Elias Allen and the Role of Instruments in Shaping the Mathematical Culture of Seventeenth-Century England

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Elias Allen (c.1588-1653) was known as the best mathematical instrument maker of his day. He lived and worked in London, creating a thriving business - he was the first English instrument maker to support himself solely through the production of instruments - and teaching his skills to many apprentices who became the core of the trade during the latter part of the century. My thesis provides a full biography of Allen, set within the framework of the community of people who were in some way connected to mathematics.

The second part of the dissertation is devoted to the most important of Allen's instruments: the Gunter quadrant and Gunter sector, and various of William Oughtred's designs - the circles of proportion, the horizontal instrument and double horizontal dial, and the universal equinoctial ring dial. These are described, their uses explained, and used as the base for discussions of the ways in which instruments influenced and were influenced by the development of mathematics. This section concludes with a catalogue of all the Allen instruments in British museums.

As well as a comprehensive literature survey of the mathematical texts printed in England during Allen's lifetime, I have given considerable time to a 'reading' of the instruments themselves - through study of the originals, through production of my own versions, through reconstructions of the methods of use, and, in the case of the sector, through computer analysis of the accuracy in use.

The conclusion of my thesis is that the mathematical culture of seventeenth-century England was far broader than that which is normally portrayed in histories of mathematics, involving a wide range of people with very different backgrounds and very different approaches to and understandings of mathematics. Above all, it is shown to be rooted strongly in a geometrical interpretation of mathematics and one which is inherently practical. In such a culture the role of instruments is fundamental and thus instrument makers like Elias Allen have a place at the heart of the mathematical community.

ELIAS ALLEN.

Apud Anglos Cantianus, iuxta Tunnbridge natus, Mathematicis Instrumentis ære incidendis sui temporis Artifex ingeniosissimus,

Obijt Londini propè finem Mensis Martij. Anno a Christo nato 1653 suæque ætatis

IV

CONTENTS

List of Illustrations

Figures

Plates

NB. All instruments are signed by Allen, unless otherwise indicated.

Acknowledgments

In preparing this thesis I have been grateful for the assistance of a great many people. Staff of the Cambridge University Library Rare Books Room, the British Library, Westminster City Archives, the Public Record Office and the Guildhall Library, Corporation of London have all aided my research. Without the help of many museum curators my catalogue of instruments would never have existed and so I would like to thank Maria Blyzinsky and Gloria Clifton (the latter for giving me access to the Project Simon database) at the Old Royal Observatory, David Bryden and Allan Simpson at the Royal Museum of Scotland, Roger Edwin of St. Andrew's University Physics Department, Stephen Johnston and Kevin Johnson at the Science Museum, John Leopold at the British Museum, Anthony North at the Victoria & Albert Museum, and Tony Simcock at the Museum of the History of Science in Oxford; I would also like to thank those private collectors who permitted me to include their Allen instruments in the catalogue. I must also thank Don Manning of the University Audio-Visual Aids unit for patiently teaching me how to improve my photographic skills, and the staff of the University Department of Engineering who reproduced the plates.

My supervisor, Jim Bennett, has been a constant source of encouragement and an essential guide in my research, particularly in the early stages of the project.

Finally, my thanks also go to my husband, Mike, for his patience and support over the last three years, for his assistance in preparing the computer program used for investigating the Gunter sector and for his amazing ability to deal with recalcitrant word processors.

This dissertation is the result of my own work and includes nothing which is the outcome of work done in collaboration.

A Note on Dating

All dates have been given according to the Old Style (Julian) system, by which the New Year was held to begin on 25th March. This was the system current in England during the seventeenth century.

Introduction

Wenceslaus Hollar is well known for his engravings of cities and towns, his portraits of the members of the court and his copies of works by famous artists, both earlier and contemporary.[1] Within this impressive portfolio there are two apparently inconsequential portraits: one of an austerely dressed divine; the other of a mathematical instrument maker in his workshop, surrounded by the tools of his trade.[2] One would presume that both men were respected within their communities and moved among relatively high social circles, since having one's portrait painted was not an everyday occurrence. The sitter for the former painting was the mathematician and cleric, William Oughtred;[3] his name and major works would be familiar to most historians of mathematics. The latter portrait[4] is a representation of the mathematical instrument maker, Elias Allen; even his *name* fails to appear in any of the standard histories of mathematics. Why is it the case that two men who seem to have had similar importance within their own communities should have received such different treatment at the hands of historians?

It could be argued that there were several instrument makers around in the seventeenth century who could have filled Allen's position and only one William Oughtred. However, this is not the way that their contemporaries viewed the situation, nor even Oughtred himself, as we can see from the introduction to his book, *The New Artificial Gauging Line or Rod*. Here he first recounts that one of his friends spoke of him as 'utterly unknowne'[5] (this was as late as 1630, when Oughtred was fifty-five); later he speaks of Allen as 'a man well knowne and esteemed by all men of his art for

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[1] Information on Wenceslaus Hollar has been taken from Katherine S. Van Eerde, *Wenceslaus Hollar. Delineator of His Time* (Charlottesville, 1970) and Richard Pennington, *A descriptive catalogue of the etched work of Wenceslaus Hollar* (Cambridge, 1982).
[2] Hollar also engraved the gunner and writer, Nathaniel Nye for the frontispiece of Nye's *The Art of Gunnery* (London, 1647). For more information on all these portraits see Appendix 1.
[3] For a reproduction of this portrait see C.K. Aked, 'William Oughtred - An Early Horological Expositor' in *Antiquarian Horology*, 13 (1981), p.193.
[4] In fact an engraving by Hollar of an earlier painting by his friend and colleague, Hendrik van der Borcht.
[5] Oughtred, *The New Artificial Gauging Line or Rod* (London, 1633), p.6.

his skilfulness in making instruments in metal' and mentions him 'being in the company of some gentlemen of good quality and worth' as if this was not unusual for the maker.[6] The latter comment occurs within a story telling how it was only through Allen's defence of Oughtred's method of gauging against that of the well-respected mathematician and Gresham professor, Edmund Gunter, that the Company of Vintners accepted Oughtred's design for a gauging line. We are left with no doubt that Allen possessed at least equal standing with Oughtred amongst his contemporaries.[7]

In his portrait Oughtred is shown carrying a book in his hand, but has no other props; Allen, meanwhile, is surrounded by a plethora of instruments, some complete and some still in the process of manufacture - he has no book, no scroll, no manuscript. The different accessories hint at the easiest ways to approach the two men: Oughtred is accessible through his own writings and those of his contemporaries, making him a relatively available subject for the scholar - it is hardly surprising to find that his biography was written almost eighty years ago.[8] Allen wrote no books and the only known example of his handwriting is a signature in the Clockmakers' Company records (plate 1). Some factual information is supplied by church registers, guild records and contemporary mathematical texts. However, the main evidence for Allen's importance to the mathematical community rests in the instruments which have survived to the present, examples of the very instruments which lie scattered on his bench in the portrait.

It would seem reasonable to infer from the two portraits that seventeenth-century mathematics was both a textually and an instrumentally based subject. Much attention has been paid to the texts by historians of mathematics, but little to the instruments. Nevertheless, these artefacts do not just have antiquarian value, but can, through careful study, shed light on whole new aspects of seventeenth-century mathematics which might never have occurred to the researcher otherwise. In the study

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[6] Ibid., p.10.
[7] However, it is important to note the context within which this was written: Oughtred is unlikely to have been known to the Vintner's Company, whereas Gunter as a Gresham professor and Allen as an instrument maker would have been. Oughtred was not a nonentity in terms of his mathematical learning.
[8] Florian Cajori, *William Oughtred: A great seventeenth century teacher of mathematics* (Chicago, 1916).

of material culture, the history of science in general still has much to learn, but the situation in the history of mathematics is worse than in any other area of the sciences. [9] It has been all too easy to assume that mathematics was then, as now, primarily concerned with ideas (although even in modern 'pure mathematics' this is not necessarily true). That this was certainly not the case in seventeenth-century mathematics, I hope to show in the following pages. Concentrating on ideas as most important tears mathematics from its social and historical setting: we do not learn to *whom* the ideas were important. I hope to show that study of the instruments can yield much information about the activities of the 'mathematical community', the types of people who were involved and the ways in which they understood mathematics - information which is not supplied by written texts because it was accepted as trivially obvious or was a subconscious part of the seventeenth-century conception of mathematics.

It is clear even from the mathematical books written during this period that instruments played a very large part in the everyday practice of mathematics. Of over one hundred and eighty texts published between 1590 and 1653, forty-five were written primarily about an instrument or instruments, and numerous others make more than passing reference to instruments. These are statistics which cannot be ignored. They undeniably point to a society in which an instrumental approach to mathematics was very widely accepted as the norm and few mathematicians worked exclusively in the abstract. It is true that the seventeenth century was a time when there were various heated discussions about the place of instruments within the teaching of mathematics and within the practical aspects of the subject, but the very fact that these discussions occurred at all is an indication of the importance of instruments.

In this dissertation my main concern is to paint as full a portrait of the mathematical instrument maker Elias Allen as possible and to present a detailed

[9] The standard histories of mathematics include Florian Cajori, *A History of Mathematics* (New York, 1919); Dirk J. Struik, *A Concise History of Mathematics* (second revised edition, New York, 1948); J.F. Scott, *A History of Mathematics. From Antiquity to the Beginning of the Nineteenth Century* (London, first edition 1958, second edition 1960, reprinted 1969); David M. Burton, *The History of Mathematics. An Introduction* (Boston, 1985).

discussion of the types of instrument which he manufactured within his workshop. However, I hope to use the central topic to raise some more general issues connected to the history of mathematics, the way in which research in this field is conducted at present and how it could be enriched for the future. I believe that there are various ways in which study of an instrument maker is pertinent to wider questions in the history of mathematics.

With an instrument maker it is perhaps more apparent (although no more true) than with a 'great mathematical thinker' that it is important to think of the individual with reference to the community within which he or she lived and worked. Allen was the first identifiable English instrument maker to be able to support himself solely through making and selling instruments. The reasons lying behind such success cannot be found purely by study of the individual life but must be sought within the wider community which affected the state of Allen's trade. Thus it is immediately obvious that a biography of Allen must be extended to consider the mathematical culture of England, in particular that part of it centred in London.

Therefore, I will begin by setting the scene in the mathematical community of the first half of the seventeenth century. By the term 'mathematical community' I wish to imply that group of people who were connected with mathematical practices or study in any way. It is intended to be inclusive of practitioners such as surveyors, navigators and astronomers; instructors in mathematics ranging from small-time teachers to the Savilean professors; makers of mathematical instruments and booksellers who published mathematical texts; and, besides these, the large number of amateur mathematicians among the gentry and nobility, and those of these classes who patronised the mathematical sciences. Of course, we must avoid viewing this seventeenth-century 'culture' as a single, static entity. When I speak of 'the mathematical culture of seventeenth-century England' I do not wish to infer that this was a single thing which remained unchanging throughout the hundred years. Clearly it did not: each individual inhabited a slightly different world and had a slightly different concept of 'the mathematical', and the main concerns of mathematics in 1700 were decidedly altered from those of 1600. Nevertheless, individual people's views of mathematics did overlap to a great extent and it is due to this fact that this shifting and many-faceted culture can be taken as a whole.

In discussion of the whole mathematical community, I hope that it will become more obvious why it is important to study the development of mathematics as a community discipline. Each branch of the community had its part to play and if one branch of that community, or even (in some cases) one individual, was omitted the whole structure of the community would be affected. In turn, the concerns of the community affected the way in which individuals carried out their roles within the group. Having considered the community as a whole I will turn to focus specifically on Elias Allen. In presenting the details of his life, I will consider the part which the mathematical community played in shaping his career and the ways in which he directed those influences to his own ends. I will also use the events of his life to show the ways in which an individual can partially remould the surrounding culture, and why it is that, from certain viewpoints, makers like Allen are as important to the history of mathematics as the Euclids and the Newtons, the Gausses and the Gödels, despite their apparent insignificance to the modern scholar.

Following on from the study of Allen himself, I will consider some of the major categories of instruments which formed the base of Allen's trade. At this point I will develop the ways in which instruments can be used as a research resource in their own right - how they can be 'read' as 'texts' - and how detailed analysis of individual pieces can lead to a richer understanding of the mathematical culture of the time. Instruments can often illuminate quite different areas from those which texts do: even those texts which discuss the instruments themselves. Material artefacts can bring up issues which are played down in the books on mathematics or which are simply not mentioned at all. Therefore, in these chapters I hope to use the knowledge gained from the study of specific instruments (supplemented by what information is available within the literature) to point out various general characteristics of the seventeenth-century mathematical culture and the role that instruments played within that culture. That is not to say that I will ignore the texts: part of the 'reading' of instruments involves learning how to think in the language of the culture in which the instruments are created, and here we can be helped by studying the mathematical books produced. Thus research employing instruments should be combined with what has been learned from primary and secondary textual sources in order to provide a more rounded overall picture, a 'thicker' description. The section on Allen's instruments will conclude with a catalogue of the surviving examples held in museum collections in Britain.

With the role of the instruments more clearly defined, it will be easier to comprehend the importance of the instrument maker to the whole, and to understand why Elias Allen was as highly regarded as William Oughtred and why it may not be such a surprise to find his likeness in Hollar's portfolio.

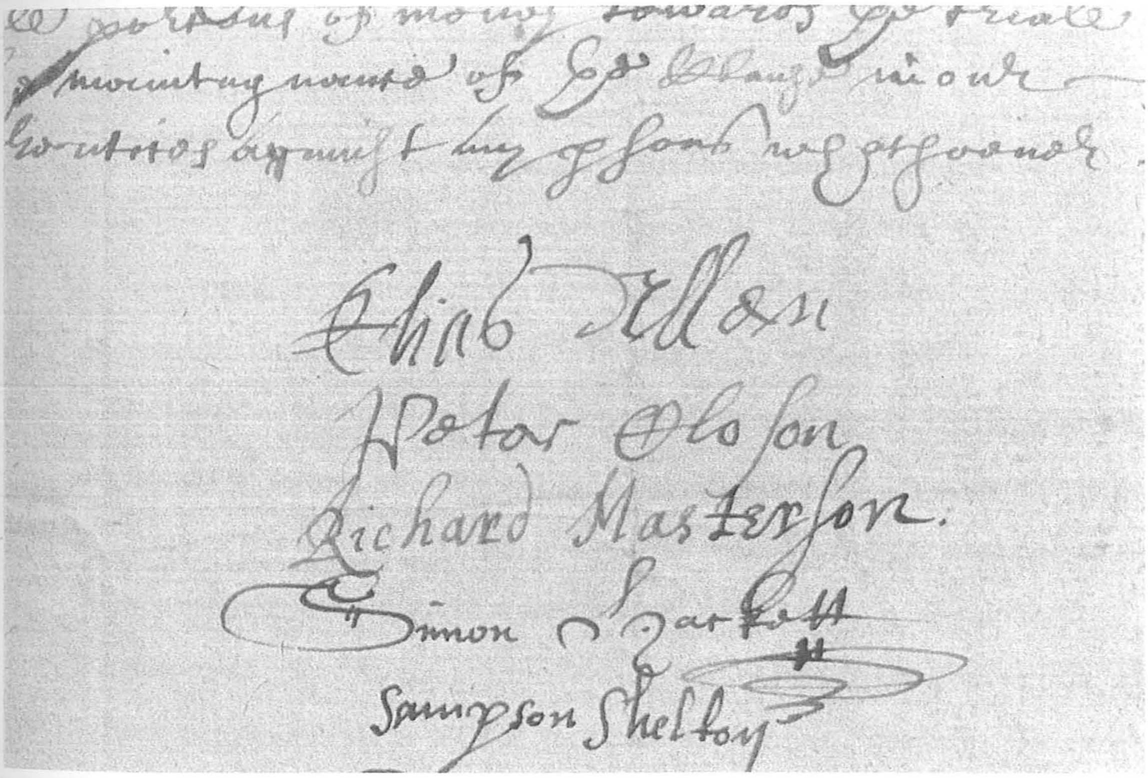

Plate 1: Elias Allen's signature in the Court Minutes of the Clockmakers' Company, Guildhall Library MS 2710, vol.1, between f.18 and f.19

CHAPTER ONE

Elias Allen's setting: the mathematical culture

History is primarily the study of people of the past but, more than this, it is the study of people within the context of a community. Thus it is important to give a broad outline of the setting within which Allen's life was played out, before the focus is narrowed to the individual. The function of the present chapter will be to provide that setting. I have chosen to concentrate on the mathematical culture of which Allen was a part, since it was within this area that his instruments would have had their greatest impact, and therefore I will be concerned mainly with the people, the institutions, the practices, the ideas, the books and the artefacts which were most clearly involved in this field.

One major way in to the world of seventeenth-century mathematics is through the study of contemporary written texts although, as I have noted, we must be aware of the fact that, by themselves, the texts will not provide the whole picture (or even, necessarily, the greater part of it). In seeking to understand the nature of the mathematical culture of the time, I undertook a literature survey, confining myself to what could be defined as 'mathematical' texts (see below, for my criteria in defining the limits of this survey). One obvious purpose was to search for references to Elias Allen in order to establish who were his direct contacts within the community, who recommended his work, and any incidental details of his life which arose from these references, which could be used to flesh out what was already known. However, the texts also lead to questions concerning the ways in which people approached mathematics, teaching practices, the role of mathematical instruments in the culture, the main areas of interest and so forth. The information gathered from these texts reveals much about the sphere of 'the mathematicals' in this period.[1]

This chapter will present the reasoning used in selecting the books and the ways the field was limited by subject and date; some general comments will be made on the areas of interest which were brought to light through this survey, with some statistical analysis of the data generated by my research; finally, I will highlight some of the different areas of the mathematical community, as they are presented both in the light of this study and also through the secondary literature covering this time. I am very much aware that a great deal of scholarly work has been devoted to the study of English mathematics and the mathematical community of London during this period, beginning with Taylor's invaluable (if persistently frustrating) *Mathematical Practitioners of Tudor and Stuart England*,[2] and that there is no value in a mere regurgitation of these works. *My* aim is to provide a framework within which to place Elias Allen.

The Printed Mathematical Texts of Seventeenth-Century England

In carrying out any literature survey it is clear that limits must be set; the problem arises in determining how to establish those limits. I chose to restrict myself roughly to the period of Elias Allen's life[3] and to concentrate on books which could be described as 'mathematical' in seventeenth-century terms. My referents for the term 'mathematical' arose largely from the contemporary use of this adjective in phrases such as 'the mathematical sciences', 'mathematical practitioners', and 'mathematical instrument maker'. Thus I accepted the following as mathematical:

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[1] This type of study is also affected by the contemporary culture of reading and the use of rhetoric in writing. I have not discussed these two areas but there has been a good deal of recent literature on the subjects. The former is dealt with in Roger Chartier, *The Order of Books*, translated by Lydia G. Cochrane (Cambridge, 1994) and Adrian Johns, *Wisdom in the concourse: natural philosophy and the history of the book in Early Modern England* (PhD thesis, Cambridge, 1992); the latter is approached in Alan G. Gross, *The Rhetoric of Science* (Cambridge, Massachusetts, 1990), Marcello Pera & William Shea (eds.), *Persuading Science: the Art of Scientific Rhetoric* (Canton, Massachusetts, 1991) and John C. Briggs, *Francis Bacon and the Rhetoric of Nature* (Cambridge Massachusetts, 1989).
[2] Cambridge, 1954.
[3] c.1588-1653.

arithmetic, geometry, algebra, trigonometry and logarithms among the more theoretical aspects of mathematics; astronomy, navigation, surveying (both civil and military), geography, gunnery, dialling, accounting/book-keeping, gauging, and other forms of mensuration - these all fell within the field of practical mathematics; books on instruments relating to any of the above subject areas; books on general mathematics and on what could be described as 'mathematical curiosities;'[4] books of mathematical tables (such as astronomical tables, tables of interest &c.). I chose to ignore those natural philosophical books with no mathematical content (although these were often written by people who were deeply involved in the mathematical community) and astrology books which did not have a mathematical content. I also avoided almanacs and similar publications: this was more due to the pragmatic constraints of time than anything else - since thousands of almanacs were published during the course of the seventeenth century it was simply impossible to include them within the bounds of a much wider survey.[5]

My survey began by a search through the *Short Title Catalogues* of Pollard & Redgrave and of Wing for all those titles likely to be related to 'the mathematical'. I also added any which I discovered to be mathematical texts but with titles that did not necessarily betray their content. I cannot claim to have produced an exhaustive list of *all* the books written on mathematical subjects but I believe that I have covered a very high proportion of them.

My chronological limits were roughly defined by Elias Allen's lifespan: therefore, I concentrated on reading those books which were published between 1590 and 1653 inclusive. These books formed the core of my study: I also considered a few of the major texts prior to this date which are recommended in contemporary

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[4] These latter including the 'Think of a number...' style of conundrum.
[5] Of course there is a great deal of information to be gained from almanacs which would be worth extensive investigation. The best source for information on almanacs is Bernard Capp, *Astrology and the Popular Press. English Almanacs 1500 - 1800* (London, 1979). The information on almanacs with is incorporated in the graph of mathematical books published (see below) has been taken from the 'Bibliography of English Almanacs' in this book (pp.347-386).

reading lists, because these clearly influenced mathematicians during the early seventeenth century. Among these books were such staples as Recorde's text books on arithmetic, geometry and astronomy; Billingsley's edition of Euclid, with the famous *Mathematicall Praeface* of John Dee and Robert Norman's *The newe attractive*. At the other end of the period, I gathered a list of titles from Wing's catalogue for the years 1654 - 1700 so that some kind of idea could be formed of the continuing trends in the development of the mathematical sciences and their applications. This latter group must obviously be treated with some care since a title does not necessarily reveal the contents of the book and no doubt I have misappropriated some books and have omitted others which should have been included.[6]

A preliminary discussion of the survey

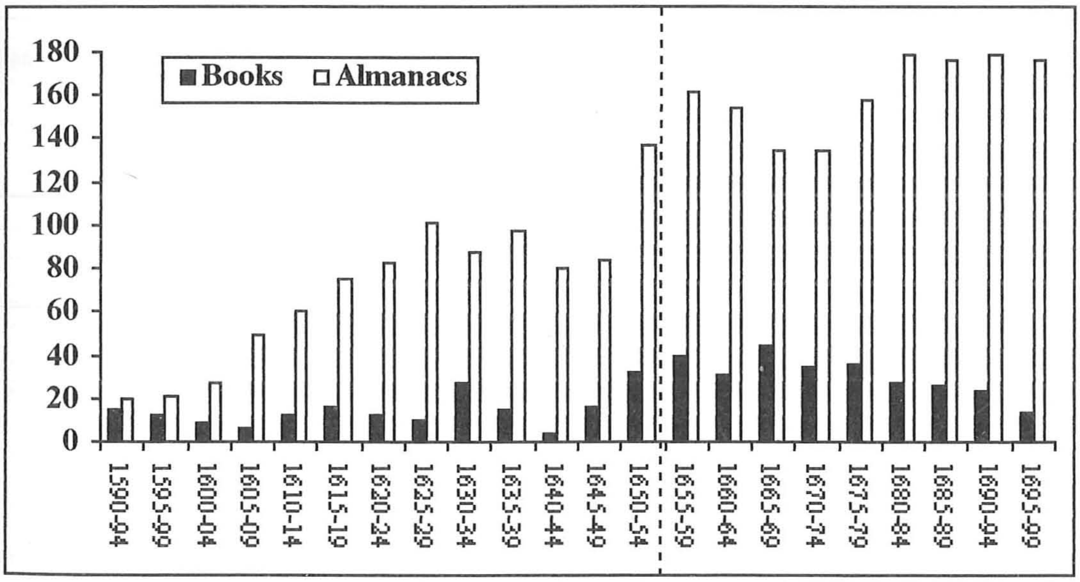

Figure 1: Mathematical texts produced between 1590 and 1699

[6] I used the British Library catalogue here, as well as the Wing catalogue, since the former supplies longer titles than the latter.

Over the 110-year period studied over four hundred and sixty new books on mathematical subjects were published in Britain, all, bar a handful, in London. Of these, approximately one hundred and eighty first appeared in Allen's lifetime. It can be seen from the accompanying graph that the number of mathematical books being produced increased considerably in the second half of the century, but the fluctuations are extensive and the peak came with the surge in popularity of the mathematical sciences and natural philosophy around the time of the foundation of the Royal Society. It is unsurprising to find that the years of the Civil War were a lean time for the mathematical book trade. The figures also reflect a growing number of books coming off the presses in general: according to Hirst "By 1640 most provincial towns had a bookseller, often more than one. The number of works printed annually in England between 1600 and 1640 rose from 259 to 277; thereafter it soared."[7]

In subjecting the books from the first half of the century to more detailed statistical analysis I asked various questions. The main ones were:

(1) What is the main focus of this book?

(2) What aspects of mathematical theory are expounded?

[7] Derek Hirst, *Authority and Conflict. England 1603 - 1658* (London, 1986), p.95. Figures can be obtained for the total number of books published before 1640 by summing the lists in the Chronological Index of Pollard & Redgrave. However, there is no such index in Wing; in order to give some rough idea of numbers I have taken the number of books in the British Library catalogue (CD-ROM version) which were published in London, Edinburgh, Oxford and Cambridge (the four main publishing cities in Britain in the seventeenth century). The results are displayed in the table below. [STC = Pollard and Redgrave, *Short Title Catalogue*, BL = British Library catalogue]

Years	1590-1594	1595-1599	1600-1604	1605-1609	1610-1614	1615-1619	1620-1624	1625-1629	1630-1634	1635-1639
STC	1420	1540	1829	2116	2247	2515	2979	2648	3263	3153
BL	688	721	827	1029	1035	1006	1158	1094	1306	1407

Years	1640-44	1645-49	1650-54	1655-59	1660-64	1665-69
BL	5970	5518	3563	3752	3780	1717

Years	1670-74	1675-79	1780-84	1685-89	1690-94	1695-99
BL	2597	3084	5087	4042	3956	3782

(3) What areas of practical mathematics are included in this book?

(4) What was the intended readership of the book?[8]

(5) If the book was written after 1614 (the year of publication of Napier's *Mirifici Logarithmorum Canonis descriptio*), does it make mention of logarithms?

(6) How many editions did the book have?

(7) Does the book refer to Elias Allen at any point?

The answers to these questions were used to develop a broad sense of the trends in and the structure of the mathematical culture of the period, insofar as these are revealed within the printed texts. Some of the results were expected; some confirmed standard interpretations of mathematics at this time; some were very surprising indeed.

Turning first to the question of the main focus of the books we find that nearly a quarter (24%) of them are devoted to mathematical instruments in some way. The other major areas of interest seem to have been navigation (11%), arithmetic (10%), astronomy (7%), dialling (6%) and trigonometry (5%). Those which are mainly concerned with mathematical theory, with little application to practical mathematics, amount to less than a sixth (15%) of the total. There is a very clear emphasis on the practical aspects of mathematics and on the use of mathematical instruments, and it is interesting to draw this from a literature survey, where one might expect ideas to dominate.

The results from the question concerning the theoretical content of the book are even more striking. Eighty-three of the books do not address mathematical theory to a great extent at all. Of these, over a quarter are books about instruments and one fifth are books on navigation; most of the tables of interest fall into this category as

[8] The answer to this question was mainly drawn from material in the prefaces and title-pages of the books studied. This is, of course, something of a simplification: the implied readership, even if clearly stated, is not necessarily the intended readership which in turn does not always coincide with the actual readership.

well. Among the remaining books the subjects of geometry, trigonometry, arithmetic and logarithms are fairly evenly spread (thirty-nine texts, thirty-one, thirty-four and twenty-eight respectively); algebraic theory is discussed in just six books. This is a great contrast to the presentation of the seventeenth century in most histories of mathematics as the era of the emergence and triumph of algebra. While it is true that algebras slowly became more popular during the latter half of the century, before 1653 Thomas Harriot's *Artis Analyticae Praxis* was the only book printed in Britain dedicated solely to the discussion of this branch of mathematics. The dominance of geometry (both theoretical and practical) is very clear indeed, particularly when trigonometry is considered as a subsection of geometry. This dominance will be a constantly recurring theme throughout the course of my thesis, since it is a central feature of the mathematical culture of this period and must therefore be acknowledged as such and not played down.

The answer to the question 'What areas of practical mathematics are included in this book?' reveals the following information: the most popular area of interest (among writers of mathematical texts at least) appears to have been astronomy (covered in nearly a third of the books). Navigation also figures largely (almost a quarter), as do dialling, surveying and accounting (15%, 15% and 12% respectively), and it is interesting to see that the order astronomy, navigation, surveying, follows the history of the introduction of mathematical techniques and instruments to these areas - astronomy earliest, navigation second and surveying bringing up the rear.

It is not always easy to be certain about the intended readership of a book, although explicit indications are sometimes given in titles and prefaces, while the content of the text is often a clue in other cases. The most frequently recurring categories are navigators, surveyors and gentlemen: the targeting of navigators and surveyors is unsurprising, given the large numbers of texts dealing with navigation and surveying, but the assumed audience among the gentry and nobility is notable. It is perhaps indicative of an increasing interest in mathematics observed among the gentry at this time; this is a subject to which I will return at various points in my discussion.

The data obtained from answering questions (5) and (6) can be dealt with swiftly. Thirty-nine out of one hundred and nineteen books published after 1614 make use of logarithms, thus demonstrating the significant effect which this innovation had on the mathematical community in the years immediately following their inception (see also Figure 2, page 45). In assessing the popularity of the books according to the number of reprints or editions which they had, fairly clear trends are discovered. Most books appeared only in one or two editions but there were others which were reprinted or even republished time after time. The two categories most often represented here are books on navigation and arithmetics. As an extreme example, both Richard Norwood's *The Sea-Mans Practice* (1637) and Edmund Wingate's *Arithmetique made easie* (1630) (which includes a large section on the use of logarithms and the application of the logarithmic rule) were issued at least twenty times and did not disappear from the booksellers' shelves until the middle of the eighteenth century.

A striking number of books mention Elias Allen. Of some hundred texts published before his death, nineteen make reference to Allen, more than twice as many as mention John Thompson, the most prominent of the instrument makers working in wood at this time.[9] Other instrument makers appear in one or two books, but no more. There is therefore clear evidence from the printed mathematical texts of the period that Allen was seen as the pre-eminent maker, at least among the book-writing section of the community.

There were two further exercises which I took up in the conclusion of this statistical analysis. One was to construct a table (see following page) summarising the connections between the mathematical theory covered in a book and the areas of

[9] There are also twelve advertisements for Allen in the almanacs of the period (see Appendix 3).

practical mathematics which were also addressed. Unsurprisingly, arithmetic finds its most common application in accounting and book-keeping; trigonometry is most often used in navigation and astronomy; algebra hardly finds any application in practical mathematics at all. Geometry dominates (especially when trigonometry is incorporated within it) and is clearly the most important part of mathematical theory when related to applications (apart from in accountancy).

Theory / Application	None	Arithmetic	Geometry	Trigonometry	Logarithms	Algebra
Navigation	22	7	12	13	10	
Astronomy	27	8	17	16	10	
Surveying	13	4	17	4	3	
Dialling	13	5	8	5	6	
Geography	5	5	8	1	4	
Accounting	8	14			4	
Gunnery	1	2	3	1		
Ship-building	1	1	1			
Cosmography	3	2	3			
Gauging	6	4	2	1	4	
Military Order		5	1		4	1
Music		1	1			
Optics		1	1			
Architecture		1	1			
Statics	1	1	1			
Mechanics	1	1	1			
Mensuration	4	1	1		1	
Time-keeping	3					
None		10	4	7	8	4

The second exercise was to total the books over five year periods from 1590 through to 1699. This included looking at the major subject categories separately - instruments, astronomy, navigation, surveying, dialling, gauging, tables of interest, arithmetic, geometry, trigonometry, logarithms and algebra (see Figure 2, page 45).[10]

It is immediately clear that books on instruments retained their appeal throughout the century, as, to a lesser extent, did those concerning navigation. Arithmetics made more impact in the second half of the century, when they were produced in large numbers. There was also a growth in interest in algebras after 1660. Dialling attracted significantly more interest in the second half of the century with around fifteen books being produced after 1650 as compared to six before this date. It appears that surveying and gauging as well became more popular towards the end of the century. On the other hand, logarithms featured prominently in the twenty years following the announcement of their discovery by John Napier, but after this they apparently ceased to be of interest. In a similar fad, tables of interest and conversion came off the presses in fairly substantial numbers through the middle of the century but were no longer being produced in any quantity by 1680.

Using texts to explore the mathematical culture

Having subjected the mathematical texts to some fairly detailed statistical analysis it is time to turn our attention to the community which created them. For it goes without saying that these books did not exist in isolation: they were written by particular individuals, for the edification of many different people, within the context of a broad group of scholars and craftsmen, gentlemen and traders, all of whom could be described in one way or another as playing out significant roles in the mathematical culture. The authors of the books which I have surveyed came from many different backgrounds, but all were bound together by the common interest in

[10] It must be stressed that the allocations for books published after 1653 may not be accurate, since they were made on the basis of the short titles and nothing else; this also explains the increase in numbers for the category "Other" in the second half of the century - books I was unsure about were simply relegated to this category along with the subjects which did not figure prominently in the survey as a whole.

things mathematical. The complex structure forming the mathematical culture of seventeenth-century England can be divided, to a certain extent, into subsets, as long as it is acknowledged that the boundaries are extremely vague. My concern is simply to describe the setting for the study of Elias Allen by sketching out the staging, props and cast of the drama in which he has a central role.

I have taken five broad categories of authors of the mathematical texts. There are what might be termed the 'academic mathematicians' - the men who had a university training and who devoted a large proportion of their time to the study of mathematical theory. There are also mathematical teachers - the numerous mathematicians who drew most of their income from private tuition in mathematics. Then there are the mathematical practitioners - the surveyors, navigators, gunners and others, who used mathematics in practising their various professions. Another subset is made up of the gentleman amateurs who pursued an interest in mathematics, but who did not depend upon it for a living. The makers of mathematical instruments form a natural fifth group.

The scene of our play is essentially London. Although other towns and cities may have a part in this narrative, I believe that it must be made clear from the beginning that the hub of the mathematical culture of this period was the capital city. The role of the universities in seventeenth-century English mathematics has been discussed extensively in recent years and Mordecai Feingold's *The Mathematician's Apprenticeship* presents a good deal of evidence to counteract the perceived dearth of mathematical teaching at Oxford and Cambridge.[11] Nevertheless, it cannot be denied that the vast majority of mathematical learning and practice was centred in London, and it should become clear that this spatial compactness of the mathematical community had a very important effect on the way in which mathematics developed in England over this period.

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[11] Cambridge, 1984. For other discussions of the place of science and mathematics in the universities of this period see Robert G. Frank, Jr., 'Science, Medicine and the Universities of Early Modern England' in *History of Science*, xi (1973), pp.194-216 & pp.239-269, and Mark H. Curtis, *Oxford and Cambridge in Transition 1558-1642* (Oxford, 1959), pp.227-260.

The 'academic mathematicians'

Let us begin with the 'academic mathematicians'. This is the group which has been accorded the most time and discussion in previous research and which figures most prominently in the standard histories of mathematics. John Napier, Henry Briggs, William Oughtred, Edmund Gunter, Sir Henry Savile, Thomas Hariot - their names are well known to most people who have more than a passing interest in the history of mathematics. These were the university-educated who devoted much of their energy to the pursuit of mathematical theory and who held the few chairs in mathematical subjects which were available at this time. They were the most dispersed set geographically, as few of them lived permanently or for prolonged periods in London - Napier rarely left Scotland; Oughtred's home was Albury parsonage in Surrey - and the university towns (particularly Oxford) figured to some extent in their later careers. Yet it was Gresham College in London which became their centre of gravity; perhaps because it was the long-time residence of Henry Briggs, a central figure in this set.

Gresham College was the brainchild of the Elizabethan financier, Sir Thomas Gresham, adviser to the Queen and founder of the Royal Exchange. Gresham believed that London needed an educational establishment which would satisfy the requirements of the merchant class of the city. His thoughts were not purely those of a wealthy philanthropist, wishing to facilitate the instruction of the masses; nor did he intend to create a university of London in conflict with the ancient seats of learning at Oxford and Cambridge (although the two universities did partially view the new college as a rival). Realising the need for England to develop and rely upon its own resources, and taking the view that economic security could not be established without widespread education, he made provision in his will for the foundation of a college which might provide the necessary environment for trade and commerce to flourish.

Although Gresham died in 1579, for various reasons[12] the college was not established until 1597 and although professors were appointed at that time, the lectures did not immediately begin in any coherent form. Gresham had made provision for seven chairs - one each in medicine, law, rhetoric, divinity, astronomy, geometry and music - and lectures were to be given twice weekly, once in Latin and once in English. The stipend of each professor was to be fifty pounds per annum.

Thus Gresham College was the first institution in England to provide professorships in mathematics. The nature of the lectures which the professors were required to give was fairly strictly regulated by the constitution of the college and it is immediately obvious that the committee had the needs of a practical community in mind when they drew up the ordinances relating to the lectures in astronomy and geometry:

> 'Touching the matter of the said solemn lectures, the geometrician is to read as followeth, *viz.* every Trinity term arithmetique, in Michaelmas and Hilary terms theorical geometry, in Easter term practical geometry [i.e. surveying]. The astronomy reader is to read in his solemn lectures, first the principles of the sphere, and the theoriques of the planets, and the use of the astrolabe and the staf, and other common instruments for the capacity of mariners; which being read and opened, he shall apply them to use, by reading geography and the art of navigation in some one term of the year.'[13]

Edward Brerewood was chosen as the first professor of astronomy; his counterpart in geometry was Henry Briggs. The former appears to have made very little impact within the mathematical community but the latter took a major part in the affairs of mathematical London.

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[12] The main ones being the residence of his wife, until her death, in the building designated for the new college, and the internal wranglings of the Mercers' Company.

[13] Ward, *The Lives of the Professors of Gresham College* (London, 1740), viii.

Briggs was not a prolific writer, his only published work being a posthumous table of logarithms,[14] but he had an extensive network of contacts which drew together most of the 'academic mathematicians' who were his contemporaries. He held office at Gresham College for twenty-five years, before taking up the newly-created Savilean Chair of Geometry at Oxford, and during this time his rooms in Gresham became a centre for discussion and correspondence. He had contact with Napier also, and was so impressed by the latter's work on logarithms that he made the long journey to Scotland in order to learn more. He was a mentor of Edmund Gunter, pressing the young mathematician's suit when the Gresham astronomy chair became vacant after Brerewood's death, and finally securing it for him after the incumbency of Thomas Williams. Briggs also had extensive correspondence with William Oughtred and it was through him that Oughtred first made acquaintance with Gunter.

After Briggs' departure for Oxford in 1619, the centre for discussion moved to the astronomy professor's rooms where Edmund Gunter had recently taken up residence. The focus of activity remained here when Henry Gellibrand succeeded Gunter and was in turn followed by Samuel Foster in 1636. The latter vacated his post the following year (according to Ward, he was ejected for failing to kneel at communion) but was reinstated in 1641 and his time in office, as described by Adamson, was perhaps the heyday of mathematics and natural philosophy at Gresham College:

> '[Foster's rooms were a] haven for scientists of all parties and a centre of instruction and discussion for all who cared to attend and Professor Taylor has asserted that "...it was in Foster's chamber that Royalist mathematicians and scientists expelled from the universities used to meet after his lectures to discuss new instruments, new experiments and hypotheses...".'[15]

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[14] Henry Briggs, *Logarithmicall Arithmetike. Or Tables of Logarithmes...with a plaine description of their use in Arithmetike, Geometrie, Geographie, Astronomie, Navigation, &c.* (London, 1631).

[15] Adamson, *The Foundation and Early History of Gresham College London, 1596 - 1704* (PhD Thesis, Cambridge, 1976). Unfortunately the more accessible 'The Royal Society and Gresham College 1660-1711' by Adamson in *Notes and Records of the Royal Society of London*, 33 (1978-79), pp.1-21, does not deal with the period under consideration here.

After Foster's death the influence of Gresham declined. Adamson has shown that the standard of the teaching supplied by the Gresham lectures (when they happened at all) was very much a hit-and-miss affair. The professors had learnt early on that the supervisory committee of the College had very little power to enforce the rules governing the teaching offices, and so it was purely due to the dedication of some of the early holders of the geometry and astronomy chairs that the College became such a centre for London's mathematicians. Once the mathematical professorships of Oxford and Cambridge were created,[16] the chairs of Gresham lost much of their appeal and tended to be occupied by less eminent scholars. An important result of the movement of the centre of gravity towards Oxford was that the close contact with the world of practical mathematics (so prevalent in London) was eroded for a time, the links only being firmly re-established with the creation of the Royal Observatory at Greenwich in 1675.

Whilst it might be expected that these 'academic mathematicians' would be the ones who would have least to do with practical mathematics, with mathematical instruments and with their makers, this is not found to be the case for most of the individuals in the group. John Napier, although better known for his work on logarithms, at an earlier stage in his attempts to find easier calculating techniques developed the counting rods which still bear his name - Napier's bones - and which were popular in the seventeenth century as one of the best means for simplifying the processes of arithmetic. Thomas Hariot was an astronomer of note, dedicating much time to the development of the telescope and to the discussion of observations made with navigational instruments such as the cross-staff. Briggs collaborated with Edward Wright at great length to produce more accurate observational tables for the second edition of the latter's *Certain Errors in Navigation.* Edmund Gunter was a prolific designer of instruments for use in mathematical calculation and navigation: he is famous for his sector, his quadrant and his rule, the last being the first device to

[16] The Savilean chairs in Oxford in 1619, the Lucasian chair of mathematics in Cambridge in 1663.

incorporate the new logarithmical methods of computation. Henry Gellibrand devoted much time to dealing with the problem of magnetic variation of the compass which had plagued navigators for so long; it was he who first drew attention to the fact that the variation was not only affected by position but had a secular element as well.[17] He is remembered by Aubrey for a 'fine sundial'.[18] The same author's very brief note on Foster is largely concerned with dialling, for which skill Foster was apparently widely known. Aubrey remarks that Foster

> 'was professor at Gresham College, London: where, in his lodging, on the wall in his chamber, is, of his own hand drawing, the best sundial I do verily believe in the whole world. Among other things it shows you what o'clock 'tis at Jerusalem, Gran Cairo etc. It is drawn very skilfully.'[19]

William Oughtred, while having his main research interests in algebra, nevertheless found time to develop several instruments, which became popular largely through his friendship with Elias Allen: these were the circles of proportion (for logarithmic and other calculations), the horizontal instrument and double horizontal dial and the universal equinoctial ring dial.

With such a great interest in practical mathematics and in mathematical instruments, one begins to wonder whether there were any members of the mathematical community who were averse to the use of instruments and who might look down on those who devoted time to developing instrumental techniques. The importance of instruments even to the learned section of the community certainly fits with the information gathered from the study of the mathematical texts - a significant proportion of the books written by this group of people were actually related to the instruments which they had designed, rather than to the more theoretical aspects of

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[17] Evidence for secular variation had first been provided by Gunter when he repeated the experiments of Edward Borough at Limehouse. However, it is unclear whether Gunter was aware of the significance of the differences in values or whether he simply assumed that Borough's measurements were wrong. See *De Sectore et Radio* (London, 1623), p.66.
[18] John Aubrey, *Brief Lives*, ed. Richard Barber (Woodbridge, 1982), p.117.
[19] Ibid., p.114.

mathematics. Indeed it comes as something of a surprise to find that anyone might be condemned for trifling with instruments, and perhaps we can see Aubrey's account of Gunter's encounter with Sir Henry Savile in a new light.

> '[Savile] sent for Mr Gunter from London, (being of Oxford University) to have been his professor of Geometry: so he came and brought with him his sector and quadrant, and fell to resolving of triangles and doing a great many fine things. Said the grave knight, "Do you call this reading of Geometry? This is showing of tricks, man!" and so dismissed him with scorn, and sent for Briggs from Cambridge.'[20]

Presumably Briggs was more circumspect than Gunter in his advocacy of instruments and practical geometry. At any rate, he was granted the post and Gunter was not.

The books of Gunter and Oughtred bear witness to a fairly close relationship between Elias Allen and both these men. Allen certainly provided the frontispiece for Gunter's work on the sector - the engraving is identical to that found on many of Allen's Gunter sectors. He was also a provider of Gunter quadrants and it seems that Gunter's instrument designs generally found a faithful realisation in the maker's workshop. The relationship with Oughtred was even closer: Oughtred often referred to Allen as his friend and the various instruments which the mathematician had created were presented to a wider public partly in order to increase trade for the instrument maker. Oughtred used Allen's workshop as a point for depositing and collecting mail, and the proximity of Allen's premises to Arundel House (Oughtred's London residence, by virtue of his position as William Howard's tutor) would have enabled a ready interaction between the two. It has already been noted that Oughtred

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[20] Ibid., p.279. Aubrey's accounts must, of course, be taken with a certain amount of circumspection: for reference to this anecdote see Willmoth, *Sir Jonas Moore* (Woodbridge, 1993), p.3 and Stephen Johnston 'Mathematical Practitioners and Instruments in Elizabethan England' in *Annals of Science*, 48 (1991), pp.342-3 and note 88.

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was prepared to accept suggested alterations to instrument designs from Allen.[21] Thus we find Allen firmly linked to the higher ranks of the mathematical community, despite his position as a craftsman. The relationship was not simply one of the artisan fulfilling the commissions of scholars (although that he did this is witnessed by the instruments produced for the Savilean professor of astronomy, John Greaves); Allen was respected as a man whose opinion was worthy of attention.

The mathematical teachers in London

In the list of authors drawn up from the literature survey, it is in fact not the 'academic mathematicians' but those whom I have termed the 'mathematical teachers' who dominate. This is hardly surprising when the needs of this section of the community are considered. Writing and publishing textbooks would have provided them with an easy way to supplement their unstable income. The types of book which they wrote are also a reflection of their role: they were the most prolific authors of arithmetics and geometry primers, and they produced numerous books on instruments, in which they generally advertised their willingness to instruct the readers further if the latter were prepared to repair to such and such an address for tuition. Seth Partridge's self-praise is typical: he declares himself capable of offering instruction in arithmetic (whole numbers and fractions, decimals, roots, astronomical fractions, algebra, arithmetical rods [i.e. Napier's bones] and the arithmetical jewel of William Pratt), geometry (principles, gauging, surveying, use of the plane table, circumferentor, theodolite, circular scale, quadrant, semicircle, peractor, sector, circles of proportion and Wingate's lines of proportion), trigonometry (use of logarithmic trigonometric tables, measuring of heights etc., doctrine of triangles), navigation (including the use of instruments, maps and charts), cosmography (use of

[21] The relationship between Oughtred and Allen is described at greater length in the following chapter.

globes, the armillary sphere, the astrolabe and Blagrave's mathematical jewel) and dialling (both fixed and instrumental).[22]

Despite the number of books which this section of the community produced, they are one of the most elusive subsets of the mathematical culture. Little can be drawn out of their books about their backgrounds or their means of support, beyond their self-identification on title pages as teachers or professors of mathematics, or as 'mathematical practitioners'. They are not often mentioned in contemporary anecdotal sources nor in institutional records. Nevertheless, there is sufficient evidence in the contemporary literature and more recent historical studies to be able to make some general remarks about them as a group and about the different ways in which their activities as teachers of mathematics were viewed by those around them.

The demand for mathematical education was of relatively recent origin. The great champion of the importance of mathematics to trade, navigation, surveying and other pursuits had been the astrologer, alchemist and mathematician, John Dee. His vociferous support for all things mathematical had raised the awareness of the upper classes at least to the utility of mathematics for performing everyday tasks such as regulating accounts.[23] Meanwhile English naval expansion increased the number of people requiring instruction in mathematical navigation. It was to meet the needs of such clients that the mathematical teachers began to proliferate in London towards the end of the sixteenth century.

Of course the Gresham lectures were available as sources of mathematical tuition for any who wished to attend. However, the professors could not always be relied upon to appear at the scheduled time and there were many who either did not wish to stoop to attending public lectures or who valued the individual attention obtained through private tuition. Those who had the money to pay a private mathematics teacher ran the risk of receiving a far less adequate knowledge of

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[22] Seth Partridge, *Rabdologia: or The Art of numbring by Rods* (London, 1648), final page.
[23] See Dee's *Mathematicall Praeface to the Elements of Geometrie of Euclid of Megara*, published in Henry Billingsley's 1570 edition of Euclid.

mathematics than they would have obtained through the lectures, but they also placed themselves in an environment where they were readily able to question and to raise points which they did not understand. One-to-one discussion also made teaching the use of instruments much easier, and such tuition often featured prominently in the advertisements of the 'professors of mathematics'.

Nevertheless, the private teaching of mathematics was clearly not a lucrative business since most of the men offering their services in such a capacity derived some part of their livelihood from other sources. It has already been observed that many published books on mathematics; they also pursued other professions to supplement these supplies. Thus Thomas Bretnor practised as a physician and wrote almanacs; John Tapp was a bookseller for many years; Ralph Handson held the office of auditor for casting up the accounts of the Court of Chancery; Charles Saltonstall became a mathematics teacher only after an active life as a captain in the merchant navy; William Leybourne began life as a printer, before turning his hand increasingly to mathematics; many also undertook work as surveyors.

The standard of tuition also varied enormously and this is reflected in the remarks made by their contemporaries. According to Taylor,

> 'A glowing account was given by an almanack-maker, G. Gilden, of the position of practical mathematics in London in 1616. "Never were there better or nearer helps to attain [mathematical knowledge] than at this present, in this City", he declared, pointing out that excellent text-books, methodical instruction by learned professors, and exact instruments were at everyone's disposal.'[24]

Apparently this was a great change from the situation twenty years previously when William Barlow had lamented the lack of decent teachers of mathematics. His astonishment in finding one good teacher (John Godwyn) led him to praise this man at great length:

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[24] Taylor, *Mathematical Practitioners*, p.58.

'A man vnskilfull in the Lattin tongue, yet hauing proper knowledge in Arithmetike, and Land-measuring, in the vse of the Globe, and sundry other Instruments: And hath obteined, partly by his owne industrie, and by reading of English writers (whereof there are many very good) and partly with conference with learned men, (of which hee is passing desirous) such ready knowledge and dexteritie of teaching and practising the groundes of those Artes, as (giuing him but his due) I haue not beene acquainted with his like. *And great pitie it is that in so populous a place, many such were not employed* [my italics]:'[25]

Gilden's sanguine view of the state of mathematics teaching was not shared by the scholar, Sir Francis Kynaston. Although prepared to admit that there were indeed many teachers available in London and that some at least were worthy of their profession, he was alarmed by the variability in competence of these teachers, remarking that

'diverse strangers professe to teach sundry, or rather all the liberall arts and sciences of which many have been found to be upon examination, and triall egregiously ignorant, whereby our youth loose both their tyme, and money:'[26]

Kynaston was also of the opinion that too many of these professors of mathematics taught their pupils 'rather for gain then any other respect:'.[27]

Opinion was not only divided as to the standard of the teaching offered by the professors of mathematics: the manner in which they proceeded in their teaching was a subject which was much discussed and argued over. Here the main issue was whether instruments should form a part of the didactic process or not. Some of the more theoretically-minded mathematicians and scholars felt that introducing instruments too early in the education of an untrained mind would distract attention from the theoretical aspects of mathematics. They believed that teachers of

[25] William Barlowe, *The Navigators Svpply* (London, 1597), sig.K2 recto-verso.
[26] Sir Francis Kynaston, *The Constitutions of the Musæum Minervæ* (London, 1636), sig.¶¶1 verso.
[27] Ibid.

mathematics should provide a sound grounding in mathematical theory before ever an instrument was shown to the scholar and that any teacher who ordered his curriculum differently was undermining the proper approach to mathematics. This view is clearly seen in the Dedicatory Epistle of Oughtred's *Circles of Proportion*, where he states categorically

> 'That the true way of Art is not by Instruments, but by Demonstration: and that it is a preposterous course of vulgar Teachers, to begin with Instruments, and not with the Sciences, and so in-stead of Artists, to make their Scholers only doers of tricks, and as it were Iuglers: to the despite of Art, losse of precious time, and betraying of willing and industrious wits, vnto ignorance, and idleness. That the vse of Instruments is indeed excellent, if a man be an Artist: but contemptible, being set and opposed to the Art.'[28]

This criticism was probably largely directed at Richard Delamain who had recently crossed swords with Oughtred in a priority dispute concerning the authorship of various instruments.[29] Delamain certainly treated the passage as a personal attack responding vehemently and at great length. He took exception to being called a vulgar teacher and a doer of tricks and pointed out that this reflected extremely badly on those members of the gentry and nobility who made use of private tutors.[30] Delamain argued for his position by saying that many who took an interest in mathematics would be daunted by too much theory at an early stage and that instruments were useful aids for introducing mathematical principles:

> 'And me thinkes in this queasy age, all *helpes* may bee used to procure a *stomacke*, all *bates* and invitations to the declining studie of so noble a *Science*, rather then by rigid Method and generall *Lawes* to scarre men away. All are not of like disposition, neither all (as was sayd before) propose the same end, some resolve to *wade*, others to put a *finger* in onely, or wet a *hand*:

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[28] William Oughtred, *The Circles of Proportion...* (London, 1632), sig.A3 verso.
[29] See Chapters Two and Five for discussion of this dispute.
[30] See Richard Delamain, *Grammelogia, or the Mathematicall Ring* (London, 1633), sig.A7 verso.

> now thus to tye them to an obscure and *Theoricall* forme of teaching, is to crop their hope, even in the very bud, and tends to the frustrating of the profitable uses, which they now know, and put to service, and to the hindering of them in their further search, in the *Theoricall* part, which otherwise they would apply themselves unto: being catched now by the sweet of this *Instrumentall bate*; which debarring would not onely injure the *Studeous* but also cause the *Mechanicke workemen* of these *Instruments*, to goe with thinner *clothes*, and leaner *cheekes*.'[31]

Delamain's hope was that, in the course of studying instruments, his pupils might be drawn into a greater desire to learn more of the mathematical theory underpinning them. He was also realistic enough to accept that there were those who would continue to search out teachers willing to provide tuition in the use of particular mathematical instruments and nothing more. It was hardly the fault of the tutors if their pupils demanded the knowledge of instrumental application while refusing to be instructed in mathematical theory. Turning away such clients would have meant a loss of income, something that few of the independent mathematical teachers could afford. The same point has been made in recent years by A.J. Turner in his article on the education of gentlemen:

> 'In general, gentlemen preferred to take the easy way out. Concerned with immediate practical matters, to be able to use the necessary instrument seemed enough.'[32]

Delamain's reference to the 'Mechanicke workemen' hints at the close connections between the teachers of mathematics and the instrument makers and Elias Allen was no exception in this respect. Many of the mathematical professors indulged in the designing of instruments as a side pursuit and numerous of their books are on the subject of these instruments. It is hardly surprising to find that Allen often

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[31] Ibid., sig.A8 recto-verso. Incidentally, Allen's portrait shows him with good-quality clothes and well-rounded cheeks.

[32] A.J. Turner, 'Mathematical Instruments and the Education of Gentlemen' in *Annals of Science*, 30 (1973), p.58.

featured as the recommended instrument maker in their texts. Allen's workshop was given as the place to purchase Delamain's horizontal quadrant, Speidell's mathematical scale, Wyberd's lunar dial and sets of Napier's bones (discussed by Barton and Partridge).[33] Allen's connections with this section of the mathematical community were extensive and doubtless it was often his instruments that were used in the course of these men's instruction of their pupils, particularly when those pupils came from the wealthier ranks, who could more easily afford brass instruments.

The mathematical practitioners

The category of mathematical practitioners overlaps with the preceding one, since mathematical teachers often supplemented their pedagogical work with practical application of their mathematical skills. This subset of the mathematical community is relatively well represented in the list of printed texts, and it is unsurprising that the subjects of their writings are usually the professions which they pursued and/or instruments which related to these professions. The most common themes were navigation and surveying, although some books were written by gunners on the application of mathematical techniques to the art of warfare. (These include William Eldred's *The Gunner's Glasse* (1646), Nathaniel Nye's *The Art of Gunnery* (1648) and John Babington's *A Short Treatise of Geometrie* (1635) which was written mainly for young gunners.) Of these, the books on navigation were by far the most popular, often running through several editions before becoming obsolete. Apart from navigators, surveyors and gunners, the title of mathematical practitioner could also be applied to architects, accountants, fortifications experts, civil engineers, shipwrights and cartographers.

This is not to say that by any means all of those who practised these trades were versed in mathematical methods. Indeed many of those who *did* know

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[33] See Delamain, *The making, description and vse of...a Horizontall Quadrant* (London, 1630), facing title page; John Speidell, *A Geometricall Extraction...* (London, 1616), sig.A4 recto-verso; John Wyberd, *Horologiographia Nocturna* (London, 1639), p.14; William Barton, *Arithmetike Abreviated* (London, 1634), p.20; Partridge, *Rabdologia*, pp.3-4.

something of the usefulness of mathematical application to their professions lamented that there were a great number who plied their trade ignorantly or inefficiently because they attempted to rely on rules of thumb and estimations of magnitudes and sizes. This was particularly the case in surveying, traditionally the most conservative and least receptive of the professions to which mathematicians had attempted to apply new methods. For centuries the business of surveying had been carried out using the traditional instruments of rod and chain and there was a great resistance to new-fangled mathematical techniques. Even when surveyors did try new instruments they often had little idea of how to use them properly and thus were even more of a liability to their clients than they had been previously.

Both Arthur Hopton and Aaron Rathborne took up this issue at various points in their writing on surveying. Hopton commented:

> 'I know that there be bookes extant treating of the art of measuring ground... but they be lame and defective, euen as a number of our surueyors be, that thrust themselues into businesses without ability to performe'.[34]

Meanwhile Rathborne drew attention to the number of people who attempted to appear knowledgeable of mathematical surveying and of the application of instruments despite not being sufficiently trained in these areas:

> 'simple and ignorant persons...who hauing but once obserued a Surueyor, by looking ouer his shoulder, how and in what manner he directs his sights, and drawes his lines thereon; they presently apprehend the businesse, prouide them of some cast Plaine Table, and within small time after, you shall heare them tell you wonders, and what rare feats they can performe;'.[35]

It is little wonder that the practice of surveying was not held in particularly high regard.[36]

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[34] Hopton, *Baculum Geodeticum* (London, 1610), sig.A3 recto.
[35] Rathborne, *The Surveyor* (London, 1616), preface.
[36] A more detailed account of seventeenth-century surveying is given in J.A. Bennett, 'Geometry and Surveying Early Seventeenth-Century England' in *Annals of Science*, 48 (1991), 345-354. The application of the cross-staff to surveying is discussed in John Roche, 'The cross-staff as a surveying instrument in England 1500-1640' in Sarah Tyacke (ed.) *English Map-Making* 1500-1650 (London, 1983), pp.107-111.

The situation was generally better concerning the application of mathematics to navigation, though there was a similar resistance to the use of new techniques.[37] Navigators were not generally averse to the use of instruments in taking latitude measurements for calculating their position: indeed, mariner's astrolabes and cross-staffs were a standard part of the equipment of any ship which did more than ply coastal waters, and such innovations as John Davis' backstaff were welcomed warmly. However, the introduction of Mercator charts, and the use of these charts for more accurate positional calculations to supplement the results of dead reckoning, were viewed with some suspicion by a significant number of pilots and masters. Many navigators felt that the traditional plain chart was sufficient for their needs, and that the only way to ensure relative safety during their voyages was to stick to the tried and tested method of latitude sailing. Mercator sailing was viewed with particular hostility because it demanded the application of trigonometry and this kind of mathematical manipulation was far beyond the training which most navigators had received. For this reason various writers presented new methods for dealing with Mercator sailing which circumvented the most complicated parts of trigonometry, and it was here that the application of logarithms to trigonometrical ratios proved most effective. Some of the new calculational instruments were developed in order to reduce the complications of mathematical navigation, although the extent to which these instruments were employed by the average navigator is unclear.[38]

Hence, while the advantages of using mathematics more extensively were trumpeted by a section of the practitioners of those professions which could be based on the practical use of geometry and astronomy, the introduction of mathematical techniques was a slow process. It gained in momentum through the course of the

[37] The history of navigation in this period is well represented by Waters, *The Art of Navigation in England in Elizabethan and Early Stuart Times* (London, 1958)J.B. Hewson, *A History of the Practice of Navigation* (Glasgow, 1951) and E.G.R. Taylor, *The Haven-finding Art* (London, 1956).
[38] See further discussion in Chapter Four.

seventeenth century but only as the new techniques were shown to be of value to the average practitioner, and as the general mathematical literacy of this class increased (a situation aided by the increasing number of teachers offering adult tuition in mathematics). The introduction of new instruments helped, although it was some time before designers accepted the fact that the most popular instruments were those which were simplest to use and required the least knowledge of mathematics. The many complicated tools which appeared in the early years of the century played little more part than to illustrate the skill of their creators; they never found a market among the more down-to-earth practitioners.

Since it was this group of people who had the most need for instruments it is to be expected that Elias Allen would have readily formed connections among the navigators and surveyors of London. There is certainly evidence for this from the texts: Allen was recommended by both Rathborne and Hopton for the various instruments which they designed.[39] He also supplied compass needles for the expedition of Thomas James in search of the Northwest passage,[40] and it would appear that he had a steady trade in Gunter sectors, judging by the number still extant (although how many of these were purchased for use at sea is unclear). At least two peractors (a form of surveying quadrant developed by Rathborne) made by Allen survive[41] and one of these shows marks of having been used for practical purposes. There is also a plane table alidade in the collection of the National Maritime Museum, and a mariner's astrolabe in the Physics Department at St. Andrew's University, though this latter instrument was almost certainly never taken to sea and may have been commissioned originally for a gentleman's collection.

The Gentlemen Amateurs

A small proportion of the printed mathematical texts of the seventeenth

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[39] See Hopton, *Speculum Topographicum* (London, 1611), sig.Ee recto; Rathborne, *The Surveyor*, p.131.
[40] Thomas James, *The Strange and Dangerovs Voyage of Captaine Thomas Iames...* (London, 1633), sig.Q1 verso.
[41] Cf. Catalogue, pp.261, 273.

century were the work of people who might be described as gentlemen amateurs. These were men from the nobility and gentry, genuinely interested in mathematics but not earning their livelihood through study, teaching or practical application of mathematics or any other profession. Their numbers gradually increased through the century as the mathematical sciences became fashionable.[42] They dabbled in whatever interested them in the sphere of mathematics, but were more likely to have broader interests (for instance, in natural philosophy and natural history) than those at the hub of the mathematical culture. It was largely from this class of people that the outer ring of the Royal Society was to be formed.

As with other sections of the community some of the texts are instruction manuals for instruments which had been designed by their authors: John Blagrave's books generally lie in this category. Blagrave had an estate at Swallowfield, near Reading and devoted his time to developing such instruments as the mathematical jewel (a form of universal astrolabe), the 'baculum familiare' (a surveying staff) and various aids to dialling.[43] Edmund Wingate was another author who is best known for the design of an instrument, in this case a logarithmic rule, which he discussed in great detail both in *The Use of the Rule of Proportion* and in his hugely popular arithmetic.[44] The other main area treated in printed texts by members of the upper classes is the issue of the need for better education in mathematics and the possibility of constructing new schools and academies which would supply mathematical teaching as an important part of the curriculum. Among unpublished works a large quantity of manuscript tracts on dialling were produced during this century, the majority of which came from the pens of gentlemen amateurs:[45] dialling appears to

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[42] See A.J. Turner, 'Mathematical Instruments'.
[43] John Blagrave, *The Mathematical Iewel* (London, 1585); *Baculum Familliare* (London, 1590); *Astrolabium vranicum generale* (London, 1596); *The Art of Dyalling* (London, 1609).
[44] Edmund Wingate, *The Use of the Rule of Proportion* (London, 1645) (originally published in Paris as *Usage de la Reigle de Proportion* in 1624); *Arithmetique made easie* (London, 1630 and many subsequent editions). Biographical information about Wingate and further study of his rule can be found in A.J. Turner, '"Utiles pour les calculs": the Logarithmic scale rule in France and England during the seventeenth century' in *Archives Internationales d'Histoire des Sciences*, 38 (1988), pp.252-270.
[45] See in particular the Lewis Evans collection at the Museum of the History of Science in Oxford.

have been a subject which fascinated amateur mathematicians throughout the seventeenth and eighteenth centuries.

Although relatively few books were written by this group of people they appear to have been a target audience for other authors of mathematical texts and are often addressed on the title pages of the books.[46] Thus Thomas Blundeville speaks of his work on astronomy, *The Theoriques of the seuen Planets...* as required reading for '...all Gentlemen that are desirous to be skilfull in Astronomie' among others; both Thomas Hylles and Nicholas Hunt stress how important arithmetic is to the gentleman wishing to obtain a rounded education; and John Wilkins recommended his writing on mechanical devices to any landed gentry who had mines on their property or who were concerned about drainage.[47] Apart from these specific references to the gentry, the ubiquitous phrase 'those who are studeous in the mathematics'[48] was almost certainly intended to be directed as much at the gentleman amateur as at the mathematical practitioner. It certainly appears that there was a growing interest in mathematics among the leisured classes which was swiftly exploited by those who earned their living through teaching mathematics and also through the production of mathematical texts and instruments.

This new dilettante interest probably had its root in the steady popularisation of the mathematical disciplines from the end of the previous century onwards. Gentry who were connected with navigation would have had knowledge of the increased application of mathematical techniques to sailing. Meanwhile the prominence of John Dee in Elizabeth's court and his championing of the mathematical sciences no doubt had their effect on other members of the court, which probably filtered down to those who aspired to appear well educated. And in the universities the trend was

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[46] Although once again we must be careful of these indications of possible readership. A book which was addressed to gentlemen may well have been aimed not only at this class but also at those who aspired to be members of the gentry; it is extremely likely that such people would have read these books.

[47] Blundeville, *The Theoriques of the seuen Planets...* (London, 1602), title page; Thomas Hylles, *The Art of vulgar arithmeticke...* (London, 1600), title page; Hunt, *The Hand-Maid to Arithmetick Refined* (London, 1633), title page; Wilkins, *Mathematical Magick* (London, 1648), sig.A4 verso.

[48] See, for example, Edmund Gunter, *De Sectore et Radio*, title page.

(albeit slowly) towards a greater openness to the value of mathematics. Little by little, accomplishment in mathematics, alongside other branches of knowledge, came to be a status symbol among the landed gentry. The well-educated gentleman was soon expected to know Euclid as well as the literary classics, to be as proficient in the application of the celestial and terrestrial globes as in playing upon musical instruments, as able to construct a sundial as to parse a sentence in Latin.[49]

Mathematics was prominent in many of the schemes for the creation of new academies which were the preoccupation of numerous commentators on the state of English education. These men were worried that too many of the sons of the upper classes were being sent abroad to complete their education and so were being influenced overmuch by foreign attitudes and ideas. They felt that it was necessary to fill the gap in the indigenous supply of teaching, and that establishing academies which would train young boys (and, occasionally, girls) between childhood and university was the means by which to accomplish this purpose. Schemes ranged from the relatively abstract (such as Henry Peacham's *The Compleat Gentleman* and John Aubrey's ideas for a school for young gentlemen) to the detailed exposition of John Dury's *The Reformed School* which presented all the necessary details for the organisation of a school for the completion of the young gentleman's secondary education.[50] Sir Francis Kynaston and his associates even reached the position of gaining letters patent from the King, buying a plot of land and printing the constitutions for their academy:[51] unfortunately, the Civil War intervened and the Musæum Minervæ never became a reality. All of these schemes acknowledged the importance of mathematics in the training of a gentleman, and lessons in arithmetic, geometry and astronomy featured prominently on all the syllabuses. It appears that

[49] See A.J. Turner, 'Mathematical Instruments' for further information on this subject.
[50] Henry Peacham, *The Compleat Gentleman* (London, 1622), modern edition by Virgil B. Heltzel (New York, 1962); Bodleian Library Ms Aubrey 10 (see also J.E. Stephens (ed.), *Aubrey on Education. A hitherto unpublished manuscript by the author of* Brief Lives (London, 1972) and A.J. Turner, 'Mathematical Instruments', pp.51-88); John Dury, *The Reformed School* (London, 1650, published by Samuel Hartlib).
[51] Sir Francis Kynaston, *Constitutions.*

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51 Sir Francis Kynaston, *Constitutions.*

mathematics was no longer viewed as a subject which could be safely left for undergraduate teaching, but rather as an important part of the school curriculum.

With the mathematical sciences and their applications becoming an increasingly popular study among the gentry, it was inevitable that instrument makers would seek to take advantage of this new market, and Elias Allen was no exception. In fact, a great many of the surviving examples of his work seem to have been intended as items for purchase by customers from the gentry or the nobility. Many of them are exquisitely crafted and show little sign of wear and tear, suggesting that they were bought as items for inclusion in gentlemen's collections. Pieces such as the astronomical compendia and the Gunter quadrants are clearly designed for the dilletante amateur mathematician rather than the mathematical practitioner.[52] It was almost certainly through his willingness to exploit this burgeoning market that Allen was able to support himself so well. There is also textual evidence that his instruments were in demand from gentlemen; indeed, one friend of Oughtred complained that Allen's new gauging rod was so popular that he had been unable to purchase one.[53]

The mathematical instrument makers

At last we come to the category to which Elias Allen belongs. In this subsection of the community approaching through the authors of the texts provides us with singularly little information. The only instrument makers to publish anything during the period surveyed were Christopher Brookes, John Prujean and Anthony Thompson, all of whose writings appeared after 1650. However, the instrument makers are represented in the texts to a certain extent. After all, with so many books concerned with explaining instruments of various kinds, there was a need to indicate to the reader where these instruments might be purchased, and so various of the better

[52] See the discussion in Chapter 3.
[53] William Robinson to William Oughtred, June 11th, 1633, reproduced in Stephen Rigaud, *Correspondence of Scientific Men of the Seventeenth Century* (Oxford, 1841), pp.17-18.

known and more highly respected instrument makers are mentioned. In the course of the literature survey I made the acquaintance (if somewhat briefly in some cases) of Elias Allen, John Allen, John Thompson, Anthony Thompson, Ralph Greatorex, Walter Hayes, John Bleighton, Thomas Browne, John Prujean, Christopher Brookes and Charles Whitwell. However, these are only a part of the body of artisans who devoted some or all of their time to making mathematical instruments, and so it is fortunate that there are other places to turn for information concerning this group. First and foremost there are the guild records of the companies of which they were members; there are also the instruments themselves.

The English trade in mathematical instruments was still in its infancy at the beginning of the seventeenth century, the establishment of a workshop (c.1544) by the émigré Thomas Gemini having been the first real step in creating this trade. Through the second half of the sixteenth century there were only a handful of craftsmen who produced any mathematical instruments (Humphrey Cole, Augustine Ryther, John Reade, James Lockerson, John Reynolds, James Kynvyn, Christopher Paine, Charles Whitwell, Emery Molyneux, John Bull, Francis Cooke and Christopher Jackson being the main representatives)[54] a reflection of the fact that the mathematical sciences were only just beginning to arouse interest on a large scale. The next century was to witness the flowering of England's mathematical culture and the concomitant foundation of a thriving community of instrument makers to satisfy the demand for instruments both old and new.

As mathematical instrument making was so undeveloped at this point there was no central guild to which the instrument makers were attached. Consequently they were members of various of the companies, representatives being found among the grocers (Whitwell, Allen and others), the joiners (the Thompsons, and Thomas

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[54] See Taylor, *Mathematical Practitioners*, G. L'E Turner 'Mathematical Instrument Making in London in the Sixteenth Century' in Sarah Tyacke (ed.), *English Map-Making*, pp.93-106, and D.J. Bryden, 'Evidence from Advertising for Mathematical Instrument Making in London, 1556-1714' in *Annals of Science*, 49 (1992), pp.301-336, although note that Bryden casts doubt on the status of Francis Cooke (p.307, n.32).

and John Browne), the goldsmiths (Humphrey Cole), and numerous other guilds. The lack of strict guild regulation allowed a substantial degree of variation in the quality of instruments and of the discipline in the makers' workshops, particularly with regard to the number of apprentices taken. When the Clockmakers' Company was founded in 1632, several attempts were made by the new guild to gather in the instrument makers and to enforce more rigid policing of the trade, but this appears to have had relatively little effect. Makers joined the new company if they felt that it would be useful to them; otherwise they took very little notice of attempts at coercion. This independence of the instrument makers no doubt enabled a greater freedom in their relationships with their customers and the instrument designers, and a greater opportunity to manipulate the market.

In the early years of the century, makers tended to specialise in either metal or wooden instruments, perhaps as a result of the different backgrounds from which their trade had originated (i.e. a joiner would more naturally develop a trade in wooden instruments, while a craftsman who had a trade ancestry in engraving might tend towards working with metal). Early advertisements for instrument suppliers usually name one brass worker and one joiner, and so Elias Allen's name is often linked with that of John Thompson, the pre-eminent producer of wooden instruments. However, as the century progressed, more and more instrument makers began to work in several different media, the object presumably being to corner as great a share of the market as possible. In the light of this diversification of business it is the more remarkable that Allen was able to establish a thriving trade while restricting his working materials to metals.

Of course, success in instrument-making depended not only on craft skill but also on the ability to make the most of advertising one's business. This subject has recently been discussed at length by David Bryden,[55] so I will limit myself here to making a few general points. It appears that the earliest means of advertisement was

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[55] Bryden, 'Evidence'.

largely through word of mouth or through informal advertisements in books. Generally, when authors spoke of new instrument designs they would include a short note (usually an integral part of the text) to the effect that 'this and all other instruments for...can be purchased from such and such a maker at such and such an address'. In other works, illustrations of the instruments often carry inscriptions holding the information as to where and from whom the object can be bought. Clearly it was very important for instrument makers to make the most of their links with writers of texts about instruments, and this was a further factor in creating a very close-knit mathematical community. These were the main kinds of advertising upon which Allen relied.

Gradually, as the century progressed, instrument makers began to produce their own trade cards and advertisements. Walter Hayes took advantage of his sale of mathematical texts to paste his own advertisements into them declaring his ability to supply a wide variety of mathematical aids:

> 'Whosoever hath or Shall have Occation for all or any of these Instruments Mentioned in this Booke or any Other for the Mathematicall Practice Ether in Silver Brasse or Wood may Bee Exactly furnished by Walter Hayes, At the Crosse Daggers in Moore fields Neere Bethlem Gate London.'[56]

John Prujean was still more forward in publishing a catalogue of his wares, in order to advertise his services to the mathematically inclined in Oxford.[57] A generation later, Edmund Culpeper had designed an illustrated trade card for informing potential clients about the full scope of his workshop.[58] However, the general situation throughout the seventeenth century, particularly in the first fifty years, was that

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[56] Quoted in Bryden, 'Evidence', p.326.

[57] John Prujean, *Notes of mathematical instruments made and sold by Jean Prujean in Oxon.* (London, 1653). This is the title and date as given in the second edition of D. Wing, *A Short Title Catalogue of Books Printed in England...1641-1700* (New York, 1972-1988), entry P. 3884, but see also Bryden's comments in 'Made in Oxford: John Prujean's 1701 Catalogue of Mathematical Instruments' in *Oxoniensa*, LVIII (1993), p.266. In the latter work it is shown that the only dated copy of the work comes from 1701, published in an edition of Richard Holland's *Globe Notes*. Similar publications survive but are undated.

[58] See Bryden, 'Evidence', pp.323-325.

instrument makers worked to commission and advertisements were simply a means for making potential customers aware of suitable artisans to approach for particular instruments.

A further way in which instrument makers appear to have made themselves more easily accessible to their buyers was through the location of their workshops. They tended to congregate in certain areas and this is vividly seen in the clustering shown on Taylor's map of workshop sites. Many of these were in the vicinity of Gresham College, but there were also groups on Tower Hill, around St. Paul's and down the length of Fleet Street and the Strand, even as far as Charing Cross. This may simply have been the natural development as the result of craft succession, with workshops being passed down from master to apprentice, but may have been instrument makers taking advantage of the fact that certain areas of London became known as good sources of high-quality instruments. Certainly the number of workshops around Gresham College would seem to be related to their proximity to an institution which, among other things, devoted time to instruction in the use of mathematical instruments.

The artefacts produced by this group of craftsmen were almost all directly related to mathematics. Instruments of natural philosophy were largely a thing of the future, and the construction of telescopes remained at a very rudimentary level until the second half of the century, when the application of telescopic sights became viable through the use of crosswires and micrometer screw gauges. However, the number of different mathematical instruments rose rapidly from the latter part of the sixteenth century and throughout the seventeenth. A relatively restricted range of standard pieces (astrolabes, armillary spheres, astronomical quadrants, sundials, cross-staffs and backstaffs, nocturnals, theodolites, plane tables, astronomical compendia and the like) expanded to a vast array of tools for easing the life of the user of mathematics. Many new instruments were produced for use in surveying (though lifespans in this area tended to be short); there was a proliferation of scales to aid measurements of various kinds; new instruments for dialling were introduced in a large number of the books on that subject; instruments for making astronomical observations became gradually more sophisticated as new methods for dividing scales were devised. In particular, calculational instruments abounded: some of these, such as the arithmetical jewel, Napier's bones and the Gunter sector, were designed to simplify the standard techniques of arithmetic; others were developed specifically to take advantage of the new invention of logarithms - the various forms of logarithmic rule and eventually the slide rule. New instruments opened up the world of mathematics to a wider public than had yet been prepared to approach such a complicated study. Numbers were not the natural playthings of most ordinary people and many of the instruments were designed with the express purpose of making mathematics and its applications more accessible.

Such, in broad outline, was the mathematical community of the early seventeenth century. The major players in this drama have all been presented through the study of the mathematical texts of the period. However, brief mention must be made of those who are relevant to the mathematical culture but who only appear in the background; in particular, patrons of the mathematical sciences, who are only mentioned in the dedications which were a standard constituent of books of this period. These men and women facilitated the expansion of the mathematical sciences through their financial support and their recommendations of those members of the mathematical community whom they felt worthy of their patronage. They included Henry Percy, the Ninth Earl of Northumberland (who gathered a group of scholars, including Thomas Hariot, under his wing), Sir Walter Raleigh and Thomas Howard, the Earl of Arundel (perhaps more famous for his love of the arts, but also a aid measurements of various kinds; new instruments for dialling were introduced in a large number of the books on that subject; instruments for making astronomical observations became gradually more sophisticated as new methods for dividing scales were devised. In particular, calculational instruments abounded: some of these, such as the arithmetical jewel, Napier's bones and the Gunter sector, were designed to simplify the standard techniques of arithmetic; others were developed specifically to take advantage of the new invention of logarithms - the various forms of logarithmic rule and eventually the slide rule. New instruments opened up the world of mathematics to a wider public than had yet been prepared to approach such a complicated study. Numbers were not the natural playthings of most ordinary people and many of the instruments were designed with the express purpose of making mathematics and its applications more accessible.

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As I pointed out earlier, the distinctions which I have made in order to present the community are to a certain extent artificial, there being a great deal of overlap between the different categories. The most incongruous grouping perhaps is that of the mathematical practitioners, who could also be found teaching their skills as mathematical tutors or sometimes making the instruments which they designed. Occasionally the instrument makers themselves gave instruction in the use of their instruments. Again, the line between 'academic mathematicians' and gentlemen amateurs is occasionally blurred. All these blurrings, overlaps and interweavings, however, simply indicate the close-knit nature of this mathematical culture. It was a community where names at least were known to most other members, manuscript treatises (such as Gunter's Latin treatise on the sector) were circulated within subsets of the community and various locations became popular venues for informal meetings to discuss new instrument designs or topical subjects, such as the discovery of logarithms or the knotty problem of magnetic variation (particularly after the phenomenon of secular variation became known). There were no sharp distinctions between 'pure' and 'applied' mathematics, nor between the mathematics of the academy and that of the mechanician who employed mathematics to ply his trade.

So much for the supporting characters in the narrative: we now turn to the central figure of this play - Elias Allen. Of course, the play in which he takes centre stage is just one of a whole series of dramas and those who only appear in cameo

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[59] Northumberland and Raleigh's roles as patrons are discussed in Robert Hugh Kargon, *Atomism in England from Hariot to Newton* (Oxford, 1966). Arundel's support for the sciences is charted in David Howarth, *Lord Arundel and his Circle* (New Haven, 1985).

roles in this story will walk into the wings and out onto the stage of a different play where they are the main focus of attention. What I hope to do in the course of the remaining chapters is to establish that the story of Elias Allen is not merely a sideshow in the mathematical culture of seventeenth-century England, and that attention to this drama will help us to interpret other parts of the overall history in new ways.

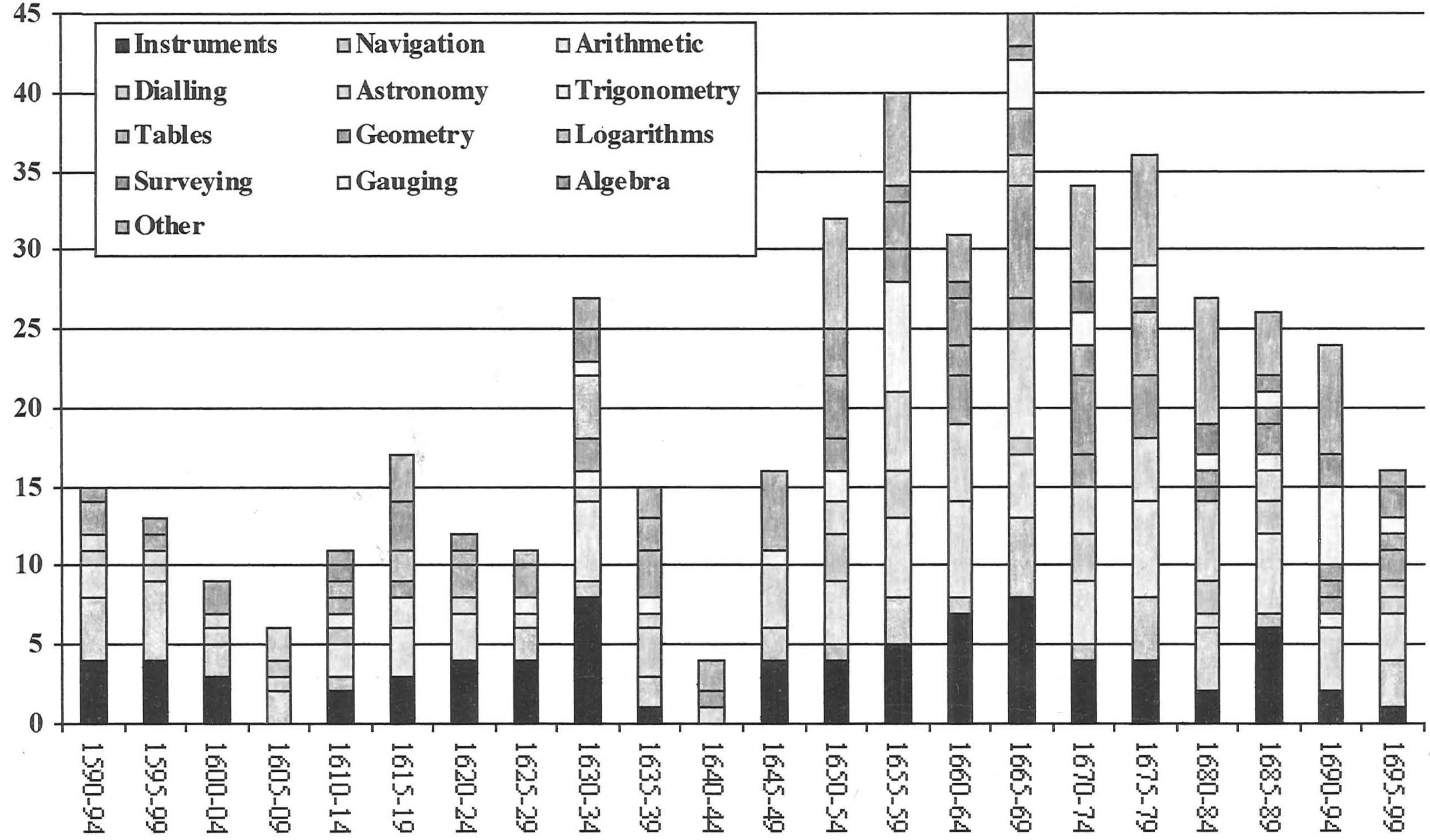

Figure 2: Main Subjects of Mathematical Books Published Between 1590 and 1699

Chapter Two

Elias Allen's Life

Of the beginning of Elias Allen's life very little is known. The inscription beneath his portrait states that he was born in the vicinity of Tonbridge but the exact village is unknown. It is possible that he came from the parish of Ashurst which lies a few miles away: he later gave a sundial to the village, which still stands in the churchyard, and so it seems likely that he had a close connection with the place, but unfortunately, none of the church records have survived from this period. The will of a Tonbridge surveyor, Henry Allen, survives,[1] recording an Elias Allen as one of the witnesses, and the combination of the town and the occupation of the testator (an occupation closely linked with the mathematical instrument trade) makes it plausible that this is the same Elias as the instrument maker. More than this, we cannot say.

However, something can be said regarding the county in which Allen grew up.[2] Kent held a major strategic position through its location between London and the continent of Europe and by virtue of the many ports along the Thames estuary where ships were unloaded and loaded in passage to or from the capital. It was the source of a large proportion of the food consumed in London, particularly wheat, but also malt, meat, fruit and fish. The forests of the Weald were also culled to satisfy the metropolitan demand for kindling wood. Thus the county had very strong links with London and many of its connections with the rest of the country were filtered through the capital.

At the end of the sixteenth century Kent was the third or fourth wealthiest county in England and held a similar position in terms of population. There were about 130,000 people, of whom the majority were farmers. In the area around Tonbridge the

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[1] Public Record Office, PROB 11/152, ff.262-3.

[2] Most of the following information is drawn from C.W. Chalklin, *Seventeenth Century Kent. A Social and Economic History* (London, 1965) and Peter Clark, *English Provincial Society from the Reformation to the Revolution: Religion, Politics and Society in Kent 1500-1640* (Hassocks, 1977).

predominant form of farming was dairying and beef production. The cloth industry and iron production (making use of the fuel provided by the Wealden forests) played an important part in the Kentish economy as well.

The area was also known for the strength of the nonconformist tradition. Religious nonconformity was established early in Kent and flourished particularly strongly in the Weald. This may have been due to the comparative isolation of the forest settlements allowing a certain freedom of expression, or to the strong presence of Protestant refugees from the continent who had escaped the religious wars of the 1560s. Whatever the case, it would seem likely that the Allen family belonged within this tradition - Elias appears not to have been a common name before the Reformation but it did then become something of a favourite in nonconformist circles.

Just as mysterious as Allen's precise birthplace is the year of his birth. Tantalisingly, his portrait's inscription speaks of him having died 'at the age of', leaving a blank where the figures would have been included. However, a rough date can be given by calculating back from the beginning of his apprenticeship, in 1602. The standard age of new apprentices was fourteen, implying that he would have been born in 1588. Apprentices were taken at other points, varying from the ages of twelve to seventeen, so the birth date can be set, with a fair degree of certainty, between 1585 and 1590. It would be a pleasant coincidence if the instrument maker had been born in 1588: 'Armada year' gave the first clear indication of the vital importance of a strong navy and hence of the need for navigational instruments which would ease the lot of the mariner.

Allen's craft descent

According to the record provided by Elias Allen's application for freedom of the Grocers' Company[3], he was bound to the London instrument maker Charles Whitwell

[3] Cf. p.55.

in the final year of Elizabeth's reign. This evidence corroborates the tentative claim for the connection made by Eva Taylor in *The Mathematical Practitioners of Tudor and Stuart England,* where she notes that the two instrument makers had very similar addresses. Even without this evidence, the similarity in style of engraving (which is remarkably close) would be a strong indication of a relationship.

Charles Whitwell had been apprenticed to Augustin Ryther (or Rider, according to the Company records) on 17th December 1582, and served under his master for eight years, being granted his freedom on 10th November 1590.[4] Ryther was one of the earliest of the indigenous instrument makers of London and divided his time between this area of craftsmanship and that of engraving maps.[5] His apprentice followed in his footsteps, though rather better known for instruments than was Ryther. Of Whitwell's maps there are few surviving examples: a map of part of Asia, one of Jerusalem, copies of John Norden's map of Surrey and one of Philip Symonson's map of Kent.[6] Other engravings produced by Whitwell include various illustrations for mathematical texts. There are beautiful examples in William Barlowe's *The Nauigator's Supply* (1597) and Hood's *The making and Use of the Geometrical Instrument Called a Sector* (1598). Whitwell not only made the engravings, but also guaranteed to provide the instruments from the stock in his shop. His work as an instrument maker is rather better represented than his maps, which seems to indicate a shift in emphasis from map-making to instrument-making which was to arrive at its

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4 Wardens' Accounts of the Grocers' Company, Guildhall MS 11,571, vol.7.

5 His only known surviving instruments are a theodolite, signed, and dated 1590, and now in Florence, a universal equinoctial dial, dated 1588, in the Science Museum and an astronomical compendium, dated 1588 (see M. L. Righini Bonelli, *Il Museo di Storia della Scienza a Firenze*, p.182; Sotheby's (London), 23/10/85, lot 331; the Science Museum sundial was acquired in the 1980s). His extant maps include five engravings for *Saxton's Atlas of England and Wales*, a copy of Saxton's *Large Map of England and Wales*, a *Bird's Eye Plan of Oxford*, a *Bird's Eye Plan of Cambridge*, three maps in Wagenaer's *Mariner's Mirror*, and the *Armada Plates*, engraved after the drawings of Robert Adams, Surveyor of Works to Queen Elizabeth (see A.M. Hind, *Engraving in England in the Sixteenth and Seventeenth Centuries* (Cambridge, 1952-55), vol.1, p.138).

6 G. T. Minadoi, *The History of the Warres betweene the Turkes and the Persians* (1595); Christianus Adrichomius, *A briefe description of Hierusalem...*(1595) (these are referred to in Hind, *Engraving in England*, vol.1, p.224); John Norden, *Map of Surrey* (c.1604) (copies in British Library and Royal Geographical Society Library); Philip Symonson, *Map of Kent*, (1596) (copy in Royal Geographical Society Library). (Information taken from Joyce Brown, *Mathematical Instrument-Makers in the Grocers' Company 1688-1800* (London, 1979).)

conclusion in this dynasty in the work of Elias Allen (who appears on extant evidence never to have produced any maps).

Whitwell, in the inscriptions accompanying the portrayals of his instruments on the title pages of these books, gives an indication of his address. He speaks, in Hood's book, of 'dwelling without Temple Barre against S. Clements Church', while his advertisement in the Barlowe states that,

> 'The instruments are made by Charles Whitwell, over agaynste Essex howse, maker of all sortes of mathematicall instruments, and the graver of these portaytures'.[7]

Essex House was one of the great mansions situated west of the city walls and along the river where it wound north and east from Westminster to the City. Essex House and Arundel House between them occupied much of the land between the Strand Lane and the Temple, with Essex House fronting onto the Strand opposite St. Clement Dane's Church. According to John Stow,[8] Essex House was so called because 'of the earl of Essex lodging there', but previous to that it had been referred to as Leycester house, 'because Robert Dudley, earl of Leycester, of late new built there'. There may well have been a connection between Whitwell and the younger Robert Dudley (son to the earl): certainly, when Dudley (who designed instruments himself) departed for Florence in about 1605, he took with him several of Whitwell's instruments, which are now housed in the Museum of the History of Science there.

While there are no known surviving examples by Whitwell of Hood's sector, or the various instruments described in *The Nauigators Supply*, there is clear evidence that Whitwell was indeed a 'maker of all sorts of mathematicall instruments'. The Florence Museum boasts a quadrant and an astrolabe, signed and dated 1595, and two nautical hemispheres.[9] A pendant sundial with perpetual calendar (dated 1593 and following the design of Nathaniel Torporley) is in the Museum of the History of Science at

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[7] Thomas Hood, *The making and Use of the...Sector*, title page; William Barlowe, *The Nauigator's Supply*, title page.
[8] Stow, *The Survey of London* (London, 1598).
[9] Bonelli, *Museo di Storia*, pp.162, 164, and 170.

Oxford, along with a universal dial of 1606. The National Maritime Museum has an astronomical compendium (1600) as does the Whipple Museum (1604), and the British Museum holds a folding dial.

Whitwell took numerous apprentices between 1593 and 1610: William Wrightson on 25th December 1593; Joshua Silvester on 25th December 1594; John Smythe on 1st May 1596; Abraham Barton on 24th June 1602; Carye Wolriche on 24th June 1608. As well as these apprentices (none of whom appear in the Grocers' Company records applying for freedom), Thomas Woodall (freed 27th April 1604) was never formally bound, and the indenture papers for Elias Allen are not extant.[10]

The Court Minutes of the Grocers' Company for 11th August 1606 record that

> 'This day it is agreyd that Charles Whitwell grocer shall have the 50^{l} for ii years w^{ch} his brother Robert Whitwell deceased latelie had. And William Whitwell and George Budd salters are alowed his sureties.'[11]

Fifty pounds was a large sum of money at this period and it is unclear why the money was granted to Whitwell. It is a completely different order of magnitude from the sum of forty shillings granted to Allen in 1649 when he was short of money. Thus it is unlikely to have been a loan to aid a struggling business. Perhaps Whitwell's business was so successful that the Company were willing to advance the capital for the purchase of materials required for the workshop such as engraving plates. In this case the loan would reflect the prosperity of Whitwell's business and the obvious reliability of the instrument maker to repay the loan.[12]

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10 See Joyce Brown, *Mathematical Instrument-Makers*. I am much indebted to this work for providing valuable information. Other sources for instrument makers in the London livery companies are M.A. Crawforth, 'Instrument Makers in the London Guilds' in *Annals of Science* 36 (1979), pp.1-34 and Joyce Brown, 'Guild Organisation and the Instrument-Making Trade, 1550-1830: the Grocers' and Clockmakers' Companies', *Idem* 44 (1987), pp.319-378.

11 Guildhall MS 11,588, vol.2.

12 That metal plates were valuable is clear from some of the evidence provided from the inventories of map-sellers from the latter part of the seventeenth century: the inventory of Thomas Jenner's property included 117 cwt of old copper plates valued at £41.16s.10d; that of Philip Lea's estate included 16 cwt of copper to a value of £132.8s. (See Sarah Tyacke, *London Map-Sellers 1660-1720* (Tring, 1978), p.118 & p.122.

Whitwell was buried in St. Clement Dane's on 23rd January 1610 (OS);[13] the administration of his effects was sought by and granted to his widow, Sarah Whitwell, on 1st February 1611(OS).[14] Allen's application for his freedom in the Grocers' Company had been made some five months earlier, on 17th September 1611.

Apprenticeship

Freedom of the guild had to be sought by Elias Allen so that he might have the right to trade within the walls of the city of London, and to own a workshop. By the beginning of the seventeenth century this freedom could be achieved in one of three ways. It was quite possible to pay for one's freedom as long as the applicant was over the age of twenty-one. Alternatively, for those whose families were already members of one guild, the option of freedom through patrimony was available. This meant that once the child of a member of a guild reached the age of twenty-one, he or she could apply to become free of the parental guild, regardless of what that guild was or the trade which the new member was intending to ply. This was a relatively common method of gaining one's freedom if the family were already involved in the guild system. By far the *most* common way of obtaining the right to set up a trading place in London was to gain freedom through apprenticeship. Even those who had parents in the trade which they would pursue in the future were likely to serve an official apprenticeship in order to obtain a mastery of the craft.

Apprentices mostly came from the large number of artisan families already involved in trade in London. However, it was also common to find fatherless boys being pressed into the trades, or for yeomen from the surrounding counties to send their sons to London to learn a craft which might lead to an enhanced standing in the long term (a job in London was a much-coveted goal for many people from rural areas or small towns). Even the younger sons of gentlemen were sometimes sent to London to learn a trade for their livelihood; the rank of merchant, despite the association with

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[13] City of Westminster Archives, St. Clement Dane's Burial Registers, vol.1 (1588-1638/39), loc. cit.
[14] Public Record Office, PROB 6/8, f.6.

manual work, was not one which was necessarily looked down upon by the gentry of this period, since the more important merchants in the city bore considerable power and influence in the capital.[15]

The laws governing the training and keeping of apprentices were still very strict at this time. The Statute of Apprentices, passed in 1562, had ordained that every practitioner of a craft had to serve an apprenticeship of at least seven years, and many of them were bound by their masters for even longer terms. During this period master and apprentice had certain legal obligations to each other, which had to be kept in order for the contract of the apprenticeship to be held valid. These varied slightly from guild to guild, but the following description from Sir Thomas Smith's *De Republica Anglorum* of 1565 provides a good illustration of the kind of behaviour which was expected:

> 'whatsoever the apprentice getteth of his owne labour, or of his masters occupation or stocke, he getteth to him whose apprentice he is, he must not lie foorth of his masters doores, he must not occupie any stocke of his owne, nor mary without his masters licence, and he must doe all servile offices about the house, and be obedient to all his masters commaundementes, and shall suffer such correction as his master shall thinke meete, and is at his masters cloathing and nourishing, his master being bounde onely to this which I have saide, and to teach him his occupation, and for that he serveth, some for vij. or viij. yeres, some ix. or x. yeres, as the masters and the friends of the young man shall thinke meete or can agree:'[16]

Indentures also usually gave rules regarding more specific behaviour which might sully the Company's reputation. For instance, the Clockmakers' Company decreed:

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[15] This is made clear by Richard Grassby's essay, 'Social Mobility and Business Enterprise in Seventeenth-century England' (in Donald Pennington and Keith Thomas (eds.), *Puritans and Revolutionaries* (Oxford, 1978), pp.355-380): 'Apprenticeship lists, registers of freemen, indentures, and family papers reveal substantial numbers of sons of country, urban and professional gentry in business. Although less numerous than yeomen and husbandmen - the two other main status categories - they constituted a significant proportion of apprentices of non-mercantile origins.' (p.356)

[16] Smith, *De Republica Anglorum* (London, 1583); modern edition by L. Alston (Cambridge, 1906), p.137. Smith was one of the Secretaries of State to Elizabeth I.

> 'He shall not play at cards, dice, tables, or any other unlawful games, whereby his said Master may have any loss.... He shall not haunt taverns or play-houses, nor absent himself from his said Master's service day or night unlawfully:'[17]

The strictures applied to the apprentice were quite severe but not unreasonable if he was going to be in a position to be useful to his master and to acquire the necessary knowledge of his craft. In return for his faithful service the obligations of his master were not inconsiderable. The master, it is true, gained the service of an increasingly skilled workman for nothing in terms of wages, but he was nevertheless required to supply board and lodging and teaching as payment. The apprentice essentially became a part of the master's family.

These were the typical terms under which an apprentice was bound. At the end of his term, an apprentice could gain his freedom if he had the consent of his master and the approval of the court of the guild, and was prepared to pay for entry into the guild. Many apprentices opted to remain in their masters' service for a further period, moving up to the rank of journeyman, whether they had taken their freedom or not. While the position of freeman in a guild had its own privileges and opportunities, it also carried its own burdens and obligations, and many artisans were loath to take those responsibilities upon themselves until they were sure of being able to establish themselves sufficiently well to earn a living. Thus the date recorded for freedom did not necessarily imply the recent completion of an apprenticeship, nor did the continuance of a former apprentice in the workshop of his master imply a lack of skill. Many who had no desire to own their own workshop, were happy to remain in the relatively safe position of journeyman to another, and masters were often glad to have the resource of several skilled pairs of hands to assist them in their work. It was often the case that it was more prestigious to work as a journeyman for a well-known and respected master than to struggle to set up a workshop of one's own.

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[17] Quoted in Brian Loomes, *The Early Clockmakers of Great Britain* (London, 1981), p.12.

Presumably Allen's experience of apprenticeship was very much the same as that of other young boys bound into the crafts. All that can be said with certainty is that by the fourth year of his term he was already showing signs of the mastery of his profession, which was to flourish in the future. The National Maritime Museum collection includes a bronze sundial of Allen's making, dated 1606.[18] It is a simple enough instrument, consisting of a circular hour scale and hour lines radiating out from an attractively shaped gnomon. The signature shows Allen having not as yet settled into a particular style, nor even having decided on the spelling of his name. The lettering runs "Elias Allin fecit 1606' with a very foursquare "E" and yet an elegant, flowing "A" which is more normally found on his instruments in association with a curly, epsilon-style "E". The whole is a witness both to the precocity of Allen's talent and the able teaching of his master.

The next mention of Allen's name in relation to his instrument-making comes five years later. In an advertisement for his work in Arthur Hopton's *Speculum Topographicum: or the Topographical Glasse*,[19] it is said that 'The Glasse is made in brasse, in blacke Horse-ally, neere Fleetebridge, by Elias Allin.'[20] It would appear from the address that Allen had by this point moved out from Whitwell's workshop and set up his own establishment some distance away; it is certain that he had gained a reputation for himself as an able maker of brass instruments at quite a young age, and while still not a freeman of the City.[21]

Some knowledge of the intervening years can be gained from the Parish Registers of St. Bride's Church, Fleet Street, the parish to which Black Horse Alley belonged.[22] The burial register contains an entry for the interment of 'Richard Allin son to Elias Allin' on 18th December 1608. There is no record of the baptism of this child either in the church registers or in the International Genealogical Index (London

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18 See Cat. no. X1.
19 London, 1611.
20 Hopton, sig.Ee recto.
21 This reference occurs in the year preceding Allen's freedom which came in 1612 (see note 24 below).
22 References are taken from St. Bride's Parish Registers, Guildhall MSS 6536, 6537, 6538.

entries), but that he was born at least as early as July 1607 is certain, since his younger brother Charles was baptised on 24th April 1608. The first daughter, Sara, was born in January 1609 (OS) and baptised on the 21st of that month. Charles only survived until shortly after his second birthday - his name appears in the burial registers on 5th May 1610. I have been unable to find any reference to the marriage of Elias Allen either in the International Genealogical Index or in the Parish Registers of either St. Bride's or St. Clement Dane's. However, his wife's name - Elizabeth - is known through records of the baptisms of some of the younger children. If Richard was a legitimate child the marriage must have happened no later than October 1606 and it is possible that this event was the reason for Allen's removal from Whitwell's workshop to his own premises.

At about the same time as the appearance of his advertisement in Hopton's book, Allen applied for freedom of the Grocer's Company. This is the first time that we find his name officially documented in the Company records:

> 'This day the humble suyte of Elias Allen for his freedom whoe allegeth to have served Charles Whitwell grocer deceassed (in his lief time using tharte [sic] of a Mathematician) by the space of nine yeres by indenture of apprenticehood is by this Corte referred to theaxamincon [sic] and Consideracon of the Wardens.'[23]

The examination and consideration of the wardens was evidently a lengthy process: it was not until ten months later (on 7th July) that Allen finally obtained his freedom, on the payment of three shillings and fourpence, and gained the right to trade within the City of London.[24]

Guild Membership

When Elias Allen obtained his freedom in 1612 he came under the jurisdiction of the Grocers' Company and bound himself to the regulations which this entailed. It

[23] Grocers' Company Records, Court Minute Books, MS 11,588, vol.2, f.659.
[24] Wardens' Accounts of the Grocers' Company (Guildhall MS 11,571, vol.1): 'Elias Allen late apprentice to Charles Whitwell entred and sworne the viith of July 1612.'

was these strict regulations which kept many men working as journeymen in their masters' workshops, rather than striking out on their own. The Grocers' Company, as any other City Company, looked after its members and was prepared to protect them legally and financially, but it expected something from them in return. The main requirement of each freeman was that he paid the quarterage of the company (usually of the order of about fourpence a year) every three months, for the privilege of being a member.

Apart from this tax levied by the guild, the freeman was required by company law to attend the company feasts (for which he had to pay) and to submit to the discipline of the company court and to searches of his property. The main purpose of these was to check that the craftsman was producing goods of sufficient quality and was not trying to sell wares which had not been approved by the company. Although the regulation of products was important in the main companies it is unlikely that it had such a great effect on the instrument makers, who traded in many companies whose hierarchies knew little about their work and so probably could not have policed thorough searches of their property.

The last main restriction on guild members concerned the number of apprentices which they were allowed to bind. Generally only one or two apprentices were allowed for a quite considerable time after the craftsman gained his freedom, while he established his trade and consolidated his own knowledge.[25] Thus Allen took on Edward Blayton as his apprentice on the same day that he became free of the guild; he was already well-established in his business, and presumably wanted to begin the

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[25] An example of the kind of stipulations governing the binding of apprentices is to be found in the Clockmakers' Company, as described in Loomes, *Early Clockmakers*. The Clockmakers were unusually strict compared to other guilds at this time, but it gives an indication of the regulations set concerning apprentices:

> 'The number of apprentices permitted to any member varied over the years, is difficult to pin down, and in any case was frequently exceeded either with official consent or without it. A member of the Clockmakers' Company was limited to one apprentice for the first five years of his Freedom, and to two when the first one had completed his five years. However, it was quite possible to break this rule so long as a fine was paid - you could do almost anything with official approval if you agreed to pay a big enough fine. The alternative was to go ahead and do it anyway, but if you were found out then you would still have to pay a fine. Sometimes second apprentices were allowed within the five-year term; later a third one or even more.'

process of training skilled assistants as soon as possible, and to expand his workshop in the process. Judging by the dates of freedom of the next two, John Blighton (perhaps a relation of Blayton) was bound shortly after this, and then there was a gap of about three years until Thomas Shewswell was taken on. It is certain that John Allen[26] was bound to his master on 25th June 1617, and that Henry Sefton entered the Allen workshop on 16th August 1620, to replace Blighton, who gained his freedom on that same day.[27]

Allen's early years as a freeman of the city

As has been mentioned, Elias Allen's first trading address, according to Hopton's *Speculum Topographicum*, was in Black Horse Alley. However, he moved back into the parish of St. Clement's at some point before 28th November 1613, on which day 'Elsabeth Allyn daughter of Ellyas' was baptised in the church.[28] According to an advertisement in John Speidell's *Geometricall Extraction* of 1616, Allen was living 'ouer against *S^t^. Clements* Church in the Strand'.[29] The similarity of this address to that of Whitwell suggests that Allen took over his master's workshop at some point during the five years following Whitwell's death, and one would suspect that it would have been fairly early in this period.

Once again, we know relatively little about Allen's life at this time. He took his first apprentices, Edward Blayton and John Blighton, the latter apparently proceeding to a flourishing business of his own, if we can judge by the number of apprentices he himself took.[30] The addition of Shewswell and John Allen brought the total to four - a large number of apprentices for a young master to have at one time; in fact, it was

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26 Named by Taylor as Elias Allen's son. However, John would have been to old for this to have been the case (born when Elias was about fifteen) and besides, later evidence indicates that Elias' sole surviving issue at the time of his death (when John Allen was still alive) was his daughter Elizabeth. Allen was a common enough name for it to be possible that there was no family connection between the two. Equally well, John may have been a nephew or a young cousin.

27 Full details of all Allen's apprentices are to be found in Appendix 2. The information with respect to binding and freedom dates comes from the Wardens' Accounts of the Grocers' Company (Guildhall MS 11,571) and the List of Apprentice bindings in the Clockmakers' Company (Guildhall MS 3939).

28 St. Clement Dane's Baptism Registers, vol.1, loc. cit.

29 Speidell, *A Geometricall Extraction* (London, 1616), sig.A4 verso.

30 See below, p.83.

probably more than was strictly allowed, and one can only conjecture as to how Allen managed to keep all of his trainees. Perhaps the fact that he lived outside the city walls meant that he was free to make some of his own rules; perhaps the Grocers' Company had simply grown lax over apprenticeship rules by this point.

Meanwhile, the young Allen family struggled to increase. Sara's burial was recorded on 6th January 1613 (OS)[31] and a third son, Elias, was buried two days after his baptism, which took place on 28th May 1615. An Abraham Allen, son of Elyas was baptised in St. Margaret's, Westminster, on 23rd November 1616: he probably belonged to the same family.[32] Mary's life was little longer than Elias' - baptised on 23rd October 1618 and buried five days later. The baptism record of Henry on 12th December 1619 has the first mention of Elizabeth as the name of Allen's wife; she is also recorded in the entries for the baptisms of the last two children. These were a second Sarah, christened on 4th February 1620 (OS), and a final daughter, whose first name is illegible (but may possibly have been Allyson), baptised in St. Olave's, Silver Street (near the Guildhall), on 22nd September 1622.[33] Sarah seems to have been an ill-fated name for the Allens: the burial record of their penultimate daughter on 7th September 1624 is the last entry relating to the family in the St. Clement Dane's registers for several years.

During this period the young master craftsman was beginning to establish himself as the main supplier of brass and silver instruments and to make links with various London-dwelling mathematicians. As has been indicated already, he was recommended by Arthur Hopton (a surveyor and acquaintance of Edward Wright) and John Speidell (a professional teacher of applied mathematics). Allen was also recommended in the major surveying textbook to be published at this time: Aaron Rathborne's *The Surveyor*. In this book we learn that 'the making of [all brass

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[31] St. Clement Dane's Burial Registers, vol.1, loc. cit. All references in this paragraph are taken from the St. Clement's Registers unless otherwise stated.
[32] *Memorials of St. Margaret's Church Westminster*, ed. A.M. Burke, (London, 1914), p.94. Elias was an uncommon enough name for this to be correct: the only other entries for Elias Allen as a father in the International Genealogical Index around this time clearly refer to the instrument maker.
[33] St. Olave's, Silver Street, Parish Registers, Guildhall MSS 6534 & 6534A, loc. cit. It is not clear why two of the baptisms occurred in churches other than St. Clement's.

surveying instruments] are well known to M. *ELIAS ALLEN* in the Strand'.[34] Rathborne was known to be friends with both Speidell and also the Gresham professor of geometry, Henry Briggs. Allen was clearly creating a niche for himself among the mathematical community of his day and beginning to establish connections which would bring him into contact with the most important mathematicians of the time.

The instruments which survive from this period are a somewhat eclectic set. Those which are dated are a mariner's astrolabe of 1616 and an astronomical compendium presented to James I in the following year.[35] Both pieces show Allen's mastery of his trade and his skill in engraving. The mariner's astrolabe is unusual in carrying Allen's signature stamped into the metal in capital letters, rather than in the flowing script which we are used to seeing on his work. The other instruments which probably date from this period are the plane table alidade and the sector in the collection of the National Maritime Museum.[36] The alidade betrays a certain shakiness in the signature and the unusual combination of upright Roman 'E' and flowing 'copperplate' 'A' reflects the style of the 1606 sundial. The sector is an interesting piece: it betrays some of the features of a Gunter sector but does not follow the design illustrated in that mathematician's book on the instrument of 1623. This may have been an instrument designed by Allen at the behest of someone in possession of Gunter's earlier Latin manuscript[37] and was made circa 1620. The reason for the eclecticism may simply be due to the selective nature of the surviving instrumental record, but it might be an indication of Allen producing instruments for whoever would commission him, whether they were mariners, surveyors, astronomers or gentlemen amateurs. It was a time when Allen was attempting to create a foundation upon which his trade could flourish, and to create connections which would last into the future. To this end he would have been likely to be ready to cater to the needs of anyone who was prepared to

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34 Rathborne, *The Surveyor* (London, 1616), p.131.
35 Cat. nos. X5 and A1.
36 Cat. nos. X3 and S5.
37 Cf. Gunter's comment in *De Sectore*, p.143.

purchase his wares, and to bide his time before dictating the type of instruments which were to become the staple trade of his workshop.

Dominance in the trade

During the third decade of the seventeenth century Allen appears to have consolidated his position in the mathematical community and established himself as an indispensable part of that structure. His fame seems to have grown considerably during this time and he formed solid links with the major mathematicians, who then acted as important agents for him, recommending him to their mathematically-minded friends and supplying him with designs for new instruments. In particular, Allen gained greatly from his association with Edmund Gunter and William Oughtred. The instruments which these two mathematicians devised form the core of the extant collection of Allen's work and, while vagaries of survival must be taken into account, this would indicate that it was these instruments that were most often purchased from Allen's workshop and which constituted the heart of his trade.

Allen's association with Gunter may well have quite early origins. As I have commented already, the National Maritime Museum sector seems to bear the signs of the instrument maker's attempts to produce a sector following Gunter's Latin manuscript on the instrument which had been circulating among the mathematical practitioners of London. Perhaps it was through such instruments as these that Allen first came to Gunter's notice; alternatively it may have been Allen's links with John Speidell, who was a long-standing friend of Henry Briggs, the latter being a close friend of Gunter himself. Gunter had been living in London since 1615, when he became incumbent of St. George's, Southwark. Four years later his second application for the Gresham chair of astronomy was successful and he took up residence in the College. Thus Gunter was resident in London from early in Allen's career as a master craftsman and may well have formed a link with him at this time.

The first concrete evidence of Allen's association with Gunter dates from 1623. It is a Gunter sector (now in the collection of the Science Museum) which carries the legend 'Elias Allen fecit 1623'.[38] This instrument is well-nigh identical in form[39] to the engraving in Gunter's famous work on the sector, *De Sectore et Radio*, which, in the first edition, is inscribed:

> 'These instruments are wrought in brasse by Elias Allen dwelling without Tempel barre ouer against S^t Clements Church: and in wood by Iohn Thompson dwelling in Hosiar lane'[40]

and which appeared in the same year.[41] The engraving is clearly the work of Allen and was presumably produced in close collaboration with the author. From this time onwards, judging by the surviving evidence, the sector became a staple constituent of Allen's stock-in-trade. The Gunter quadrant, another instrument featured in *De Sectore et Radio*, is also represented in the catalogue of Allen's works: there is one in a private collection and the Whipple Museum has two examples.

Allen's association with William Oughtred probably began later than that with Gunter. Although Oughtred was friendly with Briggs from well before 1618, according to his own accounts, and first made acquaintance with Gunter at that time, he did not stay long in London, being rector of the parish of Albury in Surrey. It was through this position that he came to the notice of the Earl of Arundel, who had a residence in Oughtred's parish. Arundel appointed Oughtred as tutor to his son, Lord Howard, and, as a result, the mathematician came to be a part of the London circle of mathematicians, since he began to spend significant portions of time at Arundel House in the Strand, from the mid-1620s. It is probable that his close association with Allen dates from his residence at the Earl's house, which was only a few minutes' walk from the instrument maker's workshop. What is certain is that, by 1627, a strong link had been forged between the two, sufficient for Allen to turn to Oughtred for advice in

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38 Cat. no. S2.
39 The only difference is that the names of the lines are omitted from the actual instrument although they appear in the engraving.
40 Edmund Gunter, *De Sectore*, opening plate.
41 Or possibly early in 1624 - the dating of the first edition of this book is very confused, with 1623 appearing on some title pages and 1624 on others.

constructing a New Year's gift for the King. (This incident formed, according to Oughtred, part of the origin of the major dispute which arose between himself and Richard Delamain, in which Allen played a significant part and of which more will be said later.)

Allen's link with Oughtred was to prove a very fruitful one, providing him with several of his most important instruments - the horizontal projection, the equinoctial ring dial and the circles of proportion. The connection was no doubt fostered by one of Allen's apprentices, Christopher Brooke(s), becoming Oughtred's son-in-law, and the two appear to have remained close throughout the remainder of Allen's life. Certainly, we find that the link remained sufficiently strong for Oughtred's pupil, Jonas Moore, to be resident at Allen's house in 1649[42] and for Oughtred to be recommending Allen as a maker of his instruments as late as 1652, in the second edition of *The Description and Use of the Double Horizontall Dyall.*

These connections with Gunter and Oughtred must have done much to increase Allen's standing in the community. Commissions from such important mathematicians to produce instruments from their designs raised his status, and provided him with new items to sell in his workshop, which was rapidly becoming an indispensable part of the mathematical community of London.

At the same time as Allen was forming links with the professional mathematicians and their associates, he was also building up his workshop. John Blighton and Thomas Shewswell were replaced by new apprentices - Henry Sefton and Robert Davenport (the latter bound on 25th March 1623). John Allen appears to have decided to remain a member of the establishment through this decade, and no doubt rose to the rank of journeyman at this time. A further two apprentices were taken by Allen in 1629: Christopher Brooke was bound on 21st August of that year and Edward Winckfeild (sic) was added to the roll on 29th September. The increasing number of

[42] Cf. Moore, *Moore's Arithmetick* (London, 1650), sig.A6 verso.

people working within Allen's walls indicates that the quantity of instruments which he was selling was increasing at the same time, and all the evidence seems to point to a burgeoning trade for the master.

Through this period we also find indications of Allen's good business sense, of which he made full use in order to establish himself and to draw trade into his workshop. Apart from his engraving in Gunter's book there is little by way of printed advertising from this period, although this was clearly something which had benefited Allen in the past and was to do so again in the future. However, this may have more to do with the fact that there was something of a dearth in the number of mathematical books published during this decade in Britain compared with the 1610s and 1630s. Yet Allen more than made up for the lack of printed advertisement by being ready to take his chances as and when they appeared: he was equally well prepared to form links with people of Royalist leaning as with those of a Puritan persuasion. Thus, while being an associate of the non-conformist Henry Gellibrand and of Henry Briggs and Edmund Gunter (who both had leanings towards the lower end of the Church), he was also very close to Oughtred, who was a High Churchman, remaining a staunch supporter of the Royalist cause throughout his life.[43]

More importantly, Allen made sure that he had links with the court. The astronomical compendium of 1617 (Cat. no. A1) was followed by another one for Charles I, made in 1632 (Cat. no. A2). Allen was also said by Oughtred to have been 'sworne his Majesties servant'[44] at some time before the autumn of 1627 and he seems to have kept a lookout for new and original designs which he could construct for the King's amusement. Such a link with the court was very important for Allen since it

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[43] Information on the churchmanship of the Gresham professors has been taken from Adamson, *The Foundation and Early History of Gresham College*; Oughtred's views are well documented in Cajori, *William Oughtred.*

[44] Oughtred, *To the English Gentrie...The just Apologie of WIL: OVGHTRED, against the slaunderous insimulations of RICHARD DELAMAIN* (London, 1633), sig.B4 verso. I have found no reference to this appointment in the state papers and Turner's reference (in 'William Oughtred, Richard Delamain and the Horizontal Instrument in Seventeenth Century England' in *Annali dell'Istituto e Museo di Storia della Scienza di Firenze*, anno vi (1981), p.110, note 42) to an entry for 26th February 1629 must be treated with caution: the entry speaks only of 'one Allen', and Allen was a common surname.

was a useful piece of publicity to be able to describe himself as making instruments which the King himself used. This kind of connection would have been a major selling-point for his work and would have attracted the patronage of other members of the court circle who were interested in mathematics. The expense of brass instruments no doubt meant that Allen had to cultivate a market among the aristocracy and gentry, and in this he clearly succeeded.

Allen's manipulation of events to improve his position is nowhere more obvious than in the dispute which raged between Oughtred and his former pupil, Richard Delamain, for several years and which embroiled the instrument maker. This dispute has been discussed at length by Cajori in his biography of Oughtred, by Turner in his 1981 article on the horizontal instrument and by Bryden in his bibliographical study of Delamain's work, and has been touched upon by numerous other historians.[45] Unsurprisingly, these writings have tended to focus on the two mathematicians and their wranglings. It is not my intention to consider the main issues of the controversy here but rather on Allen's role in the dispute.[46] This has gained little attention, but the dispute in fact provides a remarkable insight into the part which the instrument maker played in the events, his position in the community and the way in which he used the dispute to his own advantage.

The quarrel between Oughtred and Delamain began in earnest only in the early 1630s, but its origins date back to about 1627 according to Oughtred's account of the

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[45] Cajori, *William Oughtred*; A.J. Turner, 'William Oughtred...'; Bryden 'A Patchery and Confusion of Disjointed Stuffe: Richard Delamain's Grammelogia of 1631/33' in *Transactions of the Cambridge Bibliographical Society*, 6 (1974), pp.158-166. See also E.G.R. Taylor, *Mathematical Practitioners*; A.J. Turner, 'Mathematical Instruments'; Frances Willmoth, *Sir Jonas Moore: Practical Mathematics and Restoration Science* (Woodbridge, 1993), pp.48-49.

[46] The question of priority of invention is discussed in Chapter Five. Questions of priority were an important issue in the seventeenth century, hence the bitterness which was felt by those who believed that their designs or theories had been stolen by contemporaries.

proceedings.[47] This was the point when Allen asked Oughtred for suggestions about a possible New Year gift for the King. Oughtred replied:

> 'I have heard his Majesty delighted much in the great concave Dyall at White-hall: and what fitter Instrument could he have then my Horizontall, which was the very same represented in flat? and that I would upon the backeside set the theorics of the Sun and Moone. And so by help of both sides Eclipses might be calculated with great facility.'[48]

This was an instrument which Oughtred claimed to have devised some twenty-five years earlier, to have presented to the Bishop of Winchester who ordained him, and to have shown in detail to Briggs and Gunter while on a visit to Gresham College. Indeed Gunter had later published a representation of the horizontal projection (unattributed to any designer) in *De Sectore et Radio*,[49] which seems (from Oughtred's tone in the *Apologie*) to have led to a significant cooling of relations between the two mathematicians.

Elias Allen was pleased with Oughtred's suggestion and began work on the instrument at once. However, he soon ran into difficulties and asked Oughtred for help, which the mathematician supplied in a letter of 3rd December, 1627. In this letter Oughtred

> 'taught him the uses of the Instrument especially the Horizontall: and afteward the fabric or delineation of it: and how to find the semidiameters and centers of the severall circles both great and lesser, and the way to divide them. Which letter Master *Allen* yet keepeth:...and which *Delamain* confesseth he saw.'[50]

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[47] It should be noted here that the vast majority of the evidence for the events comes from the books of the two protagonists and so should be treated with a certain amount of caution. Oughtred's point of view is expressed in his *To the English Gentrie...The just Apologie*. Despite its title it was probably intended for those who were interested in buying the circles of proportion or the horizontal instrument, as it appeared in the 1633 edition of *The Circles of Proportion*.
[48] Oughtred, *To the English Gentrie...*, sig.B4 verso.
[49] Gunter, *De Sectore*, p.65.
[50] Oughtred, *To the English Gentrie...*, sig.B4 verso.

The next section of Oughtred's *Apologie* is worth quoting at length because it is here that we meet with Allen's greatest involvement in the dispute:

> 'For some good tract of time after this, when I was now in my Lords [i.e. Arundel's] service, and *Delamain* frequented my chamber: One day after he was gone downe: another man came up and told me, that *Delamain* was in Master *Allens* shop showing unto diverse a little Instrument in brasse of a triangular or rather harpe-like forme, with which he could performe all the questions of the Globe for any part of the world, and make Dialls, and describe Countries, and carry Mines under the earth as farre as betweene Temple barre and Westminster, and such like wonders, which I knew impossible for any such Instrument to performe.'

Oughtred was surprised at this, since Delamain had made no mention of the instrument earlier in the day, but his informant was adamant and Oughtred decided to go and see for himself:

> 'I came to *Elias Allens* shop; but [Delamain] was gone. I told *Elias Allen* what I had heard: and said I would goe to his house, and see it. I came to his house pretending some other occasion. He shewed me a great quadrant of Gemma Frisius he had begunne: and after that a quarter of the Analemma: which I viewing told him that the Meridians were falsely drawne.... Well, at last I asked him for the strange Instrument he had shewed: and would not be answered but he must needs shew it me: which with much tergiversation he did. Tush said I, this is nothing but halfe my Horizontall which he also acknowledging: I asked who drew it? my selfe said he. Is it possible said I that you that cannot make the Analemma, should draw this projection? Doe you know the use of it? Yes said he: I have written some notes of the uses of it: and shewed me some papers: which I looking upon saw the very notes I had declared in my letter to Master *Allen*: but here and there the words disguised after his owne apprehension.'

On his homeward journey, Oughtred called in again at Allen's workshop and said:

> 'I pray answer me a question, but answer me truely. He perceiving what I meant to aske, prevented me with these words, indeed I did: he had the letter of me a whole fortnight, almost as soone as you sent it: and I beleeve he writ it out: for the summer following, unknowne to me, he got my servant to make it for him: for which I was angry. The rest of this businesse let Master *Allen* himselfe tell you.'[51]

Such is Oughtred's account of the proceedings. Delamain, for his part, claimed that he took the projection directly from Gunter's account in *De Sectore et Radio* but offered no explanation as to the origin of his knowledge of the use of the instrument (only the very briefest of descriptions is provided by Gunter). However, it appears that no significant break between Oughtred and his former pupil occurred at this time, since Oughtred goes on to mention that he later discussed the proofs of Delamain's book on the horizontal projection with him, during the latter part of 1630.[52] This followed on from William Forster (another pupil)'s persuasion of Oughtred to allow him to publish translations of notes which the mathematician had written on both the horizontal projection and the circles of proportion (Oughtred's circular slide rule). It would seem that Oughtred was assuming that Delamain would delay the publication of his book until Forster's translation had appeared.

It was only with the pre-emptive appearance of Delamain's *Grammelogia* (his version of the circles of proportion) in January 1630 (OS) and the *Making, description and vse of a small portable Instrument for ye Pocket (or according to any Magnitude) in form of a mixt Trapezia thus Called a Horizontall Quadrant* in 1632 that the final breakdown of relations occurred. The publication of the *Grammelogia* had already embittered relations, since Delamain succeeded in obtaining royal patronage of his book and a monopoly on his version of the instrument, thus forcing Oughtred to alter his

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51 Ibid., sig.B4 verso - C1 recto.
52 Ibid., sig.C2 verso.

design. Delamain's second book was the last straw and prompted a vitriolic attack by Forster in his preface to *The Circles of Proportion and the Horizontall Instrvment*. This in turn led eventually to the bitter wrangling in the second edition of the *Grammelogia* and in *The just Apologie*.

Apart from the evidence in Oughtred's account, the close connection which Allen had with the whole incident is witnessed by the advertisements for his work which appear in all three of the books. The advertisement in the *Horizontal Quadrant* runs 'This Instrument (or any other for the Mathematicall arts) are made in Silver, or Brasse by Elias Allen or John Allen neare the Savoy in the Strand'[53]. That in the first edition of the *Grammelogia* states that 'This *Instrument* is made in Silver, or Brasse for the Pocket, or at any other bignesse, over against Saint *Clements Church* without *Temple Barre*, by Elias Allen.'[54] Allen does not have a specific advertisement in *The Circles of Proportion and the Horizontall Instrvment* but the fact that Oughtred had been at pains to put work in the path of Allen by allowing these tracts to be published at all is advertisement enough for the instrument maker.[55] Interestingly, the second edition of the *Grammelogia* refers solely to *John* Allen: 'These Instruments are made in Siluer or Brasse by Iohn Allen neare the Sauoy in the Strand'.[56]

We encounter Allen at almost every turn in the path. The reason for the appearance of Oughtred's instruments in the first place seems largely to have been in order to increase trade for the maker - Oughtred says of the circles of proportion that he decided

> '...if [Forster] would take the paines to translate some rules I had written into English, we would bestow upon *Elias Allen* (if he shall thinke they may bee

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[53] Delamain, *The Making and Vse of...a Horizontall Quadrant* (London, 1632), facing title page. NB. the address near the Savoy presumably only refers to John Allen, as witnessed by the advertisement to the second edition of the *Grammelogia*.
[54] Delamain, *Grammelogia* (London, 1630 (OS)), p.22.
[55] '...when at *William Forsters* request I was contented to give way that he might publish them, I had not the least thought to be seene or acknowleded [sic] by them: but only to gratify and doe some good to *Elias Allen*,' (Oughtred, *To the English Gentrie...*, sig.B3 recto).
[56] John Allen had moved out of the workshop by late 1629 and set up his own business above the Savoy, according to an advertisement in Daniel Browne's New Almanacke and Prognostication for...1630 (London, 1630). See Bryden, 'Evidence from Advertising', p.309. However, he did not receive his freedom from Elias Allen until 11th January 1631 (OS).

> beneficiall to him) both those Circles of proportion, and also another Instrument,...at my comming up to London in Michaelmas Terme following [1630], to attend my service, I did accordingly make a most free donation to *Elias Allen* by the ingagement of my promise'.[57]

It was almost certainly through Allen that Delamain came to know of the uses of the horizontal projection, and it may possibly have also been by this route that Delamain was able to begin his work on his circular slide rule in late 1630: that at least is what Oughtred would like to have us believe and what he strongly implies in his attack. It was in Allen's workshop, during a discussion among mathematicians gathered there, that Delamain's horizontal quadrant first appeared in public, made, apparently, by Elias Allen's 'servant'. It was Allen who saved Oughtred's book on the circles of proportion by suggesting an alternative form which would not violate the monopoly granted to Delamain, as Oughtred readily admits:

> 'to come at last to a conclusion concerning the Instrument called the Circles of Proportion, as it is set forth, not having, as I have said, the one halfe of my intention upon it; nor with a second moveable circle and a thread; but with an opening Index at the center (if so be that bee cause enough to make it to bee not the same, but another Instrument) for my part I disclaime it: it may goe seeke another Master: which for ought I know, will prove to be *Elias Allen* himselfe: for at his request only I altered a little my rules from the use of the moveable circle and the thread, to the two armes of an Index.'[58]

The very close link between Allen and Oughtred is clearly seen here. Allen had obviously gained much in terms of new instrument designs from his friend and also advice about how to make and use the instruments. Although he was slow at first to realise the designs as instruments they rapidly became a valuable source of income for him, particularly the horizontal instrument, judging by the number of times that this projection appears on extant sundials and also on the back of versions of the circles of

[57] Oughtred, *To the English Gentrie...*, sig.C3 verso - C4 recto.
[58] Ibid., sig.D1 recto.

proportion of Allen's making. However, we see that Allen was not averse to making use of opportunities for further custom as and when they became available. He was careful not to lose the favour of Delamain despite the increasingly acrimonious relationship between the mathematical practitioner and his former teacher, and appears to have remained on good terms with Delamain at least until after the publication of the tract on the Horizontal Quadrant. The lack of advertisement for Allen in the second edition of the *Grammelogia* suggests that matters had gone too far for the instrument maker to be able to retain the connection. However, by this point the affair had probably created sufficient publicity to boost his sales considerably, and Delamain continued to advertise the work of Allen's journeyman, John Allen, who was perhaps the 'servant' referred to in Oughtred's account.

Whether Allen was truly unaware of the making of the original horizontal quadrant for Delamain, as he apparently claimed to Oughtred, is impossible to decide. It would seem unlikely that an instrument of this complexity (with which even the master had had problems when he first encountered it) could be produced by a 'servant' without any assistance and without his master being aware of any untoward activity. One could conjecture that there was a certain amount of collaboration between Allen and one of his more advanced apprentices or journeymen (perhaps John Allen) and that Allen gave his reply to Oughtred in the way that would be least likely to rock that relationship and lead to a cessation of commissions from that eminent mathematician. Whatever the case, Oughtred was prepared to accept Allen's explanation of events, and not to allot any blame to the maker. In fact, the only time that Oughtred makes any depreciative comment about Allen is when he suggests that it would have been better if Allen had been a little quicker to finish his work on the horizontal instrument, rather than leaving it at such a point that Delamain felt it to be neglected by instrument maker and mathematician alike. Oughtred continued to patronise Elias Allen and the latter's workshop and reputation grew and flourished.

'Chief of the Mathematical Instrument Makers'

We have already seen from the Oughtred-Delamain controversy that by 1630 Elias Allen's workshop was certainly being used by those interested in mathematics as meeting-place for discussion and for demonstrating new inventions[59] as well as for purchasing the latest instrumental innovations. There are various mentions of the workshop 'over against St. Clement's Church' in letters from this period, mostly to or from Oughtred. Thus William Robinson wrote to Oughtred in 1633,

> 'I have light upon your little book of artificial guaging, wherewith I am much taken, but I want the rod, neither could I get a sight of one of them at the time, because Mr. Allen had none left. The nature of this book requires instrumental operation, and therefore is well accommodated thereto. I forgot to ask Mr. Allen the price of one of them, which if not much I would have one of them.'[60]

Writing in 1657, the botanist, John Beale, commented to Samuel Hartlib that 'above 20. yeares agoe, I was with Elias Allen over against Clements Church, whilst hee made the Ring-Diall, universall for all Climats.'[61]

By 1642, and surely well before this date, Allen's address was being used as a clearing-house for letters, as witness one from William Price to Oughtred from 2nd June of that year:

> 'Sir, I have been beholding to Mr. Elias Allen for the conveyance of this letter; and if you will vouchsafe me the favour, at your best leisure, to return me two or three lines in answer, and cause it to be left with Mr. Allen for me, I shall rest very thankful for the courtesy'.[62]

That Allen's workshop continued to be a casual meeting place is confirmed by a letter from John Twysden, again to Oughtred:

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59 See Oughtred's description of Delamain's display of his horizontal quadrant (reproduced on p.66).
60 Rigaud, *Correspondence*, pp.17-18.
61 Beale to Hartlib, 14 August 1657, Royal Society, Boyle Letters 7.5.
62 Rigaud, *Correspondence*, p.60.

'The result of my work, according to both your rules, you will find hereto annexed, in which I am bold to beg your judgment where my fault may be, or a farther explication of the same, being assured of your civility and goodness, and glad also to snatch any occasion to renew that little acquaintance I formerly have had of you by our casual meeting at Mr. Allen's in the Strand, by whose means I send this, and who will do me the favour to return your answer hereunto, if you shall please to send it to him'.[63]

Meanwhile, Allen's advertisements in books began to pick up once more.[64] There was the spate of notices in the publications of Delamain and Oughtred in the early 1630s, and these were followed by the second edition of *De Sectore et Radio* in 1636. In 1634, William Barton's *Arithmeticke Abreviated* had carried the information that Napier's bones were 'made in Brasse by Mr. *Elias Allen*, over against St. Clements Church, without Temple Barre'[65] and John Wyberd's *Horologiographia Nocturna* recommended Allen's shop (where 'all sorts of Mathematicall Instruments and also horizontall Sunne-Dyalls in brasse' were produced) as the best place to obtain a horizontal dial for adaptation to a lunar dial.[66]

It would probably be true to say that Allen was no longer urgently in need of the extra publicity which came to him through these advertisements. His trade was blooming and he received some important commissions during this period. Apart from the astronomical compendium which he made for the King (probably through his own initiative) he also produced a circles of proportion, with Oughtred's horizontal projection on the back, which was presented to St. John's College, Oxford, by George Barkham. In 1631 he supplied Thomas James' expedition to search for the Northwest Passage with a set of compass needles.[67] An octagonal sundial was given by Allen to

[63] Ibid., p.68.
[64] For a full list of advertisements for Allen's instruments, see Appendix 3.
[65] Barton, *Arithmeticke Abreviated* (London, 1634), p.20.
[66] Wyberd, *Horologiographia Nocturna* (London, 1639), p.14.
[67] James, *The Strange and Dangerovs Voyage of Captaine Thomas Iames* (London, 1633), sig.Q1 verso.

the parish of Ashurst, near Tonbridge in Kent (from which comes the suggestion that this was the place of his birth) in 1634. Three years later he made a six-foot astronomical quadrant for use by the astronomer John Greaves, at that time professor of geometry at Gresham College but later to take up the post of Savilean Professor of Astronomy in Oxford. This last piece is the only evidence for large-scale work by Elias Allen: the vast majority of the surviving pieces are all under two feet in dimension and many are considerably smaller, measuring only a few inches across.[68]

One further point of interest relating to this period comes from a passing comment made by Robert Hooke in the course of one of his lectures many years later. In a discourse on navigation and the measurement of degrees of longitude he mentions that 'the Standard Foot we now use was since that time [1635] agreed upon by a Club of our Mathematical Instrument-makers, of whom Mr. Elias Allen was the chief'.[69] Here we have further evidence of the important position which Allen held in the mathematical community of London. Not only that, but we are shown that his fame was to outlive him by many years - this particular lecture was given in 1683.

It is hardly surprising that with such a formidable reputation now as an instrument maker, Allen began to turn his mind to the question of advancement within the political system of the merchants of London. There may have been relatively little chance of an opening in the large Grocers' Company, particularly for a master who was not a practising grocer, and so it may be for this reason that Allen made a move towards the newly-formed Company of the Clockmakers.

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68 For more details of these instruments see Cat. nos. A2, C2, X2 and X6.

69 Hooke, *The Posthumous Works of Robert Hooke* (London, 1705), p.457. We have no further information either about this instrument makers' club (perhaps a rather informal group), or about the standardisation of the foot.

Allen in the Clockmakers' Company[70]

The Clockmakers of London had been petitioning the King for permission to found their own company for more than ten years. In 1622 they were turned down by James in their search for a charter, but their plea to Charles in 1629 met with more success: on 22nd August 1631 a charter was issued by the King, and the Worshipful Company of Clockmakers came into existence. The guild had been established in order to support the interests of the growing number of clockmakers and watchmakers in the capital. They also saw themselves as the natural home of the instrument makers, since the making of mathematical instruments was more akin to the construction of clocks than to any other craft in the city. Some instrument makers were glad to be able to be connected to a company which was closer to their trade than the goldsmiths, the joiners or the grocers, and readily took the chance to become brothers of the new company (this was a kind of associate membership - freedom of two guilds at once was not permitted). However, others resented the pressure placed on them to switch their allegiance to the new guild, particularly since the Clockmakers' Company was considerably more rigid in its regulations and their observance than the older guilds which had grown lax over the years.

Although Elias Allen had had no public part in the establishment of the new company, he was very ready to take advantage of its existence once it had been founded. He was well known in his own trade as the best maker in brass to be found and had few worries over sustaining the seniority of his position. In his own Grocers' Guild he had little chance of gaining any real power: the Grocers were one of the largest livery companies in London, and the ruling elite were almost exclusively drawn from the bona fide grocers. However, in becoming attached to the Clockmakers' Company Allen, with his formidable reputation, was in a good position to earn himself some political influence within the government of the merchant class of the city.

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[70] Most of the information relating to this company is taken from Loomes, *Early Clockmakers*.

By tradition, entry into the livery of a company was guarded by the higher echelons of the guild. It was the task of the ruling council of the company to choose who could be granted the honour to become a steward of the company and thus to place their feet on the first rung of the ladder. This could be something of a mixed blessing: it was indeed a privilege to be selected as a steward and to have the possibility of ascending to the position of assistant and perhaps even higher. However, the stewards were expected to pay for and organise the company's Annual Feast (this was the main purpose for which they were appointed). Stewards served for one year and it was from their ranks that the assistants of the guild were selected.

The ten or so assistants were the most junior members of the court of the company and were expected to pay £6 13s 4d for the privilege. However, the office of assistant was held for life, thus guaranteeing a continuing influence on the government of the guild. The most senior assistants went on to become Wardens in the Company and eventually to hold the position of Master. These senior offices were held for a year at a time, in rotation, by appointing an assistant to the office of Junior, Middle and Senior Wardenships in turn, and then to the post of Master, which he would hold for a year, after which he would return to the rank of Assistant once more. This does not mean that Assistants automatically ascended to the higher ranks: some were never offered these posts, and remained at the lower end of the council for the rest of their working lives.

As has been said, the appointment to the post of Steward or Assistant was something of a mixed blessing since the financial outlay was considerable. However, the appointment, once made, was not considered to be optional, and some members were forced to pay out sums of money that they could ill afford to lose, while others went to quite some lengths to avoid having to take on the extra responsibilities. Nevertheless, for those who could afford it, a place on the council was an opening to a great deal of authority and influence in the guild.

Elias Allen became an assistant in the Clockmakers' Company on 3rd October 1633,[71] just two years after its creation. This was despite the fact that he appears never to have become an official member of the guild.[72] On 18th January 1635 (OS) he took on the responsibilities of the Renter Warden or Treasurer[73] and this was followed a year and a day later by promotion to the Mastership (a post which he in fact held for eighteen months, until 29th July, 1638).[74] Within six years of the establishment of the new company, the best mathematical instrument maker in London had reached the highest position in a guild to which he did not officially belong, and no doubt he retained his influence on the ruling council from that time on. It is a measure of his importance within the trade that he could have risen so high so fast, and it is at this time that we see the master maker at the height of his powers.

Allen also made use of the Clockmakers' Company to obtain more apprentices. Although it was forbidden by company law for associate members who were not freemen to bind apprentices directly, the option was available for apprentices to be bound to one master and then turned over to another master to learn their trade. This second master did not have to be a member of the company, as long as he was a freeman of the City, by virtue of his membership of another guild. In this manner, Allen took on four apprentices over and above those he already had bound to him through the Grocers' Company. Ralph Greatorex was bound on 25th March 1639 and Edward Grimes almost two years later (15th March 1640 (OS)), through Thomas Dawson and Richard Masterson respectively, while the good offices of Thomas Alcock provided Allen with two further apprentices - Withers Cheney and John Prujean - on 13th April and 16th May 1646. It is perhaps noteworthy that these three associates of

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71 Minutes of the Clockmakers' Company, Guildhall MS 2710, vol.1, f.10: 'This Court Mr Elias Allen & Mr Peter Closon were admitted and sworne Assistants & did faithfully promise to pay 6^{l} 13^{s} 4^{d} each of them the 10th of Novem. next following y^{e} date hereof unto Mr John Harris renter Warden appointed by this Court to receive it.'

72 The company minutes have no record of Allen's entry into the guild.

73 These responsibilities apparently included bearing the brunt of disgruntled Guild members' annoyance: 'The same day [18th Jan 1635 (OS)] Thomas Hill a dyall maker at y^{e} Tower had 4 pocket dyalls Cutt & did affront Mr Allen with opprobrious words.' (Minutes, op. cit.)

74 Minutes of the Clockmakers' Company, vol.1, f.17: 'The Court aforesaid were in nominacon for M^{r} for y^{e} yeare ensuing Mr Charleston Mr Allin & Mr Harris & Mr Allen was freely elected M^{r}.'

Allen all held important posts early in the history of the Clockmakers' Company. Thomas Alcock became an assistant in 1638 and a Warden in 1645, although he never succeeded to the Mastership; Thomas Dawson was a petitioner for the original charter and became a free brother of the company shortly after its foundation; Richard Masterson became an assistant at the same time as Allen, rose to the office of Warden in 1637 and held the Mastership in 1642.[75] The Thomas Dawson mentioned here may well have been the same man who married Elias' daughter, Elizabeth, on 28th March 1630.[76]

Through his association with the Clockmaker's Company, Allen established a position of power and authority for himself, which presumably aided the growth of his trade. He also created craft successions for himself in two guilds, both of which would give rise to eminent makers during the eighteenth and nineteenth centuries.

A pictorial tribute

By the late 1630s Elias Allen was an important man indeed. His workshop thrived, he was on good terms with some of the greatest mathematicians of the period, and he wielded considerable power within his adoptive company. To crown his success he had his portrait painted, an unusual event indeed for an instrument maker. The painting was the work of Hendrik van der Borcht and although the original has not survived, an engraving was made of it in 1660 (possibly 1666[77]) by the celebrated artist, Wenceslaus Hollar; this is the sole source of our knowledge of Allen's personal appearance.

The picture shows a middle-aged man seated at his work bench with the tools of his trade strewn around him. He is wearing very austere clothes - a dark coat, buttoned down the front is trimmed with simple white cuffs and a broad, plain, white collar - which might indicate a Puritan background, or possibly a political stance favouring the

75 Information from Loomes, *Early Clockmakers*.
76 St. Clement Dane's Marriage Registers, vol.1, loc. cit.
77 It is impossible to decipher the final digit from the engraving.

Parliamentarians. His straggling hair is cut well above the shoulder; his forehead is largely bald, although he sports a thick, close-cropped beard and an expansive moustache beneath his prominent (even bulbous) nose. His eyes are narrowed and there is no hint of a smile on the lips: instrument-making is clearly a serious business.

Allen's right forearm rests on the bench in front of him and he is pictured holding a pair of dividers. The remainder of the bench is occupied with a tight clutter of instruments. We notice a horizontal sundial, complete with ornamental gnomon, and an equinoctial ring dial of Oughtred's design. The other items are all parts of a Rathborne peractor which Allen is in the process of assembling. His left hand supports the circumferentor base, which rests against his chest; the horizontal dial obscures most of the altitude quadrant (the peractor proper) and its alidade (with sights as yet incomplete); the sights for the circumferentor and the ball and socket joint for fitting the circumferentor to its staff lie on the bench. On the wall behind Allen's left shoulder hang a quadrant, with several simple degree scales inscribed on it, and a sector. Beneath these, a rack supports a variety of chisels and other tools. The back wall is completely plain and devoid of drapes of any description.

The existence of the Hollar engraving (along with the portrait of Oughtred mentioned in the introduction) is almost certainly a direct result of the links of all these people with Arundel House, which lay so close to Allen's workshop, and with the Earl of Arundel himself, Thomas Howard. Howard was Oughtred's most prominent patron and it was at Arundel House that Oughtred resided when he was in London. The Earl had also gathered both van der Borcht and Hollar under his wing when on his travels in the German states. He had recruited them to his service, and by 1637 they were both established at Arundel House, van der Borcht as curator of the Earl's extensive art collection and Hollar with a commission to produce engravings of the pictures forming the collection. Van der Borcht held his post until 1642 and so it is likely that the portrait of Allen was made at some point in these six years; this would certainly fit with the appearance of Allen as a middle-aged man, and also with the fact that by this time he was sufficiently well established to commission a portrait.

Civil War and its aftermath

At this point, the comfortable, well-organised world in which Elias Allen lived was invaded by the chaos of political events. Civil war broke out in 1642 and, while it was possible to avoid taking sides in the conflict and the roar of battle was a distant rumble for much of the population, nobody could escape completely from the consequences of the struggle between King and Parliament. Trade suffered and food shortages were a common occurrence. In London, perhaps, life may have continued with some semblance of normality, since the city remained in the hands of the Parliamentarians throughout the war. In fact, the greatest unrest in the capital had come with the demonstrations and riots of 1640 and 1641, and relative peace marked the war years themselves, the most obvious effects of the conflict here being the never-ending demand for money to support the war effort and the crippling lack of coal during the winter of 1643 and of other essential supplies at various periods.

The story of Allen's workshop is broken by a long gap during the war. This is unlikely to stem from a direct involvement in the conflict: despite the possible Puritan sympathies indicated by his portrait and his Christian name, Allen's connections with the court and with other aristocratic patrons probably left him in a rather ambivalent position regarding the political events. Rather the silence is more likely to be due to the dearth of mathematical texts and of the lack of demand for instruments other than those which had military applications. Such instruments do not seem to have formed any major part in Allen's work and there appears to be no surviving evidence of gunner's quadrants or calipers being produced in his workshop. If little mention is made here of one of the focal points of seventeenth-century English political history it is simply because the effect on Elias Allen is essentially impossible to gauge. What *is* known is that these years were marked by personal tragedy. Firstly, his wife died in 1642 and was buried in St. Clement's on 6th December. Five years later his last remaining son, Henry, was entered in the burial registers, on 10th June 1647.[78]

Nevertheless, Allen's workshop appears to have continued to function during the war. Edmund Wingate's *The Use of the Rule of Proportion* of 1645 (one of the very few mathematical texts to be published during the decade) reports that the instrument 'is perfectly made in Brasse by *Elias Allain*, at the signe of the Blackmoore without Temple-Barre, London'.[79] The trade was sufficiently stable for Allen to take on two more apprentices the following year and a further advertisement for the instrument maker's wares appeared in Seth Partridge's *Rabdologia* in 1648.

Hence it comes as something of a shock to find that Allen was short of money. The minutes of the Court of the Clockmakers' Company from 15th January, 1648/9 record that Allen

> 'made great complaint of his necessity and want of money hee stood in att that tyme; And in regard it did appeare to the Courte that hee had formerly done many good offices and services for the Companye; the Companye were contented and did Order that the Renter Warden should yssue forth unto him Fourty Shillinges out of the Companyes Stock'.[80]

The reasons for this necessity are unclear. Perhaps trade was beginning to pass from Allen to other instrument makers, perhaps the continuing unsettled political climate (in particular the upheavals in London caused by the Levellers and their followers) had kept the gentry market away from the shops of the capital, perhaps the general lack of money had led to a decrease in demand for items which might appear as something of a luxury at a time when even essentials had become scarce. Whatever it was, even fame could not keep the best instrument maker of his day from a decrease in fortune and an unavoidable plea for charity from his adoptive company.

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[78] St. Clement Dane's Burial Registers, loc. cit. I have assumed that Abraham and Allyson (if that was her name) had already died by this point (although I have been unable to trace burial details for these two), since Elizabeth was the sole surviving child at the time of Allen's death.
[79] Wingate, *The Use of the Rule of Proportion*, final page.
[80] Minutes of the Clockmakers' Company, Guildhall MS 2710, vol.1, f.42.

The Company was further importuned by Allen in 1651 for payment of legal fees in a case concerning his apprentice, Edward Grimes: unfortunately the records do not provide details of the nature of the trial.[81] However, there is circumstantial evidence that Allen's daughter, Elizabeth, may have been involved. Her surname was certainly Grimes when she was granted the administration of Allen's estate and the Parish Register of St. Benet, Paul's Wharf, records the marriage of Elizabeth Dawson and Edward Greymes on 6th January 1652 (OS).[82]

It seems that Allen continued in trade right up until the time of his death. As late as 1652 his skill was still being promoted by his friend, William Oughtred.[83] The account of the equinoctial ring dial which was included in the second edition of Oughtred's book on the horizontal dial concludes with the information that

> 'These Instrumentall Dials are made in brasse by *Elias Allen* dwelling over against *S^t^. Clements* Church without Temple Barre, at the signe of the *Horseshooe* neere *Essex* Gate'.[84]

The specificity of this address (which must have been known to the majority of the mathematical community at this point) perhaps indicates that the equinoctial ring dial was designed for a wider dilettante market who were not necessarily directly connected with the world of the mathematicians and mathematical practitioners at all.

Allen died the following year. According to the inscription below his portrait it must have been between 25th and 31st March, since the date is give as the end of March 1653 and the change of year was marked by Lady Day (25th March). He was

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81 13th June 1651; Minutes of the Clockmakers' Company, Guildhall MS 2710, vol.1.
82 St. Benet's Marriage Registers (1619-1715), Guildhall MS 5716, loc. cit.
83 It is perhaps of some interest that it was then four years since a book had carried an advertisement for Allen (Partridge's *Rabdologia*) and the only one to follow Oughtred's was in a new edition of Gunter's *Works* in 1653. Perhaps writers were beginning to turn to the new generation of makers.
84 Oughtred, *The Description and Use of the Double Horizontal Dyall* (London, 1652), sig.X8 verso.

buried in the churchyard which had been so close to his home for fifty years, on the 1st April.[85] He was probably about sixty-five years old.

He appears to have left no will, but the administration of his estate, which was accorded to his daughter, survives. It reads as follows:

> 'The thirteenth Day [of January] Le[tte]rs of Ad[ministrati]on issued out to Elizabeth Grimes al[ia]s Allen the n[atu]rall & lawfull Daughter & only child of Elias Allen late of ye p[ar]ish of Clement Danes in the County of Midd[lesex] dec[ease]d to Ad[minis]ter the goods, ch[att]ells & Debts of ye [said] dec[ease]d shee being first sworne truly to Ad[minis]ter it.'[86]

The workshop itself was probably taken over by Ralph Greatorex. Certainly Greatorex appears to have received his master's mantle of authority: he is implicated in a letter as being the new conveyor of mail to William Oughtred, and in a publication of 1651 he was already being recommended as a maker, alongside Walter Hayes, one of John Allen's former apprentices.[87] He filed for his freedom in November 1653.

Allen's Craft Succession

Ralph Greatorex was only one of a large number of Allen's former apprentices who set up workshops which grew and flourished through the latter half of the century. Indeed it is as much in the legacy which Elias Allen left behind in the form of his craft succession as in the witness of his own work and reputation that we see the achievement of the greatest instrument maker of the early seventeenth century. At least seven of his apprentices became master-craftsmen and plied their trade successfully for many years to come. These were John Blighton, John Allen, Robert Davenport, Christopher Brooke(s), Ralph Greatorex, Withers Cheney and John Prujean. (While Thomas Shewswell, Henry Sefton and Edward Winckfeild were freed we have no

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[85] St. Clement Dane's Burial Registers, vol.2, loc. cit. The entry describes Allen as 'the Famous Mathamatticall instrum[en]t maker'.

[86] Public Record Office, PROB 6/28, f.83.

[87] Richard Stokes to William Oughtred, 6th February 1654 (OS) in Rigaud, *Correspondence*, p.82; Oughtred, *The Solution of all Sphæricall Triangles...Published with the consent of the Author by Christopher Brookes* (Oxford, 1651), title page.

further knowledge of them, unless Shewswell was the Shaswell who set himself up as a compass maker in London.[88]) The work which survives from these men shows them to have inherited the artistry of their renowned master, and is a tribute to his skill as a teacher as well as a maker. Four of them remained in London while the other three took their Allen-trained skills further afield - two to Oxford and one to Edinburgh.[89]

John Blighton established his workshop in Tower Street near the Bull Head Tavern according to the advertisement in Henry Bond's book on gauging.[90] He worked in silver and brass and was recommended as a maker of Thomas Browne's spiral 'slide-rule', among other instruments. He took many apprentices and remained active until at least 1654 and possibly longer than this, although it is difficult to be certain, since much of the knowledge about him is owed to the details of his apprentice bindings; those after 1654 may refer to his son, freed on 9th August of that year and also called John.

John Allen presumably worked for several years as a journeyman for Elias, not taking his freedom until January 1631 (OS). By this date he had already established a place for himself not far from his master, 'neare the Sauoy in the Strand'. He almost immediately took on as apprentice Walter Hayes, bound to him on 22nd February 1631 (OS) and freed ten years later, but there are no records of any other apprentices bound to him. Surviving items of his work include two Gunter sectors (Oxford Museum of the History of Science and the Science Museum), two double horizontal dials (Centre National d'Histoire des Sciences, Brussels and the Maidstone Museum and Art Gallery) and an example of Delamain's Horizontal Quadrant (Musées Royaux d'Art et d'Histoire, Brussels), this last being a rare instrument indeed - there are only two other known examples according to Turner. [91] It is interesting that, although he was admitted as a Brother to the Clockmakers' Company, this was not until 1653:[92] perhaps

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[88] The Shaswell reference comes from the Project Simon database at the National Maritime Museum.
[89] Allen's craft succession in the Clockmakers' Company is displayed in Brown, 'Guild Organisation', p.14, although this does not include John Prujean.
[90] Henry Bond, *A New Booke of Gauging* (London, 1634), p.12.
[91] Mary Holbrooke, *Science Preserved* (London, 1992); A.J. Turner, 'William Oughtred...', p.124.
[92] Loomes, *Early Clockmakers*.

this is a remaining hint of friction between himself and his master, that he did not make the move until after Elias Allen's death.

Robert Davenport was freed on 25th November 1633, and nothing further was heard of him until 1647 when he was granted permission to trade in the city of Edinburgh. Whether he remained in the Scottish capital for long is unknown: it is unlikely, since the city appears not to have created a large market for the sale of mathematical instruments. He produced at around this time a small version of the circles of proportion with the horizontal instrument on the back, which appears to be the only extant example of his work.[93]

Christopher Brooke, as has already been mentioned, may well have been the Christopher Brookes who married the daughter of William Oughtred, thus forging a further link between Allen and the mathematician priest. Brookes was freed in 1639 and later moved to Oxford where, according to his own description, he became a manciple of Wadham College. As well as plying his trade as an instrument maker he wrote a book on the use of a quadrant he had devised and published a treatise of Oughtred's on the subject of spherical trigonometry.[94]

Ralph Greatorex, as has been said, probably took over Allen's workshop after his master's death. Although he did not take his freedom until late 1653 he would appear to have had an independent establishment from at least as early as 1651 since the title page of Oughtred's treatise on spherical trigonometry speaks of him as 'at the *Adam* and *Eve*' in the Strand. He took five apprentices between 1654 and 1658 (surely more than was permitted to him by the Company rules!), of which three are known to have been freed. He was appointed an Assistant in the Clockmakers' Company in 1666 but excused a mere two months later because he was likely to be 'going to sea'. Greatorex is the most famous of Allen's apprentices but, as Tony Simcock remarks,

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93 Information from Bryden, 'Scotland's earliest surviving calculating device: Robert Davenport's Circles of Proportion of c. 1650' in *The Scottish Historical Review*, 55 (1976), pp.54-60.
94 Brookes, *A New Quadrant* (London, 1649); Oughtred, *The Solution of all Sphæricall Triangles*.

> 'Paradoxically, this fame is based not on the survival of his instruments but on references to him in the writings of his intellectual acquaintances, particularly Samuel Pepys.'[95]

The instruments which do survive are a double horizontal dial in a private collection and two equinoctial ring dials (one in the Oxford Museum and one in a private collection), but contemporary references show him to have worked not only in the area of mathematical instruments but also to have produced some of the new instruments of natural philosophy, such as air pumps. He died at some point in the 1670s.

Withers Cheney, though not known through any extant instruments, was clearly a maker of some importance during his lifetime. He is said to have worked in Fleet Street as a 'waxchandler and free clockmaker'[96] and he bound eight apprentices between 1659 and 1690, at least three of which attained their freedom (Thomas Feilder proceeding to become Master of the company in 1715). He became an Assistant in 1682 and a Warden in 1691 but in October 1695 he refused the Mastership, apparently on account of his residence out of town.

John Prujean was the last of Allen's apprentices to set himself up in business and, like Christopher Brookes, he took his trade to Oxford, in March 1663 (OS), where he remained until his death in 1706. He wrote various pamphlets on the instruments which he manufactured, several of which survive in the collection of the Oxford Museum of the History of Science (various quadrants, an astrolabe and a planispheric nocturnal and analemma). It appears that he took no apprentices.[97]

One further member of Allen's craft succession is worth singling out for mention. This is Walter Hayes, the only known apprentice of John Allen, who essentially filled the same position in the mathematical community in the second half of the seventeenth century which Allen had occupied in the first half. He had an

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95 Simcock, 'An Equinoctial Ring Dial by Ralph Greatorex' in Anderson, Bennett & Ryan, *Making Instruments Count* (Aldershot, 1993), pp.201-215.
96 Loomes, *Early Clockmakers*.
97 More information on Prujean is provided in D.J. Bryden's 'Made in Oxford: John Prujean's 1701 Catalogue of Mathematical Instruments' in *Oxoniensa*, LVIII (1993), pp.263-264.

exceptionally busy workshop, taking fourteen (if not fifteen) apprentices during a working life which spanned at least forty-five years. His work is advertised in numerous books from the period and many examples of his instruments remain to demonstrate its very high quality. Apart from producing many instruments, and stocking quantities of mathematical textbooks for sale, he was very active in both the Clockmakers' Company (which he joined in February 1667 (OS)) and also in the Grocers' Company. In the former he became an Assistant in 1670, Warden in 1679 and Master in 1680, continuing to attend the Council until 1687; in the latter he was admitted to the Livery in 1668, became a Steward in 1670, an Assistant in 1681 and a Warden in 1686. Thus in terms of craft politics he rose to even greater heights than his illustrious predecessor.

The extent to which Elias Allen influenced the instrument trade even after his death has been shown above by the study of various individuals, but there are also some interesting statistical comments which can be made. His fourteen apprentices had at least twenty-five apprentices of their own and these in turn bound thirty-seven young men into the trade of instrument making. Meanwhile the record of their standing in the Clockmakers' Company is impressive. Taking Loomes' work as a source of information (which unfortunately does not go beyond the end of the seventeenth century), totals can be made of the number of apprentices who became freemen of the Company and those who became Assistants, Wardens and Masters on the Council; the procedure can be repeated for Allen's craft succession within the Company. Of 1323 apprentices freed overall, 131 became assistants, 85 became wardens and 72 rose to be Master of the Company. Within the Allen dynasty the figures are 27 freemen during the seventeenth century, 6 assistants, 5 wardens and 4 masters. The proportion is significantly higher for that particular succession than for the Company as a whole.

We can also look down through the lists at some of the names which appear in later years among the makers who have direct links back to Elias Allen. As I have said, Walter Hayes and Ralph Greatorex held influential positions during the latter part of the seventeenth century within the mathematical community, and Greatorex also moved in the circles of the newly established world of the mechanical philosophers. Hayes' most famous apprentice was Edmund Culpeper, renowned for his microscopes, but it was through Nathaniel Anderton (freed in 1669) that the succession eventually led down both to George Adams and his son (important makers of instruments of natural philosophy in the eighteenth century) and to the well-known Troughton dynasty, who supplied astronomical instruments to observatories throughout Europe, America and the British Empire. Meanwhile John Blighton, one of Allen's earliest apprentices, was the forerunner of the great firm of Spencer, Browning and Rust, who flourished in the eighteenth and nineteenth centuries, first through producing navigational instruments in bulk, and later through development of the spectroscope for the large-scale market. Through them Allen's craft succession extended unbroken into the twentieth century.

When Elias Allen began to trade as an independent instrument maker, most makers did not consider instrument making as a career in itself and were normally involved in surveying and in engraving maps as well. Allen took advantage of the increased interest in mathematics at the beginning of the seventeenth century and was able to find a space for himself which allowed him to concentrate purely on the area in which he excelled - the making of mathematical instruments in metal. In Allen's association with mathematicians such as Gunter and Oughtred we see the first glimmerings of the symbiotic relationship between mathematician and maker which was to dominate English astronomy of the eighteenth century, when George Graham, Jesse Ramsden and the Troughtons worked in partnership with the Astronomers Royal and were members of the Royal Society. He truly deserves the title accorded to him in the inscription adorning his portrait (itself made by no mean engraver): 'In his time, the most talented artist in the engraving of mathematical instruments of brass.'

CHAPTER THREE

Studies in the Output of Elias Allen's Workshop

(1) The Gunter Quadrant

Through the biographical study of Elias Allen something may be inferred of the man's character. He comes across as a clear-thinking, hard-headed, shrewd businessman, who made full use of his connections to bolster and expand his trade and his hold on the market. However, such characteristics in themselves are not enough to explain why he established such a thriving business and was apparently able to support himself purely through the construction of mathematical instruments.[1] An important reason for Allen's success rests in his consummate skill as an artisan. It was for this that he was known and respected throughout the mathematical community: if his instruments had been of poor quality, no amount of salesmanship would have drawn people to his premises to commission instruments or to purchase whatever may have been available ready-made. Other reasons have been presented in the preceding chapters. A fuller understanding can only come through the study of the instruments themselves and therefore it is time that we turned to consider the products of Allen's skill, the instruments which have survived to the present and which are material witnesses to their maker's ability. The next three chapters will be concerned with discussion of a variety of the instruments most strongly represented in the extant corpus: Gunter quadrants and sectors, and various instruments designed by William Oughtred - the circles of proportion, the horizontal instrument, the double horizontal dial and the universal equinoctial ring dial.

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[1] There are no known references to Allen obtaining income from other sources. The survey in Oxfordshire which is mentioned in Taylor, *Mathematical Practitioners*, was the work of another Elias Allen who was dead by 1637, according to the record of a payment made to his widow in May of that year. (Public Record Office, E/403/2756, f.44)

Why study instruments?

The study of instruments has all too often been ignored in researching the history of mathematics. It is perhaps not immediately obvious how in-depth study of these objects can add significantly to the information provided by texts. They may be mute witnesses to the skills of their creators and be useful for learning about techniques of engraving or the processes employed in dividing scales, yet what can they say about mathematics? It is now ten years since Willem Hackmann addressed the issue of the study of instrumentation in the history of science in general and wisely remarked that

> 'The mainstream historian of science could argue that the kind of studies described so far [mostly concerned with production techniques, makers and workshops] might produce interesting facts, but that these have no relevance to understanding scientific progress. But surely any attempt to place makers and their instruments in a social and intellectual framework must aid us in this endeavour. Scientific progress is not simply built on the labours of the giants such as Galileo, Newton and Einstein. Neither scientists nor instrument-makers are homogeneous groups.'[2]

He concluded that 'knowledge in the history of science, like that in science itself, comes from many quarters.'[3] Instruments are a very valuable source of information about the progress of science and mathematics and their nature in any particular era, and this information is revealed by various different approaches to studying them. I have aimed to make use of many of these approaches in dealing with each type of instrument, in order to gain the most information possible from them, relating to both Allen's skills and practices as a maker and also the mathematical culture of the day, and what it owed to these particular instruments.

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[2] Willem D. Hackmann, 'Instrumentation in the theory and practice of Science: Scientific Instruments as Evidence and as an Aid to Discovery' in *Annali dell'Istituto e Museo di Storia della Scienza di Firenze*, Anno X (1985), p.97.
[3] Ibid., p.115.

Approaches to studying instruments

(i) The standard 'antiquarian' approach

The most obvious method of learning from instruments in research is through what Hackmann describes as 'detailed morphological and taxonomic studies'[4] - i.e. the description of the different kinds of instruments and their classification into types (by maker or by function or by material, etc.). These are most useful for the identification of specific pieces or for the assignation of unsigned instruments to particular makers or workshops. Such concerns might be classified as being at the more 'antiquarian' end of what may be learnt from instruments and in this thesis they are employed mainly in the catalogue of Allen's work as represented in British museums. Little more will be said about them at this point.

(ii) Through construction

The second method of approach is through the construction of the instruments. In making replicas of the originals one quickly becomes aware of the types of problems encountered in producing a high-quality mathematical instrument. This was not a question which was purely the concern of the instrument makers, since there is material evidence that numerous amateur mathematicians were eager to try their hands at making their own instruments.[5] Designers of instruments were usually aware of this propensity and thus discussed at length in their texts the methods by which the instruments were constructed (often followed by the fallback comment, 'or it can be purchased in brass from X and in wood from Y').[6] Of course, we must be aware that some of the problems of making instruments have been considerably ameliorated for

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[4] Ibid., p.87.

[5] For example, there is an amateur brass Gunter quadrant in the Whipple Museum of Science in Cambridge (Accession no. 1765) and also a brass horizontal instrument with a logarithmic slide rule by a fellow of Trinity College (Accession no. 1029). See D.J. Bryden, *The Whipple Museum of the History of Science, Catalogue 6. Sundials and related instruments* (Cambridge, 1988), Cat. nos. 281 and 340). Anthony Turner mentions a horizontal quadrant by Pigot in 'William Oughtred...', p.124, which is thought to be an amateur instrument.

[6] E.g. William Barton in *Arithmeticke Abreviated* describes how to make a set of Napier's bones in pasteboard, stuck onto wooden blocks 'or you may have them made in Brasse by Mr. *Elias Allen*, over against St. Clements Church, without Temple Barre, and by divers other Instrument-makers' (p.20).

the modern researcher: complex calculations involving long multiplication and division, trigonometric functions, logarithms and square roots are easily performed with a pocket calculator; in the seventeenth century they could take many hours of tedious pen and paper work with sets of tables. Yet there are others which can still be as taxing today as they were when the instruments were originally devised.

(iii) Through use

Learning how to use the instruments involves both the instruments and the texts which describe them, and it is clear that a manual such as *De Sectore et Radio*[7] or *The Description and Use of the Double Horizontall Dyall*[8] can only be fully interpreted when one has the instrument to hand and can follow the instructions manually.[9] One of the main concerns of these chapters on instruments will be to present a brief description of the various ways in which they can be used to perform calculations or to take readings. This procedure will highlight various issues which are not immediately obvious from the study of the original texts, but which are relevant in coming to an understanding of seventeenth-century mathematics.

General questions

The other main concern will be to point up areas of interest which are not covered by the 'antiquarian' studies or by the 'instruction manuals'. These topics are too diverse to be classified under one particular heading, although they could generally be described as relating to the instruments' role in the surrounding culture. They tend to be specific to particular types of instrument and raise such questions as 'For whom were these instruments intended?', 'How useful were they in fulfilling their declared

7 Edmund Gunter, 1623.
8 William Oughtred, 1636.
9 Although the presence of carefully engraved plates in texts may have aided interpretation of the writing. There also appears to have been a fairly common practice of selling paper pulls of instruments. For instance, John Sellar and Philip Lea both sold paper versions of instruments at the end of the seventeenth century, and the quadrants from their workshops can all be traced to engraved plates in standard works of the time. The Oxford Museum of the History of Science has recently acquired a paper pull of a Henry Sutton planisphere of 1659 (sold at Christie's, on Thursday 26th September, 1991; lot 83).

purposes?' and so forth.[10] Because of the variety of issues that are at stake here, it is of little value discussing further in the abstract: I will simply deal with the different themes as and when they arise.

The Gunter Quadrant

A Brief History of Horary Quadrants

Horary quadrants first appeared in Europe in the twelfth century, although they had been developed among the Arabs before this time. Their purpose was to tell the time by taking observations of the altitude of the sun and converting this into the time of day once adjustments had been made for the declination of the sun and for the latitude. The quadrant plate carried a table of dates and declinations in order to calibrate a bead on the plummet string, and the bead, once calibrated, would show the time by its position among the hour lines inscribed on the plate, when a reading of the sun's altitude was taken. These hour lines represented unequal hours[11] and the quadrant was a universal instrument, usable in any latitude.[12]

New quadrants were developed during the sixteenth century which were based on single astrolabe plates and hence were only for use in one latitude. They worked within the system of equal hours, rather than unequal hours. This is the class of quadrant to which the Gunter quadrant, which was one of the most sophisticated, belongs. A simpler version, carrying only the hour lines and a declination scale, was

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[10] This is an area in which ethnographers and archaeologists work as a matter of course, but it has yet to make a great impact upon the conventional historian. The historian who is not bounded by printed and written evidence will uncover a wealth of new insights which will help to open new avenues of research and increase our understanding of the past.

[11] The unequal hours system reckoned twelve hours from sunrise to sunset, each one of the same duration on any one day, but varying in length with the time of year. It was the most common hour system until well into the middle of the fourteenth century, at which point it began to be displaced by various equal hour systems.

[12] A more detailed discussion of the early history of the horary quadrant can be found in F. Maddison, 'Medieval Scientific Instruments' in *Agrupament de Estudos de Cartografia Antiga* (serie Separatas), 30 (1969), pp.13-14 and pp.16-17.

described by Thomas Fale in his book on dialling.[13] It was not an independent instrument, since it required the use of tables in order to find the place of the sun in the zodiac for a particular date (the Gunter quadrant has the relevant date scale inscribed upon it), but its main function was the same as that of Gunter's instrument.[14]

Description of the Quadrant

The Gunter quadrant was first described in an appendix to Edmund Gunter's book, *De Sectore et Radio*, where he gave its function as being 'for the more easie finding of the houre and Azimuth'.[15] Apart from the hour lines necessary for horary calculation, it carried a horizon line (for calculation of sunrise and sunset), solar azimuth lines (which can be employed to find the bearing of the sun and hence act in lieu of a compass), a shadow square (for use in surveying) and, usually, a nocturnal on the reverse side for finding the time at night. It was thus something of a mathematical compendium, involving various aspects of practical mathematics. It remained a popular instrument throughout the seventeenth century, with examples extant both by professional instrument makers and also by enterprising amateurs, following the details for construction laid down by Gunter in his text. Indeed, the Gunter quadrant continued to be described in print and to be produced by instrument makers until the nineteenth century.

The front of the quadrant (see Figure 3) carries a degree scale (BC) around its outer edge. Also inscribed on the face of the quadrant, concentric with the degree scale, are a date scale (LM), the celestial equator (IJ) and the tropics (represented by the same arc, EF). Between the equatorial and tropical arcs a declination scale (EI) is set along the left-hand edge, and arcs representing the ecliptic (EJ), the horizon line (EK),

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[13] Fale, *Horologiographia* (London, 1593). The construction and use of this quadrant are described on sig.N4 verso - sig.O4 verso (the pages are numbered but the pagination is confused).

[14] There is room here for further research into the history of horary quadrants which should provide evidence for the reasons why the Gunter design became far more popular than any of the other quadrants being produced.

[15] Gunter, *De Sectore*, p.187.

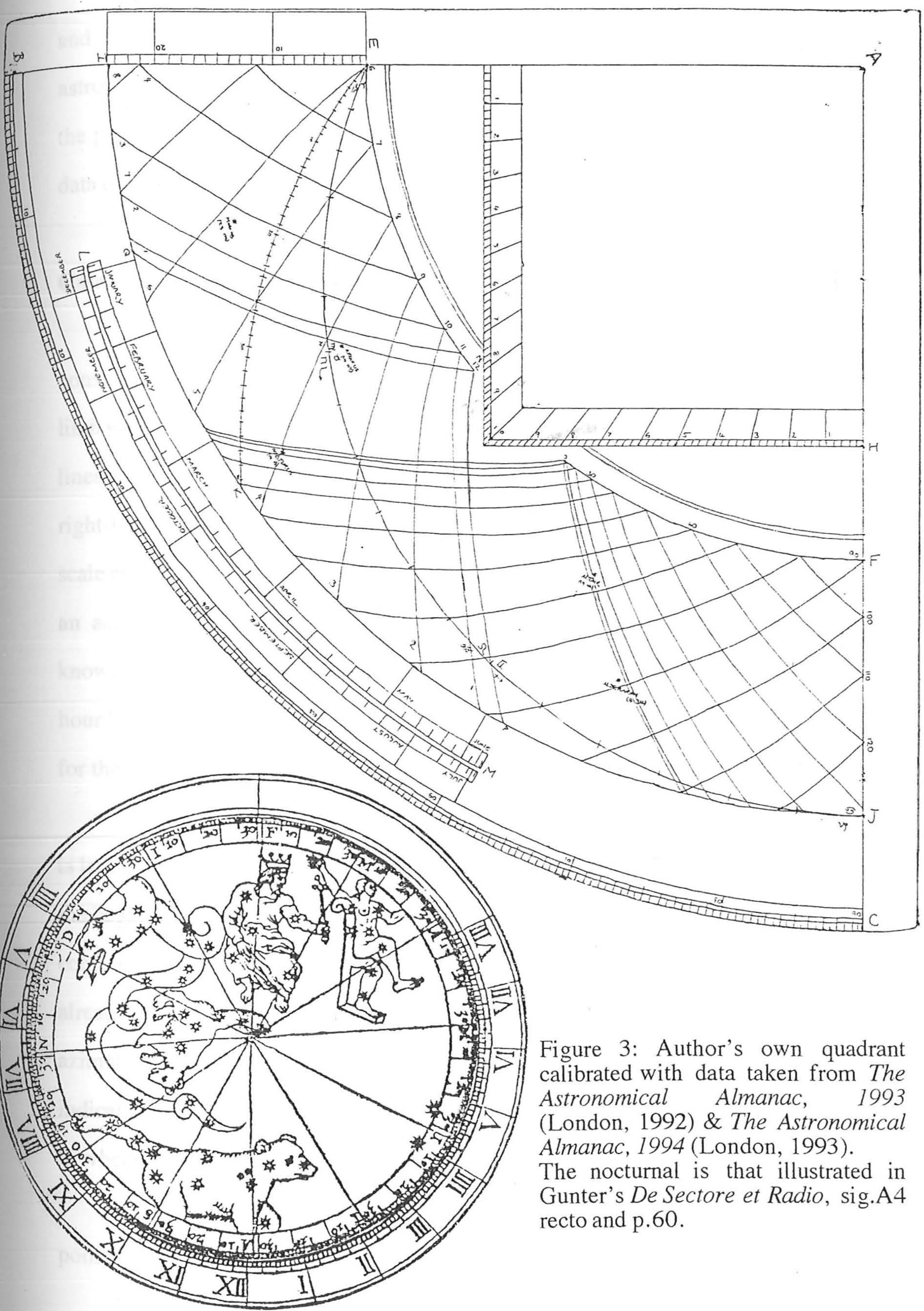

Figure 3: Author's own quadrant calibrated with data taken from *The Astronomical Almanac, 1993* (London, 1992) & *The Astronomical Almanac, 1994* (London, 1993).
The nocturnal is that illustrated in Gunter's *De Sectore et Radio*, sig.A4 recto and p.60.

and both hour lines and azimuth lines are traced out. Thus the quadrant, like the astrolabe, carries a projection of the celestial sphere onto the celestial equator. Most of the primary uses of the quadrant involve obtaining from these lines various pieces of data relating to the sun's motion.

Construction of the Quadrant

The instructions given by Gunter are generally extremely clear and make the construction of the quadrant a relatively simple affair. Dividing the degree scale on the limb was a standard procedure and the date scale, equator, tropics, ecliptic and horizon lines are straightforward to lay off, being arcs of circles whose centres either lie on the right-hand radial edge of the quadrant or are concentric with the limb. The declination scale can be defined in terms of distances from the quadrant apex and marked off with an accurate rule. Anyone possessing accurate declination tables and a working knowledge of trigonometry would be in a position to calculate the points at which the hour lines and azimuth lines intersected the tropic and equator and the declination circles for the beginning of Taurus and Gemini.

However, the means for the actual inscribing of the hour lines and azimuth lines is less clear: Gunter instructs the maker to draw 'occult parallels' to the equator through the beginning of the zodiacal signs of Taurus and Gemini[16] and to use these parallels, in conjunction with the tropic line and the equator to plot out the points which have already been calculated. He then laconically states that the particular hour line or azimuth line should be traced out as a 'line crossing through these points'[17] with no indication of how a smooth curve is to be obtained. The reader is not even informed as to whether the lines are circular arcs or not.

Fortunately, the corresponding passage in Fale's work is more expansive at this point: having used the same method as Gunter to obtain the major points on the hour

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[16] By this Gunter is referring to the tracing of very fine circle arcs, parallel to the equator, which are to be used for construction purposes only.
[17] Gunter, *De Sectore*, pp.197, 200.

lines he instructs the maker of the quadrant to 'search out the centre [of the circle whose circumference passes through the points] (by the 5. Proposstion [sic] 4. Euclid) and ioyne these three markes into one arke...'[18] Presumably Gunter assumed that his readers would be aware that the hour and azimuth lines were circular arcs and that Euclid provided a method for finding the common centre of a set of three points on such an arc. Gunter's practice of providing a fourth point of reference would then be a means for ensuring that the centre had been correctly ascertained.

Using the Gunter quadrant

(1) Finding the time

The primary function of the quadrant is in measuring the time of day. This brings our attention to the hour lines inscribed in the left part of the quadrant, between the equatorial and tropical arcs. Apart from the 12 o'clock line (NP and NQ), the hour lines always represent two times of day, one each side of 12. Thus the 11 o'clock line is also the 1 o'clock line and so on. During the summer months the daylight hours are indicated by the hour lines running from top left to bottom right and the night hours by those going from top right to bottom left. This order is reversed in the winter. Thus midday is represented by the arc NP in the summer months but by NQ in the winter months.[19]

(a) Rectifying the bead for the date: The bead (which should move on the plummet thread (attached at A) with sufficient friction to prevent slippage) is rectified

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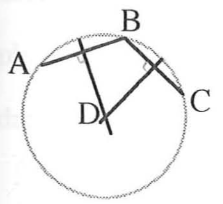
[18] Fale, *Horologiographia*, sig.O2 verso.



The fifth proposition of the fourth book of Euclid's elements states that for three points A, B and C, not on a straight line, the centre of the circle which passes through all three points is at the intersection, D, of the perpendicular bisectors of the lines AB and BC.

[19] This arrangement is a result of the way in which the stereographic projection has been transformed in order to fit it into the quadrant.

by laying the thread taut on the quadrant so that it passes through the date, and adjusting the bead until it lies on the 12 o'clock line.

(b) Finding the solar altitude: The degree scale, in conjunction with the sights (set along the right-hand limb) and the plummet, can be used for measuring the altitude of anything towards which the user directs the quadrant, though the main purpose is for taking the altitude of celestial objects. The quadrant is lifted up vertically (indicated by the plummet hanging freely but against the surface of the quadrant) and the user adjusts the quadrant until, when looking through both sights, he can see the object of interest. The point of the degree scale against which the thread of the plummet hangs provides the angle of altitude of the object. In this case we take the altitude of the sun, though because of the danger of looking directly at the sun, it is not sighted directly but, rather, the instrument is adjusted until the sun's rays pass through the pinhole of the upper sight and fall on the lower sight.

(c) Reading off the time: The rectified bead will be seen among the hour lines at the time of day. Hence, if on 25th July we rectify the bead and, sighting the sun, find its altitude to be 24°, this brings the bead to lie on the 5 o'clock/7 o'clock line. It is left up to the observer to decide whether this indicates that the time is 5pm or 7am![20]

Clearly the reverse process will give the solar altitude from the time. Similar calculations with the night hours will yield the depression of the sun below the horizon. We can also find out the times of the beginning of daybreak and end of twilight (traditionally taken as occurring when the sun is 18° below the horizon).

Either date or place of the sun in the ecliptic (obtainable from tables) can be used: the latter is generally more accurate because the date corresponds exactly to the place of the sun only one year in every four; however, one is more likely to know the date, and this is sufficiently accurate given the overall accuracy of the quadrant.

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[20] If the observer is really unsure whether it is morning or afternoon a second reading taken about a quarter of an hour later will confirm the hour of the day. NB. All examples are worked for a quadrant calibrated for modern data and a latitude of 52°N (the latitude of Cambridge).

<u>(2) Finding the right ascension</u>

The thread is laid out against the sun's place on the ecliptic (the point of the zodiacal circle in which the sun is to be found) for that date, and where it crosses the degree scale it will indicate the solar right ascension. So, when the sun is in the 10th degree of Taurus, the thread will show the right ascension to be 37°30′. This point on the ecliptic line also represents the 20th degree of Leo, the 10th degree of Scorpio and the 20th degree of Aquarius.[21] However, the solar right ascension in these cases will not be 37°30′: for Cancer, Leo and Virgo the right ascension is found by subtracting the measured amount from 180°; for Libra, Scorpio and Sagittarius the measured amount is added to 180°; for Capricorn, Aquarius and Pisces the reading must be subtracted from 360°. In this example the values are 142°30′, 217°30′ and 322°30′ respectively.

The process can obviously be applied in reverse to obtain the zodiacal place of the sun from the solar right ascension, as long as similar precautions are taken for knowing which quarter of the year is intended.

<u>(3) Finding the declination</u>

From the place of the sun in the ecliptic we can also obtain the declination. Taking our previous example of the 10th degree of Taurus, we lay the thread as before and move the bead along the thread until it reaches the ecliptic line. The thread is then moved to the left-hand edge of the instrument and the solar declination is the point where the bead lies on the declination line: 14°30′ N (as the sun passes through Taurus in the summer months of the year). The reverse operation can be performed to calculate the place of the sun when the declination is known.

[21] The reason for the different numbers of degrees for different signs arises from the nature of the transformation of the astrolabe projection onto the quadrant.

(4) Finding the meridian altitude

One use of the calendrical scale is for finding the meridian altitude of the sun. This can be obtained by laying the thread across the current date (e.g. 25th July) and finding that this brings the thread across the degree scale at 57°20′ - the meridian altitude for the day. Again, the process can be performed in reverse: having measured the meridian altitude of the sun the date can be obtained, providing it is known which half of the year is intended.

(5) Using the horizon line

As yet no mention has been made of either the horizon line or the azimuth lines. The horizon line represents the observer's horizon projected onto the celestial equator and its main functions are in the calculation of sunrise and sunset and the bearing of the sun at these times. The latter is found by rectifying the bead for the day and then moving the thread until the bead touches the horizon line, where it will indicate the amplitude of the sun's rising from the East and that of its setting from the West (whether to North or South depends on the time of year: North in summer, South in winter). At the same time as the amplitude is read off, note can be taken of where the thread cuts the degree scale. This value is known as the ascensional difference of the sun and, when converted into hours (15° for each hour), it will give the time, before or after six o'clock, of the sun's rising or setting. As an example, let us suppose that it is the 15th November. The bead is rectified and moved to its crossing with the horizon line. This is at 32°30′, indicating that the sun rises 32°30′ South of East and sets 32°30′ North of West on this day. The ascensional difference is 26°30′ which converts to 1 hour 46 minutes: sunrise is at 7:46am; sunset at 4:14pm.

(6) Using the azimuth lines

The main purpose of the azimuth lines is to find the bearing of the sun if the solar altitude is known (or to carry out the reverse operation if the direction of the sun is known). Having rectified the bead for the date and measured the solar altitude, the thread is moved until it crosses the degree scale at the *complement* of the altitude.[22] At this point the bead will show the azimuth (bearing of the sun from the meridian) by its position among the azimuth lines (whether East or West of South depends on whether it is morning or afternoon). As with the hour lines there is one set of lines for the summer months (those running from top right to bottom left) and one for the winter (running from top left to bottom right). Thus, on the morning of 16th May, if the altitude has been measured at 30°, the complement of this is 60° and the rectified bead lies at approximately 83° in the azimuth lines, indicating that the bearing of the sun is 83° East of South (or 7° South of East). In this manner the quadrant can be used to carry out the functions of the compass and also to calculate the magnetic variation of a compass.

(7) Finding the time at night

The Gunter quadrant was often engraved with points representing five bright stars (Markab, the wingtip of Pegasus; Aldebaran, the eye of Taurus; Regulus, the heart of Leo; Arcturus in Boötes and Altair, the heart of Aquila)[23], sometimes more, for the purpose of finding the time at night. Only stars with Northern declinations were chosen so that one or other of them might be visible in the Northern latitude for which the quadrant was designed. They could be used for finding the time at night in a manner related to that of finding it in the daytime. The manner of working is best shown by an example. The altitude of Arcturus (right ascension 14 hours 14 minutes or 213°30′, declination 19°20′) is measured on 20th May and found to be 49°30′, the star being in the West. The bead is set to the star's mark on the quadrant and the thread moved to the altitude. Proceeding as if for the measurement of time by the sun, the place of the bead is noted in the hour lines. It does in fact lie on the 10 o'clock/

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[22] The complement must be used because the azimuth lines are engraved on the right-hand of the quadrant, solely because of the presence of the hour lines on the left.
[23] Current astronomical identities are α Pegasi, α Tauri, α Leonis, α Böotis and α Aquilae respectively.

2 o'clock line, and since Arcturus is in the West, the afternoon time of 2 o'clock is taken. Next the sun's right ascension for that day (58°) is converted into hours (3 hours 52 minutes) and subtracted from the right ascension of the star to give 10 hours 22 minutes. This value added on to 2 o'clock gives the time of night: 22 minutes past midnight.

(8) Using the quadrat

This brings us to the end of the astronomical functions on the front of the quadrant. It remains to mention the quadrat, and the nocturnal on the reverse. The quadrat's main application was in surveying (either civil or military): it was used to find a height or distance at one or two observations.[24] For instance, suppose that the height of a certain tower is required, and the quadrant is raised so that the top of the tower is seen through both sights. The position of the thread on the scale of the quadrat is noted and a calculation of proportion is made, depending on the thread's place. Thus, if the thread lies in the scale to the left of centre (known as the parts of right shadow) then, as 100 is to the parts cut by the thread, so is the distance between the observer and the tower to the height of the tower.[25] However, if the thread lies to the right of centre (in the parts of contrary shadow) then, as the parts cut by the thread are to 100, so is the distance to the height.[26]

If the distance cannot easily be measured then two observations must be made. If possible these are taken so that the first observation has the thread cutting the quadrat at 100 parts and the second has the thread cutting the quadrant at 50 parts of right shadow, in which case the distance between the two stations will be the height of the tower. If this is not possible then two observations are taken which give the thread cutting the parts of right shadow and the distance between the two stations is measured.

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[24] For further information on the development of the quadrat (or shadow square as it was also known), see E.R. Kiely, *Surveying Instruments: their history and classroom use* (New York, 1947), pp.75-77 & pp.150-154.

[25] In modern terms, 100 : parts cut by the thread = distance between observer and tower : height of tower.

[26] In modern terms, parts cut by the thread : 100 = distance : height of tower.

The formula to obtain the height is that as the difference in parts of right shadow, multiplied by 100, is to the product of the two observations, so the distance between the two stations is to the height of the tower.[27]

(9) Using the nocturnal

Finally, we come to the nocturnal which Gunter recommended to be placed on the back of the quadrant. It consists of a rotatable disc showing the circumpolar constellations and with the months of the year set round the edge, mounted on a circle engraved on the back of the quadrant, which carries the circle of hours. Its use is very straightforward: the user observes which stars are near the meridian and rotates the nocturnal until the position of the stars in the sky is copied on the instrument (the 12 o'clock line is the meridian line). The date will now be positioned against the time of night.[28]

In search of a market for the quadrant

As has been said before, the Gunter quadrant is a compendious instrument and thus it is not easy to identify what was the intended market or what its use was perceived to be. It has numerous applications - the degree scale combined with the plummet measures altitudes, the hour lines supply a means for time-telling, the shadow square is of the type appearing on quadrants used in surveying, the nocturnal is another device for finding the hour. Each application taken in isolation might indicate a particular user as favoured above any other, but when considered more carefully the arguments are not well supported. The degree scale in combination with the sights and

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27 In modern terms, (difference in parts of right shadow) x 100 : product of the two observations = distance between the two stations : height of tower.
It is probably due to the complexity of this formula, that Gunter carries out his calculation in two steps, the first involving supposing a height for the tower and hence calculating the corresponding distance between the stations, and the second comparing this answer with the real distance, to obtain the true height. For more details see Gunter, *De Sectore*, p.216.

28 The early history of the nocturnal is given in Maddison, *Medieval Scientific Instruments*, pp.30-35 and a discussion of the instrument's use is provided by Clare Vincent & Bruce Chandler: 'Nighttime and Easter Time: The Rotations of the Sun, the Moon, and the Little Bear in Renaissance Time Reckoning' in *The Metropolitan Museum of Art Bulletin*, XXVII (1968-69), pp.372 - 384 (but especially pp.375-379).

the plummets provides a means of measuring altitudes, but hand-held quadrants were by no means accurate enough for detailed astronomical work and would surely have been a poor cousin in surveying to the peractor and altazimuth theodolite, these latter being set on rigid stands which kept them firmly in the vertical plane. The hour lines when linked to the altitude-observing functions, the calendar scale and the ecliptic circle reveal the time, and it might seem logical that as a development of the earlier horary quadrants this instrument was intended as a time-telling device. Yet it is worth noting that the common pocket sundial fulfilled the same office rather more simply, and if it was not so accurate as the quadrant (by virtue of its size) this was not a matter of great importance in a world where knowledge of the time to the nearest minute was unlikely to be viewed as a necessary requirement. The azimuth lines *could* be used as an alternative to the magnetic compass (and might indeed be more accurate since they did not require any compensations for magnetic variation) but the slowness of the procedure militated against this application - it could hardly be used for steering a ship. The shadow-square is a surveyor's tool, but the questions concerning use of a hand-held tool arise again, and those surveying quadrants which were used by practising surveyors tended to be unencumbered by any other engraving in order to maximise the area available for marking out the shadow square. Finally there is the nocturnal: when this is first described in Gunter's text, in the course of his excursus on the sector, it is noted as being 'for the vse of Sea-men':[29] however, the other applications of the quadrant would have little worth for the navigator, particularly since the time-telling aspects were invalid in any latitude other than that for which the quadrant was made.

We are forced to conclude that this instrument is not designed for significant practical use. With its compendious range of applications it seems to fit the needs of a jack of all mathematical trades, of the dabbler who knew a little bit about everything but was not an expert in anything. The question arises whether there is other evidence to corroborate such a supposition, and perhaps the first place to turn is the book in which the quadrant is described. *De Sectore et Radio* is primarily concerned with the

[29] Gunter, *De Sectore*, p.60.

construction and use of the two instruments mentioned in the short title - the sector and the cross-staff (as developed and modified by Gunter). These are essentially instruments for practical application and extensive sections of the book concern their use in navigation, surveying and dialling. Two further instruments are included in the text: a cross-bow 'for the more easie finding of the latitude at Sea'[30] (which Gunter seems to have intended as an alternative to the cross-staff and back-staff, but which appears never to have become popular enough to be found in wide-scale use) and the quadrant. The first three of these instruments have clear practical applications: one would assume that this was true of the last but study of the quadrant has seemed to imply that it was not.

Gunter's style of writing offers very few clues to assist our enquiries. The book contains neither dedication nor letter to the reader, which were the common places for authors to wax lyrical about their intentions in writing and the target of their discourses. The main text is a simple exposition of the functions of the instruments with no explicit indication of the intended audience. Perhaps some hints can be gleaned from the engraving on the title page of the work depicting the use of the various instruments discussed in the text. This shows an ordinary seaman employing the cross-bow, a more highly-ranked man making use of the cross-staff and two gentlemen utilising the sector and the quadrant. The implication is that the quadrant is for the consumption of the gentleman mathematician.[31] It would appear to be the case that Gunter found himself writing a text on his most important instrument designs and decided to make it the vehicle for presenting all the instruments which he had developed: thus the quadrant is included with the instruments of a more utilitarian nature (although such a description of the sector must be treated with care - see the following chapter) despite its lack of usefulness to the practical mathematician.

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[30] Ibid., p.97.

[31] A similar use of the title page iconography is found in Aaron Rathborne's *The Surveyor* (1616). In this engraving the author's opinion of the superiority of the altazimuth theodolite over the plane table is made very clear. For a full discussion of the engraving see J.A. Bennett, 'Geometry and Surveying Early Seventeenth-Century England'.

As an instrument falling into the field of the amateur, the quadrant could serve two purposes. It carried functions relating to various aspects of practical mathematics and was thus a possible resource for teaching the student of mathematics to understand the practical applications of the discipline to astronomy, surveying and navigation. It is clear from contemporary texts that much use was made of instruments for aiding instruction in practical mathematics and for explaining visually concepts which were difficult to grasp without the assistance of illustrations and models. This was particularly the case in astronomy, and a quadrant could be used as a didactic aid for discussing the important parts of the celestial sphere and their roles in astronomical calculations.[32] However, one could argue that such issues were better covered (though undoubtedly at greater cost) by implementation of a globe or of an armillary sphere: three-dimensional models are the best option when dealing with three-dimensional entities.

Another possibility is that the instrument functioned as a status symbol, declaring the possessor's knowledge of mathematics. Both construction and use of the Gunter quadrant (particularly the former) required a considerable grasp of the principles of mathematics (especially of trigonometry) and so a gentleman who aspired to be seen as an authority on mathematics might very well have a quadrant as an indication of wisdom and understanding in the field. I do not intend to imply that Gunter created his quadrant purely to serve as a status symbol of the wealthy, but that the instrument came to be associated with amateur mathematicians and with the collections of the gentry and nobility, rather than as something for utilitarian purposes.

The three Allen quadrants (Cat. nos. Q1 - Q3) which I have had occasion to study are beautifully-crafted examples of the genre. They all bear remarkably few signs of wear and tear, especially when compared with those instruments which have been

[32] *Vide* Henry Peacham, *The Compleat Gentleman*, p.73: 'I had rather you learned these principles of the sphere by demonstration and your own diligence, being the labour but of a few hours, than by mere verbal description, which profiteth not so much in mathematical demonstrations.'

exposed to the elements.[33] The larger of the Whipple instruments was clearly destined for a gentleman's possession, and with a radius of more than a foot would quite possibly have been too heavy and unwieldy to supply accurate measurements (it certainly was not an instrument for keeping in the pocket!). The privately-owned quadrant is also very carefully engraved, with the same evidence of Allen's considerable skill in the high quality of the figures on the nocturnal. Meanwhile there are some extant examples of instruments produced by gentlemen amateurs themselves. The evidence again seems to be in favour of the quadrant as a symbol of status and learning within the field of the mathematical sciences.

One thing is clear, and that is that Gunter quadrants and other similarly designed quadrants retained their popularity through the seventeenth century: there are many examples of this kind of single-latitude horary quadrant dating from the period and it continued to be mentioned as a common instrument in the encyclopædias of mathematical instruments which appeared towards the end of the century and at the beginning of the next. If these quadrants were indeed mostly produced largely for gentleman amateurs then this would support the claim for a general rise in interest in mathematics among the leisured classes during the seventeenth century. It is has already been noted (page 36) that commentators on education were increasingly stressing the importance of mathematical studies, both theoretical and practical, within the school curriculum of the upper classes and these commentators usually assumed that instruments would play a major part in any training in mathematics. In his unpublished treatise on education Aubrey lists the Gunter quadrant among the instruments which should form part of the equipment of the academy and in the course

[33] See, for example, the double horizontal dial (Cat. no. D3) and also the peractor (Cat. no. X4).

of his section on mathematics presents it as an aid for familiarising the student with trigonometry.[34]

The popularity of the quadrant also corresponds with the increase in the number of gentry at this time: it is well-known that the early seventeenth century was a period of expansion for this class. As Hirst has commented,

> 'The relative ease of access to gentility meant that the numbers of gentry increased fast in a period when both the profits from land and the population as a whole were multiplying. There were for example seventy-eight more gentry families in Lincolnshire in 1634 than there had been in 1562, and over England as a whole gentry numbers probably tripled between 1540 and 1640, from 5,000 to 15,000. In this sense at least it is possible to talk of the "rise of the gentry", one of the old subjects of historians' controversy.'[35]

Members of this class were generally eager to increase their status in the eyes of their neighbours and one way to a rise in status was through the acquisition of knowledge. Mathematics was just one of the many areas in which this hunger for knowledge might have sated itself. Of course, beautifully crafted instruments were not just a symbol of the owner's intellectual abilities but also revealed an eye for aesthetics and a understanding of high culture. In this sense the instruments from gentlemen's collections are part of the evidence for the growing tendency for making collections of fine art.[36] Nevertheless, the relative austerity of the attractiveness of mathematical instruments is a constant reminder that these artefacts were originally intended to be functional and not purely decorative.

As a worker in brass and other metals, Elias Allen would no doubt have been keen to foster the interest in mathematics and mathematical instruments which had newly arisen on a relatively wide scale among the upper classes. His wares would

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[34] Bodleian MS Aubrey 10, f.109 and f.30.
[35] Derek Hirst, *Authority and Conflict*, p.14. The controversy is mapped in J. Hexter, 'Storm over the Gentry' in his *Reappraisals in History* (1961).
[36] For instance, the art collection of the Earl of Arundel, discussed in David Howarth, *Lord Arundel and his Circle*.

probably have been too expensive for the average professional mathematical practitioner and so alternative markets must have been valuable to Allen. His reputation was not only based on his avowed skill in dividing scales and producing accurate working tools, but also on the artistry of his engraving as a whole. The Gunter quadrant was the kind of instrument designed to appeal to the gentleman and this was presumably why Allen produced these instruments, alongside the astronomical compendia and universal equinoctial ring dials which are also likely to have been destined for the upper class pocket.

The Gunter quadrant has been revealed as something of an enigma in the history of seventeenth-century mathematics. However, it does serve as a reminder of various aspects of this mathematical culture: the increasing involvement of gentlemen amateurs; the importance of instruments in the teaching of mathematics and explanation of principles; and the strength of the visual and tactile aspects of the mathematical culture in this period. Mathematics was not simply a subject for mental or verbal investigation and calculation: it was solidly grounded in the physical world and most students of mathematics found that understanding came most easily through explanation of principles in conjunction with instrumental demonstration. The quadrant is one of many witnesses to the continuing role of instruments in teaching, and in pointing up the importance of instruments we are reminded of the relevance of the instrument makers to this mathematical community.

CHAPTER FOUR

Studies in the Output of Elias Allen's Workshop

(2) The Gunter Sector

Among Elias Allen's extant work, the Gunter sector stands out as one of the two most commonly represented instruments (the other being the astronomical compendium). Although there are only two Gunter sectors in the main section of my catalogue which are actually signed by Allen (Cat. nos. S1 and S2)[1], there are numerous other examples which are attributable to Allen by the style and quality of the engraving. It would appear that these sectors constituted a significant proportion of the output of the Allen workshop. Allen's handiwork is also evident in the illustration of the sector which forms the frontispiece of the book in which it is described and its use is explained - Gunter's *De Sectore et Radio*; this engraving carries an advertisement (in Allen's hand) for the maker as provider of the sector: 'These instruments are wrought in brasse by Elias Allen dwelling without Tempel barre ouer against S^{t} Clements Church: and in wood by Iohn Thompson dwelling in Hosiar Lane'.[2]

So, the Gunter sector appears to have been a mainstay of Allen's trade. The reasons for the popularity of this instrument and its role in the mathematical culture will be the subject of the present chapter.

A History of the Sector

The sector was first introduced at the end of the sixteenth century when designs were produced by both Galileo and Thomas Hood. Galileo's instrument was

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[1] Note also the signed sectors auctioned in 1971 and 1972 - see Catalogue, p.272.

[2] Edmund Gunter, *De Sectore et Radio* [1623], and see Figure 4. Some versions of the advertisement add 'And by Nathaniell Gos dwelling at Ratclif'. The history of the first edition of this book seems to have been rather complicated with several different issues being produced, some dated 1623 and some dated 1624, with various different title pages appearing in the different versions. (See the comments on this topic by David Bryden in 'Evidence from Advertising', p.315, particularly footnote 74.). Gunter writes in his conclusion to the section on the sector that 'If I finde this [i.e. the work on the sector] to giue you content, it shall incourage me to do the like for my *Crosse-staffe*, and some other Instruments.' (p.143); however, there is no evidence of the books on the sector ever having been published separately from the rest.

developed sometime between 1597 and 1599 and was intended for military use as well as general calculation; it appeared in print in *Le operazione del compasso geometrico e militare* in 1606.[3] Hood's version was published in 1598 in a book whose title gives a good summary of the possible uses:

> *The Making and vse of the Geometricall Instrument, called a Sector. Whereby many necessarie Geometricall conclusions concerning the proportionall description, and division of lines, and figures, the drawing of a plot of ground, the translating of it from one quantitie to another, and the casting of it vp Geometrically, the measuring of heights, lengths and breadths may be mechanically performed with great expedition, ease, and delight to all those, which commonly follow the practise of the Mathematicall Arts, either in Suruaying of Land or otherwise.*[4]

The main application of this instrument was clearly intended to be surveying, where problems of proportion (which the sector was designed to solve) played an important part. Hood's book carries an advertisement for Charles Whitwell as maker of his instrument and hence Allen, as Whitwell's apprentice, would have been well-versed in the nature of such instruments before he began his work for Gunter. Hood describes his sector as a 'mathematicall instrument consisting of 2. feete, one moueable, an other fixed, making an angle, and of a circumferentall Limbe'[5] which presumably is the reason for its name, since it would resemble a circle sector when open.[6] It carried sights on the two 'feet' and was therefore as much a surveying instrument as a calculational device.

It was about 1606 that Gunter wrote a Latin manuscript detailing the uses of his version of the sector. This was a considerably more sophisticated instrument than the

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[3] An English translation is available: *Galileo Galilei: Operations of the Geometrical and Military Compass 1606*, translated and with an introduction by Stillman Drake (Washington, 1978). The introduction has an excellent exposition of the history of the sector. Another account of the development of the instrument is given in A.N.B. Garvan 'Slide Rule and Sector' in *Actes du dixieme Congres International d'histoire des sciences* (Ithaca, 1964), vol. i, pp.397-400.
[4] Hood, London, 1598.
[5] Hood, *The Making and vse of the...Sector*, f.1 recto.
[6] The definition of 'sector' with reference to a circle had been used in Billingsley's 1570 translation of Euclid's *Elements*, book III, definition ix.

earlier ones, being able to work through problems concerned with two- and three-dimensional bodies and also to deal with questions of trigonometry. Apart from the scales necessary for these calculations it also carried lines connected with questions about the Platonic solids and for comparisons of metallic bodies of different composition and weight or magnitude. The first printed (English) version did not appear until 1623, seventeen years or so after its initial conception, and the impetus for getting the book into print seems to have been (according to Gunter) that too many people who knew no Latin were buying the instrument and attempting to use it having had no instruction in its operation.[7] This book was, of course, *De Sectore et Radio* or *The Description and vse of the Sector. The Crosse-staffe and other instruments. For such as are studious of Mathematicallpractice.*

The Gunter sector at this point carried the following scales: the Line of Lines, the Line of Superficies, the Line of Solids, the Line of Sines, the Line of Tangents, the Line of Secants, the Meridian Line, the Line of Quadrature, the Line of Segments, the Line of Inscribed Bodies, the Line of Equated Bodies, the Line of Metals, the Line of Inches and the Line of Lesser Tangents. The circular limb of Hood's sector and the sights had disappeared. A short brass strut, pivoted at the end of one arm was used in conjunction with the Line of Lesser Tangents and incidentally fulfilled the function of providing additional friction when the instrument was open to small angles.

De Sectore et Radio enjoyed a long publishing life: a new edition appeared in 1636 and again in 1653, now as part of *The Works of Edmund Gunter*; three further editions appeared in 1662, 1673 and 1680. This longevity was linked to the continuing popularity of the Gunter sector (and other forms of the instrument). It was extensively modified as the century progressed: later Allen instruments incorporated lines for performing logarithmic calculations and Foster's version of the instrument removed the Meridian Line and added a Line of Versed Sines. Other sectors of the later seventeenth

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[7] See *De Sectore*, p.143. The Allen sector in the National Maritime Museum appears to be something of a 'halfway house' between a Hood sector and a Gunter sector, and was perhaps commissioned from the maker by someone who was unclear as to what the Gunter manuscript was describing. See Cat. no. S5; a similar instrument was auctioned at Sotheby's in 1988 - see Catalogue, p.272.

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century carried only the trigonometrical functions, apart from the Lines of Lines, Superficies and Solids.

In whatever form it appeared, however, the sector was praised as an instrument of great utility and it had an established place in the encyclopædias of mathematical instruments. Writers of these works at the turn of the century spoke of the instrument as 'generally useful in all the practical parts of the Mathematicks,...and particularly contrived for Navigation, Surveying, Dialling, Astronomy, Projection of the Sphere, &c.'[8] Edmund Stone remarked in his revised edition of Samuel Cunn's exposition of the sector: 'Among the Multitude of Mathematical Instruments that have been invented, the SECTOR...claims a principal Place, and ever since the Invention thereof, has been had always in the greatest Esteem by the Ingenious of *the Mathematical Kind*, but more especially with those who busy themselves in the practical Parts of that *Learning*.'[9]

Using the Gunter sector

A Gunter sector consists of two arms pivoted on a flat circular hinge (akin to that of a modern folding carpenter's rule) and engraved with various scales (listed above). All the scales apart from the Line of Tangents, the Line of Secants, the Meridian Line, the Line of Inches and the Line of Lesser Tangents appear in pairs, with matching lines on each arm, and radiate from the centre of the hinge. The Lines of Tangents and Secants are set along the outer edge of one side of the instrument in such a way that, when the instrument is completely opened out, the Line of Tangents forms a complete scale from the end of one arm to the end of the other, with the Line of Secants running parallel to the second half of the Line of Tangents. The Meridian Line appears in a similar position on the other side of the instrument. The Lines of Inches and Lesser Tangents are set along the outer rims of the arms.

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[8] James Moxon and Thomas Tuttell, *The Definition, Explanation, Nature and Meaning of the Principal Mathematical Instruments* in Joseph Moxon, *Mathematicks made Easie, or a Mathematical Dictionary* (London, 1701), p.18. Cf. Harris, *Lexicon Technicum: or, an Universal English Dictionary of Arts and Science* (London, 1704).

[9] Cunn, *A New Treatise of the Construction and Use of the Sector* (London, 1729), sig.A2 recto. There is no good history of the development of the sector at the present time, and there is much need for one. In particular it would be useful to study the way in which the Gunter design was developed, both before the publication of the text, and through the rest of the seventeenth century.

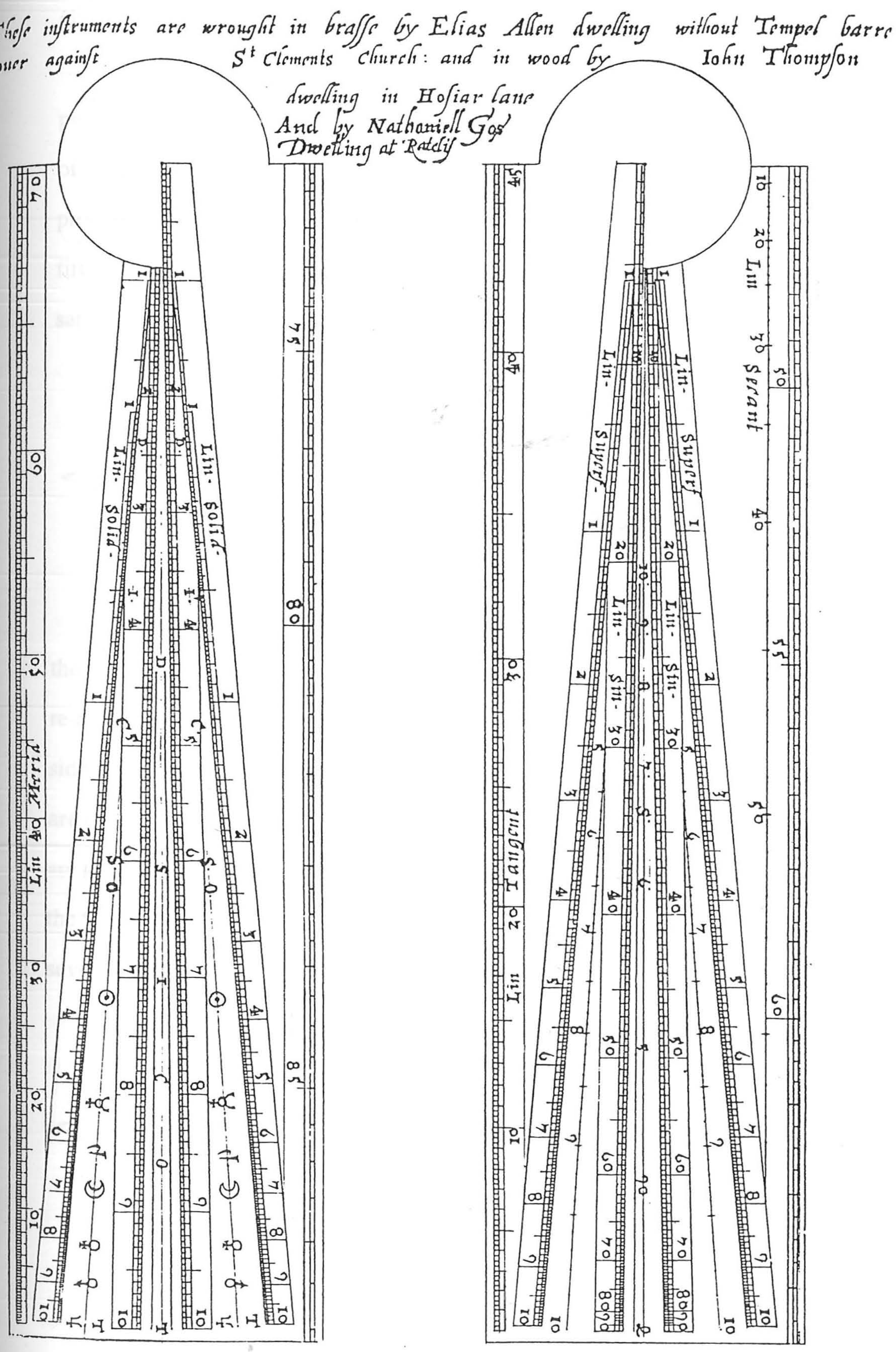
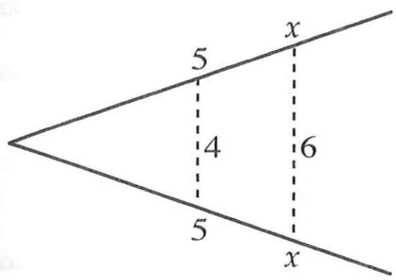


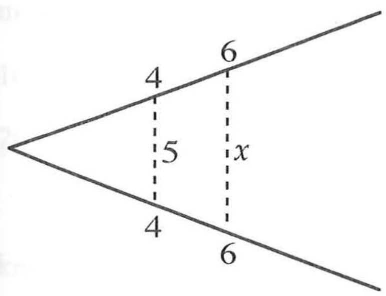
Figure 4: Engraving of a Gunter sector, from the 1623 edition of *De Sectore et Radio*

(1) The Line of Lines

Calculations involving the sector generally revolve around problems of direct proportion or, as Gunter expresses it: 'generally the vse hereof consists in the solution of the *Golden rule*, where three lines being giuen of a known denomination, a fourth proportionall is to be found.'[10] For instance, the three numbers given might be forty, fifty and sixty and the problem would be to find a fourth number which would be in the same ratio to sixty as fifty is to forty.[11] This can be done in one of two ways.



By the first method, a pair of dividers is opened out until it stretches from zero to four on the Line of Lines (which is an ordinary numeric scale, calibrated from zero to ten) on one side of the sector, and the sector is then opened out until the two ends of the open dividers can be placed in the marks for five in the Lines of Lines on the two limbs of the opened sector. At this point, the dividers are re-adjusted until the feet can be stretched from zero to six on the Line of Lines on one side of the sector. Keeping the sector open to the same angle all the time the dividers are moved along the sector with one foot on each limb of the instrument until the feet are standing in the same position on the two Lines of Lines. This position will supply the fourth proportional number - seventy-five (though indicated on the Line of Lines as seven and a half).



Alternatively, the dividers can be opened in the first instance to five units of the Line of Lines and then used to open the sector by being placed in the marks for four in the Line of Lines. This time the dividers must be readjusted by setting one foot in the six mark on one limb and the other in the six mark on

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[10] Gunter, *De Sectore*, p.9.
[11] In modern terms, to find x, where $x : 60 = 50 : 40$.

the other limb, while keeping the sector open at a constant angle. If one foot of the dividers is now placed in the hinge end of the Line of Lines the other will fall on the Line of Lines at the position of the fourth proportional - seven and a half.

The two methods can be used interchangeably according to which is the most useful in a given situation: both are dependent on the properties of similar triangles. Making use of this basic principle of proportion, the Line of Lines can be used to multiply two numbers together (here the third known number is one); to increase or decrease a line according to a given proportion; to divide a line into a specific number of equal parts, and so on.

The Lines of Superficies, Solids, Sines, Tangents and Secants are all functions of the Line of Lines, as will become more obvious as each line is studied in turn. They are used to perform in a single operation what might take several steps of calculation and searching through mathematical tables.

(2) The Line of Superficies

This line is used to deal with questions relating to areas, square numbers and square roots (problems relating to the latter two matters will also involve the Line of Lines). The term 'Superficies' was the standard one used for a two-dimensional geometric object in this period. The Line of Superficies is divided into one hundred unequal parts, although the instrument is usually marked "1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10". Here the first "1" does indicate "1 part" but the remaining numbers represent 10, 20, 30, 40, 50, 60, 70, 80, 90 and 100 respectively.

One main function of the Line of Superficies is to find the side of a square of known area (i.e. a square root), which operation is performed in conjunction with the Line of Lines. It would proceed as follows: for a square of known area, the dividers are opened out to measure the value of that area along the Line of Superficies, bearing in mind that the whole line represents one hundred for areas less than or equal to one hundred; ten thousand for areas between one hundred and ten thousand; one million for areas between ten thousand and one million; etc. This opening of the dividers will yield the length of the side on the Line of Lines when one foot is placed at zero, bearing in mind that the whole of *this* line represents ten for an area less than or equal to one hundred; one hundred for areas between one hundred and ten thousand; one thousand for areas between ten thousand and one million.[12]

Another example of using the Line of Superficies is to find the proportion between two similar superficies. Here, one side of the greater plane figure is measured with dividers and the sector is opened until the feet of the dividers rest in the points of 100 in the Line of Superficies. Next the like side of the smaller superficies is measured with the dividers and the dividers are moved along the two Lines of Superficies until the feet are resting in the equivalent points in the two lines. These points will provided the number which, when placed over 100, gives the fraction which the smaller superficies is of the larger (e.g. if the points were at 40, then the proportion would be two to five).

The Line of Superficies can also be used to add or subtract like superficies, to make a square equal in size to a given superficies (and hence to find its area) and to increase or decrease a superficies in a given proportion (very useful to a surveyor for rescaling estate plans). To add two like superficies, find the proportion between the two shapes (as already shown) and add the two numbers together; for instance, if the proportion is five to two the sum will be seven. The result gives the proportion which the new shape must have in relation to the larger of the old - here it will be seven to five, and the length of one side is found by reversing the process of calculating proportions.[13]

The method for finding the area of a given superficies varies depending on its shape. If it is a rectangle, the mean proportional (defined below) of the two sides will be the length of the side of the square required; if it is a triangle the mean proportional

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[12] This is rather longwinded but it is the way in which the calculation would have been understood in the seventeenth century when geometrical methods were far more familiar than algebraic ones.

[13] That is, the length of the side of the greater superficies is measured with the dividers and the sector is opened until the feet of the dividers rest in the points of 50 (in this case) in the Line of Superfices. The distance measured across between the points of 70 will be the length of side of the new shape.

of the perpendicular height and half the base is the side of the required square; for all other polygons the shape must be divided up into triangles and the areas totalled at the end of the calculation. The mean proportional of two lines is the square root of the product of the two lines and it is calculated as follows: the longer line is measured in the Line of Lines with a pair of dividers and the sector is opened out until the feet of the dividers rest in this value on the two Lines of *Superficies*. Then the shorter line is measured with the dividers in the Line of Lines and the distance between the two arms of the sector when measured across from this second value on the two Lines of *Superficies* is the mean proportional.

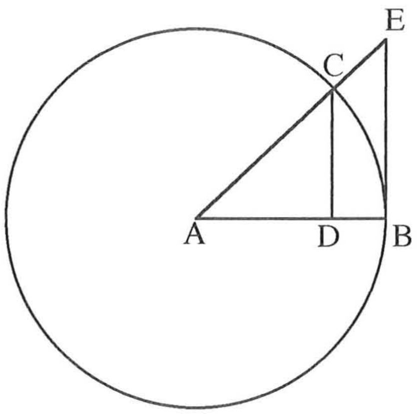
(3)The Line of Solids

This performs the identical operations for volumes and solids which the Line of Superficies carries out for areas. It can also be used for obtaining the side of a cube of known volume (i.e. a cube root). The line is calibrated into a thousand unequal parts, although the instrument is generally marked "1, 1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10". Here, the numbers represent 1, 10, 100, 200, 300, 400, 500, 600, 700, 800, 900 and 1000 parts respectively. It must be remembered when using the line for calculating cube roots that the correct representation of the whole line must be used. Thus, in finding the side of a cube of volume less than or equal to one thousand the whole of the Line of Solids will represent one thousand; it will represent one million for volumes between one thousand and one million; one thousand million for volumes between one million and one thousand million, etc., and the whole Line of Lines will represent ten for volumes less than or equal to one thousand; one hundred for volumes between one thousand and one million, and so forth.

The Line of Solids can also be used for certain weight-related questions. For instance: If a bullet of 27lb has a diameter of 6 inches, what diameter does a 125lb bullet have? This problem can be solved in the following manner: the dividers are opened out to span 6 in the Line of Lines and then the sector is opened until the feet of the dividers stand in the points of 27 in the Lines of Solids. Keeping the sector open at this angle, the dividers are opened until the feet can be placed in the points of 125 on the Lines of Solids. This opening of the dividers will give the answer when measured in the Line of Lines: the diameter of the larger bullet must be 10 inches.

(4) The Lines of Sines, Tangents and Secants

The next natural group of scales on the sector comprises those lines relating to trigonometry. For all the trigonometric functions it is important to remember that they were taken as lengths rather than the decimal ratios to which we are accustomed today. The most appropriate way to explain this is with a diagram:



For a radius, AB (also referred to as the 'whole sine'), and angle BAC,

Sine BAC is the length CD

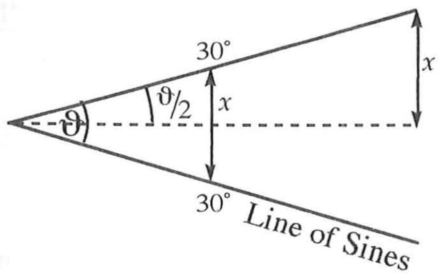
Tangent BAC is the length BE

Secant BAC is the length AE.

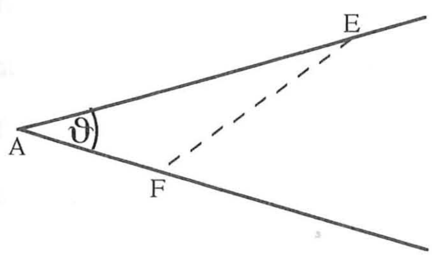
When numbers rather than lengths were required the whole sine was not given as one but as ten thousand or one hundred thousand or one million, etc. (as decimals were uncommon) and the trigonometrical functions were listed accordingly. Thus, for the angle of 60° with the radius taken to be 100,000, the sine value would be given as 86,603, the tangent as 173,205 and the secant as 200,000. The Line of Sines is delineated on the sector according to the understanding that the radius under consideration is the Line of Lines; it is numbered in degrees (as are the Lines of Tangents and Secants) and so opening the dividers from 0 to the angle BAC along the Line of Sines will give the length CD. If a numerical value is required the dividers must be transferred to the Line of Lines. The fact that the base radius is the Line of Lines also explains why the Line of Tangents and Line of Secants extend along both limbs of the instrument, forming a complete scale when the sector is opened out: the tangent will be larger than the radius for angles greater than 45°, and the secant of an angle is always greater than the radius. Thus secants are measured from the beginning of the Line of *Tangents* up to the relevant point in the Secant scale.

The basic use of the Line of Sines is to find the sine of an angle *as a length* for a given radius of a circle. The dividers are opened out to this radius and then the sector is opened until the feet of the dividers stand in the points of ninety on the two Lines of Sines. The distance between any pair of numbers (e.g. fifty and fifty) on the Lines of Sines will be the sine of that number. If the sine is required as a number rather than as a length this involves a simple comparison of the angle as measured in the Line of Sines (by extending the dividers between zero and the angle) with the Line of Lines (setting one foot of the dividers in the zero and noting to where the other foot reaches). This is the way in which Tangents and Secants have to be obtained, since there is only one Tangent and one Secant Line on the instrument, and the values are often greater than that of the radius. If either function is required as a length relative to a particular radius, then it must be scaled in proportion, using the Line of Lines as previously shown.

Application of the Line of Sines to problems in planar trigonometry is done directly by setting up triangles on the sector. For instance, in a certain triangle two sides and the angle between them are known; the third side is required. First the sector is opened to the correct angle, which is most easily shown in diagrammatic form:



For angle ϑ, sin $1/2\vartheta$ is measured with dividers in the Line of Sines. Then the sector is opened out so that the feet of the dividers stand at 30° in the Line of Sines. [Since sin $1/2\vartheta = x$.]



Now, having measured the angle, the two known lengths (here AE and AF) are noted in the Line of Lines. The dividers are stretched from E to F and the third side can then be measured in one of the Lines of Lines.

If the information required was the angle AFE then this would require application of the formula, as the sine of the known angle (ϑ) is to the opposite side (EF) so is the sine of the required angle to the opposite side (AE). In order to calculate the sine of the angle required the value of sine ϑ must be taken from the Line of Sines with the dividers and the feet of these set in the Line of Lines at the points of the length EF when measured along the Line of Lines from the hinge. The distance across between the points for the length AE as measured in the Line of Lines will be the sine of the required angle and this angle can be found by measuring that distance along the Line of Sines.

Where spherical trigonometry is involved, problems are solved by means of various rules of proportion. For example, in a right-angled triangle it might be required to find a side, given the base and the angle opposite to the required side. The rule applied in this case is that as the whole sine (or radius) is to the sine of the base, so the sine of the opposite angle is to the sine of the required side.[14] The dividers are opened out to the length of the base (in the Line of Sines) and the sector is opened until the feet of the dividers stand on 90 in the two Lines of Sines. If the dividers are now adjusted so that they stand in the Lines of Sines at the quantity of the known angle, they will be open to the length of the required side, when measured in the Line of Sines.

(5) The Meridian Line

The Meridian Line is the last of the major lines (as Gunter saw it) on the sector, and it does not form part of a natural group, although its use is often combined with the trigonometric functions of sine and tangent. Its purpose is for drawing out parallels of latitude on a Mercator chart and also for performing various navigational calculations. If the size of the chart required is such that a degree of longitude at the equator measures the same as a hundredth part of the Line of Lines (i.e. as 0.1 on the Line of

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[14] In spherical geometry lengths were generally indicated by the sine of the angle which they subtended at the centre of the sphere. The formula used here would be given in modern terms as

sine 90° : sine base = sine opposite angle : sine required side.

Lines), then the parallels of latitude can be marked down along the Meridians directly from the Meridian Line. However, for any other size of chart, the distances between the parallels have to be increased proportionately, by using the Line of Lines.

The kind of calculations which this line performs most usefully are those which involve longitudes - questions which cannot be resolved by plane trigonometry on an ordinary chart, but which are reduced to plane trigonometry by a Mercator chart or a Meridian line. Thus such a question as the difference in longitude of a ship's two positions from one noontide to the next can be ascertained, given the rumb (measured from the Meridian) along which the ship has travelled and the initial and final latitudes. Here, the necessary rule of proportion is that as the radius is to the tangent of the rumb from the meridian, so is the difference of the latitudes (measured in the Meridian Line) to the difference of the longitudes (measured in the Line of Lines). The dividers are extended to the tangent of the rumb (in the Line of Tangents) and used to open out the sector by placing their feet in the points of 90° in the Line of Sines. With the sector kept at this angle, the dividers are extended along the Meridian Line until one foot rests in each of the known latitudes, and this quantity is measured in the Line of Lines. Finally the dividers are re-adjusted once more so that the two feet stand in the two Lines of Lines at the quantity which has just been calculated. The amount to which the dividers have been opened is the difference of longitude in degrees (measured in the Line of Lines). By this means, longitudes can be obtained without recourse to a chart.

The remaining Lines on the sector (those of Quadrature, Segments, Inscribed Bodies, Equated Bodies, Metals and Lesser Tangents) do not fall into any particular groupings but perform various self-contained functions without reference to other lines (except for the Line of Metals, which is linked with the Line of Solids).

(6) The Line of Quadrature

The function of the Line of Quadrature is basically to find regular polygons of equal area. So, given a hexagon of a particular size, if the dividers are opened to the length of the hexagon's side and their feet placed in the points of 6 on the Lines of Quadrature on the open sector, the equivalent lengths for 10, 9, 8, 7, 5, and Q will give respectively the sides for the decagon, nonagon, octagon, heptagon, pentagon and square of equal area. The distance from S to S is the radius of a circle of equal area, while the distance from 90 to 90 gives the quarter-circumference of this circle.

(7) The Line of Segments

The Line of Segments performs one very simple function - it is used to divide a circle of a given diameter into two segments according to a given proportion of area. Thus, for instance, if the proportion required is 60 : 40, the dividers are set to the diameter and used to open the sector by being placed on the points of 10 in the Lines of Segments. The distance between the points of 6 will be the length of the necessary chord to draw across the circle in order to give the desired division.

(8) The Line of Inscribed Bodies

The Line of Inscribed Bodies is marked "D, S, I, C, O, T". "S" stands for the semidiameter (radius) of the sphere in which the bodies are to be inscribed. The other letters indicate the five Platonic or regular solids (dodecahedron, icosahedron, cube, octahedron, tetrahedron). The sole function of this line is to supply the length of side of the regular bodies which can be inscribed inside a given sphere, so that all vertices of the polyhedron touch the surface of the sphere.

(9) The Line of Equated Bodies

The Line of Equated Bodies is marked "D, I, C, S, O, T" where "S" now represents the *diameter* of a sphere, but all the others are as for the Line of Inscribed Bodies. The purpose of this Line is to enable the calculation of the sides of the regular solids which have an equal volume with a sphere of a particular diameter. Thus it performs the equivalent service for regular solids which the Line of Quadrature supplies for regular polygons.

(10) The Line of Metals

Along the Lines of Metals we find the symbols "☉, ☿, ♄, ☽, ♀, ♂, ♃" which represent gold, mercury, lead, silver, copper, iron and tin respectively. Gunter describes their use as being 'to giue a proportion between these seuerall metalls, in their magnitude and weight, according to the experiments of *Marinus Ghetaldus* in his booke called *Promotus Archimedes*.'[15] They are used in conjunction with the Line of Solids and their function is fourfold: having like bodies of several metals and equal weight, given the magnitude of one to find the magnitude of the rest; having like bodies of several weights and equal magnitude, given the weight of one to find the weight of the rest; having a body of one metal, to make a like one of another metal and equal weight; having a body of one metal, to make a like body from another metal, according to a given weight. The first operation is carried out by taking the given magnitude out of the Line of Solids with the dividers and using them to open the sector in the points of that metal in the Line of Metals. The parallels for the other metals will give the necessary magnitudes when measured in the Line of Solids. The other functions are fulfilled in similar ways.

(11) The Lines of Inches and of Lesser Tangents

Finally we come to the lines along the outer rims of the sector. The Line of Inches needs no further explanation: its function is purely one of measurement. However, there is also a Line of Lesser Tangents which is used to draw out hour-lines for sundials on any specified plan and also, in conjunction with the strut of the sector, to measure the altitude of the sun. The strut is attached, at the end furthest from the hinge, to the arm of the sector which does *not* carry the Line of Lesser Tangents, by means of a pivot screw; for use with the Line of Lesser Tangents it is rotated until it is perpendicular to the arm to which it is attached and the sector is then closed so that the

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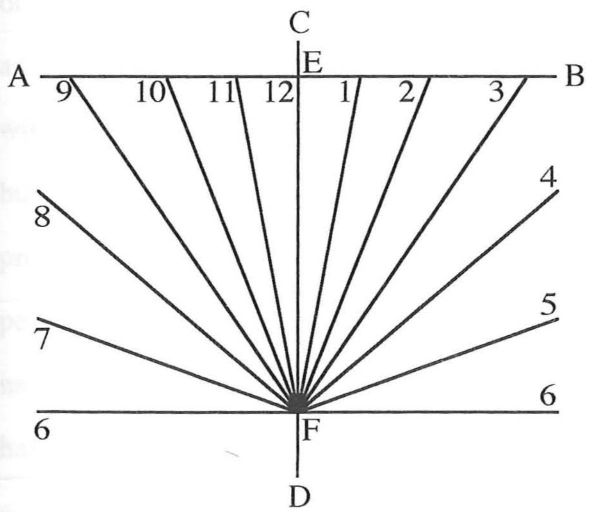
[15] Gunter, *De Sectore*, p.133.

strut protrudes, perpendicular to the Line of Lesser Tangents. The protruding length of the strut is equal to the distance from 0 to 45° in the Line of Lesser Tangents. If the sector is held up with the arms vertical and the strut pointed towards the sun, the shadow of the lower edge of the strut will fall on the Line of Lesser Tangents and show the altitude of the sun in degrees.[16]

The Line of Lesser Tangents can be used to lay out dials on any plane surface. I will use a horizontal plane as an example, but similar rules apply for other kinds.

First, a line (AB) is drawn to represent the horizon, crossed (at E) by a perpendicular line (CD) for the Meridian or North-South line (this is also the 12 o'clock line). Next the dividers are opened out to stretch from 0 to 15° in the Line of Lesser Tangents and this distance is marked from E towards both A and B.[17] The same procedure is repeated for 30°, 45°, 60° and 75°. Then the secant of the complement of the latitude is obtained by opening the dividers so that one foot is placed at the complement of the latitude in the Line of Lesser Tangents and the other is extended to the far end of the strut (i.e. that not attached to the sector). This length is marked off along the Meridian from E to F and a line is drawn parallel to AB at this point. This line is the 6 o'clock line of the dial. Lines are drawn from F through the points marked off along AB and these supply the remaining hour lines of the dial.



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[16] Parallax is not a problem because the extent to which the shadow stretches along the Line of Lesser Tangents is dependent only on the position of the lower edge of the strut.

[17] Clearly, the diagram I have supplied only shows a part of the line AB as it would appear in the actual construction of the dial - I have followed Gunter (see *De Sectore*, p.138) in choosing to curtail the diagram to the area which would appear on the completed dial.

Even a very brief summary of the uses of the sector demonstrates the multitude of applications which are possible with this instrument and its usefulness to all kinds of areas of mathematics. It is little wonder that it was praised by the encyclopædia writers for its all-round excellence and utility. In contrast to the Gunter quadrant this is clearly an instrument which has many functions for the practical mathematician, and yet it shares some features with the quadrant: it is a multifaceted instrument which has roles in various different mathematical sciences, it very obviously demonstrates the importance of geometrical approaches in mathematical calculations, and there are certain of its functions which are of less practical use than others. Having covered the manner of the sector's use, there are still numerous issues which arise regarding its application and the points the instrument raises concerning the nature of the mathematical culture within which it was used. There is not space here to deal with all these issues in detail but I will consider briefly the themes of geometrical presentation of mathematical problems and of the place of the Platonic solids within English mathematics of this period, before concentrating at greater length on the role of the Gunter sector in navigation - a subject which is perhaps not quite as straightforward as some historians have supposed it to be.

Geometry as the language of mathematics

The sector is at least as important a witness to the significance of geometry to seventeenth-century English mathematics as the Gunter quadrant, if not more so. The very nature of the calculations which are performed on it is firmly rooted in a geometrical understanding of and approach to mathematics: all problems are resolved by reducing them to problems of proportion and then by the construction of similar triangles followed by the comparison of the lengths of sides of these triangles. It is far more common to consider values in terms of lengths than in terms of numbers: for instance, when Gunter deals with the uses of the Line of Lines he devotes eight sections of the chapter to questions concerning straight lines and their relationships to each other and only two to numbers (which he speaks of as being analogous to the propositions concerning lines and therefore solvable by the same method). The same is true when he comes to problems relating to areas and volumes. Even the fact that the Lines are named Line of Lines, Line of Superficies, Line of Solids (rather than Line of Numbers, Line of Square Numbers, Line of Cube Numbers, for instance) indicates the propensity for taking a geometric approach. As I have already noted above, the tendency to think of square and cubic roots in terms of geometric shapes was very strongly established in the understanding of these concepts and continued to be so throughout the century, despite the development of algebraic methods.

The fundamentally geometric approach utilised by the sector does not end with the three basic Lines: problems of planar trigonometry are most often dealt with by simple construction of the triangle in question through opening out the sector to represent the required shape. The Lines of Sines, Tangents and Secants are all constructed on the foundation of thinking of the trigonometrical functions in terms of lengths related to the radius of a circle. The purpose of the Meridian Line is to aid in the reduction of spheres to planes in order to be able to solve questions of navigation more easily. The Line of Segments is used in conjunction with drawn circles, and the calculations concerning inscribed and equated bodies all lead to answers in terms of lengths which can only be converted into numbers by application of the dividers to the Line of Lines. Whichever way we attempt to use the sector we are constantly confronted by the undeniably geometric nature of the mathematics within which this instrument belongs.

The evidence for the enduring appeal of the sector throughout the seventeenth and eighteenth centuries is thus important in bearing witness to the continued understanding of mathematics in geometrical terms, even as algebraic methods were developed and algebra became a firmly established branch of mathematics. It is interesting that the sector retained its place as a calculating instrument long after the logarithmic rule had appeared and become a relatively widely used instrument, and this may have been due to the fact that the sector offered a more obviously geometric approach to numbers and mathematics in general. It seems that this was a society which found it very much easier to understand numbers in terms of geometric entities than as symbols written on a page: a line twice as long as another one gave a better sense of the nature of two than the numeral, 2; similarly it was easier to see how the side of a square would have some kind of direct relationship to its area, than to talk in abstract terms of square roots.

This strongly geometric and visual approach to mathematics once again brings us back to the important role of instruments in aiding the understanding of mathematics and their function as teaching aids. Instruments provide opportunity for exploring the world of mathematics in a directly physical way which may be more easily assimilable than a purely verbal discussion of the concepts. Some instruments, such as armillary spheres, globes, astrolabes and the Gunter quadrant have more obvious didactic roles than others (which were intended primarily for direct application to various fields of practical mathematics). However, even the Gunter sector could be used as a very simple teaching aid for demonstrating the properties of similar triangles and the way in which the resolution of questions of direct proportion revolves around the fundamental nature of similar triangles.

The sector as witness to neo-Platonic thought?

The relevance of many of the scales appearing on the Gunter sector is immediately obvious: the Lines of Lines, Superficies and Solids allow the rapid solution of calculations involving lengths, areas and volumes; the Lines of Sines, Tangents and Secants resolve trigonometrical problems and, when used in conjunction with the Meridian Line, can be applied to navigational problems concerned with the Mercator projection; the Line of Lesser Tangents enables the construction of sundials and the measuring of solar altitude. However, Gunter also supplied a further five scales: the Lines of Quadrature, Segments, Metals, Inscribed Bodies and Equated Bodies.

Gunter himself clearly saw these scales as less important than the others, remarking

> 'The lines of *lines*, of *superficies*, of *solids*, of *sines*, with the laterall lines of *tangents* and *meridians*, whereof I haue hitherunto spoken, are those which I principally intended: that little roome on the *Sector* which remaineth, may be filled vp with such particular lines as each one shall think conuenient for his purpose. I haue made choise of such as I thought might be best prickt on without hindring the sight of the former, viz. lines of *Quadrature*, of *Segments*, of *Inscribed bodies*, of *Equated bodies*, and of *Mettals*.'[18]

Nevertheless the implication is that these lines must have been connected to areas of mathematics which were of general interest to the mathematical practitioners of the period, since, otherwise, there would be no reason at all to inscribe them on the instrument. It is also worth noting that inspection of the instrument without reference to *De Sectore et Radio* does not make it immediately obvious that the 'extra' Lines are significantly less important than the other scales, and consideration of some of the 'dispensable' Lines may raise new insights. I propose to concentrate here on the Lines relating to the Platonic bodies - the Line of Inscribed Bodies and the Line of Equated Bodies.

The Platonic (or regular) solids had long been an area of interest to mathematicians and natural philosophers. Discovery of these bodies was generally ascribed to the Pythagoreans, but their most common name arose from Plato's introduction of them into his cosmological treatise, *The Timaeus*. Here, four of the five solids are ascribed to the four elements - the cube to earth, the tetrahedron to fire, the octahedron to air and the icosahedron to water; the dodecahedron was held to have a

[18] Gunter, *De Sectore*, p.127.

particular affinity to the heavens since it came closest in form to the sphere.[19] In the following century the final part of Euclid's *Elements of Geometry* was devoted to a discussion of the regular solids and to a demonstration that there were only five such solids which could be formed. It was these two works which supplied the foundation for Renaissance fascination with the Platonic solids and the continuing interest through the sixteenth and seventeenth centuries. Renaissance treatment of the subject matter appeared in Luca Pacioli's *De Divina Proportione*[20] and Charles de Bovelles' *De mathematicis corporibus*[21] where the link to the Platonic tradition of the solids as the elemental bodies was very strong.

The most important English writers on the subject were Thomas Digges in his *Pantometria* and John Dee in his commentary on the 1570 edition of Euclid's *Elements* produced by Henry Billingsley.[22] Digges was concerned only with study of the geometrical properties of the bodies but Dee also discussed the application of the bodies within the Platonic cosmology. However, the main emphasis of both was on the place of the regular polyhedra at the very heart of geometry. Dee spoke of them as 'the ende and perfection of all Geometry, for whose sake is written whatsoeuer is written in Geometry' and also remarked that

> '...these are the thinges of all other entreated of in Geometrie, most worthy and of greatest dignitie, and as it were the end and finall entent of the whole arc of Geometrie, and for whose cause hath bene written, and spoken whatsoeuer hath hitherto in the former bookes [of the *Elements*] bene sayd or written.'[23]

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19 Plato, *The Timaeus*, 53C - 57D. Plato's use of the regular solids is discussed in detail by Cornford in *Plato's Cosmology. The* Timaeus *of Plato translated with a running commentary by Francis Macdonald Cornford* (London, 1937).
20 Venice, 1509, with illustrations by Leonardo.
21 Paris, 1511. See the discussion of this work in P.M. Sanders, 'Charles de Bovelles's Treatise on the Regular Polyhedra (Paris, 1511)' in *Annals of Science*, 41 (1984), pp.513-566.
22 Thomas Digges, *A geometrical practical treatize named Pantometria* (London, 1591); Euclid, ed. Henry Billingsley, *The Elements of Geometrie, of the most auncient Philosopher Euclide of Megara* (London, 1570).
23 Euclid, ed. Henry Billingsley, *The Elements of Geometrie*, f.311 verso.

Kepler's use of the solids in his cosmological text, *Mysterium Cosmographicum* (Tübingen, 1596), no doubt helped to retain interest in the solids, and his work was certainly known to the more dedicated students of contemporary astronomy. Similarly the fascination with dialling among the mathematicians of seventeenth-century England was an aid in keeping an awareness of regular polyhedra - various horologiographical works discussed the construction of dials on the faces of the Platonic solids, no doubt in order to demonstrate the knowledgeability and skilfulness of the author. Overall it is clear that study of the regular solids was seen as the acme of the pursuit of geometry, and a grasp of their properties and relationship to each other was an indication of one's complete mastery of this branch of mathematics.

Thus in the Lines of Inscribed Bodies and Equated Bodies we find little that belongs in the sphere of practical mathematics and this raises the question of the intended market for the sector. These are scales which reflect the mathematician's interest in abstract problems and are relevant to the end of the mathematical community which was more concerned with theoretical rather than practical mathematics. For whatever reason, there was seen to be a need to make this instrument attractive not only to those who required the more utilitarian scales for day-to-day calculations but also to produce something which would appeal to those who dabbled in mathematics purely as an intellectual pursuit. A market for the sector was obviously being sought among the gentlemen amateur mathematicians as well as among those who would wish to make use of the instrument in day to day calculations or as part of their navigational equipment. No doubt Elias Allen kept this in mind when designing his instruments and it may be one of the reasons why his Gunter sectors continued to display the 'less useful' scales even when other writers were developing sectors which concentrated on the applications most relevant to navigation and other areas of practical mathematics.[24]

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[24] Witness Cat. no. S4 where the scales become extremely cramped together in order to accommodate the logarithmic functions as well as all the standard lines found on the Gunter sector.

Investigating the usefulness of the sector in navigation

I have already touched on the application of the Gunter sector to navigation. This was a use which Gunter himself encouraged and discussed at great length in his user's guide to the instrument, it was an area which was frequently referred to by commentators on the sector throughout the seventeenth and early eighteenth centuries, and it has been widely assumed by recent historians of navigation that the Gunter sector was of great importance in the development of mathematical navigation at this period. This latter view is nowhere more clearly propounded than in Waters' monumental study of Early Modern navigation, *The Art of Navigation in England in Elizabethan and Early Stuart Times*, where he writes:

> 'by describing...how to avoid, by use of the Sector, the tedious mathematical calculations involved in the solution of plane and spherical triangles in the period before logarithms were invented, [Edmund Gunter] first made arithmetical navigation a practical proposition at sea.'[25]

The received assessment is that such an instrument greatly reduced the rigours of calculation and therefore removed the terrors of mathematical navigation; it was quick and easy to use, and yet retained sufficient accuracy to render tenable results for the navigator. Whilst this may well be true, the assertion has not been supported in recent times by practical investigation of the navigational use of the sector: for this reason, I decided to make such a study myself.

The most obvious way in which to do this is by performing a set of calculations, typical of those which a navigator would have employed in the course of a normal day at sea to establish his position according to the stipulations of Mercator sailing. Mercator's projection had been rendered useful at the very end of the sixteenth

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[25] Waters (London, 1958), p.366. This argument has been accepted in J.A. Bennett, *The Divided Circle* (Oxford, 1987), p.62, and in Harriet Wynter & Anthony Turner, *Scientific Instruments*, (London, 1975), p.64. Jean Randier, *Marine Navigational Instruments*, translated John E. Powell (London, 1980) includes the sector as a matter of course.

century through the detailed explanation of the Mercator chart and its use given in Edward Wright's *Certaine Errors in Navigation.* However, this book only gave details of how to find a ship's position given that the navigator was already in possession of an accurate Mercator chart, and all calculations were carried out by rulers and pairs of compasses, in conjunction with the graduated scale of latitudes on the Mercator chart. Although Wright declared that his table of meridional parts could be used in conjunction with the Canon of Triangles [i.e. trigonometry] to carry out navigational calculations, he felt that it was sufficient for sailors' needs simply to demonstrate how these calculations could be performed with the use of a Mercator chart, a rule and a pair of dividers:

> '...the Mariner shall not need to trouble himselfe any further herewith [with trigonometry], but only to cast vp his accounts vpon the chart truly made (as before is shewd) which of al other is most fit & ready for his ordinarie vse. Now therefore it may be sufficient, onely to shewe how the former Problemes may mechanically be performed vpon the nauticall planisphaere before described [the Mercator projection].'[26]

It was not until trigonometry spread outside the handful of accomplished mathematicians that position could be calculated with reference solely to the last known location. The most accessible work on the subject was that of Pitiscus, which only became available in English after 1600. In 1614 a translation by Ralph Handson appeared,[27] to which was appended Handson's own explanation of arithmetical navigation, the first to be provided for the English navigators. From this time it became possible for relatively adept navigators to calculate their position at sea, using the trigonometrical methods. 1623 saw the publication of Gunter's book on the sector and the question arose as to which method was superior. The sector was specifically aimed

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[26] Wright, *Certaine Errors in Navigation* (London, 1599), sig.L3 verso. Wright's method is demonstrated after the trigonometrical calculations (page 140).
[27] Pitiscus, *Trigonometrie* (London, 1614).

at dealing with problems related to Mercator sailing and therefore a problem of this kind will be considered in this study.

The time and difficulty associated with performing the day's calculations by pen and paper can be compared readily with that occasioned in carrying out the same calculations by use of a Gunter sector. Of course, it must be ensured that the arithmetical methods used are the same as those which would have been available to a seventeenth-century navigator and hence all arithmetical operations have been based on John Tapp's *The Pathway to Knowledge*, an arithmetic manual first published in 1613.[28] In order to carry out the calculations with a sector, I made a replica of an early Gunter sector, based on the Allen instrument held in the Whipple Museum of Science in Cambridge (Cat. no. S1). The instrument was made out of card and then attached to a standard hinged carpenter's rule to produce a nine-inch instrument. Markings for the more esoteric lines on the sector (such as the Line of Meridional Parts, the Line of Quadrature, the Line of Equated Bodies, etc.) were set down from measurements taken from the original. However, for the Lines of Lines, Superficies, Solids, Sines, Tangents and Secants the markings were calculated afresh in order to ensure their accuracy. This instrument was then used in conjunction with a pair of dividers to carry out the calculations.

The base for the information employed in this exercise was taken mainly from a hypothetical log laid out in Norwood's *The Seamans Practice*. The voyage concerned was from Somers Island in the Bermudas to England and the hypothetical situation was the following:

It is 24th February 1632.[29] The ship's position was last calculated three days ago as latitude 34 degrees 25 minutes North and longitude 2 degrees 38 minutes East of Somers Island. Since that time the ship has run a course bearing East-North-East, half a point North [i.e. 062 degrees] by the compass. For the last two days the variation of

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[28] This work was chosen as it was one of the most recent arithmetics available when *De Sectore* appeared.
[29] The year 1632 was chosen as solar declination tables for this year were readily available in Norwood's *Trigonometrie* (London, 1631).

the compass has been calculated at 8 degrees West of North. Today the bearing of the sun at sunrise was given by the compass as 15 degrees 30 minutes South of East, and the meridian altitude of the sun (measured with the cross-staff) was found to be 46 degrees 38 minutes. The following must be calculated: the latitude of the ship; the variation of the compass; the overall distance travelled (and the distances travelled North and East) in order to make comparison with the dead reckoning results; the longitude of the ship.

The calculations are given in full in order to give an idea of the time taken to carry them out and the arithmetical complexity involved. I have set out the pen and paper calculations first and append to them a discussion of the techniques involved and the problems raised. Following this I have treated the instrumental procedures in the same sort of way.

The pen and paper calculations

1) The calculation for the latitude[30]

Meridian altitude of the sun as taken by the cross-staff - 46 degrees 38 minutes.

This must be adjusted for the necessary corrections: parallax of larger[?] staff[31] - 1 degree 35 minutes; surplus of horizon - 5 minutes; semidiameter of the sun - 16 minutes.

Total adjustment - 1 degree 56 minutes. Hence the actual meridian altitude of the sun is 44 degrees 42 minutes.

Elevation of equinoctial plane = meridian altitude +/- declination.

For 24th February in a northerly latitude the declination is added on.

[30] This was taken from Thomas Hariot's method as supplied in Brit. Mus., Add. Ms. 6788.

[31] 'Back-staff' is the term given in Taylor's reproduction of this calculation in *The Haven-finding Art* (London, 1956) on page 221. However, there seems to be an anomaly here since the back-staff was designed in order to remove the problems of parallax caused by use of the cross-staff. The original manuscript is, in fact, very unclear at this point. The word which has been rendered 'backe' by Taylor is a very hasty interpolation and is even more difficult to read than the rest of the manuscript. However, comparison with other parts of the manuscript brought me to the conclusion that the word was not 'backe'; it could possibly be 'larger' which would seem to make considerably more sense.

From tables of solar declination it is found that the declination for 24th February 1632 is 5 degrees 42 minutes South.

Therefore the elevation of the equinoctial plane = 44d 42´ + 5d 42´ = 50d 24´

	44d 42´ +
	5d 42´
	50d 24´

Latitude is the complement of the elevation of the equinoctial plane

Therefore latitude is 39 degrees 36 minutes.

2) <u>The calculation of the variation of the compass</u>[32]

This is done by calculating the true amplitude of the sun and comparing it with that recorded by the compass.

From Handson's rules it is known that as the Radius is to the secant of the latitude, so is the sine of the declination of the sun to the sine of its amplitude. In other words, the sine of the amplitude is found by multiplying the secant of the latitude by the sine of the declination and dividing the product by the radius.

Secant of the latitude	129784
Sine of the declination	9932
	259568
	389352
	1168056
	1168056
	1289014688

Therefore sine of the amplitude = 12890 $^{14688}/_{100000}$

Therefore amplitude (from trigonometrical tables) is 7 degrees 24 minutes (South of East because it is before the vernal equinox and the latitude is northerly).

Difference between the actual amplitude and the measured amplitude is 15d30´ - 7d24´ which is approximately 8 degrees West of North, as before.

Therefore the true course of the ship is eight degrees further West than East-North-East, half a point North.

True course is NE 54 degrees.

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[32] This and all the following pen and paper calculations are taken from Handson's appendix to his translation of Pitiscus' *Trigonometry*.

3) The calculation of the distance travelled in the previous three days

According to Handson's rules, as the radius is to the secant of the angle between the meridian and the rhumb, so is the difference in latitudes (in miles) to the distance. In other words, the distance is found by multiplying the difference in latitudes by the secant of the angle between the meridian and the ship's course and dividing the product by the radius.

Difference in latitudes	39d 34′ -
	34d 25′
	5d 11′

Difference in miles equals 5 multiplied by 60 plus 11

equals 300 plus 11

equals 311 miles.

Secant of 54d0′	170130
Difference in latitudes	311
	170130
	170130
	510390
	52910430

Divide by radius to give 529 $^{10430}/_{100000}$

Therefore the distance travelled is 529 miles.

4) The calculation of the longitude

According to Handson's rules, as the difference in longitude is to the difference in meridional parts[33] for the given latitudes, so is the tangent of the angle between the meridian and the rhumb to the radius multiplied by 10. In other words, the longitude is calculated by multiplying the difference in meridional parts by the tangent of the angle between the meridian and the ship's course and dividing the

33 Tables of meridional parts are taken from Wright, *Certaine Errors*.

product by ten times the radius. [For an explanation of meridional parts see Appendix 4.]

Meridional parts for 39d36´	25967
Meridional parts for 34d25´	22078
Difference	3889

Tangent of 54d0´

```
     137638
       3889
    1238742
   1101104
  1101104
  412914
  535274182
```

Hence the difference in longitude = $535\,{}^{274182}/_{1000000}$

The difference in degrees = $^{535}/_{60}$

```
  5
 535|8
 60
```

The difference in longitude is 8 degrees 55 minutes.

The ship is sailing Eastwards. Hence to find the new longitude simply add 8d55´ to the previous longitude, which was 2d38´ East of Somers Island.

The new longitude is found to be 11 degrees and 33 minutes East of Somers Island.

5) Calculation of the distance travelled North

This is simply the difference in latitude, given in miles. Hence the distance travelled North is 311 miles.

6) Calculation of the distance travelled East

Mean latitude of ship is 37 degrees.

From Handson's rules, as the miles in a degree of meridian are to the miles in a degree of latitude at parallel 37 degrees, so is the radius to the sine of the complement of the latitude. In other words, the miles in a degree of latitude can be found by multiplying the sine of the complement of the mean latitude by sixty (the number of miles in a degree of meridian) and dividing the product by the radius. This result is then multiplied by the difference in longitude to give the distance travelled East. This can be shown as:

$$\text{distance travelled East} = \frac{\text{sine of } 53 \times 60}{\text{radius}} \times \frac{535}{60}$$

which is simplified to:

$$\text{distance travelled East} = \frac{\text{sine of } 53 \times 535}{100000}$$

sine of 53 degrees

```
   79864
     535
  399320
 239592
399320
42727240
```

dividing this product by the radius gives a distance travelled East of 427 miles.

The total time taken for these calculations was approximately fifty minutes.

Notes on the calculations performed

All the trigonometrical calculations are ordered in such a manner that the only division that is necessary is by the radius. This greatly reduces the complexity of calculation. For instance, if the variation of the compass had been found by using the sine of the complement of the latitude (rather than the secant of the latitude), the calculation would have run as follows (using seventeenth-century methods of long division):

as the sine of the amplitude is to the radius, so is the sine of the solar declination to the sine of the complement of the latitude. In other words, the sine of the amplitude is found by multiplying the sine of the solar declination by the radius and dividing the product by the sine of the complement of the latitude.

sine of the declination multiplied by the radius = 9932 x 100000

= 993200000

sine of the complement of the latitude 77051

The division would look something like this:

```
     61
   16942
   625876
   886987
  22279821
  9920000000|12890
  770511111
   7705555
    77000
     777
      7
```

This gives the result that the sine of the amplitude is 12890 $^{12610}/_{77051}$ which yields the same amplitude as before.

It can be appreciated that such arithmetic would have been a daunting prospect to most navigators, particularly since it is very difficult to check through these long division calculations for mistakes, whereas the multiplications can easily be reworked and examined for errors. Hence the attitude of most modern historians can be understood - that arithmetical navigation was an exceedingly complicated and difficult process. However, when the problems are reduced to ones which only involve division by the radius, the whole operation of long division is removed and, with it, much of the terror of calculation, although it is still relevant to say that most navigators were not versed in any sort of mathematical approach to navigation.

The other point to be noted is that the use of the tables of meridional parts removes the need to utilise any spherical trigonometry in the calculation of longitude. These meridional parts come from tables used for marking off Mercator charts and hence are connected to a plane representation of the world rather than a spherical one. Thus plane trigonometry can be used, but latitude scales must be transformed into meridional parts in order to compensate for the distortion of the Mercator projection when plotting position.

Calculations by Wright's Method

In order to do these calculations I simply drew out on a sheet of graph paper the equator and meridian lines required for the exercise and constructed a latitude scale according to Wright's method.

1) Calculation of the latitude

This is carried out as before.

2) Calculation of the variation

Since no trigonometry is to be used, the variation has to be found by measuring the bearing of the sun at noon (very difficult to ascertain the exact time when this measurement should be made) or by measuring the bearing of the sun at a point in the morning and then again in the afternoon at the point when the sun is at the same altitude which it had attained when the morning measurement was made. In the latter case, the mean bearing will give the compass bearing for the sun at noon. Now the variation can be obtained by determining the difference between the compass bearing of the sun at noon and South. From this value of the variation the actual course of the ship is determined as before.

3) Calculation of the distance travelled

First the difference in latitudes is calculated: 5°11′. Using a pair of dividers, this amount is measured out along the degree scale on the equator. A parallel to the equator is drawn at this distance from the equator, making sure that the parallel drawn crosses the relevant rhumb (here 054°) from the windrose on the equator. Now the distance from the centre of the windrose to the crossing point of rhumb and parallel is measured with the dividers. This amount is set along the equatorial degree scale to determine the distance travelled (it is converted from degrees into leagues by multiplying by 20). [Calculated as 527 miles.]

4) Calculation of the longitude

With the dividers the difference in latitudes is measured on the graduated meridian scale at the side of the chart (the scale of meridional parts) - i.e. one foot of the dividers is placed at latitude 34°25′ and the other is extended to latitude 39°34′ on the meridian scale. Keeping the dividers open to this distance, they are moved along the equator until one foot rests on the equator and the other foot rests on the rhumb line (so that the dividers are perpendicular to the equator). The distance along the equator from the windrose to the base of this perpendicular gives the longitude difference. [Calculated as 8°55′ - the exact result from the trigonometrical calculations.]

5) Calculation of the distance travelled East

First of all the mean latitude is calculated and at this latitude the difference in longitude is marked out on the chart as two meridians. Next the length of a degree of latitude at this point is set on the dividers (measuring from 36°30′ to 37°30′). Finally, the number of times the dividers can be fitted into the distance along the parallel between the two longitudes is sought out. This will give the distance in scores of leagues, but will obviously not have a very high degree of accuracy. [Calculated as approximately 410 miles.]

The time taken for these calculations was approximately forty minutes.

Calculations using the sector

The calculation of latitude makes no use of the sector and thus is performed in exactly the same way as before. The other calculations are performed as follows:

1) Calculation of the variation of the compass

As the sine of the true amplitude of the sun is to the radius, so is the sine of the solar declination to the sine of the complement of the latitude. (In this case the sine of the complement of the latitude is used rather than the secant of the latitude because the Line of Sines is more convenient to use than the Line of Secants. This formula is simply a rearrangement of the earlier one used for the pen and paper calculations.)

Thus the dividers are opened out along the Line of Sines from zero to the sine of the solar declination (5d42′) and this length is then used to open out the sector at the marks in the Line of Sines for the sine of the complement of the latitude. Readjusting the dividers so that the two feet lie in the marks for the sine of ninety degrees (otherwise known as the whole sine or the radius) and laying the dividers along one Line of Sines with one foot at zero will give the true amplitude as 7 degrees 30 minutes. The rest of the calculation proceeds as before.

<u>2) Calculation of the distance travelled in the previous three days</u>

According to Gunter's rules,[34] as the sine of the complement of the rhumb from the meridian is to the radius, so is the difference of latitudes to the distance sailed.

The difference of latitudes is calculated as before and found to be 311 miles.

The complement of the rhumb is 36 degrees.

The dividers are opened to the difference of latitude, measured in the Line of Lines, and this length is then used to open out the sector at the marks in the Line of Sines for the sine of the complement of the rhumb. Readjusting the dividers so that the two feet lie in the marks for the radius and laying the dividers along one Line of Lines with one foot at zero will give the distance travelled as 525 miles.

<u>3) Calculation of the difference in longitude</u>

According to Gunter's rules, as the radius is to the tangent of the rhumb from the meridian, so is the proper difference of latitudes to the difference of longitudes. However, where the rhumb falls nearer to the equator than to the meridian it is more

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[34] These rules are taken from *De Sectore*.

convenient to use the following rule: as the tangent of the rhumb from the equator is to the radius, so is the proper difference of latitudes to the difference of longitudes. (The description of the latitude difference as 'proper' indicates that it should be read off in the Meridian Line, rather than in the Line of Lines.)

Rhumb from the equator is 36 degrees.

The dividers are opened to the tangent of 36 degrees, measured from zero to thirty-six in the Line of Tangents, and this length is then used to open out the sector at the marks in the Line of Sines for the radius. Now the dividers are reset by placing one foot in the initial latitude (34d25′) on the Meridian Line, and the other foot in the final latitude (39d36′). The reset dividers are then moved along the sector until the feet fall in the same spot in the two Lines of Lines. This point will give the longitude difference as 9 degrees. The initial longitude was 2d38′ East of Somers Island, and since the ship is still sailing East, the new longitude will be 11d38′ East of Somers Island.

4) Calculation of the distance travelled North

As noted before, this is simply the difference in latitudes - 311 miles.

5) Calculation of the distance travelled East

According to Gunter's rules, as the miles travelled East are to the difference in longitude (in minutes), so is the sine of the complement of the mean latitude to the radius.

The dividers are opened to the difference of longitude (540 minutes), measured from zero in the Line of Lines, and this length is then used to open out the sector at the marks in the Line of Sines for the radius. Readjusting the dividers so that the two feet now lie in the marks for the sine of the complement of the mean latitude (37 degrees) and laying the dividers along one Line of Lines with one foot at zero will give the distance travelled East as 431 miles.

The total time taken for these calculations was approximately 35 minutes.

Notes on the calculations performed and the use of the sector

In the course of using the sector for these calculations I became aware of various limitations of the instrument. The first was that in order for reliable results to be produced the hinge of the sector must be stiff, so that once the sector is opened out to a given angle it will stay at exactly that angle throughout the course of the operation. Any shift in the angle of the sector during the calculation will immediately bring inaccuracies into the answer. When using one side of the sector only, the instrument can be clamped with one hand while the other is used to operate the dividers (providing that the dividers available are a type that can be operated one-handed). However, most of these calculations require the sector to be turned over, in order to use scales from both sides of the instrument: the Line of Lines and the Line of Sines are on opposite sides. In these cases it is essential that the opening angle of the sector can be maintained by the friction of the instrument itself. The sector strut is useful for small angles since its presence increases the friction and therefore cuts down the movement of the sector. However, there is a noticeable drop in stiffness when the angles are too great for the strut to be functional and all the friction comes from the hinge alone.

Similarly the dividers used must stay open at the angle set in order to get accurate results, and so they must have a stiff pivot. Dividers which are operated single-handed will automatically be less likely to shift because they can be held steady by the hand operating them, but it should be noted that this is another possible source of error.

The third problem is connected to the fact that the dividers have a tendency to slip around on the sector, particularly if it is metal. With a card or wooden instrument it is possible to avoid this problem to some extent by jabbing the feet of the dividers into the material of the instrument, but this can not be done in all parts of the calculation and nor can it be done on a metal instrument (it may also cut down the accuracy of the lines over a prolonged period of time). Such problems of slippage would clearly be exacerbated by the motion of a ship at sea.

Finally, it was immediately obvious that the accuracy of the calculations was much lower than that of the calculations performed with pen and paper. The sector is too small an instrument to yield results accurate to a minute of arc or a mile in distance, whereas the pen and paper calculations make use of tables which do give angles to the minute and trigonometrical functions to an accuracy of five decimal places (in modern terms). The Line of Sines can cause particular problems when dealing with large angles since the numbers draw closer together as they approach the whole sine and the accuracy is correspondingly reduced. Similar, though less acute, problems are found in using the Line of Tangents for small angles and the Meridian Line for tropical latitudes. Of course, these problems can be reduced by repeating the calculations and taking the mean results: however, in so doing, the sector's advantage of speed is immediately removed.

Comparison of the methods of calculation

The two main areas to be taken into consideration in this comparison are those of speed and accuracy. In timing the calculations I found that the sector speeded up the process by about fifteen minutes, taking thirty-five minutes, as opposed to the fifty minutes of the pen and paper calculations. The latter calculations were done deliberately slowly and all of them were fully checked through, in order to simulate as far as possible the conditions under which a seventeenth-century navigator would have worked. The percentage difference is quite marked but what must be considered is how much difference a quarter of an hour would actually make to a navigator on board ship.

The greater speed of the instrumental method must also be weighed against the considerable drop in accuracy. As mentioned above, the sector is simply incapable of reaching the same accuracy of calculation as the written method because the tables provided for use with trigonometrical calculations give far more accurate values than can possibly be read off a sector. Calculations carried out using the sector tend to give answers accurate to about half a degree or five miles, purely because of the limits set by the size of the divisions. The accuracy for distance is considerably better than that for angles but it is still the kind of error which will make a considerable difference to the estimated position over even a short period of time.[35]

Of course, it is difficult to be exactly sure how accurate the original measurements themselves were. In accounts of voyages at this time not only latitude but also variation and longitude are often recorded to the nearest minute of arc, even though it is highly unlikely that such accuracy could have been achieved. Indeed, one ship's navigator commented that the variation of the compass could only be found to the nearest degree with any certainty:

> '...nor can any iudgement at all be made to twenty leagues thereby (that shall be infallible) the magneticall amplitude beeing so difficult to obserue truely by the Ships motion, and the Needles quickness, that a degree is scarce an error.'[36]

The measurement of the latitude was significantly more accurate than this. Accounts of Dutch voyages in search of the Northeast passage[37] appear to indicate that skilled observers could take measurements of the meridian altitude of the sun or of the Pole Star with an accuracy in terms of one or two minutes of arc when using either the astrolabe or the cross-staff. Certainly this is the accuracy to which the measurements are recorded. Modern trials of these instruments would seem to belie such confidence in the instruments: Christopher Daniel used both cross-staff and mariner's astrolabe during the reconstruction of the voyage of the *Golden Hinde* in 1974-5.[38] He estimated that the accuracy of the cross-staff was in the order of twenty nautical miles (one third of a degree) while that of the astrolabe was only in the order of thirty nautical miles

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[35] I have not found any evidence that contemporary trials were made to investigate the accuracy of the sector.

[36] Thomas Roe in Samuel Purchas, *Haklvytvs Posthumus or Purchas his Pilgrimes* (London, 1625), p.535.

[37] See the Hakluyt Society publications, volume 13 (London, 1853) - *Three Voyages by the North East*. These voyages date from the final decadeof the sixteenth century.

[38] See Stimson & Daniel, *The Cross Staff: Historical Development and Modern Use* (London, 1977).

(half a degree). However, he was able to take measurements with the cross-staff for low altitude observations which were read off the scale to an accuracy of a minute of arc. This might explain why the Dutch navigators were able to note down such accurate figures for altitude measurements: they were working in high latitudes and so the altitude of the sun would be correspondingly small. Whatever the accuracy of observational instruments, it is not surprising that latitudes, variations and longitudes were generally recorded to an accuracy of minutes of arc since the trigonometrical tables of the time would convert all results of calculations into values given to a minute of arc.

Since the issue of observational errors is a significant one with respect to the efficacy of the sector it will now be considered in greater detail. The written calculations seem to imply that position can be found to within minutes of arc. However, does such pin-point accuracy mask observational errors and thus mislead the navigator into believing that he knows his position precisely when, in fact, the errors of compass and altitude measurements could put his readings out by several degrees when carried through the various operations to calculate the longitude position? In an attempt to answer such questions I investigated the problems created by the introduction of errors into the calculations. I will first of all lay out a sample calculation to show how I arrived at my results, follow this by a short résumé of the method used and the results obtained, and finally draw some conclusions from the gathered data. (NB. Throughout the course of all these calculations it was assumed for the sake of simplicity that the trigonometrical tables and tables of declination available would all have been accurate.[39])

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[39] This is likely to have been the case for the trigonometrical tables but not, in fact, for the tables of solar declination which were often as much as a degree out. For example, Wright makes this clear in *Certaine Errors*: 'Notwithstanding the Sunne and Starres are at sea the most certain marks and guides the Nauigator hath,... Yet the Tables of declinations of the Sun & fixed Starres hitherto published, which I haue compare together and examined by obseruation, are oft times very faulty:' (Sig.O4 recto).

The sample calculation (performed by twentieth-century methods) is taken with the following set of values (reasons for this choice should become apparent later):

initial latitude: 0° (i.e. the Equator)

present latitude: 5° North[40]

bearing of sun at sunrise: East 30° North, according to the compass

course of ship: North 10° West, according to the compass

declination of sun: 11.75° North

Assuming that there are no errors involved, the following calculation can be made:

True amplitude of the sun $= \sin^{-1}$ (secant latitude x sine declination)

$= \sin^{-1} (\sec 5 \times \sin(-11.75))$ [41]

= East 11.80° North

Therefore, variation of compass = 18.20° East of North

Therefore, actual course of ship = North 8.20° East

Distance travelled = difference in latitudes in minutes x sec (actual course)

$= 5 \times 60 \times \sec 8.20$

= 303 miles

Longitude difference[42] (in minutes)

$$= \int_{y_1}^{y_2} \sec y \, dy \times {}^{180}/_{\pi} \times 60 \times \tan(\text{actual course})$$

$$= {}^{10800}/_{\pi} \times \{\ln|\tan({}^{5}/_{2} + {}^{\pi}/_{4})| - \ln|\tan({}^{0}/_{2} + {}^{\pi}/_{4})|\} \times \tan(\text{actual course})$$

$$= {}^{10800}/_{\pi} \times (\ln|\tan 47.5|) \times \tan 8.20$$

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40 The latitude is given direct (rather than the altitude of the sun at noon) because the calculation of latitude has no effect on the error.

41 Negative values have been taken for declination North.

42 For explanation of this formula see Appendix 4.

$= 43.3'$

[where y = current latitude]

Distance travelled East = longitude difference in minutes x sin (complement of mean latitude)

$= 43.3 \times \sin (90 - 2.5)$

$= 43.3 \times \cos 2.5$

$= 43.3$ miles

This gives the information for the position of the ship, provided that all the original data are accurate. The next task is to take account of the errors. These have been assumed to be in the order of one degree for the compass (i.e. half a degree either way) and one third of a degree for the cross-staff (i.e. ten minutes either way).[43] With these errors the following results are obtained when carried through the arithmetical calculations:

Variation could be anything from 17.7° East of North to 18.7° East of North.

Actual course could be anything from North 7.2° East to North 9.2° East.

Distance travelled could be anything from 282 miles to 324 miles (giving a % error of 14%).

Longitude difference could be anything from 35.5′ to 51.9′(giving a % error of 38%).

Distance travelled East could be anything from 35.4 miles to 51.9 miles (giving a % error of 38%).

When the calculations are carried through on the sector we must take into account not only the errors due to observations but also the errors derived from using the sector itself, both when setting up the instrument and when reading off the answer. These errors were estimated by eye and it was assumed that the accuracy could never be

[43] The errors have been taken from those mentioned above, pp.146-7.

greater than half a division.[44] These will vary depending on the calculation being made, but for the values in question the errors will be as follows:

(1) Variation calculation: declination will have an extra error of ± 0.125°; complement of latitude will have an extra error of ± 0.5°; the value read off will have an error of ± 0.125°.

(2) Distance calculation: complement of actual course will have an extra error of ± 0.5°; the value read off will have an error of ± 1.25 miles.

(3) Longitude calculation: meridional parts will have an extra error of ± 0.5° each; actual course will have an extra error of ± 0.125°; there will be a reading error of ± 0.125° (i.e. ± 7.5 minutes).

(4) Calculation of distance travelled East: longitude difference will have a further error of ± 0.125°; complement of mean latitude will have an error of ± 0.5°; there will be a reading error of ± 0.125 miles.

These extra errors, when incorporated into the calculations, yield the following results:

Variation could be anything from 17.4° East of North to 19.0° East of North.

Actual course could be anything from North 6.9° East to North 9.5° East.

Distance travelled could be anything from 281 miles to 326 miles (giving a % error of 15%).

Longitude difference could be anything from 18.9′ to 71.8′ (giving a % error of 121%).

Distance travelled East could be anything from 17.6 miles to 73.0 miles (giving a % error of 128%).

[44] I have not discussed the issue of instrument accuracy but relevant comments on the subject can be found in Chapman: 'The Design and Accuracy of some Observatory Instruments of the Seventeenth Century' in *Annals of Science*, 43 (1983), pp.457-471. See also, Chapman *Dividing the Circle* (Chichester, 1990) and the review of this book by D.J. Bryden in *Annals of Science*, 49 (1992), pp.396-7.

As can be seen immediately, the introduction of an instrumental means of calculation has a very large effect on the accuracy of the result. In fact, this particular set of data yields particularly bad results from the sector, but it does give an indication of the problems involved in trying to obtain a satisfactory knowledge of the ship's position if the observational measurements are inaccurate.

In order to obtain a representative sample of results a computer program (see Appendix 5) was set up to analyse the errors created by a wide range of data. The kind of values permitted were intended to reflect the calculation used as a previous example from Norwood's hypothetical log. Thus it was assumed that the lapse of time between calculations of distance and longitude was about three days. This resulted in a limit of 600 miles being set on the distance travelled since this is about the maximum distance which could be travelled in three days. Similarly it was assumed that the change in latitude was in the order of five degrees. In fact, this was given as a set alteration, simply to limit the time taken to run the program, although it would inevitably place bounds on the extent of the results obtainable.

Only values in the Northern Hemisphere were considered because results for the Southern Hemisphere will closely follow those obtained from the Northern Hemisphere. Latitudes up to 65°N were chosen (North of the Arctic Circle different calculations have to be employed since there are times when the sun does not rise or set) and it was assumed that the direction of travel was always to a greater latitude. Factors were introduced to limit variation to suitable levels for particular latitudes, which were later refined by comparison with charts of magnetic variation for the period of the early seventeenth century.[45] The error values employed were those listed above.

The program was designed to give details of the maximum error possible in each of the variation, the actual course of the ship, the distance travelled, the difference in longitude and distance travelled East or West for each set of values of initial latitude,

[45] These were taken from D.R. Barraclough, 'Spherical harmonic analysis of the geomagnetic field for eight epochs between 1600 and 1910' in *Geophysical Journal of the Royal Astronomical Society*, 36 (1974), pp.497-513.

the final latitude, the bearing of the sun at sunrise and the course according to the ship's compass. The results were then analysed and for each set of latitude values the lowest and highest maximum errors were noted down and percentage errors calculated from the expected values (those obtained if no errors were introduced). The results of this analysis are set out in the table below.

Initial and final latitudes (degrees)	0, 5	10, 15	20, 25	30, 35	40, 45	50, 55	60, 65
Variation error (degrees)	1.01	1.04	1.08	1.13	1.23	1.46	3.04
Course error (degrees)	2.01	2.04	2.08	2.13	2.23	2.46	4.04
Lowest maximum distance error (miles)	41.9	44.1	44.9	42.3	44.4	44.6	44.8
Error as % of expected value	14%	14%	14%	14%	14%	14%	15%
Highest maximum distance error (miles)	108	109	116	116	116	119	117
Error as % of expected value	19%	19%	19%	19%	19%	20%	20%
Lowest maximum longitude error (minutes)	16.5	22.2	25.3	21.2	30.8	39.0	64.1
Error as % of expected value	38%	28%	27%	37%	30%	32%	22%
Highest maximum longitude error (minutes)	102	106	121	132	151	189	246
Error as % of expected value	21%	21%	21%	21%	21%	22%	23%
Lowest maximum E/W distance error (miles)	16.4	21.7	23.3	17.9	22.7	23.8	29.6
Error as % of expected value	38%	28%	27%	37%	30%	32%	22%
Highest maximum E/W distance error (miles)	102	103	111	111	112	115	114
Error as % of expected value	21%	21%	21%	21%	21%	22%	23%

Comments on the results

(1) The lower limits on the distance travelled remain relatively constant.

(2) The upper limit on distance travelled and distance travelled East or West reaches a plateau, but this is largely due to expected values above 600 miles being disallowed.

(3) Longitude errors increase steadily the further North the ship is (as the meridians draw closer together).

(4) There is a dramatic increase in variation, course and longitude errors North of 60°. In the worst possible scenario it would be possible to over- or underestimate the longitude difference by two degrees! Even at the equator the worst longitude error could be nearly a degree away from the ship's actual position, and this is only after a few days' sailing. It is easy to see why navigators continued to rely on latitude sailing until their observational instruments improved and they had a means of checking their longitude.

The results from the lowest and highest latitudes were then recalculated for the additional errors of the sector. The outcome for the first of these calculations has already been given; the other three were as follows:

Highest maximum error values at initial latitude 0°, final latitude 5°N.

Variation could be anything from 17.4° East of North to 19.0° East of North (an error of 1.6°).

Actual course could be anything from North 56.9° East to North 59.5° East (an error of 2.6°).

Distance travelled could be anything from 510 miles to 633 miles (an error of 123 miles; 22% error).

Longitude difference could be anything from 330′ to 654′ (an error of 324′; 67% error).

Distance travelled East could be anything from 327 miles to 656 miles (an error of 329 miles; 67% error).

Lowest maximum error values at initial latitude 60° North, final latitude 65° North.

Variation could be anything from 15.8° East of North to 22.5° East of North (an error of 6.7°).

Actual course could be anything from North 7.0° West to North 14.7° West (an error of 7.7°).

Distance travelled could be anything from 281 miles to 333 miles (an error of 52 miles; 17% error).

Longitude difference could be anything from 63.6′ to 196.3′ (an error of 132.7′; 108% error).

Distance travelled West could be anything from 29.0 miles to 92.8 miles (an error of 63.8 miles; 113% error).

Highest maximum error values at initial latitude 60° North, final latitude 65° North.

Variation could be anything from 29.9° East of North to 32.5° East of North (an error of 2.6°).

Actual course could be anything from North 57.0° West to North 60.6° West (an error of 3.6°).

Distance travelled could be anything from 511 miles to 651 miles (an error of 140 miles; 24% error).

Longitude difference could be anything from 902′ to 1268′ (an error of 366′; 34% error).

Distance travelled West could be anything from 385 miles to 595 miles (an error of 210 miles; 42% error).

It is immediately clear from these results that sectors introduce a very significant further error over and above the errors induced by observational inaccuracies. It appears that the problems are less acute in higher latitudes than at the equator but they are still not negligible. It is still safer to use the misleading accuracy of pen and paper calculations than to resort to an instrument.

Over against these considerations must be set the advantages of the sector. It was faster to use than traditional methods of calculation and its form may have been more amenable to sailors already used to instruments for taking measurements and celestial observations. Indeed, at the time when the sector was introduced it would have been very rare for any navigator to pursue rigorous mathematical calculations.

For many pilots the sector supplied an alternative to dead reckoning rather than to mathematical methods of Mercator sailing. In such a situation it is easier to see how the application of the sector would have been an improvement on earlier techniques. Perhaps, the sector would then have been viewed in the light of an advertisement for the new mathematical techniques available: once a navigator had been convinced that the sector supplied better and safer results than dead reckoning, he might have been more prepared to accept that it would be worth while attempting to master the complexities of the mathematical calculations themselves. A sector in this instance would not be a competitor with mathematical methods but would be a means of facilitating their more widespread use; thereafter the sector would still continue to have a role in educating novice navigators and introducing them to mathematical navigation. Whether or not they then decided to continue to use the sector rather than pen and paper calculations would depend on their conception of the relative advantages of one method over the other.

It is also difficult to know to what extent navigators were aware of the inaccuracies introduced by the sector. That they did know that *observational* inaccuracies were a problem is clear, but it may not have occurred to them that instrumental methods of computation would have introduced further errors into the process of navigation. They may well have been aware that larger instruments yielded better results but there is no reason to assume that they would have known that the accuracy of the sector was significantly inferior to mathematical calculation. The unanswered question is that of how navigators weighed the pros of speed and convenience of the sector against the cons of inaccuracy (to the extent that they were aware of this problem). The answer may lie in log books of the period if details are provided of inventories and methods of navigational calculation, but this is an area which is as yet unexplored.[46] It is a very likely scenario that calculations were made on

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[46] One place to start might be India House which holds the records of the East India Company. However, very few log books survive from the first half of the century, and so the information may not be forthcoming.

slates which were then wiped clean, and that only the final result was recorded, with no indication of the method.

All in all, the introduction of the sector into navigation seems to have been something of an ambivalent event. The advantages of speed and simplicity are balanced by severe problems of inaccuracy. Yet the sector seems to have had an enduring appeal - it continued to be produced throughout the century and later versions were specifically directed towards trigonometrical problems (and presumably, therefore, towards navigators) with the Lines of Superficies, Solids, Metals, Inscribed Bodies, Equated Bodies, Quadrature and Segments being removed. The Meridian line was also eliminated as its use could now be supplied by means of the logarithmic scales which were transferred from the Gunter scale onto the sector. Later commentators on its use were often particularly keen to stress the application of the sector to navigation: for instance, Samuel Cunn[47] in his exposition on the instrument declares:

> 'From what hath been said it is abundantly evident that the Sector is sufficient for all the Calculations necessary for Navigation, particularly if you call to mind how easily and pleasantly the astronomical Problems are solved by it. I mean that of finding the Amplitude and the Azimuth. For the former no more than one Opening of the Compasses is necessary, and the latter but two.'[48]

The analysis of the sector's utility in navigation presented here is only the beginning of a much longer study which could be carried out on the instrument and its role in seventeenth-century navigation. Further avenues could be explored: the effect which different sizes of sector had on the accuracy of the results obtained in calculations could be investigated, or the bounds placed on the values of latitude could

[47] Cf. p.112. Taylor's *Mathematical Practitioners* describes Cunn (fl.1714-22) as a teacher of mathematics, a land-surveyor and quantity surveyor, and the designer of a new sector, which was made by Thomas Heath.
[48] Samuel Cunn, *A New Treatise*, p.212.

be extended to explore what happens with ships sailing on different courses. There is also the question of the increased accuracy of the sector over the traditional methods of dead reckoning, which has only been hinted at briefly in this discussion. However, I hope that the foregoing presentation has demonstrated the sorts of ways in which detailed investigation and application of an instrument to mathematical problems can reveal unexpected information which changes the way we view issues related to historical mathematical cultures. This chapter on the sector has covered some widely differing subjects and raised points concerning all kinds of areas of the mathematical sciences. Questions concerning the people involved in using particular mathematical instruments, concerning the foundations of mathematical thinking in the seventeenth century, and concerning the sometimes ambivalent role of instruments in practical mathematics have all arisen from study of a single instrument. There are surely similar studies to be done on other artefacts of the mathematical culture which will disclose otherwise unsuspected information.

CHAPTER FIVE

Studies in the Output of Elias Allen's Workshop

(3) Instruments designed by William Oughtred

Elias Allen's most fruitful relationship with a mathematician was that with William Oughtred. Oughtred was always ready to put custom Allen's way and to recommend the maker's services to his friends. He also gave Allen first option on his various instrument designs, most of which were indeed published in order to provide the maker with increased trade. Oughtred's designs included a gauging rod, the universal equinoctial ring dial, a slide rule, the circles of proportion, the horizontal instrument and the double horizontal dial (which shared many features with the horizontal instrument), all of which were made and marketed by Elias Allen. All except the gauging rod and the slide rule are known through surviving examples of Allen's work, the most popular (during Allen's lifetime) seeming to be the universal equinoctial ring dial and the double horizontal dial, although the ring dial's appeal was much more enduring.[1] The present chapter will begin by considering the circles of proportion, the horizontal instrument and the double horizontal dial, all of which are closely connected with each other - the two latter by virtue of the similarities of their features and functions and the two former through the fact that in the three surviving examples of the circles of proportion known to me (two by Allen and one by his apprentice, Robert Davenport) the horizontal instrument appears on the reverse of the instrument. The instruments are also linked in being the artefacts at the centre of the controversy between Oughtred and Delamain, which I will discuss later in the chapter. After that, I will turn to a consideration of the popular universal equinoctial ring dial.

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[1] That Allen made the gauging rod is evident from William Robinson's letter to Oughtred, complaining that he was unable to obtain one, despite his interest (quoted above, p.71). The slide rule is mentioned in Forster's preface to Oughtred's *Circles*, as '*two Rulers* of that sort, to be vsed by applying one to the other, without any compasses:' (sig.A3 verso).

The Circles of Proportion

The circles of proportion were devised by William Oughtred as a calculational tool making use of the new theory of logarithms. In Oughtred's original design the instrument consisted of two rings pivoted on the same centre, the inner one of which could be rotated over the outer, with a thread attached to the centre which could be laid across the various concentric scales inscribed on the rings. Thus it was essentially a circular slide rule. However, due to Richard Delamain's plagiarism of the design (according to Oughtred's account of the affair) and the patent which he gained from the King for it, Oughtred was forced to modify his arrangement. At the suggestion of Elias Allen, he restructured the instrument as a single circular plate engraved with a series of concentric scales, some for simple arithmetical calculations, some for trigonometric functions. The moving parts were now provided by two radial index arms, joined by a friction-tight pivot at the centre of the disc; the arm incorporating the pivot was termed the antecedent arm, the other index was termed the consequent arm. This is the form in which the known surviving examples of the instrument appear, and since it was modified at Allen's suggestion it could be viewed as being as much the instrument maker's creation as the mathematician's. Oughtred, at least, was prepared to give the credit to Allen as inventor.[2]

There are two basic ways in which the instrument can be used. Firstly, either of the arms can be used as a rule laid across the circles to read off the values of the sines and tangents of given angles or to find the logarithm of a number. Secondly, the two arms can be used together to solve problems based around the rules of proportion (see below for a more extensive discussion of this point).

For the first main function of the instrument, the most important scale is the scale of numbers which provides the value of the sines and tangents of various angles

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[2] In *The just Apologie of WIL: OVGHTRED, against the slaunderous insinuulations of RICHARD DELAMAIN...* (published in the 1633 edition of *The Circles of Proportion and the Horizontal Instrument*), Oughtred declares that this design 'may go seeke another Master: which for ought I know, will prove to be Elias Allen himselfe: for at his request only I altered a little my rules from the use of the moveable circle and the thread, to the two armes of an Index.' (sig.D recto). See Plates 28 - 30 for illustrations of the circles. This design does have an advantage for manufacture in that the scales only have to be divided once, and that the demanding skill of making one disc which will turn within a ring is not required.

directly. Thus, for instance, if the sine of 40° is required, the bevelled edge of one of the index arms is laid against 40 in the scale of sines (the outermost scale) and the sine is read off in the scale of numbers (the fourth scale from the rim of the instrument): 0.642. Oughtred recommends taking the radius as 1,000, so the sine of 40° would actually be read as 642. When seeking for sines between 6° and 90° and tangents between 6° and 45° the first and second scales respectively are used, and the scale of numbers is read from 100 to 1,000. The third ring is used for tangents between 45° and 84° and necessitates reading the scale of numbers as 1,000 to 10,000. Tangents above 84° are read from the sixth scale and the value of the scale of numbers is now 10,000 to 100,000; the seventh and eighth scales are for dealing with sines and tangents of angles less than six degrees and in this case the scale of numbers is set from 10 to 100.

A single arm is also used to find logarithms of numbers. These can be read by bringing one of the indexes to the relevant number in the scale of numbers, at which point the logarithm can be read off in the scale of equal numbers (the fifth ring from the edge). Thus the logarithm of 7 is found to be 0.845.[3] For numbers higher than 10 an integer must be added dependent on the number of digits in the number. This will be 1 if there are two digits in the number, 2 if there are three digits, 3 if there are four, etc.[4] Thus log 70 is 1.845, log 700 is 2.845, log 7000 is 3.845 and so on.

The scale of equal numbers is also used for finding square and cube roots. The logarithm of the number is obtained in the same manner as before and is then halved for a square root, or divided by three for a cube root. The index is moved to the resulting value in the scale of equal numbers, and the answer read off from the scale of numbers. Hence, if we want to find the square root of 625 we read off log 625 as 2.796 and divide this by two to get 1.398. We look for 0.398 in the scale of equal numbers,

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[3] The use of decimal notation is not anachronistic here, since Oughtred frequently used decimals in his writing.

[4] In modern terms, the addition of an integer is the same as multiplying the original number by that power of ten indicated by the integer.

which has 2.5 as its antilogarithm, and so the antilogarithm of 1.398 will be 25 (the root of 625).

As has been said, the other way of using the instrument is based on the golden rule of proportion: that four numbers A, B, C and D are in proportion if A is to B as C is to D.[5] Owing to the nature of logarithms, the scales (apart from that of equal numbers) are calibrated in such a way that if the antecedent arm is set at A and the consequent at B, movement of the antecedent to C will move the consequent to D (such that A, B, C and D are in proportion). Oughtred carefully sets out all the possible configurations of this rule, so that the operation in any situation is immediately clear and does not have to be worked out. Thus he gives four versions of the rule:

Rule 1: A is to B as C is to D

Rule 2: C is to D as A is to B

Rule 3: B is to A as D is to C

Rule 4: D is to C as B is to A

This is followed by instructions concerning which rule to apply depending on which term is unknown: if D is unknown, Rule 1 is used, if B, Rule 2, etc. For each rule the procedure is to set the antecedent arm to the first term, the consequent arm to the second, and then move the antecedent arm (which automatically moves the consequent arm, due to the friction-tight pivot) to the third term, revealing the fourth (unknown) term against the consequent in its new position.

The most fundamental application of this rule is to questions about geometric progressions, where B:A is the ratio of the progression (if this is an integer then clearly A = 1), C is a particular term, and D is the next term in the series. A geometric progression can be written out easily if the ratio and first term are given. The antecedent and consequent arms are set at A and B respectively in the scale of numbers. The antecedent arm is moved to the first term and the consequent is now at the second

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[5] In modern terms, A:B = C:D or AD = BC.

term. Then the antecedent arm is moved to the second term and the consequent reveals the third term, etc.

Multiplication, division and raising to powers are all specific forms of this rule, and so are readily carried out by application of one or other of Oughtred's rules. For multiplication, A is taken as 1, B and C as the two numbers to be multiplied and D as the number sought; Rule 1 is applied. Thus to multiply 37 by 240 the antecedent arm is set at 1, the consequent at 37 (noting that the consequent has one more digit than the antecedent). The antecedent is moved to 240 and the consequent lies at 8880 (again read as 1 digit or power of 10 higher). If, during this operation the consequent had passed beyond the beginning of the scale, then the answer would have been two digits or powers of ten greater.

For division, Rule 3 is used with A as 1, B as the divisor and D as the dividend, yielding C as the quotient. For finding a square the antecedent is set to 1, and the consequent to the number in question; when the antecedent is revolved to that number, the consequent reveals the square. Keeping this angle between the arms, if the antecedent is moved to the square, the consequent will now give the cube. This can be repeated ad infinitum, as long as one is careful to remember that each time the consequent passes the beginning of the scale, the result is increased by a power of 10.

For trigonometrical calculations, Oughtred's rules must be followed carefully. Thus, for instance, it is given that as the radius (whole sine) is to the sine of the solar longitude so the sine of the sun's greatest declination is to the sine of the sun's declination at that longitude.[6] Suppose that the longitude is unknown, i.e. B, we use Rule 2, setting the antecedent to the angle of the greatest solar declination in the scale of sines and the consequent to the angle of solar declination at the longitude. If the antecedent is moved to 90° (corresponding to the radius) the consequent arm will yield the angle of the sun's longitude.

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[6] In modern terms, sine 90° : sine (solar longitude) = sine (greatest solar declination) : sine (solar declination at that longitude).

The illustration in *The Circles of Proportion* shows a nocturnal at the centre of the circles and this appears both on the Whipple version of the instrument (Cat. no. C1) and on the Davenport circles in the Edinburgh collection.[7] It can be used to find the time at night in two ways, one by use of the Pole Star and a star in the Great Bear (Aliot, or ε Ursa Majoris), the other by use of the circle of stars in the innermost ring of the disc.

Using the first method, we set the index to the date and note where it crosses the hour circle. Then, turning so that we can view Polaris and Aliot at the same time, we hold up the instrument by the ring and adjust the index so that its position on the instrument is parallel to the line between the two stars.[8] The point at which the index crosses the hour circle now, added to the value gained from the date, will give the hour, though clearly if the result is greater than twelve, then we must subtract twelve from the answer to obtain the time.

Using the second method it is necessary to have previously found the meridian of the place and set two markers along the meridian line. Standing at the northernmost of the two markers and looking towards the south, we determine whether any of the twelve stars listed in the inner ring is over the meridian, or note which ones are to either side. The index is set to the star or point which was over the meridian marker and we note where it crosses the hour circle. As for the first method, if this value is added to the hour shown by the date, it will reveal the time of night.

Using the Horizontal Projection

As has been said, Oughtred's horizontal projection (projection of the celestial sphere onto the plane of the horizon for a specific latitude) appears on both the horizontal instrument and the double horizontal dial. The only major difference

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[7] Accession no. 1972-252.

[8] In Oughtred's description he says that the pole star should be sighted through the central hole and that the index should be rotated until it points to Aliot where it appears over the rim of the instrument (Oughtred, *Circles*, p.109). However, the size of the central hole on the surviving examples would render this operation problematic, hence I have suggested the above method as the actual practice.

between the two in appearance (apart from the obvious one that the dial is equipped with a gnomon) is that whereas the horizontal instrument has an index arm carrying the solar altitude scale, on the double horizontal dial this function is provided by a radial line engraved on the dial itself, in conjunction with use of a pair of compasses or dividers (clearly the gnomon would interfere with movement of the index arm, and so this can not be used). The main use of the instrument is for questions relating to solar astronomy and to time-keeping. As several of the functions are the same for the two instruments, these will be discussed first.

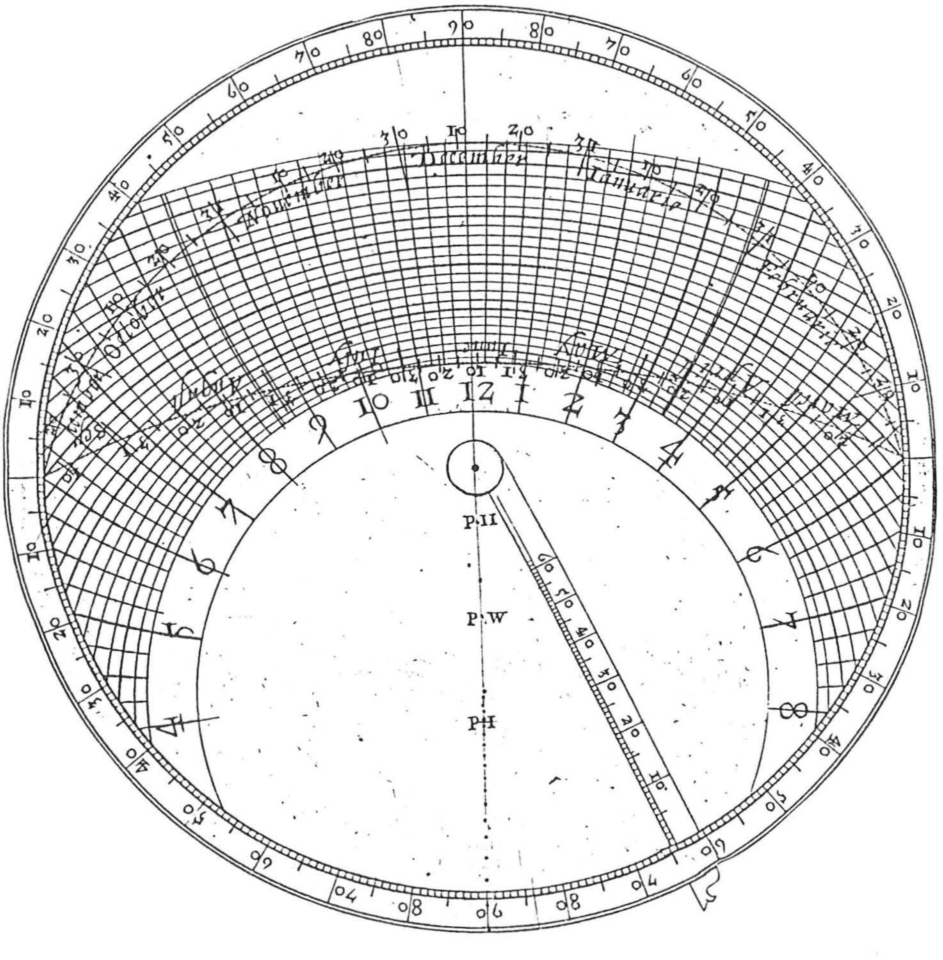

Figure 5: The horizontal instrument

<u>1) Finding the declination of the sun on a particular day</u>

The date is found on the ecliptic scale and the declination line which passes through this point gives the declination of the sun. This parallel of declination also depicts the sun's course for that day (known as the diurnal arch). Hence, for 10th October we find that the solar declination is 10°S, according to Figure 5,[9] and that the sun traces out the path shown by this parallel, from 17° S of E at sunrise to 17° S of W at sunset.

<u>2) Finding sunrise and sunset</u>

These are found from where the diurnal arch meets the degree circle (which represents the horizon). The compass bearing can be read off immediately from the intersection; the hour of sunrise or set is indicated by which hour line crosses the parallel of declination at this point. For 10th October, sunrise is at approximately 6:55am, sunset at 5:05pm. From these values the length of day and night can be calculated for that date. The hours of daylight are simply twice the time of sunset (10 hours and 10 minutes in our example) while the night lasts twice as long as the hour of sunrise (13 hours and 50 minutes here).

At this point the use of the two instruments diverges slightly and so each will be considered separately. Turning first to the double horizontal dial,[10] we continue with our pursuit of its astronomical functions.

<u>3) Finding the true place of the sun on the dial</u>

With the dial correctly positioned the hour is found by use of the normal dial around the rim of the instrument. The place where this hour line on the horizontal projection is cut by the shadow of the vertical edge of the gnomon will reveal the place

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[9] Taken from Oughtred, *Circles*, p.112. This represents an instrument calibrated for 51°30′ N - the latitude of London.
[10] See Plate 25 for an illustration of this instrument.

of the sun in the projection at that moment in time. This point will also reveal which parallel of declination is the diurnal arch for that day and so tracing the declination line to its intersection with the ecliptic will reveal the date if this is unknown (two dates will be yielded, but the assumption is that the user will at least be aware of the season of the year!).

4) Finding the azimuth of the sun

With the dial correctly aligned, the place where the shadow of the gnomon crosses the horizon ring will give the azimuth of the sun at that moment in degrees from East or West (the complement of the angle will give the azimuth of the sun from the meridian).

5) Finding the altitude of the sun at noon

The declination parallel is sought for the day in question and the intersection of this parallel with the meridian line is noted. A pair of compasses or dividers is extended from this point to the centre of the dial; keeping the compasses open to the same extent and one foot remaining at the centre, the other foot, when placed on the solar altitude scale, will give the meridian altitude of the sun.

6) Finding the hour if the altitude is known

Having measured the altitude of the sun with a suitable instrument, a pair of compasses is extended from the centre of the dial to the value of the altitude in the solar altitude scale. Keeping one foot of the compasses in the centre and the open angle of the compasses constant, the compasses are rotated until the other foot touches the diurnal arch of the sun for that day. The hour line crossing the diurnal arch at that point indicates the time.

7) Finding the degrees of the sun below the horizon at a particular time

The declination parallel for the date is found and the equivalent declination parallel on the other side of the equator is found (e.g. for 10th October the relevant parallel will be 10°N). Now we must mark where this second parallel crosses the appropriate hour line (being careful to remember that for night hours the 12 represents midnight and the hours to the left of the meridian are evening hours, those to the right are morning hours) and a pair of dividers extended from the centre to this intersection will provide the degrees of the sun below the horizon when transferred to the solar altitude scale.

8) Finding the length of twilight

This was taken to extend from sunset to the time when the sun was 18° below the horizon in the evening, and the equivalent time before sunrise in the morning. The declination parallel for the date is found and the corresponding night parallel taken. One foot of the compasses is placed at the centre of the dial and the other at 18° on the solar altitude scale. Keeping one foot at the centre, the compasses are rotated until the other foot touches the night parallel and this will give the time of the beginning of twilight in the morning, or of the end of twilight in the evening, from the hour line which crosses the night parallel at this point. (For 10th October this is found to be 7pm and 5am.)

9) Finding the declination of a wall

This is the angle between the wall and the parallel of latitude, and it is necessary to know its value in order to construct a dial on the wall. First a board with a straight edge must be taken and a line drawn on it, perpendicular to the edge. If this board is set against the wall and held parallel to the ground with the dial upon it, and the dial is then rotated until both the ordinary dial and the horizontal projection show the same time, the angle between the meridian line on the dial and the perpendicular line on the board will be the declination of the wall.

Returning to the horizontal instrument, these further uses can be made of it:

3) Finding the solar altitude, the true place of the sun on the instrument and the hour

With a projecting pin placed through the centre and the instrument held up by the suspension ring so that the edge is directed towards the sun, the shadow of the pin is cast across the degree scale and the number of degrees which the shadow cuts in the degree scale will be the altitude of the sun. If this value is noted on the index arm and the arm is moved until the altitude coincides with the diurnal arch (on the East side of the meridian for morning hours and the West side for the afternoon) the intersection of the two will be the true place of the sun in the projection at that moment, and the hour line which crosses this intersection point will indicate the time.

4) Finding the right ascension of the sun

This is the angular distance of the sun from the first point of Aries, measured eastward along the celestial equator. The date is found on the ecliptic, and the hour line which passes through this point is traced to its intersection with the equatorial line. If a ruler is laid along the line passing through this point and the pole of the world (indicated by the point "PW" on the instrument) it will cut the degree scale at the right ascension for that date. For the summer months this number is measured anticlockwise round the degree circle from the vernal equinox; for the winter months it is measured clockwise round the degree circle from the autumnal equinox and 180° is added to the result. Thus for 28th December, for instance, the hour line of 1:15 intersects with the ecliptic at this date and when a ruler is laid across from the pole of the world, through the intersection of the equinoctial line and the 1:15 hour line it meets the degree circle at

18° West of the meridian. Reckoning round from the autumnal equinox and adding 180° this gives the right ascension for that day as 288° from the first point of Aries.

5) Finding the longitude of the sun in the ecliptic

A ruler is laid across from the point marked PI for the summer months and from the point marked PII for the winter months, through the date and cuts the degree circle at the sun's place in the ecliptic. In order to calculate the longitude of the sun we must reckon from the West through the meridian to the East point and back again, allowing 30° for each sign (Aries, Taurus, Gemini, Cancer, Leo, Virgo out and Libra, Scorpio, Sagittarius, Capricorn, Aquarius, Pisces back again). Hence if we want to know the longitude of the sun on 5th November, we lay the ruler across from PII through the date and find that it cuts the degree circle at 53°, which will give the longitude as the 23rd degree of Scorpio.

6) Finding the azimuth of the sun

The index is moved to the true place of the sun in the instrument and we mark where it cuts the horizon. The number of degrees from here to the meridian line gives the azimuth from the south at that moment. By reversing the process, if the azimuth is known, the solar altitude and the hour can be found.

7) Finding the time at which the sun is in the East (summer months only)

The index arm is moved so that it crosses the horizon circle at the East and where it crosses the diurnal arch for that day gives the time (by observing which hour line passes through this point). The corresponding value on the index arm provides the solar altitude when the sun is in the East. This procedure can be extended by analogy to find the height of the sun at any time of day (simply move the index to the intersection of the diurnal arch and hour line and note the number on the index scale).

8) Finding the meridian for the place of use

Once again the true place of the sun on the instrument at the time must be found and a projecting pin must be placed through the hole in the centre of the instrument. The instrument is now held horizontally and rotated until the shadow of the pin falls on the true place of the sun (i.e. along the edge of the index arm). The meridian line of the instrument will now run North-South.

9) Finding the depth of the sun below the horizon and the times of twilight

These functions are performed in a very similar way to that used with the horizontal dial, except that the index arm replaces the compasses and solar declination scale.

10) Finding the declination of a wall

The board is constructed as for the double horizontal dial but now a thread and plummet are held up in such a way that the shadow falls across the perpendicular drawn on the board. This represents the azimuth of the sun at that time and must be drawn on the board. At the same time the altitude of the sun is found with the instrument and the azimuth is calculated (operations 3 and 6), and so the meridian can be drawn onto the board (the intersection of the azimuth line and the perpendicular having supplied the centre of the circle). Given these construction lines the angle of declination can be measured.

Further applications of the horizontal instrument in dialling are detailed at great length in Oughtred's book but are too numerous to enter into at this point. Suffice it to say that the amount of the text dedicated to discussions of dialling is typical of the interest in the subject during the seventeenth century.

Instruments as focuses for dispute: the Oughtred-Delamain controversy

Elias Allen's involvement in the argument which flared up between William Oughtred and his former pupil Richard Delamain has already been considered at length in a previous chapter; the questions related to the use of instruments as didactic aids have also been raised in discussion of the mathematical community. However, one area which has not yet been considered is the issue of plagiarism and of intellectual property as it was perceived by members of the seventeenth-century mathematical culture. Since this seems to have been the major cause of the dispute between Oughtred and Delamain, I intend to consider it now, and the way in which instruments could become bones of contention, fought over by the rivals for their design, the sole beneficiaries of the quarrels normally being the instrument makers, who could make the most of the increased publicity for the instruments.

As has been said, it was during the preparation of William Forster's translation of Oughtred's manuscripts on the circles of proportion and the horizontal projection that both of Delamain's books appeared, giving details of the two instruments and claiming a monopoly on the design of the circles of proportion. The account of the 'mathematical ring' or 'grammelogia' (as Delamain termed the circles) made it quite clear that this was the author's own invention, which he had laboured to conceive ever since the publication of Gunter's design for a logarithmic rule and which he had finally invented in February 1629 (OS), some eleven months before the appearance of the book. However, in the case of the horizontal quadrant, Delamain was at pains to point out (in the *first* edition of the treatise on its use) that, although he had taken the design straight from the description and diagram in *De Sectore et Radio*, he had since become aware that the originator of this particular stereographic projection was none other than his mentor, Oughtred. In so writing, Delamain felt that he had discharged his necessary duties to the older mathematician.

However, Oughtred was not prepared to let matters rest at this point and he allowed Forster to raise the complaint in his preface, that another author (unnamed, but clearly Delamain is inferred) had precluded Oughtred's publication by producing his own and had thus drawn attention away from the true designer of the instruments. The grievance which Oughtred seems to be raising is not that Delamain stole his designs (though his comments on being forced to produce a different version of the circles of proportion in order to circumvent the monopoly granted to Delamain betray a considerable amount of bitterness)[11] but that he did not grant Oughtred first right of presentation of his own instrument to the general public. The reference seems to have been more clearly linked to the horizontal projection; indeed the fact that Delamain's *Grammelogia* had been in print for over a year without Oughtred having complained about plagiarism suggests that he felt that he had less of a case for arguing that Delamain had done him a disservice over this instrument as well.

What may have been intended on the part of Oughtred to have been simply a sharp reminder to Delamain (who had apparently been well aware that Forster was in the process of preparing a translation of Oughtred's manuscript on the horizontal projection) of the privileges of the designer to first publication was taken by the latter as a vehement attack against his integrity and in the second edition of the *Grammelogia* he presented his opinion of Oughtred's and Forster's actions in no uncertain terms. Of course, it is highly probable that Delamain's feelings that he had been harshly treated were fuelled by the numerous unjust rumours which (according to his account) were circulating with respect to himself. He now refused to acknowledge Oughtred's prior claim to the instruments at all and took the position that the invention had been

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[11] A monopoly was the means by which a patent of invention was granted at that time. The system of patents in the seventeenth century was very different from that currently employed as Christine MacLeod explains in *Inventing the Industrial Revolution: The English patent system, 1660-1800* (Cambridge, 1988): 'Over the last two centuries, the word 'patent' has come to have a precise and technical meaning: a grant of monopoly powers over the commercial exploitation of an invention for a limited period. This conceals its origin. William Blackstone, the jurist, writing in 1768, was still familiar with the broader understanding of the word, in which a patent for invention was by one type of royal 'letters patent':

"'The king's grants are also a matter of public record... These grants, whether of lands, honours, liberties, franchises or ought besides are contained in charters, or letters patent that is, open letters, litterae patentae, so called because they are not sealed up, but exposed to open view, with the great seal pending at the bottom: and are usually directed by the king to all his subjects at large."

'Thus letters patent were simply the document by which special privileges were conferred. Grants for invention were a relatively late arrival on the administrative stage, and were regarded as just one more instrument of royal policy. This meant that they were recorded indiscriminately on the patent rolls, among grants of land, office, honours, and other perquisites in the royal gift.' (p.10) completely his own, his only aid having been Gunter's writing on both the horizontal projection and the inscription of logarithmic scales on a ruler.

Oughtred's reply followed rapidly, his *Just apologie* appearing in print hard on the heels of the *Grammelogia*. This time the reproof was not simply restricted to the untimely publication of the horizontal quadrant but to a wider attack on Delamain's underhand methods, and it is here that Oughtred claims explicitly that Delamain had stolen the design for the horizontal instrument directly from Elias Allen, at the same time as strongly implying that the circles of proportion had been plagiarised as well. Perhaps Oughtred was also still smarting from Gunter's lack of acknowledgement in *De Sectore et Radio* of his authorship of the horizontal projection. Neither Oughtred nor Delamain appears in a very good light at the close of the dispute and there is no doubt that their relationship came to an acrimonious end.

However, this heated dispute is valuable to the historian in revealing some of the ways in which the mathematical culture of the period viewed intellectual property. It becomes very clear from the accounts of both Oughtred and Delamain that it was fairly common practice for manuscripts about new methods and new types of instrument to circulate throughout the community without constraint, and that discussion about designs and theories was wide-ranging and happened under all sorts of circumstances. The impression gained is of an open and trusting community where knowledge is freely shared and ideas are pooled. Gunter's manuscript concerning the sector enjoyed a wide distribution before it appeared in print and Oughtred appears to have had no quarrel with Allen for allowing Delamain to read about the details of the horizontal projection. Nor is Delamain vilified for having had a horizontal quadrant prepared for him by somebody in the workshop: it appears that as long as due acknowledgement of the originator of an instrumental design or mathematical method was given then justice was felt to have been done.

Nevertheless there *were* unwritten rules, and it is obvious that one of these concerned the priority of putting things into print. Oughtred was not the only mathematician to feel the sting of seeing his own work on the booksellers' shelves before he had time to publish it. Edward Wright, who prepared the first explanation of how to apply the Mercator chart to navigation, was galled in having his publication pre-empted by Iodocus Hondius (a well-known cartographer) and made certain to stress in his preface that he had had assistance 'neither of *Mercator*, nor any man els' in deducing the necessary calculations for laying out the Mercator chart.[12] He also brought the reader's attention to a letter which Henry Briggs had received from Hondius, in which the latter apparently expressed his contrition: 'I haue written to *M. Wright* in excuse of my self, I am verie sorie that he is angrie with me for that cause.'[13] Wright was leaving no ambiguity as to his views on being precluded by another writer.

A similar story was told by William Pratt, the inventor of the 'arithmetical jewel', an instrument for facilitating calculations and thus removing the need for pen and paper. Details of this invention first appeared in 1617 in John Harper's *The Iewel of Arithmetick*, although Harper nowhere claimed the instrument as his own design. Shortly thereafter Pratt's book appeared, bound with a version of the same instrument: *The Arithmeticall Jewell: or The vse of a small Table; whereby is speedily wrought, as well all Arithmeticall workes in whole Numbers, as all fractionall operations, without fraction or reduction*. Pratt's grievance concerning the plagiarism is cleverly expressed in the preface of the book:

> 'I was likely of late (courteous Reader) to haue lost mine onely Sonne, on whose birth and breed I had bestowed much care and cost; but on inquirie, found him at last by strangers fire; and fearing that [it] might in time bee reputed base, or at least wise marred by ill education, I haue heere taken it home, intreating your kinde testimony that I father it as mine owne, with full assurance (if not blinded with ouerweening affection) that he will heereafter proue no lesse pleasing to me, then profitable to his natiue Country. Yet must I needs acknowledge the modesty of my Compater or Competitor, to be farre exceeding

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[12] Edward Wright, *Certaine Errors in Navigation*, sig.¶¶4 verso.
[13] Ibid., sig.¶¶¶ recto.

hirs, who in presence of King *Salomon* laid claime to her neighbours childe, for that (as I haue heard by those who haue seen and read his harmeless workes) he hath not once exprest the Author of this Instrument; whether to bee himselfe, or some other man: And had hee not giuen way to his friend, *I.C.* idely to iniure mee in his harsh and hobling rimes, I could well haue borne the rest. I must likewise confesse, that I freely acquainted him with all the ordinary vse & operations of this my Table; but of the maine and principall (as to worke fractionall operations in whole numbers) I gaue him no light at all, as may well appeare by his laborious Rules in that kinde. But if by divulging his booke before me; he hath either disparaged the excellent vse of this my Table, or (which I little feare) preuented my hopes and expectation, and with *Iaakob* bereft *Esau* both of birthright and bleßing: The bleßing of *Isachar* goe also with him, whereunto I leaue him.'[14]

Although Pratt did not name his competitor, it seems clear from the similarity of the instruments described that Harper was the person in question. The inference is that Pratt was quite happy for knowledge of his instrument to be accessible through word of mouth to the rest of the mathematical community but that he was not prepared for someone else to pre-empt his *written* information on the subject. Nor was he prepared for someone else to gain the credit for the invention of his instrument. We also see the anxiety of Pratt that a poor representation of his work (which he believed Harper to have produced) would lead to the Jewel not being given the notice by other users of mathematics which he felt that it deserved.[15]

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[14] Pratt, *The Arithmeticall Jewell...* (London, 1617), sig.A4 recto-verso. The friend of Harper referred to by Pratt had written a commendatory letter to Harper which was included in the introduction and which implied that the invention was Harper's; the latter made no attempt to refute his friend's implication. The 'blessing of Issachar' refers to Jacob's blessing of his sons in Genesis, chapter 49 (verses 14-15): 'Issachar is a rawboned donkey lying down between two saddlebags. When he sees how good is his resting place and how pleasant is his land, he will bend his shoulder to the burden and submit to forced labour.'

[15] This debate is also discussed in Turner, 'Mathematical Instruments', pp.83-84 and in D.J. Bryden, 'The arithmeticall jewell or Jewell of Arithmeticke' in *Quarto - Abbot Hall Art gallery Quarterly Bulletin* 23 (1985), pp.7-14.

These three controversies all bear witness to the way in which the very openness of the mathematical community and the availability of ideas and manuscripts to a wide range of people paved the way for such disputes over ownership and bitter accusations of plagiarism. It is easy to ask why inventors did not publish their work as soon as possible and so circumvent the possibility of priority arguments. However, the assumption here is that mathematicians saw it as wise to divulge their knowledge to the whole of the reading public. In Oughtred's case it is ironic that it was precisely because he was unhappy about making his instruments available to all and sundry (in case they were used as a substitute for a solid grounding in theoretical mathematics)[16] that he was slow to publish his designs and thus found himself in the situation of seeing his instruments in print under another man's name.

These several disputes are also additional evidence for the importance of mathematical instruments to all members of the community, both those who were more deeply involved in practical mathematics and those whose interests lay more naturally with mathematical theory and the realm of proofs and demonstrations. If the instruments had been of less importance to their designers then the urge to lay first claim to invention would have been less acute.

Given Oughtred's avowed aversion to allowing the appearance of instruments at too early a stage in mathematical tuition it is interesting to note the manifest usefulness of the horizontal projection as a didactic aid. Such an instrument is ideal for introducing the concepts of astronomy to a novice and for demonstrating the way in which the sun's path across the heavens varies with the season. It is the epitome of the extremely visual and physical approach to mathematics which was so prevalent among the mathematical practitioners of the period. It is also interesting that the other main function of the horizontal projection is to reduce the intricacies of the spherical

[16] This kind of approach to mathematics with a belief in the primacy of mathematical theory was also found in the works of Thomas and William Bedwell (see Johnston, 'Mathematical Practitioners', pp.320-330).

trigonometry required for astronomical calculations to an instrumental form where the answers can readily be found by application of scales to lines traced on the instrument, or even just by searching for the intersection between particular curves, representing the circles of the celestial sphere. Thus it provides a means for people unlearned in mathematical astronomy to find the answers to questions relating to the movement of the sun, so undermining Oughtred's position that only those who had a full knowledge of the theoretical methods for obtaining the same results should be permitted to make use of the shortcuts which instruments offered.

The circles of proportion is just such another aid for calculation, involving no prior knowledge of the nature of logarithms (although the manner of its construction can be used to demonstrate their properties). It is a calculational facilitator which obviates the need for searching through log tables. Perhaps it is noteworthy that the eventual design preserved the link with that other popular calculational tool, the sector, through the inclusion of two radial index arms which are reminiscent of the arms of the sector and of the use of similar triangles for dealing with questions of mathematical proportion. It is also slightly more clearly linked in this form to the logarithmic rule which was used in conjunction with a pair of compasses. Slide rules were very slow to make an impact in the community perhaps because the mere conjunction of numerals did not convey a great deal to people who expected their mathematical instruments to be more evidently based on geometric methods, as they had been in the past. Certainly, there appears to have been less demand for Delamain's 'grammelogia' which was a true circular slide rule: to my knowledge there are no surviving examples of this instrument, and yet there are at least three specimens of the Allen design (this is presumably the correct appellation for it, since Oughtred would not own it as his).

The Universal Equinoctial Ring Dial

This simple time-telling device was developed by Oughtred from the astronomical ring, which was first described by Gemma Frisius and others early in the sixteenth century and which was, in turn, 'a simplified armillary sphere consisting of three or four rings, with pin-hole sights'.[17] Gemma's instrument consisted of three rings, but Oughtred modified the instrument, replacing the innermost ring with a bridge containing a central slot, in which a sliding pin-hole sight could move along a date scale. The dial was first described in print in Oughtred's 1652 account of the double horizontal dial - *The Description and use of the Double Horizontall Dyall...whereunto is added, The Description of the generall Horologicall Ring.* His description of the instrument is brief and clear:

> 'It consisteth of two brazen circles: a Diameter, and a little Ring to hang it by.
>
> 'The two circles are so made, that though they are to be set at right angles, when you use the Instrument: yet for more convenient carrying, they may be one folded into the other.'[18]

The inner (equinoctial) ring carried a scale of hours with the two sixes corresponding to the ends of the bridge when the instrument was closed. One quarter of the outer (meridian) ring was divided into ninety degrees, and, in conjunction with a tooth on the sliding shackle of the hanging ring, this scale set the instrument for the observer's latitude. The bridge was engraved with a date scale running from solstice to solstice, with six months marked on each side of the central slot. Oughtred's account mentions no further features of the instrument, but it was very common to find all the remaining sections of the meridian and equinoctial rings engraved with the names of towns or cities and their respective latitudes.

The use of the universal equinoctial ring dial is very straightforward: first the tooth on the shackle of the hanging ring is moved to the observer's latitude in the degree scale and the pin-hole of the central slider is moved into line with the date. Then

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[17] A.V. Simcock 'An Equinoctial Ring Dial by Ralph Greatorex', p.204. Gemma's design dates from 1534 and is described in P. Beausardus et al., *Annuli astronomici ...*, (Paris, 1558) and G. Frizon, *Les Principes d'astronomie & cosmographie ... Plus, est adiousté l'usage de l'anneau astronomic...*, (transl. C. De Boissiere, Paris, 1556). A lengthier discussion of different types of universal dials and astronomical rings is given in the Simcock paper. An early history of the universal equinoctial ring dial can also be found in Maddison, *Medieval Scientific Instruments*, pp.43-46.

[18] Oughtred, *The Description and use of the Double Horizontall Dyall...* (London, 1652), sig.X8 recto.

the instrument is opened out until the meridian and equinoctial rings are at right angles, and the bridge is adjusted so that the pin-hole is turned towards the time as roughly estimated by the user. Finally the dial is held up and rotated until the beams of the sun pass through the pin-hole and fall on the hour scale, indicating the time (at which point the meridian ring hangs in a north-south plane).

Although the design was not published until 1652 and the oldest surviving dated dial was made in the same year by Anthony Thompson, the existence of at least eight dials by Allen suggests that the instrument's design came from a considerably earlier period in Oughtred's life. The Allen dials are also supported by additional evidence from a letter and two pictures: the letter is that of John Beale to Samuel Hartlib to which reference has already been made;[19] the pictures are Allen's own portrait and van Dyck's portrait of the Earl and Countess of Arundel,[20] in which Lady Howard is depicted holding a universal equinoctial ring dial.[21] Allen's portrait was painted circa 1640 and the reference in the letter is to a time in the late 1630s; the best evidence, however, is provided by the van Dyck, which was painted in 1639 to mark the Earl's plans for an expedition to Madagascar (which plans were never realised). Thus the design must have been in existence by the late 1630s and Oughtred may well have developed it before that time - most of his other instrument designs came from early in his career.[22]

The other main point of note with regard to the Allen dials concerns the original ownership of the instruments, which is often hinted at by the cities whose names are

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[19] See Chapter Two, p.71.
[20] Plate 2.
[21] It is interesting that it is Lady Howard and not her husband who holds the dial and also a pair of dividers. What is also interesting is that the ring dial could not have been a standard iconographic element since it had only very recently been developed in this form. People who saw the portrait could not be expected to know exactly what the instrument was; it would probably only have been identified as some kind of mathematical tool.
[22] I have treated the question of the date at greater length in 'Dating Oughtred's Design for the Equinoctial Ring Dial' in *Bulletin of the Scientific Instrument Society*, 44 (March 1995), p.25.

engraved on the dials. These generally give some indication of the people by whom they were commissioned and the sort of market for which the dials were made. It was common for the latitude immediately to the right of the latitude quadrant to indicate either the place of manufacture or that where it was most likely to be used.[23] In fact, none of the Allen dials in museum holdings have London as the base latitude, which suggests that they were all commissioned pieces rather than ones which Allen had made without a purchaser in mind. Two of them have a base latitude of Nottingham (Cat. nos. U2 and U3) and carry very similar lists of towns which seem to point towards an owner (perhaps the same one for both instruments) from the Northeast of England with reason to travel to the coastal cities of the Low Countries, France and the Iberian Peninsula. Cat. no. U1 has a base latitude of Amsterdam and the place names range from Dublin in the West to Constantinople in the East with all the major cities of Europe mentioned: however, the presence of seven British cities suggests that the owner was an Englishman. Perhaps the most interesting of the four catalogued dials is Cat. no. U4. This has a base latitude in Lisbon and the vast majority of the cities named are towns around the coastal lands of the Mediterranean - Malaga, Majorca, Barcelona, Marseilles, Leghorn, Rome, Naples, Venice, Constantinople, Aleppo, Alexandria, Smyrna, Syracuse, Genoa and Gibraltar. The implication is that the dial was owned by a Portuguese merchant, most of whose trade was carried out within the sphere of the Mediterranean countries.

Once again, we are left with a sense of an instrument which was mainly marketed for the gentry and aristocracy, and the wealthier members of the merchant class. Universal dials were of most use to those who had occasion to travel widely and it was only the wealthy or those concerned with overseas trade who could afford to do so. This would seem to provide further evidence that Elias Allen concentrated on cultivating a sales area within the upper classes.

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[23] The concept of base latitude appears in Simcock, 'An Equinoctial Ring Dial', p.210.

In this chapter the investigation of the instruments has been carried out largely through the medium of the texts relating to them. This is a more traditional approach to research than the methods used with the Gunter quadrant and Gunter sector, yet it is still the artefacts which are at the centre of the study - these books would not exist if the instruments did not. Study of the texts in conjunction with the instruments they describe provides yet another way of researching the material culture of mathematics.

The three chapters concerning Elias Allen's work have given some indication of the ways in which instruments can be brought into research in the history of mathematics. Whilst the questions raised and the answers obtained depend greatly on the particular instrument under scrutiny, all of them have relevance to the history of mathematics in general. Instruments are not just of antiquarian interest, and there is a need for the relevance of this research approach to be recognised within the wider field of the history of mathematics and science.

As I have said, numerous different issues have been brought up by the study of different kinds of mathematical instruments. The long discussion on the sector has pointed up some of the problems of accuracy and the extent to which this was balanced against pragmatic questions of ease and speed of use. The sector is also a witness to the way in which instruments may have been used to initiate the mathematically illiterate into the apparently occult world of numbers, geometry, trigonometry and all the other subjects which constituted the arena of seventeenth century mathematics. Instruments like the sector may have had a secondary role in advertising the efficacy of mathematical methods and leading people who might have been unwilling or unable to make the leap from rules of thumb to complicated arithmetical calculations to accept that mathematics might be more approachable than they had at first feared and that there was a benefit in acquiring knowledge of the underlying theory and its applications.

The importance of instruments as didactic aids has long been recognised as a factor of the way in which mathematics was taught over the centuries, and the Gunter quadrant and Oughtred's horizontal instrument in particular are good examples of this application of instruments, although the vast majority of the instruments mentioned in this thesis *could* be used in some way or other to demonstrate mathematical principles, since by their very nature they embody these principles. The history of the disputed origin of the horizontal instrument also supplies evidence in the perennial debate over the correct methods of introducing mathematical concepts in the course of education. It is still an issue today whether material aids should be involved before the theoretical demonstrations and proofs have been discussed or whether the latter should come first and the instruments then be produced as illustrations of the applications of the theorems which have just been successfully proved. The discussion was particularly heated during the seventeenth century, and this is reflected in the amount of time which has been devoted by various modern historians to researching the debate.[24]

Finally, all the instruments can be used to investigate the markets for mathematics and to build up a fuller picture of the range of people involved in the mathematical culture of the seventeenth century. Many of the instruments which have been discussed point towards the growth of interest from amateurs with money to spend and perhaps more of a concern for an apparent show of knowledgeability in the field of mathematics than a clear competence in manipulating mathematical concepts. However, there are other instruments which were obviously intended for the market of the practical mathematicians, the people who applied mathematical formulae in the course of the everyday pursuit of their professions. The sheer quantity of types and numbers of instruments bears witness to the fact that mathematics was a growth industry in seventeenth-century England, and that instruments were right at the heart of the processes of learning and doing mathematics. It is this importance of instrumental techniques alongside the strong emphasis on visual demonstration and on geometry which will form the core of my concluding discussion, following the catalogue provided in the next chapter.

Plate 2 (on page following): Anthony van Dyck, *Thomas Howard, Earl of Arundel, and his Countess ('The Madagascar Portrait')*. 1639

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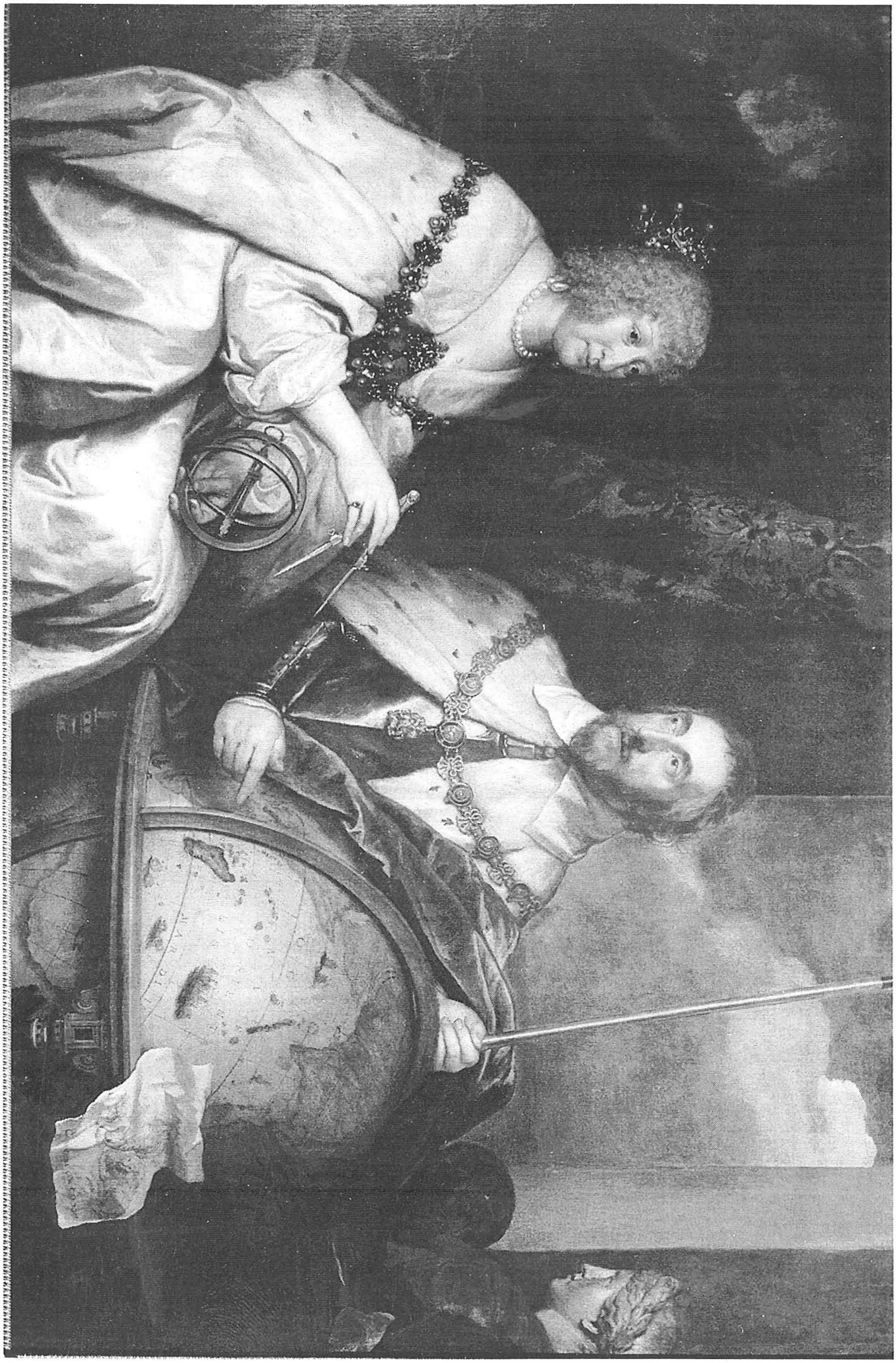
[24] See, for instance, A.J. Turner, 'Mathematical Instruments'; D.J. Bryden, 'A Didactic Introduction to Arithmetic: Sir Christopher Cottrell's Instrument of Arithmeticke' in *History of Education*, 2 (1973), pp.5-18.



Chapter Six

Catalogue of Allen's Instruments

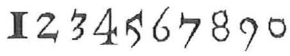
This catalogue of Elias Allen's instruments includes all those signed instruments which are held in museum collections in Britain and a few others from private collections. Several unsigned pieces have been included by virtue of the similarity of their engraving to that of the signed instruments. Appended to the main catalogue is a section detailing those instruments which may possibly have links to Allen, and any instruments which are known to be his handiwork, but which I have been unable to view at first hand. Those instruments which have been discussed at length in earlier chapters need no introduction here; any other unusual instruments are accompanied by a brief description of their use. In all cases, unusual features have been noted and discussed after the main description. A brief bibliography is given where possible.

Abbreviations

BM - British Museum, London
MHS - Museum of the History of Science, Oxford
NMM - National Maritime Museum, London
RMS - Royal Museum of Scotland, Edinburgh
SM - Science Museum, London
Whipple - Whipple Museum of the History of Science, Cambridge

Notes on Allen's style

There are various traits in Elias Allen's style of engraving which are particularly useful in ascribing unsigned instruments. The most pronounced are in the numerals where certain characteristics almost always appear. There are strong serifs on the 2, 3, 5 and 7; the 1 always has horizontal bars at top and bottom; the 4 has an elongated upright; the 6 and the 9 rarely have the circle completely closed. Perhaps the most typical figures apart from the 4 are the 8 and the 0. Allen appears to have had difficulties drawing perfect circles for numerals, and both of these figures often appear to have small horns. The ten numerals are shown below to give an indication of Allen's style and to demonstrate the basis on which the unsigned instruments were ascribed.



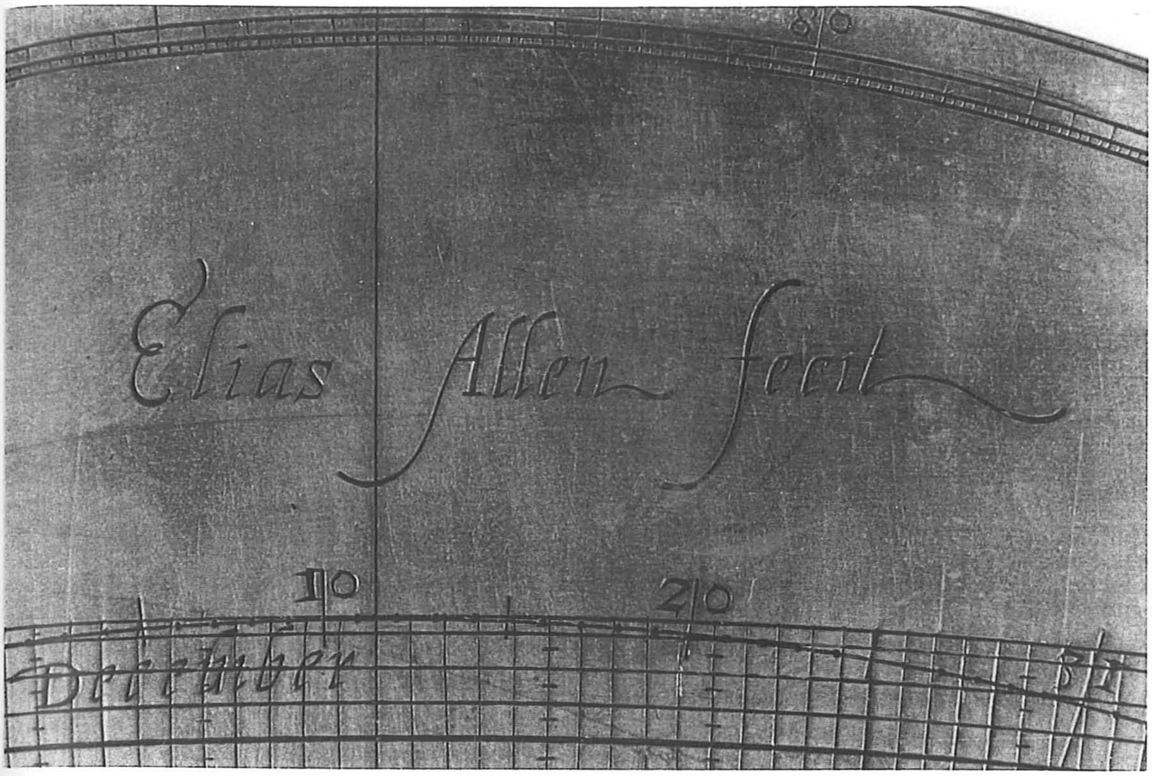


Plate 3: Italic-style signature from Cat. no. C1

Plate 4: Roman-style signature from Cat. no. Q3

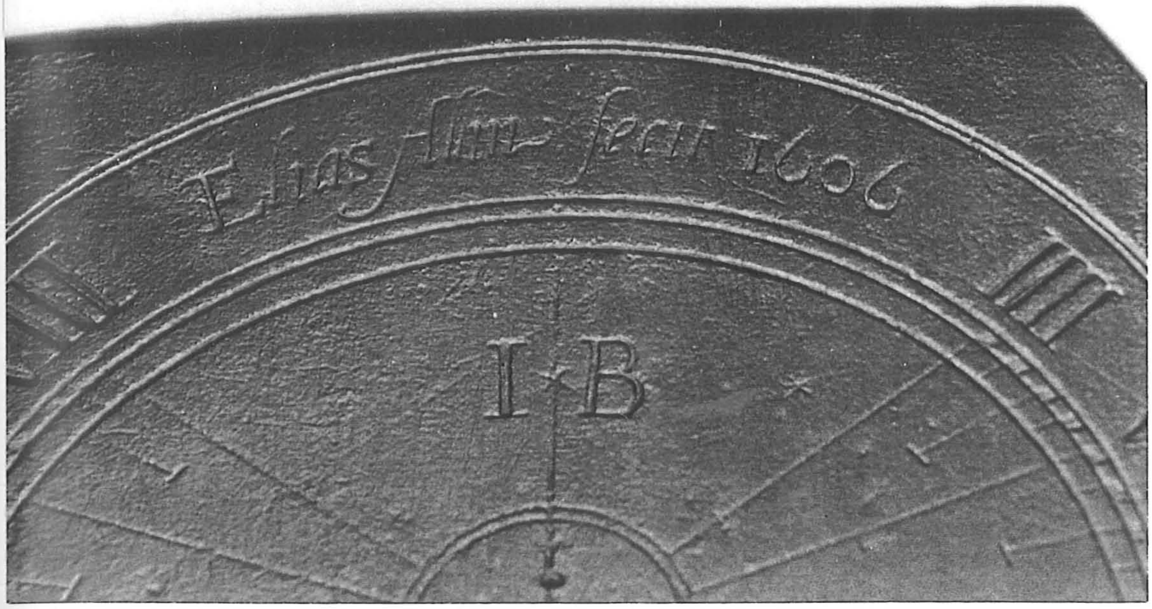


Plate 5: Signature from Cat. no. X1

Plate 6: Signature from Cat. no. X3

Sectors

The engraving of Cat. nos. S1, S2 and S3 follows the illustration (facing page 1) in the 1623/4 edition of Gunter's *De Sectore et Radio* (also by Allen), save that none of the lines are named.

S 1.

Location: Whipple (Accession no. 2819)
Plate 7

Brass Gunter sector. Two arms (each 229.5 x 29 x 6mm) pivoted on flat circular hinge (diameter 31mm), comprising three elements for one arm and two for the other. Pivoted support strut at non-hinge end (110.5 x 30 x 3mm). Recesses in both arms to accommodate closed and open positions of strut.

Recto

Radial lines, repeated on each arm, reading from inner edge:

1) Line of Inscribed Bodies in the same Sphere, marked across both arms, "D, S, I, C, O, T" [dodecahedron, sphere, icosahedron, cube, octahedron, tetrahedron].
2) Line of Lines, divided [0] to 10; numbered by 1; subdivided to 0.5, to 0.1 and to 0.05.
3) Line of Metals, marked "☉, ☿, ♄, ☽, ♀, ♂, ♃" [gold, mercury, lead, silver, copper, iron, tin]; integrated with Line of Equated Bodies, marked "D, I, C, S, O, T" [dodecahedron, icosahedron, cube, sphere, octahedron, tetrahedron].
4) Line of Solids, marked 1, 1[0], 1[00], 2[00], 3[00]...10[00]. 1 -1[0] divided to 5; subdivided to 1. 1[0] - [50] divided to 10; subdivided to 1. [50] - 1[00] divided to 10; subdivided to 2. 1[00] - 10[00] divided to 50; subdivided to 10 and to 5.

Along outer edge:

1) Meridian Line, forming continuous scale when instrument is fully open; divided 0 - 70°15′ on one arm, 70°15′ - 86°30′ on other arm; numbered by 10°; subdivided to 5°, to 1° and to 15′.

Verso

Radial lines, repeated on each arm, reading from inner edge:

1) Line of Quadrature, marked across both arms "10, 9, 8, 7, S, 6, 5, 90, Q".
2) Line of Sines, divided [0] - 90°; numbered by 10°; subdivided to 5°. 1° - 85° subdivided to 1°, and to 30′.

3) Line of Segments, divided 5 - 10; numbered by 1; subdivided to 0.1.

4) Line of Superficies, marked 1, 1[0], 2[0], 3[0]...10[0]. 1 - 1[0] divided to 5; subdivided to 1 and to 0.5. 1[0] - 10[0] divided to 5; subdivided to 1 and to 0.5.

Along outer edge:

1) Line of Tangents, forming continuous scale when instrument is fully open, divided [0] - 45° on one arm, 45° - 63° on other arm; numbered by 10°; subdivided to 5°, to 1° and to 15′.

2) Line of Secants (only with second half of Line of Tangents), divided [0] - 58°; numbered by 10°. 10° - 58° subdivided to 1°.

Outer Rim

1) Line of Inches, divided in both directions ([0] - 9 from non-hinge end and [9] - 18 from hinge); numbered by 1; subdivided to 0.5, to 0.1 and to 0.05.

2) Line of Lesser Tangents, divided [0] -75°; numbered by 10°; subdivided to 5° and to 1°.

Signature on inner rim of one arm: 'Elias Allen fecit' with straight capitals.[1]

[1] For a comparison of the two main styles of Allen's signature see plates 3 and 4.

S 2.

Location: SM (Inventory no. 1976-638)

Brass Gunter sector. Two arms (each 178 x 21.5 x 4mm) pivoted on flat circular hinge (diameter 31mm), comprising three elements for one arm and two for the other. Pivoted support strut at non-hinge end (86 x 21.5 x 1mm). Recesses in both arms to accommodate closed and open positions of strut.

Recto

Radial lines, repeated on each arm, reading from inner edge:

1) Line of Inscribed Bodies in the same Sphere, marked across both arms, "D, S, I, C, O, T".
2) Line of Lines, divided [0] to 10; numbered by 1; subdivided to 0.5, to 0.1 and to 0.05.
3) Line of Metals, marked " ☉, ☿, ♄, ☽, ♀, ♂, ♃"; integrated with Line of Equated Bodies, marked "D, I, C, S, O, T".
4) Line of Solids, marked 1, 1[0], 1[00], 2[00], 3[00]...10[00]. 1 -1[0] divided to 5; subdivided to 1. 1[0] - [50] divided to 10; subdivided to 5 and to 1. [50] - 1[00] divided to 10; subdivided to 2. 1[00] - 10[00] divided to 50; subdivided to 10 and to 5.

Along outer edge:

1) Meridian Line, forming continuous scale when instrument is fully open; divided 0 - 70°15′ on one arm, 70°15′ - 86°30′ on other arm; numbered by 10°; subdivided to 5°, to 1° and to 15′.

Verso

Radial lines, repeated on each arm, reading from inner edge:

1) Line of Quadrature, marked across both arms "10, 9, 8, 7, S, 6, 5, 90, Q" [bottom half of "7" not engraved].
2) Line of Sines, divided [0] - 90°; numbered by 10°; subdivided to 5°. 1° - 85° subdivided to 1°. 1° - 80° subdivided to 30′.
3) Line of Segments divided 5 - 10, numbered by 1, subdivided to 0.5, 0.1.
4) Line of Superficies, marked 1[0], 2[0], 3[0]...10[0]. [1] - 1[0] divided to 5; subdivided to 1 and to 0.5. 1[0] - 10[0] divided to 5; subdivided to 1 and to 0.5.

Along outer edge:

1) Line of Tangents, forming continuous scale when instrument is fully open; divided [0] - 45° on one arm, 45° - 63° on other arm; numbered by 10°; subdivided to 5°, to 1° and to 15′.

2) Line of Secants (only with second half of Line of Tangents), divided [0] - 56°; numbered by 10°. 10° - 56° subdivided to 5° and to 1°.

Outer Rim

1) Line of Inches, divided in both directions ([0] - 7 from non-hinge end and 7 - 14 from hinge); numbered by 1. 0 - 1 subdivided to fifths, to tenths, and to fiftieths. 1 - 2 subdivided to fifths and to twenty-fifths. 2 - 3 subdivided to twelfths and to twenty-fourths. 3 - 4 subdivided to elevenths and to twenty-seconds. 4 - 5 subdivided to tenths and to twentieths. 5 - 6 subdivided to ninths and to eighteenths. 6 - 7 subdivided to eighths and to sixteenths.
2) Line of Lesser Tangents, divided [0] -75°; numbered by 10°; subdivided to 5° and to 1°.

Signature on recto of hinge: "Elias Allen fecit 1623" with straight capitals.

Recto badly corroded.

S 3.

Location: RMS (Accession no. 1984-184)

Plates 8, 9

Brass Gunter sector. Two arms (each 231 x 26 x 6mm) pivoted on flat circular hinge (diameter 35mm), comprising three elements for one arm and two for the other. Pivoted support strut at non-hinge end (107 x 27 x 2.5mm). Recesses in both arms to accommodate closed and open positions of strut.

Recto

Radial lines, repeated on each arm, reading from inner edge:

1) Line of Inscribed Bodies in the same Sphere, marked across both arms, "D, S, I, C, O, T".
2) Line of Lines, divided [0] to 10; numbered by 1; subdivided to 0.5, to 0.1 and to 0.05.
3) Line of Metals, marked " ☉, ☿, ♄, ☽, ♀, ♂, ♃"; integrated with Line of Equated Bodies, marked "D, I, C, S, O, T".
4) Line of Solids, marked 1, 1[0], 1[00], 2[00], 3[00]...10[00]. 1 -1[0] divided to 5; subdivided to 1. 1[0] - [50] divided to 10; subdivided to 5 and to 1. [50] - 1[00] divided to 10; subdivided to 2. 1[00] - 10[00] divided to 50; subdivided to 10 and to 5.

Along outer edge:

1) Meridian Line, forming continuous scale when instrument is fully open; divided 0 - 70°15′ on one arm, 70°15′ - 86°30′ on other arm; numbered by 10°; subdivided to 5°, to 1° and to 15′.

Verso

Radial lines, repeated on each arm, reading from inner edge:

1) Line of Quadrature, marked across both arms "10, 9, 8, 7, S, 6, 5, 90, Q".
2) Line of Sines, divided [0] - 90°; numbered by 10°; subdivided to 5°. [0] - 85° subdivided to 1°. [0] - 80° subdivided to 30′.
3) Line of Segments, divided 5 - 10; numbered by 1; subdivided to 0.5 and to 0.1.
4) Line of Superficies, marked 1, 1[0], 2[0], 3[0]...10[0]. 1 - 1[0] divided to 5; subdivided to 1 and to 0.5. 1[0] - 10[0] divided to 5; subdivided to 1 and to 0.5.

Along outer edge:

1) Line of Tangents, forming continuous scale when instrument is fully open; divided [0] - 45° on one arm, 45° - 63° on other arm; numbered by 10° (45° also numbered); subdivided to 5°, to 1° and to 15′.

2) Line of Secants (only with second half of Line of Tangents), divided [0] - 55°; numbered by 10°. 10° - 55° subdivided to 5° and to 1° [but only three dots between 10° and 15°].

Outer Rim

1) Line of Inches, divided in both directions ([0] - 9 from non-hinge end and [9] - 18 from hinge); numbered by 1. 1 - 9 and 9 - 17 subdivided to 0.5, to 0.1 and to 0.05.
2) Line of Lesser Tangents divided [0] -75°; numbered by 10° (75° also numbered); subdivided to 5° and to 1°.

No signature. This may possibly the work of an apprentice as the workmanship is not as good as would normally be expected of Allen. That the sector comes from the Allen workshop is testified by the style of the numbers.

S 4.

Location: MHS (Orrery Collection 25-43)

Plate 10

Brass Gunter sector. Two arms (each 305 x 34.5 x 6mm) pivoted on flat circular hinge (diameter 48mm), comprising three elements for one arm and two for the other. Pivoted support strut at non-hinge end (143 x 34.5 x 2mm). Recesses in both arms to accommodate closed and open positions of strut.

Recto

Radial lines, repeated on each arm, reading from inner edge:

1) Line of Inscribed Bodies in the same Sphere, marked across both arms, "D, S, I, C, O, T".
2) Line of Lines, divided [0] to 10; numbered by 1; subdivided to 0.5, to 0.1 and to 0.05.
3) Line of Metals, marked " ☉, ☿, ♄, ☽, ♀, ♂, ♃"; integrated with Line of Equated Bodies, marked "D, I, C, S, O, T".
4) Line of Solids, marked 1, 1[0], 1[00], 2[00], 3[00]...10[00]. 1 -1[0] divided to 5; subdivided to 1. 1[0] - [50] divided to 10; subdivided to 5 and to 1. [50] - 1[00] divided to 10; subdivided to 2. 1[00] - 10[00] divided to 50; subdivided to 10 and to 5.

Along outer edge, reading from edge inwards:

1) Meridian Line, forming continuous scale when instrument is fully open; divided 0 - 70°15′ on one arm, 70°15′ - 86°30′ on other arm; numbered by 10°; subdivided to 5°, to 1° and to 15′.
2) Line of Tangents, forming continuous scale when instrument is fully open; divided [0] - 45° on one arm, 45° - 63° on other arm; numbered by 10°; subdivided to 5°, to 1° and to 15′.
3) Line of Secants (only with second half of Line of Tangents), divided [0] - 59°; numbered by 10°; subdivided to 5°. 10° - 59° subdivided to 1°.

Verso

Radial lines, repeated on each arm, reading from inner edge:

1) Line of Quadrature, marked across both arms "10, 9, 8, 7, S, 6, 5, 90, Q".
2) Line of Sines, divided [0] - 90°; numbered by 10°; subdivided to 5°. 1° - 85° subdivided to 1°. 1° - 80° subdivided to 30′.
3) Line of Segments, divided 5 - 10; numbered by 1; subdivided to 0.5 and to 0.1.

4) Line of Superficies, marked 1, 1[0], 2[0], 3[0]...10[0]. 1 - 1[0] divided to 5; subdivided to 1 and to 0.5. 1[0] - 10[0] divided to 5; subdivided to 1 and to 0.5.

Along outer edge:

1) Logarithmic scale of Numbers, forming continuous scale when instrument is fully open; divided [0.0]2 - [0.13] on one arm, [0.13] - 1[.]0 on other arm. 0.02 - 0.1 marked "2, 3, 4,..., 9, 1". [0.0]2 - [0.13] divided to 0.01; subdivided to 0.005 and to 0.001. [0.13] - 1[.]0 marked "2, 3,..., 10"; divided to 0.05; subdivided to 0.01. [0.13] - [0.]4 subdivided to 0.005 and to 0.001. [0.]4 - 1[.]0 subdivided to 0.002.[2]
2) Upper arm only: Line of Artificial Tangents, divided 7°40′ - 45°; numbered 10°, 20°, 30°, 40°, 45° and back again to 80°; divided to 10°; subdivided to 5°, to 1° and to 10′.
3) Lower arm only: Line of Artificial Tangents, divided 82°20′ - 89°; numbered 85° [actually in the wrong place - it is inscribed at 84°], 89°; divided to 1°; subdivided to 5′. This line forms continuation of scale no.2 when instrument is fully open.
4) Line of Artificial Sines, forming continuous scale when instrument is fully open; 1° - 7°30′ on one arm, 7°40′ - 90° on other arm; numbered 1°, 5° [actually in the wrong place - it is inscribed at 6°], 10°, 20°,..., 90°; divided to 5°. 1° - 80° subdivided to 1°. 1° - 75° subdivided to 30′. 1 - 55° subdivided to 5′.[3]

<u>Outer Rim</u>

1) Line of Inches, divided [0] - 12 from non-hinge end; numbered by 1. 1 - 12 subdivided to 0.5 to 0.1 and to 0.05.
2) Line of Lesser Tangents, divided [0] -75°; numbered by 10°; subdivided to 5° and to 1°.

There is no signature but this is almost certainly an Allen instrument, since the engraving style, particularly of the numbers is the same as that on signed instruments.

The presence of the logarithmic scales on this sector suggests that the instrument is later than Cat. nos. S1, S2 and S3.

Bibliography: R.T. Gunther, *Early Science in Oxford* (Oxford, 1921), vol. 1, pp.144, 382.

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[2] Here I am assuming that the full extent of the scale corresponds to one radius (1) when used in conjunction with the trigonometrical functions. However, when used for other calculations the scale might be held to run up to 10, 100 etc.

[3] The first half of this line also forms the beginning of the Line of Artificial Tangents since, for small ϑ, $\sin \vartheta$ is approximately equal to $\tan \vartheta$.

S 5.

Location: NMM (Identification no. N80-27(2) CI/S.28)

Plates 11,12

Brass sector. Two arms (each 210 x 23.5 x 4.5mm) pivoted on flat circular hinge (diameter 31mm), comprising three elements for one arm and two for the other. Pivoted support strut/rule (with scales) at non-hinge end (159 x 23.5 x 1mm). Recesses in both arms to accommodate closed and open positions of strut.

Recto

Radial lines, repeated on each arm, reading from inner edge, named on lower arm of sector as "Equal Parts" and "Solids" respectively:

1) Line of Equal Parts, divided [0] to 100, numbered by 10; subdivided to 5, to 1 and to 0.5.
2) Line of Solids, divided [0] to 10; numbered by 1; subdivided to 0.5, to 0.1 and to 0.05.

Along outer edge:

1) Lower arm: some form of scale of equal parts, marked "5,10,15" (yet with four main divisions - the first is unnumbered). First division subdivided to 0.2, to 0.1 and to 0.05.
2) Upper arm: similar type of scale with same markings but main divisions closer together (31mm as opposed to 35.5mm).

Along inner edge of lower arm:

1) Numbers 90, 100, 110, 120, 130 marked from non-hinge end; these do not appear to correspond to any marked scale. Numbers occur at 71, 93, 111, 124, 134mm from end.

Verso

Radial lines, repeated on each arm, reading from inner edge, named on lower arm of sector as "Sines" and "Superf" respectively:

1) Line of Sines, divided [0] - 90°; numbered by 10°; subdivided to 5°. 1° - 85° subdivided to 1°. 1° - 80° subdivided to 30′.
2) Line of Superficies, marked 1[0], 2[0], 3[0]...10[0]. [0.1] - [1] divided to 0.05; subdivided to 0.01. [1] - 10[0] divided to 5; subdivided to 1.

Inner Rim

1) Line of Equal Parts on both arms (same size scale as main Line of Equal Parts); numbered by 10; divided to 5; subdivided to 1.

Outer Rim

On arm holding strut:

1) Line of Meridian Parts [unnamed], divided [0] - 70°; numbered by 10°; subdivided to 5° and to 1°.

On other arm:

1) Line of Lesser Tangents [unnamed], divided [0] - [70°]; numbered by 10°. 10° - 70° subdivided to 5° and to 1°. Circular hole for gnomon at hinge end.

Strut Recto

1) Scale of Equal Parts numbered 1 - 6 (but with seven main divisions). First division divided transversally to 0.01.

Strut Verso

Unequal scales, reading from outer rim:

1) Hour scale, marked, 12, 1, 2, 3,..., 12; divided to 1; subdivided to 30 minutes. 1 -11 subdivided to 10 minutes and to 5 minutes. 12.30 - 1 and 11 - 11.30 subdivided to 10 minutes.
2) Degree scale, divided 90° - [0] - 90°; numbered by 10°; subdivided to 5°. 85° - 85° subdivided to 1°.[4]

Rectangular holes for sights at non-hinge end of each arm.

On verso: two threaded round holes on lower arm. One near hinge, one halfway down.

Signature on outer rim of arm with Lesser Line of Tangents: 'Elias Allen fecit' with straight capitals.

This sector is unusual among extant examples of Allen's sectors. It is clearly designed along the lines of the Gunter sector, but does not appear in the form which is illustrated on the title page of Gunter's *De Sectore et Radio*: the scales are differently positioned; the strut is longer and carries scales on it; there are various holes for the introduction of sights and gnomons. This probably indicates that it dates from the period before the publication from the book when, according to Gunter, people were already asking for samples of the new version of the instrument which he had developed around 1606, and described in a Latin manuscript written at about that time.

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[4] Both these scales are probably for use as a kind of protractor for constructing angles, marking off degrees or hours on a circle, or measuring angles.

Bibliography: *An Inventory of the Navigation and Astronomy Collections in the National Maritime Museum Greenwich* (London, 1983), vol. 1, p.11-15

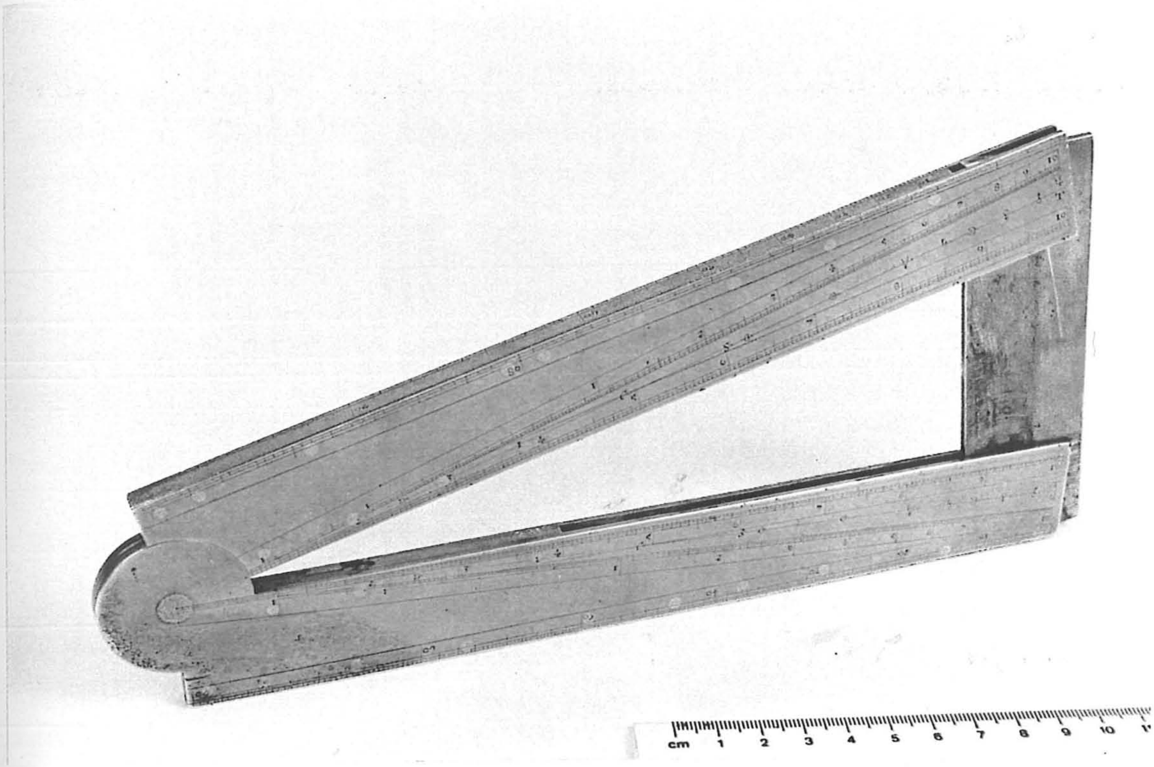

Plate 7: Cat. no. S1 (recto)

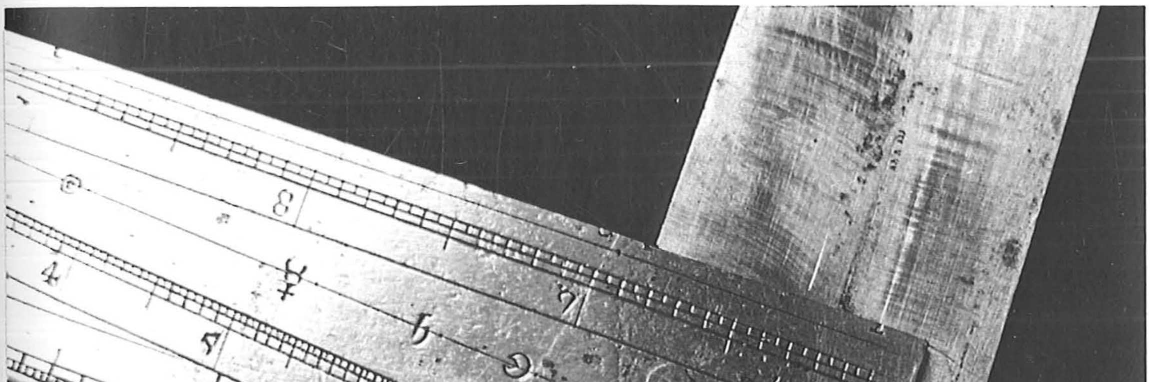

Plate 8: Detail of engraving from Cat. no. S3, showing Line of Lines, Line of Metals, Line of Solids and Meridian Line

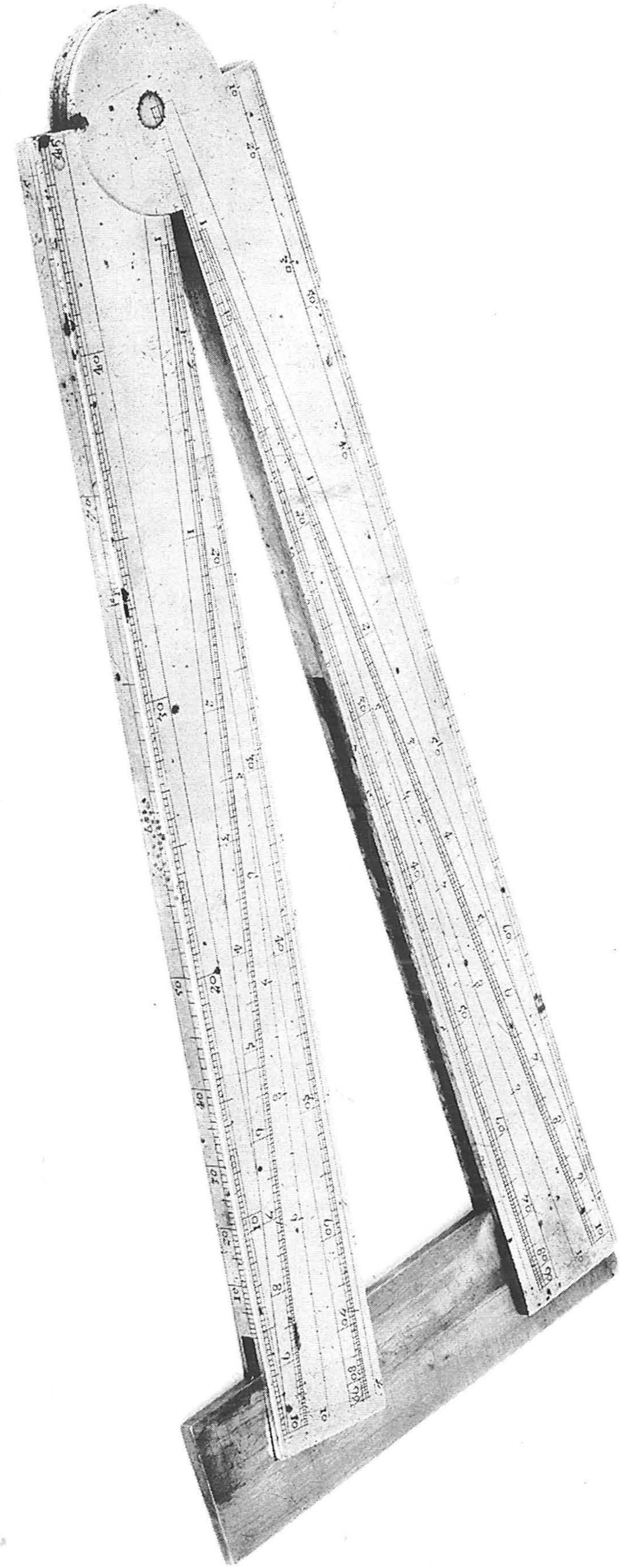

Plate 9: Cat. no. S3 (verso)

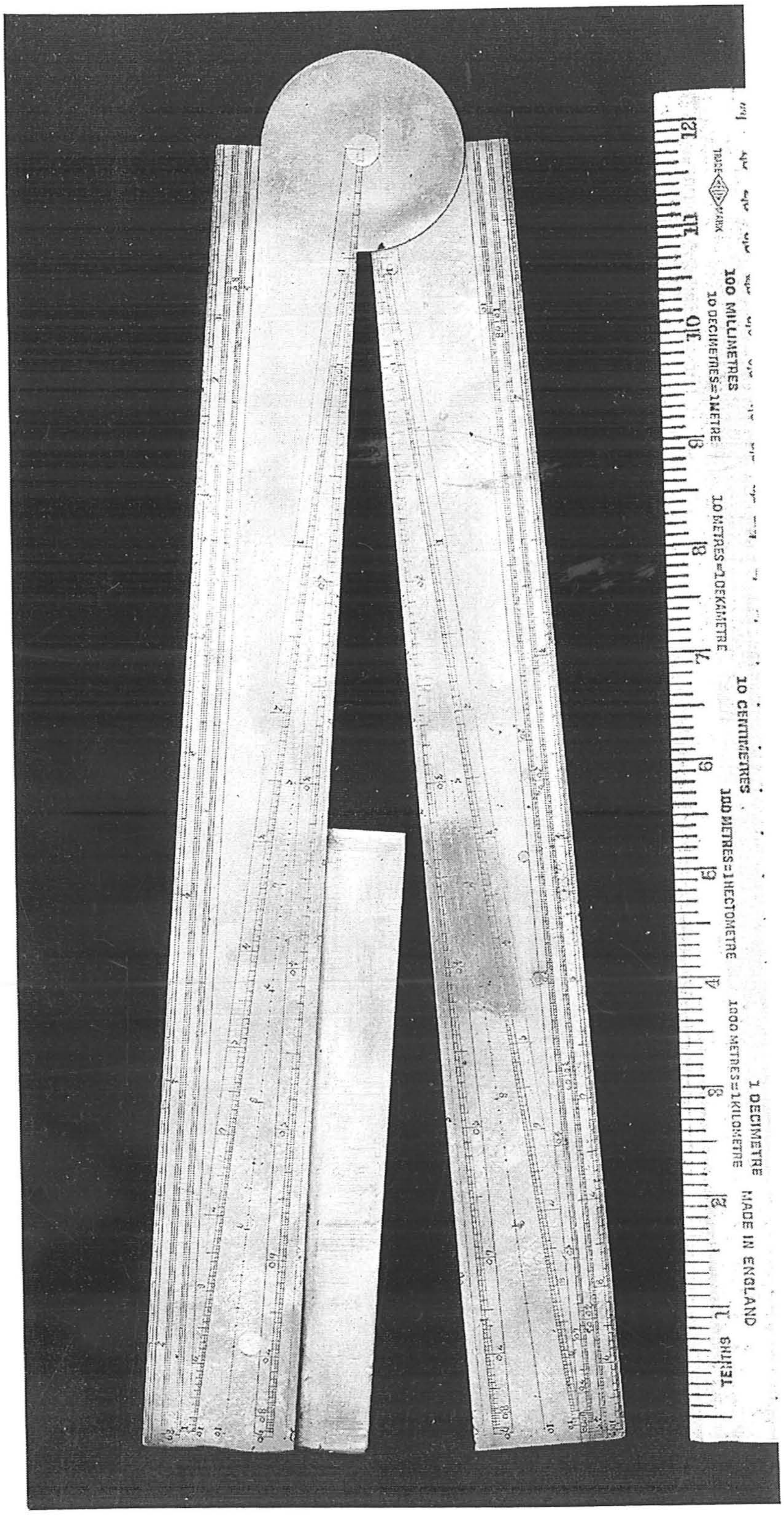

Plate 10: Cat. no. S4 (verso)

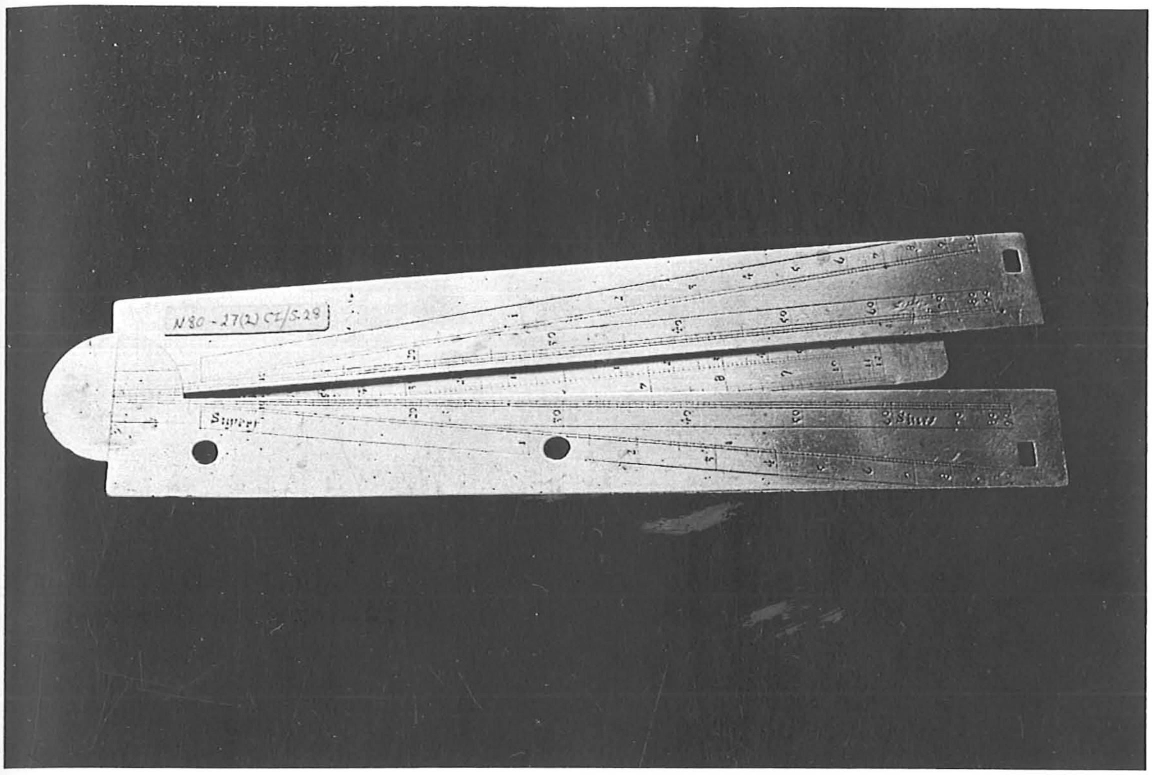

Plate 11: Cat. no. S5 (verso)

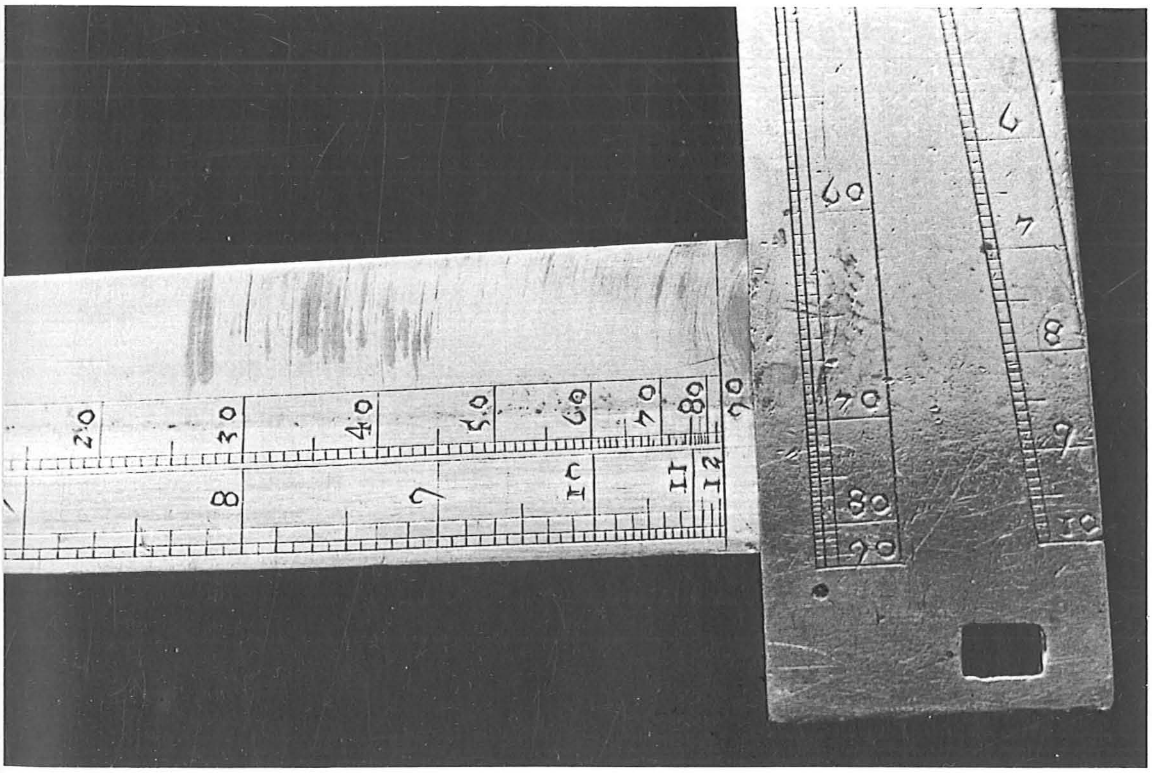

Plate 12: Detail from Cat. no. S5, showing part of strut, Line of Sines and Line of Superficies

Universal Equinoctial Ring Dials

These all follow the description in Oughtred, *The Description and Use of the Double Horizontall Dyall ...whereunto is added, The Description of the generall Horologicall Ring* (London 1652). Possible original ownership has been discussed already in Chapter 5 (page 180f.)

U 1.

Location: MHS (Identification no. 71)

Brass ring dial of Oughtred's design, diameter 87.5mm. Two concentric rings and bridge, folding flat when not in use, with suspension ring and shackle which slides in grooved rim of outer (meridian) ring. Tooth on shackle with bevelled edge on left-hand side to indicate latitude. Inner diameter of meridian ring, 77mm. Inner diameter of inner (equatorial) ring, 69mm.

<u>Recto</u>

Meridian ring:

Degree scale divided clockwise [0] - [90°]; numbered by 10°; subdivided to 5° and to 1°. 85° - 90° hidden by bridge support.

Towns with latitudes, listed clockwise "Amsterdam 52 26, Haga 52 12, Hamburg 53 43, Heilburg 49 36, Vienna 48 22, Venecia 45 15, Lisbona 38 30, Roma 42".

Equinoctial ring:

Hour circle numbered III - XII, I - VIII, XI [sic.; should be IX] clockwise; III corresponds to 45° on degree scale.

<u>Verso</u>

Meridian ring:

Towns with latitudes, clockwise from top "Copenhagen 55 43, Cracouia 50, Dantiscum [Danzig] 54 23, Emden 53 32, Colonia 50 56, Moscouia 59, Ratisbona [Regensburg] 49 9, Madrid 40 45, Neapolis 43, Constantinopl 43".

Equinoctial ring:

Towns with latitudes, clockwise from top "Cantarberi 51 10, London 51 32, Oxford 51 45, Cambridg [latitude omitted], Yorke 54, Edenburg 56, Dublin 53 30, Calles 50 45, Diep 49 40, Paris 48 50, Leon 45, Marcelle 43, Praga 50".

Inner rim of equinoctial ring: hours between 3am and 9pm, divided to hour; subdivided to 15 minutes.

Universal Equinoctial Ring Dials

These all follow the description in Oughtred, *The Description and Use of the Double Horizontall Dyall ...whereunto is added, The Description of the generall Horologicall Ring* (London 1652). Possible original ownership has been discussed already in Chapter 5 (page 180f.)

U 1.

Location: MHS (Identification no. 71)

Brass ring dial of Oughtred's design, diameter 87.5mm. Two concentric rings and bridge, folding flat when not in use, with suspension ring and shackle which slides in grooved rim of outer (meridian) ring. Tooth on shackle with bevelled edge on left-hand side to indicate latitude. Inner diameter of meridian ring, 77mm. Inner diameter of inner (equatorial) ring, 69mm.

Recto

Meridian ring:

Degree scale divided clockwise [0] - [90°]; numbered by 10°; subdivided to 5° and to 1°. 85° - 90° hidden by bridge support.

Towns with latitudes, listed clockwise "Amsterdam 52 26, Haga 52 12, Hamburg 53 43, Heilburg 49 36, Vienna 48 22, Venecia 45 15, Lisbona 38 30, Roma 42".

Equinoctial ring:

Hour circle numbered III - XII, I - VIII, XI [sic.; should be IX] clockwise; III corresponds to 45° on degree scale.

Verso

Meridian ring:

Towns with latitudes, clockwise from top "Copenhagen 55 43, Cracouia 50, Dantiscum [Danzig] 54 23, Emden 53 32, Colonia 50 56, Moscouia 59, Ratisbona [Regensburg] 49 9, Madrid 40 45, Neapolis 43, Constantinopl 43".

Equinoctial ring:

Towns with latitudes, clockwise from top "Cantarberi 51 10, London 51 32, Oxford 51 45, Cambridg [latitude omitted], Yorke 54, Edenburg 56, Dublin 53 30, Calles 50 45, Diep 49 40, Paris 48 50, Leon 45, Marcelle 43, Praga 50".

Inner rim of equinoctial ring: hours between 3am and 9pm, divided to hour; subdivided to 15 minutes.

<u>Bridge</u>

Ends at VI and VI of equinoctial ring. Double bar of width 10mm with central opening for sliding pin-hole gnomon. Bars engraved with scale of months, solstice to solstice, indicated by initial letter "D, N, O, S, A, I" on one, "I, F, M, A, M, I" on other; divided to month; subdivided to 10 days and to 5 days (except for December and June - not all five-day divisions given). Solstices at 10 Dec, 12 Jun (very approximate).

Equinoctial ring pivot allows movement through 90°. Bridge pivot allows 360° rotation.

Signature on recto of equinoctial ring: "Elias Allen fecit Lndini [sic]" with straight capitals.

Bibliography: R.T. Gunther, *Early Science in Oxford*, (Oxford, 1923), vol.2, p.152; AV. Simcock, 'An Equinoctial Ring Dial by Ralph Greatorex' in Anderson, Bennett & Ryan, *Making Instruments Count: Essays on Historical Scientific Instruments presented to Gerard L'Estrange Turner* (Aldershot: Variorum, 1993), p.201.

U 2.

Location: MHS (Identification no. 57-84/113)

Brass ring dial of Oughtred's design, diameter 86mm. Two concentric rings and bridge, folding flat when not in use, with suspension ring and shackle which slides in grooved rim of outer (meridian) ring. Tooth on shackle with bevelled edge on left-hand side to indicate latitude. Inner diameter of meridian ring, 77mm. Inner diameter of inner (equatorial) ring, 70mm.

Recto

Meridian ring:

Degree scale divided clockwise [0] - [90°]; numbered by 10°; subdivided to 5° and to 1°. 87° - 90° hidden by bridge support.

Towns with latitudes, listed clockwise "Notingham 53, Lincoln 53 30, Yorke 54, Newcastell 54 30, Barwicke 55, Edenburg 56, Dublin 53 30, Calles 50 45, Diep 49, Roan 48".

Equinoctial ring:

Hour circle numbered III - XII, I - IX clockwise; III corresponds to 45° on degree scale.

Verso

Meridian ring:

Towns with latitudes, clockwise from top "Madrit 40 45, Venecia 45 15, Roma 42 2, Paris 48 50, Orleanc 48, Haga 52 12, Amsterdam 52 26, Antwerp 51 12, Bruxells 50 45, Dunkerk 51, Hamburg 53 40, Copenhagen 55 43".

Equinoctial ring:

Towns with latitudes, clockwise from top " W Chester[5] 53 10, Darby 53, Douer 51, Cantarberi 51 10, London 51 32, Oxford 51 45, Cambridg 52 16, Couentri 52 20, Lecester 52 30, Shrowesberi 52 40, Glocester 52".

Inner rim of equinoctial ring: hours between 3am and 9pm, divided to hour; subdivided to 15 minutes.

Bridge

This is a replacement, probably nineteenth or early twentieth century.

[5] Presumably this is intended to refer to Chester.

Equinoctial ring pivot allows movement through 90°. Bridge pivot allows 360° rotation.

The instrument is unsigned, but the style of engraving is a clear indication that this is an Allen piece.

Bibliography: AV. Simcock, 'An Equinoctial Ring Dial by Ralph Greatorex' in Anderson, Bennett & Ryan, *Making Instruments Count: Essays on Historical Scientific Instruments presented to Gerard L'Estrange Turner* (Aldershot: Variorum, 1993), p.201.

U 3.

Location: NMM (Identification no. NAA70-9 D349)

Plates 13, 14, 16

Brass ring dial of Oughtred's design, diameter 64mm. Two concentric rings and bridge, folding flat when not in use, with suspension ring and shackle which slides in grooved rim of outer (meridian) ring. Tooth on shackle with bevelled edge on left-hand side to indicate latitude. Inner diameter of meridian ring, 53.5mm. Inner diameter of inner (equatorial) ring, 46.5mm.

<u>Recto</u>

Meridian ring:

Degree scale divided clockwise [0] - [90°]; numbered by 10°; subdivided to 5° and to 1°. 84° - 90° hidden by bridge support.

Towns with latitudes, listed clockwise "Notingham 53, Lincoln 53 30, Yorke 54, Newcastell 54 30, Barwick 55, Edenburg 56, Dublin 53 30".

Equinoctial ring:

Hour circle numbered III - XII, I - IX clockwise; III corresponds to 45° on degree scale.

<u>Verso</u>

Meridian ring:

Towns with latitudes, clockwise from top "Calles 50 45, Diepe 49 40, Orlea^{n}s 48, Paris 48 50, Amsterdam 52 26, Haga 52 12, Bruxells 50 45, Lisbona 38 30, Madri 40 45".

Equinoctial ring:

Towns with latitudes, clockwise from top "Canterberi 51 10, London 51 32, Oxford 51 45, Cambrdg 52 16, Lecester 52 30, Shrowesberi 52 40, Glocester 52, Bristoll 51 20".

Inner rim of equinoctial ring: hours, divided to hour; subdivided to 15 minutes.

<u>Bridge</u>

Ends at VI and VI of equinoctial ring. Double bar of width 10mm with central opening for sliding pin-hole gnomon. Bars engraved with scale of months, solstice to solstice, indicated by initial letter "D, N, O, S, A, I" on one, "I, F, M, A, M, I" on other; divided to month; subdivided to 10 days and to 5 days (except for December and June - subdivided to 10 days only). Solstices at 10 Dec, 12 Jun (very approximate).

Equinoctial ring pivot allows movement through 90°. Bridge pivot allows 360° rotation.

Signature on recto of equinoctial ring: "Elias Allen fecit" with straight capitals.

Bibliography: *An Inventory of the Navigation and Astronomy Collections in the National Maritime Museum Greenwich* (London, 1983), vol. 2, p.29-16.

U 4.

Location: BM (Registration no. 96 3-7 6; Ward Catalogue no. 184)

Plate 15

Incomplete brass ring dial of Oughtred's design, diameter 87mm. Two concentric rings, folding flat when not in use, with shackle which slides in grooved rim of outer (meridian) ring. Tooth on shackle, suspension ring and bridge missing. Inner diameter of meridian ring, 77mm. Inner diameter of inner (equatorial) ring, 71mm.

Recto

Meridian ring:

Degree scale divided clockwise [0] - [90°]; numbered by 10°; subdivided to 5° and to 1°; 85° - 90° hidden by bridge support.

Towns with latitudes, listed clockwise "Lisbona 39, Madrit 40 45, Malg^a^o 36 45, Mayorke 39 40, Seuill 37 40, Barsalonia 41 15, Toletum 39 54".

Equinoctial ring:

Hour circle numbered III - XII, I - IX clockwise; III corresponds to 45° on degree scale.

Verso

Meridian ring:

Towns with latitudes, clockwise from top "Constantinople 41, Alepo 38, Alexandr^i^a 31 10, Smyrna 38, Sira^c^usa 36, Genua 44, Milan 45, Messena 38 10, Gibralatar 36, Toletum 40, Zant [Zakinthos - an island East of Greece] 37".

Equinoctial ring:

Towns with latitudes, clockwise from top "[Lon]don 51 32, Paris 48 50, Orlean[s], Marcellia 43 10, Lyons 42, Leghorn 43 25, Florenc 43 40, Neapolis 41, Venecia 45 15, Roma 42 2".

Inner rim of equinoctial ring: hours between 3am and 9pm, divided to hour; subdivided to 15 minutes.

Equinoctial ring pivot allows movement through 90°.

Signature on recto of equinoctial ring: "Elias Allen fecit Londini" with straight capitals.

Bibliography: F.A.B. Ward, *A Catalogue of European Scientific Instruments in the Department of Medieval and Later Antiquities of the British Museum*, (London, 1981), p.66.

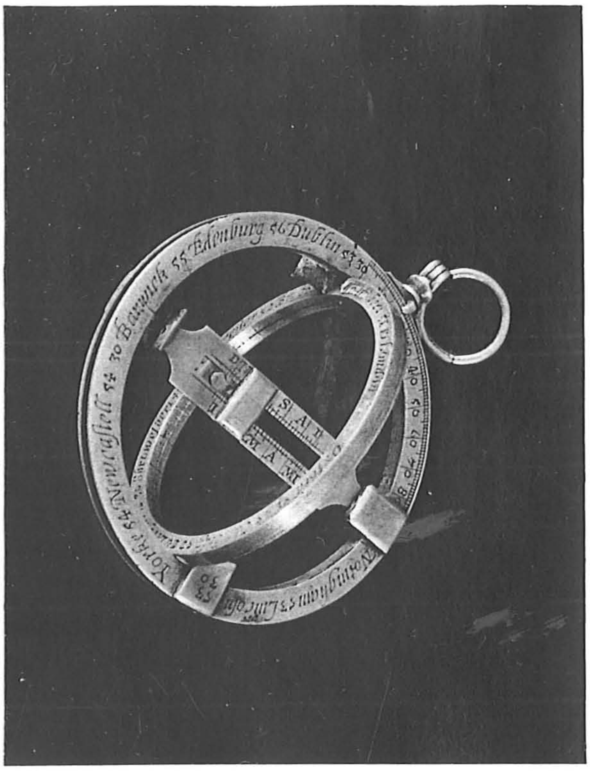

Plate 13: Cat. no. U3

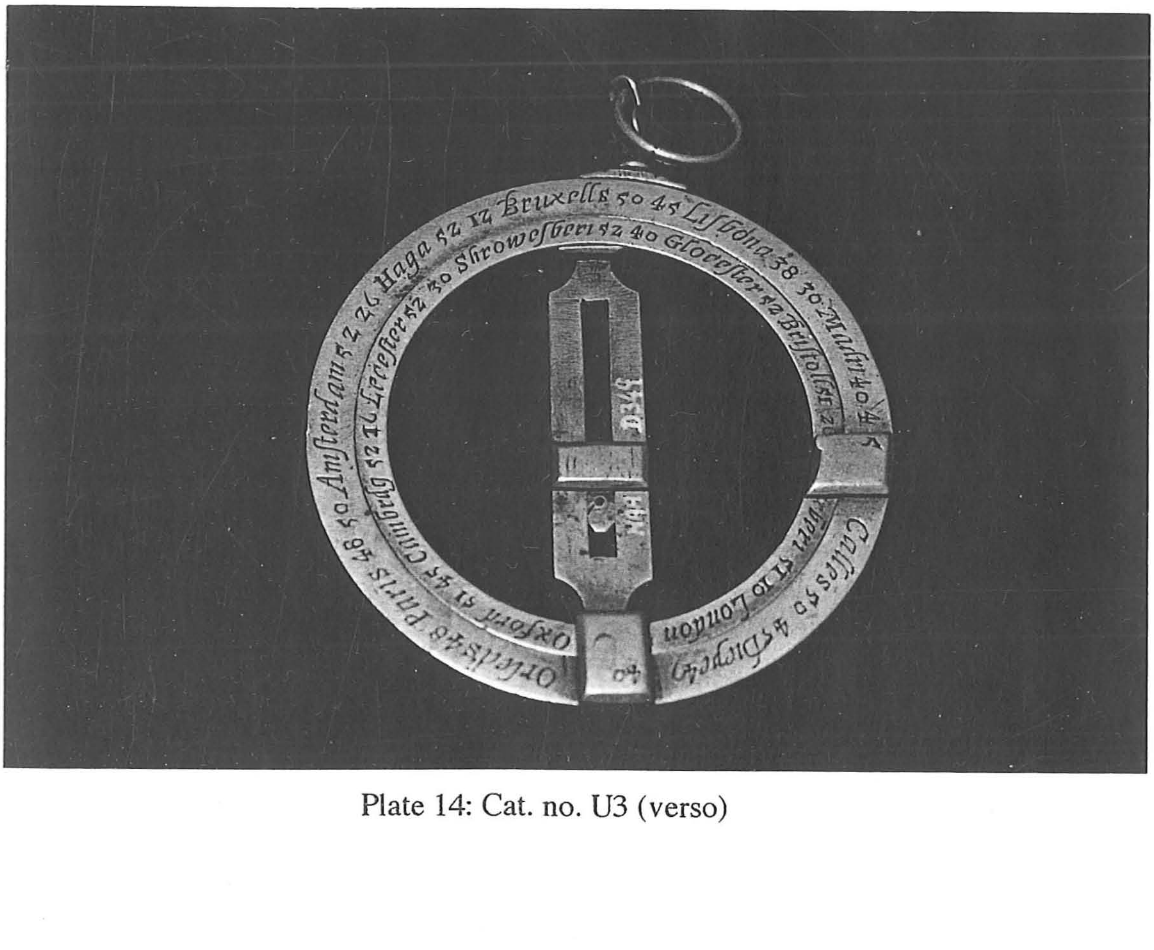

Plate 14: Cat. no. U3 (verso)

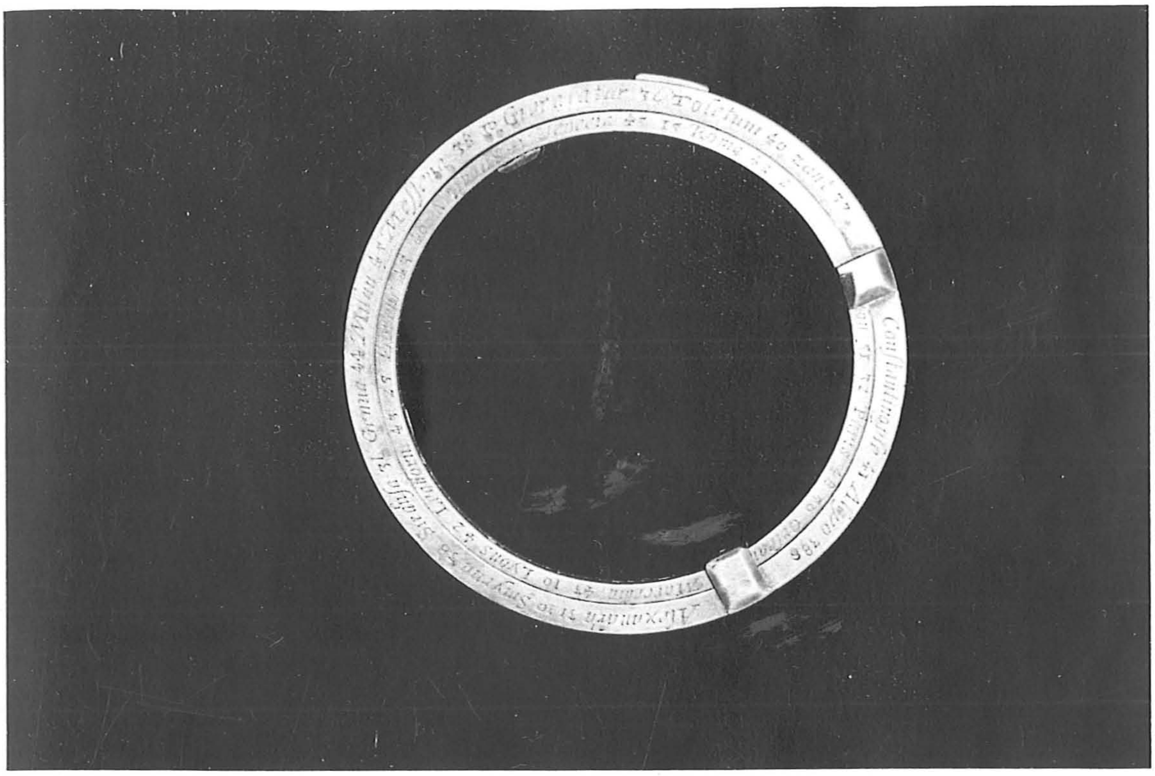

Plate 15: Cat. no. U4 (verso)

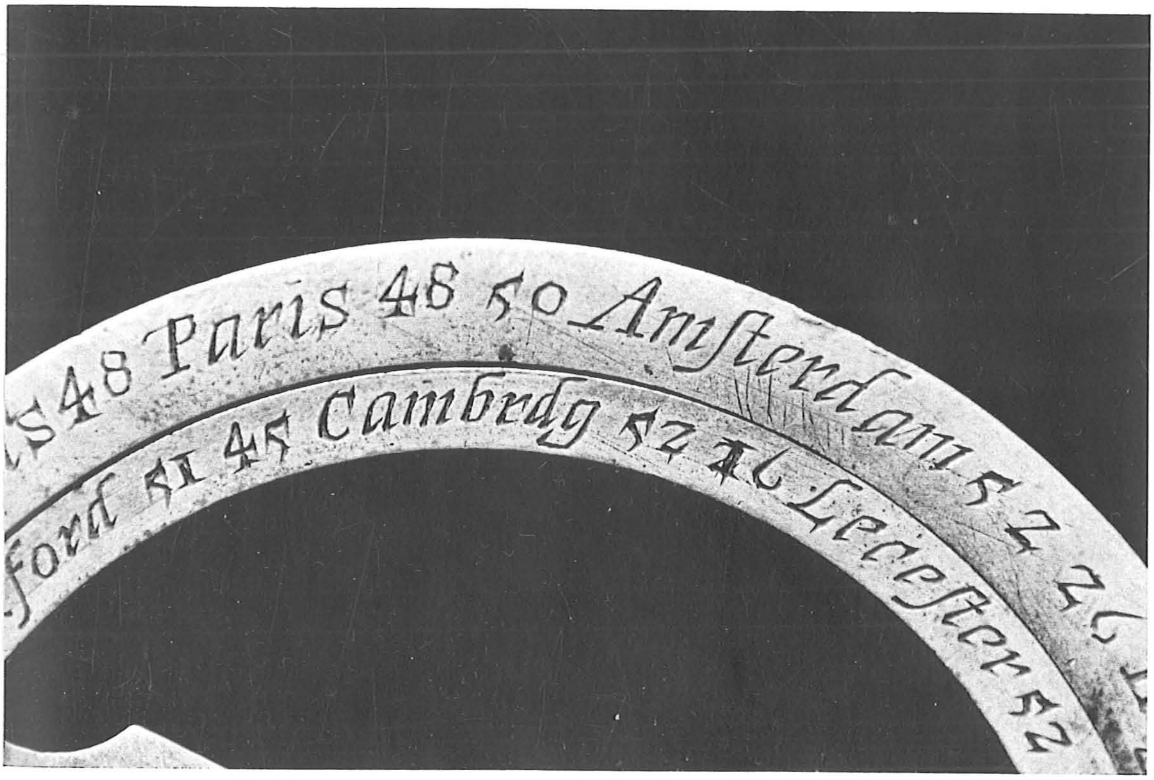

Plate 16: Detail from Cat. no. U3, showing engraving error

Astronomical Compendia

Astronomical compendia were common in the sixteenth, seventeenth and early eighteenth centuries. They varied in their combinations of instruments but generally included a compass, an equinoctial dial, a table of latitudes, a nocturnal and a lunar volvelle, thus making them usable both during the daytime and at night, in a variety of different places.

A1.

Location: Victoria & Albert Museum, London (Identification no. M51-1963)
Plates 17, 18

Gilt-brass astronomical compendium (61mm diameter x 23mm depth) with flat stem pierced by suspension ring; comprising lunar volvelle, calendar, nocturnal, universal equinoctial dial, compass, and list of towns with latitudes.

<u>Outside of lid</u>

Full royal coat of arms of James I.

Shield: quarterly, 1st and 4th quartered (1st and 4th three fleur de lys, 2nd and 3rd three lions passant); 2nd lion rampant of Scotland; 3rd harp of Ireland.

Surrounded by garter, inscribed with: "HONI SOIT QVI MAL Y PENS". Banner below shield carries motto "DIEV ET MON DRoit".

Supporters: Crowned lion and unicorn rampant.

Crest: Crown surmounted by standing crowned lion.

Decoration on hinge and clasp.

<u>Inside of lid</u>

List of towns with latitudes set in concentric circles (from edge, clockwise):

1) "London 51 32, Newarke 53 10, Newcastle 54 57, Carleele 54 55, Lecester 52 48, Cantarbury 51 12"
2) "Royston 51 56, Dancaster 53 28, Barwicke 55 20, Lancaster 53 45, Warwick 52 43, Chechester 51 0"
3) "Huntington 52 6, Yorke 53 58, Edenburgh 55 36, Darby 52 55, Northhamton 52 36, Southhamton 51 5"
4) "Grantham 52 50, Rippon 54 12, Glasquo 55 20, Notingham 52 58, Cambridg 52 10, Bristol 51 30"
5) "Lincolne 53, Durham 54 46, S Andres 56 41, Chester 52 54, Hartford 51 45, Oxforde 51 45"

Central inscription: "THE LATITVD OF CITIES AND TOWNES IN INGLAND AND SCOTLAND"

Universal equinoctial dial

Universal equinoctial dial, pivoted on plain semicircular support.

Equinoctial ring:

Recto: hours clockwise I - XII, I - XII; divided to 30 minutes.

Verso: hours anticlockwise VI - XII, I -VI in bottom half alone; divided to 30 minutes.

Shaped gnomon, with bevelled edge, attached to quadrant which is hinged to equinoctial ring and which moves through a slot in supporting semicircle. Quadrant divided both sides [4°] - 90°; numbered by 10°; subdivided to 5° and to 1°.

Compass

Compass with printed and painted paper card (very intricate detail in painting). Degree scale divided [0] - 90° - [0] - 90° - [0] from North; numbered by 10°; subdivided to 2°; coloured green. Cardinal points in blue with fleur de lys for North and cross for East; quadrantal points in red, green and gold; intermediate points in green and gold; by-points in gold. Three concentric gold rings, with by-points radiating from outermost, intermediate points from middle ring and remaining points from inner circle. Centre circle intricately painted in blue and gold. Area between central and outer circle shaded by speckling, remainder plain. Initials "R G" either side of North point between inner and middle circles. Blued steel needle, with cross for North, with central brass bearing; no crosswires. Glass plate surmounting all.

Back of case: nocturnal

Volvelle with fixed ring and rotatable discs, each with pointer (both plain) with bevelled edge on left-hand side.

Fixed ring:

months by name ("Ianuary, February, March, Aprill, May, Iune, Iuly, August, September, Octobe, Nouembe, December"); numbered by 10; divided to 2.

Outer rotatable disc:

hour circle I - XII, I - XII clockwise; divided to 30 minutes. Bevelled edge of pointer at XII; other hours have small protruding teeth.

Inner rotatable disc:

radial slit from centre following line of bevelled edge of pointer.

Inside back of case: prime and epact for 1617 - 1635[6]

Concentric circles, with divisions, reading from rim and clockwise from catch.

1) "The Prime", 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1, 2.

2) "and Epact", 3, 14, 25, 6, 17, 28, 9, 20, 1, 12, 23, 4, 15, 26, 7, 18, 29, 11, 22.

3) "begining 1617".

Back of compass: lunar volvelle

Date circle with named months clockwise (spellings as for nocturnal except "October" and "Nouember"); numbered by 10; divided to 2. Solstices on XII line.

Degree scale divided by zodiacal signs ["♈, ♉, ♊, ♋, ♌, ♍, ♎, ♏, ♐, ♑, ♒, ♓"]; divided to sign; numbered by 10°; subdivided to 2°. Beginning of Aries at 10 March. Other equinox at 13 September.

Hour circle: I - XII, I - XII clockwise, divided to 30 minutes. XII line runs from hinge to clasp.

Compass circle: 16 points named by initial; by-points marked by single line.

Rotatable disc with projecting shaped pointer (with bevelled edge on left-hand side) and lunar month scale numbered [0] - [29]$^{1}/_{2}$ by 2; divided to 1.

Rotatable disc with projecting shaped pointer (with bevelled edge on left-hand side). Aspectarium (diagram showing the aspects of the planets - conjunction, opposition, trine, quadrature and sextile): lines radiating from base of pointer through geometric shapes (triangle, square and star - if these lines were repeated round the circle they would form an equilateral triangle, square and regular hexagon, all inscribed within the circle). Circular aperture near edge of disc displaying the moon's phase.

Signed on equinoctial ring: "Elias Allen fecit" with straight capitals.

Bibliography: Claud Blair, 'A Royal Compass-dial' in *The Connoisseur*, December 1964, pp.246-248.

[6] These numbers are used for calculating the date of Easter, which is linked to the lunar calendar; the prime is used to calculate the date of the Paschal full moon (the first full moon after the vernal equinox); the epact is the age of the moon on 1st January. Either number, in conjunction with the dominical letter (which denotes the date of the first Sunday in the year), can be used to calculate Easter, the first Sunday after the Paschal full moon.

A 2.

Location: SM (Identification no. 1883-135)

Illustrated in Brown, *Mathematical Instrument-makers*, plates 7a, 7b, 7c

Brass astronomical compendium (80mm diameter x 20mm depth) with round stem pierced by suspension ring; comprising lunar volvelle, calendar, nocturnal, universal equinoctial dial, compass, and list of towns with latitudes.

Outside of lid

Royal coat of arms of Charles I.

Shield: quarterly, 1st and 4th quartered (1st and 4th three fleurs de lys, 2nd and 3rd three lions passant); 2nd lion rampant of Scotland; 3rd harp of Ireland.

Surrounded by garter, inscribed with: "HONI SOIT QVI MAL Y PENSE". Banner below shield carries motto "DIVE ET MON DROIT".

Supporters: Crowned lion and unicorn rampant.

Crest: Crown surmounted by standing crowned lion.

Rim has tooled pattern:

Inside of lid

List of towns with latitudes set in concentric circles (from edge, clockwise):

1) "Cantarberi 51 20, Winchester 51 0, Oxford, 51 45, Northamton 52 10, Buckingham 52 0, Shrowesbere 52 30, Newcastell 54 30, Edenburghe 55 56"
2) "London 51 32, Salesbery 51 10, Cambridg 52 16, Colchester 52 0, Bedford 52 20, Herefoord 52 10, Durham 54 20, Striuiling [presumably Stirling] 56 15"
3) "Chechester 50 30, Plimouth 50 20, Huntington 52 30, Ipswich 52 10, Hartford 51 50, Glocester 51 45, Lancaster 54 10, Falkland 56 15"
4) "Arrundel 50 35, Excester 50 40, Warwick 53 0, Norwich 53 0, Pembrook 52 10, Couentre 52 45, Notingha: 53 10, Dunbar 56 0"
5) "Douer 51 10, Bristoll 51 20, Lecester 52 50, Yarmou 53 5, Denbigh 53 20, Lichfild 53 5, Lincoln 53 20, Carlill 54 50"
6) "Rye 51 0, Wells 51 30, Derby 53 10, Hull 53 40, S^t Dauids 52 0, Bath 51 40, York 54 0, Barwi: 55 0"

Universal equinoctial dial

Universal equinoctial dial pivoted on decorated semicircular support.

Equinoctial ring:

Recto: hours clockwise I - XII, I - XII; divided to 30 minutes.

Verso: hours VI - XII, I - VI in bottom half alone (but all twenty-four hour-divisions marked); divided to 30 minutes in numbered half.

Shaped gnomon, with bevelled edge, attached to quadrant which is hinged to equinoctial ring and which moves through a slot in supporting semicircle. Quadrant divided anticlockwise [5°] - 90°; numbered by 10°; subdivided to 5° and to 1°.

Compass

Compass with printed and painted paper card. Degree scale divided [0] - 360° clockwise from North; numbered by 10°; subdivided to 5° and to 1°. 0 - 90° and 180° - 270° coloured yellow, remainder coloured green. Wind ring divided into 64, delineated white, black, white, black, etc. Compass points shown by isosceles triangles split symmetrically, painted gold on left-hand side, coloured on right (cardinal points in blue, quadrantal points in red, intermediate points in green; by-points shown only by lines). Central gold circle. North point indicated by gold fleur de lys. Blued steel needle, with arrowhead for North, cross for South, surmounted by octagonal brass pyramid; no crosswires. Glass plate [damaged] surmounting all.

Back of case: nocturnal

Volvelle with fixed ring and rotatable discs, each with shaped pointer with bevelled edge on left-hand side.

Fixed ring:

months by name ("Ianuarie, Februarie, March, Aprill, May, Iune, Iuly, August, September, October, Nouember, December"); numbered by 10; divided to 2.

Outer rotatable disc:

hour circle I - XII, I - XII clockwise; divided to 30 minutes. Bevelled edge of pointer at XII; other hours have small protruding teeth.

Inner rotatable disc:

radial slit from centre following line of bevelled edge of pointer.

Inside back of case: Paschal calendar for 1630 - 1656[7]

Concentric circles, with divisions, reading from rim and clockwise from catch.

1) "Epact", 6, 25, 14 [covering 2 divisions], 3, 22, 11, 29 [2 divisions], 18, 7, 26, 15 [2 divisions], 4, 23, 12, 1 [2 divisions], 20, 9, 28, 17 [2 divisions], 6, 25, 14, 3 [2 divisions], 22, 11, 29, 18 [2 divisions], 7, 26.
2) "Anr: nu:" 6, 5, 4 [2 divisions], 3, 2, 1, 19 [2 divisions], 18, 17, 16, 15 [2 divisions], 14, 13, 12, 11 [2 divisions], 10, 9, 8, 7 [2 divisions], 6, 5, 4, 3 [2 divisions], 2, 1, 19, 18 [2 divisions], 17, 16.
3) "An: dom:", 1656, 1652, 1648, 1644, 1640, 1636, 1632 [coincide with double divisons in circles 1 and 2].
4) "Paschal", 16, 5, _, 13, 2, _, 10, _, 18, 7, _, 15, 4, _, 12, 1, _, 9, _, 17, 6, _, 14, 3, _, 11, _, 19, 8, _, _, _, _, _, _, _. ["_" indicates a blank division.]
5) "Let dom", C, D, E, F, G, A, B, C, D, E, F, G, A, B, C, D, E, F, G, A, B, C, D, E, F, G, A, B, C, D, E, F, G, A, B, C.
6) "Dies", 25 [5 divisions], 31 [6 divisions], 5 [5 divisions], 10 [5 divisions], 15 [5 divisions], 20 [5 divisions], 25 [5 divisions] (numbers at right-hand end of space).
7) "mens", "MARTIVS" [up to 31 of circle 6], "APRILIS".

Back of compass: lunar volvelle

Date circle with named months clockwise (spellings as for nocturnal); numbered by 10; divided to 5; subdivided to 1.

Degree scale divided by zodiacal signs, divided to sign; subdivided to 10°, to 5° and to 1°. Beginning of Aries at 10 March. Other equinox at 13 September.

Hour circle: I - XII, I - XII clockwise, divided to 30 minutes. XII line runs from hinge to clasp.

Compass circle: 16 points named by initial; by-points marked by single line.

Rotatable disc with projecting shaped pointer (with bevelled edge on left-hand side) and lunar month scale numbered [0] - [29]$^1/_2$ by 2; divided to 1.

Rotatable disc with projecting shaped pointer (with bevelled edge on left-hand side). Aspectarium (diagram showing the aspects of the planets - conjunction, opposition, trine, quadrature and sextile): lines radiating from base of pointer through geometric shapes (triangle, square and star). Circular aperture near edge of disc displaying the moon's phase.

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[7] For calculating the date of Easter. "Anr: nu:" refers to the prime or 'golden number'; "Paschal" gives the date of the Paschal full moon; "Let dom" is the dominical letter. See n.6 above.

Hinge decorated front and back with angels. Clasp also decorated.

Signed on equatorial ring: “Elias Allen fecit” with straight capitals.

Bibliography: Joyce Brown, *Mathematical Instrument-Makers in the Grocers' Company 1688-1800* (London, 1979)

A3.

Location: MHS (Lewis Evans 233)

Plate 19

Silver astronomical compendium (51.5mm diameter x 13mm) with decorated suspension ring; comprising moon dial, nocturnal, universal equinoctial dial, compass and list of towns with latitudes.

Outside of lid: nocturnal[8]

Volvelle with fixed ring I - XII, I - XII anticlockwise; hours divided to 30 minutes (with barred lines); subdivided to 15 minutes.

Rotatable disc: months marked by initial; numbered by 10; divided to 5; subdivided to 1. Engravings of six circumpolar constellation figures (Ursa Major, Ursa Minor, Draco, Cepheus, Cassiopeia, Auriga)[9] with individual stars. Lines from centre through month divisions. Projecting pin, to assist turning of nocturnal, set on line between June and July.

Inside of lid

List of towns with latitudes set in concentric circles (from edge, clockwise):

1) "Cantarberi 51 10, Newcastell 54 30, Midelburg 51 30, Hedelburg 49 36, Ratisbona [Regensburg] 49 9, Constantinopel 43"
2) "London 51 32, Barwick 55, Antwerp 51 12, Ingolstat 48 42, Brandenburg 52 30, Ierusalem 32 10"
3) "Oxford 51 45, Edenburg 56, Amsterdam 52 26, Norenburg 49 26, Luxenburg 49 50, Neapolis 40 42"
4) "Cambridg 52 16, Aberdien 57 20, Bruxells 50 45, Hamburg 53 43, Lunenburg 53 36, Madrid 40 45"
5) "Notingham 53, Glosgow 57, Dunkerke 51, Magdenburg 51 15, Franckford 50, Lisbona 38 30"
6) "Lincoln 43 30, Dublin 53 30, Colona 50 56, Lubeca 53 58, Vienna 48 22, Venecia 45 15"
7) "Yorke 54, Paris 48 50, Wesel 51 30, Brema 53 8, Praga 50 6, Roma 42 2"

Centre: Coat of arms, shield engraved with fruit tree.

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[8] This is very similar in style to the nocturnals on the Gunter quadrants (Cat. Nos. Q1, Q2 and Q3).

[9] This constellation actually lies farther away from the pole and inverted from the position shown by Allen. It has been inverted (retaining the correct position of the two stars in the head of the constellation) in order to fill a gap in the star map.

Universal equinoctial dial

Universal equinoctial dial pivoted on decorated semicircular support.

Equinoctial ring:

Recto: hours clockwise I - XII, I - XII; divided to 30 minutes.

Verso: hours VI - XII, I - VI in bottom half alone (but all twenty-four hour-divisions marked); divided to 30 minutes in numbered half.

Shaped gnomon, with bevelled edge, attached to quadrant which is hinged to equinoctial ring and which moves through a slot in supporting semicircle. Quadrant numbered anticlockwise [0, 10°] - 90° by 10°; divided to 10°; subdivided to 5° and to 1°.

Compass

Compass with printed and painted paper card. Cardinal points in blue with gold heads (fleur de lys for North, cross for East, arrowhead for South, spade for West); quadrantal points in red; intermediate points in green; by-points in white. [Only cardinal points are named in full; quadrantal points named as "North W" etc.; other points named by initials only.] Three concentric gold rings with by-points radiating from outermost, intermediate points from middle ring and remaining points from inner circle. Blued steel needle with arrowhead for North, cross for South; mounted on central brass octagonal pyramidal bearing; crosswires. Glass plate surmounting all.

Back of instrument: lunar volvelle

Hour circle: I - XII, I - XII clockwise, divided to 30 minutes. XII line runs from hinge to clasp.

Compass circle: 16 points named by initial; divided to $22^1/_2$°.

Rotatable disc with projecting shaped pointer (with bevelled edge on left-hand side) and lunar month scale numbered [0] - [29] $^1/_2$ by 2; divided to 1.

Rotatable disc with projecting shaped pointer (with bevelled edge on left-hand side). Aspectarium (diagram showing the aspects of the planets - conjunction, opposition, trine, quadrature and sextile): lines radiating from base of pointer through geometric shapes (triangle, square and star). Circular aperture near edge of disc displaying the moon's phase.

Suspension ring: This is probably later; it is stamped with the name "COLE".[10]

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[10] Ambrose Heal, *The London Goldsmiths, 1200 - 1800* (Cambridge, 1935, reprinted 1972), lists various Coles (jewellers, silversmiths, goldsmiths etc.), all active at the very end of the seventeenth century or the first half of the eighteenth century who could easily have been responsible for supplying this ring.

Signed on equinoctial ring: "Elias Allen fecit Londini" with straight capitals.

Judging by the towns listed, it would seem likely that this compendium belonged to an English merchant who had considerable trade with the Hanseatic league, or else to a member of the nobility who had occasion to travel widely in the Low Countries and the German states.

A4.

Location: Private collection[11]

Plates 21, 22

Brass astronomical compendium (82mm diameter x 25mm depth) with spherical stem pierced by suspension ring; comprising lunar volvelle, nocturnal, universal equinoctial dial, compass and list of towns with latitudes.

Outside of lid: nocturnal

Volvelle with fixed ring and rotatable discs, each with pointer (outer one shaped, inner one plain) with bevelled edge on left-hand side.

Fixed ring:

months by name ("Ianuari, Februari, March, Aprill, May, Iune, Iuly, August, September, October, Nouember, December"); numbered by 10; divided to 5; subdivided to 1.

Outer rotatable disc:

hour circle I - XII, I - XII clockwise; divided to 30 minutes. Bevelled edge of pointer at XII; other hours have small protruding teeth.

Inner rotatable disc:

radial slit from centre following line of bevelled edge of pointer.

Inside of lid

List of towns with latitudes set in concentric circles (from edge, clockwise):

1) "Cantarberi 51 10, Couentri 52 20, Notingham 53 0, Yorke 54 0, Edenburg 56 0, Amsterdam 52 26, Calles 50 45, Lisbona 38 30"
2) "London 51 32, Lecester 52 30, WChester[12] 53 10, Newcastell 54 30, Aberdien 57 20, Antwerp 52 15, Diep 49 40, Madrid 40 43"
3) "Oxford 51 45, Shrowesberi 52 40, Lincoln 53 30, Barwicke 55 0, Glasquo 56 10, Bruxells 50 45, Paris 48 50, Venecia 45 15"
4) "Cambridg 52 16, Glocester 52 0, Darby 53 0, Carlil 54 40, Dublin 53 30, Haga 52 12, Orleanc 48 0, Roma 42 2"

Universal equinoctial dial

Universal equinoctial dial pivoted on semicircular support.

Equinoctial ring:

Recto: hours clockwise I - XII, I - XII; divided to 30 minutes.

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[11] In present owner's possession for the last twenty years; previous owner - private collector in France.

[12] Presumably this is intended to refer to Chester.

Verso: hours VI - XII, I - VI in bottom half alone; divided to 30 minutes.
Shaped gnomon, with bevelled edge, attached to quadrant which is hinged to equinoctial ring and which moves through a slot in supporting semicircle. Quadrant divided anticlockwise [5°] - 90°; numbered by 10°; subdivided to 5° and to 1°.

Compass

Compass with printed and [somewhat carelessly] painted paper card.[13] Degree scale divided [0] - 360° clockwise from North; numbered by 10°; subdivided to 5°, to 2° and to 1°; coloured yellow. Wind ring divided into 64, coloured yellow, black, yellow, black, etc. Compass points shown by isosceles triangles split symmetrically. Cardinal points: N and E orange on left-hand side, yellow on right; S and W yellow on left-hand side, orange on right. Quadrantal points red. Intermediate points: NNW and SSE maroon; WNW and ESE beige; WSW and ENE blue; SSW and NNE green. By-points shown only by lines. Central brown circle. North point indicated by black fleur de lys. Blued steel needle, with arrowhead for North, cross for South; brass crosswires; both surmounted at centre by brass octagonal pyramid. Glass plate surmounting all.

Back of instrument: lunar volvelle

Date circle with named months clockwise (spellings as for nocturnal); numbered by 10; divided to 5; subdivided to 1.
Degree scale divided by zodiacal signs; divided to sign; subdivided to 10°, to 5° and to 1°. Beginning of Aries at 10$^1/_2$ March. Other equinox at 13 September.
Hour circle: I - XII, I - XII clockwise, divided to 30 minutes (with barred lines). XII line runs from hinge to clasp.
Compass circle: 16 points named by initial, by-points marked by single line.
Rotatable disc with projecting [to outer edge of dial] shaped pointer (with bevelled edge on left-hand side) and lunar month scale numbered [0] - [29]$^1/_2$ by 2; divided to 1.
Rotatable disc with projecting [to outer edge of dial] shaped pointer (with bevelled edge on left-hand side). Aspectarium (diagram showing the aspects of the planets - conjunction, opposition, trine, quadrature and sextile): lines radiating from base of pointer through geometric shapes (triangle, square and star). Circular aperture near edge of disc displaying the moon's phase.

Signed: "Elias Allen Fecit" with straight capitals.

Bibliography: Sotheby's (London), Catalogue of Sale 27th March, 1972, lot 105.

[13] The printing on the card is identical to that found on Cat. no. A2, indicating that they were produced from the same printer's-plate.

A5.

Location: BM (Registration no. 63 9-29 2; Ward Catalogue no. 362)

Plates 20, 23

Brass astronomical compendium (60mm diameter x 218m depth) with flat stem pierced by suspension ring; comprising lunar volvelle, nocturnal, universal equinoctial dial, compass and list of towns with latitudes.

Outside of lid: nocturnal[14]

Hour Circle, divided anticlockwise 1-12, 1-12; numbered by 1; subdivided to 15 minutes.

Circle of Months: named months ("Ianuarie, Februa:, March, Aprill, May, Iune, Iuly, August, September, October, Nouember, December"); numbered clockwise by 10; divided to 5.

Circle of Stars: "Luc ♈ ", "Ext ala", "Os peg", "vultur", "Cap oph", "Lanx bor", Spi ♍ ", Can ♌ ", "Cor ♌ ", "Can min", "Seg ori:", Ocu ♉ ".[15]

Index arm with bevelled edge on left-hand side and shaped pointer.

Inside of lid

List of towns with latitudes set in concentric circles (from edge, clockwise):

1) "Cantuarie 51 5, Edenburgum 56 0, Marsila 43 0, Neapolis 41 0, Amsterdamum 52 26, Heidelberga 49 22"
2) "Londinum 51 32, Diblinum 53 30, Madritum 41 0, Venecia 45 15, Antuerpia 51 16, Brunsuaga [Brunswick] 52 29"
3) "Oxonium 51 45, Burdegala [Bordeaux] 45 30, Florencia 43 5, Vienna 47 45, Bruxela 50 48, Brandenburga 52 23"
4) "Cantibrid: 52 16, Caletum [Calais] 50 40, Seuila 37 0, Geneua 45 30, Dantiscum [Danzig] 54 20, Norimberg 49 24"
5) "Eboracu: [York] 54 0, Lutecia [Paris] 48 50, Lisbona 40 0, Roma 42 5, Haga 52 5, Praga 50 5"

[14] This is the nocturnal illustrated in Oughtred, *Circles of Proportion* (engraving facing page 1).

[15] The bright star in the head of Aries, the wing tip of Pegasus, the mouth of Pegasus, the heart of the Vulture, the head of Ophiuchus, the North balance, Spica (in Virgo), the tail of Leo, the heart of Leo, the Little Dog, the latter shoulder of Orion, the eye of Taurus.

Universal equinoctial dial

Universal equinoctial dial pivoted on decorated semicircular support.

Equinoctial ring:

Recto: hours clockwise I - XII, I - XII; divided to 30 minutes.

Verso: hours VI - XII, I - VI in bottom half alone; divided to 30 minutes in numbered half.

Shaped gnomon, with bevelled edge, attached to quadrant which is hinged to equinoctial ring and which moves through a slot in supporting semicircle. Quadrant numbered anticlockwise [8°] - 90° by 10°; divided to 10°; subdivided to 5° and to 1°.

Compass

Compass with printed and painted paper card.[16] Degree scale divided [0] - 90° - [0] - 90° - [0]; numbered by 10; subdivided to 2°; coloured green. Cardinal points in blue with shaped heads (fleur de lys for North, cross for East, arrowhead for South, spade for West); quadrantal points in red, intermediate points in green, by-points in brown.[17] [Only cardinal points are named in full; quadrantal points named as "North W" etc.; other points named by initials only.] Three concentric rings (painted brown) with by-points radiating from outermost, intermediate points from middle ring and remaining points from inner circle. Steel needle, blue for North end, grey for South; mounted on central brass circle; no crosswires. Glass plate surmounting all.

Back of compass: lunar volvelle

Date circle with named months clockwise (spellings as for nocturnal except "Februari", "September", "Nouembe"); numbered by 10; divided to 2.

Degree scale divided by zodiacal signs; divided to sign; subdivided to 10° and to 2°. Beginning of Aries at 10 March. Other equinox at 13 September.

Hour circle: I - XII, I - XII clockwise, divided to 30 minutes. XII line runs from hinge to clasp.

Compass circle: 16 points named by initial; by-points marked by single line.

Rotatable disc with projecting shaped pointer (with bevelled edge on left-hand side) and lunar month scale numbered [0] - [29]$^1/_2$ by 2; divided to 1.

Rotatable disc with projecting shaped pointer (with bevelled edge on left-hand side). Aspectarium (diagram showing the aspects of the planets - conjunction, opposition,

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[16] The printing on this card is identical to that found on the Charles Whitwell compendium in MHS and on Cat. no. A3 (except that the latter does not carry a degree scale). This suggests that they were all produced from the same printer's-plate.

[17] The last colour may be the result of ageing; the same is probably true for the concentric rings.

trine, quadrature and sextile): lines radiating from base of pointer through geometric shapes (triangle, square and star). Circular aperture near edge of disc displaying the moon's phase.

Signed on equinoctial ring: "Elias Allen fecit" with straight capitals.

The Latin names for the towns and the relative lack of British towns in the list might indicate a continental owner.

Bibliography: Bibliography: F.A.B. Ward, *A Catalogue of European Scientific Instruments in the Department of Medieval and Later Antiquities of the British Museum*, (London, 1981), p.126.

A 6.

Location: BM (Registration no. 1926 10-16 8; Ward Catalogue no. 363)
Plate 24

Incomplete brass astronomical compendium (59mm diameter x 218m depth) with lunar volvelle, part of nocturnal and list of towns with latitudes. Compass and equinoctial ring dial missing.

Outside of lid: nocturnal

Date circle with named months ("Ianuari, Februari, March, Aprill, May, Iune, Iuly, August, September, Octo:, Noue: [these two months are either side of the hinge and so there is insufficient room for the complete name], December"); divided to 10; subdivided to 2.
Hole in centre of lid.[18]

Inside of lid

List of towns with latitudes set in concentric circles (from edge, clockwise):

1) "Cantuaria 51 0, Edenburgum 56 0, Burdegala [Bordeaux] 45 50, Lisbona 39 40, Venecia 45 15, Heidelberga 49 35"
2) "Londinum 51 32, Dublinum 53 30, Marsilia 43 0, Florentia 43 5, Vienna 47 45, Brunsuaga [Brunswick] 52 30"
3) "Oxonia 51 45, Caletum [Calais] 50 40, Madritum 41 0, Neapolis 41 0, Geneua 45 30, Bruxela 51 25"
4) "Eboracu: 54 0, Lutecia [Paris] 48 50, Seuila 37 0 , Moscoua 55 30, Roma 42 5, Haga 51 20"

Lunar volvelle

Date circle with named months clockwise (spellings as for nocturnal except "Februar:", "October", "Nouember"); numbered by 10; divided to 2.
Degree scale divided by zodiacal signs; divided to sign; subdivided to 10° and to 2°. Beginning of Aries at 10 March. Other equinox at 13 September.
Hour circle: I - XII, I - XII clockwise, divided to 30 minutes (with barred lines). XII line runs from hinge to clasp.
Compass circle: 16 points named by initial; by-points marked by single line.
Rotatable disc with projecting shaped pointer (with bevelled edge on left-hand side) and lunar month scale numbered [0] - [29]$^1/_2$ by 2; divided to 1.

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[18] This nocturnal was presumably of the same type as that on Cat. no. A4.

Inner rotatable disc missing, thus allowing spiral of lunar representation to be seen. There is no signature (presumably it would have been on the equinoctial dial) but the style of the engraving marks this instrument as a product of the Allen workshop.

The Latin names for the towns and the relative lack of British towns in the list might indicate a continental owner.

Bibliography: Bibliography: F.A.B. Ward, *A Catalogue of European Scientific Instruments in the Department of Medieval and Later Antiquities of the British Museum*, (London, 1981), p.126.

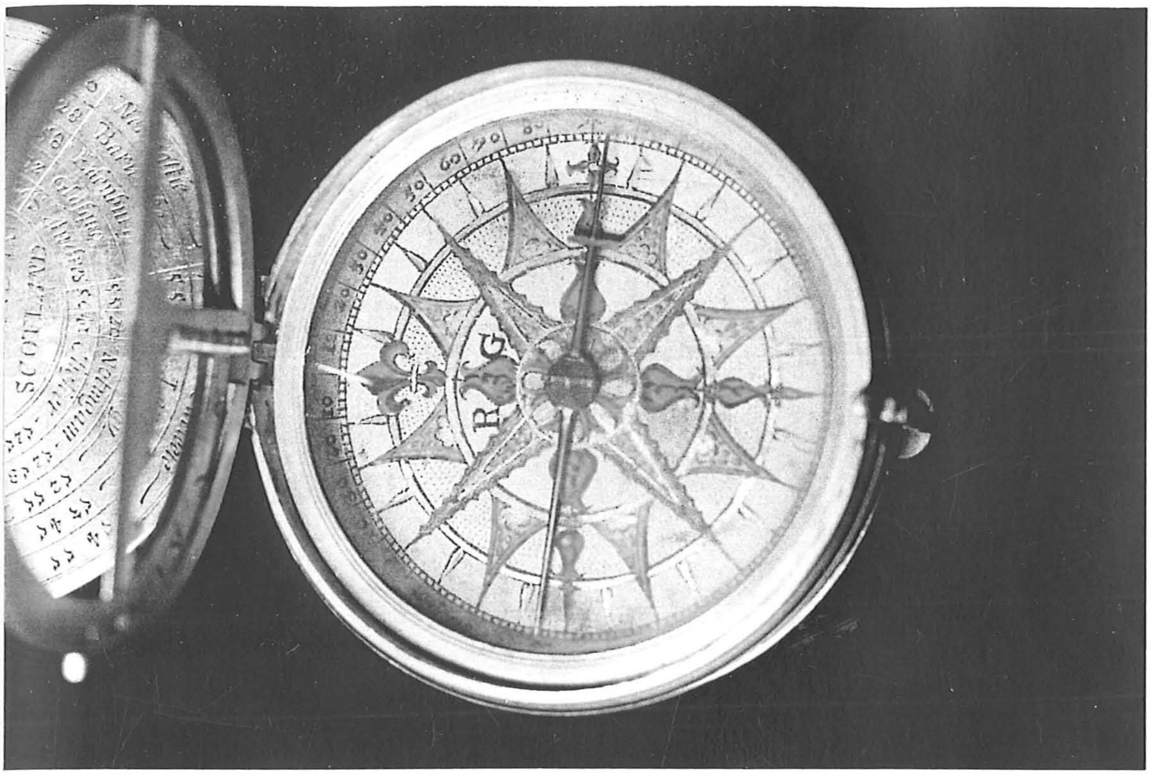

Plate 17: Cat. no. A1 (compass)

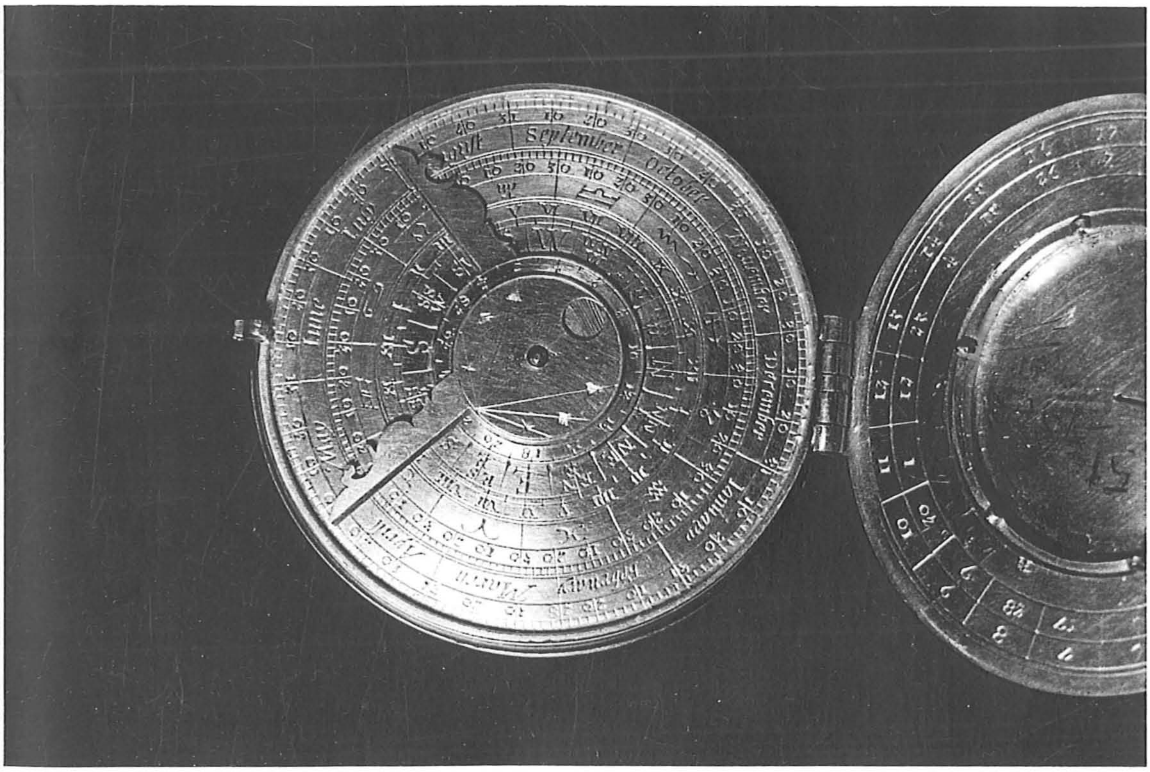

Plate 18: Cat. no. A1 (lunar volvelle)

Plate 19: Cat. no. A3

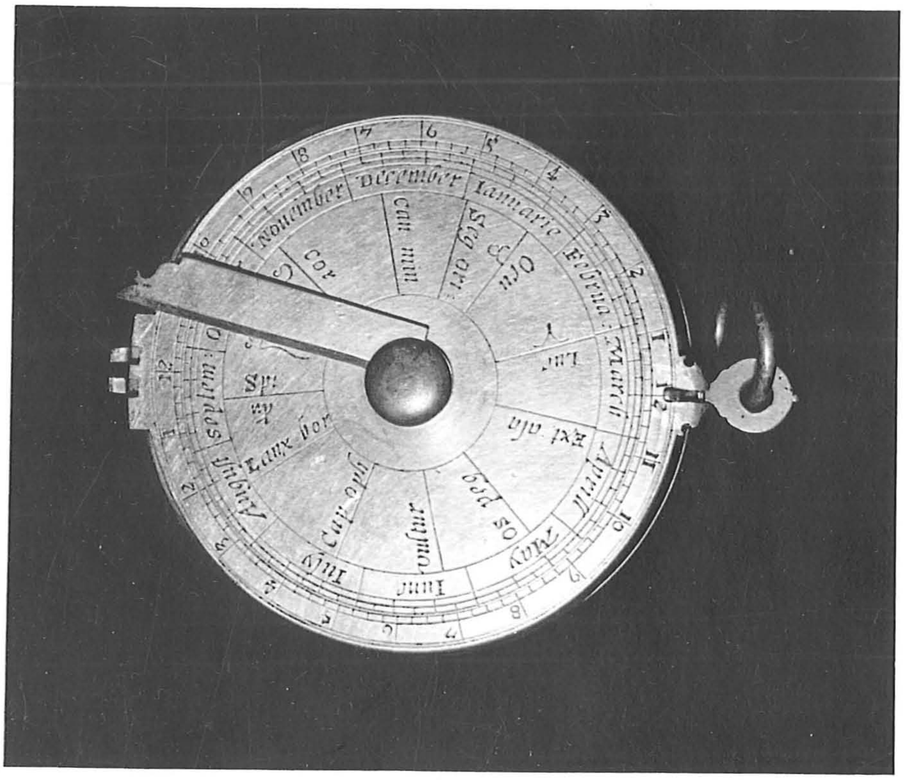

Plate 20: Cat. no. A5 (nocturnal)

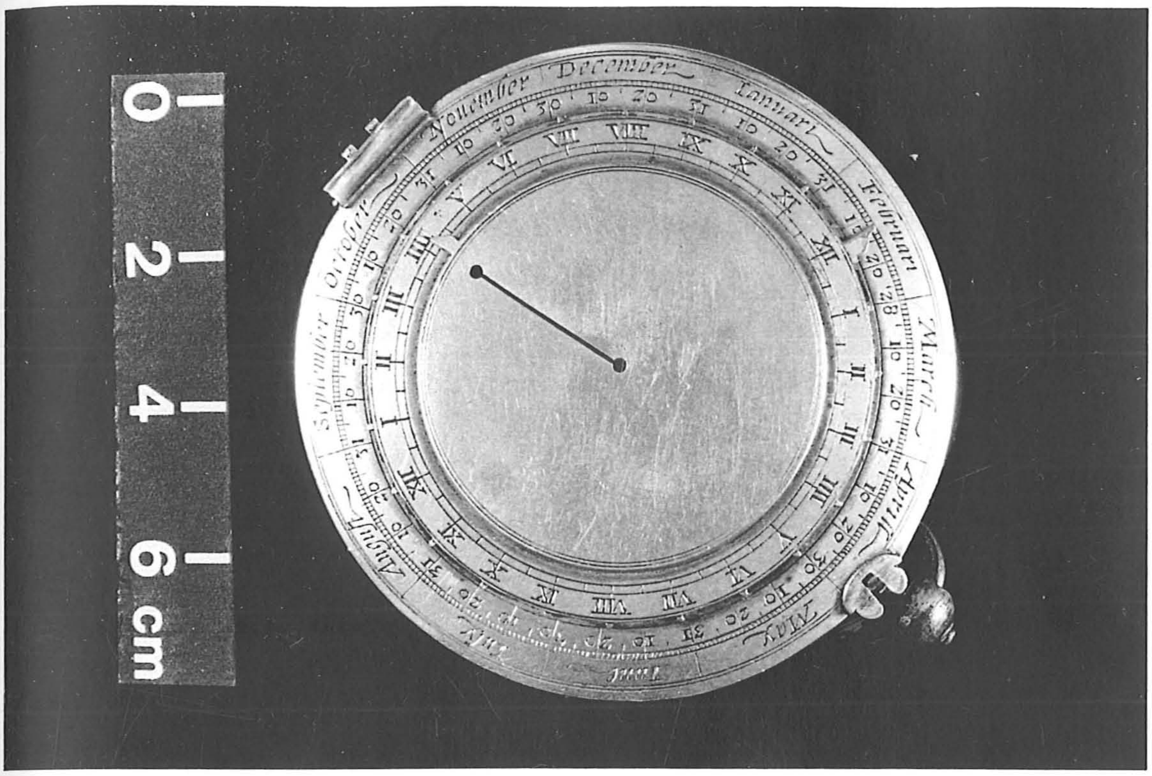

Plate 21: Cat. no. A4 (nocturnal)

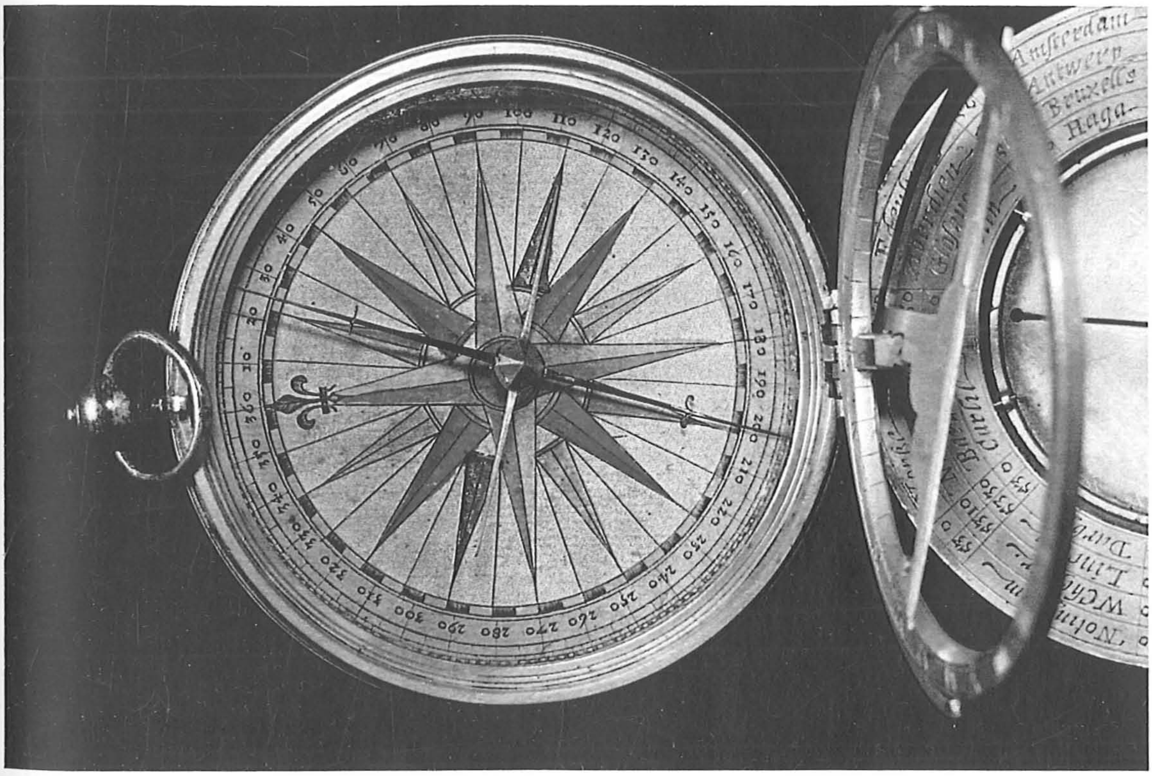

Plate 22: Cat. no. A4 (compass)

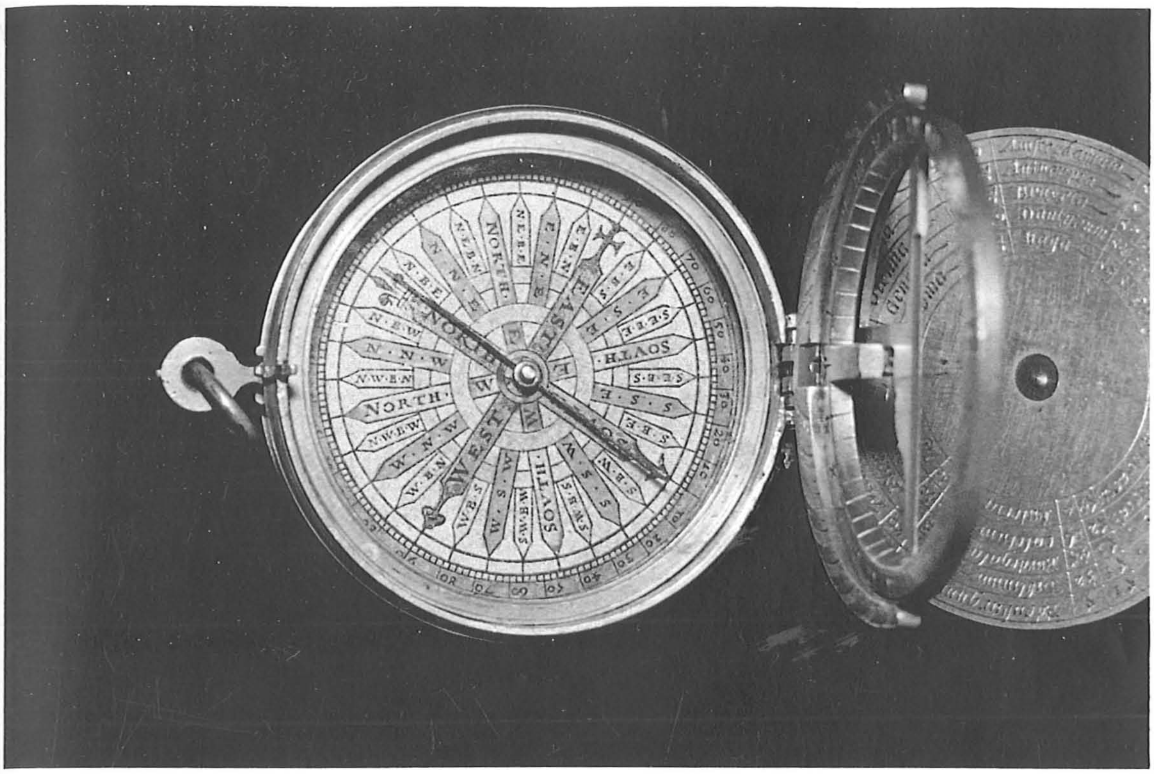

Plate 23: Cat. no. A5 (compass)

Plate 24: Cat. no. A6 (lunar volvelle (incomplete))

Double Horizontal Dials

These are all examples of the design produced by Oughtred and described in *The Description and Use of the Double Horizontall Dyall* (London, 1652).

D1.

Location: SM (Identification no. 1921-693)
Plate 25

Brass octagonal dial for latitude $51^1/2$°N;[19] side length 105mm (as if cut from square of side 260mm); brass gnomon.

Following concentric circles engraved (working from rim towards centre):

1) Minutes circle, unnumbered; divided to 30 minutes; subdivided to 15 minutes and to 3 minutes. Around meridian line there is a gap (width 3.5mm) between 60 of previous hour and 0 of next hour.[20]
2) Hour circle, numbered IIII - XII, I - VIII clockwise; divided to 30 minutes (with barred lines); subdivided to 15 minutes.
3) Continuation of hour line divisions with numbering 12, 1 - 12, 1 - 4. The words "A Moone Diall" engraved between 4 and 12. (NB. Hour lines only extend across circles 1 - 3.)
4) Degree circle, divided clockwise from meridian line 90° - [0] - 90° - [0] - 90°; numbered by 10°; subdivided to 5°, and to 1°.

Within degree circle is set Oughtred's horizontal projection:
Lines of declination, uncalibrated, tropic to tropic through equator; divided to 10°; subdivided to 2°.
Circle of ecliptic with named months ("Ianuari, Februari, March, Aprill, May, Iune, Iuly, August, September, October, Nouember, December"); numbered by 10; divided to 5; subdivided to 1. Equinoxes at 10 March and 13 September; solstices at $11^1/2$ December and 11 June.
Hour lines, perpendicular to lines of declination and only drawn between the Tropics; numbered (in Arabic numerals) by hour; divided to 15 minutes. Delineated approx. 3:45am - 8.15pm at summer solstice; 8.15am - 3.45pm at winter solstice. Along

[19] The latitude was calculated from the point on the solar altitude line which is at the same distance from the centre as the equatorial line where it passes across the meridian line. The value on the altitude line would give the solar altitude at midday on the equinox, and the complement of this value is the latitude.

[20] This probably indicates that one edge of the gnomon shadow is to be used for the morning hours and the other edge for the evening hours while the hour of noon is indicated when the shadow of the gnomon falls exactly within the gap in the minute scale.

equator hour lines numbered, right to left, [0] - 12 and back (left to right) to 24. [NB. Mistakes at 7 and 17: 7 is in 12 - 24 line and 17 is in 1 - 12 line.]
Above Tropic of Capricorn, following arc of degree circle: various stars named with their right ascensions: "Pleides * 3^h 1/2, Bulls eye * 4^h 1/4, Great Dog * 6^h 1/2, Lions heart * 9^h 3/4, [new line] Vindemiatrix * 12^h 1/4, Arcturus * 14, ♏ heart * ^{h} 16".
Between hours and gnomon (following arc of declination lines):

1) "Age ☽" [so described at both ends]; numbered 5 - 25 by 1; divisions either side of each number, which carry through into scale 2.
2) "Houre" [so named at both ends]; numbered "8, 9, 10, 11, 12, 12 1/2, 1, 2, 2 1/2, 3, 4, 5, 6, 6 1/2, 7, 8, 9, 10, 11, 12 1/2, 1"
3) Continuation of list of stars: "the Goate * 4^h 3/4, Harp Star * 18^h 1/2, Vultur * 19^h 1/2, Fomahant * 22^h 1/2, Marchab * 22^h 1/2".

Construction line on meridian line with centres from which circles of declination have been drawn.

Line marked for solar altitude runs from 50° on lower left quadrant of degree scale to centre.[21] Divided [0] - 62°; numbered by 10°; subdivided to 5° and to 1°.

Simple brass gnomon (width 3mm): central bevelled vertical edge for use with horizontal projection; conventional polar gnomon with shaped tip for use with main dial.
Circular screw holes at points of octagon and either side of gnomon for attachment to base.
Signature in hour circle: "Elias Allen fecit" with italic capitals.

Bibliography: A.J. Turner, 'William Oughtred, Richard Delamain and the Horizontal Instrument in Seventeenth Century England' in *Annali dell'Istituto e Museo di Storia della Scienza di Firenze*, anno vi (1981), pp.99-125.

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[21] This line, in conjunction with a pair of compasses, would have been used with the horizontal projection.

D2.

Location: NMM (Identification no. D379 A72-14)

Plate 26

Brass octagonal dial for latitude 51°N;[22] side length 125mm (as if cut from square of side 309mm); brass gnomon.

Following concentric circles engraved (working from rim towards centre):

1) Minutes circle, numbered 10, 20,..., 60 clockwise for each hour; divided to 30 minutes; subdivided to 5 minutes and to 1 minute. Around meridian line there is a gap (width 4mm) between 60 of previous hour and 0 of next hour.[23]
2) Hour circle, numbered IIII - XII, I - VIII clockwise; divided to 30 minutes (with barred lines); subdivided to 15 minutes. (NB. Hour lines only extend across hour and minute circles.)
3) Compass circle with all thirty-two compass points named ("N, NBE, NNE, NEBN, NE, NEBE, ENE, EBN, E" etc.). South corresponds with XII line.
4) Degree circle, numbered clockwise 40° - 0 - 90° - 0 - 40° (although scale extends to 45° corresponding to NE and NW points); divided to 10°; subdivided to 5° and to 1°.

Within degree circle is set Oughtred's horizontal projection:

Lines of declination, uncalibrated, tropic to tropic through equator; divided to 10°; subdivided to 2°.

Circle of ecliptic with named months ("Ianuari, Februari, March, Aprill, May, Iune, Iuly, August, September, October, Nouember, December"), numbered by 10; divided to 5; subdivided to 1. Equinoxes at 10 March and 13 September; solstices at 11 1/2 December and 11 June.

Hour lines perpendicular to lines of declination and only drawn between the Tropics; numbered (in Roman numerals) by hour, divided to 15 minutes. Delineated approx. 3:45am - 8.15pm at summer solstice; 8.15am - 3.45pm at winter solstice.

Construction line on meridian line with centres from which circles of declination have been drawn.

Very faint line marked for solar altitude runs from "NE" point of degree circle to centre.[24] Numbered 10, 20, 3[0] [nothing more is visible], divided to 10°; subdivided to 5° and to 1°.

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[22] See footnote 19.
[23] See footnote 20.
[24] See footnote 21.

Simple brass gnomon (width 4mm): central bevelled vertical edge for use with horizontal projection; conventional polar gnomon for use with main dial.
Circular screw holes at points of octagon for attachment to base.

Signature in hour circle: "Elias Allen fecit" with italic capitals.

Bibliography: A.J. Turner, 'William Oughtred, Richard Delamain and the Horizontal Instrument in Seventeenth Century England' in *Annali dell'Istituto e Museo di Storia della Scienza di Firenze*, anno vi (1981), pp.99-125; *An Inventory of the Navigation and Astronomy Collections in the National Maritime Museum Greenwich* (London, 1983), vol. 2, p.29-3.

D3.

Location: MHS (Identification no. 43-5)

Plate 27

Brass circular dial for latitude 51°N;[25] diameter 315mm; brass gnomon.

Following concentric circles engraved (working from rim towards centre):

1) Minute circle, numbered 10, 20,..., 60 clockwise for each hour; divided to 10 minutes; subdivided to 5 minutes and to 1 minute. Around meridian line there is a gap (width 3.5mm) between 60 of previous hour and 0 of next hour.[26]
2) Hour circle, numbered IIII - XII, I - VIII clockwise; divided to 30 minutes (with barred lines); subdivided to 15 minutes. Hour lines only extend across hour and minute circles.
3) Compass circle with all thirty-two compass points named ("N, NBE, NNE, NEBN, NE, NEBE, ENE, EBN, E" etc.). South corresponds with XII line.
4) Degree circle, numbered clockwise 40° - 0 - 90° - 0 - 40°; divided to 10°; subdivided to 5° and to 1°.

Within degree circle is set Oughtred's horizontal projection.

Lines of declination, uncalibrated, tropic to tropic through equator; divided to 10°, subdivided to 2°.

Circle of ecliptic with named months ("Ianuarii, Februari, March, Aprill, May, Iune, Iuly, August, September, October, Nouember, December"); numbered by 10; divided to 5; subdivided to 1. Equinoxes at 10 March and 12½ September.

Hour lines perpendicular to lines of declination and only drawn between the Tropics; numbered (in Roman numerals) by hour; divided to 15 minutes. Delineated approx. 3:45am - 8.15pm at summer solstice; 8.15am - 3.45pm at winter solstice.

Construction line on meridian line with centres from which circles of declination have been drawn.

Simple brass gnomon (width 3mm): central bevelled vertical edge for use with horizontal projection; conventional polar gnomon for use with main dial.

Three round threaded screw holes for attachment to base. One in degree circle (next to "N"); remaining two in compass circle (between "SW" and "SWBW" and between "ESE" and "SEBE").

Signature in hour circle: "Elias Allen fecit" with italic capitals.

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[25] See footnote 19.

[26] See footnote 20.

The engraving is in very poor condition, presumably due to prolonged outdoor exposure. This has completely obliterated the solar altitude scale.

Bibliography: A.J. Turner, 'William Oughtred, Richard Delamain and the Horizontal Instrument in Seventeenth Century England' in *Annali dell'Istituto e Museo di Storia della Scienza di Firenze*, anno vi (1981), pp.99-125.

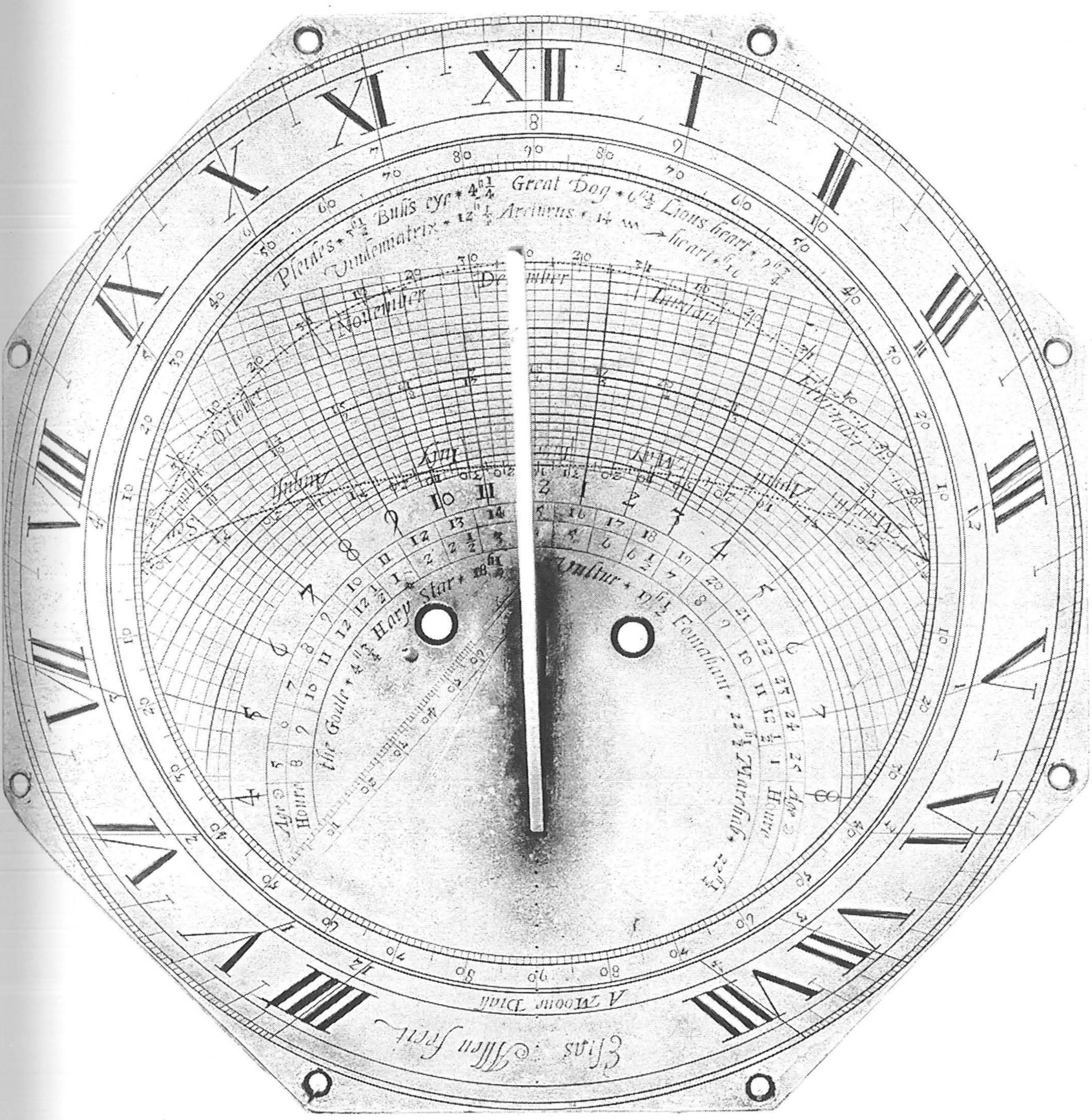

Plate 25: Cat. no. D1

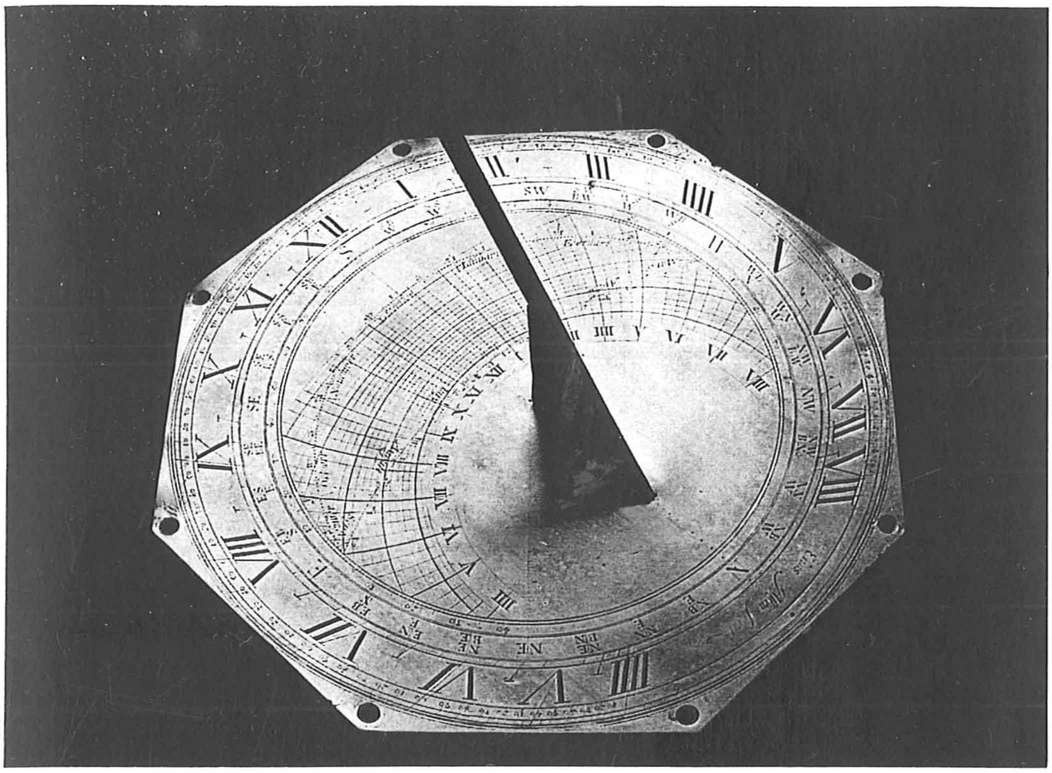

Plate 26: Cat. no. D2

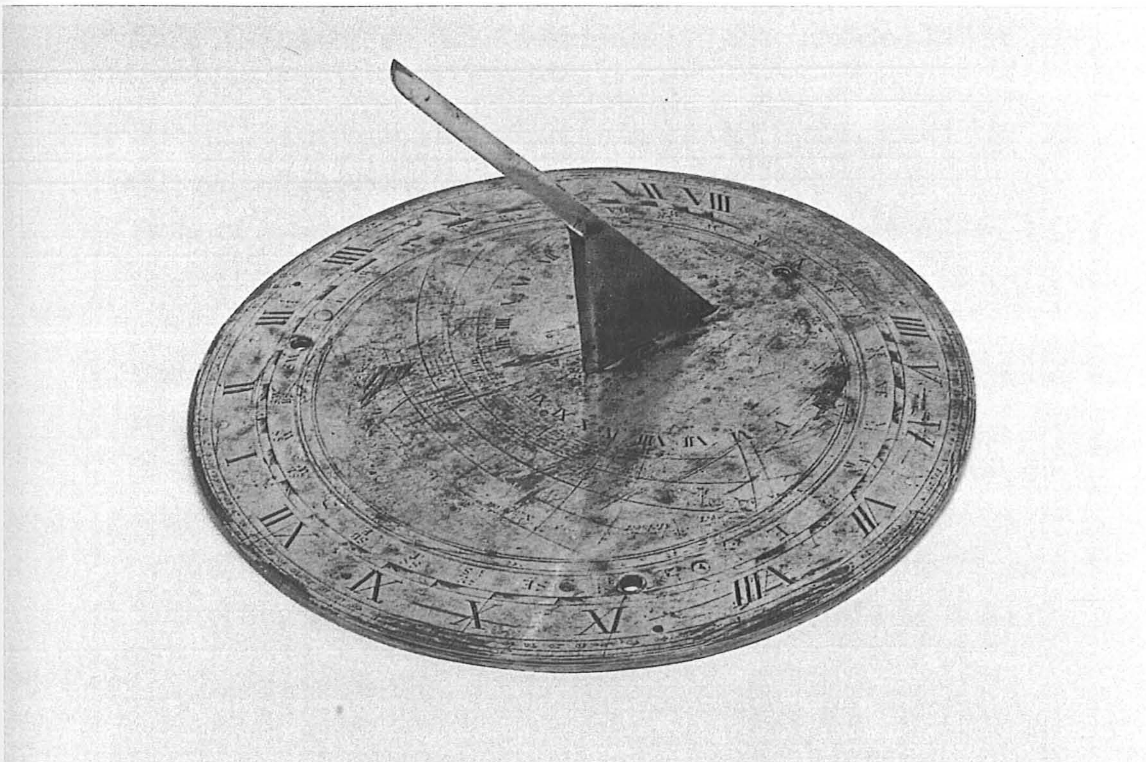

Plate 27: Cat. no. D3

Circles of Proportion and Horizontal Instruments

The two known examples combine these two instruments and so are considered together. They are realisations of the instrument described in Oughtred's *The Circles of Proportion and the Horizontal Instrument* (London, 1632). The engraving closely follows the illustrations in this book, which are clearly also Allen's work.[27]

C1.

Location: Whipple (Accession no. 833)
Plates 3, 28, 29, 32

Circular brass disc with suspension ring and shackle. One side (circles of proportion) with double index arm, other side with single. Diameter 314mm; thickness 2mm.

Recto

Circles of proportion of Oughtred's design, with the following concentric scales engraved on disc (working from rim towards centre):

1) Scale of Sines, divided anticlockwise 6° - 90°; numbered by 10°. 6° - 60° subdivided to 5°, to 1°, to 30′ and to 5′. 60° - 70° subdivided to 5°, to 1°, to 30′ and to 10′. 70° - 80° subdivided to 5°, to 1° and to 30′. 80° - 85° subdivided to 5° and to 1°.
2) Scale of Tangents, divided anticlockwise 6° - 45°; numbered 10°, 20°, 30°, 40°, 45°; subdivided to 5°, to 1°, to 30′ and to 5′.
3) Scale of Tangents, divided anticlockwise 45° - 84°; numbered 45°, 50°, 60°, 70°, 80°, 84°; subdivided to 5°, to 1°, to 30′ and to 5′.
4) Scale of Numbers, divided anticlockwise 1 to 10; numbered by 1. 1 - 7 subdivided to 0.5, to 0.1, to 0.05 and to 0.01. 8 - 10 subdivided to 0.5, to 0.1, to 0.05 and to 0.02.
5) Scale of Equal Numbers, divided anticlockwise 1 to 10; numbered by 1; subdivided to 0.5, to 0.1, to 0.05 and to 0.01.
6) Scale of Tangents, divided anticlockwise 84° - 89°20′; numbered by 1°; subdivided to 30′ and to 5′.
7) Scale of Tangents, divided anticlockwise 40′ - 6°; subdivided to 30′ and to 5′.
8) Scale of Sines, divided anticlockwise 40′ - 6°; subdivided to 30′ and to 5′.

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[27] Facing page 1 for the circles of proportion, facing page 113 for the horizontal instrument.

Nocturnal:

Circle of Hours, divided anticlockwise 1-12, 1-12; numbered by 1; subdivided to 15 minutes.

Circle of Months with named months ("Ianuarie, Februarie, March, Aprill, May, Iune, Iuly, August, September, October, Nouember, December"); numbered clockwise by 10; divided to 5 days; subdivided to 1.

Circle of Stars; stars mentioned are "Luc ♈", "Ext ala", "os peg", "vultur", "Cap oph", "Lanx bor", "Spi ♍", "Can ♌", "Cor ♌", "Can min", "Seg ori", "Ocu ♉".[28]

Two index arms with bevelled edges, moving together with friction-tight adjustment in a flat hinge; marked "S, T, T, N, E, T, T, S" to correspond to scales 1 - 8.

Verso

Oughtred's horizontal instrument[29] for 52°N.[30]

Degree circle divided 90° - 0° - 90° - 0° - 90°; numbered by 10°; divided to 5°; subdivided to 1° and to 15′.

Lines of declination, uncalibrated, tropic to tropic through equator; divided to 1°; subdivided to 30′ on 20-minute and 40-minute time lines.

Circle of ecliptic with named months (as on recto); numbered by 10; divided to 5; subdivided to 1. Equinoxes at 10 March and 12 September.

Hour lines perpendicular to lines of declination and only drawn between the Tropics; numbered by hour; subdivided to 4 minutes. Delineated 3:44am - 8.16pm at summer solstice; 8.16am - 3.44pm at winter solstice.

Construction line down centre of instrument with centres from which circles of declination have been drawn. Three named points: "PII", "PW", "PI".

Single index arm with bevelled edge, marked for solar altitude towards centre 0 - 62°; numbered by 10°; divided to 5°; subdivided to 1° and to 30′.[31]

Signed on verso: "Elias Allen fecit" with italic capitals.

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[28] The bright star in the head of Aries, the wing tip of Pegasus, the mouth of Pegasus, the heart of the Vulture, the head of Ophiuchus, the North balance, Spica (in Virgo), the tail of Leo, the heart of Leo, the Little Dog, the latter shoulder of Orion, the eye of Taurus.

[29] This is a projection of the upper hemisphere on the plane of the horizon.

[30] See footnote 31 below.

[31] When the index arm lies along the meridian line, the equator coincides with 38° of solar altitude, thus indicating a latitude of 52°.

Bibliography: A.J. Turner, 'William Oughtred, Richard Delamain and the Horizontal Instrument in Seventeenth Century England' in *Annali dell'Istituto e Museo di Storia della Scienza di Firenze*, anno vi (1981), pp.99-125; D.J. Bryden, *The Whipple Museum of the History of Science, Catalogue 6. Sundials and related instruments* (Cambridge, 1988), Cat. no. 339; R.T. Gunther, *Early Science in Cambridge* (Oxford, 1937), pp.35 & 183.

C2.

Location: MHS (Identification no. 24-32)

Plates 30, 31

Circular brass disc with broken shackle. Single index arm/alidade on circles of proportion side.[32] Diameter 465mm; thickness 2mm.

Recto

Circles of proportion of Oughtred's design, with the following concentric scales engraved on disc (working from rim towards centre):

1) Scale of Sines, divided anticlockwise 6° - 90°; numbered 6°, 7°, 8°, 9°, 10°, 15°, 20°,..., 80°, 90°; divided to 10°. 6° - 65° subdivided to 5°, to 1°, to 30′ and to 5′. 65° - 80° subdivided to 5°, to 1°, to 30′ and to 10′. 80° - 85° subdivided to 5°, to 1° and to 30′.
2) Scale of Tangents, divided anticlockwise 6° - 45°; numbered 6°, 7°, 8°, 9°, 10°, 15°, 20°,..., 45°; divided to 10°; subdivided to 5°, to 1°, to 30′ and to 5′.
3) Scale of Tangents, divided anticlockwise 45° - 84°; numbered 45°, 50°, 55°,..., 80°, 81°, 82°, 83°, 84°; subdivided to 5°, to 1°, to 30′ and to 5′.
4) Scale of Numbers, divided anticlockwise 1 to [10]; numbered by 1; subdivided to 0.5, to 0.1, to 0.05 and to 0.01.
5) Scale of Equal Numbers, divided anticlockwise [0] to 10; numbered by 1; subdivided to 0.1, to 0.05 and to 0.01.
6) Scale of Equal Numbers (set back to back with Scale 5), divided anticlockwise [0] - 50 and then 60 - 100 on same scale [i.e. 60 corresponds to 10, etc.]; numbered by 10; subdivided to 5, to 1, to 0.5 and to 0.1.
7) Scale of Tangents, divided anticlockwise 84° - 89°20′; numbered by 1°; subdivided to 30′ and to 5′.
8) Scale of Tangents, divided anticlockwise 0°40′ - 6°; numbered by 1°; subdivided to 30′ and to 5′.
9) Scale of Sines, divided anticlockwise 0°40′ - 6°; numbered by 1°; subdivided to 30′ and to 5′.

[32] This arm is probably meant to be mounted on the side with the horizontal instrument.

Within the Scale of Sines is the Rojas projection of the celestial sphere.

Degree scale divided 90° - [0] - 90° - [0] - 90°; numbered by 10°; subdivided to 5°, to 1° and to 30′.

Equator divided 0 - 90° (centre to circumference), 90° - 270° (back again across centre to far side), 270° - 360° (back again to centre); numbered by 10°; subdivided to 1°.

Ecliptic inclined at 23 1/2°; calibrated by zodiacal sign; divided to 10°; subdivided to 5° and to 1°.

Lines of celestial latitude (parallel to equator) uncalibrated 0 - 70° by 1°.

Meridians/hour circles drawn across parallels of latitude, numbered left to right [12], 11, 10,..., 1, [0] and back again, at 25°N and 25°S. Slip at right-hand end of southern scale down first to 26°S and then to 27°S. Lines drawn for every 20 minutes (or 5°).

Verso

Oughtred's horizontal instrument[33] for 52°N.[34]

Degree circle divided 90° - [0] - 90° - [0] - 90°; numbered by 10°; subdivided to 5°, to 1° and to 15′. Also hours marked clockwise 4 - 12, 1 - 8; numbered by 1; subdivided to 20 minutes and to 4 minutes. Gap in scale at 12 (between 60 of previous hour and 0 of next hour) [4am and 8pm correspond to 55 1/2°S].

Lines of declination, uncalibrated, tropic to tropic through equator; divided to 1°.

Circle of ecliptic with named Latin months ("Ianuaius, Februarius, Martius, Aprilis, Maius, Iunius, Iulius, Augustus, September, October, Nouember, December"); numbered by 10; divided to 5; subdivided to 1. Equinoxes at 10 March and 13 September; solstices at 11 1/2 December and 11 1/2 June.

Hour lines perpendicular to lines of declination and only drawn between the Tropics; numbered by hour, 4 - 12 - 8; subdivided to 4 minutes. Delineated 3:44am - 8.16pm at summer solstice, 8.16am - 3.44pm at winter solstice.

Construction line down centre of instrument with centres from which circles of declination have been drawn. Three named points: "PII", "PW", "PI".

Below projection: inscription "GEORGIVS BARKHAM; FILIVS D. IOHANNIS
COMMENSALIS; BARKHAM S.T.P
DONO DEDIT A° 1635"

[33] See footnote 29.

[34] See footnote 35 below.

Below this is an engraving of a coat of arms; surmounted by bust with tasselled cap. Shield has seven vertical divisions, the second, fourth and sixth of which are cross-hatched at top and bottom. Motto: "RECTA, CERTA"
Index arm (width 12.5mm, diameter of central circle 25mm, thickness of non-bevelled edge 1.5mm) with bevelled edge and shaped ends; extending across whole of disc; marked for solar altitude on bevelled edge of each bar towards centre 0 - 62°; numbered by 10°; divided to 5°; subdivided to 1°.[35] Raised rectangles (13 x 3.5 x 1mm) at 30° for erection of sights [missing].

Signed on verso: "Elias Allen fecit" with italic capitals in space at top of projection.

The so-called Rojas projection of the sphere was probably Arabic in origin. It derives from Ptolemy's analemma and is an orthographic or orthogonal projection of the celestial sphere onto the colura of the solstices. For further details see the article 'Hugo Helt and the Rojas Astrolabe Projection' by Francis Maddison in *Junta de Investigações do Ultramar* (Coimbra, 1966).

Bibliography: R.T. Gunther, *Early Science in Oxford* (Oxford), vol.1 (1921), pp.147-156, vol.2 (1923), pp.140-142; A.J. Turner, 'William Oughtred, Richard Delamain and the Horizontal Instrument in Seventeenth Century England' in *Annali dell'Istituto e Museo di Storia della Scienza di Firenze*, anno vi (1981), pp.99-125.

[35] When the index arm lies along the meridian line, the equator coincides with 38° of solar altitude, thus indicating a latitude of 52°.

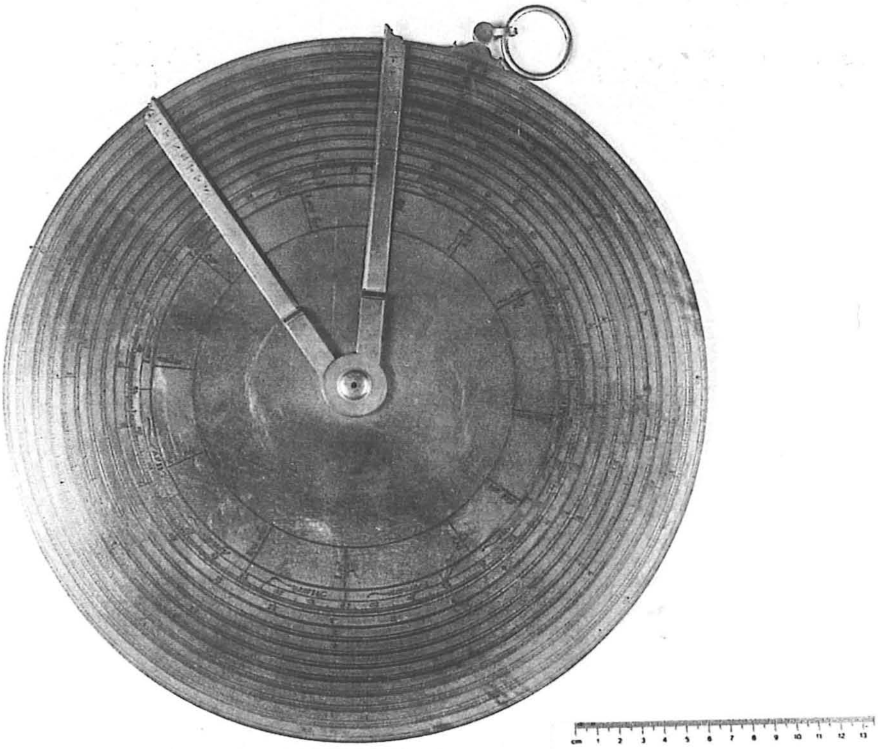

Plate 28: Cat. no. C1 (circles of proportion)

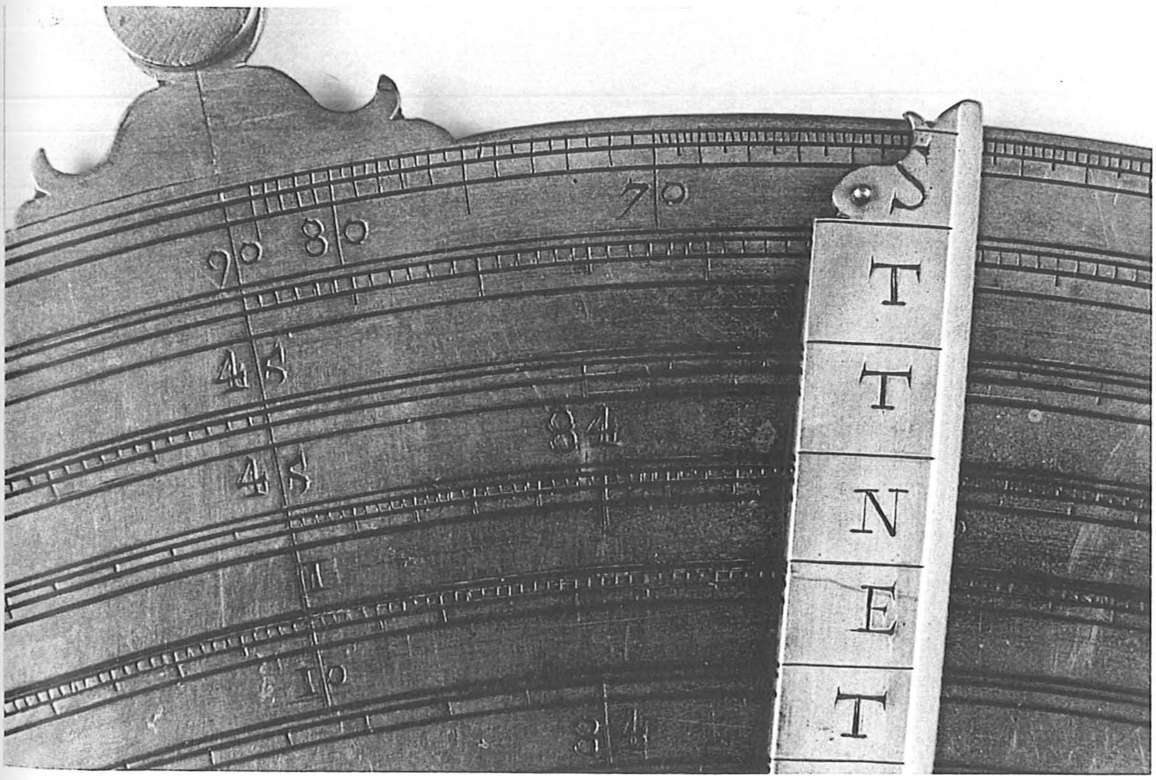

Plate 29: Detail from Cat. no. C1, showing scales and index arm from circles of proportion

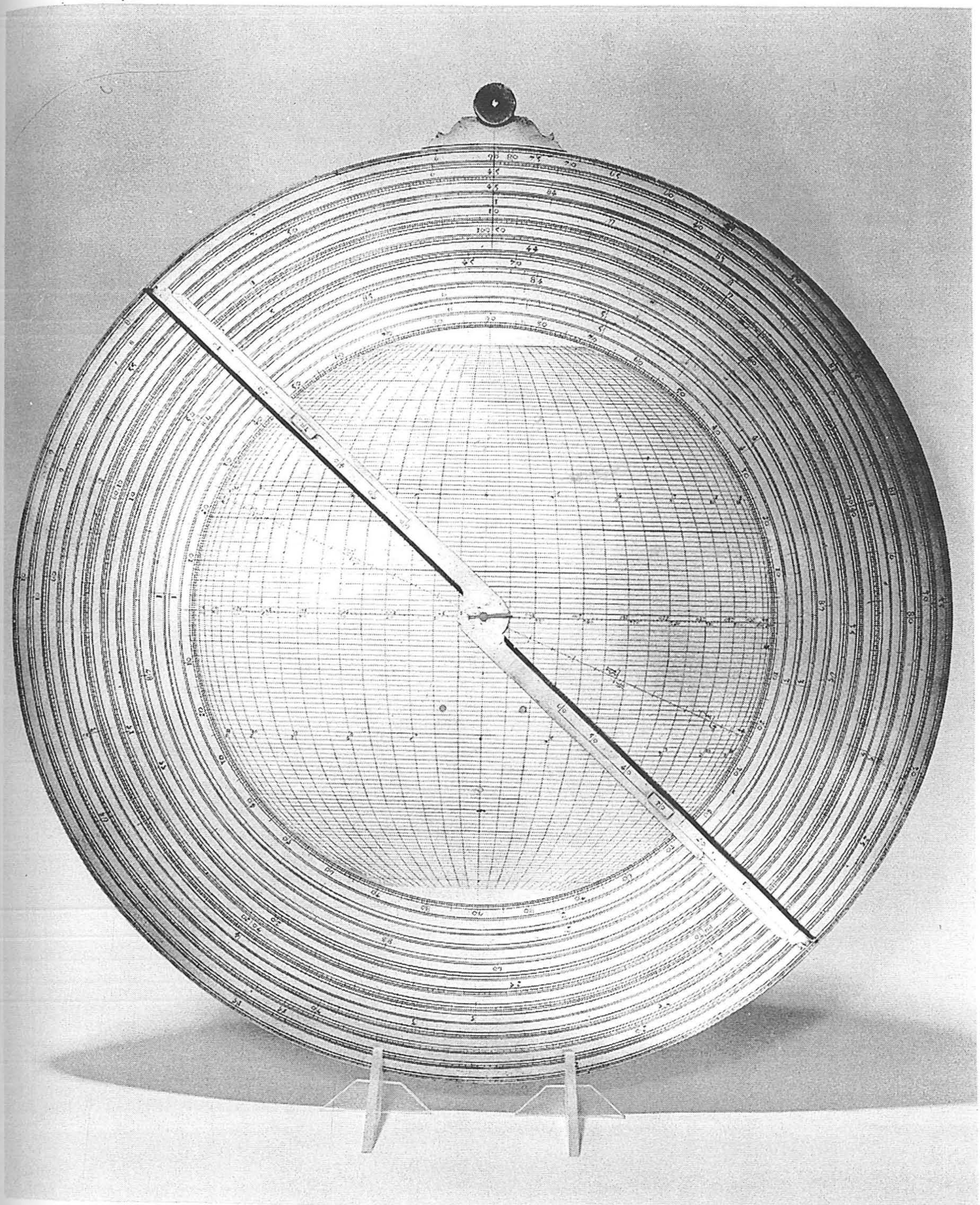

Plate 30: Cat. no. C2 (circles of proportion)

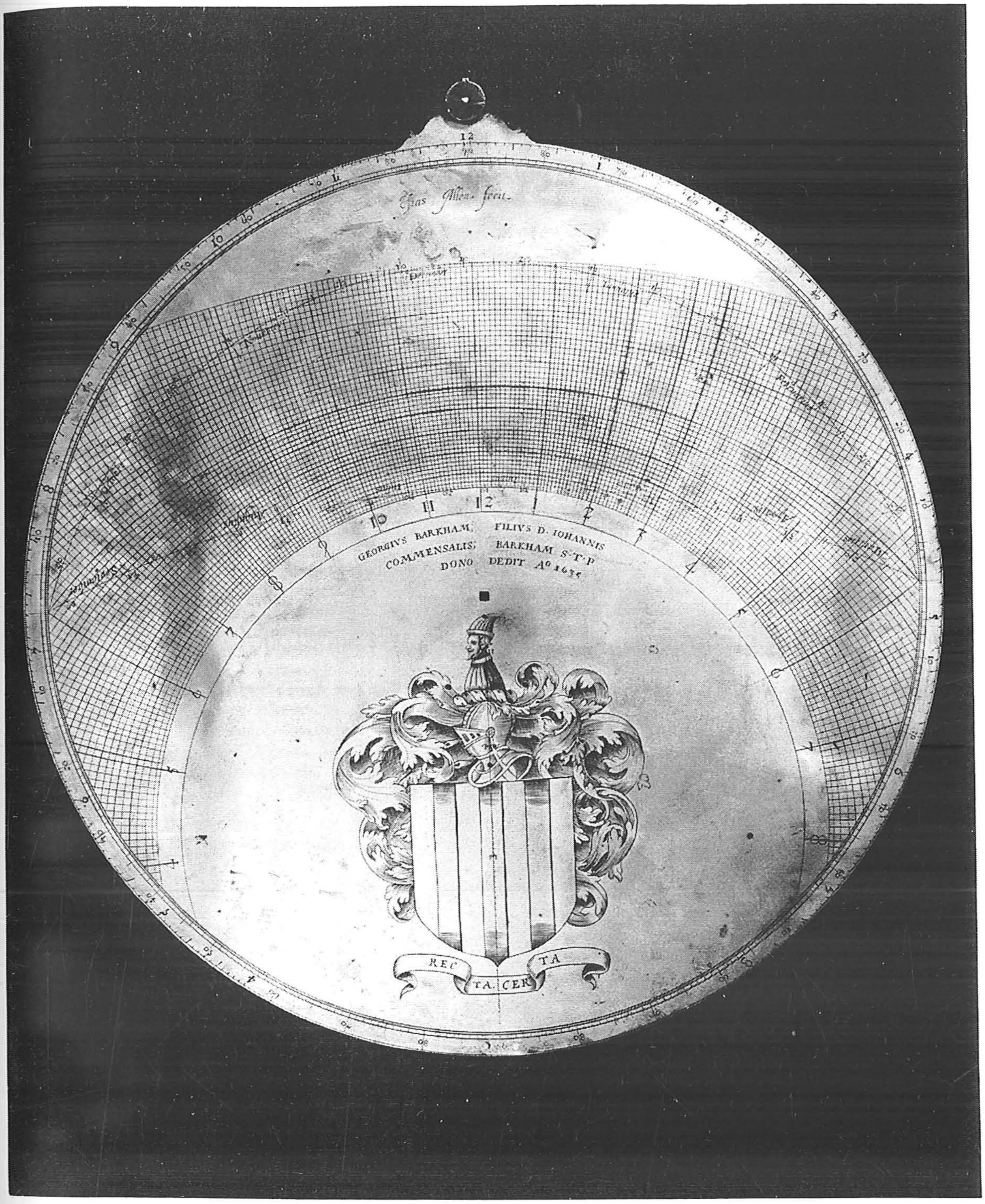

Plate 31: Cat. no. C2 (horizontal instrument)

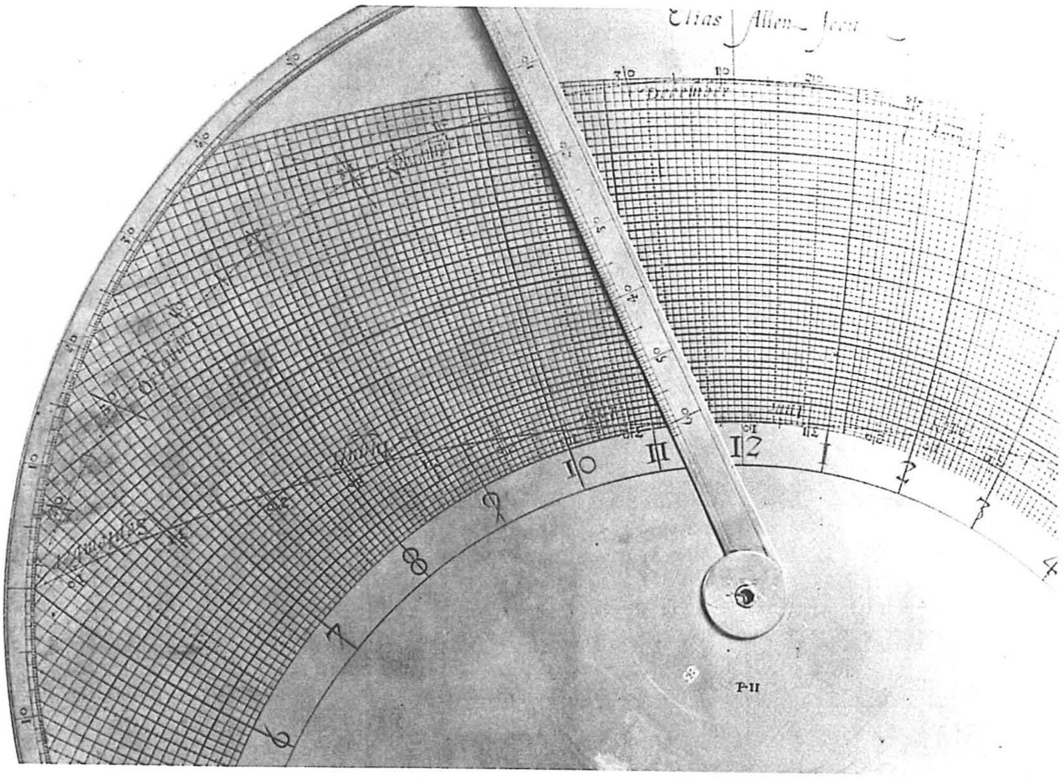

Plate 32: Detail from Cat. no. C1, showing part of horizontal projection and solar altitude scale

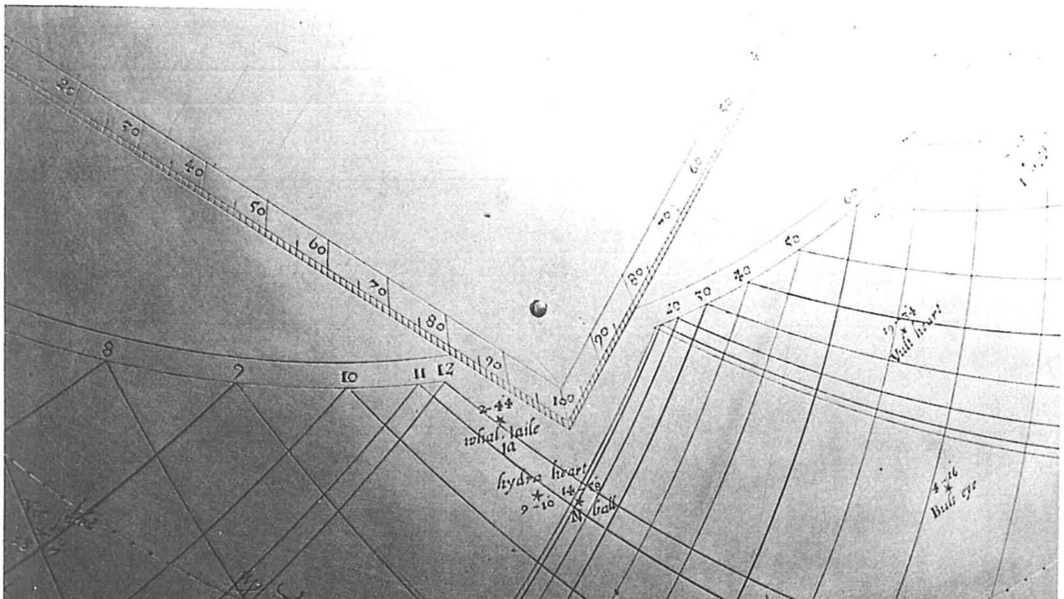

Plate 33: Detail from Cat. no. Q1 (recto)

Gunter Quadrants

These all follow the illustration in Gunter's *De Sectore et Radio*, page 188. Cat. no. Q1 even includes the letters referred to in the description of the construction of the instrument.

Q1.

Location: Whipple (Accession no. 1026)

Plates 33, 34

Brass Gunter quadrant for latitude 52°N. Length of side 329mm. Radius 317mm.

Recto

Degree scale along edge, divided [0] to 90°; numbered by 10°; subdivided to 5°, to 1°, to 30′ and to 10′.

Calendar arch set between 14°30′ and 61°30′, with equinoxes at 10 March and $12^{1}/2$ September. Months marked by initial; days divided to 10; subdivided to 5 and to 1, except around solstices (divided to 10 only).

Arcs for equator and tropics limit projection of hours, azimuths and horizon line.

Declination scale down left-hand radial edge, divided [0] - [23°30′]; numbered by 10°; subdivided to 5°, to 1° and to 20′.

Projection of ecliptic, calibrated by zodiacal signs ["♈, ♉, ♊, ♋, ♌, ♍, ♎, ♏, ♐, ♑, ♒, ♓"]; each sign divided to 10°; subdivided to 5° and to 1°.

Horizon line divided [0] - 40°; numbered by 10°; subdivided to 5° and to 1°.

Curved, numbered lines representing whole hours on left-hand side of instrument; summer daylight hours 4 - 12, 1 - 8, winter daylight hours 6 - 12, 1 - 6. [9am summer line was incorrectly engraved at first.]

Curved azimuth lines at 10°-intervals from 0 - 120° on right-hand side of instrument; numbered by 10° from 20°.

Major stars placed on instrument with their right ascensions:

"peg wing 23-55′"; "Vir spike 13-7′"; "Arcturus 14-0′"; "Lions heart 9-48′"; "S ball 14-32′"; "whal. taile 2-44′"; "hydra heart 9-10′"; "N ball 14-58′"; "Buls eye 4-16′"; "Vult heart 19-34′"; "L[ittle] dogg 7-22′"; "great dog 6-30′".[36]

Shadow square with linear scales 0 - 100 along each of two sides of square; 100 corresponds to 45°; numbered by 10; subdivided to 5 and to 1.

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[36] These values for the right ascensions of the stars lie between those provided in the 1623 edition of Gunter's *De Sectore* (p.190) and reprinted in the edition of 1636 (p.113, p.233), and the updated values for 1670 listed in the third edition of 1653 (p.113, p.233). (See Bryden, *Sundials and Related Instruments* (Cambridge, 1988), Cat. no. 280.)

Only one sight remaining, carrying both slit and pinhole sights.[37]

Verso: nocturnal

Volvelle with fixed ring, numbered I-XII, I-XII, anticlockwise; hours divided to 15 minutes; subdivided to 5 minutes.

Rotatable disc: months marked by initial; numbered by 10; divided to 5; subdivided to 1. Engravings of five circumpolar constellation figures (Ursa Major, Ursa Minor, Draco, Cepheus, Cassiopeia) with individual stars. Lines from centre through month divisions. Projecting pin (to assist turning of nocturnal) set on line between May and June.

Signed on recto beneath tropic line, to left of calendar arch: 'Elias Allen fecit' with straight capitals.

Bibliography: Derek J. Price, 'The Early Observatory Instruments of Trinity College, Cambridge' in *Annals of Science*, 8 (1952), pp.1-12; D.J. Bryden, *The Whipple Museum of the History of Science, Catalogue 6. Sundials and related instruments* (Cambridge, 1988), Cat. no. 280.

[37] This seems to be of poorer quality than the rest of the instrument - perhaps a later addition?

Q2.

Location: Whipple (Accession no. 1764)

Plate 35

Brass Gunter quadrant for latitude 52°N. Length of side 88.5mm. Radius 83mm.

Recto

Degree scale along edge divided [0] to 90°; numbered by 10°; subdivided to 5°, to 1° and to 30′.

Calendar arch set between 14°30′ and 61°30′, with equinoxes at 9 March, 12 September. Months marked by initial; days divided to 10; subdivided to 5.

Arcs for equator and tropics limit projection of hours, azimuths and horizon line.

Declination scale down left-hand radial edge, divided [0] - [23°30′]; numbered by 10°; subdivided to 5° and to 1°.

Projection of ecliptic, calibrated by zodiacal signs; each sign divided to 10°; subdivided to 5° and to 1°.

Horizon line divided [0] - 40°; numbered by 10°; subdivided to 5° and to 1°.

Curved, numbered lines representing whole hours on left-hand side of instrument; summer daylight hours 4 - 12, 1 - 8, winter daylight hours 6 - 12, 1 - 6. Also dotted lines representing half-hours.

Curved azimuth lines at 10°-intervals from 0 - 120° on right-hand side of instrument; numbered by 10° from 20°.

Major stars placed on instrument with their right ascensions:[38]

"W[ing] peg 23 54"; "Arctu 58" [should be 13 58]; "L[ion's] heart 9 48"; "B[ull's] eye 4 15"; "Vult heart 19 33".

Shadow square with linear scales 0 - 10 along each of two sides of square; 10 corresponds to 45°; numbered by 1; subdivided to 0.2.

Pinhole sights set between 90° line and edge of instrument.

Verso: nocturnal

Volvelle with fixed ring, numbered 1-12, 1-12, anticlockwise; hours divided to 30 minutes; subdivided to 15 minutes.

Rotatable disc: months marked by initial; numbered by 10; divided to 5; subdivided to 1. Engravings of five circumpolar constellation figures (Ursa Major, Ursa Minor, Draco, Cepheus, Cassiopeia) with individual stars. Lines from centre through month divisions. Projecting pin (to assist turning of nocturnal) positioned near edge at beginning of May.

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[38] Taken from table in Gunter, *De Sectore*, p.190.

Signed on recto beneath tropic line, to right of calendar arch: 'Elias Allen fecit' with italic capitals.

The nocturnal is very similar in style to the woodcut illustration in *De Sectore et Radio* sig.A4 recto. The numerals, letters and stars are not in Allen's style (and they are also dissimilar from those on the front of the instrument), suggesting a different maker for the nocturnal - perhaps one of Allen's apprentices.

Bibliography: R.T. Gunther, *Early Science in* Cambridge (Oxford, 1937), p.188; D.J. Bryden, *The Whipple Museum of the History of Science, Catalogue 6. Sundials and related instruments* (Cambridge, 1988), Cat. no. 279.

Q3.

Location: Private Collection

Plates 4, 36, 37

Brass Gunter quadrant for latitude $51^1/_2$°N. Length of side 145mm. Radius 139mm.

Recto

Degree scale along edge divided [0] to 90°; numbered by 10°; subdivided to 5°, to 1° and to 30′.

Calendar arch set between 15° and 62°, with equinoxes at 10 March and $12^1/_2$ September. Months marked by initial; days divided to 10; subdivided to 5.

Arcs for equator and tropics limit projection of hours, azimuths and horizon line.

Declination scale down left-hand radial edge, divided [0] - [23°30′]; numbered by 10°; subdivided to 5° and to 1°.

Projection of ecliptic, calibrated by zodiacal signs; each sign divided to 10°; subdivided to 5° and to 1°.

Horizon line divided [0] - [40°]; numbered by 10°; subdivided to 5° and to 1°.

Curved, numbered lines representing whole hours on left-hand side of instrument; summer daylight hours 4 - 12, 1 - 8, winter daylight hours 6 - 12, 1 - 6.

Curved azimuth lines at 10°-intervals from 0 - 120° on right-hand side of instrument; numbered 30°, 60°, 90°, 100°, 110°, 120°.

Major stars placed on instrument with their right ascensions:[39]

"W[ing] peg 23 54"; "Arctu: 13 58"; "H[eart] Lion 9 48"; "Buls eye 4 15"; "Vultures heart 19 33".

Shadow square with linear scales 0 - 10 along each of two sides of square; 10 corresponds to 45°; numbered by 1; subdivided to 0.2.

Pinhole sights set over 90° line (raised to allow plummet string to reach 90°).

String with spherical bead and plummet; latter consists of cylinder surmounting cone with spheroidal mounting for string attachment.

Verso: nocturnal

Volvelle with fixed ring, numbered 1-12, 1-12, anticlockwise; hours divided to 15 minutes.

Rotatable disc: months marked by initial; numbered by 10, divided to 5; subdivided to 1.[40] Engravings of six circumpolar constellation figures (Ursa Major, Ursa Minor, Draco, Cepheus, Cassiopeia, Auriga) with individual stars (major ones indicated by

[39] See footnote 38.

[40] There is one mistake here - although December is engraved with 31 days the final marking is "30" not "31".

open centre). Lines from centre through month divisions. Projecting pin (to assist turning of nocturnal) positioned near edge at approximately 19 April.

Signed on recto beneath tropic line, to left of calendar arch: 'Elias Allen fecit' with straight capitals.

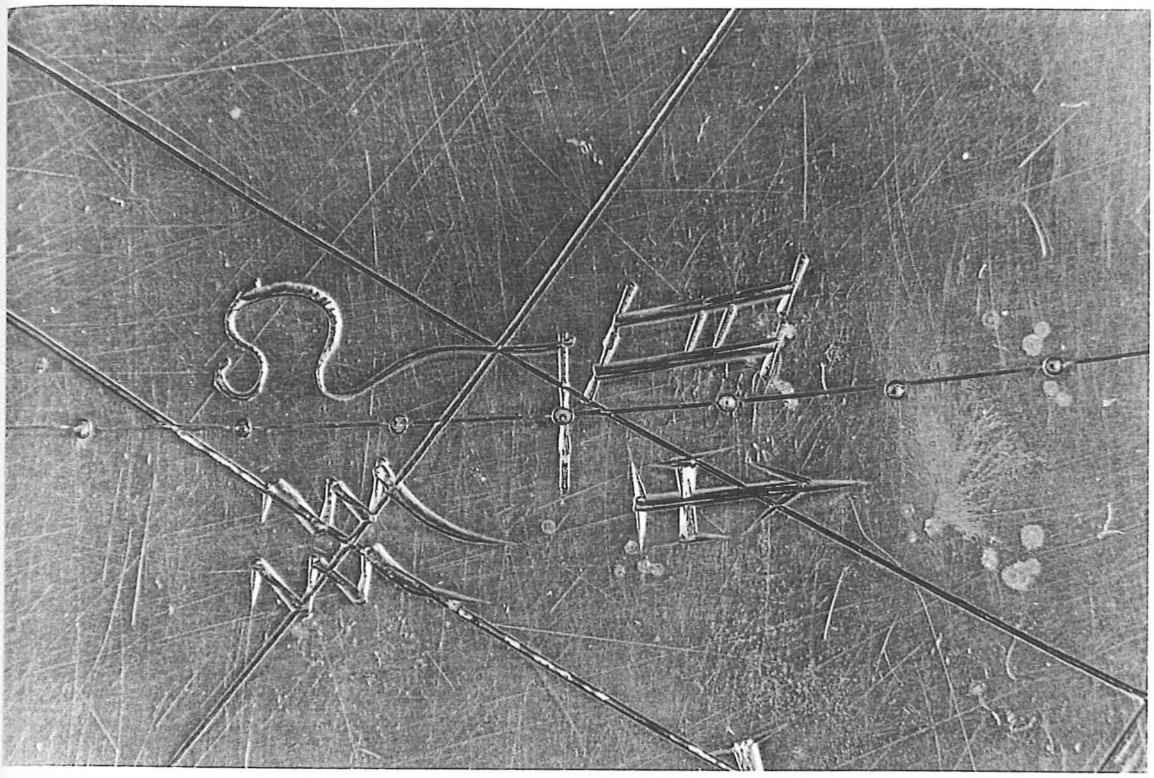

Plate 34: Detail of engraving from Cat. no. Q1, showing zodiacal signs from ecliptic arc

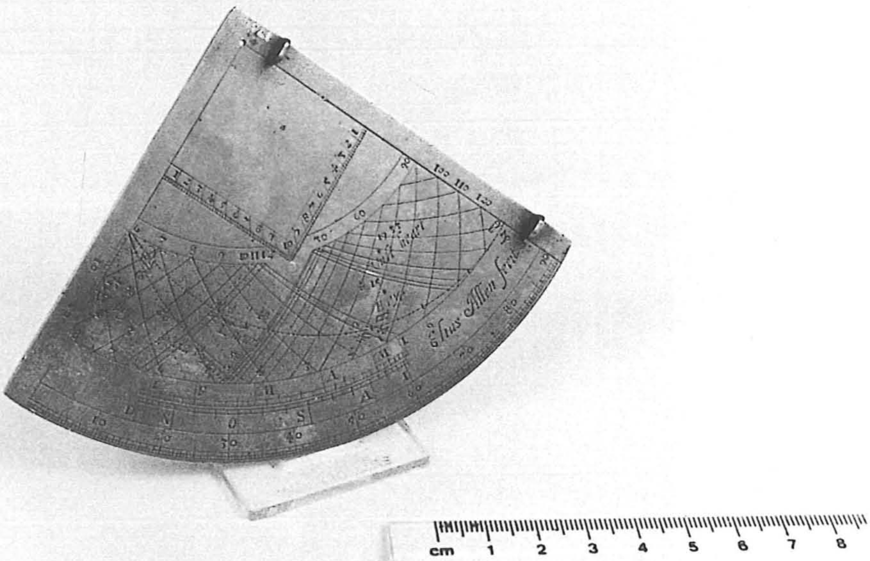

Plate 35: Cat. no. Q2 (recto)

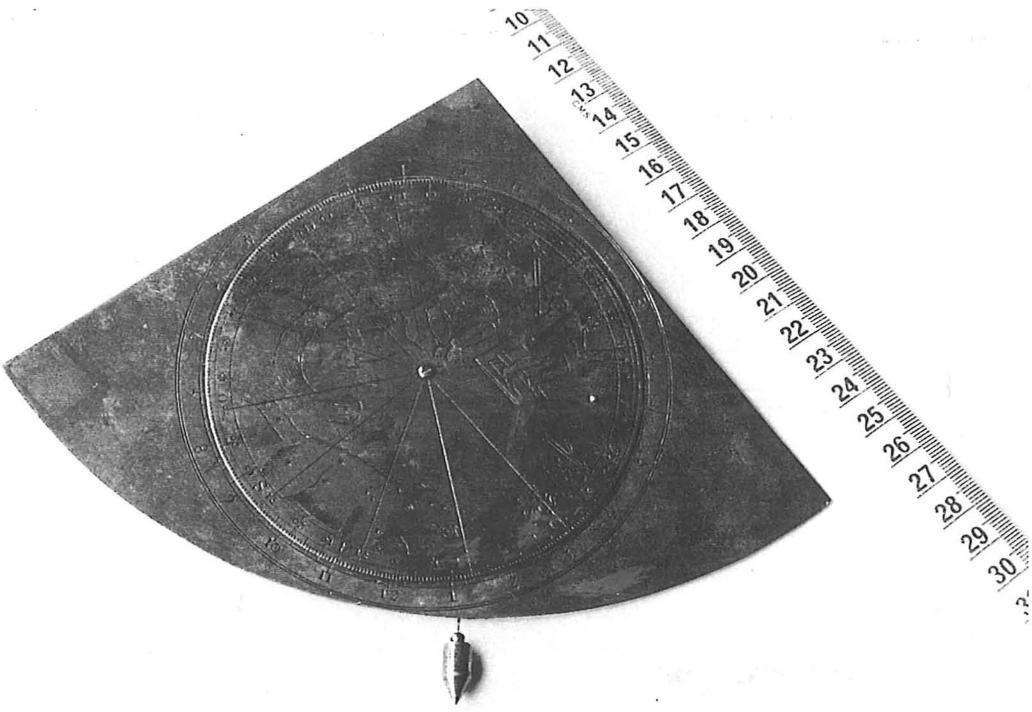

Plate 36: Cat. no. Q3 (verso)

Plate 37: Detail from Cat. no. Q3 (verso), showing engraving of Cepheus

Other Instruments

X 1.

Horizontal Sundial

Location: NMM (Identification no. D387/A80-13)

Plates 5, 38

Bronze garden dial for latitude 51°N, octagonal with sides measuring alternately 78mm and 35mm (as if cut from square of side length 127mm).

Double engraved circle, diameter 125mm.
Hour circle, numbered clockwise IIII - XII, I - VIII.
Minutes circle, divided to hours; subdivided to 30 minutes, (with barred lines), to 15 minutes and to $7^1/2$ minutes.
Double engraved circle centred on crossing-point of VI and XII hour lines, diameter 21.5mm. Only VI and XII hour lines extend inside this circle.
"I B" marked with a star on projection of XII line.[41] Another faint asterisk is observable in right upper 'quadrant' between meridian line and IIII hour line.
Shaped perpendicular gnomon of height 64mm.
Screw holes at midpoints of short sides of octagon, for attachment to base.

Signature in hour circle: "Elias Allin fecit 1606" with straight E, italic A.[42]

It should be noted that there are various unusual features in the signature on this instrument: the mixture of the style of capitals; the spelling of the surname; the perfect '0' which Allen generally appeared to have great difficulty in writing. These combined with the early date (four years into Allen's apprenticeship) possibly raises some doubts as to the authenticity of the instrument. However, it may simply be that the instrument maker had not yet settled into a natural pattern for signing his works.

Bibliography: *An Inventory of the Navigation and Astronomy Collections in the National Maritime Museum Greenwich* (London, 1983), vol. 2, p.29-66

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[41] Probably the initials of the owner.
[42] Compare with Cat. no. X3.

X 2.

Horizontal dial

Location: Churchyard of Ashurst Church, Kent.

Plates 39, 40

Bronze octagonal dial; side length 85mm (as if cut from square of side 205mm); bronze gnomon. For latitude 51° North, as measured from the style.

Hour circle, numbered IIII - XII, I - VIII clockwise; divided to 30 minutes (with barred lines); subdivided to 15 minutes.

Inscription: "ELIAS ALLEN MADE THIS
DIALL AND GAVE IT TO
THE PARISH OF ASHURST
AÑO DOMINI 1634"

Shaped gnomon, height 85mm.

Nails at points of octagon for attachment to base.

Bibliography: Claud Blair, 'A Royal Compass-dial' in *The Connoisseur*, December 1964, pp.246-248.

X 3.

Alidade/Scale

Location: NMM (Identification no. N80-27(3) SI/A7)

Plates 6, 43

Brass rectangular rule (509 x 42 x 3.5mm) with bevelled edge.

<u>Recto</u>

Six scales of differing lengths, though all numbered [0] to 100 by 10 and all with one extra division at the beginning which is subdivided to 5 and to 1.

The scales are marked 24, 20, 16, 12, 11, 10, finishing at bevelled edge. These are for division of inches into 24ths, 20ths, etc. up to 10ths.

Raised rectangular strips (25 x 8.5 x 1.5mm) at each end, for attachment of sights [missing].

<u>Verso</u>

Plane Scale numbered [0] - 31, by 1. Divided vertically into 10 (numbered 2, 4, 6, 8). Divided horizontally by 1 [which is 0.5 inches] with punches in vertical 5 line to show 0.5. Line right across rule at 15.

Transversal scale in division before 0. Numbered 2, 4, 6, 8 both horizontally and vertically. Punches on vertical 5 line halfway between each diagonal. Divided to 0.1; subdivided to 0.01 by transversals.

Signature on recto at far end from scales: "Elias Allen Fecit" with straight "E", italic "A".

The unusual mixture of capitals perhaps indicates an early instrument. The same combination is also found on the bronze garden dial, Cat. no. X1. It seems that Allen may well have experimented with various different signatures before he settled on the two alternatives which appear on most of his instruments (one with straight capitals, the other with italic capitals). The alidade is similar to those which are found on plane tables, and so it is likely that this was the original use of the instrument.

Bibliography: *An Inventory of the Navigation and Astronomy Collections in the National Maritime Museum Greenwich* (London, 1983), vol. 2, p.30-21.

X 4.

Peractor

Location: Private collection

Plates 41, 42, 44

Brass peractor[43] consisting of quadrant (with alidade mounted on one radial edge) pivoted between fork arms of vertical brass stand for attachment to circumferentor. Total height 420mm. Length of quadrant side 165mm. Radius 146mm.

Quadrant recto

Degree scale: [0] - 90°, numbered by 10°; divided to 5° (with ⁜ motif); subdivided to 1° and to 30′.

Linear scale down both radial edges, numbered [0] - 50 by 5; divided to 1. These two scales form the boundaries of a sinical quadrant: '5'-lines engraved more deeply than others.

Quadrant verso

Plain. Scraped flat but not polished.

Alidade

Rectangular brass rod, 265mm x 8mm x 5mm. Sights at ends, 49mm height x 5mm x 1.5mm. Each sight is pierced by a pinhole and an open circle: the latter has its centre marked by a shaped pointer. The sight at the upper end of the alidade has the pinhole above the open circle; the positions are reversed on the lower sight. No other markings.

Stand

Fork arm recto: linear scale identical to edge scales of quadrant; bevelled edge on right-hand side.

Below quadrant, stand is pierced by open circle for plummet bob (not present, but guideline for string visible on pivot arm verso).

Below this the stand has a horizontal bar with a foot at each end for attachment to horizontal circumferentor with central compass (circumferentor not present).

Quadrant pivot screw decorated with simple curlicue.

No signature.

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[43] Follows illustration in Aaron Rathborne's *The Surveyor*, p.131.

It should be noted here that the numeral engraving is not Allen's work. The original attribution to Allen was made by Professor Gerard Turner,[44] on the basis that no other workshop would have been producing this instrument at this period. Corroboration for the attribution is found in the fact that the ✝ motif on the degree scale and some of the numerals on the scales are of a type with engraving on the plain table alidade from the National Maritime Museum, which *is* signed. It would seem likely that the numerals were the work of an apprentice.

The peractor was a surveying instrument used for measuring the heights of trees, buildings, etc. When it was attached to a circumferentor it could supply all the functions of an altazimuth theodolite.

[44] Private communication to owner.

X 5.

Mariner's Astrolabe

Location: Department of Physics, University of St. Andrew's

Plates 46, 47

Brass ring (diameter 400mm, width 19mm) with quadrants perforated to reduce air resistance. Suspension ring. Alidade with sights.

Degree circle, divided clockwise from shackle 90° - [0] - 90° - [0] - 90°. Upper right quadrant and both lower quadrants numbered by 10° on inner rim of ring; subdivided to 4°, to 1°, to 30′ and to 15′. Upper left quadrant numbered by 10° on outer rim of ring; subdivided to 5° and to 1°; subdivided to 10′ by transversal scale.

Alidade with shaped ends; width 26mm, thickness 6mm; bevelled edge on left-hand side; diameter of central circle 50mm.
Slit and pinhole sights on alidade: 46mm wide, 215mm apart.

Signature: "ELIAS ALLEN FECIT 1616" all in block capitals. This signature is unusual in having been stamped into the metal rather than engraved.

Bibliography: Alan Stimson, *The Mariner's Astrolabe* (Utrecht, 1988), p.78; R.G.W. Anderson, *The Mariner's Astrolabe: an exhibition at the Royal Scottish Museum, Edinburgh* (Edinburgh, 1972), pp.28-9.

X 6 - X 9.

Savilean Instruments

Location: MHS

These instruments are so named because they were made for John Greaves, Savilean professor of astronomy at Oxford from 1643 until his removal by the Parliamentarians in 1648. Prior to this period he was professor of geometry at Gresham College (1631-1643). Although only the large astronomical quadrant (X6) is signed, I have included the other three instruments since they date from the same period and so may have some connection with Allen. X6 is the only surviving example of a large-scale Allen instrument.[45]

X 6.

Astronomical Quadrant (Identification no. 36 - 4/1)

Plate 48

Brass astronomical quadrant from the Radcliffe Observatory, Oxford. Scale and main limbs remaining. Radius 1982.5mm. Width of scale 69mm.

Degree scale, divided [0] - 90° in both directions; numbered by 5°; subdivided to 1°, to 30′ and to 6′; subdivided by transversal scale across ten concentric arcs to 0.6′.

Signed "Elias Allen F[I]ecit Londini 1637" with straight capitals.

X 7.

Astronomical quadrant (Identification no. 36 - 4/3)

Brass astronomical quadrant, radius 617mm.

Only the scale remaining (and this is badly damaged).

Degree scale, numbered left to right [0] - 90° by 10° (inner rim); numbered right to left [0] - 90° by 1 (outer rim); divided to 10°; subdivided to 5°, to 1°, to 30′ and to 10′; subdivided by means of transversal scale to 1′.

[45] Further research into Greaves' commissioning of the instruments might provide some clue concerning the origin of the unsigned instruments.

X 8.

Astronomical Sextant (Identification no. 36 - 4/2)

Astronomical sextant consisting of brass scale with iron limbs (detached); radius 1867mm; width of scale 37mm.

Degree scale, divided left to right [0] - 64°; numbered by 1; subdivided to 10′; subdivided by transversal scale to 1′.

The numbering on this scale is quite unlike Allen's, and so even if he did engrave the scale the numbering must surely have been done by someone else.

X 9.

Astronomical Sextant (Identification no. 36 - 4/4)

Astronomical sextant consisting of brass scale with iron limbs.

Degree scale: too eroded to detect any markings except the very faint lines of the transversals and concentric arcs.

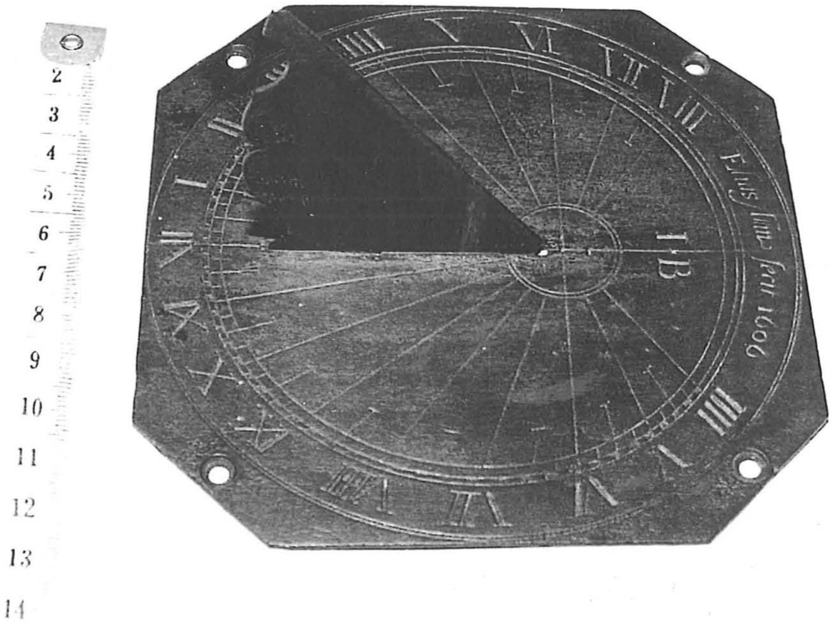

Plate 38: Cat. no. X1

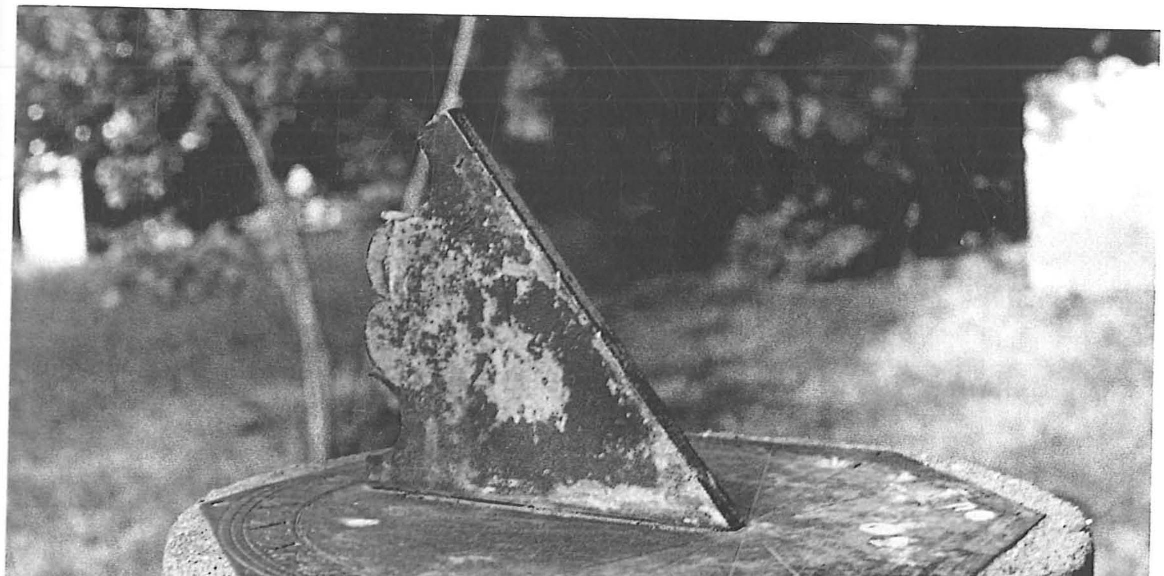

Plate 39: Cat. no. X2

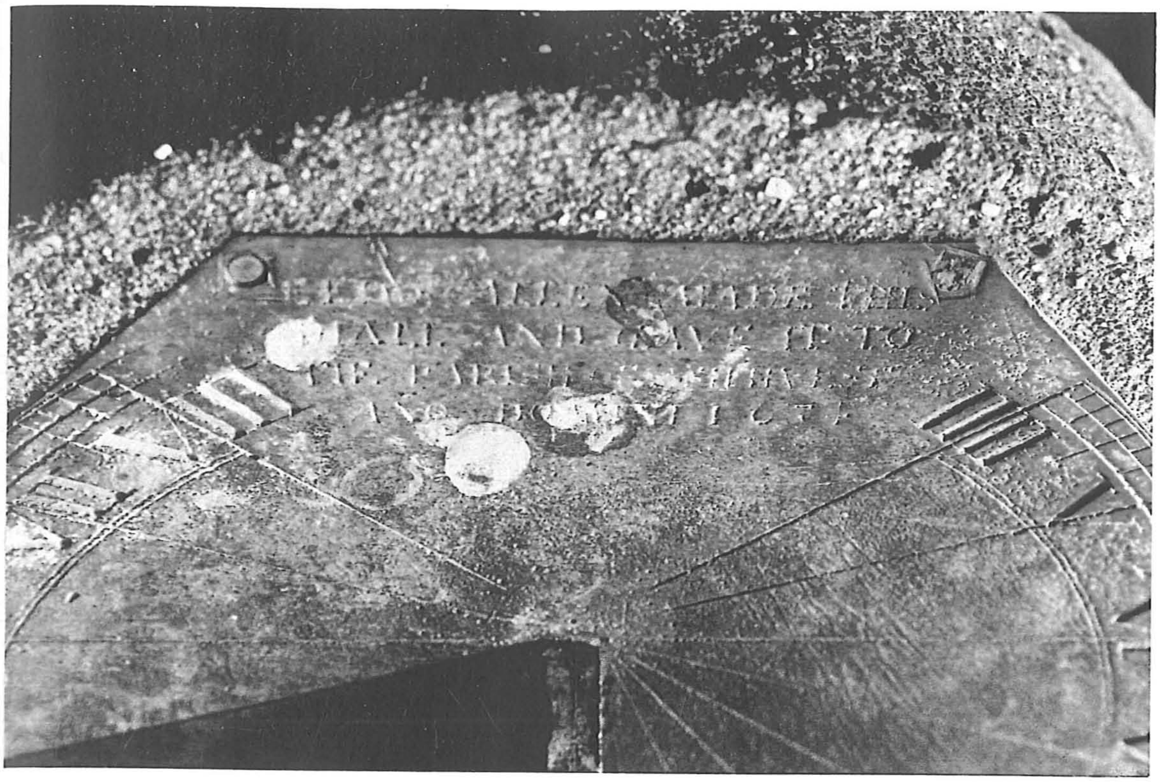

Plate 40: Detail from Cat. no. X2, showing inscription

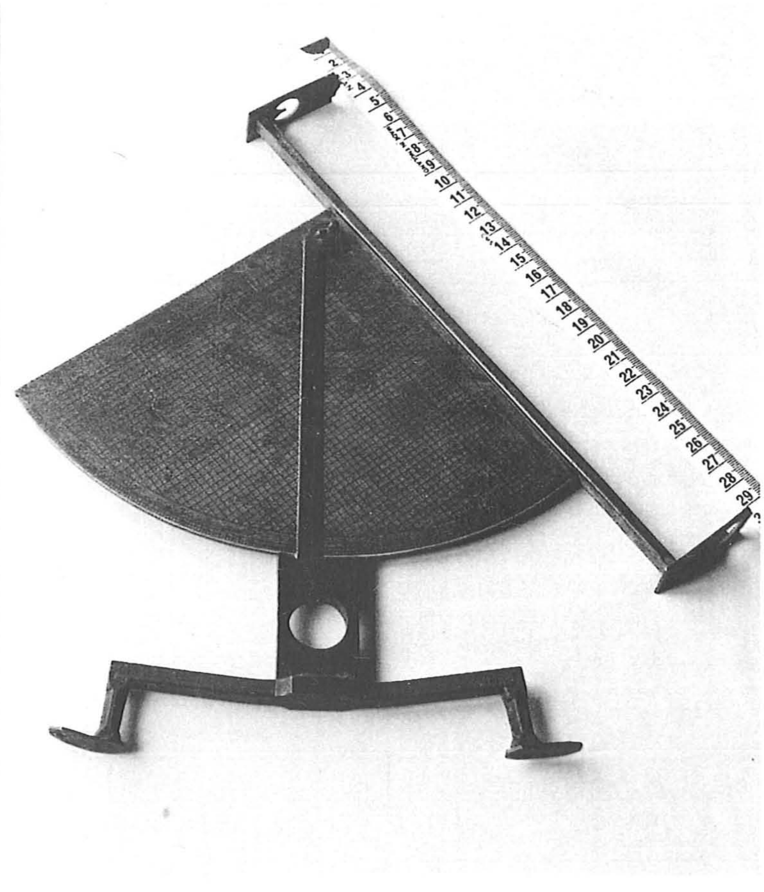

Plate 41: Cat. no. X4 (recto)

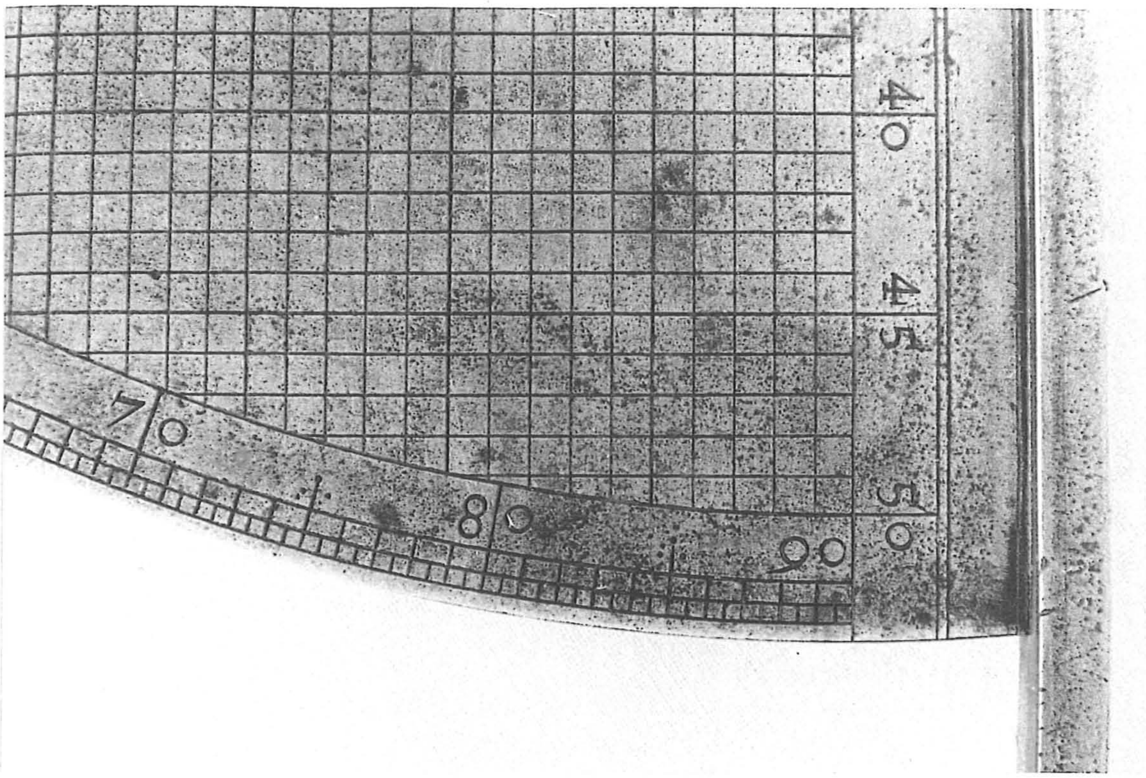

Plate 42: Detail of scale from Cat. no. X4 (peractor)

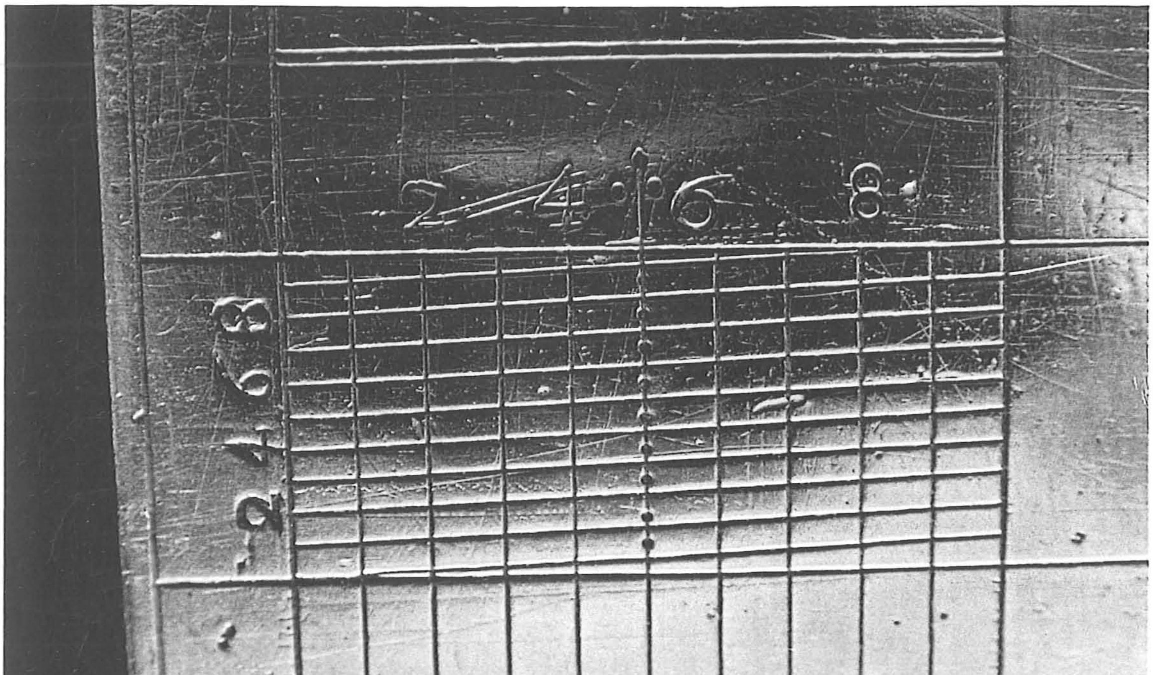

Plate 43: Detail of scale from Cat. no. X3 (alidade)

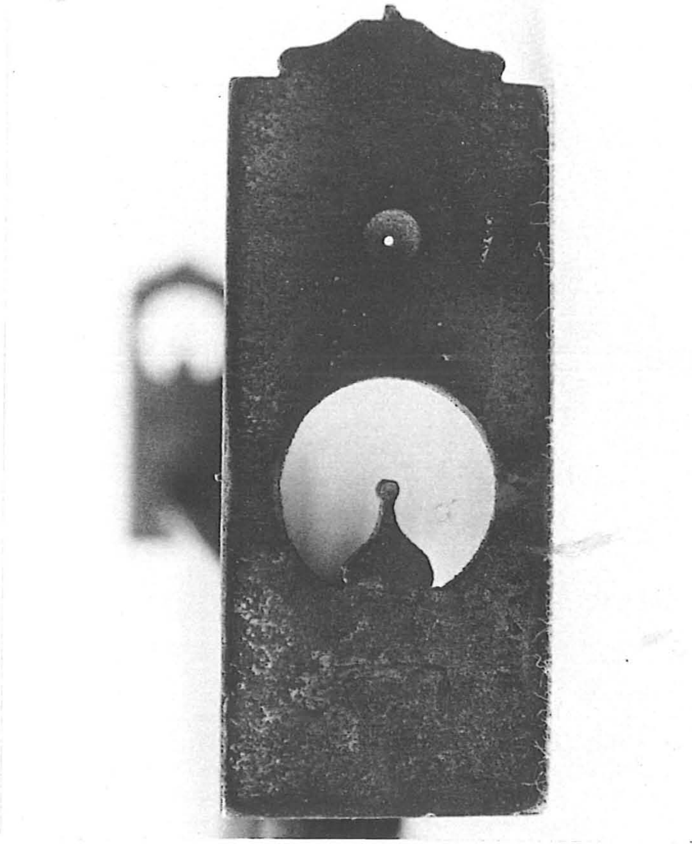

Plate 44: Sight from Cat. no. X4

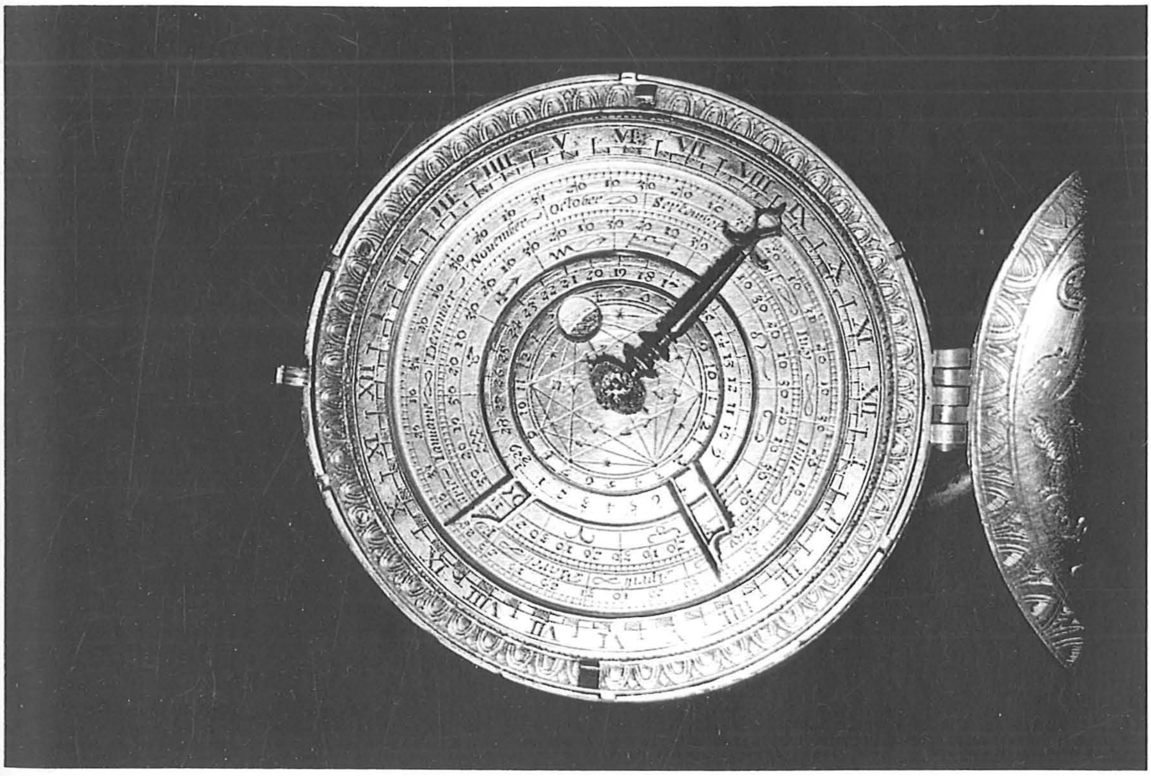

Plate 45: Lunar volvelle (possibly Allen) from watch by H. Roberts (BM)

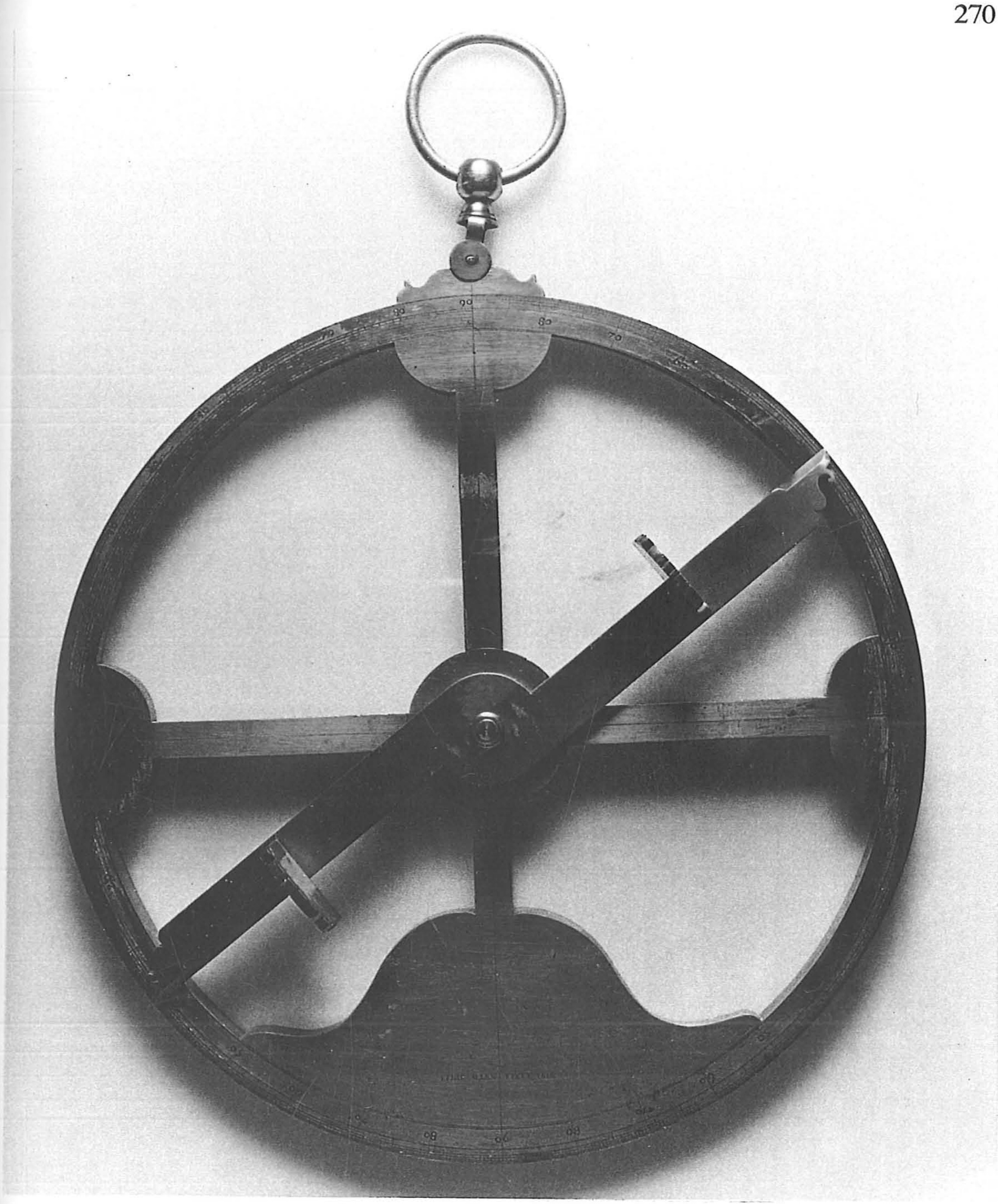

Plate 46: Cat. no. X5

Plate 47: Detail of signature from Cat. no. X5

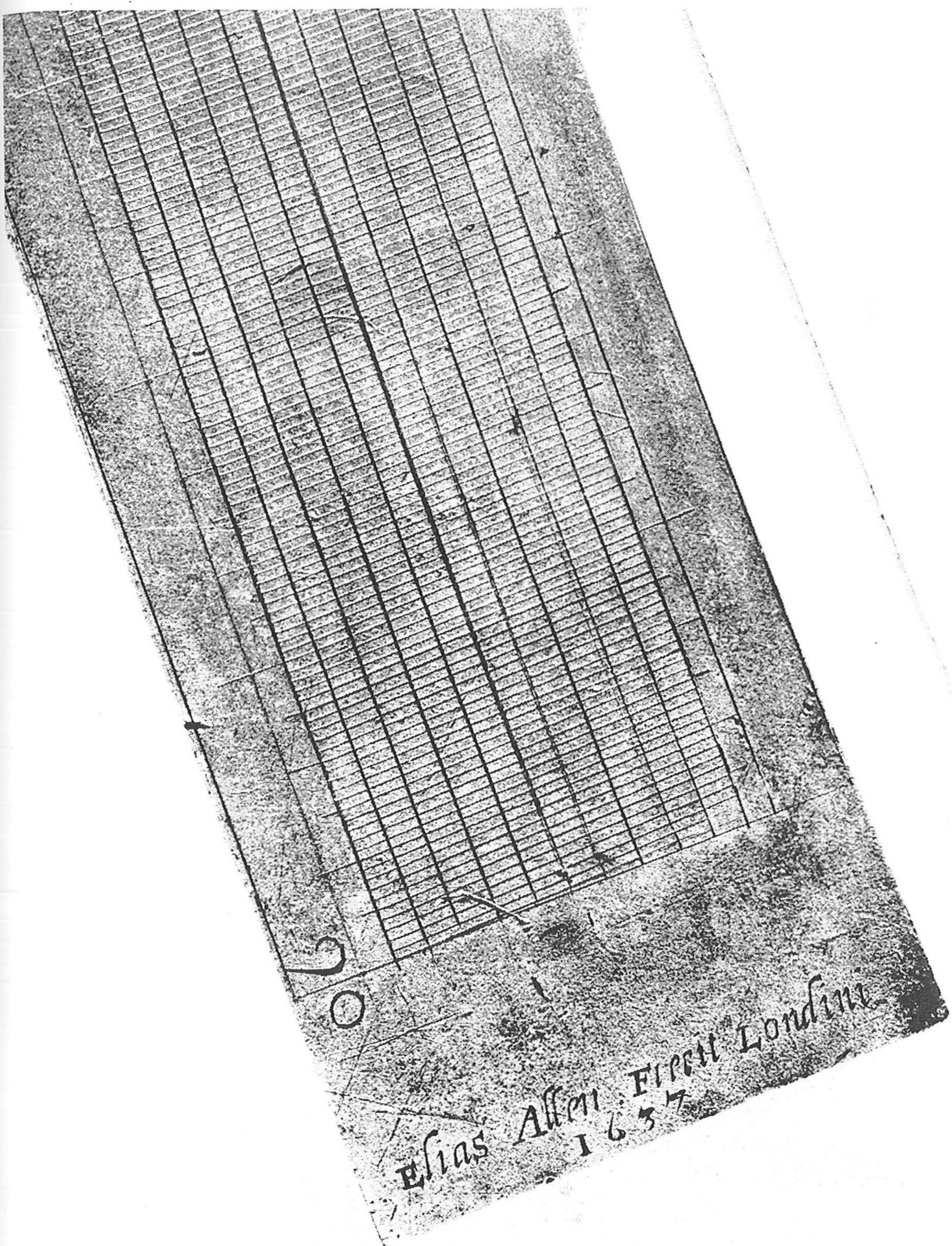

Plate 48: Detail of scale from Cat. no. X6

Other instruments by, or possibly attributable to, Allen

Most of these are taken from listings in auctioneers' catalogues. These catalogues are a valuable source of information on instruments since they provide details of pieces which have been sold to private owners and are therefore not available for study. They are often also the most readily available source for instruments which have been bought by museums but do not appear in any published catalogues of museum holdings. However, one must approach the auction catalogues with care - attributions are not always correct and there are the possibility of fakes appearing such as the one mentioned below. It is advisable to check photographs if possible to compare the style with known instruments.

1) Sectors. Seven unsigned Gunter sectors have been through the salesrooms in the last fifteen years,[46] and appear from the illustrations to follow Allen's engraving style. At least four of these were definitely different instruments - it is impossible to tell from the photographs whether the remaining three were. Two signed Gunter sectors have also been sold.[47] An unusual Allen sector was sold at Sotheby's in 1988:[48] it was similar to the NMM sector (Cat. no. S5), although even simpler in style and closer to the continental sectors of this period; it carried the same hour and degree scales on an elongated strut as appear on Cat. no. S5.

2) Universal Equinoctial Ring Dials. Four signed dials have been auctioned in recent years.[49] One had a replacement bridge; one was signed 'Elias Alleen' but was definitely genuine. Apart from these, it may be worth noting that Lewis Evans recorded a silver universal equinoctial ring dial in what is now the MHS collection, which was unsigned but which he attributed to Allen: unfortunately, the dial was sold in 1952.

3) Astronomical Compendia. Four compendia have been sold (apart from that already listed as Cat. no. A4) - all signed and all different.[50] That sold in 1962 had the same compass plate as Cat. no. A1 and latitude tables for 30 towns, all named in Latin. The

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[46] Christie's (London), 3/2/82, lot 3; Christie's (South Kensington), 18/7/85, lot 219; Christie's (South Kensington), 20/8/87, lot 193; Sotheby's (London), 3/10/88, lot 214; Tesseract, Catalogue 30 (Fall, 1990), no. 26; Tesseract, Catalogue 37 (Summer, 1992), no. 40; Sotheby's (London), 7/10/94, lot 117.
[47] Sotheby's (London), 12/7/71, lot 193; Christie's (London), 7/6/72, lot 106.
[48] Sotheby's (London), 3/10/88, lot 247.
[49] Sotheby's (London), 30/11/59, lot 109; Sotheby's (London), 27/3/72, lot 73; Sotheby's (London), 14/6/79, lot 72; Christie's (New York), 17/12/86, lot 137.
[50] Sotheby's (London), 26/6/62, lot 230; Christie's (London), 30/3/77, lot 104; Sotheby's (London) 7/7/78, lot 19; Christie's (South Kensington), 14/3/85, lot 224.

compendium sold in 1977, although being mainly gilt-brass, had a silver equinoctial ring; it featured a lunar volvelle, table of latitudes, compass, nocturnal and equinoctial dial. The following year a compendium was sold complete with its original gold-stamped leather case and engraved with the initials W.B. which may well be those of the original owner; it had the same accoutrements as the previous compendium, but the compass plate was the same as Cat. nos. A3 and A5. Finally, the compendium sold in 1985 - gilt-brass with nocturnal, table of latitudes, equinoctial dial, compass (same plate as Cat. nos. A2 and A4, and a replacement needle) and lunar volvelle.

4) Double horizontal dials. Two double horizontal dials have appeared at auctions - one circular and made, unusually for Allen, of bronze;[51] one octagonal and with a broken gnomon.[52] Another octagonal dial is present in a private collection, having been on display in the Whipple Museum for several years. Apart from these, an Allen dial was sold at Christie's in 1985[53] - it was not described as a double horizontal dial, but the photograph showed the altitude scale which was used with the Oughtred projection, suggesting that it would be such a dial.

5) A signed octagonal brass dial (width 195mm) was auctioned by Christie's in 1978.[54]

6) A very unusual instrument was advertised in a Christie's catalogue from 1985,[55] which, although unsigned, was sufficiently similar to Allen's other work for there to be no reasonable doubt that this was one of his pieces. The instrument was a brass nocturnal following the style of the lunar volvelles appearing on many of Allen's astronomical compendia, with a Gunter quadrant engraved on the reverse. The monogram "*F* " inscribed on the quadrant was probably that of the original owner. The instrument is 90mm in diameter, and is calibrated for 50°30′ N.

7) A brass 'snuff-box type' of 'equatorial' dial, signed 'Elias Allen 1627' was listed in the John C. Tomlinson collection.[56]

8) A complete signed peractor and circumferentor is held at the Louvre Museum in Paris. It was earlier in a private collection.[57]

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[51] Christie's (South Kensington), 29/9/94, lot 424.
[52] Sotheby's (London), 7/10/94, lot 424.
[53] Christie's (South Kensington), 18/7/85, lot 222.
[54] Christie's (London), 8/3/78, lot 49.
[55] Christie's (South Kensington), 17/10/85, lot 279.
[56] Information from private correspondence.
[57] Illustrated in Guye & Michel, *Time and Space* (London, 1970), figure 282.

9)The British Museum possesses a watch by H. Roberts (probably of pre-1615 date) which carries a lunar volvelle in brass and silver very similar to those appearing on Allen compendia, although the engraving of some of the 1s does not follow Allen's style - they appear as 1 [Plate 45].

10) A signed ring dial which appeared twice in consecutive years in the auctioneers is not genuine, although it could be an early instrument with the signature added later.[58]

These additional instruments, if all by Allen, increase the number of known instruments by twenty-eight.

[58] Sotheby's (London), 11/6/85, lot 309, and 18/6/86, lot 119.

Conclusion

The mathematical culture of seventeenth-century England was exceedingly rich and varied. It did not consist of a few individuals dabbling in mathematical pursuits or isolated geniuses wrestling with profound theoretical problems. Rather it was a world in which many different people were involved and in which instruments and practical techniques were as important as ideas, the formulation of theorems, and the demonstration of mathematical proofs. Developments in mathematics over the century were at least as common in the instruments and techniques as in the theory. Indeed, new concepts and theories were themselves often incorporated immediately into the instrumental application of mathematics - theoretical and practical mathematics were inseparably intertwined. The changes and development of mathematics were driven largely by the concerns and interests of practical mathematicians.

The particular way in which mathematics developed in England was very much a result of the kind of marketing it received in the second half of the sixteenth century. Central to the push for widespread education of the literate classes in the subject of mathematics was the influential figure of John Dee.[1] Dee's championing of the mathematical sciences is well known through his *Mathematicall Praeface* and it is clear from his writing that he was very strongly concerned with practical mathematics and the economic advantages of a knowledge of mathematical techniques and applications. A large proportion of the *Praeface* was taken up with the roles of geometry and arithmetic in practical arts; not just the more obvious ones of navigation, astronomy, architecture and the like, but also some unusual applications such as 'hypogeiodie', by which means the course of mine shafts could be traced on the earth's surface and thus disputes concerning ownership of underground wealth could be resolved. His stress was firmly

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[1] Two good general reference books for Dee are Nicholas H. Clulee, *John Dee's Natural Philosophy: Between Science and Religion* (London, 1988) and William H. Sherman, *John Dee. The Politics of Reading and Writing in the English Renaissance* (Amherst, 1995). The *Mathematicall Praeface* is treated in Allan Debus' introduction to his facsimile edition of the *Praeface* (New York, 1975).

on the usefulness of mathematics to extend the greatness of the commonwealth of England. Dee was also concerned that the advantages of mathematical techniques should be made available to as wide an audience as possible and this was why he was so willing to give his support to the production of an English version of Euclid, thus removing the bar which had cut off those unlearned in Latin from advancing their knowledge in other areas.

Dee was not the only person to encourage the extensive promulgation of mathematics. One of the reasons for Thomas Gresham's urge to found a London college was to increase the learning of the merchant classes. His was an entirely pragmatic approach - he believed that the economy of the state would be greatly improved if a better education was provided for those involved in managing the economy and in promoting trade. Thus, in the context of Gresham College, mathematics was to be taught for increasing the knowledge of a broad range of merchants and craftsmen in order that they would be better able to pursue their trades thereafter and so would be of greater use to the state as a whole.

With such attitudes as those of Dee and Gresham fuelling the broad introduction of mathematical learning into England it is hardly surprising that practical mathematics was the dominant force during the seventeenth century and that mathematics developed with an emphasis on the technical rather than on more scholarly avenues. In this environment it is easy to understand why Gunter was surprised to find his application of instruments scorned by Henry Savile: for the vast majority of people involved in the mathematical culture, instruments were part and parcel of mathematical practice and it was unusual to meet mathematicians who were hostile to instrumental methods.

Above all, the seventeenth-century mathematical culture involved a vast network of different people with different abilities, different needs, different interests and different contributions to the structure of seventeenth-century mathematics. These people inevitably had different understandings of mathematics and different awarenesses of the extent to which their work or leisure pursuits or intellectual studies were dependent on mathematics. Nevertheless, they were dependent on each other for mutual education and for learning more about specific instruments, ideas and applications. Mathematics extended far beyond the sphere of the academic mathematicians - the producers of new theories and ideas (which are the kind of things which tend to excite the historian of mathematics) - to a large group of people which increased considerably as the century progressed and as more and more people came to rely upon mathematics during the course of their daily lives. 'Mathematicians' came from many different walks of life: some were academically trained, but many more had learnt their mathematics in the course of their professional careers and were therefore far more concerned with using mathematical techniques than with theory in the abstract.

I do not wish to imply that mathematical theory is unimportant and that the development of new concepts and theories should be made a sideline in the history of mathematics. Seventeenth-century mathematics in England would certainly have been the poorer if the likes of Napier and Oughtred had not lived. Nevertheless, their contributions to mathematics must be seen within the context of the wider mathematical culture. Any immediate impact of their work almost invariably came as the result of its relevance to practical mathematical arts. Logarithms, in particular, were lauded precisely because they made life easier for mathematical practitioners whose professional tasks involved long and complicated arithmetical calculations. If a theorem was useful at a practical level it was rapidly adopted and passed into the mathematical culture - *vide* the proliferation of logarithmic calculating devices and the appearance of logarithmic scales on standard instruments soon after the publication of Napier's texts; if the practical applications were less obvious, the theorem was more likely to be left on a shelf until such time as it could be utilised effectively.

It could be said that the mathematics of this period in England had little to do with new theoretical research and was more concerned with a steady spread of knowledge; that the stress was on the education of an ever-widening group of people. However, this sense of consolidation over innovation is only apparent when theory is the subject under consideration. Although most members of the community were very conservative when it came to theoretical methods they were more receptive to new instruments. This may have been due to the fact that the general concept of using instruments was an old one, whereas algebraic methods were completely unknown territory and were less easily adaptable to instrumental techniques than was geometry.

Whatever the reason for this conjunction of conservatism with respect to theory and willingness to accept instrumental innovation, there is incontrovertible evidence for the seventeenth century being a time of amazing expansion in mathematical instrument making. Designs for new instruments appeared regularly and makers were kept busy satisfying the requirements of those who, having read about the latest developments in the material culture of mathematics, were eager to possess these mechanical aids to the mathematical arts. While many of the instruments were based on theoretical principles which had long been known, their actual design and development constituted original research just as much as the investigation of logarithms or the development of methods for solving complicated algebraic equations. Similarly, the production of better astronomical tables and logarithms and trigonometric ratios supplied to greater and greater degrees of accuracy may not seem to be a great step forward in the development of mathematics, but it was these kinds of innovations which were most highly sought after and most urgently required by the mathematical community of the time.

As I have pointed out in the discussion of the literature survey, there was a *very* strong emphasis on geometry and geometrical methods in the mathematics of seventeenth-century England. This is hardly surprising when the most important text book was still Euclid's *Elements of Geometry*,[2] which not only grounded the student in the principles of geometry but also taught arithmetic topics (such as mathematical progressions and the rule of proportion) by geometric means. Questions relating to square and cube roots were often discussed by the representation of the numbers in geometrical form: it appears that a quantity made more sense when presented as the

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[2] This is evident from the number of citations of the work in books on geometry written in the century, and also from the number of editions published in England (it had eight different editors between 1651 and 1700).

length of the side of a physical square or cube. It is also worthwhile remembering that questions such as these often arose from the desire to deal with practical problems: the ability to find cube roots was of vital importance to shipbuilders trying to determine how fluid displacement and cargo capacity related to linear dimensions.

Diagrammatic methods were extremely important didactically and were an integral part of the teaching process. Furthermore, instruments were incorporated into mathematical tuition as a form of manipulable diagram (with the paper instruments set into texts supplying a half-way house between the two) - although, as we have seen, this inevitably led to pedagogical wrangles about when it was correct to introduce instruments into a student's mathematical instruction. In general, teaching was weighted towards ensuring that the student was sufficiently skilled in the practice of the mathematical arts and so instruments were brought in at an early stage of the process. Advocates of more theoretically-grounded courses were relatively few in number.

In such a world it is small wonder that instruments played such an important part. While the design of these mathematical instruments required the application of complicated theoretical procedures as often as not (for instance, the knowledge of spherical trigonometry in order to produce astronomical projections), using the instruments did not. For solving mathematical problems or carrying on work based on the application of mathematical techniques, all that was necessary was an understanding of how to manipulate the instrument in the relevant situations, and how to interpret the results obtained through instrumental application. An initial technophobia could be overcome through repeated use of the instrument and the skills required generally depended on a lower degree of literacy than those for performing pen and paper mathematical calculations. The proliferation of instruments throughout the seventeenth century is witness to the demand from a broad range of people for a kind of mathematics which could be readily assimilated and practically applied.

If instruments were so important in this mathematical community, then the same must hold true for the instrument makers. It was all very well for mathematicians to design new and better instrumental aids for mathematical applications, but if there were no skilled artisans to produce the artefacts themselves then the original designs would be useless. Makers also supplied reliable information about what was and was not practicable in the design and construction of instruments. Thus it is clear that, as facilitators at least, the mathematical instrument makers formed a vital part of the community. They supplied the necessary aids for the application of mathematics within the numerous arts which relied upon mathematical techniques - astronomy, navigation, surveying, architecture, gunnery, banking, engineering, shipbuilding, etc. - and thus rendered the whole field of practical mathematics possible.

Elias Allen was the archetype of a good instrument maker. He was not hesitant to make suggestions for the improvement of instrument design; he presided over a workshop which was large enough to satisfy the requirements of a wide range of customers and his fame guaranteed a good outlet for innovations; he trained his apprentices well, thus ensuring the continuing production of high-quality instruments. Above all, his engraving was accurate and his signature on an instrument was a reassuring stamp of quality.

In concentrating on the development of new ideas and concepts, and in considering only the careers of the brightest intellectual stars, traditional approaches to the history of mathematics risk denying seventeenth-century English mathematics much of the richness and diversity which makes it such an exciting field to study. We cannot ignore the social, political and economic concerns which helped shape the development of mathematics through the century. There is an extraordinary breadth to this mathematical community: conceptual developments jostle with practical needs; professional mathematicians (whether academics or practitioners) stand shoulder to shoulder with dilettante dabblers. Through all we see the importance of the material aspects of mathematics and the significance of mathematical instruments: inevitably Elias Allen and his fellow mathematical instrument makers stand at the heart of the community.

APPENDIX 1

The Hollar Portraits

Wenceslaus Hollar's collection of portraits includes three of members of the mathematical community of the seventeenth century. The subjects are William Oughtred, Nathaniel Nye and Elias Allen. These portraits are all detailed in Richard Pennington's *A descriptive catalogue of the etched work of Wenceslaus Hollar 1607-1677* (Cambridge, 1982) which is a new edition of the catalogue published by Gustav Parthey in 1853 (*Wenzel Hollar. Beschreibendes Verzeichnis seiner Kupferstiche*). According to this catalogue, copies of each of the portraits are present in each of the four main Hollar collections - the British Museum, the Royal Library in Windsor, the Hollar collection in Prague and the Fisher collection in the Thomas Fisher Rare Books Library in Toronto. Copies of the Allen portrait are also in the National Maritime Museum and the Science Museum.

Portrait of William Oughtred

The mathematician is depicted in plain black clerical garb with a white collar. A scull cap hides his hair, but he wears a short white beard. In his left hand, which is resting on a table in front of him, he holds an octavo volume. The background is very simple, further contributing to the austere nature of the portrait. Beneath the portrait are inscribed the words 'GULIELMUS OUGHTRED *Anglus* ex Academia *Cantabrigiensi* An°. Ætat. 73. 1646.'

The portrait was engraved from a painting by Hollar and was used as the frontispiece to the 1647 edition of Oughtred's *Key of the Mathematicks*. It is referred to in one of John Aubrey's manuscripts: 'Mr Winceslaus Hollar did draw old M^r Oughtred's picture and etched it: they say that no picture in black and white could be more like a man: there is a cheerful aire visible.' [Bodley MS. Aubrey 4, p98 verso]

Portrait of Nathaniel Nye

Nathaniel Nye was a gunner in the royal army at Worcester. This engraving was made in 1644, when he was twenty years old. The young man has fair, shoulder-length hair and is clean-shaven. He wears a deep, lace-edged, white collar over a slashed doublet. On a column to the right of him hangs a quadrant (presumably a gunner's quadrant); an armillary sphere stands on the floor in front of the column, but behind Nye. Beneath the portrait is written 'The true Effigies of Nathaniell Nye Mathematician'; in the lower left of the picture are the words 'W: Hollar delin: et fecit Aqva forti Londini, 1644'.

This portrait also forms a frontispiece to a book, this time Nye's *The Art of Gunnery*, which first appeared in 1647 and had two further editions.

The Allen portrait has been described elsewhere (see Chapter Two). The only addition to this description is that provided in George Vertue's *A Description of the Works of the Ingenious Delineator and Engraver Wenceslaus Hollar* (second edition, 1759). Here Allen is said to be 'in his Hair, Whiskers and Beard, Band, in his right hand a Pair of Compasses, before him a great Variety of mathematical Instruments'. The engraving is taken from a painting by van der Borcht whose only other known portrait was of second Earl of Bristol, when Lord Digby.

There are some interesting differences between the portraits of Oughtred and Nye and that of Allen. The Allen portrait was painted by someone other than Hollar while the other two were solely the work of Hollar. The engraving was also made after Allen's death and so it could not have been used for advertising purposes for the instrument maker: there is no evidence that it was ever part of a book. However, the likenesses of Oughtred and Nye appeared as frontispieces for their works. Thus, while there are easy explanations for the existence of the portraits of these two, there is no obvious one for Allen's portrait. It may have been commissioned by the Earl of Arundel for some reason, or Allen may simply have been a friend of both painter and engraver.

APPENDIX 2

Details of Apprentice Bindings

Apprentice	Date bound	Date freed	Company
Edward Blayton	7/7/1612		Grocers
John Blighton		16/8/1620	Grocers
Thomas Shewswell		29/7/1623	Grocers
John Allen	25/6/1617	11/1/1631	Grocers
Henry Sefton	16/8/1620	11/1/1631	Grocers
Robert Davenport	25/3/1623	25/11/1635	Grocers
Christopher Brooke(s)	21/8/1629	3/9/1639	Grocers
Edward Winckfeild	29/9/1629	3/9/1639	Grocers
George Cooke	5/8/1635		Grocers
Ralph Greatorex	25/3/1639	25/11/1653	Clockmakers
Edward Grimes	15/3/1640		Clockmakers
Withers Cheney	13/4/1646	20/4/1657	Clockmakers
John Prujean	16/5/1646		Clockmakers
William Jordan	7/3/1649		Grocers

NB. All dates here are in Old Style

APPENDIX 3

Advertisements for Allen's Workshop

1611

'You may haue any of the Instruments in this booke made of wood, in Hosier lane, neere Smithfield in London, by Iohn Tomson.

'The Glasse is made in brasse, in blacke Horse-ally, neere Fleetebridge, by Elias Allen.'

(Arthur Hopton, *Speculum Topographicum*, sig.Ee recto.)

1612

'You may have [Hopton's topographical glasse] exactly made in brasse, without Temple-barre. Where also you may have my Staffe in brasse, free from warping or yeelding, with such other projectments thereon, as could not be placed weel upon the same in wood,....'

(Arthur Hopton, *An Almanac and Prognostication*, sig.B2 recto - verso.)[1]

1616

[The instrumental form of Hopton's Clavis Mathematica improved by Gunter is] 'made in *brasse* by M. *Elias Allen* over against S. Clements Church'

(Thomas Bretnor, *A New Almanac and Prognostication*, sig.A3 verso.)[2]

Advert for 'Elyas Allen' and John Thompson.

(G. Gilden, *A New Almanac and Prognostication*, sig.B2 recto - verso.)

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[1] This could be the almanac writer Edward Pond who had a shop 'at the Globe, a little without Temple bar, between the Bull head and the mermaid Tavernes', where 'instruments are made and to be sold.' See Pond, *An Almanac* (1612), sig.A2 recto.

[2] I am indebted to David Bryden for the advertisements taken from almanacs.

'The making of this Instrument [the peractor] and the rest in brasse are well known to M. *ELIAS ALLEN* in the Strand; and of those in wood to M. *IOHN THOMPSON* in Hosyer Lane.'

(Aaron Rathborne, *The Surveyor*, p.131.)

'a Mathematicall Scale...which with all other Mathematicall instruments, are made by my louing friends *M^r^ Elias Allen*, ouer against St. Clements Church in the Strand, in Brasse; and *M^r^. Iohn Tomson* in Hosier lane by Smithfield (in Wood) and may also both in Wood and Braße be had, with the instructions thereof by me at my house:'

(John Speidell, *A Geometricall Extraction*, sig.A4 recto-verso.)

1617

'All Instruments fitting the Mathematicks in general are made in brasse by my loving friend, M. Elias Allen, at the Bull head over against S. Clements Church-yard without Temple bar, and in wood by my kinde neighbours, M. Iohn Thompson and Nathanael Gosse, at their shops in Hosiar Lane.'

(Thomas Bretnor, *A New Almanac and Prognostication*, sig.A3 verso.)

1618

'All Mathematical Instruments are made in brasse by my kind friend master *Elias Allen* at the Bull's head over against Saint *Clements* Church in the Strand, the which I thought good to let the ingenious Practitioner know in regard there be som teachers about this towne, which for their owne private gaine will fobbe them off with trash instead of good ware, and so in stead of good dealing they are abused and speed ill.'

(Thomas Bretnor, *A New Almanac and Prognostication*, sig.C8 verso.)

'*All other sorts of* Mathematicall *Instruments, they are very exactly made in Copper by Maister* Elais Allen *at his house over against* S. Clements *Church without* Templebar*: And in Wood by my very good friends M.* Iohn Thomson, *and M. Nathaniell Gosse, at their Shoppes in* Hosier-lane *neere* West Smith-field.'

(G. Gilden, *A New Almanac and Prognostication*, sig.B2 verso.)

1619

'*All other sorts of* Mathematicall *Instruments, they are very exactly made in Copper by Maister* Elais Allen *at his house over against* S. Clements *Church without* Templebar*: And in Wood by my very good friend M.* Iohn Thomson.'

(G. Gilden, *A New Almanac and Prognostication*, sig.B2 verso.)

1623

'These instruments are wrought in brasse by Elias Allen dwelling without Temple barre ouer against S^t^ Clements Church: and in wood by Iohn Thompson dwelling in Hosiar lane And by Nathaniell Gos Dwelling at Ratclif.'[3]

(Gunter, *De Sectore et Radio*, facing page 1.)

1624

'There are also many new invented excellent Instruments for the *Mathematicks* made in *London* by my very good friend M. *Iohn Thomson*, in *Hosier-Lane* neere *Smithfield*, and in Copper by master *Elias Allen* at his house over against S. *Clements Church* without Temple Barre.'

(G. Gilden, *A New Almanac and Prognostication*, sig.B2 recto.)

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[3] The reference to Gos is not included in all the impressions.

1625

'...as also many new invented Instruments for the Mathematicks made in London in Wood, by my very good firiend M. *Iohn Thomson*, in *Hosier Lane* nere Smithfield, & in Copper by M. *Elias Allen* at his house over against S. *Clements Church*, without *Temple Barre.*'

(G. Gilden, *A New Almanac and Prognostication*, sig.B2 recto.)

1626

'...as also many new invented Instruments for the Mathematicks made in London in Wood, by my very good firiend M. *Iohn Thomson*, in *Hosier Lane* nere Smithfield, & in Copper by M. *Elias Allen* at his house over against S. *Clements Church*, without *Temple Barre.*'

(G. Gilden, *A New Almanac and Prognostication*, sig.B2 verso.)

1627

'These and all other mathematicall Instruments are made with the newest inuentions, in brasse by Master *Elias Allen* in the Strand, and in wood by Master *Iohn Tomson*, in Hosier-lane.'

(John Speidell, *A Breefe Treatise on Sphaericall Triangles*, p.43.)

1630

'This *Instrument* is made in Silver, or Brasse for the Pocket, or at any other bignesse, over against Saint *Clements Church* without *Temple Barre*, by *Elias Allen.*'

(Richard Delamain, *Grammelogia*, p.22.)

1632

'Printed for Elias Allen maker of these and all other Mathematical Instruments, and are to be sold at his shop ouer against St Clements Church without Temple-barr.'

(William Oughtred, *The Circles of Proportion and the Horizontall Instrument*, title page.)[4]

'This Instrument (or any other for the Mathematicall arts) are made in Silver or Brasse by Elias Allen, or Iohn Allen neare the Savoy in the Strand.'

(Richard Delamain, *The making, description and use of...a Horizontall Quadrant*, facing title page.)

1633

'These Instruments are made in brasse by *Elias Allen* over against St. *Clements* Church without Temple-barre: where also those who are desirous may bee instructed thereof: and such as shall have occasion may have vessels gauged.'

(William Oughtred, *The New Artificial Gauging Line or Rod*, p.[42].)

1634

'...you may have them [Napier's bones] made in Brasse by Mr. *Elias Allen*, over against St. Clements church, without Temple Barre, and by divers other Instrument-makers.'

(William Barton, *Arithmeticke Abreviated*, p.20.)

1636

'These and all other Mathematicall Instruments are made in Brass by Elias Allen dweling with out Tempel barr a gainst St Clements Church'

(Edmund Gunter, *De Sectore et Radio*, facing title page.)

---

[4] Exactly the same advertisement appeared on the 1639 edition.

1639

'...have it made in brasse (which may be done by M^{r}. *Elias Allen* dwelling without *Temple* Barre, over against S^{t}. *Clements* Church, *London*, who maketh all sorts of Mathematicall Instruments and also horizontall Sunne-Dyalls in brasse)'

(John Wyberd, *Horologiographia Nocturna*, p.14.)

1645

'Note, that this Instrument is perfectly made in Brasse by *Elias Allain*, at the signe of the Blackmoore without Temple-Barre, London: And in wood by *John Tompson* and *Anthony Tompson* in Hosier Lane.'

(Edmund Wingate, *The Use of the Rule of Proportion*, sig.E10 verso.)

1648

'[the Napier's bones] are ready made in Wood, by Master *John Thompson* in Hosier lane neere Smithfield, who makes all kinde of Mathematicall Instrument, and also by M^{r}. *Anthony Thompson* in Gresham Colledge, and by M^{r}. *Thomas Browne* at the Globe neere Aldgate. In Silver or Brasse they are made by M^{r}. *Elias Allen*, over against S^{t}. Clements with out Temple-Barre.'

(Seth Partridge, *Rabdologia*, pp.3-4.)

1652

'These Instrumentall Dials are made in brasse by *Elias Allen* dwelling over against *S^{t}. Clements* Church without Temple Barre, at the signe of the *Horseshooe* neere *Essex* Gate.'

(William Oughtred, *The Description and Use of the Double Horizontall Dyall*, sig.X8 verso.)

1653

'These and all other Mathematicall Instruments are made in Brass by Elias Allen dwelling at ye Hors-Shoo without Temple barr neer St Clement's Church, and also by Walter Hayes, at the Cross-daggers in Moor-fields neer Bethelem-gate, and in Wood by Anthony Tompson in Hosyer-lane neer Smith-field.'

(Edmund Gunter, *The Works of Edmund Gunter*, facing title page.)

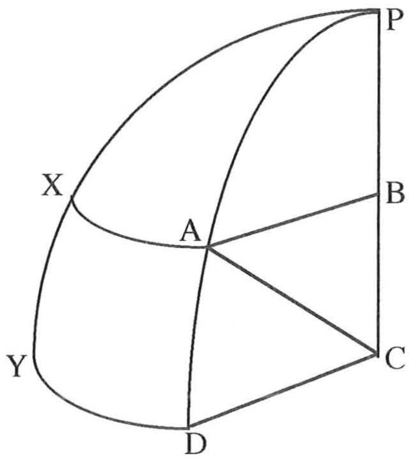
APPENDIX 4

Meridional parts

In order to reduce the terrestrial globe to a plane chart with all the meridians parallel the latitude scale must be stretched on an ever-increasing base, the further from the equator the parallel is. The ratio between a unit of longitude (constant on this projection) and a unit of latitude is calculated by the following procedure:

The proportion between the distance apart of two meridians at the equator is to their distance apart in any parallel of latitude as the radius of the equator is to the radius of the small circle of latitude.

i.e. YD: XA:: DC: AB
But DC = AC
Therefore YD: XA:: AC: AB
Therefore $\frac{YD}{XA} = \frac{AC}{AB}$
= cos ACB
= sec ACD
= sec latitude
Therefore YD = XA sec latitude



Meridians are parallel in this projection

Therefore a degree of longitude is the same in any latitude

Therefore longitude scale increases in proportion to secant of latitude

Therefore to preserve correct proportion, latitude scale must also increase proportional to secant of latitude.

Therefore 1′ of latitude = 1′ of longitude x secant latitude

Therefore distance of any parallel of latitude from the Equator = sum of secants of all arcs of one minute between equator and that parallel.

This was the basis for Wright's tables of meridional parts.[1]

[1] Details of this calculation were taken from Hewson, *A History of the Practice of Navigation* (Glasgow, 1951), pp. 33-35.

In order to obtain more accurate values of the meridional parts we must consider what happens as the unit of longitude is reduced towards zero. At the limit the sum becomes an integral and so

$$\text{distance of parallel y from equator} = \int_0^y \text{secant } y \, dy$$

Therefore to find the difference in meridional parts of two parallels the following formula is used:

difference between parallel y_2 and parallel y_1

$$= \frac{10800}{\pi} \int_{y_1}^{y_2} \text{secant } y \, dy$$

$$= \frac{10800}{\pi} \left[\ln \left| \tan \left(\frac{y}{2} + \frac{\pi}{4} \right) \right| \right]_{y_1}^{y_2}$$

$$= \frac{10800}{\pi} \left(\ln \left| \tan \left(\frac{y_2}{2} + \frac{\pi}{4} \right) \right| - \ln \left| \tan \left(\frac{y_1}{2} + \frac{\pi}{4} \right) \right| \right)$$

[The factor of $10800/\pi$ is necessary in order to convert the result of the integration from radians into minutes of arc.]

This was the formula used in the computer program and all other calculations performed for use in the error analysis.

APPENDIX 5

The computer program (with explanatory notes)

```
10 LET PI = 3.141592654
20 FOR W = 0 TO 60 STEP 10
30 LET X = W + 5
40 FOR Y = -90 TO 90 STEP 10
50 FOR Z = -90 TO 90 STEP 10
60 PRINT W, X, Y, Z
70 LET VER = 0: LET NBER = 0: LET DTER = 0
80 LET LDER = 0: LET EWER = 0: LET FLAG = 0
90 FOR D = -23.5 TO 23.5 STEP 5.875
100 LET VMIN = 1000: LET VMAX = -1000: LET FLAG2 = 0
110 LET NBMIN = 1000: LET NBMAX = -1000
120 LET DTMIN = 1000: LET DTMAX = -1000
130 LET LDMIN = 2000: LET LDMAX = -2000
140 LET EWMIN = 1000: LET EWMAX = -1000
150 FOR F0 = -1 TO 1 STEP 2
160 FOR F1 = -1 TO 1 STEP 2
170 FOR F2 = -1 TO 1 STEP 2
180 FOR F3 = -1 TO 1 STEP 2
190 LET E0 = F0/6: LET E1 = F1/6: LET E2 = F2/2: LET E3 = F3/2
200 REM Variation
210 LET V = (SIN(D*PI/180)/(COS((X + E1)*PI/180)))
220 LET V = Y + E2 - (ATN(V/(SQR(1-V*V))))*180/PI
230 IF FLAG2 = 0 THEN
        LPRINT V; ",";
        LET FLAG2 = 1
    ENDIF
240 IF V<VMIN THEN
        LET VMIN = V: LET V0M = F0: LET V1M = F1: LET V2M = F2            LET
V3M = F3
    ENDIF
250 IF V>VMAX THEN
        LET VMAX = V: LET V0P = F0: LET V1P = F1: LET V2P = F2
        LET V3P = F3
    ENDIF
260 REM New Bearing
270 LET NB = Z + E3 - V
280 IF NB<NBMIN THEN
        LET NBMIN = NB: LET NB0M = F0: LET NB1M = F1: LET NB2M = F2
        LET NB3M = F3
    ENDIF

290 IF NB>NBMAX THEN
        LET NBMAX = NB: LET NB0P = F0: LET NB1P = F1: LET NB2P = F2
        LET NB3P = F3
    ENDIF
300 REM Distance travelled
310 LET DT = 60*(5 - E0 + E1)/(COS(NB*PI/180))
320 IF (ABS(V)>(15*X/65 + 20)) OR (ABS(NB)>90) OR (ABS(DT)>800) THEN
        GOTO 500
        ELSE LET FLAG = 1
    ENDIF
```

```
330 IF DT<DTMIN THEN
        LET DTMIN = DT: LET DT0M = F0: LET DT1M = F1: LET DT2M = F2
        LET DT3M = F3
    ENDIF
340  IF DT>DTMAX THEN
        LET DTMAX = DT: LET DT0P = F0: LET DT1P = F1: LET DT2P = F2
        LET DT3P = F3
    ENDIF
350 REM Longitude Difference in Minutes
360 LET LD = 10800*(LOG(TAN(((X+E1)/2+45)*PI/180)) -
                    LOG(TAN(((W+E0)/2+45)*PI/180)))*TAN(NB*PI/180)/PI
370 IF LD<LDMIN THEN
        LET LDMIN = LD: LET LD0M = F0: LET LD1M = F1: LET LD2M = F2
        LET LD3M = F3
    ENDIF
380  IF LD>LDMAX THEN
        LET LDMAX = LD: LET LD0P = F0: LET LD1P = F1: LET LD2P = F2
        LET LD3P = F3
    ENDIF
390 REM Distance travelled East or West
400 LET EW = COS((X + W + E0 + E1)*PI/360)*LD
410 IF EW<EWMIN THEN
        LET EWMIN = EW: LET EW0M = F0: LET EW1M = F1: LET EW2M = F2
        LET EW3M = F3
    ENDIF
420  IF EW>EWMAX THEN
        LET EWMAX = EW: LET EW0P = F0: LET EW1P = F1: LET EW2P = F2
        LET EW3P = F3
    ENDIF
500 NEXT F3: NEXT F2: NEXT F1: NEXT F0
510 IF VER<(VMAX - VMIN) THEN
        LET VER = VMAX - VMIN: LET AV0M = V0M: LET AV1M = V1M
        LET AV2M = V2M: LET AV3M = V3M: LET AV0P = V0P
        LET AV1P = V1P: LET AV2P = V2P: LET AV3P = V3P:  LET AVD = D
        LET AVMAX = VMAX: LET AVMIN = VMIN
    ENDIF
520 IF NBER<(NBMAX - NBMIN) THEN
        LET NBER = NBMAX - NBMIN: LET ANB0M = NB0M
        LET ANB1M = NB1M: LET ANB2M = NB2M: LET ANB3M = NB3M
        LET ANB0P = NB0P: LET ANB1P = NB1P: LET ANB2P = NB2P
        LET ANB3P = NB3P: LET ANBD = D: LET ANBMAX = NBMAX
        LET ANBMIN = NBMIN
    ENDIF
530 IF DTER<(DTMAX - DTMIN) THEN
        LET DTER = DTMAX - DTMIN: LET ADT0M = DT0M
        LET ADT1M = DT1M: LET ADT2M = DT2M: LET ADT3M = DT3M
        LET ADT0P = DT0P: LET ADT1P = DT1P: LET ADT2P = DT2P          LET
ADT3P = DT3P : LET ADTD = D: LET ADTMAX = DTMAX                      LET
ADTMIN = DTMIN
    ENDIF
540 IF LDER<(LDMAX - LDMIN) THEN
        LET LDER = LDMAX - LDMIN: LET ALD0M = LD0M
        LET ALD1M = LD1M: LET ALD2M = LD2M: LET ALD3M = LD3M
        LET ALD0P = LD0P: LET ALD1P = LD1P: LET ALD2P = LD2P
        LET ALD3P = LD3P : LET ALDD = D: LET ALDMAX = LDMAX
        LET ALDMIN = LDMIN
    ENDIF
```

```
550 IF EWER<(EWMAX - EWMIN) THEN
        LET EWER = EWMAX - EWMIN: LET AEW0M = EW0M
        LET AEW1M = EW1M: LET AEW2M = EW2M: LET AEW3M = EW3M
        LET AEW0P = EW0P: LET AEW1P = EW1P: LET AEW2P = EW2P
        LET AEW3P = EW3P:  LET AEWD = D: LET AEWMAX = EWMAX
        LET AEWMIN = EWMIN
    ENDIF
560 NEXT D
570 IF FLAG = 0 THEN
        GOTO 720
    ENDIF
580 LPRINT:
590 LPRINT "W="; W; " X="; X; " Y="; Y; " Z="; Z
600 LPRINT "V:"; VER; " AT D="; AVD
610 LPRINT "V MAX= "; AVMAX; " AT "; AV0P; ","; AV1P; ","; AV2P; ","; AV3P;
". VMIN = "; AVMIN; " AT "; AV0M; ","; AV1M; ","; AV2M; ",";          AV3M
620 LPRINT "NB:"; NBER; " AT D="; ANBD
630 LPRINT "NB MAX= "; ANBMAX; " AT "; ANB0P; ","; ANB1P; ","; ANB2P;
        ","; ANB3P; ". NBMIN = "; ANBMIN; " AT "; ANB0M; ","; ANB1M; ",";
        ANB2M; ","; ANB3M
640 LPRINT "DT:"; DTER; " AT D="; ADTD
650 LPRINT "DT MAX= "; DTMAX; " AT "; ADT0P; ","; ADT1P; ","; ADT2P;  ",";
ADT3P; ". DTMIN = "; ADTMIN; " AT "; ADT0M; ","; ADT1M; ",";          ADT2M;
","; ADT3M
660 LPRINT "LD:"; LDER; " AT D="; ALDD
670 LPRINT "LD MAX= "; LDMAX; " AT "; ALD0P; ","; ALD1P; ","; ALD2P;  ",";
ALD3P; ". LDMIN = "; ALDMIN; " AT "; ALD0M; ","; ALD1M; ",";          ALD2M;
","; ALD3M
680 LPRINT "EW:"; EWER; " AT D="; AEWD
690 LPRINT "EW MAX= "; AEWMAX; " AT "; AEW0P; ","; AEW1P; ",";
        AEW2P; ","; AEW3P; ". EWMIN = "; AEWMIN; " AT "; AEW0M; ",";
        AEW1M; ","; AEW2M; ","; AEW3M
700 LPRINT: LPRINT
710 INPUT Q$
720 NEXT Z: NEXT Y: NEXT X
```

Notes on the programme

Line 20: W is the initial latitude of the ship.
Line 30: X is the current latitude of the ship.
Line 40: Y is the bearing of the sun at sunrise - negative values are for sunrise North of East and positive values are for sunrise South of East, with 0° being due East.
Line 50: Z is the course of the ship according to the compass - negative values lie between West and North, and positive values between North and East, with 0° being due North.
Lines 70 and 80: VER, NBER, DTER, LDER, EWER are the maximum error values (set initially to 0) for variation, actual course, distance travelled, longitude difference and distance travelled East or West respectively. FLAG is a marker to ensure that the calculations are only carried out for sensible values of variation, actual course and distance travelled. See note on line 320 for further details.
Line 90: D is the solar declination.
Lines 100 to 140: These lines set initial values for the minimum and maximum values of variation, actual bearing, distance travelled, longitude difference and difference travelled East or West. FLAG2 is a second marker; for its use see note on line 230.
Lines 150 to 180: These variables F0, F1, F2 and F3 set up the signs for the observational error values and are used to keep information on the printout clearer (see further in lines 610ff).
Line 190: E0, E1, E2 and E3 are the actual observational error values.

Lines 210 and 220: These lines calculate the variation at a particular set of values for W, X, Y, Z, D, F0, F1, F2 and F3.
Line 230: For every declination at a particular set of W, X, Y, Z this line will cause the first calculated value of the variation to be printed out. The marker, FLAG2 is then reset so that no further values of variation are printed out for that declination. The purpose of this exercise was to provide a rough record of the variation values being used, so that any which were untenable for a particular latitude could be detected. [Variation values were checked against maps of magnetic variation for 1600 and 1650 to ensure that they were feasible - maps taken from Barraclough 'Spherical harmonic analysis of the geomagnetic field for eight epochs between 1600 and 1910' in *Geophysical Journal of the Royal Astronomical Society*, 36 (1974), pp. 497-513.]
Line 240: This line compares the variation calculated with the currently stored minimum value of the variation; if the most recently calculated value is less than that stored, the new value will be stored, along with the signs of the errors for that value of variation.
Line 250: This line carries out a similar process for the stored maximum value of variation.
Line 260: 'New Bearing' refers to the actual course of the ship.
Lines 270 to 290: These are the calculations relating to the actual course of the ship which correspond to the calculations carried out for the variation.
Lines 310 to 340: The calculations related to the distance travelled.
Line 320: This line dispenses with non-feasible values: the variation is not permitted to be greater in magnitude than a latitude-related quantity (this is only a very basic relation between latitude and variation - the actual feasible values of variation have to be checked more carefully from the printout); the actual bearing is not allowed to be greater in magnitude than 90 because the latitude of the ship is increasing and therefore the course can never have a South component in it; the maximum allowed value of distance travelled was set at 800 miles (in order to give some error leeway over and above the stipulated maximum of 600 miles). If one of the values does not fit these criteria then the programme moves straight on to the next value of the errors (see line 500). Otherwise the marker, FLAG, is reset to show that is has been possible to carry out a full set of calculations for that particular set of W, X, Y, Z. If this has not been possible the programme will proceed to the next value of Z (see notes on line 720) without printing any results.
Lines 360 to 380: The calculations related to the longitude difference.
Lines 400 to 420: The calculations related to the distance travelled East or West.
Line 500: This line sets the computer to run through the loop of calculations again for the next set of observational error values.
Lines 510 to 550: These lines deal with the stored value of the maximum errors in the different variables. The maximum error at a given declination is given as the difference between the stored maximum value and the stored minimum value. The stored value of the maximum error is compared with the difference between the current maximum and minimum values and if the latter is greater it is stored in place of the previous stored value. At the same time the maximum and minimum values (and their accompanying error signs) and the relevant declination are stored.
Line 560: The programme now runs through this whole loop again for the next value of the declination.
Line 570: If, by this point, no calculations have been made on the distance, longitude difference and distance travelled East or West (shown by the marker being still set at 0) then nothing is printed out and the programme proceeds to the next value of Z.
Line 590: This line prints out the particular set of values of W, X, Y and Z.
Lines 600 and 610: These lines print out first of all the maximum error value of the variation and give the declination at which this maximum error was obtained. Then the relevant maximum and minimum values of variation are printed out with their corresponding error signs. Error signs were used rather than the actual errors because a printout such as '-1, 1, 1, -1' is much more easily assimilated than '-0.16666667, 0.16666667, 0.5, -0.5'.
Lines 630 to 690: In these lines the other variables are given the same treatment in the printout as the variation was given.

Line 710: This line was included to prevent the computer from getting ahead of the printer (which only has a limited amount of memory). Each time a set of results is printed out the return key on the keyboard must be pressed before any further calculations are made.
Line 720: The whole string of calculations is repeated, first for the next value of Z (until Z reaches 90), then for the next value of Y (until Y reaches 90) and finally for the next value of X.

A typical printout would look like this:

-6.911064, -12.81003, -18.70747, -24.60396, -30.5, -36.39604, -42.29253, -48.18997, -54.08894
W=0 X=5 Y=-30 Z=-40
V: 1.012737 AT D= -23.5
V MAX= -5.898326 AT -1,1,1,-1. V MIN= -6.911064 AT -1,-1,-1,-1
NB: 2.012737 AT D= -23.5
NB MAX= 47.41106 AT -1,-1,-1,1. NB MIN= 45.39833 AT -1,1,1,-1
DT: 107.9849 AT D= -11.75
DT MAX= 624.9729 AT -1,1,-1,1. DT MIN= 516.988 AT 1,-1,1,-1
LD: 102.4065 AT D= -11.75
LD MAX= 537.5406 AT -1,1,-1,1. LD MIN= 435.1341 AT 1,-1,1,-1
EW: 102.3091 AT D= -11.75
EW MAX= 537.0291 AT -1,1,-1,1. EW MIN= 434.72 AT 1,-1,1,-1

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Court Minutes of the Grocers' Company, MS 11,588, vol.2
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Printed Books

All of the mathematical texts included in the literature survey are listed, even if not cited in the text. Books were published in London, unless otherwise indicated. I have included a list of the books first published between 1654 and 1700 for reference, but it must be remembered that this may be a slightly misleading list since it was compiled purely on the basis of titles. Some of the posthumous publications which appeared after 1653 have been included in the first list since they were written before that date, and so were covered in the literature survey. Following these two lists I have included all the other sixteenth, seventeenth and eighteenth century books which I consulted in the course of my work.

Books first published between 1590 and 1653

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Thomas Addison, *Arithmetical Navigation* (1625)
John Aspley, *Speculum Nauticum. A looking glasse for sea-men* (1624)
John Babington, *A Short treatise of geometrie* (1635)
John Bainbridge, *An Astronomicall Description of the late Comet* (1619)
William Barlowe, *A Breife Discovery of the Idle Animadversions of Marke Ridley...vpon a Treatise entituled, Magneticall Aduertisements* (1618)

William Barlowe, *Magneticall Aduertisements: or divers pertinent obseruations, and approued experiments concerning the nature and properties of the Load-stone* (1616)

William Barlowe, *The Nauigators Supply* (1597)

William Barton, *Arithmeticke Abreviated. Teaching the Art of Tennes or Decimals.... Shewing the use also of Napiers Bones* (1634)

William Bedwell, *Mesolabium Architectonicvm. That is, A most rare, and singular Instrument* (1631)

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John Blagrave, *The Art of Dyalling* (1609)

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Thomas Blundeville, *A briefe description of universal maps and cards and their use* (1636)

Thomas Blundeville, *M. Blundeville his exercises...which Treatises are verie necessarie to be read and learned of all...that are desirous to haue knowledge as well in Cosmographie, Astronomie, and Geographie, as also in the Arte of Navigation* (1594)

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