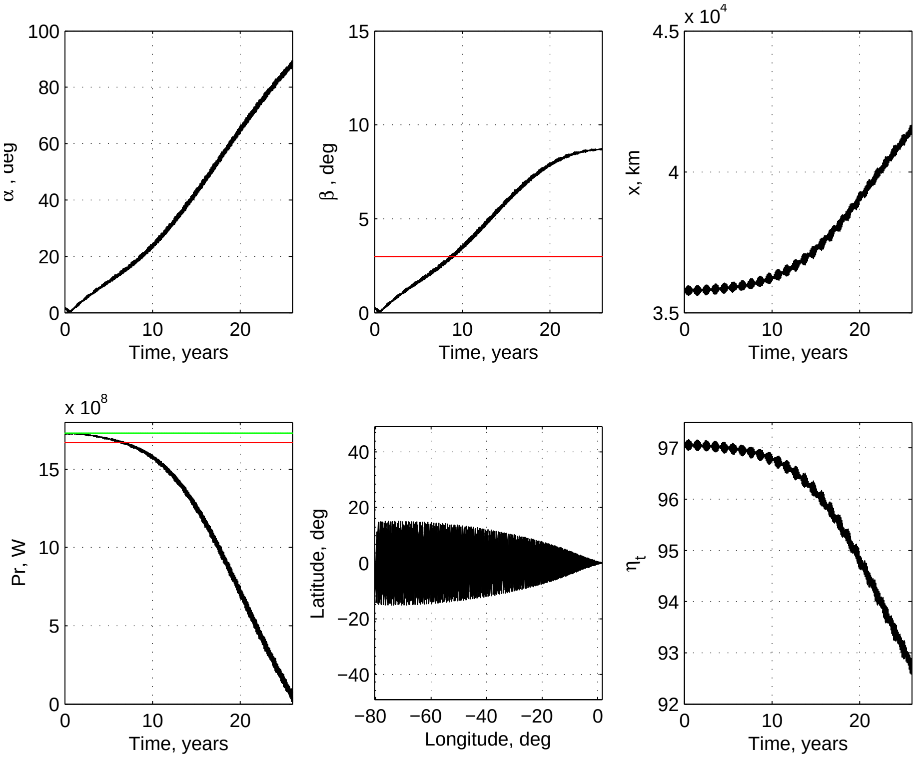

Figure 11. Long-term variation in SPS performance parameters for cylindrical SPS initially in GEO. The red line in the middle-top figure represents the β angle limit of 3°, and is exceeded after ~ 9 years. The green line in the bottom left is the steady power from controlled GEO and the red line is the minimum power from GLP.