# 1 **Natural Giesekus fluids: shear and extensional behaviour of food**  2 **gum solutions in the semi-dilute regime**

- 3 M.D. Torres · B. Hallmark ·L. Hilliou· D.I. Wilson
- 4

### 5 **Abstract**

6 The shear and extensional behaviour of two aqueous gum solutions, namely (i) 1-20 g/L guar gum <sup>1</sup> and (ii) κ/ι-hybrid carrageenan solutions (5-20 g/L), are shown to exhibit Giesekus-fluid behaviour 8 when in the semi-dilute regime. In this regime a common set of Giesekus fluid parameters described 9 both shear and extensional behaviour. A new analytical result describing the extension of a Giesekus 10 fluid in the filament stretching geometry is presented. This also gave reasonable predictions of the 11 Trouton ratio. Higher concentration guar solutions, in the entangled regime, yielded different 12 Giesekus fluid parameters for extension to those for simple shear. The extensional data for all 13 concentrations of both gums collapsed to a common functional form, similar to that reported for cake 14 batters<sup>2</sup>; the limits of the new filament thinning expression provide insight into this behaviour.

- 15
- 16 **Keywords** Concentration regimes · Extensional · rheology · Guar gum · Giesekus fluid · κ/ι-
- 17 hybrid carrageenan gum · Shear rheology · Trouton ratio

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M.D. Torres

L. Hilliou

D.I. Wilson

Department of Chemical Engineering and Biotechnology, New Museums Site, University of Cambridge, Pembroke St, Cambridge, CB2 3RA, UK.

Department of Chemical Engineering, University of Santiago de Compostela, Lope Gómez de Marzoa St, Santiago de Compostela, E-15782, Spain.

B. Hallmark  $(\boxtimes)$ 

Department of Chemical Engineering and Biotechnology, New Museums Site, University of Cambridge, Pembroke St, Cambridge, CB2 3RA, UK. e-mail address: bh206@cam.ac.uk

Institute for Polymers and Composites/I3N, University of Minho, Campus de Azurém, 4800-058, Guimarães, Portugal.

Department of Chemical Engineering and Biotechnology, New Museums Site, University of Cambridge, Pembroke St, Cambridge, CB2 3RA, UK.

### 19 **Introduction**

20 Many liquid-based foods exhibit complex rheological behaviour as a result of their multi-phase 21 nature; examples include emulsions (liquid-liquid), dense suspensions (solid-liquid), polymer 22 solutions, bubbly liquids and foams (both gas-liquid). The presence of a long-chain polymer or units 23 of dispersed phase gives rise to a wide range of microstructures and deformation behaviours<sup>1</sup> that are 24 desired in the product. The use of such materials is increasing, both in the design and formulation of 25 new products and in supporting the replacement of traditional components such as fat and sugars in 26 existing formulations.

27

28 Rheological testing is regularly employed to establish and quantify the deformation behaviour of food 29 materials for materials characterisation and quality control purposes. It is also required for process 30 engineering studies of food systems, where rheological measurements are allied with computational 31 fluid dynamics (CFD) simulations of equipment in the design of new equipment and/or to confirm 32 whether a new formulation can be processed on an existing multi-product line. Identification of the 33 appropriate constitutive equation, and its associated parameters, is a critical step in this process. Food 34 processing devices, such as filling nozzles and mixers, impose a mixture of linear and extensional 35 shear on the material <sup>2</sup> and it is important that the constitutive equation used in the CFD calculations 36 represents both modes of deformation properly.

37

38 Whereas devices for measuring shear viscosity have been available for a long time, commercial 39 instruments for measuring extensional viscosities accurately and independently of shear contributions 40 are relatively new. Extensional rheometers, such as the CaBER, the FiSER and the Cambridge 41 Trimaster monitor the necking of extensionally-strained fluid filaments as a function of time, which  $42$  allows rheological parameters to be calculated<sup>3</sup>. In the absence of direct measurements, CFD 43 calculations have had to make assumptions about the relationship between extensional and shear flow 44 behaviour, such as the magnitude of the Trouton ratio, *Tr*, which is the ratio of the extensional and 45 shear viscosities at a given shear rate<sup>4</sup>. For a Newtonian fluid in uniaxial extension,  $Tr = 3<sup>4</sup>$ . The value

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46 of *Tr* is rarely known *a priori* for complex foods as these are usually non-Newtonian and the 47 microstructural features that give rise to non-Newtonian behaviour will give rise to different 48 dependencies on shear rate<sup>1</sup>. It is therefore important to establish whether there exist any cases of non-49 Newtonian behaviour which fit a non-Newtonian constitutive equation for both linear and extensional 50 shear behaviour, so that measurements of one deformation mode can be used to predict the other with 51 reasonable confidence, thereby reducing the amount of rheological testing.

52

53 This paper demonstrates that the Giesekus constitutive equation  $<sup>5</sup>$ , which was originally developed to</sup> 54 describe the shear behaviour of polymer solutions, gives a very good description of the shear and 55 extensional rheology of aqueous guar gum and κ/ι-hybrid carrageenan gum solutions. One of the 56 attractive features of the Giesekus equation is that it is available as standard in many commercial CFD 57 software packages.

58

59 Guar gum is a galactomannan and one of the most cost effective natural neutral hydrocolloids due to 60 its ready availability and ease of manufacture by extraction from *Cyamopsistetragonolobus* seeds. 61 This long-chain polysaccharide biopolymer is highly polydisperse, has a semi flexible random coil 62 conformation composed of a linear mannan backbone bearing side chains of a single galactose unit, 63 and contains a mannose to galactose ratio of  $\sim$  1.6-1.8:1. Guar gum is widely used in food and other 64 applications as a thickener and rheology modifier. Aqueous solutions are shear thinning, and several 65 studies  $6-9$  have demonstrated that its non linear shear rheology can be described very well by the 66 . Cross model <sup>10</sup>. The Cross model, however, is a generalised Newtonian model and hence does not 67 provide an *a priori* estimate of extensional behaviour.

68

69 Carrageenans are natural linear polysaccharides extracted from red seaweeds (*Gigartinales,*  70 *Rhodophyta*) and are extensively used as thickeners, gelling, texturing, suspending or stabilising 71 agents<sup>11</sup>. Most carrageenophyte seaweeds produce  $\kappa/\iota$ -hybrid carrageenans, and alternative algal 72 resources for carrageenan production are needed to cope with the steadily increasing demand for food

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 $73$  texturing agents<sup>11</sup>. They are natural polyelectrolyte copolymers comprising blocks of κ or *ι*-74 carrageenan disaccharide units randomly distributed along the chain, together with minor amounts of 75 non-gelling disaccharide units (biological precursors). The relative amount of κ or ι-depends on the 76 plant biology and the biopolymer extraction procedure. The chemical structure of κ/ι-hybrid 77 carrageenans has a direct impact on its gel properties as reported recently  $12-14$ .

78

The extensional rheology of guar gum solutions has been studied widely $8,15,16$ , whereas there are no 80 studies to date of aqueous κ/ι-hybrid carrageenan solutions. Bourbon *et a*l. employed a CaBER device 81 to study a narrow range of guar gum concentrations (0.39-0.97  $g/L$ ) and compared their results with FENE 82 constitutive models. We have recently presented measurements <sup>1</sup> of the extensional rheology of guar 83 gum solutions obtained using the Cambridge Trimaster filament stretching device  $17$  over a wider 84 range of concentrations (1-20 g/L), crossing the transition from the semi-dilute to the entangled 85 regime. It will be shown that the Giesekus equation provides a good description of both linear and 86 extensional shear behaviour of both these data sets and new sets obtained for κ/ι-hybrid carrageenan 87 solutions.

88

89 The Giesekus model has been widely used to describe the shear response of solutions of synthetic 90 . polymers  $18,19$ . It has received little attention in the literature on food polymer solutions, even though it 91 is able to account for the both the low and high shear rate plateau viscosities and the shear-thinning 92 behaviour reported for many viscous food liquids. It has also been used to describe the extensional 93 viscosity of synthetic polymers  $3,20,21$ . Anna and co-workers  $3$  fitted a multi-mode Giesekus model to 94 data obtained from filament stretching of Boger fluids consisting of high molecular weight 95 polystyrene dissolved in styrene oil: this required the solution of a set of coupled ordinary differential 96 equations. This paper presents the derivation of a simple analytical expression that predicts the time-97 dependent viscoelastic filament thinning of a single-mode Giesekus fluid. This is found to give a good 98 description of the experimental data obtained for the guar gum and κ/ι-hybrid carrageenan gum 99 solutions. We are not aware of this result being presented previously elsewhere. It allows the

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100 extensional behaviour to be compared with that expected from (linear) shear measurements. The 101 ability to predict extensional behaviour from shear flow measurements is tested and shown to depend 102 on whether the solution is semi-dilute and unentangled or whether the solution exists in the entangled 103 regime.

104

### 105 **Experimental**

106 Sample preparation

107 Commercial guar gum was supplied by Sigma-Aldrich (batch no. 041M0058V, India). Theκ/ι-hybrid 108 carrageenan gum was extracted from *M*. *stellatus* seaweed using the method described by Hilliou and 109 co-workers <sup>13</sup> Aqueous solutions of guar gum (reported by Torres and co-workers<sup>1</sup>) and κ/ι-hybrid 110 carrageenan gum over a wide range of concentrations (1-20 g/L) were prepared following procedures 111 established in the literature<sup>7,12</sup>. The guar gum was dispersed in tap water by stirring at 1400 rpm 112 overnight on a magnetic hotplate stirrer (VMS-C4 Advanced, VWR, UK) at room temperature, 113 between 19 °C and 21 °C, to ensure complete hydration of the gums. The aqueous κ/ι-hybrid 114 carrageenan gum solutions were prepared by dissolving the gum in NaCl 0.1 M solutions, in order to 115 fix the ionic strength. Some air was incorporated into the solution during stirring and deaerated 116 samples of the continuous phase were obtained by centrifugation at 2250 rpm (500 *g*) for 5 min. All 117 samples were, at minimum, duplicated.

118

### 119 Rheology measurements

120 Steady shear measurements were performed on a Bohlin CVO120HR controlled-stress rheometer 121 (Malvern Instruments, Malvern, UK) using sand-blasted parallel plates (25 mm diameter and 1 mm 122 gap) to prevent wall slippage. The normal force generated by the flow between plates was also 123 measured. Measurements of axial thrust were used to estimate the normal stress difference,  $N_1 - N_2^4$ ; 124 calibration of this device indicated that reliable axial thrust data could be acquired for thrusts greater than  $9.8\times10^{-3}$  N. Samples were loaded carefully to ensure minimal structural damage, and held at rest 126 for 5 min before testing to allow stress relaxation and temperature equilibration. A thin film of a

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127 Newtonian silicone oil (viscosity 1 Pa s) was applied to the exposed sample edges to prevent 128 evaporation. All measurements were made under isothermal conditions (20 ºC) and, at minimum, 129 duplicated.

130

131 Extensional measurements were investigated using the Cambridge Trimaster, a high-speed filament 132 stretch and break-up device  $17$ . The apparatus consists of two cylindrical 1.2 mm diameter stainless 133 steel stubs which are moved vertically apart at speed with high spatial precision. Measurements 134 reported here featured an initial gap spacing of 0.6 mm, final gap spacing of 1.5 mm, and piston 135 separation speed of 75 mm  $s^{-1}$ . The initial filament diameter,  $D_1$ , was measured once the stainless steel 136 stubs had reached their final position; this parameter was strongly dependent on the fluid properties 137 and was often slightly different for each experiment. The filament stretching and thinning profiles 138 were monitored using a high speed camera (Photron Fastcam 1024 PCI) which allows the diameter of 139 the filament midpoint to be measured to within 1 µm at a rate of 6000 frames per second. The device 140 did not feature a force transducer so separating forces were not recorded. All experiments were 141 performed at least in duplicate in an air-conditioned room at 20 °C. Error bars corresponding to 142 experiment variation of repeated steady shear and extensional tests are plotted where the measurement 143 uncertainty was greater than the symbol size. Further information about the used protocols can be 144 found in a previous publication  $\frac{1}{1}$ .

145

146 **Theory** 

- 147 Giesekus model for shear behaviour
- 148 The total stress in a fluid, **σ**, is written as
- 149  $\sigma = -p\mathbf{I} + \tau$

 $150$  (1),  $(1)$ ,  $(2)$ 151 where  $p$  is the hydrostatic pressure and **I** the identity tensor. In the Giesekus constitutive equation  $5.22$ ,

152 the shear stress, **τ**, is modelled as

153 
$$
\frac{\tau}{\lambda} + \frac{\partial \tau}{\partial t} + \mathbf{v} \nabla \tau - ((\nabla \mathbf{v})^T \tau + \tau (\nabla \mathbf{v})) = \frac{\eta_0}{\lambda} \dot{\gamma} - \frac{a}{\eta_0} \tau \cdot \tau
$$

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159

$$
\dot{\gamma} = (\nabla \mathbf{v}) + (\nabla \mathbf{v})^{\mathrm{T}}
$$

 $161$  (3).

162 Expressions that describe the shear rate dependence of the shear viscosity as a function of shear rate 163 for a single-mode Giesekus fluid have been presented in the literature<sup>5</sup>. These expressions have been 164 successfully applied to describe the flow curve of aqueous solutions of rod-like micelles for the 165 surfactant system cetyltrimethylammonium<sup>20</sup>, and are used here to describe the steady, one-166 dimensional, shear response of guar gum and κ/ι-hybrid carrageenan gum solutions of varying 167 concentrations. In summary,

168 
$$
\eta(\dot{\gamma}) = \frac{\eta_0(1-n_2)}{1+(1-2a)n_2}
$$

 $169$  (4).

170 Here the dimensionless terms *n*<sub>2</sub> and Λ are given by

171 
$$
n_2 = \frac{1 - A}{1 + (1 - 2a)A}
$$

 $172$  (5),

173 
$$
A = \sqrt{\frac{\sqrt{1 + 16a(1 - a)\lambda^2 \dot{y}^2} - 1}{8a(1 - a)\lambda^2 \dot{y}^2}}
$$

 $174$  (6).

175 For steady shear at steady shear rate, Eq. (2) implies that first normal stress difference, *N*1, is of the 176  $form^{24,25}$ 

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177 
$$
N_1 = 2\lambda \eta_0 \frac{n_2(1 - a n_2)}{\lambda^2 a (1 - n_2)}
$$

 $178$  (7).

179 Over some concentration ranges, the second normal stress difference,  $N_2$ , has been observed to 180 be $N_2 \sim N_1 / 10^{26}$ . For this work, however, the analytical form of the second normal stress difference 181 for a Giesekus fluid was used<sup>25</sup>

182 
$$
N_2 = \frac{-aN_1}{2} \frac{(1-n_2)}{(1-an_2)}
$$

 $183$  (8).

184 Equations (7) and (8) were then used to estimate the magnitude of the stress difference,  $N_1$  -  $N_2$ , 185 measured in the steady shear experiments.

186

#### 187 Derivation of a simple expression for the extensional behaviour of a Giesekus fluid

188 The derivation of the extensional thinning of a filament of cylindrical thread of a Giesekus fluid follows similar lines to that of Entov and Hinch<sup>27</sup>, in their analysis of the filament breakup of dilute 190 polymer solutions using a finitely extensible nonlinear elastic (FENE) model <sup>28</sup>. Other authors, notably 191 the McKinley group at MIT, have derived simple expressions that describe viscoelastic filament 192 breakup using the FENE-P model<sup>3</sup> along with models involving coupled ordinary differential 193 equations that describe the filament breakup of Giesekus fluids  $29$ . This section presents the derivation 194 of a simple expression that predicts time-dependent viscoelastic filament thinning of a single-mode 195 Giesekus fluid.

196

197 The first simplification made is to assume that extensional deformations only occur in the axial and 198 radial directions within the liquid filament, and that the divergence of the extra stress tensor is zero. 199 This allows Equation (2) to be simplified to a pair of ordinary differential equations, which are a 200 function of the axial and radial components of the stress within the filament, *viz*.

201 
$$
\frac{\tau_{zz}}{\lambda} + \frac{\partial \tau_{zz}}{\partial t} - \tau_{zz}\dot{\gamma}_{zz} = \frac{\eta_0}{\lambda}\dot{\gamma}_{zz} - \frac{a}{\eta_0}\tau_{zz}^2
$$

 $202$  (9),

203 
$$
\frac{\tau_{rr}}{\lambda} + \frac{\partial \tau_{rr}}{\partial t} - \tau_{rr}\dot{\gamma}_{rr} = \frac{\eta_0}{\lambda}\dot{\gamma}_{rr} - \frac{a}{\eta_0}\tau_{rr}^2
$$

$$
204
$$
 (10).

205 The boundary conditions on the liquid filament assume that any end effects, where the liquid filament 206 may contact a surface, are negligible resulting in zero axial stress. This assumption is commonly used 207 when deriving expressions for extensional thinning<sup>30</sup> but strictly speaking requires experimental 208 validation. Alternative assumptions can include an exponential increase in normal stress for 209 viscoelastic fluids<sup>31</sup>. Furthermore, it is assumed that surface tension,  $\alpha$ , alone is responsible for the 210 radial stress that causes the filament to thin, with the radial stress being assumed to be equal to the 211 Laplace pressure. These boundary conditions can be derived from Equation (1) and give

$$
\sigma_{zz} = 0 = -p + \tau_{zz}
$$

$$
213 \tag{11}
$$

$$
\sigma_{rr} = -p + \tau_{rr} = -\frac{2\alpha}{D}
$$

 $215$  (12).

216 Equation (11) allows the unknown hydrostatic pressure, *p*, to be calculated, which can then be 217 substituted into Equation (12); this leads to an expression that relates the radial and axial components 218 of the stress to the surface tension and to the filament diameter. It is now assumed that the axial 219 extensional stress is the dominant stress in the problem giving the following relationship between this 220 stress, the surface tension and filament diameter, *D*.

$$
\tau_{zz} \approx \frac{2\alpha}{D}
$$

 $222$  (13).

223 In order to calculate the variation of the filament diameter as a function of time, the strain rate in the 224 radial direction can be related to the change in velocity in the radial direction velocity, *i.e.*

$$
\gamma_{rr} = 2 \frac{\partial \mathbf{v}_r}{\partial r}
$$

226 (14). 227 This, in turn, can then be related to the filament diameter, giving <sup>2</sup> . 4 1 2 2 2 *rr r D D r t D t D t* γ ∂ ∂ ∂ ∂ ∂ = = = ∂ ∂ ∂ ∂ ∂ ∂ <sup>228</sup> 229 (15). 230 Integration of Equation (15) with respect to the filament diameter yields a simple ordinary differential 231 equation in terms of radial direction shear rate; . d d 4 *<sup>D</sup> zz <sup>D</sup> t* γ 232 = − 233 (16). 234 The radial and axial components of the strain rate can then be related by remembering that the 235 deformation is a uniaxial extension. The expressions for axial extension rate, Equation (16), and 236 approximate axial tensile stress, Equation (13), can now be substituted into the axial component of the 237 Giesekus expression, which was shown in Equation (9). The resulting expression is 2 0 2 2 0 2 d 1 8 d d 4 4 2 dt d d *D D a D D D t D t D* <sup>α</sup> <sup>α</sup> <sup>α</sup> <sup>η</sup> α <sup>λ</sup> <sup>λ</sup> <sup>η</sup> − + + = − <sup>238</sup> 239 (17).

240 Some manipulation and reorganisation of the terms in Equation (17) result in a single, non-linear, 241 ordinary differential equation that relates the rate of change of filament diameter to the physical 242 properties of the fluid:

243 
$$
\frac{dD}{dt} = -\alpha \left( \frac{\eta_0 D + 2a\lambda \alpha}{2\eta_0 D + 3\alpha \lambda} \right)
$$

$$
244 \tag{18}
$$

245 Integration of Equation (18), subject to the initial condition that at  $t = 0$  the initial filament diameter 246 is $D_1$ , yields

247 
$$
\frac{\alpha\lambda(3-4a)}{\eta_0}\ln\left(\frac{\eta_0 D + 2a\lambda\alpha}{\eta_0 D_1 + 2a\lambda\alpha}\right) + 2(D-D_1) = \frac{-\alpha t}{\eta_0}
$$

 $248$  (19).

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249 An analogous derivation for an upper convected Maxwell (UCM) fluid<sup>32</sup> makes the assumption that 250 viscous stresses are negligibly small when compared to elastic stresses. For the condition that both 251  $a = 0$  and  $\eta_0 = 0$ , Equation (19) simplifies to the expected UCM result for extensional filament thinning where  $D'_{D_1} = \exp(-t'_{3\lambda}).$ 252 thinning where  $D'_{D_1} = \exp(-t/3\lambda)$ . Equation (19) relates the filament diameter implicitly to time, 253 making it more straightforward to calculate the variation in time as a function of a given filament 254 diameter, rather than *vice versa*. Equation (19) can also be expressed in non-dimensional form as

255 
$$
(4a-3)\ln\left(\frac{(D_{D_1})+2a\lambda\alpha}{1+2a\lambda\alpha}\right)\frac{2\eta_0D_1}{\alpha\lambda}\left(D_{D_1}-1\right)=\frac{t}{\lambda}
$$

256

 $257$  (20). 258 Equation (20) highlights that there are two contributions to the characteristic timescale for the thinning 259 of the liquid thread. The first term on the left hand side contains the Giesekus mobility parameter, 260 which relates to the anisotropic relaxation of the polymer chains. The second term, which would also 261 arise from the Upper Convected Maxwell constitutive equation, contains terms that relate to the 262 longest relaxation time, zero shear rate viscosity and surface tension alone.

263

264 In the absence of direct measurements of extensional force (as in the case of the Trimaster device used to collect the extensional data presented here), it is possible to estimate the extensional viscosity,  $\eta_E$ , 266 of a fluid undergoing a uniaxial extension using  $17$ .

267 
$$
\eta_E = (2X - 1) \frac{-\alpha}{dD/dt}
$$
  
268 (21).

269 In this expression, *X* is a coefficient that accounts for the deviation of the filament shape from a 270 uniform cylinder due to inertia and gravity; in other studies this has been assumed to be roughly 0.7  $271$   $17,33$ . Combining Equation (18) with Equation (21) yields the following estimate of the extensional 272 viscosity of a Giesekus fluid:

$$
\eta_{E} = \eta_{0} \left( 2X - 1 \right) \frac{3 + 2\eta_{0} D/\alpha \lambda}{2a + \eta_{0} D/\alpha \lambda}
$$

 $274$  (22).

275 Equation (22) allows the extensional viscosity to be estimated relatively simply for a Giesekus fluid. 276 Other methods<sup>34</sup> of estimating the extensional viscosity for FENE fluids are described in the literature; 277 these involve interpolation between the asymptotic and perturbation solutions that respectively 278 describe the limiting cases of extensional viscosity a large and small extension rates.

### 279 Statistical analysis

280 The parameters of the models considered were determined from the experimental data with a one-281 factor analysis of variance (ANOVA) using PASW Statistics (v.18, IBMSPSS Statistics, New York, 282 USA). When the analysis of variance indicated differences among means, a Scheffé test was 283 performed to differentiate means with 95% confidence  $(p< 0.05)$ .

284

### 285 **Results and discussion**

#### 286 *Concentration regimes*

287 Both the guar gum and carrageenan solutions were found to be shear thinning and highly viscoelastic,

288 with each property being strongly dependent on concentration. The effect of concentration on the guar 289 gum solutions was investigated in our previous paper  $1$  and the results are summarised here alongside 290 new results for the  $\kappa$ /*ι*-hybrid carrageenan solutions.

291

292 The dynamic response of the guar gum solutions to oscillatory shear testing indicated a transition from 293 being predominantly viscous, at concentrations below  $\sim$  5 g/L to exhibiting a significant elastic 294 response at higher concentrations. This transition was not observed in the  $\kappa/\iota$ -hybrid carrageenan gum 295 solutions, which indicated predominantly viscous behaviour over the frequency range investigated 296 (data not reported) for all concentrations tested. This suggested that the carrageenan solutions were in 297 the semi-dilute regime, while the higher concentrations of guar gum were subjected to significant 298 entanglements.

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300 Another observation suggesting that the threshold of the entangled regime for the guar gum solutions 301 lay near 10 g/L was that the measured surface tension deviated from the Szyszkowski equation  $35$  at 10 302 g/L, indicating that the solution was exhibiting behaviour analogous to that associated with critical  $303$  aggregation concentration  $36$ . This is consistent with other reported studies of guar sum solutions 304 <sup>37,38</sup> In contrast, the concentration dependence of surface tension of κ/ι-hybrid carrageenan gum  $305$  solutions was satisfactorily fitted to the Szyszkowski equation<sup>35</sup> over the whole concentration range 306 studied (see Appendix, Figure A.1).

307

308 In the semi-dilute coil overlap region, the reduced viscosity,  $\eta_{red}$ , is expected to follow the Martin 309 equation<sup>39</sup>,  $viz$ .

$$
\frac{\eta_0 - \eta_s}{c \eta_s} = \eta_{red} = [\eta] \exp(k[\eta]c)
$$

 $311$  (23)

312 where  $\eta_s$  is the solvent viscosity, *c* the solution mass concentration, [ $\eta$ ] the intrinsic viscosity and *k* the 313 Huggins parameter. In this region the coil overlap parameter,  $c[\eta]$ , is expected to lie between  $1 \leq c[\eta]$ 314 <10. The zero shear rate viscosity values were used to estimate  $\eta_{red}$  and are plotted against *c* in Figure 315 1(*a*). The guar gum data fit Equation (23) reasonably for *c*≤ 5 g/L but deviate strongly from the trend 316 at higher concentrations. Linear regression yielded  $\eta$  = 1.09±0.02 L/g and  $k = 0.84\pm0.07$ , which 317 compare favourably with the values for similar gums<sup>40</sup>, namely  $[\eta] = 1.06L/g$ , and  $k = 1.07$  at 24°C 318 and 0.73 at 45°C. Other studies of guar gum and similar materials  $4.7$ ,8 have reported similar behaviour. 319 The κ/ι-hybrid carrageenan gum data showed a good fit to Equation (23) over the whole concentration 320 range studied, giving  $\eta$  = 1.05±0.01 L/g and  $k = 0.34\pm0.03$ . The  $\eta$  values are consistent with those 321 previously reported for  $\kappa$ /ι-hybrid carrageenan gum solutions <sup>41</sup>. Values of  $k$  < 0.5 indicate systems 322 where the solvent-polymer interactions are favoured over polymer-polymer interactions<sup>39,40</sup>. Namely, 323 the polyelectrolyte nature of the κ/ι-hybrid carrageenan gum is mirrored in Figure 1(*b*). The double

<sup>299</sup> 

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324 logarithmic plot confirms the expected power law behaviour for polyelectrolytes polymers 325 approaching the theoretical one,  $\eta_0 / \eta_s \sim k' c^{15/4}$ , for semi dilute-entangled regime of polyelectrolyte 326 solutions<sup>42</sup>, with *k*' values of 0.12.

327

### 328 Shear thinning behaviour

329 The shear flow data for both gums gave a good fit to the Cross model  $1$ for the guar gum data, 330 carrageenan results provided in the Appendix, Figure A.2, but this model does not provide insight into 331 extensional behaviour. Figure 2(*a*) shows that the Giesekus model (Equation (4)) gives a good 332 description  $(R<sup>2</sup> > 0.988)$  of the shear flow data for guar gum solutions in the semi-dilute regime 333 (*c* = 1-5 g/L). There is an increasingly poorer fit in the entangled regime as the concentration increases 334 ( $\geq$  10 g/L). In contrast, Figure 2(*b*) shows that the k/1-hybrid carrageenan gum solutions exhibited 335 Giesekus-fluid behaviour over the whole concentration range studied.

336

337 Equation (4) has three adjustable parameters: the zero shear rate viscosity, the Giesekus mobility 338 parameter and the relaxation time;  $\eta_0$  was taken from the experimental data at the lowest shear rate 339 studied,  $0.01 \text{ s}^{-1}$ . The remaining two parameters, *a* and  $\lambda$ , were fitted to the experimental data by a 340 least squares algorithm and the results are presented in Figure 3. Several workers<sup>43,44</sup> have reported 341 that although the theoretical range of the mobility parameter is  $0 \le a \le 1$ , physically realistic solutions 342 are only obtained for the range 0<*a*<0.5. An upper limit of 0.5 was hence used here.

343

344 The fit in Figure 2(*a*) obtained for higher concentrations for guar gum solutions is arguably less good 345 than at lower concentrations, due to the limitation of having a single relaxation time. Figure 2(*a*) also 346 shows a marked increased in apparent viscosity with concentration. The highest concentration in these 347 tests, 20 g/L, corresponds to a polymer volume fraction of 0.055 and Figure (*a*) shows an increase of 348 six orders of magnitude in  $\eta_0$  for a 20× increase in guar gum concentration. A similar trend is seen 349 with the k/t-hybrid carrageenan, Figure 2(*b*), albeit with a smaller increase (four orders of magnitude) 350 over the concentration range.

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376 *N*2values of κ/ι-hybrid carrageenan gum solutions approached a common value at high shear rates.

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377 Figure 4 also shows that the normal stress differences predicted by Equations (7) and (8), gives very 378 good agreement for the guar gum solutions in the semi-dilute regime and all the κ/ι-hybrid 379 carrageenan gum solutions tested. It should be noted that the lines in Figure 4 are computed using the 380 parameters obtained from fitting the Giesekus model (Equation (4)) to the shear stress data, not the 381 measured normal force. The values of the model parameters  $\eta_0$ ,  $\lambda$  and  $\alpha$  are those given in Table 1.

382

#### 383 Filament stretching behaviour

384 Plots of filament stretching data such as those in Figure 5 exhibit features observed in other studies, 385 namely an initially linear decrease in ln  $D/D_1$  with time followed a steep reduction in  $D/D_1$  as the 386 filament approaches rupture at time  $t_F$ .

387

388 The guar gum and κ/ι-hybrid carrageenan gum data were compared with existing models for 389 extension of single- and multi-mode FENE materials  $3.27$  and gave a poor fit. In contrast, the single-390 mode Giesekus model, Equation (2), gives good agreement for both the guar gum solutions at lower 391 concentrations and all the κ/ι-hybrid carrageenan gum solutions tested. It should be emphasised that 392 lines in Figure 5are computed with Equation (19), using the parameters obtained from fitting the 393 Giesekus model (Equation (4)) to the data from shear experiments (see Figure 2): the values of the 394 model parameters  $\eta_0$ ,  $\lambda$  and  $a$  are those given in Table 1. The values of the liquid-air surface tension, 395  $\alpha$ , in Equation (19) for guar gum are those previously reported<sup>1</sup>, while the values for the  $\kappa$ /ι-hybrid 396 carrageenan solutions were determined for this study using the same protocol (see Appendix, Figure 397 A.1).

398 The agreement between the experimental data and the prediction for  $\kappa/\iota$ -hybrid carrageenan gum 399 solutions prepared at 5 g/L (Figure 5(*a*)) and for the semi-dilute guar gum solutions prepared at 1 g/L 400 and 2 g/L (Figure 5(b)) is excellent ( $R^2$  > 0.990). The filament rupture time, when  $D/D_1$  approaches 401 zero rapidly, is predicted quite well for the 5  $g/L$  case but the evolution in  $D/D_1$  prior to this deviates 402 noticeably from the model. These results indicate that Equation (19) is able to describe the extensional

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403 behaviour of the semi-dilute guar gum solutions with parameters obtained from shear flow tests. The 404 converse is expected to follow, that shear flow behaviour could be predicted using extensional tests. 405 Extensional tests have the advantage of requiring very small volumes of liquids, however shear flow 406 devices are able to assess the influence of phenomena such as wall slip; this could not be predicted 407 from extensional testing alone.

408

409 Figure 5(*c*) shows progressively less good agreement between the experimental data and the model 410 predictions as guar gum concentration increases within the entangled regime. At 10  $g/L$ , where 411 entanglement is apparent, and there is an associated increase in the magnitude of the elastic response  $\frac{1}{2}$ , 412 Equation (19) does not give a good prediction of the extensional behaviour when the parameters 413 obtained from Equation (4) are used. The fit is even poorer with higher concentrations.

414

415 The derivation of Equation (19) involves the assumption that the axial stress dominates over the radial 416 stress; guar gum solutions that have a significant elastic response will also have significant first 417 normal stress differences<sup>1</sup>, possibly invalidating this assumption. This may be the cause of the failure 418 of Equation (19) to fit the data at high concentrations. It may also be the case, however, that the 419 constitutive model needs to be revised in order to capture the phenomena associated with the 420 extension of the guar gum solutions, such as using a multi-mode formulation. The approach taken here 421 was to allow the mobility parameter, *a,* and relaxation time, λ, to differ from the values obtained from 422 the shear flow data in Figure 2.  $D_1$ ,  $\eta_0$ ,  $\eta_\infty$  and  $\alpha$  are not altered. Figure 6 shows that Equation (19) 423 gives a very good fit  $(R^2 > 0.992)$  to the data at 5 g/L and 10 g/L if the mobility parameter and 424 relaxation time are allowed to differ from those obtained from fitting to the linear shear data in Figure 425 2(*a*). The revised values of *a* and  $\lambda$  are reported alongside those obtained from shear flow testing in 426 Table 3. The revised values of *a* are smaller, 0.63 for 5 g/L and 0.92 for 10 g/L, while the relaxation 427 parameter decreases by over an order of magnitude to give similar values (29ms for 5 g/L *cf*. 23ms for 10 g/L). These results are consistent with those previously reported for a wormlike micellar solution<sup>45</sup> 429 and for different commercial thickeners<sup>46</sup>.

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430

431 Comparing the guar gum shear data in Figure 2(*a*) with the extensional data in Figure 6, it can be seen 432 that the extensional response becomes substantially independent of solution concentration at *c*>~10 433 g/L. In contrast, the shear flow apparent viscosity increases by approximately two orders of magnitude 434 between  $c = 10$  g/L and 20 g/L. These results indicate that different physical mechanisms govern the 435 deformation response and it is unlikely that a simple, single-mode, model will cease to describe both 436 behaviours. A multi-mode formulation is likely to be needed for the entangled regime.

437

### 438 Time-concentration superposition and rupture time

439 An alternative means of describing the extensional behaviour of guar gum and κ/ι-hybrid carrageenan 440 gum solutions, seemingly independently of solution regime, is to use time-concentration superposition 441 as previously reported<sup>1,2</sup>. In this approach the non-dimensional filament diameter is plotted against 442 normalised time, where time is normalised by the filament rupture time. The plots demonstrate that 443 κ/ι-hybrid carrageenan gum solutions (Figure 7(*a*)) and guar gum solutions in both the semi-dilute 444 and entangled regime (Figure 7(*b*))collapse onto a single master curve; analogous behaviour has been 445 reported for other biopolymeric solutions, such as cellulose  $47$ , where self-similar filament thinning 446 behaviour is observed when the time is normalised by the polymer relaxation time. The parameter in 447 this semi-empirical model is the filament rupture time, which increases monotonically with gum 448 concentration and is less sensitive to concentration at higher values:  $t_F$  approaches an asymptote in the 449 entangled regime, in a similar fashion to the mobility parameter (see Figure 3(*b*)). Figure 8 shows that 450 the two parameters,  $t_F$  and  $a$ , are closely correlated.

451

452 Insight into this behaviour can be gained from examining Equation (20). As the non-dimensional filament diameter,  $\frac{D}{D_1}$ *D* 453 filament diameter,  $\frac{D}{D_1}$ , tends to zero, the time, *t*, tends to the filament rupture time, *t<sub>F</sub>*, *viz*:

$$
(4a-3)\ln\left(\frac{2aA}{1+2aA}\right) + \frac{2}{A} = \frac{t_F}{\lambda}
$$

 $455$  (25),

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456 where  $0 - 1$ *A D* λα η 457  $A=$  $458$  (26) 459 Close examination of Equation (25) reveals that the filament rupture time, *tF*, actually decreases with 460 increasing mobility parameter but increases with increasing zero shear rate viscosity. The trend shown

461 in Figure 8 can be explained by the fact that both the mobility parameter and zero shear rate viscosity 462 increase with increasing solution concentration; in this case the increase in *tF* due to the increasing 463 solution viscosity outweighs the decrease in *tF* due to increasing mobility parameter.

464

465 This result suggests that the filament rupture time can be estimated from the shear flow tests. This is 466 tested by evaluating the left hand side (LHS) of Equation (25), using the parameters listed in Table 1, 467 and comparing the result with the  $t_F$  values divided by the relaxation time. One could also multiply the 468 LHS by  $\lambda$  to give a comparison of estimated against experimental times. Figure 9 shows that the 469 equality is obeyed for the κ/ι-hybrid carrageenan gum solutions over the entire concentration range 470 investigated, and the guar gum solutions in the semi-dilute regime. The predictive relationship does 471 not hold for the guar gum solutions in the entangled regime.

472

473 Equation (25) also provides a prediction of filament rupture time if the product *aA* is large so that the 474 argument of the log term approaches unity. This gives:

$$
t_F \approx \frac{2\lambda}{A} = \frac{2\eta_0 D_0}{\alpha}
$$

 $476$  (27)

477 Substitution of this result into Equation (20) yields

478 
$$
(4a-3)\ln\left(\frac{\frac{t_F}{\lambda}\left(\frac{D}{D_1}\right)+4a}{\frac{t_F}{\lambda}+4a}\right)-\frac{t_F}{\lambda}\left(\frac{D}{D_1}\right)=\frac{t_F}{\lambda}\left(\frac{t}{t_F}-1\right)
$$

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486 For the case where *A* is large and the mobility parameter *a* = 0.5 (the upper limit suggested in the 487 literature  $43,44$ ), it can be shown that Equation (20) predicts a rupture time longer than that given by 488 Equation (27), namely:

$$
t_F \approx \frac{3\lambda}{A} = \frac{3\eta_0 D_0}{\alpha}
$$

490 (29).

491 For the guar gum solutions tested in the entangled regime, the condition of *aA* being large is not met, 492 hence Equation (27) could not be used to predict the filament rupture time. With  $c = 10$  g/L, however, 493 the mobility parameter  $a = 0.5$  and the product  $aA = 28.5$ ; Equation (29) predicts a rupture time of 494 125ms which compares reasonably with the experimental value of 111ms.

495

496 Trouton ratio

497 The Troutonratio<sup>48</sup> quantifies the relationship between the shear and extensional viscosities. For a 498 uniaxial extensional flow, the Trouton ratio, *Tr*, is given by

$$
\text{Tr} = \frac{\eta_E(\dot{\varepsilon})}{\eta(\sqrt{3}\dot{\varepsilon})}
$$

 $500$  (30),

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501 where the extensional and shear viscosities,  $\eta_E$  and  $\eta$  respectively, are evaluated at the Hencky strain 502 rate,  $\dot{\varepsilon}$ , and shear rate equivalent to  $\sqrt{3}\dot{\varepsilon}$ . The Hencky strain for cylindrical filament thinning can be 503 evaluated from  $17$ 

504

$$
505 \t\t \t\t \varepsilon = 2\ln\left(\frac{D_0}{D(t)}\right)
$$

 $506$  (31).

507 The *Tr* values for the guar gum solutions in the semi-dilute guar gum regime, evaluated at different 508 Hencky strains, are compared with the Giesekus model predictions (calculated using  $\eta_E$  from Equation 509 (21) and  $\eta$  from Equation (4)) in Figure 10. The same comparison for  $\kappa$ /*ι*-hybrid carrageenan gum is 510 shown in the Appendix, Figure A.3. *Tr* is always >3, the value expected for a Newtonian fluid, 511 confirming that studies of the type presented here are required if these materials are to be simulated 512 reliably. There is broad agreement, over four orders of magnitude, between the experimental values 513 and the model predictions. Good quantitative agreement is again evident with small *c*, at the lower end 514 of the semi-dilute regime, and greater deviation evident at the upper end of the semi-dilute regime for 515 guar gum solutions. The qualitative trend is captured for both  $c = 5$  g/L and 10 g/L, and the 516 quantitative differences are close enough to give reasonable estimates. There is significant 517 disagreement between the experimental values of *Tr* and the model predictions for the entangled 518 regime for guar gum solutions, where  $c = 15 \text{ g/L}$  and 20 g/L. The origin of this disagreement can be 519 seen by examining the differences in the values of the mobility parameter and relaxation time 520 according to whether the fitting was carried out with shear or extensional data; these are given for the 521 upper end of the semi-dilute regime in Table 3. When making the transition to the entangled regime, 522 the dynamics of the polymer solution are not sufficiently described by one characteristic timescale 523 alone; both reputation dynamics and unentangled chain stretching are contributory factors. The single-524 mode formulation of the Giesekus fluid used in this work cannot capture both of these contributions, 525 and is hence unable to predict the response of the fluid in the entangled regime. Good quantitative 526 agreement was obtained for the κ/ι-hybrid carrageenan gum solutions (see Appendix, Figure A.3).

527 These results provide confidence in using the expressions developed here as a reasonable starting 528 point to describe both the shear and extensional response of these gum solutions, and possibly other 529 biomacromolecular materials, in the semi-dilute regime.

530

#### 531 **Conclusions**

532 A simple expression is presented describing the extensional behaviour of a Giesekus fluid undergoing 533 filament stretching. This expression, and one for simple shear reported previously by Giesekus<sup>5</sup>, was 534 used to analyse sets of shear and extensional rheological data reported for aqueous solutions of guar 535 gum<sup>1</sup> and new experimental data obtained for  $\kappa$ /ι-hybrid carrageenan gum solutions at concentrations 536 ranging from 1-20 g/L. The range of guar gum concentrations studied found to span the transition 537 from the semi-dilute to entangled regime, based on measurements of the intrinsic viscosity and surface 538 tension.

539

540 A single set of Giesekus parameters was found to provide a good quantitative description of both 541 simple and extensional shear behaviour for guar gum solutions in the semi-dilute regime and all the 542 κ/ι-hybrid carrageenan gum solutions. The parameters were dependent on gum concentration, and the 543 Trouton ratio deviated strongly from the Newtonian value of 3. These findings indicate that the 544 Giesekus model can be used to describe the rheology of these materials in CFD simulations. It also 545 suggests that simple shear tests can be used to give a reliable estimate of their extensional behaviour.

546

547 At higher guar gum concentrations, both the simple and extensional shear behaviour gave poorer fits 548 to the expressions for a single term Giesekus fluid. For solution concentrations towards the upper limit 549 of the semi-dilute regime  $(c = 5 \text{ g/L}$  and 10 g/L) the expressions still fitted the data well, but with 550 separate mobility parameters and relaxation times for simple and extensional shear modes. This could 551 be attributed to the effect of chain overlap on extension over simple shear. At the highest 552 concentration studied the expressions did not describe the data sets well.

553

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### 564 **Nomenclature**

## 565 **Roman letters**



566

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### 568 **Greek letters**





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### 684 **Table captions**

685 **Table 1.** Parameters obtained by fitting Equation (4) to guar gum and κ/ι-hybrid carrageenan solutions

686 shear data in Figure 2.

687

**Table 2.** Values of power law factor,  $K^*$ , and power law exponent, *n*, required to estimate the 689 relationship between material properties, *f*(*c*), and solution concentration.

690

- 691 **Table 3**.Parameters obtained by fitting Equation (19) to guar gum solution extensional data in
- 692 Figure 6 compared against those obtained using the shear data shown in Figure 2.

Solution concentration	Mobility parameter	Relaxation time	Surface tension	Zero shear rate viscosity	Initial filament	$R^2$ value $\left( -\right)$
$\cal C$	$\boldsymbol{a}$	$\lambda$	$\alpha$	$\eta_0$	diameter	
(g/L)	$\left( \cdot \right)$	(s)	(N/m)	(Pa s)	$D_1$	
					$(\mu m)$	
Guar gum						
$\mathbf{1}$	$0.00035 \pm 0.00001$ <sup>d</sup>	$0.090 \pm 0.003$ <sup>f</sup>	$0.0685 \pm 0.0002$ <sup>a</sup>	$0.0029 \pm 0.0002$ <sup>f</sup>	$205 \pm 2^{f}$	0.993
$\overline{2}$	$0.0026 \pm 0.0002$ <sup>c</sup>	$0.150 \pm 0.002$ <sup>e</sup>	$0.0675 \pm 0.0003^b$	$0.022 \pm 0.003$ <sup>e</sup>	$280 \pm 3^e$	0.992
5	$0.051 \pm 0.002^b$	$0.878 \pm 0.002$ <sup>d</sup>	$0.0675 \pm 0.0002^b$	$0.480 \pm 0.004$ <sup>d</sup>	$430 \pm 1^d$	0.988
10	$0.50 \pm 0.00^a$	$2.37 \pm 0.01$ <sup>c</sup>	$0.0674 \pm 0.0001^b$	$5.87 \pm 0.05$ <sup>c</sup>	$477 \pm 3^c$	0.950
15	$0.50 \pm 0.00^a$	$14.5 \pm 0.1^b$	$0.0673 \pm 0.0001^b$	$89.5 \pm 0.2^b$	$482 \pm 1^{b}$	0.891
20	$0.50 \pm 0.00^a$	$18.2 \pm 0.2^a$	$0.0672 \pm 0.0001^b$	$550 \pm 12^a$	$487 \pm 2^a$	0.886
κ/ι-hybrid carrageenan gum						
5	$0.0029 \pm 0.0004$ <sup>d</sup>	$0.312 \pm 0.003$ <sup>d</sup>	$0.0725 \pm 0.0002$ <sup>a</sup>	$0.040 \pm 0.002$ <sup>d</sup>	$316\pm1^{b}$	0.995
10	$0.021 \pm 0.002$ <sup>c</sup>	$2.33 \pm 0.05$ <sup>c</sup>	$0.0714 \pm 0.0002^b$	$0.381 \pm 0.003$ <sup>c</sup>	$319 \pm 1^{a,b}$	0.996
15	$0.18 \pm 0.01^b$	$8.24 \pm 0.04^b$	$0.0709 \pm 0.0002$ <sup>c</sup>	$3.85 \pm 0.04^b$	$321 \pm 1^a$	0.993
20	$0.495 \pm 0.00^a$	$13.55 \pm 0.06^a$	$0.0703 \pm 0.0002$ <sup>d</sup>	$14.1 \pm 0.1^a$	$324 \pm 2^a$	0.990

694 **Table 1.** Parameters obtained by fitting Equation (4) to guar gum and κ/ι-hybrid carrageenan solutions shear data in Figure 2.

 $\beta$ <sup>†</sup>Data are presented as mean  $\pm$  standard deviation. Data values in a column with different superscript letters are significantly different at the  $p \le 0.05$  level. 696

**Table 2.**Values of power law factor,  $K^*$ , and power law exponent, *n*, required to estimate the relationship between material properties,  $f(c)$ , and solution





699 Units of  $K^*$  are [parameter]  $(g/L)^{-n}$ 

700

701





705  $\dagger$ Data are presented as mean  $\pm$  standard deviation. Data values in a column with different superscript letters are significantly different at the  $p \le 0.05$  level.



# 707 **Figure captions**

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764

762

763 *(b)* 

765 **Figure 1** Effect of guar gum (circles) and κ/ι-hybrid carrageenan gum (triangles) concentration on 766 reduced viscosity. Data points for guar gum are computed from experimental  $\eta_0$  data from  $767$  Torres and co-workers<sup>1</sup>. Dashed lines are fits of Equation (23)to the first three data points 768 for guar gum solutions and to the whole set of data for κ/ι-hybrid carrageenan gum 769 solutions.


773 **Figure 2** Comparison of the measured apparent viscosity of (*a*) guar gum solutions (reproduced from Torres and co-workers<sup>1</sup>) and (*b*) κ/ι-hybrid carrageenan gum solutions with the Giesekus 775 model, Equation(4). Concentration: 1g/L (open grey circles); 2g/L (open black triangles); 776 5g/L (open grey squares); 10g/L (open black diamonds); 15 g/L (dashes) and 20 g/L (crosses). 777 Dashed loci indicate model fit, parameters in Table 1.

779 (*a*)



784 (*c*)





**Figure 3** Effect of concentration on Giesekus model parameters. (*a*)  $\eta_0$ ; (*b*) *a* and (*c*)  $\lambda$  for guar gum and  $\kappa$ /*i*-hybrid carrageenan gum. Dashed loci shows the power law trend line, Equation (24), and κ/ι-hybrid carrageenan gum. Dashed loci shows the power law trend line, Equation (24), 788 fitted to the data for solutions in the semi-dilute regime, parameters given in Table 1. 789 Symbols: circles – guar gum solutions, triangles - κ/ι-hybrid carrageenan gum solutions.

790





*(b)* 

**Figure 4** Normal stress differences (*N*1-*N*2) measured for aqueous (*a*) κ/ι-hybrid carrageenan gum and (*b*) guar gum solutions prepared at different polymer concentration. Lines show the difference between Equations (7) and (8) fitted to the data for studied solutions. Concentration: 2 g/L 799 (open black triangles); 5 g/L (open grey squares); 10 g/L (open black diamonds); 15 g/L 800 (dashes) and 20 g/L (crosses). The dashed horizontal line corresponds to the noise floor of 801 the normal force transducer.







809

810 **Figure 5** Comparison of measured non-dimensional filament diameter with Giesekus model, 811 Equation (19) for (*a*) κ/ι-hybrid carrageenan gum solutions, (*b*) guar gum solutions, semi-812 dilute regime; (*c*) guar gum solutions, entangled regime. Concentration: 1 g/L (open grey 813 circles); 2 g/L (open black triangles); 5 g/L (open grey squares); 10 g/L (open black 814 diamonds); 15 g/L (dashes) and 20 g/L (crosses). Dashed loci show predictions from 815 Equation (19) using the parameters obtained from steady shear studies given in Table 1.



818 **Figure 6** Fitting of guar gum solution filament stretching data in Figure 4(*c*) to Equation (19) when

- 819 Giesekus model parameters *a* and λ are allowed to differ from those obtained from fitting
- 820 shear flow data (Figure 2(*a*)). Dashed loci model fit with parameters given in Table 3.



826 **Figure 7** Time-concentration superposition of the filament stretching data in Figure 5, where time is 827 normalised by the experimental filament rupture time: (*a*) κ/ι-hybrid carrageenan gum and 828  $(b)$  guar gum (reproduced from Torres and co-workers<sup>1</sup>).



831 **Figure 8** Relationship between filament rupture time, obtained from extensional testing, and mobility 832 parameter obtained from shear flow data. Symbols: circles – guar gum, triangles - κ/ι-hybrid 833 carrageenan gum solutions.



840 gum and (*b*) guar gum solutions. Open circles – guar gum in semi-dilute region; solid 841 circles – guar gum in entangled regime; diamonds – κ/ι-hybrid carrageenan. Labels indicate 842 gum concentration.



844 **Figure 10** Effect of Hencky strain on Trouton ratio, for different guar gum concentrations. Symbols

845 show experimental data reported by Torres and co-workers<sup>1</sup>. Solid loci represent predictions 846 of the Trouton ratio *via* Equation (4) and Equation (22)

### 848 **Appendix**

## 849 *Surface tension*

The surface tension between the κ/ι-hybrid carrageenan gum solutions and air at 21°C was determined experimentally using the sessile drop method with a Kruss Drop Shape Analyser 100 device. Values reported are the mean from at least ten measurements. The influence of κ/ι-hybrid carrageenan gum 853 concentration was satisfactorily fitted to the Szyszkowski equation<sup>35</sup>:

$$
854 \qquad \frac{\alpha}{\alpha_0} = 1 - s_1 \ln \left( 1 + \frac{c}{s_2} \right) \tag{A.1}
$$

855 where  $\alpha_0$  is the surface tension of the solvent, *c* the concentration of the surfactant and  $s_1$ and  $s_2$  are fitting 856 parameters.



857

858 **Figure A.1.** Effect of κ/ι-hybrid carrageenan gum concentration on surface tension relative to water. 859 Solid trend line shows Equation (A.1) fitted to the κ/ι-hybrid carrageenan gum data with 860 parameters  $s_1 = 0.0135$  and  $s_2 = 0.0018$  g/L.

#### 862 *Cross model*

- 863 The experimental flow curves for κ/ι-hybrid carrageenan gum samples were satisfactorily fitted  $(R<sup>2</sup>$
- 864 0.990, standard error < 0.0023 Pa s) to the Cross-Williamson model, Eqn. (A.2),

865 
$$
\frac{\eta_{app}}{\eta_0} = \frac{1}{1 + k_c \gamma}
$$
 (A.2)

866 where  $\eta_0$  is the zero-shear rate viscosity,  $k_c$  is the time constant and *n* is the flow index.

867



868

870 **Figure A.2.** Flow curves of representative aqueous κ/ι-hybrid carrageenan gum solutions prepared at 871 different concentrations. Symbols: 5 g/L (squares); 10 g/L (diamonds); 15 g/L (dashes) and 20 872 g/L (crosses). Dashed loci indicate model fit obtained using Cross model, Equation (A.2), 873 parameters in Table A.1.

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874





 $\text{a}^{\dagger}$   $\text{b}^{\dagger}$  Data are presented as mean  $\pm$  standard deviation. Data values in a column 878 with different superscript letters are significantly different at the  $p \le 0.05$ level.  $\begin{array}{c} 879 \\ 880 \end{array}$ 

## 881 *Trouton ratio*



882

883 **Figure A.3.** Effect of Hencky strain on Trouton ratio, for different κ/ι-hybrid carrageenan gum 884 concentrations. Symbols: 5 g/L (squares); 10 g/L (diamonds); 15 g/L (dashes) and 20 g/L 885 (crosses). Solid loci represent predictions of the Trouton ratio *via* Equation (4) and Equation 886 (21)



Figure 1a. Effect of guar gum (circles) and κ/ι-hybrid carrageenan gum (triangles) concentration on reduced viscosity. Data points for guar gum are computed from experimental  $\eta_0$  data from Torres and co-workers<sup>1</sup>. Dashed lines are fits of Equation (23) to the first three data points for guar gum solutions and to the whole set of data for κ/ι-hybrid carrageenan gum solutions. 87x79mm (300 x 300 DPI)



Figure 1b. Effect of guar gum (circles) and κ/ι-hybrid carrageenan gum (triangles) concentration on reduced viscosity. Data points for guar gum are computed from experimental  $\eta_0$  data from Torres and co-workers<sup>1</sup>. Dashed lines are fits of Equation (23) to the first three data points for guar gum solutions and to the whole set of data for κ/ι-hybrid carrageenan gum solutions. 83x76mm (300 x 300 DPI)



Figure 2a. Comparison of the measured apparent viscosity of guar gum solutions (reproduced from Torres and co-workers<sup>1</sup>) with the Giesekus model, Equation (4). Concentration: 1 g/L (open grey circles); 2 g/L (open black triangles); 5 g/L (open grey squares); 10 g/L (open black diamonds); 15 g/L (dashes) and 20 g/L (crosses). Dashed loci indicate model fit, parameters in Table 1. 70x59mm (300 x 300 DPI)



Figure 2b. Comparison of the measured apparent viscosity of κ/ι-hybrid carrageenan gum solutions with the Giesekus model, Equation (4). Concentration: 1 g/L (open grey circles); 2 g/L (open black triangles); 5 g/L (open grey squares); 10 g/L (open black diamonds); 15 g/L (dashes) and 20 g/L (crosses). Dashed loci indicate model fit, parameters in Table 1. 79x75mm (300 x 300 DPI)



Figure 3a. Effect of concentration on Giesekus model parameter  $\eta_0$  for guar gum and  $\kappa$ /ι-hybrid carrageenan gum. Dashed loci shows the power law trend line, Equation (24), fitted to the data for solutions in the semidilute regime, parameters given in Table 1. Symbols: circles – guar gum solutions, triangles - κ/ι-hybrid carrageenan gum solutions. 165x144mm (300 x 300 DPI)



Figure 3b. Effect of concentration on Giesekus model parameter a for guar gum and κ/ι-hybrid carrageenan gum. Dashed loci shows the power law trend line, Equation (24), fitted to the data for solutions in the semidilute regime, parameters given in Table 1. Symbols: circles – guar gum solutions, triangles - κ/ι-hybrid carrageenan gum solutions. 65x54mm (300 x 300 DPI)



Figure 3c. Effect of concentration on Giesekus model parameter λ for guar gum and κ/ι-hybrid carrageenan gum. Dashed loci shows the power law trend line, Equation (24), fitted to the data for solutions in the semidilute regime, parameters given in Table 1. Symbols: circles – guar gum solutions, triangles - κ/ι-hybrid carrageenan gum solutions. 169x153mm (300 x 300 DPI)



Figure 4a. Normal stress differences  $(N_1-N_2)$  measured for aqueous  $\kappa/1$ -hybrid carrageenan gum solutions prepared at different polymer concentration. Lines show the difference between Equations (7) and (8) fitted to the data for studied solutions. Concentration: 2 g/L (open black triangles); 5 g/L (open grey squares); 10 g/L (open black diamonds); 15 g/L (dashes) and 20 g/L (crosses). The dashed horizontal line corresponds to the noise floor of the normal force transducer. 158x128mm (300 x 300 DPI)



Figure 4b. Normal stress differences  $(N_1-N_2)$  measured for aqueous guar gum solutions prepared at different polymer concentration. Lines show the difference between Equations (7) and (8) fitted to the data for studied solutions. Concentration: 2 g/L (open black triangles); 5 g/L (open grey squares); 10 g/L (open black diamonds); 15 g/L (dashes) and 20 g/L (crosses). The dashed horizontal line corresponds to the noise floor of the normal force transducer. 160x126mm (300 x 300 DPI)





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Figure 5b. Comparison of measured non-dimensional filament diameter with Giesekus model, Equation (19) for guar gum solutions, semi-dilute regime. Concentration: 1 g/L (open grey circles); 2 g/L (open black triangles); 5 g/L (open grey squares); 10 g/L (open black diamonds); 15 g/L (dashes) and 20 g/L (crosses). Dashed loci show predictions from Equation (19) using the parameters obtained from steady shear studies given in Table 1. 149x123mm (300 x 300 DPI)

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Figure 6. Fitting of guar gum solution filament stretching data in Figure 4(c) to Equation (19) when Giesekus model parameters a and λ are allowed to differ from those obtained from fitting shear flow data (Figure 2(a)). Dashed loci – model fit with parameters given in Table 3. 148x122mm (300 x 300 DPI)



Figure 7a. Time-concentration superposition of the filament stretching data in Figure 5, where time is normalised by the experimental filament rupture time: κ/ι-hybrid carrageenan gum 68x54mm (600 x 600 DPI)





68x54mm (600 x 600 DPI)







Figure 9a. Comparison of the left- and right-hand sides of Equation (25) for κ/ι-hybrid carrageenan gum solutions. Labels indicate gum concentration 146x121mm (300 x 300 DPI)



Figure 9b. Comparison of the left- and right-hand sides of Equation (25) for guar gum solutions. Open circles – guar gum in semi-dilute region; solid circles – guar gum in entangled regime. Labels indicate gum concentration 146x123mm (300 x 300 DPI)



Figure 10. Effect of Hencky strain on Trouton ratio, for different guar gum concentrations. Symbols show experimental data reported by Torres and co-workers<sup>1</sup>. Solid loci represent predictions of the Trouton ratio via Equation (4) and Equation (22) 99x82mm (300 x 300 DPI)



Figure A1. Effect of κ/ι-hybrid carrageenan gum concentration on surface tension relative to water. Solid trend line shows Equation (A.1) fitted to the  $\kappa$ / $\cdot$ -hybrid carrageenan gum data with parameters  $s_1 = 0.0135$ and  $s_2 = 0.0018$  g/L. 61x46mm (600 x 600 DPI)



Figure A2. Flow curves of representative aqueous κ/ι-hybrid carrageenan gum solutions prepared at different concentrations. Symbols: 5 g/L (squares); 10 g/L (diamonds); 15 g/L (dashes) and 20 g/L (crosses). Dashed loci indicate model fit obtained using Cross model, Equation (A.2), parameters in Table A.1. 73x64mm (600 x 600 DPI)


Figure A3. Effect of Hencky strain on Trouton ratio, for different κ/ι-hybrid carrageenan gum concentrations. Symbols: 5 g/L (squares); 10 g/L (diamonds); 15 g/L (dashes) and 20 g/L (crosses). Solid loci represent predictions of the Trouton ratio via Equation (4) and Equation (21) 99x82mm (300 x 300 DPI)