Dictionary Design for Distributed Compressive Sensing

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***Abstract*—Conventional dictionary learning frameworks attempt to find a set of atoms that promote both signal representation and signal sparsity fora class of signals. In distributed compressive sensing (DCS), in addition to intra-signal correlation, inter-signal correlation is also exploited in the joint signal reconstruction, which goes beyond the aim of the conventional dictionary learning framework. In this letter, we propose a new dictionary learning framework in order to improve signal reconstruction performance in DCS applications. By capitalizing on the sparse common component and innovations (SCCI) model [1], which captures both intra- and inter-signal correlation, the proposed method iteratively finds a dictionary design that promotes various goals: i) signal representation; ii) intra-signal correlation; and iii) inter-signal correlation. Simulation results showthat our dictionary design leads to an improved DCS reconstruction performance in comparison to other designs.**

***Index Terms*— Compressive sensing, dictionary learning, distributed compressive sensing.**

I. Introduction

COMPRESSIVE SENSING (CS) is a framework that aims to reconstruct an unknown signal with a reduced number of random projections. Therefore, CS represents a convenient universal compression method that has been attracting growing interest in various applications such as wireless sensor networks (WSNs) [2] and electrocardiogram (ECG) [3], [4] acquisition. The success of CS is due to the fact that many natural signals of interest lie approximately in a union of low-dimensional subspaces in the higher-dimensional ambient space [5]. This union of subspaces can itself be described by a sparse combination of columns of a dictionary, e.g., a wavelet transform-based dictionary.

Rather than using a predefined dictionary, it is also possible to construct customized dictionaries for specific classes of signals, which result in improved CS performance [6]. With a given set of T training signals $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_T] \in \mathbb{R}^{N\times T}$ ($N \ll T$), the dictionary $\mathbf{\Psi} \in \mathbb{R}^{N\times K}$ ($N \leq K$) can be learnt by solving the following optimization problem [7]:

$$\min_{\mathbf{\Psi},\mathbf{\Theta}} \quad \|\mathbf{\Psi\Theta} - \mathbf{X}\|_F^2 + \lambda \sum_{t=1}^{T} \|\boldsymbol{\theta}_t\|_0, \tag{1}$$

where $\mathbf{\Theta} = [\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_T] \in \mathbb{R}^{K\times T}$ is composed of the set of sparse signal approximations corresponding to the training signals $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_T]$, $\|\cdot\|_F^2$ and $\|\cdot\|_0$ denotes the Frobenius norm and the ℓ_0 norm, respectively, and $\lambda > 0$ is a parameter that balances signal representation and signal sparsity induced by the dictionary. This classical dictionary learning framework only exploits the sparsity structure of signals, but additional structure can also be incorporated into the design framework. For example, Zelnik-Manor *et al.* propose a dictionary learning approach for signals with a block-sparse structure [8] and Szabo *et al.* develop a dictionary learning method that takes into consideration the overlapping group structure of a signal [9].

As a generalization of CS, distributed compressive sensing (DCS) [1], [10] exploits both correlation within a signal (often capture by sparsity in some basis) as well as correlation across signals, in order to sense and reconstruct more efficiently a collection of signals. Intra- and inter-signal correlation arise in many applications. For example, temperature signals measured by various sensors in the field are not only sparse (in some appropriate basis) but also highly correlated across sensors; and likewise ECG signals of adjacent heartbeats are also sparse (in some appropriate basis) and significantly correlated. Recent work [1], [10] has demonstrated that joint reconstruction of several correlated signals via DCS outperforms recovering each signal one by one via CS. Therefore, and in the same way that conventional dictionary learning such as the method of optimal directions (MOD) [11] and KSVD [12] can substantially improve the performance of CS, we are also motivated to study how a dictionary learning framework customized for a set of signals that exhibit both intra- and inter-signal correlation can improve the performance of DCS applications.

In this letter we propose a new dictionary learning design for correlated signals to improve DCS reconstruction performance. The proposed design captures both the intra-signal structure and inter-signal correlation, and according to our experiments outperforms dictionaries learned via conventional methods for both synthetic data and for actual ECG data.

II. Measurement and Reconstruction in DCS

In the DCS setting [1], [10], a set of signals $\mathbf{x}_i \in \mathbb{R}^N$ ($i = 1, \ldots, L$) are expressed in terms of the sparse representations

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$\boldsymbol{\theta}_i \in \mathbb{R}^K$ $(K \geq N, i = 1, \ldots, L)$ under a dictionary $\boldsymbol{\Psi} \in \mathbb{R}^{N \times K}$, as follows:

$$\mathbf{x}_i = \boldsymbol{\Psi}\boldsymbol{\theta}_i. \tag{2}$$

These representations $\boldsymbol{\theta}_i$ $(i = 1, \ldots, L)$ are also taken to obey the sparse common component and innovations (SCCI) model which captures both intra- and inter-signal correlation. In particular,

$$\boldsymbol{\theta}_i = \mathbf{z}_c + \mathbf{z}_i, \tag{3}$$

where $\mathbf{z}_c \in \mathbb{R}^K$ with $\|\mathbf{z}_c\|_0 = S_c \ll K$ denotes the common component of the sparse representation $\boldsymbol{\theta}_i \in \mathbb{R}^K$, which captures the inter-signal correlation and is common to all the correlated signals, and $\mathbf{z}_i \in \mathbb{R}^K$ $(i = 1, \ldots, L)$ with $\|\mathbf{z}_i\|_0 = S_i \ll K$ denotes the innovations component of the sparse representation $\boldsymbol{\theta}_i \in \mathbb{R}^K$, which captures the intra-signal correlation and is specific to the signal i. The joint sparsity level of signals following the SCCI model, which is exploited by the reconstruction algorithms in [1], is determined by $S_c + \sum_{i=1}^{L} S_i$.

The DCS signal measurement process is based on the computation of low-dimensional projections of each high-dimensional signal independently, as follows:

$$\mathbf{y}_i = \boldsymbol{\Phi}_i\mathbf{x}_i + \mathbf{n}_i = \boldsymbol{\Phi}_i\boldsymbol{\Psi}(\mathbf{z}_c + \mathbf{z}_i) + \mathbf{n}_i, \tag{4}$$

where $\mathbf{y}_i \in \mathbb{R}^M$ is the projections vector associated with signal i, $\boldsymbol{\Phi}_i \in \mathbb{R}^{M \times N}$ is the projections matrix associated with signal i where $M \ll N$, and $\mathbf{n}_i$ denotes the noise term for the measuring process. The signal reconstruction process in DCS involves solving the following optimization problem to jointly recover the original signal representations[1]

$$\min_{\tilde{\mathbf{z}}} \|\tilde{\mathbf{z}}\|_0 \quad \text{s.t.} \quad \|\mathbf{A}\tilde{\mathbf{z}} - \tilde{\mathbf{y}}\|_F^2 \leq \epsilon, \tag{5}$$

where $\epsilon \geq 0$, $\tilde{\mathbf{z}} = [\mathbf{z}_c^T\ \mathbf{z}_1^T\ \ldots \mathbf{z}_L^T]^T \in \mathbb{R}^{(L+1)K}$ is the extended signal representation vector, $\tilde{\mathbf{y}} = [\mathbf{y}_1^T \ldots \mathbf{y}_L^T]^T \in \mathbb{R}^{LM}$ is the extended measurements vector and $\mathbf{A} \in \mathbb{R}^{LM \times (L+1)K}$ is the extended sensing matrix given by:

$$\mathbf{A} = \begin{bmatrix} \boldsymbol{\Phi}_1\boldsymbol{\Psi} & \boldsymbol{\Phi}_1\boldsymbol{\Psi} & 0 & 0 & \ldots & 0 \\ \vdots & & & & \ddots & \vdots \\ \boldsymbol{\Phi}_L\boldsymbol{\Psi} & 0 & 0 & 0 & \ldots & \boldsymbol{\Phi}_L\boldsymbol{\Psi} \end{bmatrix}.$$

The superiority of DCS for recovering a set of correlated signals is not only demonstrated by the milder sufficient and necessary conditions for DCS reconstruction in comparison to the conditions for CS [10], but is also shown in comprehensive experiments [1].

Inspired by the fact that DCS can outperform standard CS, we next put forth a dictionary design framework that promotes the SCCI signal model for a set of correlated signals in order to improve further the performance of DCS.

III. Dictionary Learning for DCS

Given a set of T training matrices $\mathbf{X} = [\mathbf{X}_1, \ldots, \mathbf{X}_T] \in \mathbb{R}^{N \times LT}$, where each training matrix $\mathbf{X}_t = [\mathbf{x}_{1,t}, \ldots, \mathbf{x}_{L,t}] \in \mathbb{R}^{N \times L}$ $(t = 1, \ldots, T)$ is composed of L correlated signals, the dictionary learning problem for DCS can be formulated as follows[2]

$$\min_{\substack{\boldsymbol{\Psi}, \mathbf{z}_{c,t}, \mathbf{z}_{i,t} \\ (t=1,\ldots,T; i=1,\ldots,L)}} \sum_{t=1}^{T} \left(\sum_{i=1}^{L} \|\boldsymbol{\Psi}(\mathbf{z}_{c,t} + \mathbf{z}_{i,t}) - \mathbf{x}_{i,t}\|_F^2 + \lambda \left(\sum_{i=1}^{L} \|\mathbf{z}_{i,t}\|_0 + \|\mathbf{z}_{c,t}\|_0 \right) \right), \tag{6}$$

where $\mathbf{z}_{c,t} \in \mathbb{R}^K$ and $\mathbf{z}_{i,t} \in \mathbb{R}^K$ $(t = 1, \ldots, T; i = 1, \ldots, L)$ denote the common component and the innovation component of the sparse representation $\boldsymbol{\theta}_{i,t}$ associated with the signal $\mathbf{x}_{i,t}$ respectively, and $\lambda \geq 0$ denotes the weight for the joint sparsity level in the optimization problem. The traditional dictionary learning framework [11]–[13] aims to find a set of atoms that lead both to an accurate and parsimonious representation of each individual signal, assuming implicitly that the signals are drawn independently according to the model $p(\mathbf{X}|\boldsymbol{\Psi}) = \prod_{i=1}^{L}\prod_{t=1}^{T} p(\mathbf{x}_{i,t}|\boldsymbol{\Psi})$ [12]. However, the proposed formulation in (6) attempts to determine a dictionary that yields an accurate and parsimonious representation of the overall set of signals. The conventional dictionary learning can find some atoms relating to common components, while the proposed learning process enforces the SCCI model so as to reduce the joint sparsity level. In particular, by aiming to promote low joint signal sparsity, i.e., $\sum_{t=1}^{T}(\sum_{i=1}^{L}\|\mathbf{z}_{i,t}\|_0 + \|\mathbf{z}_{c,t}\|_0)$, then one expects DCS tailored reconstruction algorithms, such as (5) or the proposed joint orthogonal matching pursuit (JOMP), to achieve better performance in comparison to standard CS reconstruction algorithms.

Note that the design framework represents a generalization of the standard dictionary learning framework (this can be readily appreciated by removing the effect of the common components). Therefore, and akin to many efficient heuristic dictionary learning algorithms such as the MOD [11] and K-SVD [12] that rely on the use of iterative approaches to solve the non-convex dictionary design problem, our proposed approach to DCS dictionary design also involves two stages: joint sparse approximation and dictionary update.

A. Joint Sparse Approximation

In the joint sparse approximation stage, the dictionary $\boldsymbol{\Psi}$ is assumed to be fixed, and the common components $\mathbf{z}_{c,t}$ and the innovation components $\mathbf{z}_{i,t}$ $(i = 1, \ldots, L)$ for each set of correlated signals $\mathbf{X}_t$ $(t = 1, \ldots, T)$ can be obtained by solving the following problem:

$$\min_{\substack{\mathbf{z}_{c,t}, \mathbf{z}_{i,t} \\ (i=1,\ldots,L)}} \sum_{i=1}^{L} \|\boldsymbol{\Psi}(\mathbf{z}_{c,t} + \mathbf{z}_{i,t}) - \mathbf{x}_{i,t}\|_F^2 + \lambda \left(\sum_{i=1}^{L} \|\mathbf{z}_{i,t}\|_0 + \|\mathbf{z}_{c,t}\|_0 \right). \tag{7}$$

In principle, we could adopt a convex optimization approach, whereby the ℓ_0 norm is replaced by the ℓ_1 norm, to approximate the solution to this optimization problem. However, the resulting

[1]Note that generally the ℓ_0 norm can be replaced by the ℓ_1 norm to relax the problem as a convex problem.

[2]The traditional dictionary learning problem is formulated by using only one subscript to represent distinct signals in the training dataset. On the other hand, our learning framework involves using two subscripts: subscript i differentiates signals within the same subset and subscript t differentiates signals of different subsets. This means it is only required that each subset with L signals in the training data share the same common component.

optimization problem is not entirely suitable for the sparse approximation step in the dictionary learning process in view of its computational complexity[3]. Instead, we adopt a greedy algorithm that represents a generalization of the standard orthogonal matching pursuit (OMP) from the setting associated with the reconstruction of a single sparse signal to the setting associated with the reconstruction of multiple sparse signals that obey the SCCI model. The pseudo-code of this algorithm, which we refer to as JOMP, is described in Algorithm 1.

Algorithm 1 Joint Orthogonal Matching Pursuit

Input: A set of signals $\{\mathbf{x}_i\}(i = 1, \ldots, l)$, a dictionary $\mathbf{\Psi}$ and a positive value β.

Output: The common component $\mathbf{z}_c$ and the innovation components $\mathbf{z}_i$ $(i = 1, \ldots, L)$.

Process: Do

1) Initialize $\mathbf{r}_i = \mathbf{x}_i$ $(i = 1, \ldots, L)$, $\Omega_i = \emptyset$ $(i = 1, \ldots, L)$, $\Omega_c = \emptyset$, $\mathbf{z}_i = \mathbf{0}$ $(i = 1, \ldots, L)$ and $\mathbf{z}_c = \mathbf{0}$;
2) Calculate $\mathbf{w}_c = \sum_{i=1}^{L} \mathbf{\Psi}^T \mathbf{r}_i$, and $\mathbf{w}_i = \mathbf{\Psi}^T \mathbf{r}_i$ $(i = 1, \ldots, L)$;
3) If $\max_j |w_{c_j}| \geq \max_{i,j} |w_{i_j}|$, then $j^* = \arg\max_j |w_{c_j}|$ and $\Omega_c = \Omega_c \cup \{j^*\}$; Otherwise $\{j^*, i^*\} = \arg\max_{i,j} |w_{i_j}|$ and $\Omega_{i^*} = \Omega_{i^*} \cup \{j^*\}$;
4) Compute $[\mathbf{z}_{c\Omega_c}^T \; \mathbf{z}_{1\Omega_1}^T \; \ldots \; \mathbf{z}_{L\Omega_L}^T]^T = (\hat{\mathbf{\Psi}}^T \hat{\mathbf{\Psi}})^{-1} \hat{\mathbf{\Psi}}^T [\mathbf{x}_1^T \ldots \mathbf{x}_L^T]^T$, where

$$\hat{\mathbf{\Psi}} = \begin{bmatrix} \mathbf{\Psi}_{\Omega_c} & \mathbf{\Psi}_{\Omega_1} & \mathbf{0} & \ldots & \mathbf{0} \\ \vdots & & & \ddots & \vdots \\ \mathbf{\Psi}_{\Omega_c} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{\Psi}_{\Omega_L} \end{bmatrix};$$

5) Compute $\mathbf{r}_i = \mathbf{x}_i - \mathbf{\Psi}_{\Omega_c} \mathbf{z}_{c\Omega_c} - \mathbf{\Psi}_{\Omega_i} \mathbf{z}_{i\Omega_i}$;
6) If halting condition is true, return $\mathbf{z}_c$ and $\mathbf{z}_i$ $(i = 1, \ldots, L)$; otherwise go to step 2;

(Note that a matrix subscript with respect to a set denotes a selection of the columns of the matrix indexed by the set.)

In each iteration of the JOMP, the largest correlation between the residue $\mathbf{r}_i$ $(i = 1, \ldots, L)$ of any innovation component and columns of $\mathbf{\Psi}$ is compared with the largest correlation between the total residue $\sum_{i=1}^{L} \mathbf{r}_i$ and columns of $\mathbf{\Psi}$, and then the index of the larger element corresponding to either common component or one of the innovation components is selected. The common component and the innovation components are updated via least square estimation after the update of their support, and a new residue for each signal is calculated at the end of each iteration. The algorithm stops when a certain criterion is satisfied, for example such as reaching the maximal joint signal sparsity level or a threshold value of the residue. Note that JOMP is similar to OMP except that it involves the correlation between the total residue $\sum_{i=1}^{L} \mathbf{r}_i$ and the columns of $\mathbf{\Psi}$. This key innovation enables the JOMP to fit the SCCI model and to find the common component and the innovation components with a low joint sparsity level. We adopt the JOMP to solve the correlated signal representations not only because of its simplicity but also because it is a natural generalization of the OMP which is widely used in standard dictionary learning designs [12].

[3]For a signal with M elements whose sparse approximation has N elements with S non-zero elements $(S < M < N)$, the complexity of the OMP is $\mathcal{O}(SMN)$, which is much smaller than $\mathcal{O}(M^2 N^{1.5})$, i.e., the complexity of solving an ℓ_1 optimization problem [14].

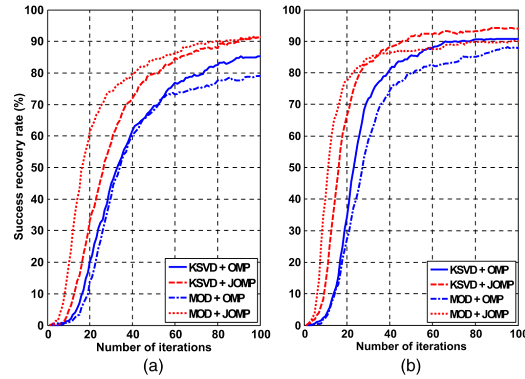

Fig. 1. Success recovery rate of the ground truth dictionary. (a) High inter-signal correlation with $S_c = 3$ and $S_i = 1$ $(i = 1, \ldots, L)$; (b) Low inter-signal correlation with $S_c = 2$ and $S_i = 2$ $(i = 1, \ldots, L)$.

B. Dictionary Update

The dictionary update approaches of either MOD or KSVD are appropriate for the proposed framework. For MOD, the common components $\mathbf{z}_{c,t}$ and the innovation components $\mathbf{z}_{i,t}$ $(i = 1, \ldots, L)$ for the correlated signals $\mathbf{X}_t$ $(t = 1, \ldots, T)$ are assumed to be fixed, and then the dictionary $\mathbf{\Psi}$ can be obtained by solving the following problem:

$$\min_{\mathbf{\Psi}} \quad \|\mathbf{\Psi} Z - \mathbf{X}\|_F^2, \tag{8}$$

where $\mathbf{Z} = [\mathbf{z}_{c,1} + \mathbf{z}_{1,1} \ldots \mathbf{z}_{c,1} + \mathbf{z}_{L,1} \ldots \mathbf{z}_{c,T} + \mathbf{z}_{L,T}] \in \mathbb{R}^{K \times LT}$. By using least square estimation, the solution is immediately given by

$$\mathbf{\Psi} = \mathbf{X}\mathbf{Z}^T (\mathbf{Z}\mathbf{Z}^T)^{-1}. \tag{9}$$

For KSVD, only one column of the dictionary and the non-zero entries in the associated row of $\mathbf{X}$ are updated each time.

In summary, our approach to solve the dictionary learning problem alternates between two steps:

- We first find the common component and the innovation components of the sparse representations with a fixed dictionary $\mathbf{\Psi}$ by using joint-sparsity-enforcing algorithms, i.e., the JOMP;
- Then we find an estimate for the dictionary $\mathbf{\Psi}$ by using either least square estimation as in MOD or using the approach of KSVD.

The proposed dictionary design not only exploits the intra-signal structure as does a conventional dictionary design, but also capitalizes on the inter-signal structure. It provides a means for DCS to efficiently reconstruct correlated signals of a certain class under a customized dictionary. In contrast, conventional dictionary designs do not promote the joint sparsity and predefined dictionaries are not specialized for a certain class of signals.

IV. Performance Results

We now compare the performance of the proposed dictionary learning approach with other dictionary learning approaches, such as MOD or KSVD, for DCS reconstructions.

A. Experiments with Synthetic Data

We first evaluate the efficacy of the algorithm in recovering an underlying pre-specified dictionary. In particular, a 20×50 random dictionary is generated with independent and identically

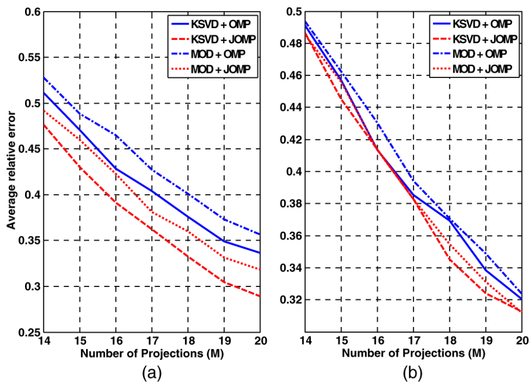

Fig. 2. DCS reconstruction performance with the usage of different dictionaries. (a) High inter-signal correlation with $S_c = 3$ and $S_i = 1$; (b) Low inter-signal correlation with $S_c = 2$ and $S_i = 2$.

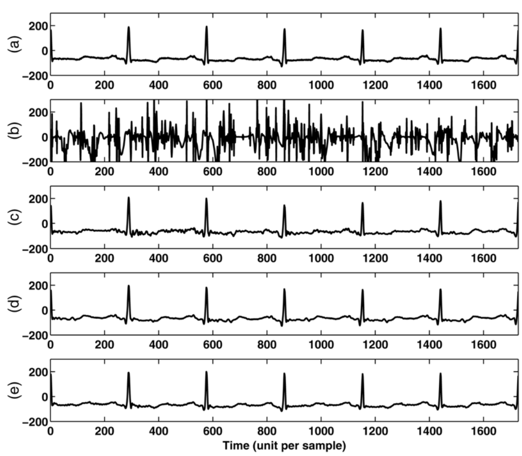

Fig. 3. A segment of ECG signal and reconstructed results ($M = 50$). (a) Original ECG signal; (b) CS reconstruction with the wavelet dictionary, and averaged relative error is 1.1304; (c) CS reconstruction with a dictionary based on intra-signal correlation, and averaged relative error is 0.1445; (d) DCS reconstruction with a dictionary based on intra-signal correlation, and averaged relative error is 0.0902; (e) DCS reconstruction with the proposed dictionary based on the SCCI model, and averaged relative error is 0.0609.

distributed (i.i.d.) Gaussian entries, followed by a column normalization. The set of training data consists of 1500 (6×250) signals, which are divided into 250 subsets, each of which is composed of 6 signals satisfying the SCCI model. The common component of signals within the same subset is created by a linear combination of S_c different dictionary atoms, with uniformly i.i.d. coefficients in random and independent locations. Similarly, innovation components of each signal within the same subset is created by a linear combination of S_i different dictionary atoms. White Gaussian noise to yield a signal-to-noise ratio (SNR) of 20 dB is added to these signals.

The convergence behaviour of different algorithms, i.e., success recovery rate of the ground truth dictionary (averaged over 10 trials) versus the number of iterations is shown in Fig. 1. It can be seen that the proposed approach always outperforms conventional approaches and is particularly effective for signals with high inter-signal correlation as shown in Fig. 1(a) than signals with low inter-signal correlation as shown in Fig. 1(b). In the experiment, conventional KSVD and MOD both using OMP take 0.46 and 0.34 seconds on average for each iteration, respectively, while 0.36 and 0.23 seconds are taken by the proposed KSVD and MOD both with JOMP, respectively[4]The reduction of running time can be explained by noting that the JOMP uses $S_c + \sum_{i=1}^{L} S_i$ iterations to find the sparse approximations for L signals, while about $\sum_{i=1}^{L}(S_c + S_i)$ iterations are required by OMP[5].

We now evaluate the quality of dictionary designs for DCS applications. We use the previous setting for training purposes and we use 1200 (6×200) signals for testing. $M = 50$ projections are taken for each test signal by applying projection matrices with i.i.d. Gaussian entries and column normalization. The test data is reconstructed via DCS with the use of various dictionary designs and compared with the original data. The performance is evaluated by using the average relative error, i.e., $\sum_{h=1}^{H} \frac{\|\tilde{\mathbf{x}}_h - \mathbf{x}_h\|_2}{H\|\mathbf{x}_h\|_2}$, where $\mathbf{x}_h$ and $\tilde{\mathbf{x}}_h$ ($h = 1, \ldots, H$) denote the hth original signal and the hth reconstructed signal in the test data set respectively. Fig. 2 shows the benefit of the proposed dictionary designs for DCS applications.

B. Experiments with ECG Data

We now evaluate the efficacy of our approach with real ECG data. We conduct preprocessing as in [4] for some ECG records taken from the MIT-BIH Arrhythmia Database set [15], which involves beat detection and period normalization. The set of training data consists of 1, 061 (6×288) segments randomly extracted from 48 ECG records, where each segment is composed of 6 adjacent heartbeats with dimension 288. A dictionary of size 288×312 that only takes into account intra-signal correlation is learned by using MOD with OMP (choosing the halting sparsity level to be 10), while the proposed dictionary design that leverages the SCCI model is learned by using MOD with JOMP (choosing the halting joint sparsity level to be 30)[6]. The learning process terminates after 50 iterations. The performance of the learned dictionaries are evaluated by using the other 1,089(6×288) segments that are not in the training set. Reduced numbers of measurements are taken from the test heartbeat signals by using random matrices whose entries are drawn i.i.d. from a Gaussian distribution and followed by column normalization. The test data is reconstructed via DCS or CS with the use of various dictionary designs and compared with the original data. We use OMP for CS reconstructions, and JOMP for DCS reconstructions. Fig. 3 demonstrates the higher reconstruction quality of the proposed dictionary design in comparison to the other designs, where the reconstructed ECG segment is one of the test segments in the MIT-BIH Arrhythmia Database set.

V. Conclusion

In this letter, we propose a novel dictionary learning approach, which captures both intra- and inter-signal correlation, for DCS applications. We have shown via experiments with both synthetic data and actual ECG data that the proposed design, provides a significant improvement to the DCS reconstruction performance.

[4]Our simulations are performed in MATLAB R2012b environment on a system with a quad-core 3.4 GHz CPU and 32 GB RAM, running under the Microsoft Windows 7 operating system.

[5]Note that the computation of one iteration of the JOMP is in general more demanding than that for OMP. For learning a dictionary with very large dimensions, the growth of computational complexity within one iteration of the JOMP becomes the dominant factor and consequently the JOMP can consume more running time in total.

[6]The JOMP considers L signals, and thus we choose a higher halting joint sparsity level for JOMP than the halting sparsity level for OMP.

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