The role of $F_L(x, Q^2)$ in parton analyses.

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Abstract

We investigate the effect of the structure function F_L in global parton analyses of deep inelastic and related hard scattering data. We perform NLO and NNLO analyses which include the reduced cross section HERA data at high y, as well as earlier direct measurements of F_L. We find that the NNLO analysis gives a better description of F_L at low x than that performed at NLO. Nevertheless the data show evidence of the need of further contributions to F_L, which may be of higher-twist origin. We study such corrections both phenomenologically and theoretically via a renormalon approach. The higher-twist corrections extracted from a successful fit to the data are in general agreement with the theoretical expectations, but there is still room for alternative theoretical contributions, particularly at low x and Q^2. The importance of future measurements of F_L is emphasized.

1   Introduction

The cross-section for deep-inelastic charged lepton$-$proton scattering depends on the two independent structure functions $F_2(x, Q^2)$ and $F_L(x, Q^2)$. The former is dominated by the quark parton distributions, and the latter, in principle, by the gluon distribution (except at high x). However, $F_2(x, Q^2)$ is found to be much larger than $F_L(x, Q^2)$. Moreover, they appear in the cross-section in the combination

$$\tilde{\sigma}(x, Q^2) = F_2(x, Q^2) - \frac{y^2}{1 + (1 - y)^2} F_L(x, Q^2), \tag{1}$$

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where $y = Q^2/xs$. The quantity $\tilde{\sigma}(x, Q^2)$ has become known as the "reduced cross-section". Since $y \ll 1$ in most of the kinematic range, $\tilde{\sigma}(x, Q^2)$ is effectively the same as $F_2(x, Q^2)$. However, at HERA, for the lowest x-values at given Q^2, the value of y can become as large as $0.7 - 0.8$, and the effect of $F_L(x, Q^2)$ becomes apparent [1]. This is seen in the data as a flattening of the growth of $\tilde{\sigma}(x, Q^2)$ as x decreases to very small values (for fixed Q^2), leading eventually to a turnover. Hence, when analysing the HERA structure function data it is particularly important to fit any theoretical prediction to the measured $\tilde{\sigma}(x, Q^2)$, rather than to model-dependent extracted values of $F_2(x, Q^2)$ [2]. Indeed, important lessons may be learned by placing particular emphasis on the data at very high y. In this paper we examine the impact of the contribution from $F_L(x, Q^2)$ in this region.

As well as this very low x HERA data, we will also study the impact of the much higher x direct measurements of $F_L(x, Q^2)$, which were made by SLAC [3], BCDMS [4] and NMC [5] by measuring the cross-section at a variety of values of y. In this region the contribution from both quarks and gluons is obviously important, as indeed it turns out to be at the low-x values. It has recently been proposed to make a direct determination of F_L at the low-x values accessible at HERA by making some measurements of the cross-sections with lowered beam energy [6]. There is also a possibility of a measurement associated with eRHIC [7]. We conclude by discussing the importance of such future measurements.

2 Perturbative Stability of $F_L(x, Q^2)$

We have recently been able to make accurate and reliable predictions of $F_L(x, Q^2)$ up to NNLO in perturbative QCD [8, 9]. The procedure is to first determine the parton distribution functions from a global fit to the available deep inelastic and related hard scattering data, without the inclusion of any F_L data. For instance, the gluon distribution is constrained at small x by measurements of $\partial F_2/\partial \ln Q^2$, and at large x by Tevatron jet data. The extracted partons are then used to predict F_L. In this way we can study the perturbative stability of this fundamental quantity as one increases the order of the calculation. The results obtained are shown in Fig. 1.[2] It is immediately clear that at NLO there is a serious problem with $F_L(x, Q^2)$ at the lowest values of x and Q^2 with it becoming (unphysically) negative. This is a reflection of the behaviour of the gluon distribution at the same order. However, the NNLO coefficient functions for $F_L(x, Q^2)$, $C_{Lg,q}^{\rm NNLO}$, turn out to be large and positive at small x (for both quarks and gluons). In detail

$$F_L = \alpha_S(C_{Lg}^{\rm LO} + \alpha_S C_{Lg}^{\rm NLO} + \alpha_S^2 C_{Lg}^{\rm NNLO} + ...) \otimes g \;\; + \;\; g \to q \tag{2}$$

where up to NLO the shape of $F_L(x, Q^2)$ is dominated by that of the partons, particularly the gluon at low x. ($C_{Lg}^{\rm NLO}$ is divergent at small x, but the $1/x$ term has a very small, negative

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[2]The NNLO variable-flavour number scheme [10] has been used in the NNLO global analysis. The details of the results are sensitive to this updated treatment of heavy flavours.
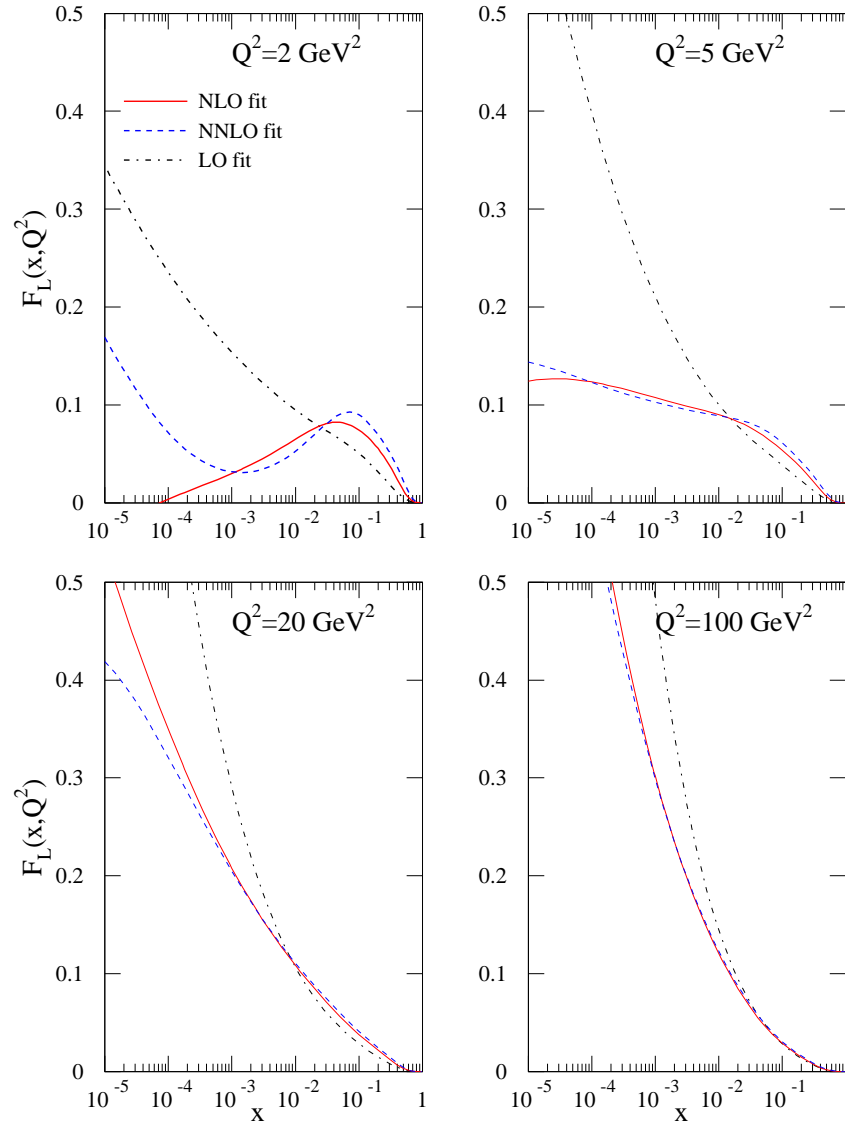
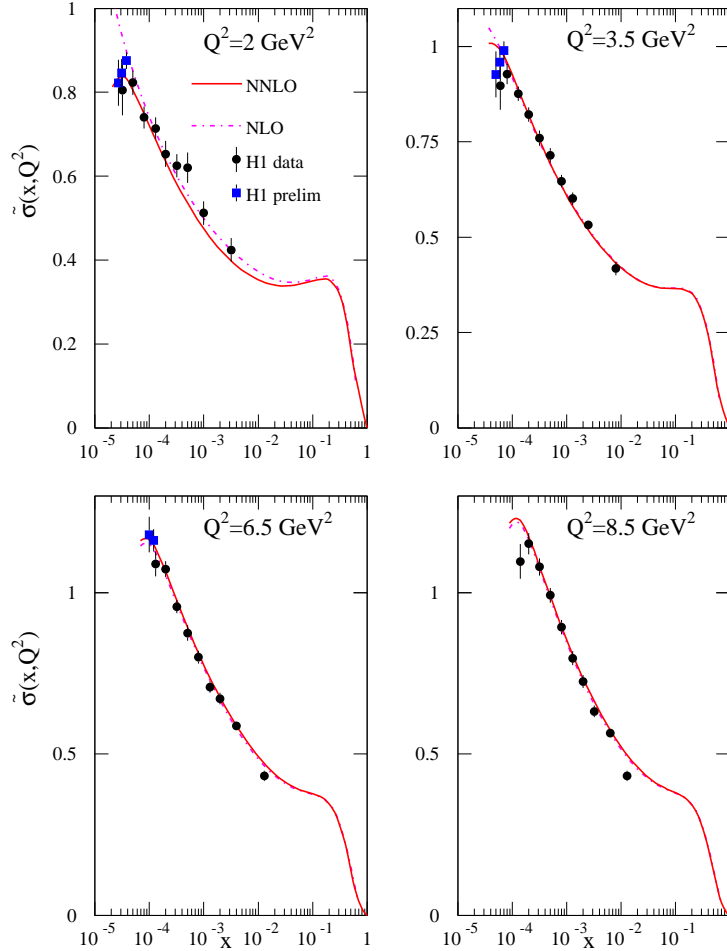Figure 1: $F_L(x, Q^2)$ predicted from the global fit at LO, NLO and NNLO.

Figure 2: The consistency check of $F_L(x, Q^2)$ for the NLO and NNLO MRST fits.

coefficient.) The NNLO longitudinal coefficient function $C_{Lg}^{NNLO}(x)$ is given by

$$C_{Lg}^{\text{NNLO}}(x) = n_f \left(\frac{1}{4\pi}\right)^3 \left(\frac{409.5 \ln(1/x)}{x} - \frac{2044.7}{x} - \cdots\right). \tag{3}$$

There is clearly a significant positive contribution at very small x, $x \ll 0.01$, and this counters the decrease in the small-x gluon. Hence, even though the gluon is even more negative at small x and Q^2 at NNLO than it is at NLO, the prediction for $F_L(x, Q^2)$ has become positive. Indeed, the effect of the NNLO coefficient functions is so important, at low x and Q^2, that $F_L(x, Q^2)$ starts to increase as x decreases below about 10^{-3}. At higher Q^2, i.e. $Q^2 \lesssim 5$ GeV^2, the NLO and NNLO predictions are not too dissimilar at small x, though the current very close agreement at $Q^2 = 5$ GeV^2 is coincidental.

This predicted increase in $F_L(x, Q^2)$ is very important for the comparison with the high-y HERA $\tilde{\sigma}$ data. The very small, or even negative, values at NLO mean that there is no turn-over in the theoretical curves to accompany that in the data, as seen in Fig. 2. However, the discrepancy is cured at NNLO, and the comparison with $\tilde{\sigma}(x, Q^2)$ is quite good. In fact, if

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the NNLO $\overline{\text{MS}}$ gluon distribution were positive definite at input ($Q_0^2 = 1 \text{ GeV}^2$) the resulting $F_L(x, Q^2)$ would be rather too large at the smallest x and Q^2 and the turnover in $\tilde{\sigma}(x, Q^2)$ would be too large. (Also the shape of $F_L(x, Q^2)$ with Q^2 at small x would be very strange, growing quickly as Q^2 decreases.) This illustrates the fact that the small-x gluon is a scheme-dependent, unphysical quantity, which in the $\overline{\text{MS}}$ scheme is very unlike the physical $F_L(x, Q^2)$.

Bearing this in mind, we attempted to construct a more "physical" definition for the small-x gluon, in a similar spirit to that we used recently for the high-x gluon [11]. Explicitly, we invented a scheme where the gluon was defined by

$$\tilde{g}^{F_L}(x, Q^2) = \left(\delta(1-x) + \frac{\alpha_S \tilde{C}_{Lg}^{\text{NLO}} + \alpha_S^2 \tilde{C}_{Lg}^{\text{NNLO}}}{C_{Lg}^{\text{LO}}} \right) \otimes g^{\overline{\text{MS}}}(x, Q^2), \tag{4}$$

where $\tilde{C}_{Lg}^{\text{NLO}}$ and $\tilde{C}_{Lg}^{\text{NNLO}}$ are functions identical to the NLO and NNLO coefficient functions in the small-x limit, but modified at high-x so that momentum is conserved in the transformation between schemes. If the exact coefficient functions were used in Eq.(4) then the "physical" definition of the gluon, $\tilde{g}^{F_L}(x, Q^2)$ would be guaranteed to be the same shape as $F_L(x, Q^2)$ at small x, and hence would have a genuine physical interpretation in this scheme. However, performing fits where we defined as input $\tilde{g}^{F_L}(x, Q_0^2)$ and transformed to the $\overline{\text{MS}}$ scheme (as we defined the high-x gluon in DIS scheme before transforming to the $\overline{\text{MS}}$ scheme in [11]) we found that $\tilde{g}^{F_L}(x, Q^2)$ still tended to be valence-like or even negative at small x. This is because the exact C_{Lg}^{NLO}, and particularly C_{Lg}^{NNLO}, do not have zero first moment, i.e. are not momentum conserving, and in fact using the real coefficients in Eq.(4) would leads to a considerably larger value of $\tilde{g}^{F_L}(x, Q^2)$ than momentum conserving functions can. Hence, we conclude that it is not easy to devise a simple scheme where the low-x gluon behaves like $F_L(x, Q^2)$, but where the interpretation in terms of parton distribution functions as probabilities is clearly maintained.

We conclude that even though the NNLO prediction for $F_L(x, Q^2)$ is much better than that at NLO, there are still problems. Perturbation theory does not seem to be converging for this quantity at low x and Q^2. Indeed, there have previously been suggestions that small-x resummations may play an important role in $F_L(x, Q^2)$ [12]. However, there also appears to be a problem at higher x. The comparison of theory to data for $F_L(x, Q^2)$ is not satisfactory for the higher-x direct measurements of $F_L(x, Q^2)$. Indeed, when we perform new global fits including the F_L data, we find χ^2 of 44 for the 36 F_L points at NLO, with a definite tendency for data to lie above theory. There is a big improvement at NNLO, with the corrections being large and positive, and the χ^2 is 36/36. However, there is still a tendency for data to lie a bit above theory. A comparison between data and theory is shown in Fig.3. In any case at high x it is likely that higher twist is an important contribution to $F_L(x, Q^2)$.

3   The inclusion of Higher Twist

The paucity of data for $F_L(x, Q^2)$, as compared with the case for $F_2(x, Q^2)$, means that it is not possible to perform the entirely phenomenological analysis of higher twist that we performed
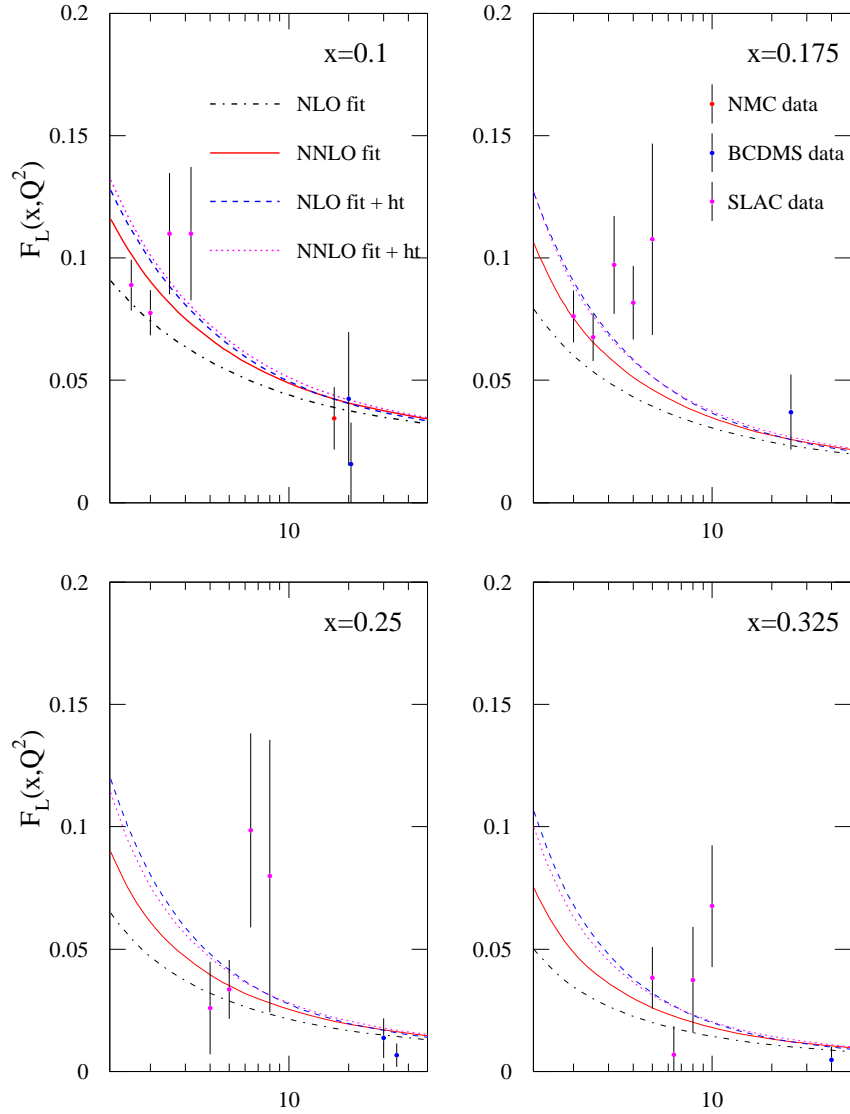
Figure 3: The comparison with the direct data on $F_L(x, Q^2)$ at NLO and NNLO; and also after including higher twist contributions as described in the text.

for the latter [13], i.e. to include a higher twist correction of the form $(c(x)/Q^2)F_2(x, Q^2)$ where we allowed $c(x_i)$ to be independent parameters representing 13 bins in x.[3] For $F_L(x, Q^2)$ the number of bins would have to be much smaller to avoid having gaps, or having only 1 or 2 points in a bin. Hence, we appeal to theoretical motivation for our choice of the higher-twist correction. For the case of the nonsinglet higher-twist contribution to $F_2(x, Q^2)$, a correction of the form $(c(x)/Q^2)F_2(x, Q^2)$ has long been suspected to be enhanced by $1/(1-x)$ at high x. It also must satisfy the Adler sum rule, and hence vanish as $x \rightarrow 0$. The renormalon calculation [15] has exactly this trend, as indeed does the phenomenological higher twist extracted by a global fit [13]. In particular there seems to be no evidence for a large contribution beyond the nonsinglet contribution, and renormalon calculations are problematical for such extra contributions [16].

For $F_L(x, Q^2)$ there is no reason to expect the same type of enhancement at high x, and also no reason for the higher-twist contribution to vanish at low x. In this case a correction of the approximate form $(c/Q^2)F_2(x, Q^2)$ is expected, where c is constant. Again the renormalon calculation is in reasonable agreement with the naive prediction [17, 15], giving a nonsinglet contribution of

$$F_L^{HT}(x, Q^2) = \frac{A}{Q^2}(\delta(1-x) - 2x^3) \otimes \sum_f Q_f^2 q_f(x, Q^2). \tag{5}$$

This, and similar expressions are often presented with $F_2(x, Q^2)$ on the right-hand side. However, it is strictly the quarks that should appear, with the coefficient functions containing the appropriate quark-squared weighting, this combination of quark distributions and charge-squared weighting then being identical to the LO expression for $F_2(x, Q^2)$. The overall normalisation A has been estimated to be $(8C_F \exp(5/3)/\beta_0)\Lambda_{QCD}^2$ [17], but this is uncertain, and its value can also vary enormously by choosing Λ_{QCD} defined at different orders. In [18] the generous NLO value[4] of $\Lambda_{QCD} = 347$ MeV was taken, giving a large higher-twist correction with $A = 0.8$ GeV^2.[5]

Here we perform global fits including both the direct data on $F_L(x, Q^2)$ and the indirect high y data from HERA, in which we allow A to be a free parameter. At both NLO and NNLO the quality of the fit to both the direct and the HERA data improves. For the direct data we get $\chi^2 = 34/36$ at NLO and one better at NNLO. The values $A = 0.36$ and $A = 0.16$ are found at NLO and NNLO respectively, each with an error of ± 0.08. The quality of the fit to direct data is shown in Fig.3, and the effect of the higher-twist contribution to $F_L(x, Q^2)$ can be seen from Fig.4. The values of A preferred by fitting only to the HERA data are larger $(A = 0.58(0.25)$ at NLO(NNLO)$)$, but this makes $F_L(x, Q^2)$ too large for the best fit to the direct data.

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[3] There is an analysis for $F_L(x, Q^2)$ similar in spirit to this in [14] which has only one $x < 0.1$ bin. It also predates the calculation of the NNLO coefficient functions for $F_L(x, Q^2)$.

[4] Note that the most recent MRST2004 values are $\Lambda_{QCD}^{NLO} = 347$ MeV and $\Lambda_{QCD}^{NNLO} = 251$ MeV.

[5] In [18] A was defined to be the coefficient of the whole higher-twist term for the first moment rather than the coefficient of the x-dependent function, hence the value of $A = 0.4$ GeV^2 was quoted.
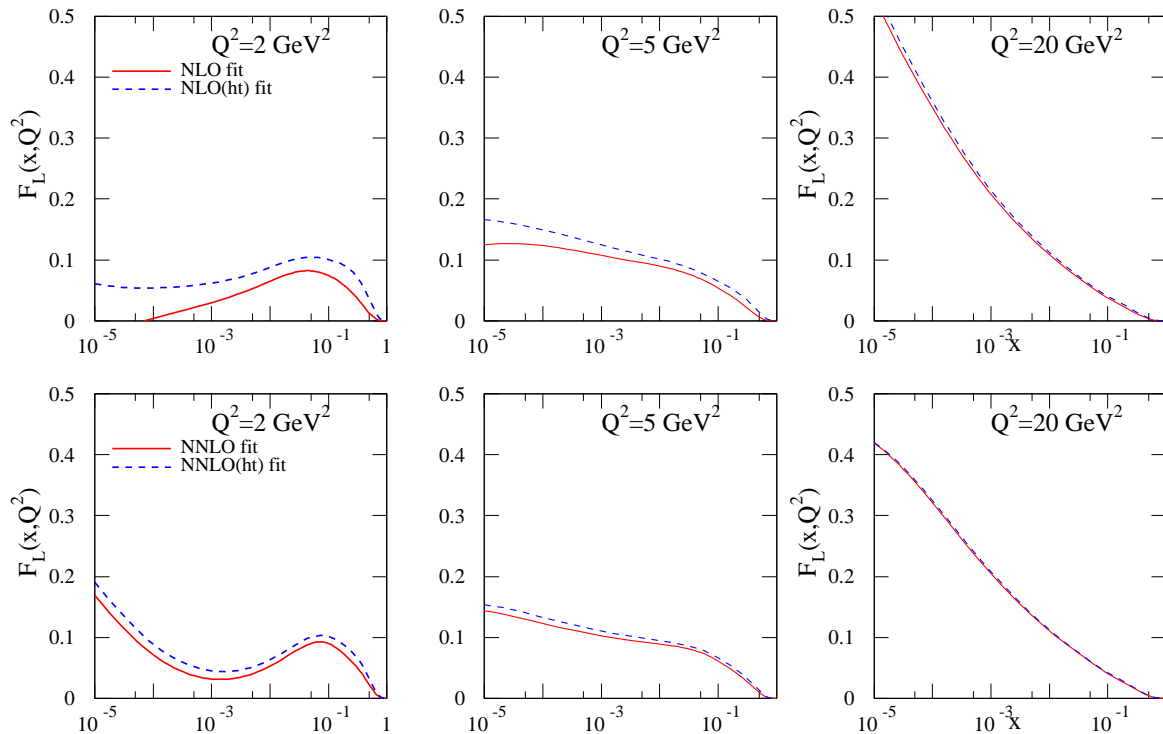
Figure 4: Predictions for $F_L(x, Q^2)$ at NLO (NNLO), with and without the renormalon correction, are shown in the top (bottom) set of plots.

We also tried a fit with a simpler form of the higher-twist correction

$$F_L^{HT}(x, Q^2) = \frac{A}{Q^2} \sum_f Q_f^2 q_f(x, Q^2). \tag{6}$$

This lead to no changes of any significance. The fit is of essentially the same quality. The value of A increases by a factor of 3, which is exactly what one gets if one takes the small-x limit of Eq.(5). This shows that except at very high x the correction in Eq.(5) reduces to that in Eq.(6) with 1/3 times the normalization. This is easily understood. In moment space the multiplicative factor in Eq.(5) is $N/(N+2)$ (using the convention in [17]), which in the small-x $(N \to 1)$ limit becomes 1/3.

We can also investigate the size of the renormalon correction by further examination of the formulae in [17]. The renormalon correction is obtained from taking the pole contributions of the inverse Borel transformation of Eq.(20) in [17]. This then gives the term in Eq. (23) in [17], but with an uncertain normalization. Alternatively one can derive the perturbative expansion in the naive non-abelianization limit by expanding Eq. (20) in powers of s, the Borel conjugate variable to Q^2, and performing the inverse transformation term-by-term. Doing this in moment space for their $N = 1$, i.e. the conventional $N = 0$ (small-x limit), we find, for $\alpha_S = 0.36$,

successive LO, NLO, NNLO,... contributions[6]

$$0.0765 + 0.052 + 0.043 + 0.043 + 0.050 + ... \quad , \tag{7}$$

i.e. the LO coefficient function for $\alpha_S = 0.36$ is 0.0765, the NLO correction in the large β_0 limit is 0.052 the NNLO contribution in the same limit is 0.043 *etc.* Hence, for $\alpha_S = 0.36$, which corresponds to $Q^2 \sim 2$ GeV^2, the NNLO and NNNLO term are roughly the same and we are led to keep three perturbative terms, with 0.043 being an estimate of the higher twist. Indeed, if one evaluates the finite part, i.e. the principal value, of Eq.(23) in [17] one gets 0.1782 (to be compared with $0.0765 + 0.052 + 0.043 = 0.1715$) as the perturbative contribution with the renormalon correction taken out, which is very consistent.

Repeating this procedure for $\alpha_S = 0.3$, i.e. $Q^2 \sim 4$ GeV^2, one gets

$$0.064 + 0.036 + 0.025 + 0.021 + 0.020 + 0.023 + ... \quad . \tag{8}$$

Now N^{3}LO is still smaller than NNLO, and perhaps N^{4}LO is representative of the overall higher-twist contribution, i.e. 0.02, which is about half of 0.043, consistent with a $1/Q^2$ dependence when Q^2 goes from $2 \to 4$ GeV^2. In this case, explicit evaluation of the principle value of Eq.(23) gives 0.1482, to be compared with 0.146 (the sum of the first four terms). Again we have consistency, but where the series should be truncated is a function of Q^2. However, the term one includes at higher Q^2 is rather small.

Since the renormalon term is $A/Q^2 \times N/(N+2)$ then at $N = 1$ our higher twist should be $A/(3Q^2)$. For $Q^2 = 2$ GeV^2 we therefore have $A/6 \sim 0.04$, that is $A \sim 0.24$. This should be compared with the values $A = 0.38$ and $A = 0.16$ obtained in our NLO and NNLO fits. However, it is clear that the NLO value is too high because it is missing significant NNLO corrections, whereas perhaps all the correction to NNLO should be higher twist. In this case the fitted value of 0.16 compares well with the approximate prediction of 0.24, especially considering that the fit to HERA data alone favours $A \sim 0.25$.

Considering the variation with N, or equivalently with x, leads to complications however. The higher-twist correction at $N = 1$ (that is the conventional $N = 0$) is $A/(3Q^2)$, whereas at larger N it tends to A/Q^2. This might suggest that at a fixed Q^2 a particular term in the series, e.g NNLO at $\alpha_S = 0.36$, increases with increasing N, saturating at 3 times its $N = 1$ value at large N. This is not the case. In fact the NNLO term is slowly varying with N, and actually decreases for very large N. For example, for $N = 6$ and $\alpha_S = 0.36$ we get

$$0.0218 + 0.0297 + 0.0404 + 0.056 + ... \tag{9}$$

i.e. the NNLO term is slightly smaller than at $N = 1$; but note that we now have no convergence. We have to go to higher Q^2 to get any evidence of convergence. If we take, for example, $\alpha_S = 0.24$ (corresponding to $Q^2 \sim 10$ GeV^2) then for $N = 6$ we have

$$0.0145 + 0.0132 + 0.0120 + 0.0111 + 0.0109 + 0.0114 + ... \quad . \tag{10}$$

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[6]To be precise, the series is the perturbative expansion of the $N = 1$ moment of the quark coefficient function. Thus, the moment of the quark contribution to F_L (which should be dominant at highish x, and is in practice important at all x at low Q^2) is the sum of the series multiplied by the moment of the quark distribution.

So at this scale there is some convergence. Compare this to the same Q^2 for $N = 1$,

$$0.0508 + 0.0233 + 0.0129 + 0.0085 + 0.0066 + 0.0061 + 0.0065 + ... \quad . \tag{11}$$

Here we would keep 5 or 6 terms in the series and the higher-twist contribution is 0.0065 (compared to $0.24/(3Q^2) = 0.008$, which is not too bad). However at $N = 6$ we would keep 4 or 5 terms and the higher twist is 0.011. So at the higher Q^2 the picture is reasonably consistent, though arguably we should treat more terms as perturbative at low x than at high x, and the higher-twist term enters at about 6th or 7th order. On the other hand, at low Q^2 we cannot say that the higher twist is roughly NNNLO. This is the case at low x, but at high x the situation is confused. If we evaluate the inverse transformation explicitly for $N = 6$ and $\alpha_S = 0.36$ we find 0.022 for the perturbative contribution, which implies keeping something like just the LO term with NLO representing the size of the higher twist, two orders lower than at lower N and x. However the size of the higher twist is then not quantitatively consistent. We conclude that the high x and low Q^2 domain is "dangerous". This is another reason, along with target mass, to avoid fitting data in this kinematic region. For $\alpha_S = 0.24$ (that is $Q^2 \sim 10$ GeV^2) the explicit integral gives 0.058 for $N = 6$, consistent with keeping the first five terms. For $N = 1$ it gives 0.0987, again roughly corresponding to the first 5 terms. Hence, as long as we stay away from high N combined with low Q^2 everything is consistent. Nevertheless, at high N and low Q^2 one can still get some idea of what is happening, i.e. the whole perturbative expansion is difficult to distinguish from higher twist, but the problem is that there is no quantitative approach applicable in this (high x, low Q^2) domain.

4 Conclusions

There is much to learn from the inclusion of the data which are both directly, and indirectly, sensitive to $F_L(x, Q^2)$ in a global fit. There is clear evidence that NLO in perturbation theory alone is not a sufficiently sophisticated approach to fit either the direct or indirect data. There is an improvement from including the recently calculated NNLO corrections, but still some significant problems. The further inclusion of a higher-twist contribution, inspired by the renormalon correction in the nonsinglet sector, is very successful, and the normalization of this contribution seems to be consistent with the theoretical expectations. However, there is still room for further theoretical corrections, particularly at low x and Q^2. There are numerous suggestions that even higher orders in perturbation theory are important here due to the large $\ln(1/x)$ terms. Indeed, the NNNLO coefficient functions calculated in $\overline{\text{MS}}$ scheme in the latter paper of [9] lead to large corrections for $Q^2 = 2$ GeV^2 and $x \sim 0.0001$, and studies including resummations suggest important modifications in this region for $F_L(x, Q^2)$, see, for example, [19, 20]. The same is true for studies using dipole models [21, 22] which contain both higher-order corrections and higher twists (different from those in the renormalon correction considered here). A variety of predictions from different theoretical approaches is shown in Fig.5. A direct measurement of $F_L(x, Q^2)$ for $Q^2 \lesssim 5$ GeV^2 would be very important in determining which theoretical approaches are most reliable, as discussed in detail in [18].
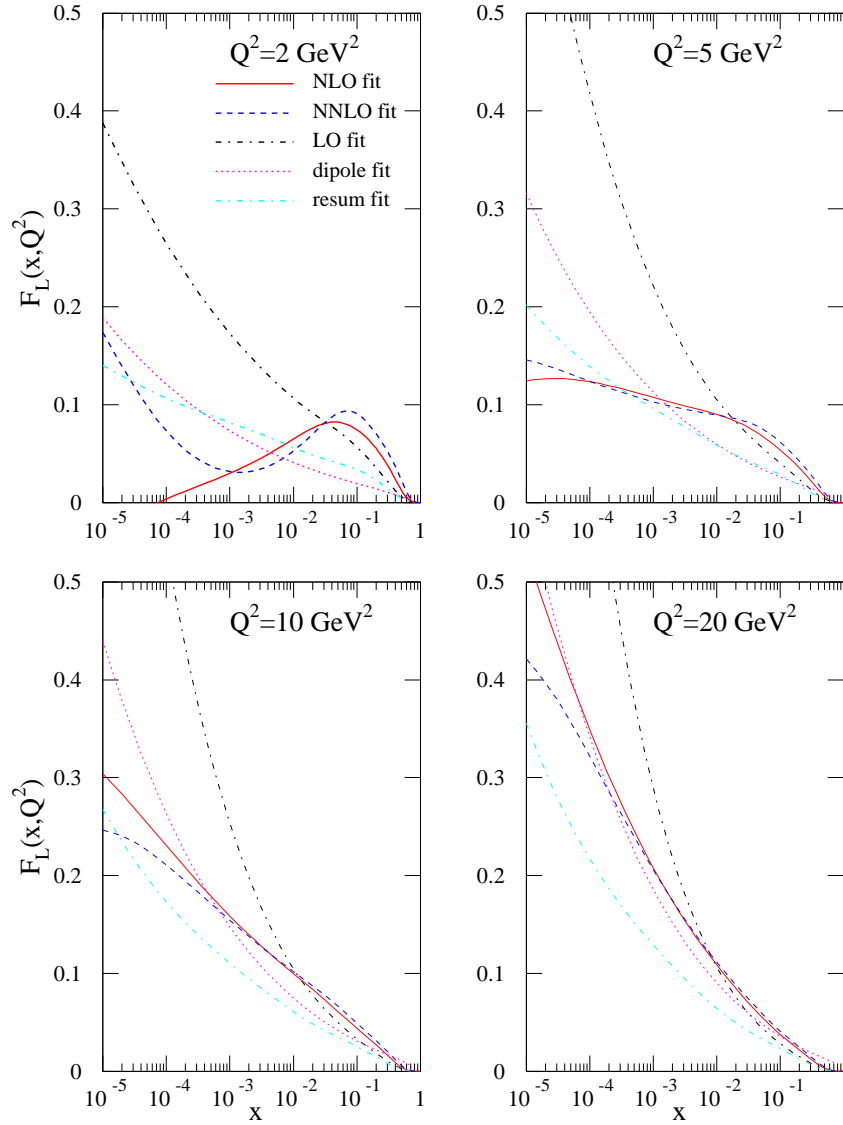
Figure 5: $F_L(x, Q^2)$ predicted from the global fit at LO, NLO and NNLO, also from a fit which performs a double resummation of leading $\ln(1/x)$ and β_0 terms, and finally from a dipole model type fit.

Acknowledgements

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