Cockshott, W.P., and Cottrell, A. (2008) Probabilistic political economy and endogenous money. In: Probabilistic Political Economy Conference, 14-17 Jul 2008, London, UK.

Copyright © 2008 The Authors

A copy can be downloaded for personal non-commercial research or study, without prior permission or charge

Content must not be changed in any way or reproduced in any format or medium without the formal permission of the copyright holder(s)

When referring to this work, full bibliographic details must be given

http://eprints.gla.ac.uk/82481

Deposited on: 09 July 2013

Enlighten – Research publications by members of the University of Glasgow
http://eprints.gla.ac.uk

Probabilistic political economy and endogenous money

P. Cockshott, A. Cottrell*

July 17, 2008

Since the foundational work of Farjoun and Machover [2], important contributions to the field of probabilistic economy have been made in [1] and in [6]. Both of the latter works assume an economy with exogenous money. In this context one naturally has conservation of money as a postulate. However it is questionable whether a capitalist economy could ever work with entirely exogenous money, and it is interesting to see to what extent probabilistic arguments can illuminate the evolution of the type of endogenous money system that characterizes contemporary capitalism.

We first argue, on probabilistic grounds, that a system with a strict conservation law on money was historically unsustainable. We then make the case that phenomena such as the formation of a rate of interest, periodic commercial crises, and the formation of a rentier class can be understood using the sort of reasoning pioneered by Farjoun and Machover.

1 The impossibility of monetary conservation

The most obvious characteristic of capital—one might call it the signature of capital—is its ability to undergo exponential growth. Marx characterized this signature using the notation $M-C-M'$, where $\Delta M = M' - M > 0$, and attempted to explain how such expansion is possible.

It could not, he argued, come about in the field of commodity exchange, since this was governed by a law of conservation of values. Echoing Newton's concept that to each action there is an equal and opposite reaction, Marx argued that to the extent that one set of people sold goods above value another set must sell below value. Thus, he argued, profit could only be explained outside of the realm of commodity exchange, by the exploitation of labour in the production process. In the capitalist factory—where Freedom, Equality and Bentham do not prevail—the working day is extended beyond the time required to produce the workers' means of subsistence, in order to provide a surplus that funds profits.

Ian Wright [5, 6] has shown that this model can be validated in computer simulations, with some very simple assumptions.

But Marx's analysis only partly answers the problem of "where the money comes from". He explained how capitalists obtained a net income from their capital, but this was only half the problem. If the capitalists follow the maxim ascribed to them by Marx—"Accumulate, accumulate! That is Moses and the prophets!"—then the signature of capital $M-C-M'$ extends into

$$M-C-M'-C'-M''-C''-M'''\dots$$

which requires exponential growth in the quantity of money. In the 19th century, the British economy, like most others, depended on precious metal for its monetary base. An exponential growth in the quantity of money implies the same sort of growth for gold stock. But if we look at historical data for the growth of

*Department of Computing Science, University of Glasgow, and Department of Economics, Wake Forest University, respectively. This is a draft paper for presentation at the conference on Probabilistic Political Economy, Kingston University, July 14–17, 2008.

the world gold stock, we find that during the 19th century it was growing at well under 1% per annum. Given that the British economy grew at over 2% a year, there was a discrepancy between the growth of gold and the growth of commodity circulation.

Since gold stocks could not grow fast enough to support the expansion of the economy, capitalists had to resort to commercial bills. An Iron Master taking delivery of coal would typically write a bill of exchange, a private certificate of debt, promising to pay within 30 or 90 days.

Payment of wages would generally have to be done in cash. Capitalists have tried at times to pay wages in tokens redeemable only at company stores ("scrip") but legislation by the state, eager to maintain its monopoly of coinage if not to defend the interests if the workers, tended to put a stop to this. Payment in cash represents a transfer from the safes of capitalists to the pockets of their employees, with a corresponding cancellation of wage debts. At the end of the week, the wage debt has been cleared to zero, and there has been an equal and compensating movement of cash.

Workers then spend their wages on consumer goods. For the sake of simplicity we assume that there is no net saving by workers so that in the course of the week all of the money they have been paid is spent. This implies that immediately after payday, the money holdings of the workers are equal to one week's wages. If these wages were paid in coin this would have set a lower limit to the quantity of coin required for the economy of function.

When workers spend their wages on consumer goods they transfer money only to those firms who sell consumer goods—shopkeepers, inn-keepers and so on. We can expect these firms not only to make up the money they had paid out in wages, but to retain a considerable surplus. The final sellers of consumer goods will thus end up with more money than they paid out in wages. From this extra cash they can afford to redeem the bills of exchange that they issued to their suppliers.

In the absence of bank credit, suppliers of manufactured consumer goods would be entirely dependent for cash on money arriving when the bills of exchange, in which they had initially been paid, were eventually redeemed by shopkeepers and merchants. The payment situation facing raw materials firms was even more indirect: they could not be paid unless the manufacturers had sufficient cash to redeem bills of exchange issued for yarn, coal, grain, etc.

Period	Stock (million troy oz.)	Annual growth (percent)
1840–1850	617.9	0.27
1851–1875	771.9	0.89
1876–1900	953.9	0.85
1901–1925	1430.9	1.64
1926–1950	2130.9	1.61
1951–1975	3115.9	1.53
1976–2000	4569.9	1.54

Table 1: Growth of the world gold stock, 1840 to 2000

The process of trade between capitalists leads to the build-up of inter-firm debt. We suggest that the total volume of inter-firm debt that could be stably supported would have been some multiple of the coinage available, after allowing for that required to pay wages. If one takes the aggregate of all firms the ideal signature of this process can be represented as:

$$M \to [C \Rightarrow (C + \Delta C)] \to M + \Delta M$$

where $[C \Rightarrow (C+\Delta C)]$ represents the production process that generates a physical surplus of commodities after the consumption needs of the present working population has been met. If there is no new issue of coin by the state then the ΔM cannot be "real money"; rather, it must be in the form of bills of exchange and other inter-firm credit.

For the capitalist class considered as a whole this should not be a problem since the ΔM is secured against the accumulated commodity surplus ΔC. There is a net accumulation of value as commodities, and accounting practice allows both the debts owed to a firm and stocks of commodities on hand to be included in the value of its notional capital. As the process of accumulation proceeds in this way the ratio of commercial debt to real money will rise. If the period for which commercial credit is extended remains fixed–say at 90 days—then a growing number of debts will be falling due each day. If these have to be paid off in money, then a growing number of firms will have difficulty meeting their debts in cash.

Consider an individual firm: what is the probability that it will not be able to meet its debts?

Let us first normalize the assets liabilities and cash of firms with respect to their turnover. We assume that, normalized to turnover, a firm's expected gross assets and gross liabilities in terms of commercial credit will follow a negative exponential distribution with a probability density of the form:

$$P(d) = Ke^{-\frac{d}{t}} \tag{1}$$

where d is debt, K is a normalization constant, and t is the firm's turnover.[1] The same distribution is assumed to apply to the debts owed to the firm. There is thus a two dimensional probability surface relating assets to liabilities as shown in Figure 1.

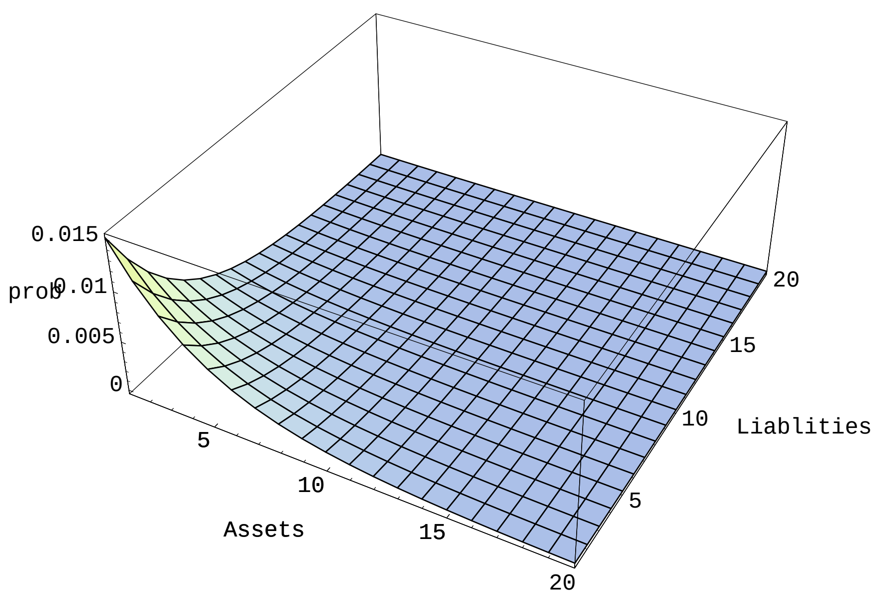

Figure 1: Conditional probability surface for assets and liabilities normalized to turnover.

We can use this probability surface to estimate the distribution of firms along the net-creditor/net-debtor axis as shown in Figure 2. The density peaks where the firm has zero net commercial debt, but the

[1]This is, of course, the Gibbs–Boltzmann distribution discussed in A. Dragulescu and V. M. Yakovenko (2000). The actual distribution may be either this or a power law but the argument that follows is robust in either case.

probability mass close to this peak is actually very small. It is more likely that firms will have either a net credit or debit balance in their dealings with other capitals. The probability distribution is symmetrical, since to every commercial debtor there corresponds a commercial debtor. It falls off steeply on either side.

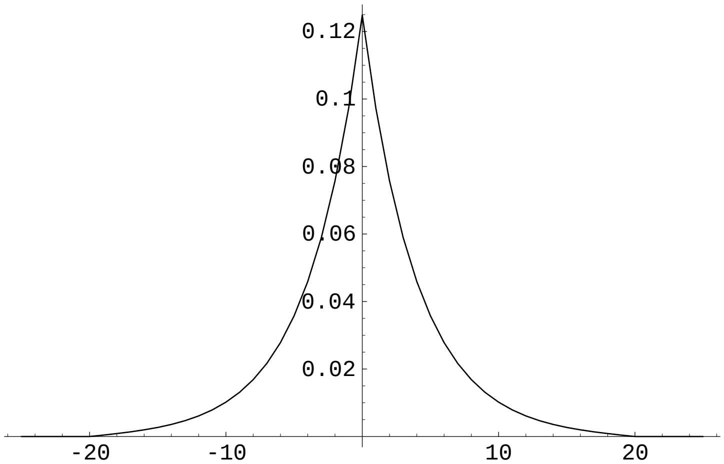

Figure 2: Plot of the probability distribution expected for firms along the net asset/net creditor axis with respect to commercial debts outstanding. The plot is zero-centered because of the symmetry of the commercial debt relationship.

We now have to consider two things:

1. How much of the debt will be falling due each month.
2. How likely is it that a firm will have insufficient cash to meet its debts at the end of the month.

It is obvious that the longer the period, p, for which commercial credit is extended, the smaller will be the amount of debt falling due each month. If it were the custom to extend 90-day credit, then $\frac{1}{3}$ of the debt would fall due each month, as opposed to all of it for a 30-day commercial credit rule.

In order to work out how likely it is that a firm will have too little money to pay its debts we need to have some model of the distribution of money balances in firms. We are provided with this from the data produced by [6] which shows that in the SA model, the probability of a firm having money holding x is given by the Pareto distribution:

$$P(x) \propto (\frac{x}{m})^{-(\alpha+1)} \tag{2}$$

where $\alpha = 1.4$ and m denotes the average money holding of a firm. Using this we can plot the probability surface $C(l,\mu)$ relating liabilities l to cash μ (Figure 3).

Firms will be unable to meet their bills if

1. They have net liabilities l.
2. The net repayments on these debts, $r = \frac{l}{p}$ where p is the period on commercial loans, is greater than the current cash balance.

Thus the probability of a firm defaulting will be given by

$$P(\text{default}) = \int_{-\infty}^{0} \int_{0}^{\infty} C(l,\mu).(\frac{l}{p} + \mu < 0)\; d\mu\; dl \tag{3}$$

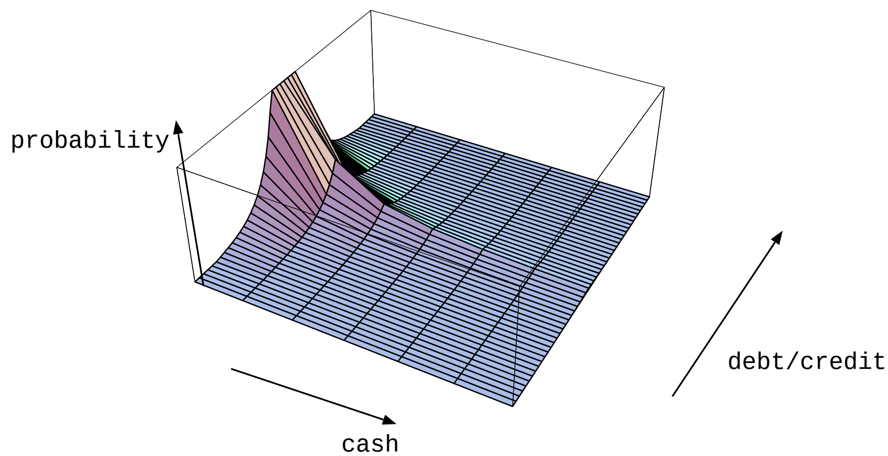

Figure 3: Plot of $C(l,m)$, the probability distribution expected for firms along the net asset/net creditor axis with respect to cash holdings.

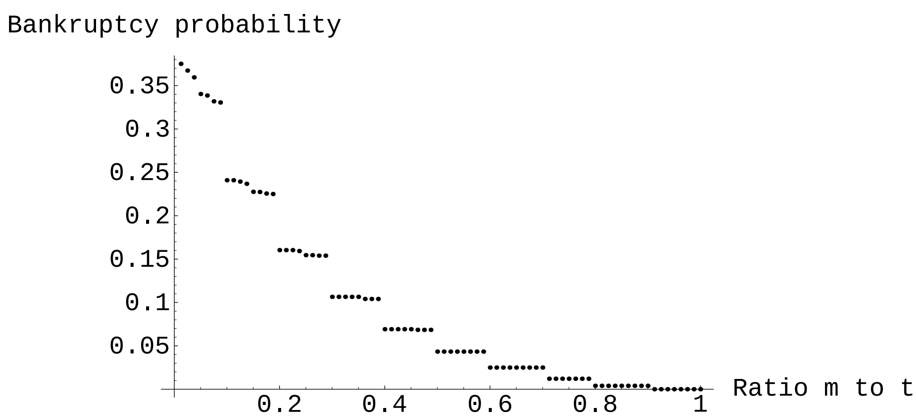

Figure 4: Plot of the fraction of firms going bankrupt in a time period as a function of the ratio of the mean money holding m in equation 2 to the mean turnover t in equation 1. The evaluation assumes a 3-month duration of commercial loans. The steps in the curve are the result of 'binning' as the functions were evaluated on a discrete grid.

However, the shape of the surface $C(l,\mu)$ will depend on the amount of money in circulation. As the probable amount of money held by each firm rises, the default probability will fall. In Figure 1 we see how the probability of bankruptcy declines as the ratio of cash to turnover rises.

It is important to note that what is being considered here is purely stochastic bankruptcy due to cash-flow fluctuations. It is quite aside from bankruptcies that may occur due to economic inefficiencies or long term increases in costs. We are looking at the bankruptcy that can hit perfectly viable firms due to random fluctuations in indebtedness.

1.1 Conclusion

This basic argument gives, we think, a means of understanding the commercial crises of 19th century capitalism under the gold standard.

The commercial cycle goes through a phase of expansion and accumulation during which the ΔM is increasing, sustained by commercial credit. As the volume of commercial credit relative to the real cash required to pay wages increases, the probability of a firm's being unable to meet obligations due to random variations in cash flow rises exponentially. Suddenly, a significant number of firms become illiquid. The process cascades as the creditors of the illiquid firms in turn run into difficulties. Whereas previously credit was treated as being as good as money, now only the real thing is trusted. The collapse of commercial credit then throws the whole economy into one of its periodic crises.

The longer term conclusion is that these crises would become progressively more severe as the long-term effects of the discrepancy between the rate of growth of the gold stock and the rate of growth of turnover make themselves felt. These crises would have impeded the continued expansion of capitalism were it not for the invention of bank money, both in its original form of privately issued banknotes[2] and in its later development as chequing accounts.

2 Probability and the formation of a rate of interest

Now let us try applying the sort of argument that we applied to firms, to banks.

A pre-capitalist banker who made loans out of his own capital could be sure that he would not face insolvency since he had no obligations to depositors. The specific feature of capitalist banking, however, is that it is based on deposit taking.[3] A deposit-taking banker is in a much more perilous position since he accepts cash over and above his own capital which he then lends out. Because he has lent out cash that was not his own capital, and is under an obligation to encash deposits on demand (or after some fixed warning period), he can easily become insolvent. The remaining cash he holds in his safe is never enough to meet his maximum obligation to his creditors. Should the day dawn on which too many of them demand their money back, he is lost, as witness the recent dolorous experience of the Northern Rock in the UK.

Suppose we model this as a stream of customers arriving at the banker's till at random intervals. Each customer either makes a deposit or a withdrawal. The customers may make a withdrawal of any amount up to their current credit balance. We further assume that in a steady state customers are as likely to make a deposit as to make a withdrawal.

The deposits and withdrawals by customers are the result of multiple independent circumstances. As such they have a noisy character analogous to the "shot noise" which sets a limit to the information capture accuracy of any camera or photo-sensor owing to the discrete photon nature of light. Shot noise is proportional to the square root of the mean number of photons arriving at each sensor during the exposure period of the camera. As a result shot noise falls as a proportion of the total signal, the more photons we

[2] These still exist in Scotland and Northern Ireland.

[3] See [3] for a sustained argument on this point.

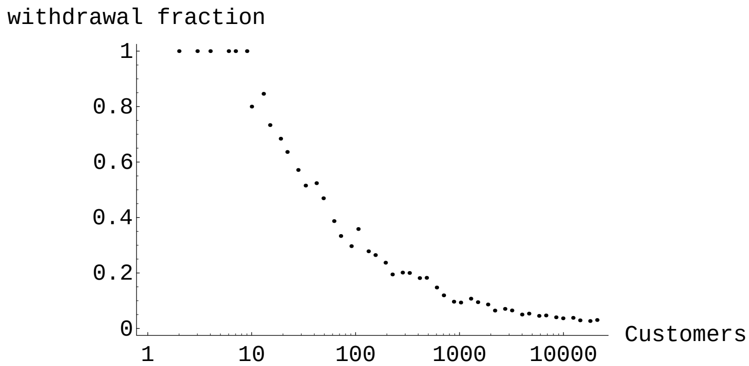

Figure 5: Plot of the largest fraction of a bank's deposits withdrawn in a single day over a 20-year period as the number of customers rises. The plot derives from a simulation in which it is assumed that customers are as likely to make deposits as withdrawals, and may withdraw any sum up to their maximum deposit. The irregular nature of the trace reflects the underlying stochastic properties of such a simulation.

can capture. A similar principle applies to banking. The more customers that a bank has, the smaller will be the proportional variation in the withdrawals from day to day.

Look at Figure 5. The horizontal axis shows the number of customers, and the vertical axis the highest proportion of the bank's deposits withdrawn in any week over a 20-year period. As the number of customers rises, the variation in the amount withdrawn in any week falls, and so too does the maximum withdrawal that can be expected. A very small bank would have to keep all its deposits in the safe as an insurance against having to pay them out, but a bank with 20,000 customers might never see more than 3.4% of its cash deposits withdrawn in any week. A bank with that number of customers could safely issue as loans 20 times as much in paper banknotes as the coin that it held in its vaults, safe in the knowledge that the probability of it ever having to pay out that much in one day was vanishingly small. Thus with a starting capital of 10 million the bank could lend out 200 million after building up its network of customers.

This creation of new paper money by the banks was the hidden secret behind the signature of capital $M \rightarrow C \rightarrow M' \rightarrow C' \rightarrow M''$.

2.1 Importance of number of depositors

The asymptotic decline of the required reserve fraction as a function of the number of customers also explains two major features of banking history:

1. The concentration of banking capital from many local to a few national (and more recently, international) banks. A large bank with more customers can afford to keep smaller reserves, thus generating a greater 'multiplier' effect on its initial capital. This makes it more profitable than a smaller bank.

2. In the early history of banking, failures and runs on banks were common, but they are now rare. As the mean size of banks rises, the probability of failure tends to decline.

The exception to the second point comes when there is a very uneven spread in the sizes of deposits made with the bank. If a large share of the bank's deposits is made by a small number of customers, then the likelihood of shot noise causing withdrawals to exceed reserves is much higher than with a bank which has lots of small customers.

Again the experience of the Northern Rock is illustrative. A major portion (around 75%) of its liabilities was not owed to ordinary depositors, but took the form of loans (equivalent to deposits in the context of this argument) made to it by other banks. Since the number of other banks depositing with it was orders of magnitude smaller than the number of ordinary customers, the risk it faced from random withdrawals by other banks was far greater than would be expected for a bank with so many customers overall. It was effectively in the position of a bank with around 100 customers. Safety would have dictated reserves of perhaps 25% of its deposits. In fact its cash reserves and deposits with the Bank of England were much lower (less than 1% of liabilities in its 2006 consolidated company accounts).

2.2 Costs of loans

The cost to a bank of making a loan is related to the likelihood that the reserves left after the loan will be too small to cover fluctuating withdrawals. If this happens the bank may lose its capital.

From a microeconomic point of view the cost is also determined by the amount of interest that a bank has to pay on the capital that it borrows, but that is the recursive case. The fixed point or macroeconomic result is provided by the previous argument since this does not presuppose the existence of the rate of interest which it seeks to explain.

Consider a random variable W which is the maximal excursion of reserves from their mean position during a year, due to random deposits and withdrawals by customers. Figure 6 illustrates this. For the sake of argument let us normalize the distribution so that one standard deviation of W is 1,000,000, as might be the case with a small bank of the 19th century. With reserves at 3 standard deviations of W, we know from the tabulation of the normal distribution that the probability of bank failure in any one year would be of the order of 0.0015. If reserves fell to 2,000,000 or 2 standard deviations then the failure probability in the year would rise to 0.026. Suppose that the banker had a capital of 5,000,000 and that he would lose his capital if the bank failed, then, if he started with reserves of 3 million, making a loan of 1 million would have an expected cost of $0.026 - 0.0015 \times 5,000,000$ or about 125,000. This expected cost sets a lower limit on the interest it would be rational for the banker to charge for the loan, namely 12.5% in this case. For safety's sake he would be likely to charge more than this.

The lower rational limit to the interest charged would vary with

1. The banker's capital: the more of his own capital is invested in the bank the more he stands to lose on failure.

2. The size of the loan, since this determines the reserve position after making the loan.

3. The size of the bank reserves measured in standard deviations of W (σ_W). The larger the reserves relative to W the lower the cost of making the loan.

Since the interest charged will be proportionate to the loan, for small loans it follows that the rational floor rate of interest should be proportional to the slope of the normal curve at the intersection between the normal curve and the level of reserves measured in terms of σ_W.

We cannot go from this to deducing what the actual rate of interest would be as a function of the ratio of reserves to deposits. For a start, the preceding argument has been about a rational floor to the rate of interest on the assumption that the banker is lending his own cash. If the marginal loan is made out of money for which the banker is himself having to pay interest, the interest that the banker himself pays would have to be added to this floor for him not to expect a loss.

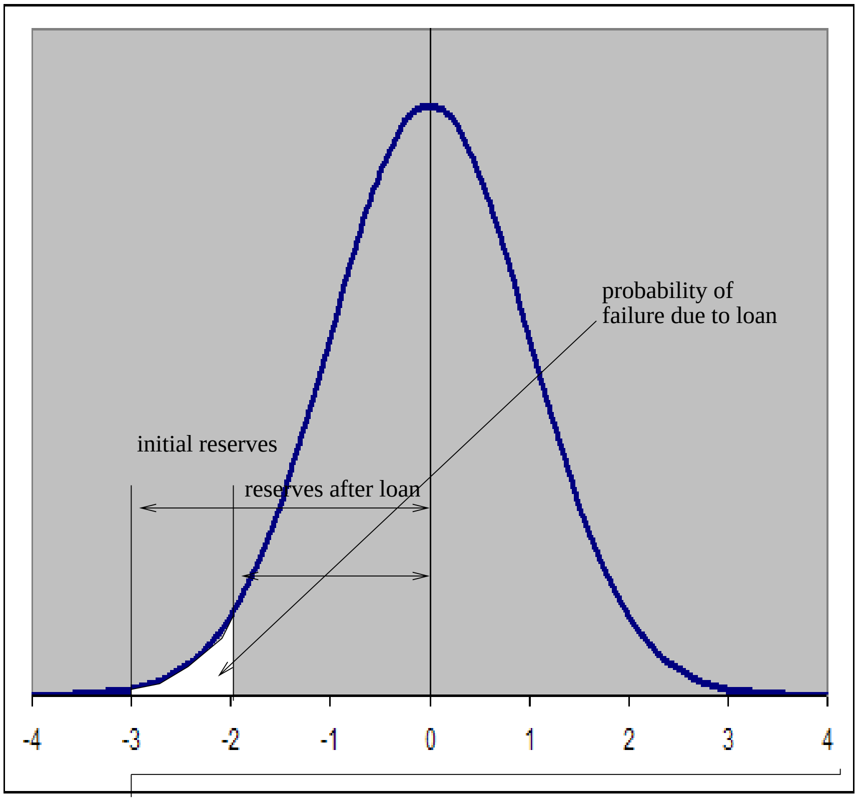

Figure 6: The plot shows the distribution W which represents the greatest excursion from the mean reserve position. The scale is both in standard deviations and in millions. If the bank initially has reserves of 3 million then a loan of 1 million will increase the probability that withdrawals will exceed reserves.

A further factor is that the expected loss, if the marginal loan turns out to be the last straw that breaks the bank at some future date, is proportional to the banker's own capital. The ratio of the banker's capital to the deposits he has accepted will, over the population of banks, be another random variable.

Beyond this we have to add the effects of ignorance. It is a conceit of economic theory to assume perfect knowledge (or knowledge of probability distributions in the present case). Nineteenth-century bankers were probably ignorant of the work of Gauss on the normal distribution, and even had they understood it they may not have had good enough records to accurately measure σ_W. We can expect that the actual rate of interest charged by banks will be randomly distributed with a mean somewhat above the rational floor, but with some bankers lending below their rational floor. Those that do lend below run the risk of failure, but this is a risk rather than a certainty. We might expect, however, that even if the bankers were not aware of a precise relationship between reserves and failure probability, trade folklore would convey to them the need to get anxious as the ratio of reserves to deposits fell. This anxiety would be reflected in higher interest rates, and there would thus still be an inverse relationship between the reserve to deposit ratio and the interest rate.

If reserves are exogenously determined, the interest rate will then vary as a result of the endogenous creation of bank money—either negotiable deposits, or issues of bearer notes in return for deposits.

3 Genesis of financial crises

Financial crises have been a perennial feature of capitalism and even occurred in highly developed slave economies.[4]

We will first explain the basic mechanism generating financial crises under capitalist banking systems, and then present a simulation model that demonstrates these mechanisms.

1. In a population of capitals there will be a dispersion of profit rates.
2. This dispersion of profit rates will be wider than the dispersion of short term interest rates, since competition equalizes the latter more effectively.
3. In consequence in a population of capitals there will be some (group L) whose rate of profit on trading operations falls below the prevailing rate of interest and others (group U) whose trading profit rate is above the prevailing interest rate.
4. It will pay group U to take out bank loans to expand their business, while group L are better served by leaving their profits in the bank. Group U are voluntary borrowers. Group L do not invest in real capital, but accumulate financial assets instead.
5. There is a further group C of firms whose trading profit is insufficient to cover the the interest on their current debt. These firms are involuntary borrowers. They tend to gradually run up bigger and bigger overdrafts until they either restructure or fail. Note that of these three groups only group U carry out real investment.
6. Over time the population of capitals polarizes into borrowers and lenders.
7. The growth of inter-firm debts mediated by the banking system will, over time, cause the reserve ratio of the banks to deteriorate.
8. Deteriorating reserve ratios force up the rate of interest.

[4] Tacitus, in the *Annals of Imperial Rome* [4], recounts a commercial crisis arising in Egypt and rapidly spreading to the Roman credit system.

Table 2: First model

Firm	Inputs			Output	Surplus
	Cons c	Labour	Var c		
1	1	2	1	3	1
2	2	4	2	6	2
Sums	3	6	3	9	3

9. This increases the portion of capitals that fall into groups L and C.
10. Every increase in group L depresses real investment, and forces more of the firms producing capital goods into group C.
11. This feeds back to step 6, causing an acceleration of the whole process until a commercial crisis ensues. Too many firms are in groups L and C and too few in group U to sustain economic growth. A recession is triggered.

4 A model illustrating financial crises

We present a model of the dynamics of a 'gas' of capitals to see if it polarizes into rentiers and industrial capitals. The general method followed is derived from Ian Wright's work.

We assume two populations of capitalists: those producing means of production, and those producing consumer goods.

In order to simplify calculations we choose the production parameters so that the mean production functions of each section are such that the outputs sell for unit value. Thus each sector will employ 1 unit of means of production and 2 units of labour to produce 3 units of physical output, whose value will also be 3.

The rate of surplus value is assumed to be 100% so that for every 2 units of labour employed the workers will get 1 in wages.

A simple system with 6 workers and two firms is as shown in Table 2.

The sum of the consumed constant capital in both firms equals the output of firm 1, and the output of firm 2 is sufficient to meet the consumption of the workers of both firms plus the consumption of the capitalists, if they spend all their capital on consumer goods.

We assume there is one bank and at time t_0 each capitalist has sufficient money in the bank to pay wages. At the outset there are no outstanding loans to the banking system, so the total money assets of the capitalists equal the total liabilities of the bank and these equal its gold reserves.

If a capitalist purchases a commodity or labour he pays with a draft on the bank, so over time his assets with the bank can be positive or negative. We model this by an account for each capitalist. A single collective account is kept for all workers.

The bank sets a rate of interest which is initially zero and which will rise as an exponential function of the liability to reserve ratio thus

$$r(t) = 0.01e^{L(t)/R(t)-1} - 1$$

The current rate of interest determines if a capitalist will invest or save the surplus left over after personal consumption. The reserves (gold) are assumed to remain constant throughout the simulation.

For firms whose rate of profit is less than the rate of interest, their mean rate of gross investment is equal to half the rate at which their capital is depreciating. If firms have a rate of profit above the rate of interest their mean gross investment rate is twice the depreciation rate.

Table 3: Second model

Dept	Inputs			Output	Surplus
	Const c	Labour	Wages		
1	2	4	2	6	2
2	2	4	2	6	2
Sums	4	8	4	12	4
		Capitalist	expenditure	by sector	
		Investment	Consumption		
	1	1	1		
	2	1	1		
	Totals	2	2		

Whenever an agent, whether it be a capitalist or a worker, attempts to purchase consumer goods or raw materials they randomly select a supplier and attempt to buy their target amount from that supplier. If the supplier has insufficient stocks in hand, the purchaser buys the remaining stock of the supplier and then another supplier is chosen at random to supply the residual. If no suppliers have any remaining stocks, the attempt to purchase fails.

We assume that capitalists always try to consume at a rate equivalent to 70% their profit. If the rate of profit they earn is lower than the rate of interest, they save the remaining 30% of their surplus. Otherwise the remainder of their surplus, along with bank loans, finances their new investment.

In order to ensure that the system can be able to expand from the start we set it up with the industries in the ratios shown in Table 3.

In this configuration all flows balance, and the economy is capable of following a growth path provided that the supply of labour is assumed to be arbitrarily elastic. It is assumed that technology does not change over the period of the simulation.

The firms will initially all be the same size and have the same initial cash assets but symmetry will be broken by two means

1. A randomized purchasing procedure, which means that it is indeterminate which firms will be left with unsold stocks or unmet orders after a purchasing round.

2. An initial small random variation in the productivity of firms around the mean.

The initial population was set to 500 firms and the simulation run for 48 time steps.

4.1 Results

The results of the simulation are shown in Figures 7, 8 and 9.

At first the economy grows exponentially. This entails a parallel growth in the loan to reserve ratio of the banking system, which in turn produces an increase in the rate of interest. Eventually the increase in interest causes a downturn in investment, precipitating a recession.

The population of firms, all of whom started out with no initial debts, polarizes into two groups. The polarization process is shown in Figure 9. The main population cluster forms around the mean rate of profit and with gearing ratios in the range -0.5 to $+0.5$. A firm with a gearing ratio of 0.5 has half of its employed capital funded by bank loans. A firm with a gearing ratio of -0.5 has net deposits with the bank equal to half its productive capital.

As time goes on the distribution assumes a tadpole shape, growing a tail of rentier capitals with very low (substantially negative) gearing ratios. For these capitals, productive resources represent only a small part of their assets. At time step five, the capital with the most extreme rentier traits had a cash to constant capital ratio of about 1/1. By the end of the simulation, however, the rentier tail includes firms whose financial assets are more than 100 times greater than their real assets.

The growth of the rentier tail is intimately linked to the change in the L/R ratio of the banking system shown in Figure 8. As the rentier tail becomes more marked, the saving by rentiers grows, pushing up the L/R ratio, and by the arguments in Section 2 this leads to higher interest rates. But higher interest rates will in turn accentuate the growth of the rentier tail.

5 Conclusions

We first developed a stochastic model of the likelihood of firms going bankrupt under a purely commodity money system, extended by commercial credit between firms. This showed that as the level of production rises faster than the stocks of gold, a threshold is reached at which stochastic bankruptcies of still profitable firms would likely precipitate a commercial crisis.

We then extended the analysis to cover firms using bank accounts as money. We showed that the rate of interest charged by banks should, on probabilistic grounds, be an increasing function of the loan to reserve ratios of the banks. We argued that banks with more customers will be more secure against random failure, and thus there will be marked advantages to scale in banking. This, we claimed, was the driving force behind the merger of small banks into larger ones. We noted that this advantage to scale is negated where banks rely not on a large number of depositors for funds, but on credit provided by a small number of fellow banks. This observation has clear relevance to the recent failure of the Northern Rock bank in Britain.

We then analysed the interaction between a banking system which creates endogenous money and a population of firms. We showed that symmetry-breaking will result in the precipitation of a class of rentiers and the polarization of the the population of capitals. This process culminates in financial crisis as the loan to reserve ratio rises and pushes interest rates too high to sustain investment.

Such a process arguably lay behind the depression of the 1930s which forced the abandonment of gold as the substance of bank reserves.

In 2008, the same process appears to be operating, except in this case the reserve base of the banking system is provided by fiat money issued by the state. The implication is that once the polarization of capital has taken hold, only ever-greater injections of fiat money into the system by central banks can sustain the viability of the credit system.

References

[1] A. Dragulescu and V. M. Yakovenko. Statistical mechanics of money. *The European Physical Journal B*, 17:723–729, 2000.

[2] Emmanuel Farjoun and Moshe Machover. *Laws of Chaos, a Probabilistic Approach to Political Economy*. Verso, London, 1983.

[3] Geoffrey Ingham. *The Nature of Money*. Polity, Cambridge, 2004.

[4] Tacitus. *The Annals of Imperial Rome*. Penguin, London, 1973.

[5] I. Wright. Simulating the law of value. *Submitted for publication, preprint at http://www. unifr. ch/econophysics/articoli/fichier/WrightLawOfValue. pdf*, 2003.

[6] I. Wright. The social architecture of capitalism. *Physica A: Statistical Mechanics and its Applications*, 346(3-4):589–620, 2005.

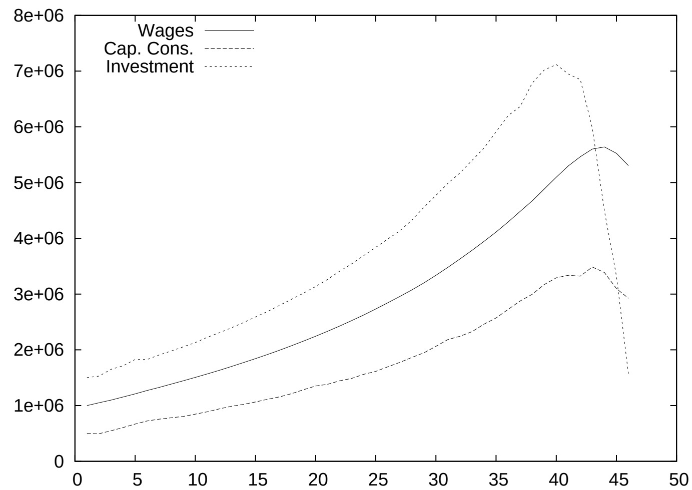

Figure 7: Plot of the evolution of wages, capitalist consumption and investment in the simulated economy. The economy at first follows an exponential growth path for all three indicators. After interval 40 there is a downturn in investment that feeds into a decline in the other indicators a few steps later.

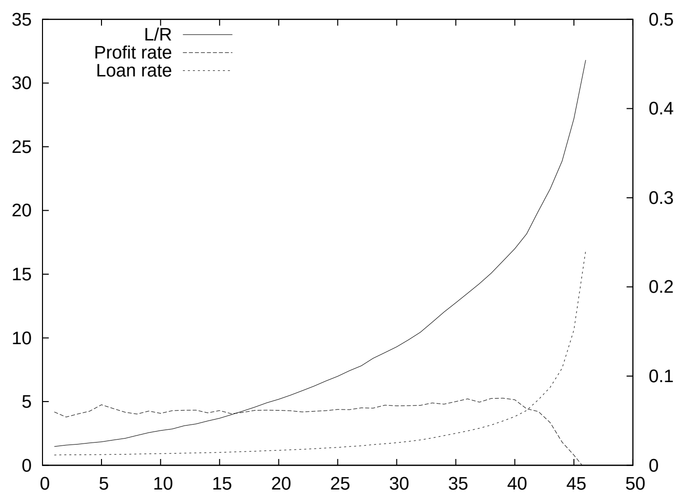

Figure 8: Plot of the evolution of the loan to reserve ratio (L/R), the interest rate paid on bank loans, and the mean rate of industrial profit. By comparing this with Figure 7 it can be seen that the downturn in investment follows a rise in the interest rate until it is equal to or greater than the mean rate of profit. This rise in the interest rate is itself a consequence of the rising L/R ratio.

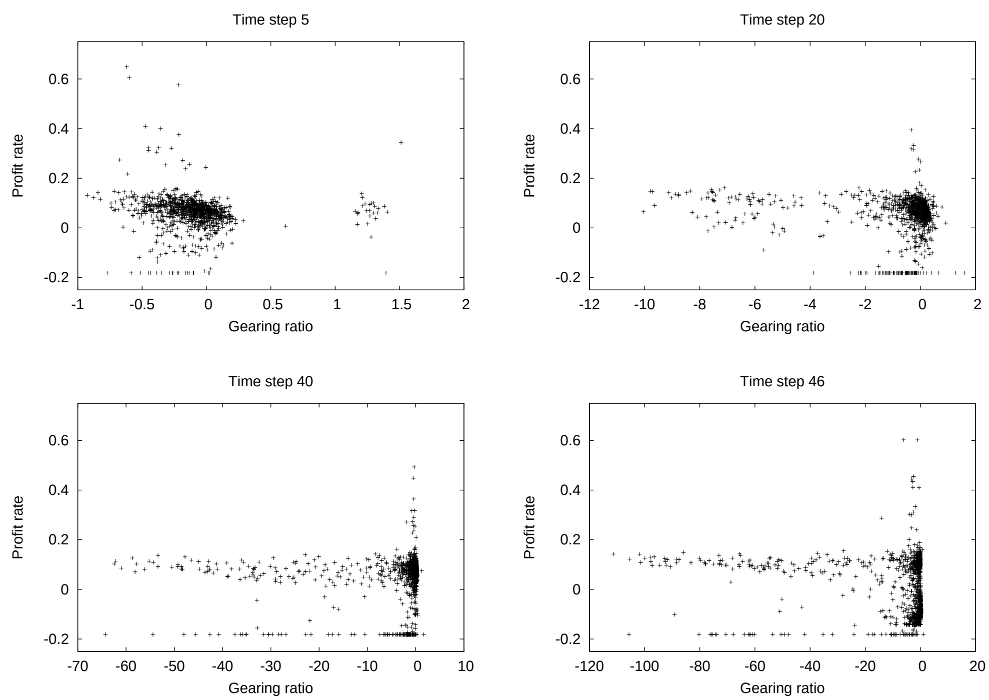

Figure 9: Plots showing the evolution of the population of firms in the space defined by gearing ratios against rate of profit. Note how an initially compact distribution develops a tadpole shape, as it grows a tail of rentiers. In the final time step the profit distribution has become bimodal, with firms producing investment goods showing losses. Note the change in scale of the *x* axis as time progresses. Note also that while a firm whose gearing ratio rises above 1 is unlikely to survive, firms with negative gearing ratios have a high survival rate.