

NLO corrections to the photon impact factor*

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We review the program of the calculation of next-to-leading order corrections to the virtual photon impact factor. Following a brief introduction we present some technical aspects for the various contributions. Recently obtained results for transversely polarised virtual photons are discussed and an outline of how infrared divergences are cancelled is given. Implications of the subtraction of leading energy logarithms are discussed.

1. Introduction

Understanding the total cross section for the scattering of two highly virtual photons, having virtualities Q_1^2 and Q_2^2 at large centre-of-mass energy s ($s \gg Q_1^2, Q_2^2$) should be in reach of perturbative QCD. The process $\gamma^*\gamma^* \rightarrow$ hadrons is considered to be an excellent testing ground for the applicability of perturbative QCD in the Regge limit [1,2]. If the energy is high enough to validate Regge asymptotics but not too high in order to suppress unitarity corrections we expect the $\gamma^*\gamma^*$ cross section to be described by the BFKL [3] equation.

To leading logarithmic accuracy (LLA) the predicted cross section, based on the BFKL equation, rises too quickly with increasing s . The situation is very different at NLO. The calculation of NLO corrections to the BFKL Kernel was initiated in [4] and finally completed in [5,6]. The corrections were first seen to be very large. However, their size is under control when additional collinear logarithms are taken into account [7,8] or when the kinematical conditions are forced to avoid these extra logarithms (rapidity vetoes) [9,10]. The NLO corrections tend to lower the power rise of cross sections to values that seem to be compatible with the data [11,12]. [13–16] seem to confirm this although the $\gamma^*\gamma^*$ -cross section is considered

using only NLO corrections to the kernel, ignoring those to the photon impact factor. These studies, however, can at best be viewed as an estimate of higher order corrections since they do not take care of higher order corrections to the coupling between external particles, virtual photons in the $\gamma^*\gamma^*$ case, and the NLO BFKL ladder. In order to make reliable predictions, being consistent to NLO, the NLO corrections to the coupling of virtual photons to the exchanged BFKL ladder, described by the impact factor, has to be taken into account. These corrections are currently under study [17–19] and the status of this work is reviewed in this contribution.

Besides $\gamma^*\gamma^*$ scattering perturbative QCD at high energies may be studied in any situation where a large rapidity gap between targets is observed and where a hard scale is involved in observing that final state (cf. Fig. 1). Prominent examples are the observation of forward jets in γ^*p collisions at HERA [20,21] or the production of Mueller-Navelet jets [22] in hadron-hadron collisions. The coupling of the BFKL ladder to the relevant jet production vertex at NLO has been finished recently [23–25].

2. The calculational program

We focus our discussion on the example of $\gamma^*\gamma^*$ -scattering which may serve as the canonical example for a scattering process in perturbative QCD at very high energy and is sometimes referred to as the gold-plated process to test BFKL dynamics.

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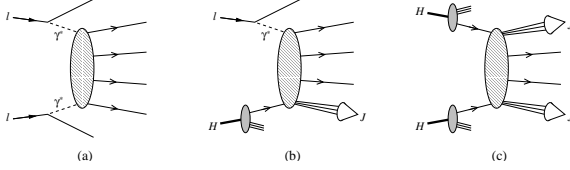


Figure 1. Hard processes in perturbative QCD at high energy.

At high energy we anticipate Regge factorization and as a result the total cross section for $\gamma^*\gamma^*$ -scattering is written as a convolution (cf. Fig. 2)

$$\sigma_{\gamma^*\gamma^*}(s) = \Phi_{\gamma^*} \otimes \mathcal{G}_\omega \otimes \Phi_{\gamma^*} \quad (1)$$

where $\mathcal{G}_\omega(\mathbf{r}^2, \mathbf{r}'^2, s_0)$ is the Green's function for the exchange of two reggeized gluons, projected into the colour singlet state, obtained as a solution of the (NLO) BFKL equation. Φ_{γ^*} is the impact factor for virtual photons under discussion. At leading order, this impact factor (Fig. 3) is calculated from cut quark box diagrams: the virtual photon splits into a $q\bar{q}$ -pair and the reggeized gluons from the t -channel couple to the $q\bar{q}$ pair in all possible ways.

At NLO we have contributions from virtual corrections to the leading order diagrams as well as contributions with an additional gluon in the

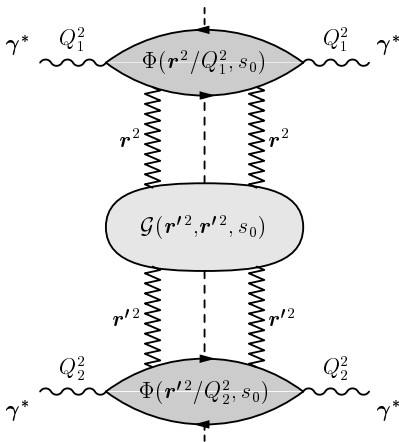


Figure 2. Regge factorization of the $\gamma^*\gamma^*$ scattering process.

intermediate state (cf. Fig. 4). We express the impact factor in terms of the particle-Reggeon vertices $\Gamma_{\gamma^* \rightarrow q\bar{q}}^{(0)}$, $\Gamma_{\gamma^* \rightarrow q\bar{q}}^{(1)}$ and $\Gamma_{\gamma^* \rightarrow q\bar{q}g}^{(0)}$, denoting the coupling of a virtual photon to Reggeon via a $q\bar{q}$ -pair at LO, NLO and with an additionally emitted gluon, respectively. These vertices result from perturbative QCD amplitudes with colour octet exchange, taken in the high energy limit and using Regge factorization. Expanding the impact factor in powers of the strong coupling, $\Phi_{\gamma^*} = g^2 \Phi_{\gamma^*}^{(0)} + g^4 \Phi_{\gamma^*}^{(1)}$ we would naively write the NLO impact factor in terms of these vertices as

$$\begin{aligned} \Phi_{\gamma^*}^{(1)} = & \int \frac{dM_{q\bar{q}}^2}{2\pi} d\phi_{q\bar{q}} 2 \operatorname{Re} \Gamma_{\gamma^* \rightarrow q\bar{q}}^{(1)} \Gamma_{\gamma^* \rightarrow q\bar{q}}^{(0)} \\ & + \int \frac{dM_{q\bar{q}g}^2}{2\pi} d\phi_{q\bar{q}g} \left| \Gamma_{\gamma^* \rightarrow q\bar{q}g}^{(0)} \right|^2. \end{aligned} \quad (2)$$

dM_i^2 and $d\phi_i$ denote the invariant mass and the phase space of the respective intermediate states $i = q\bar{q}, q\bar{q}g$.

The virtual corrections $\Gamma_{\gamma^* \rightarrow q\bar{q}}^{(1)}$ have been calculated in [17]. We have expressed all loop integrals in analytic form as an expansion in $\epsilon = (4 - D)/2$, quite in contrast to [26] where all integrals are kept as they are. For the real corrections, we considered the square of the particle-reggeon vertex $|\Gamma_{\gamma^* \rightarrow q\bar{q}g}^{(0)}|^2$, which has been calculated in [18] for longitudinally polarised virtual photons. In [19] we recently completed the real corrections by adding the contributions from transversely polarised photons. The real corrections have been considered in [27] as well, but not in a very suitable form for the task of finally evaluating the impact factor.

However, calculating the amplitudes as they are does not quite complete the task. The individual contributions are still infrared divergent and have to be combined in order to get the expected cancellation that has been shown previ-

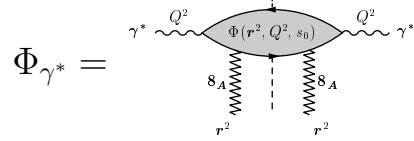


Figure 3. The γ^* impact factor.

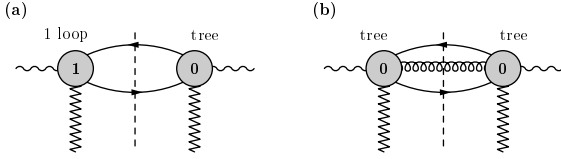


Figure 4. Contributions to the γ^* -impact factor at NLO.

ously [28]. At the same time, one has to consider the subtraction of leading logarithmic terms. These are present in the virtual corrections and proportional to the well-known LLA gluon trajectory function. In the real contribution to the impact factor they arise as the additional gluon is emitted with a large rapidity separation to the $q\bar{q}$ -pair. Both of these LLA-terms are individually infrared divergent as well as the emitted gluon becomes soft.

In [19] we consider the subtraction of infrared divergences and leading log terms in combination. We have extracted the infrared divergent contributions from real and virtual corrections and defined suitable subtraction terms. The difference of our results and the respective subtraction terms is finite upon integration over the gluon phase space. Re-adding the subtracted terms with the integrations over the gluon phase space performed, explicitly allows us to exhibit the infrared divergences as poles in ϵ and cancel them successfully against those from the virtual corrections. The final result for the NLO impact factor reads

$$\begin{aligned} \Phi_{\gamma^*}^{(1)} &= \Phi_{\gamma^*}^{(1,v)} \Big|_{\text{fin}} + C_A \Phi_{\gamma^*}^{(1,r)} \Big|_{C_A}^{\text{fin}} + C_F \Phi_{\gamma^*}^{(1,r)} \Big|_{C_F}^{\text{fin}} \\ &- \frac{2\Phi_{\gamma^*}^{(0)}}{(4\pi)^2} \left\{ \beta_0 \ln \frac{\mathbf{r}^2}{\mu^2} + C_F \ln(\mathbf{r}^2) \right\} \\ &+ \int \left| \Gamma_{\gamma^* \rightarrow q\bar{q}}^{(0)} \right|^2 \left\{ C_A [\ln^2 \alpha(1-\alpha)s_0 - \ln^2 M^2] \right. \end{aligned}$$

$$\begin{aligned} &+ 2C_F \left[8 - 3 \ln \alpha(1-\alpha)\Lambda^2 + \ln^2 M^2 \right. \\ &\left. + \ln^2 \frac{\alpha}{1-\alpha} \right] \left. \right\} \frac{dM^2}{2\pi} \frac{d\phi_{q\bar{q}}}{(4\pi)^2}. \quad (3) \end{aligned}$$

Therein, \mathbf{r}^2 is the transverse momentum of the reggeized gluon in the t channel, α is the light cone momentum fraction of the outgoing quark and the invariant mass of the $q\bar{q}$ -pair is denoted with M^2 . In the first line we have the finite remainders from the calculation from real (r) and virtual (v) corrections. The second line arises upon renormalization and includes a term proportional to β_0 , the only term depending on the renormalization scale μ . The last three lines are the finite remainders of the subtraction terms. The dependence on Λ , characterising the cone in which the emitted gluon is considered to be collinear, will cancel against a similar term, implicit in the first line.

The subtraction of the leading logarithmic terms induces a scale s_0 which can be translated into a rapidity cutoff beyond which the emitted gluon will belong to the leading logarithmic term. However, since the particular choice of this scale is arbitrary, the NLL impact factors depend on it. This dependence was irrelevant at LLA, since a change in the scale

$$\ln \frac{s}{s_0} = \ln \frac{s}{s_1} + \ln \frac{s_1}{s_0} = \ln \frac{s}{s_1} + \text{NLLA} \quad (4)$$

is of higher order w.r.t. the LLA. These NLLA terms are now taken care of and may be phenomenologically important.

3. Outlook and Conclusions

Besides the above discussion of the NLO impact factor, our calculations have the potential to give further insight into the photon wave function picture. This picture, in conjunction with the saturation model has been applied successfully to the description of both deep-inelastic and diffractive scattering cross sections at HERA, e.g. [29,30]. First steps in this direction have been done in [18], showing that an extension of the current picture to a higher $q\bar{q}g$ Fock-state of the virtual photon

is in principle possible. Further steps in this direction include a consistent treatment of infrared divergences in configuration space and remain to be done.

In order to complete the calculation of the impact factor we have to calculate the phase space integrals over the remaining infrared finite terms, defined in [19]. We will express the phase space integrals in terms of a set of standard integrals. For first phenomenological applications this might best be done numerically.

With these results we will be able to calculate the $\gamma^*\gamma^*$ cross section to NLL accuracy. In combination with the NLO jet vertex [23–25] an interesting variety of phenomenological applications of the NLO BFKL equation is now possible. Numerical results for the cross sections for $\gamma^*\gamma^*$ -scattering, production of forward jets at HERA and Mueller-Navelet jets at the Tevatron and the LHC are important goals of future work and remain to be done.

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