Multi-Pulse Train Modelling of Piezo DoD Inkjet Print-Head Response

Stephen D HOATH

University of Cambridge, IfM, 17 Charles Babbage Road, Cambridge CB3 0FS, United Kingdom

sdh35@cam.ac.uk

Abstract

Resonant oscillations set up internal fluid waves within a piezo-DoD print-head channel as a result of actuation drive pulses. Such waves will persist for some time after droplet ejection from the nozzle, and the residual wave amplitude can interfere (constructively or destructively) with all succeeding actuation drive pulses, potentially altering the speed and volume of successive droplets. As uncontrolled interference would worsen printing quality, residual waves are usually reduced by a combination of print-head design and waveform optimisation for better performance at continuous (steady-state) printing frequencies. However, the residual waves following any changes of printing frequency can influence "first" drops and short bursts of drops. Exact analytic expressions are provided here for the N-pulse burst DoD print-head response function with fixed printing frequency. The present paper explains the purpose and application of the model predictions to published piezo-driven DoD data. An examination of the effect of fluid properties, the identification of unexpected jetting behaviour and some issues with manufacturing prototype quality, tests of assumptions made in the simple model and extensions to the prediction of print-head performance using realistic complex waveforms are also discussed. An earlier shorter report, mainly introducing the multi-pulse train modelling approach and some applications within Xaar, was first presented at NIP31/DF2015.[1]

Key words

Drop-on-Demand, Inkjet, Residual Waves, Modelling, Frequency Response, First Drop, Burst and Continuous Printing, Piezo, Print-Head, Jetting, Complex Fluids, Measured Data Interpretation

Introduction

As a result of the industrial marketing push towards ever higher inkjet printing (jetting) frequencies and smaller drop sizes, and the rapid progress being made in applications of micro-electro-mechanical systems (MEMS) -based manufacturing, some drop-on-demand (DoD) inkjet print-head designs rely on piezo-actuated (driven) resonant chambers to generate liquid droplets. It is convenient to consider a single nozzle device, although the generalization to multiple nozzle print heads is both feasible and desirable. The resonant oscillations set up waves in fluid within each DoD print-head channel as a result of the actuation drive pulses[2-4]. Such waves will persist for some time in the channel after the droplet ejection from the nozzle[5] and the residual wave amplitude can interfere (constructively or destructively) with all succeeding actuation drive pulses, potentially altering the speed and volume of successive droplets.[6] As uncontrolled interference would worsen printing quality continuous printing residuals are often tamed by the combination of print-head design and waveform modifications[7-10]. Residual wave effects on the observed speed and volume of the last fluid droplet ejected by a short burst of (N) drive pulses, i.e. following distinct changes of printing frequency, and the steady-state printing condition (where N is large) are also predicted using the multi-pulse train (MPT) modelling.[1] The effect of residual waves may also change the timing of the droplet, leading to drop misplacement, while incomplete refilling[11] or overfilling of the nozzle following droplet ejection can both have very major effects on speed, direction and volume of the subsequent droplet(s) that are significantly harder to model than at lower printing frequencies without such complications for the initial conditions.

A simple approach to model the magnitudes of the (measurable) drop speed and volume from the print-head treats the response for the last pulse of the N-burst as if it was generated with the phase and magnitude of an impulse peak response but then forms the resultant sum over damped residual waves after taking account of the actual phases and decay for all (N-1) preceding impulses. This approach avoids nearly all the details of the piezo-driven wave behaviour during the waveform,

while focussing on the implications for applications of DoD printing, which inherently involve both steady state (longer term) and transient (short term) bursts of pulses. For single isolated pulses, and a range of low frequencies for steady state printing, the residual waves will have diminished before another pulse is applied. Consequently, for such low frequencies, DoD print-head response can be considered as "normal". The residual wave oscillation amplitude immediately after an impulse equals the normalised value 1, and this amplitude diminishes exponentially with elapsed time to effectively reach zero before the start of any succeeding applied drive pulse. Clearly, for an extended pulse length, lack of sufficient residual wave damping would limit the maximum repetition rate (frequency) at which overlap of the residual wave oscillations from prior pulses and succeeding pulse(s) would interfere with the wave amplitude from the nth pulse. The relative phase of the damped residual waves and the succeeding pulses determines whether the overlap enhances their summation constructively or destructively. Indeed, extended pulse waveform designs for DoD inkjet print heads are often deliberately engineered to produce droplets of a specific speed and volume simultaneously with minimum possible absolute amplitude for the residual wave. This usually raises the maximum (steady state) printing frequency range within which some specific tolerances on the jetting characteristics of the droplets, for example the allowable variations in drop speed and volume, can be maintained. As will be shown, the flat bandwidth frequency range over which piezo-driven DoD drop speeds are kept within specifications of ±5% is closely linked to the overall damping factor (Q) of the print-head's (natural) resonance frequency (f_H), at least for applied drive waveforms comprised of a single unipolar pulse.

More complicated waveform designs for specific combinations of piezo-driven DoD print-heads and fluids often aim to extend the useable frequency range beyond its flat (unipolar pulse) bandwidth. Importantly, where the desired drop speed is relatively high, suitable control of the "pinch-off time" for a fluid jet may avoid unwanted "satellite" production, by sufficiently reducing the inkjet ligament length at the required jetting speed, and simultaneously enable successful higher frequency printing. There are other technical reasons for developing more complex drive waveforms, which are outside

the scope of the present paper. Some, for example, greyscale printing, can be approached within the spirit of the present multi-pulse train model, which will be described in some detail in a later paper.

It must be emphasised that piezo-driven DoD print-heads are usually driven by complex waveforms to achieve the best performance in applications; in the present paper a simple waveform is used so that underlying responses of the piezo-driven DoD print-heads themselves might be better revealed. This aids the identification of more fundamental performance limits, and following the application of linear response theory, to more easily investigating the potential directions for beating these limits.

Residual wave models for burst and continuous printing

Earlier analytic work[3,4] on the residual wave responses for multiple coupled-channels in piezo-driven DoD arrays focussed on results in the time domain, rather than in the frequency domain as herein. Recent use of advanced control theory[8-10, 12] has now produced robust optimised DoD waveforms, a method that provides a direct route to a specific residual wave duration and frequency bandwidth.

To a certain extent, the present approach allows print-head characteristics to be inferred (reverse-engineered) from the measured frequency response for both the jetted droplet speed and volume. The exact analytic summations could easily be replaced (and extended) by numerical summations, which could then be used to construct and investigate more general DoD drive waveform tools[13]. However for the present explanatory purposes, only exact analytic results will be considered here. Furthermore, explicit results for burst, as well as continuous, printing are provided for the first time.

All approaches to exact modelling of residual waves in resonant piezo-drive DoD print-heads rely on the linearity of the equations of motion in time, which imply that wave amplitudes (perhaps from different physical sources, or events such as the drive pulse) can be simply added together by taking into account their magnitude and relative phase.[4] After a particular drive-pulse completes, an associated residual wave starts at time t=0. The previous drive pulse would have been decaying away for time t=1/f at the fixed printing frequency f, and its amplitude would have decayed through the exp(-b/f) factor while the phase of the (cosine) oscillation will have altered by the angle a/f. This

4

previous wave continues to decay, independently of the new wave, but after the particular drive pulse has finished the overall residual wave amplitude will be the sum of these two independent waves. Thus linearity suggested that the resulting residual wave after 2 pulses is $1+\cos(a/f)\exp(-b/f)$ at frequency f, after 3 pulses is $1+\cos(a/f)\exp(-b/f)+\cos(2a/f)\exp(-2b/f)$ and after N such pulses at this frequency equals $\sum[\cos(na/f)\exp(-nb/f)]$, where $\sum[]$ is the multi-pulse train summation of the nth pulse over N pulse contributions, formed by n taking the successive integer values from 0 to N-1. [1]

Equations

Evaluation of $\sum[]$ exploits the exact analytical result $(1-z^N)/(1-z)$ for complex number z, where $|z|<1$, $\mathrm{Re}\{\exp(ja/f)\}=\cos(a/f)$ and $\mathrm{Im}\{\exp(\pm ja/f)=\pm\sin(a/f)$ and identifying $\sum[z^n]$ with the re(al) part $\mathrm{Re}\{\sum[z^n]\}$. By replacement of $\cos(a/f)\exp(-b/f)$ by $z=\exp(-b/f+ja/f)$, and noting $z^*=\exp(-b/f-ja/f)$, we find that $\mathrm{Re}\{\sum[z^N]\} = \mathrm{Re}\{(1-z^N)/(1-z)\}=[\mathrm{Re}\{(1-z^N)(1-z^*)\}]/[(1-z)(1-z^*)]=[\mathrm{Re}\{1-z^*-z^N+z^Nz^*\}]/[1-z-z^*-z^*z]$, and so find $\sum[] = R(N, f) =[1-\cos(a/f)\exp(-b/f)-\cos(Na/f)\exp(-Nb/f)+\cos(a(N-1)/f)\exp(-(N+1)b/f)]/D(f)$. In the steady state limit $R(N,f) \Rightarrow R(f) = [1-\cos(a/f)\exp(-b/f)]/D(f)$, where $D(f)=[1+\exp(-2b/f)-2\cos(a/f)\exp(-b/f)]$. The rather useful finding, that denominator D(f) is independent of N, simplifies both R(N,f) and R(f). We find that this steady state result R(f) follows immediately from properties of the Z-transform. [14] [Furthermore, the properties of the Z-transform can be used for evaluating summations over terms involving $\sin(na/f)\exp(-nb/f)$ as easily as those in $\cos(na/f)\exp(-nb/f)$ within the summations for R(f).]

Characterisation of residual waves in fluid jetted by a piezo-driven DoD print-head

For a damped oscillatory wave which has a "normal" amplitude, the simple function $\cos(at)\exp(-bt)$, takes the value 1 at elapsed time t=0 and then oscillates at a frequency $f_D=a/2\pi$ and decays at the rate $b=\pi f_H/Q$, where a and b are considered to be constants. The quality factor Q for resonant DoD inkjet print-heads links the damped frequency f_D and natural frequency f_H through relationships [15] such as $f_D=f_H\sqrt{1-1/(2Q)^2}$ for a single degree oscillator, where usually Q>>½. This latter case [15] implies that $b\approx a/(2Q)$ for all the dominant and higher-order residual waves in inkjet print-heads. We avoid

choice of the oscillator model and approximating the rate of damping (b) in our derivations so that other highly (but still resonant) damped residual waves can also be included in frequency response.

Consequences of Q-factors for inkjet print-heads

Resonant inkjet print-heads excited with constant voltage unipolar pulses can have high frequency responses that show jetted drops with far higher or somewhat lower speeds than at low frequency, to an extent that depends on the Q-factor and the (dimensionless) printing frequency ratio f/f_H. Starting at low printing frequencies, if the response is normalised to unity, then for response peaks (highs) occurring at higher frequencies due to constructive residual oscillations, the (voltage) level(s) of the (unipolar) actuation pulse used to drive the piezo DoD print-head could be reduced in order to compensate this, with potential benefits to the device power consumption and lifetime. Conversely, wherever the printing frequency produces a normalised response below unity, e.g. near valleys (lows) of destructive residual oscillations, the level(s) would need to be increased for compensation, with higher device power and cooling requirements and limiting fluid viscosity. Other scenarios are possible for compensation: the pulse waveform duration or its level of complexity could be altered. Print-head operation near frequencies corresponding to peaks (or valleys) of the print-head system response can still be distinctly advantageous for some applications requiring highly stable response. However, further mention of choosing an operating frequency lies beyond the scope of this article.

Viscous fluids and inkjet print-head Q-factors

The multi-pulse and steady state printing frequency responses derived exactly for simple piezo-drive pulses in an earlier section can be interpreted in terms of the frequency f_H and damping Q-factor of the print-head and fluid combination. The chosen material and mechanical properties, the design and the physical build of the print-head might be expected to dominate the natural resonant frequency, while the damping also depends on the relevant properties of the fluid (such as density and viscosity). Non-Newtonian fluid behaviour, density, compressibility (including effects due to dissolved gases), elasticity and dynamic surface tension may alter the apparent natural frequency of

the resonance, as could inclusion of material components with extra compliance (or arising from defective bonds). Mechanical, fluidic or electrical cross-talk present between neighbouring channels would also be expected to influence the apparent damping factors dependent on which nozzles were being driven, and with what drive and phase; such considerations are avoided in the present work, as they are better included within numerical simulations more easily than in analytic forms.

Fluid properties, such as the speed of sound, significantly control the values of Q and f_D, but it is also worthwhile explaining how some additional losses might change Q-factors for print-heads. Such additional losses may be (unintended) consequences of changing the jetted fluid viscosity, e.g. while successively setting up and using different fluids or operating temperatures with a print-head.

Recall that Q-factors are a measure of stored energy ÷ power lost over an oscillation period. Notice that constant $b=\pi f_H/Q$ involves reciprocal $1/Q$, thus proportional to total power lost ÷ stored energy. So $1/Q$ = (head loss + fluid loss) ÷ storage = head loss ÷ storage + fluid loss ÷ storage = $1/Q_0 + 1/Q_V$, where $1/Q_0$ represents the head contribution and $1/Q_V$ the (viscous) fluid contribution to total $1/Q$. Fluid losses thus increase $1/Q_V$ from 0 for inviscid fluids to higher values linearly with finite viscosity.

The two (possibly simultaneous) physical contributions to the decay constant b are the magnitude of the resonant frequency f_D, and the magnitude of the total losses (including the fluid viscosity). While f_D depends on mechanical compliances for a given print-head (resonance chamber) geometry, the most obvious fluidic influence on f_D arises from speed of sound corrected for compliance of the environment[16]. For printer and fluid combinations with the same speed of sound and f_D values, comparing the observed values of the residual wave damping rate (b) found for fluids with different but known viscosities could be used to infer the values of $1/Q_0$ and $1/Q_V$ for each fluid jetted, and even to predict the effective Q-factor for other fluids to be jetted by that particular inkjet print-head. One may anticipate that more viscous fluids may have lower Q and more highly damped responses in inkjet print-heads, but direct measurements of residual waves could confirm or reject such changes. Importantly, frequency sweeps conducted for different fluids in the same print-head could

differ in a predictable fashion (higher viscosity should lead to lower Q and more damped frequency responses).

Results

An application of the MPT model has been made to data shown in Figure 3 of Derby and Reis (2003), for jetting paraffin with 0, 20vol% and 40vol % alumina powder loadings.[16] The overall reduction in drop volume, as particle loading is increased, arises from a combination of the fixed drive voltage applied to the print-head and the increasing viscosity of the fluid with loading. In the present application of the MPT model, this difference between fluids is lost to normalisation by each fluid's low frequency volumes (for which residual waves fully decay between printing drops). Exact model frequency responses computed between 1 and 20 kHz were overlaid and manually fitted to the data trends by suitable adjustments of the Q-factors and damped frequencies assumed during printing.

Figure 3 shows that at each loading highest frequency peak F is the main (m=1) resonance, as can most easily be seen by tracking the sub-harmonics (m = 2, 3, 4, …) frequencies (F/m) for the unloaded 0% data. The lowered print-head resonant frequency F as particle loading is increased agrees reasonably with the reduction in the speed of sound within the print head also computed by Derby and Reis (2003).[16] Ignoring background slopes the MPT model fits, to each fluid's normalised data, assuming Q-factor as a free parameter can reasonably approximate the large % variations seen in 0.1-0.9 nano-litre scale droplet volumes. The fits show that physical properties of the jetted inks, differing significantly between material loadings, had a major influence on the effective (damped) Q-factors, thereby distorting the frequency response spectrum. As the %volume loading increases, the fitted Q-factors were 6.8, 5.5 and 4.5. As $Q = 2\pi f_H \times$ stored energy $\div$ power loss, the speed of sound in a (loaded) fluid influences both the damped frequency (f_D) and the quality factor (Q) of the print head in the same (linear) manner. As the predicted change of the speed of sound with %volume loading[15], and hence the change to the damped frequency f_D, compared almost directly with the trends in Q factors fitted to the corresponding jetting data[16], no changes to the Q-factors could be

8

ascribed to the effects of the greatly increased viscosity as the %volume loading was increased, meaning that viscosity only influenced the drop volume because a constant drive voltage was used.

Figure 1

Figure 1. Measured drop volume, as a function of DoD printing frequency, for liquid paraffin loaded with alumina powder[13] as compared in a self-consistent analysis with R(f) from the MPT model for the unloaded paraffin response. Fitted f_D and Q values vary with the speed of sound in these fluids.

Figure 2

Figure 2. Published data[24] shown as normalised √(speed x volume) are compared to R(f). See text.

Measurements for inkjet drops jetting published as normalised values of drop speed and drop volume[24] have been combined as normalised √(speed x volume) for comparison to predicted R(f) from the simple MPT model, assuming natural resonance frequency near 24.8kHz and a Q-factor of about 5. (Note that term √(speed x volume) is proportional to √(momentum) for fixed drop density. For this particular experiment the absolute values of drop speed and volume could not be disclosed, while a physical model suggested drop momentum would be prove to be more representative.) The failure of this R(f) to predict the presence of a ~16.5kHz response peak, despite good correspondence with the lower sub-harmonic peak locations at 12.4, 8.4, 6.2, 4.9kHz… their relative strengths and even the "flat background" limit B= 4.9kHz could point to the presence of additional resonances in the inkjet print-head, such as the presence of a refill mode in this device[24] or a meniscus oscillation mode.

Figure 3

Figure 3. Published data[24] shown as normalised drop momentum compared to a combined weighted response from 2 independent resonances producing frequency spectra of the form R(f). "Double f" points (open circles) are some of the data but replotted at points at twice the frequency, merely for ease of comparison with the apparent location of the other resonance in the full response. See text.

An example chosen to mimic the possible location of another independent resonance mode, which may represent refill but fails to properly represent the loss of drop volume that is inherent in refill, is shown in Figure 3 for the same data as used in Figure 2. Nevertheless, some features of the resultant weighted frequency response do appear to represent the data quite well, by assuming somewhat arbitrary choices for the relative strength, natural resonant frequency and Q-factor to achieve this. The (unpalatable) alternatives are a complete failure of the simple MPT approach, or high frequency measurement issues for imaging software or even some neglected physics relevant to inkjet printing. As a merely empirical (and hopefully unconvincing) gesture towards an alternative, some of the data in Figure 3 (and Figure 2) have been replotted at twice their correct frequencies for a comparison with the apparent location and width of the additional resonance, as they do roughly correspond. To challenge the MPT modelling approach seriously, high frequency jetting data will be examined next.

Figure 4

Figure 4. Measurements of drop speed, at 20-40kHz jetting frequency, taken for a standard XJ1001.

Figure 4 shows drop speeds at fixed 20-40kHz jetting frequency, from an independent laboratory[13], for a standard XJ1001 print-head. R(f) is shown in blue on a cubic background variation, while the data were normalized for comparison purposes to the average drop speed between 5 kHz and 6 kHz. The normalised speed range specified by $1\pm\exp(-\pi) = 100\pm4.32\%$ are also superposed on the graph.

Figure 5

Figure 5. Measured drop speed and volume data for a particular SiMEMS DoD product development print-head geometry for a wide range of printing frequencies are combined as √(momentum) and compared with response R(f) of the MPT model. These data are normalized along the frequency axis.

Figure 5 shows some product development data on DoD drop speed and volume measurements for a certain print-head geometry[1] converted into normalised √(momenta) vs. normalised frequency (i.e. kHz/f_H) for consistency with the NIP31/DF2015 conference paper. The error bars reflect the spread

of results for 10 drops measured (using a JetXpert™) at each steady state printing frequency. The comparison between the shape and magnitude of the frequency sweep data and the MPT model prediction R(f), which here shown without an underlying "background", illustrates the quality of both the data (being the harmonic mean of the drop speed and drop speed) and the model. The benchmark provided by such modelling assisted with the early elimination of production issues.[18]

Figure 6.

Figure 6. Results obtained by fitting the MPT model predictions to the single jetting channel drop speed frequency sweeps, for a bank of neighbouring channels in a development prototype device that contained known device bonding faults towards the centre of the array. The curves merely guide the eye and clearly have no intrinsic meaning between channel numbers; parameters f_H and Q have been normalised to fitted channel 2 values and interpreted in the text.

A final example of the application of the MPT model predictions returns, in a sense, to the impact of losses on Q and hence the shape of the frequency sweeps. Figure 6 shows normalised results from fitting the 2 free parameters (f_H and Q) of R(f) to frequency sweep (speed) data measured for a bank of individually actuated next-neighbour channels in a development prototype device that contained known device bonding faults towards the centre of the array. Unlike the previous case considering losses, the same fluid was jetted from each channel, and the variations of Q are much bigger than those of the resonance. The implication was the presence of other sources of losses for individual channels; subsequent IR camera inspection revealed an issue with unwanted leakage between channels that corresponded to trends shown in the fitted Q-factor. Such early successes underpinned the value of the MPT approach for benchmarking DoD response, and improved the automated assessment of jetting data sets across development design devices.[18] For some print-head designs it is reasonable to talk about Helmholtz frequency (f_H), for others it is about the wave speed and associated modes[19], but this does not detract from MPT summation of residual waves.

Complicated drive waveforms

Generalisation of this MPT approach to predict the effect of complicated drive waveforms is quite straightforward (if unwieldy): just add suitably weighted and phased combinations of the linear response R(f) found for unipolar excitation of the print-head and fluid used. For example, the explicit exact summations for the extra time-dependences of residual wave generation by ramped unipolar waveforms, as given previously[3], can be immediately handled using the MPT method for complex z. The properties of the Z-transform[14] can be applied to finite sums over decaying sine waves, and give the same denominator D(f) as was found for the sums over decaying cosine waves resulting in R(f). Thus decaying waves of any phase are handled by decomposition into cosine and sine components. Additional drive pulses with appropriate weights and phases can be used to represent bipolar pulses, grey-scale printing. Numerically summing R(f) terms, rather than exact results, has been proposed[13] so the exact mathematical results for such extensions of the simple MPT model are not given here.

Burst printing

The majority of inkjet printing applications also involve short bursts of printing (N) drops rather than steady state printing, and it was shown earlier[1] that the simple MPT model can easily provide exact predictions for the evolution of the N-burst frequency response R(f, N) with increasing N towards the steady state response R(f) that might be expected for a given print-head characterised by f_H and Q. Note that the printing frequency response curve for a single drop (i.e. N=1) does not physically exist, although the purely mathematical representation would be a straight line R(f, 1)=1 at all frequencies, reflecting the normalisation for the MPT model. Achieving near steady state requires N > Q pulses.

Figure 7 shows the N-burst frequency response R(f, N) for N=2-10 and the steady state response R(f) for the case f_H=240kHz and Q=6, while Figure 8 shows the evolution for N-burst frequency response R(f, ϕ, N) corresponding to constant phase angle ϕ=30° between the timing of the print-head drive pulse and the assumed damped cosine peak response. [Thus R(f, ϕ=0°, N) is equivalent to R(f, N).] The major changes are, as expected, between N=2 and N=3, and persist to N > 6 in both Figures.

Figure 7

Figure 7. The N-burst frequency (f) response R(f,N) and steady state response R(f) resulting from a single exponentially damped cosine oscillator having natural resonance f_H=240 kHz and Q-factor 6. These representative values lead to sub-harmonic peaks at conveniently located frequency values for steady state printing. The "flat background" frequency region defined by $f<B=f_H/Q$ (=40kHz here) corresponds to the upper and lower (dashed line) limits at 1±exp(-π) ≈100%±4.3% that just "touch" the Qth sub-harmonic peak of R(f). Approaching the steady state printing response R(f) takes N > Q.

Figure 8

Figure 8. Overall normalised response R(f, φ, N) compared with the steady state response R(f, φ) for a phase angle φ=30° and the steady state response R(f) with φ=0°, for f_H=240kHz and Q=6. See text.

Discussion

Changes predicted by the simple MPT model, between the earliest drops of N-burst printing and later continuously printed drops in steady state printing, are considered as "first drop" effects completely unrelated to the evaporation of ink[20], raised concentrations of particles at the DoD nozzle meniscus, which are often associated with (extended) non-printing periods[21], rather low printing frequencies[22], or an unmerged satellite[23]. Although the MPT model predictions are inherent to DoD inkjet printing, they could be completely masked or distorted by conditions for fluid jetting and the complicated drive waveforms intended to reduce the effect of residual oscillations on steady state printing. Although response changes are completely predictable for N-burst printing, using any waveform, there has not been previous mention of their existence in the inkjet printing literature.

In contrast to this, the phase angle to represent the release time of a drop within the cycle relevant to the excitation of a wave response in DoD printing has been analysed in some detail previously.[3] In the ($\omega_0 t^*$) phase nomenclature of Dijksman and Pierik[3], for sine waves rather than the cosine waves of the present work, the release for drop jetting speeds above thresholds imposed by the surface tension was stated to be ($\omega_0 t^*$)=2π/3. This should be equivalent to φ=($\omega_0 t^*$)-π/2=π/6=30°.

The worst case scenario envisaged is that the phase angle only applies to the timing of the response on the cosine wave but NOT on the exponential damping. Slightly weaker effects arise if the phase applies to the cosine wave AND the exponential damping term. [Both are considered in Appendix 1.] Comparisons with real response data showed only minor effects from these different phase angles.

Figure 9

Figure 9. Comparison of the time evolution of residual waves for phase $\phi=0°$ and phase $\phi=30°$, for $f_H=240kHz$ and Q=6.

Figure 10

Figure 10. Comparison of the steady state printing responses R(f) and R(f, ϕ) where phase $\phi=30°$, for $f_H=240kHz$ and Q=6. See text.

Figure 11

Figure 11. The shape of the correction Y(f,ϕ,N) after N-burst printing (N=2-10) and phase $\phi=30°$, for $f_H=240kHz$ and Q=6. See text.

Figure 9 shows the time evolution of residual waves for zero phase and for this finite phase ϕ value and Figure 10 shows the impact of either choice on the steady state printing response R(f) and R(f,ϕ) respectively. The differences between these cases are primarily shifts towards (a) higher frequency sub-harmonic peaks, (b) larger swings, and (c) higher asymmetry around these frequency peaks, and in particular steeper inclines and more gradual falls as the frequency is swept over all these peaks. The similarities are mainly in the "flat-background" region, which for $\phi=0°$ was < 40 kHz, although a finite phase angle $\phi =30°$ has slightly reduced the "flat background" region due to (b) and (c) above.

Figure 11 confirms that the shape of the correction Y(f,ϕ,N) to steady state printing response R(f) rapidly approaches steady state correction shape Y(f,ϕ), arising from the finite phase angle $\phi =30°$, and the number of pulses N increases from 2 to 10. In Figures showing frequency sweeps the single pulse (N=1) response is omitted because there can be NO frequency spectrum without a repetition!

Conclusions

Multi-pulse train (MPT) model predictions for burst and steady-state piezo-DoD inkjet printing at a fixed frequency are given in both cases by exact but quite straightforward analytical results assuming impulse responses for each pulse and linear summation of wave amplitudes. Extensions of the MPT model results to far more complex waveforms, such as commonly used for such DoD print-heads, involve combining further but similar terms and (as usual) taking into account their relative phase.

Some key assumptions underlying the use of a simple impulse response for the MPT model have been examined by comparison with other published model approaches to computing residual wave amplitudes and phases. In particular, phase angle effects cause a frequency-dependent modification of the MPT model which only appear for higher frequency jetting, and appear quite small provided the drop speed is well above the threshold for droplet production. [See Appendix]

Drop measurement data for steady state printing, recorded over a range of frequencies by research teams using different types of resonant piezo-DoD print-head driven by simple unipolar waveforms, are well described by MPT with just 2 adjustable parameters: natural frequency and quality factor.

Changes (in jetted droplet speed and volume) between the first (few) drop(s) and later continuously printed drops appears in the MPT model as a "first drop" effect unrelated to the evaporation of ink or other effects on the fluid in or on the resting meniscus at the nozzle exit immediately beforehand.

Self-consistent data analyses using MPT showed that loaded fluid properties can alter DoD printing frequency spectra, by reducing the effective resonant frequency and Q, although additional effects on Q arising from significant viscosity changes were not detected. The direct application of the simple MPT model response for a unipolar "push" waveform driving a resonant piezo-DoD inkjet print-head to jet particle loaded fluids, gave a quantitative understanding of the jetting data[16].

An additional (unexpected?) resonant response in a measured frequency sweep[24] was identified by linearly combining the MPT model predictions for the major peaks with an independent MPT model term for the additional response, thus enabling this to be characterised for future research studies.

Higher frequency jetting response data obtained using simple waveforms were also fitted by MPT model predictions. The results of comparisons between different piezo-DoD inkjet print-heads can also assist with future product design, formulation of waveforms for improved performance (drop speed, drop volume and frequency range) and indications of jetting sensitivity to fluid properties.

Appendix

The MPT response $R(f,\phi,N)$ that corresponds to residual waves out-of-phase by the angle ϕ (which may itself be a function of frequency f) requires an evaluation of $\sum[\cos(\phi+an/f)\exp(-bn/f)]$ over the usual range of n from 0 to N-1. For reasons of space, the full derivation for the steady state printing response $R(f,\phi)$ is shown, whereas the final result for the burst printing response $R(f,\phi,N)$ is quoted.

As before, substitute for the n-pulse term $z^n=\exp(-nb/f+jna/f)$ getting complex $\sum z^n=1/(1-z)$, but then combine $(1-z^*)$ and the complex multiplier $\exp(j\phi)$ accounting for the phase angle, before taking the real part and dividing the result by $D(f)$ and $\cos(\phi)$ to find $R(f,\phi)$. This result is phase-shifted from the usual $R(f)$ since $R(f,\phi)=R(f)\{1-\tan(\phi)\exp(-b/f)\sin(a/f)/[1-\exp(-b/f)\cos(a/f)]\}$ equals $R(f)$ only when $\phi=\pi m$ (m=0,1…). Phase angles given by $\phi=(m+\frac{1}{2})\pi$ for m=0,1… are unphysical because they correspond to adding up residual waves of zero magnitude [since $\exp(-bt)\sin(at)=0$ at t=0] following each pulse. Such waves are irrelevant anyway, because the observed DoD inkjet response (drop momentum, speed and volume) is well-modelled by others[4,18] to correspond with a phase $|\phi|<\pi/2$.

The N-pulse burst frequency spectrum is found similarly by forming $R(f,\phi,N)=R(f)+Y(f,\phi,N)$, with $Y(f,\phi,N)= -(\tan(\phi)e^{-b}\sin(a)+e^{-Nb}\{\cos(Na)-e^{-b}\cos((N-1)a)-\tan(\phi)[\sin(Na)-e^{-b}\sin((N-1)a)]\})/D(f)$, and where the a & b variables in the function arguments were substituted for A/f and B/f, respectively.

Damping term e^{-Nb}{} within $Y(f,\phi,N)$ for $f <$ "flat bandwidth frequency" reduces below $e^{-\pi} = 4.32\%$ as N increases above ca. Q pulses, irrespective of the phase angle ($0 \leq \phi < \pi/2$). However any non-zero ϕ introduces additional frequency distortions to the simplest form $R(f)$, derived assuming phase $\phi=0$, even in the limit of steady state printing, through additional oscillating term $- \tan(\phi)e^{-b}\sin(a)/D(f)$.

The phase angle ϕ can be ANY function of frequency, but provided ϕ remains constant during a particular printing burst, the general result $R(f,\phi,N)$ will hold. Since this should also remain true for steady state printing, predicted frequency sweep shapes can be compared with observed response to deduce changes of phase angle as a function of frequency.

Print bursts of int(Q) drops below flat bandwidth frequency are sufficient to approach the simplest form $R(f)=\{1-\exp(-b/f)\cos(a/f)\}/D(f)$ for DoD frequency response first presented at NIP31.[1] Ignoring phase angle ϕ also leads to the N-burst frequency response derived before[1]. This N-burst response for zero phase shift can be expressed in the form $R(f,N)=R(f)\{1-\exp(-NB/f)(\cos(Na)-e^{-b}\cos((N-1)a))\}$.

Acknowledgements

References

1.   S.D. Hoath, "A Simple Model For DoD Inkjet Frequency Response", Proc. IS&T's 31st Intl. Conference on Digital Printing Technologies (IS&T, Springfield, VA 2015) pp. 8-12

2.   T. Bogy and F. Talke, "Experimental and theoretical study of wave propagation phenomena in drop-on-demand ink jet devices," *IBM J. Res. Dev.*, 28, 314–321 (1984).

3.   J.F. Dijksman, "Hydrodynamics of small tubular pumps," J. Fluid Mech., 139, 173–191 (1984).

4.   J.F. Dijksman and A. Pierik, Inkjet Printing Technology for Digital Fabrication (I.M. Hutchings and G.D. Martin (Eds), Wiley, 2013) Ch. 3 "Dynamics of Piezoelectric Print-Heads", pp. 65-66.

5.   J.R. Castrejón-Pita, S.D. Hoath, A.A. Castrejón-Pita, N.F. Morrison and I.M. Hutchings, "Time-resolved Particle Imaging Velocimetry within the Nozzle of a Drop-on-Demand Print-head", J. Imaging Sci. Technol. 56, 1-6 (2012).

6.   M.-J. van der Meulen, H. Reinten, F. Dijksman, D. Lohse and M. Versluis, "Experimental methods for retrieving flow information from within inkjet nozzles", J. Imaging Sci. Technol. 61?, (2016).

7.   H. Wijshoff, "The dynamics of the piezo inkjet print-head operation", Physics Reports 491, 77-177 (2010).

8.   A. Khalate, X. Bombois, R. Babuška, H. Wijshoff, and R. Waarsing, "Performance improvement of a drop-on-demand inkjet print-head using an optimization-based feedforward control method," Control Engineering Practice, 19, 771–781 (2011).

9.   A. Khalate, X. Bombois, G. Scorletti, R. Babuška, S. Koekebakker, and W. de Zeeuw, "A Waveform Design Method for a Piezo Inkjet Print-head based on Robust Feedforward Control", IEE, J. Microelectromechanical Systems, 99, 1-10 (2012).

10.   N. Morita, A. Khalate, A. van der Buul, and H. Wijshoff, Fundamentals of Inkjet Printing: The Science of Inkjet and Droplets (S.D. Hoath (Ed.), Wiley-VCH 2016) Ch. 3

11. J.F. Dijksman, P.C. Duineveld, M.J.J. Hack, A. Pierik, J. Rensen, J.E. Rubingh, I. Schram and M.M. Vernhout, "Precision inkjet printing of polymer light emitting displays", J. Mater. Chem. 17, 511-522 (2007).

12. A. Khalate, X. Bombois, G. Scorletti, R. Babuška, S. Koekebakker, and W. de Zeeuw, "A Waveform Design Method for a Piezo Inkjet Print-head based on Robust Feedforward Control", J. Microelectromechanical Systems 21, 1365-1374 (2012).

13. I. Reinhold, private communications (2015).

14. J.R. Ragazzini and L.A. Zadeh, The analysis of sampled-data systems, Trans. Am. Inst. Elec. Eng. 71 (II): 225–234 (1952).

15. University of Cambridge, Department of Engineering, Mechanics Data Book (2000), pp6-12; 4.6 Harmonic response of a linear second-order system 4.6.3 case (c).

16. C. Ainsley, N. Reis and B. Derby, Freeform fabrication by controlled droplet deposition of powder filled melts, J. Mater. Sci. 37, 3155-3161 (2002); B. Derby and N. Reis, Inkjet Printing of Highly Loaded Particulate Suspensions, MRS Bull., 28 (11), 815–818 (2003).

17. S.D. Hoath, W.-K. Hsiao, S. Jung, G. Martin, I.M. Hutchings, N.F. Morrison, and O.G. Harlen, Drop Speeds from Drop-on-Demand Ink-Jet Print Heads, J. Imaging Sci. Technol., 57, 10503-1-10503-11 (2013).

18. T. Cruz-Uribe and M. Dorrestijn (private communications).

19. A reviewer's comment.

20. Theo Driessen and Roger Jeurissen, in Stephen D. Hoath (Ed), Fundamentals of Inkjet Printing: The Science of Inkjet and Droplets (Wiley-VCH, Weinheim, 2016). Chapter 4

21. H. Dong, W.W. Carr, and J.W. Morris, Visualization of drop-on-demand inkjet: Drop formation and deposition, Rev. Sci. Instrum., 77, 085101 (2006).

22.   K.-S. Kwon, H.-S. Kim, and M. Choi, Measurement of inkjet first-drop behavior using a high-speed camera, Rev. Sci. Instrum., 87, 035101 (2016)

23.   A. Famili, S.A. Palkar, and W.J. Baldy, First drop dissimilarity in drop-on-demand inkjet devices, Phys. Fluids, 23, 012109 (2011).

24.   G.D. Martin, unpublished analysis of holographic images. See also Kye-Si Kwon, Lisong Yang, Graham D. Martin, Rafael Castrejón-Garcia, Alfonso A. Castrejón-Pita and J. Rafael Castrejón-Pita, in Stephen D. Hoath (Ed), Fundamentals of Inkjet Printing: The Science of Inkjet and Droplets (Wiley-VCH, Weinheim, 2016). Chapter 12. Visualisation and Measurement, pp 320-325.

25.   K.-S. Kwon, M.-H. Jang, H.Y. Park, and H.-S. Ko, An inkjet vision measurement technique for high-frequency jetting, Rev. Sci. Instrum., 85, 065101 (2014). Figure 18.