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$m_{T\text{Gen}}$: Mass scale measurements in pair-production at colliders

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We introduce a new kinematic event variable $m_{T\text{Gen}}$ which can provide information relating to the mass scales of particles pair-produced at hadronic and leptonic colliders. The variable is of particular use in events with a large number of particles in the final state when some of those particles are massive and not detected, such as may arise in R-parity-conserving supersymmetry.

1 Introduction

When LHC experiments like ATLAS [1, 2] and CMS [3] begin to look for signs of supersymmetry (or any other models with large numbers of new particles), one of the first things they will want to probe is the mass scale associated with these new particles, if there are any. Single particles produced in narrow s-channel resonances will in general be easy to spot (one would hope) and will not be considered further here. The fun begins when new particles are produced in pairs in non-resonant processes, in particular those in which some particles go unobserved. Examples within the context of R-parity conserving supersymmetry might include processes like $gg \to \tilde{g}\tilde{g}$, $gq \to \tilde{g}\tilde{q}$, $q\bar{q} \to \chi_2^0\chi_1^0$ or $q\bar{q} \to \tilde{l}\bar{\tilde{l}}$.

In this paper we present a new event variable, "$m_{T\text{Gen}}$", which is designed to measure the mass scale(s) associated with any new particle(s) which might be *pair* produced at future colliders. On an event-by-event basis, $m_{T\text{Gen}}$ supplies a lower bound for the mass of either of the two particles which were pair produced and whose decay products were observed, under the assumption that the event was indeed of that kind. The intention is that over *many* events a histogram of $m_{T\text{Gen}}$ would reveal one or more edge structures whose upper endpoints would correspond to the masses of the particles which were being produced in large numbers. The $m_{T\text{Gen}}$ variable takes as input (1) the reconstructed momenta of the observed particles in each event, and (2) the masses of any unobserved particles which are taken to have been produced in the decays of the primary particles. Though $m_{T\text{Gen}}$ can benefit from information regarding particle identification, this is not a requirement.

It is not the intention of this paper to discuss how $m_{T\text{Gen}}$ performs when particle momenta are poorly measured, nor to discuss issues of finite precision or acceptance. Demonstration of the performance of $m_{T\text{Gen}}$ in environments representative of future colliders is the subject of current ongoing work. This paper seeks primarily (1) to provide a source of documentation for the definition of the $m_{T\text{Gen}}$ variable, and (2) to document in the Appendix an analytic closed form approximation for m_{T2} which is valid for events with very little initial state radiation and which is needed to demonstrate that $m_{T\text{Gen}}$ can be calculated efficiently.

2 $m_{T\text{Gen}}$

As far as $m_{T\text{Gen}}$ is concerned, an event at a future collider is a set O containing n_O observed Lorentz four-momenta: $O = \{o_i^\mu : i = 1, \ldots, n_O\}$. Although quantum interference means that terms like "initial state" and "final state" cannot really be applied to the momenta in O in a well defined way, it is nevertheless common and expedient when analysing real events to treat some momenta as if they were the result of the decays from the pair of partons produced in the primary $2 \to 2$ process used in the matrix element, and to treat other momenta as if they were the result of initial state radiation (ISR). We will therefore divide the observed momenta into two non-overlapping sets $F = \{f_i^\mu : i = 1, \ldots, n_F\}$ (for momenta supposedly from the central $2 \to 2$ pair production process) and $G = \{g_i^\mu : i = 1, \ldots, n_G\}$ (for momenta supposedly from initial state radiation). In practice this might be done by assigning all four-momenta whose transverse momenta are greater than some threshold and whose rapidity is sufficiently central to F, while placing anything else in G. The value of $m_{T\text{Gen}}$ for each event will inevitably depend to some extent the exact nature of the cuts used, and potential biases would have to be investigated in specific cases. Most event variables (e.g. thrust and sphericity) have similar second order dependence on cuts. The endpoint structures which are the eventual target of any $m_{T\text{Gen}}$ investigation, however, are expected to be particularly insensitive to these cuts. Individual events where misassignments are made will either be swept below the endpoint by the minimisation procedure (when momenta are omitted from F), or will be smeared above the endpoint (when ISR with unusually large transverse momentum is added to F in error).

In a real event, we do not know from which "side"[1] of the event any particular observed particle has come. If we *did* know from which side each particle had come, we could use m_{T2} [4, 5] to place a lower bound on the mass of the two hypothesised outgoing primary particles. For a given event, even though we do not know the "correct" side assignments, there is nothing to prevent us trying *all possible side assignments*, evaluating m_{T2} for each of them, and then reporting the lowest value of m_{T2} so obtained. This is in fact how $m_{T\mathrm{Gen}}$ is defined.

2.1 Definition of $m_{T\mathrm{Gen}}$

$m_{T\mathrm{Gen}}$ is defined to be the smallest value of m_{T2} [4, 5] obtained over all possible partitions of momenta in F into two subsets α and β – each subset representing the decay products of a particular "side" of the event. Recall that m_{T2} is itself defined in terms of $\mathbf{p}_T^\alpha$ and m_α (respectively the transverse momentum and invariant mass of one side of the event), $\mathbf{p}_T^\beta$ and m_β (respectively the transverse momentum and invariant mass of the other side of the event), and χ (the mass of each of the unobserved particles which are supposed to have been produced on each side of the event) as follows:

$$\begin{aligned} m_{T2}^2 \quad & (\mathbf{p}_T^\alpha, \mathbf{p}_T^\beta, \not{\mathbf{p}}_T, m_\alpha, m_\beta, \chi) \equiv \\ \equiv \quad & \min_{\not{\mathbf{q}}_T^{(1)} + \not{\mathbf{q}}_T^{(2)} = \not{\mathbf{p}}_T} \left[\max \left\{ m_T^2(\mathbf{p}_T^\alpha, \not{\mathbf{q}}_T^{(1)}; m_\alpha, \chi),\ m_T^2(\mathbf{p}_T^\beta, \not{\mathbf{q}}_T^{(2)}; m_\beta, \chi) \right\} \right] \end{aligned} \tag{1}$$

where

$$m_T^2(\mathbf{p}_T^\alpha, \mathbf{p}_T^{\chi_1^0}; m_\alpha, \chi) \equiv m_\alpha^2 + \chi^2 + 2(E_T^\alpha E_T^{\chi_1^0} - \mathbf{p}_T^\alpha \cdot \mathbf{p}_T^{\chi_1^0}) \tag{2}$$

in which

$$E_T^\alpha = \sqrt{(\mathbf{p}_T^\alpha)^2 + m_\alpha^2} \qquad \text{and} \qquad E_T^{\chi_1^0} = \sqrt{(\mathbf{p}_T^{\chi_1^0})^2 + \chi^2} \tag{3}$$

and likewise for $\alpha \longleftrightarrow \beta$. With the above definition (in the case $\chi = m_{\chi_1^0}$), m_{T2} generates an event-by-event lower bound on the mass of the particle

[1] We will use the term "side" to refer to the division of the particles in F into two groups, depending on which of the two outgoing primary particles they descend from. An event, then, is an object with two sides, and possibly also some initial state radiation. The term "side" is not meant to suggest that the momenta of a particular side are in some way spatially correlated (e.g. in one hemisphere). Indeed, if the two primary partons were scalars produced at threshold, then the decay products of each "side" would be completely intermixed.

whose decay products made up either of the two sides of the event, under the assumption that the event represents pair production followed by decay to the visible particles and an unseen massive particle on each side. When evaluated at values of $\chi \neq m_{\chi_1^0}$ the above properties are retained approximately (see [4, 5]). There exist events which allow this lower bound to saturate, and so (in the absence of background) the upper endpoint of the m_{T2} distribution may be used to determine the mass of the particle being pair produced.

We caution the reader to avoid the trap of mistakenly concluding that $m_{T\mathrm{Gen}}$, as defined above, is a function of purely transverse (and not also longitudinal) momenta of the visible particles in F. On the contrary, the definition above makes use of the z-momentum of every visible particle. Although this z-momentum plays no part in forming the transverse *momentum*, $\mathbf{p}_T^\alpha$ or $\mathbf{p}_T^\beta$, of either side in any partition, it nonetheless can play a significant role in forming each side's *invariant mass*: m_α or m_β. (This caution is not unique to $m_{T\mathrm{Gen}}$, but is equally relevant for any situation in which m_{T2} is used where a side consists of an "effective" particle composed of two or more real particles.) We note that it is possible to define a "Truly Transverse" form of $m_{T\mathrm{Gen}}$, which we shall denote "$m_{TT\mathrm{Gen}}$" by requiring, before evaluation begins, that each input four-momentum be individually longitudinally boosted to a frame in which its z-component of momentum is zero. In effect this throws away each particle's z-momentum, does not change its transverse momentum, and reduces its energy so as to keep the particle's mass invariant. It is equivalent to evaluating $m_{T\mathrm{Gen}}$ in a "transverse" Minkowski space with (1+2) dimensions rather than the usual (1+3) dimensions.

3 Discussion

m_{T2} has been used in the definition of $m_{T\mathrm{Gen}}$, rather than a function of the invariant masses of the two sides of the event, as it is vital to account for the energy-momentum of unobserved particles. If an event has a non-zero total transverse momentum, then either there were unobserved particles in the event, or some momentum was mis-measured (or lost or created), or indeed a combination of both. m_{T2} takes these unobserved particles into account in hadron colliders. However, see the comment in the next section regarding use in lepton colliders.

3.1 Regarding specialisations

There is room for some slight specialisation of $m_{T\mathrm{Gen}}$ to the particular problem in hand. $m_{T\mathrm{Gen}}$ need not always be calculated in exactly the same way.

For example, if it were desirable to suppose that neither side of an event could decay *entirely* into invisible particles, then one might wish to impose the additional constraint that F contain *at least two* momenta, and that one should only consider partitions of F into *non-empty* subsets. Events with fewer than two momenta in F could either be ignored or given $m_{T\text{Gen}}$ values of 0.[2]

In an alternative specialisation, one could suggest constraining the partitions of F in such a way as to allow only those which meet certain requirements in terms of conserved quantum numbers. For example, one might choose to reject assignments which make the absolute value of the charges on either side of the event greater than 1 or veto events where the total charge in F is more than some fixed value.[3] One might try to restrict on the basis of lepton number, arguing from the standpoint of lepton universality (though this might be hard given the possibility of unobserved neutrinos).

Finally, if $m_{T\text{Gen}}$ were to be used at a lepton collider where the momentum of the centre of mass was known to a reasonable precision, it would be sensible to replace m_{T2} (which is a variable designed for hadron colliders and so uses only transverse quantities in order to be insensitive to longitudinal boosts) with a variable analogous to m_{T2} but designed to make use of z-momenta.

3.2 Avoiding over-specialisation

Though one can tailor $m_{T\text{Gen}}$ to the requirements or assumptions of a particular investigation, it should be pointed out that the "philosophy" of $m_{T\text{Gen}}$ is to avoid such specialisation where unnecessary.

Most events that are actually of the pair-produced type we are concentrating on will have a large number of partitions. One and only one of those partitions is correct, and it will produce an m_{T2} value that is appropriate (i.e. bounded above by the mass of the initially produced particle). All other (i.e. all wrong) partitions will result in two or more particles from different "sides" of the event being assigned to the same side. These wrong partitions

[2]When investigating particular classes of R-parity violating supersymmetric models, an alternative specialisation might be appropriate. Here it might be desirable to suppose that none of the decay products on either side of the event could go undetected (be invisible). In this case one might consider evaluating not m_{T2} in the definition of $m_{T\text{Gen}}$ but instead the larger of the two *invariant masses* of each side of the event. Note, however, that if this were done, one would have to satisfactorily address the question of where any observed missing transverse momentum had come from. Under the supposition at work here, large missing transverse momenta would indicate either large measurement error, or an event incompatible with the assumptions, and might suggest that the event should be disregarded or recalibrated.

[3]For example, at the LHC the charge of primary interaction should be -1, 0 or $+1$.

thus *tend* to have have large m_{T2} values, if only for the reason the union of particles from opposite sides of the event tends to yield four-momenta with very large invariant masses. Most wrong partitions therefore lead to m_{T2} values larger than that generated by the "correct" partition, and so excluding large numbers of these "bad" partitions often has no effect on the eventual value of $m_{T\text{Gen}}$.

By avoiding complex specialisations, one can make the variable to a large extent insensitive to a detector's ability (or inability) to distinguish particle types. This can make the variable useful for example during early running when the detectors' abilities to determine particle identification may not be well understood.

3.3 Use as a cut variable

Although m_{T2} was originally proposed as a variable for measuring particle masses, it can also be used as a "cut variable" intended to separate certain new-physics signals from Standard Model backgrounds.[4] This success is in part due an accidental conspiracy of three effects. (1) m_{T2} tends to small values for back-to-back QCD-like events. (2) m_{T2} tends to small values when the missing transverse momentum is small (which it is in much of QCD). (3) m_{T2} tends to small values when the missing momentum is parallel to one of the visible particles fed to m_{T2}. This can easily happen in QCD when there are neutrinos in a jet, or a single jet is mismeasured through inadequate containment in a detector or passage through a crack region. By accident rather than by design, therefore, m_{T2} tends to shift badly measured and Standard Model events away from large values, which are where the endpoints of the distributions containing new physics are expected to be found.

$m_{T\text{Gen}}$ shares those features of m_{T2}, as it is always bounded above by m_{T2} for the correct partition. $m_{T\text{Gen}}$ may therefore also be expected to find a role as a "cut variable", distinguishing events by their inherent mass scale, and able to focus on better-measured events.

3.4 Comparison with other mass-scale variables: M_{Eff}

In the past it has been suggested that a good starting point for the determination of the mass scale of these new particles is the "effective mass" distribution [6, 7]. There are a number of slightly different definitions of M_{Eff}

[4] When used in this way the parameter χ which represents the mass of the stable invisible particle is typically set to zero. This is of course correct to a very good approximation for the neutrinos which produce the missing momentum in Standard Model events.

and the phrase "mass scale" (a comprehensive list and comparisons between them may be found in [8]) but a typical definition of $M_{\rm Eff}$ would be

$$M_{\rm Eff} \;=\; \not{p}_T + \sum_i p_{T(i)}, \tag{4}$$

in which $\not{p}_T$ is the magnitude of the event's missing transverse momentum and where $p_{T(i)}$ is the magnitude of the transverse momentum of the i-th hardest jet or lepton in the event.

All definitions of $M_{\rm Eff}$ are motivated by the fact that new TeV-scale massive particles are likely to be produced near threshold, and so by attempting to sum up the visible energy in each event, one can hope to obtain an estimate of the energy required to form the two such particles. Broadly speaking, the peak in the $M_{\rm Eff}$ distribution is regarded as the mass-scale estimator.

3.4.1 Problems with $M_{\rm Eff}$

Although the effective mass is a useful variable, and simple to compute, it has a few undesirable properties:

$M_{\rm Eff}$ can be sensitive to the beam energy and the proton's parton distribution functions (pdfs). This is primarily because the desired correlation between $M_{\rm Eff}$ and the mass scale relies on the assumption that the particles are produced near threshold. While it is true that the cross sections will usually *peak* at threshold, they can have significant tails extending to $\sqrt{\hat{s}}$ values considerably beyond the threshold value. As a result, the $M_{\rm Eff}$ distribution is broad, having a width similar to its mean. This smearing means that it is very hard to make *precise* statements about the mass scale from $M_{\rm Eff}$ alone. In contrast, the endpoints of $m_{T\rm Gen}$ are "precision measurements" in the sense that, up to the statistical error, the experimental resolution, and the systematics from mis-assignments between F and G, the endpoint of an $m_{T\rm Gen}$ edge has a direct interpretation as the mass of the underlying generated particles.

The smearing due to pdfs described above means further that in the upper tails of the $M_{\rm Eff}$ distribution one has little chance of distinguishing the different contributions arising from *distinct* pair-production processes. There, the components of the $M_{\rm Eff}$ distribution are expected to blur together. With $m_{T\rm Gen}$ it is possible, though not guaranteed, that one will see multiple edges or changes in gradient in the upper parts of the spectrum allowing the masses of more than one high mass particle to be determined with precision. The extent to which this will work in practice is likely to depend on the extent to which the $m_{T\rm Gen}$ distribution is itself smeared by mis-assignments between F and G. Secondary edges at lower values of $m_{T\rm Gen}$ may, if not obscured by

Standard Model backgrounds, also carry precision information on the masses of lighter particles.

In fairness to $M_{\rm Eff}$, one should point that $m_{T\rm Gen}$ has its own weaknesses. Most obviously, it is a number of orders of magnitude more costly[5] to compute than $M_{\rm Eff}$, for what could turn out to be little gain. Secondly, due to its m_{T2} dependence, $m_{T\rm Gen}$ has an explicit dependence on the hypothesised mass(es) χ of any invisible decay products. This makes it more complicated to handle, as it is should strictly be termed an *event function* (of χ) rather than an *event variable*. It should be said in defence of $m_{T\rm Gen}$, however, that it is good that this dependence on χ is explicit and plays a precise role in offsetting the $m_{T\rm Gen}$ endpoints to locations that have a clear physical interpretation. Though the $M_{\rm Eff}$ variable itself is simpler to compute, lacking χ dependence, the need to consider χ cannot be escaped when using $M_{\rm Eff}$ to draw conclusions about mass scales [8]. Thirdly and finally we note that $m_{T\rm Gen}$, as canonically defined, may be more sensitive to high rapidity ISR than $M_{\rm Eff}$ (see section 3.5). Having said this, the *truly transverse* form, $m_{TT\rm Gen}$, should be much more tolerant of ISR while retaining the theoretical properties of $m_{T\rm Gen}$.[6]

3.5 Example $m_{T\rm Gen}$ distributions

Simulations have been performed for several different supersymmetric particle spectra, including the Snowmass points[9], for proton-proton collisions at LHC centre-of-mass energy of $\sqrt{s}$ =14 TeV. The `HERWIG`[10, 11, 12] Monte Carlo generator was used to produce inclusive unweighted supersymmetric particle pair production events. Final state particles (other than the invisible neutrinos and neutralinos) were then clustered into jets by the longitudinally invariant k_T clustering algorithm for hadron-hadron collisions[13] used in the inclusive mode with $R = 1.0$ [14]. Those resultant jets which had both pseudo-rapidity ($\eta = -\ln\tan\theta/2$) satisfying $|\eta| < 2$ and transverse momentum greater than 10 GeV/c were used to calculate $m_{T\rm Gen}$ and $M_{\rm Eff}$.

In figures 1, 2, and 3 we show the distributions which would be obtained for several different spectra if it were possible to accurately assign all visible momenta to the correct category F or G (i.e. “interesting final state momenta” versus “initial state radiation”). The `HERWIG` initial state radiation

[5]Though see more positive view of this in section 3.7.

[6]Despite having $m_{TT\rm GEN} \le m_{T\rm GEN}$ on an event-by-event basis, the position of the upper kinematic *endpoint* of the $m_{TT\rm Gen}$ distribution should be the same as the endpoint for $m_{T\rm Gen}$ (although with reduced statistics at the endpoint) provided that it is kinematically possible for the decay products of each side to be produced with vanishing relative rapidity.

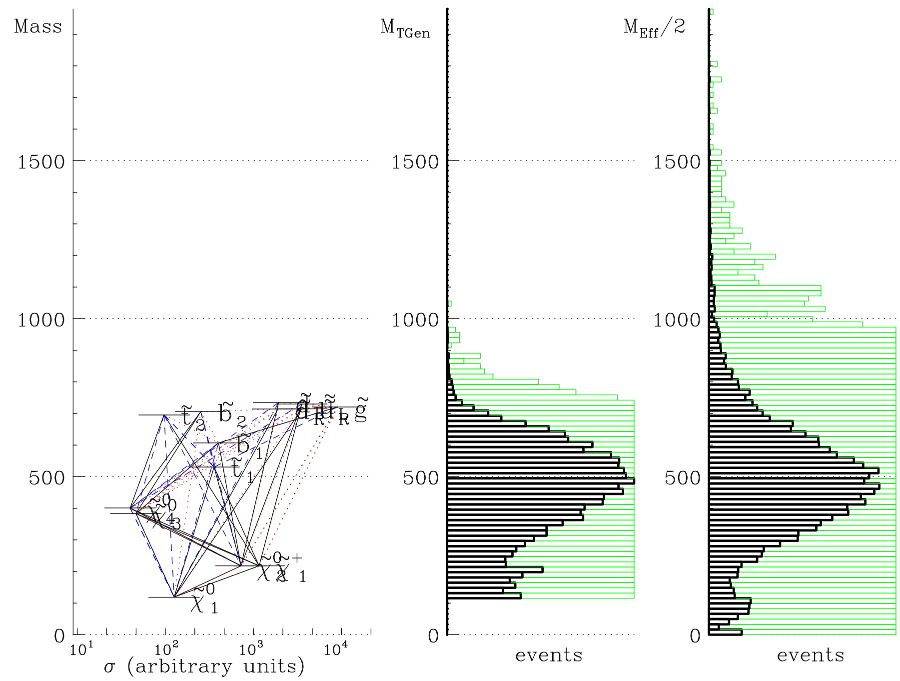

Figure 1: On the left hand side is a graphical representation of the susy mass spectrum of Snowmass point 4. The vertical positions of the particles indicate their masses. The horizontal positions of the centres of the bars indicate the relative LHC production cross-section (arbitrary units). The lines joining particles indicate decays with branching fractions in the following ranges: greater than 10^{-1} solid; $10^{-2} \to 10^{-1}$ dashed; $10^{-3} \to 10^{-2}$ dotted. The middle plot shows the distribution of our variable, $m_{T\text{Gen}}$, with $m_{T\text{Gen}}$ increasing vertically to ease comparison with the spectrum. The right hand plot shows the distribution of another variable, $M_{\text{Eff}}/2$, where M_{Eff} is defined in (4). In both the $m_{T\text{Gen}}$ and the M_{Eff} plots, the lighter shading shows the histograms with the number of events multiplied by a factor of twenty, so that the detail in the upper tail may be seen.

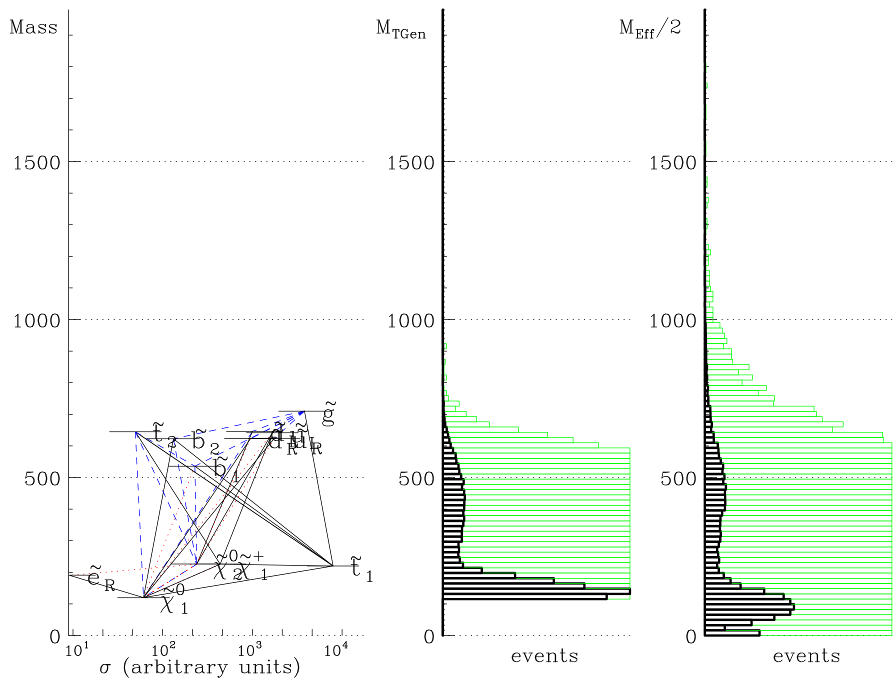

Figure 2: As for figure 1, but for Snowmass point 5.

and underlying event have been switched off, and the parameter χ which is required to calculate $m_{T\mathrm{Gen}}$ has been set to the mass of the lightest supersymmetric particle. The missing transverse momentum has been calculated from the negative vector sum of the momenta of the fiducial jets for reasons of computational efficiency as described in section 3.7.

It can be seen that the upper edge of the distributions gives a very good indication of the mass of the heaviest pair-produced sparticle. Distributions from a variety of different supersymmetric points show similar behaviour. This means that the position of the upper edge of the $m_{T\mathrm{Gen}}$ distribution can be used to find out about the mass scale of any semi-invisibly decaying, heavy, pair-produced particles.

Note that we have deliberately not used any information about the identity of the observed particles, so we do not know from this plot alone whether the particles produced were squarks, gluinos, or indeed something completely different. But we do have a very good indication that there is a particle being pair-produced, and subsequently decaying to a mixture of visible and invisible particles, and we have a good information about the mass scale at which this particle (or these particles) may be found.

Furthermore, in all three cases a change in slope can be observed at lower masses due to significant pair production of lower-mass particles (chargino and/or neutralino pairs for figure 1 and 3 or stop pairs for figure 2). Therefore it is possible in principle to extract from this distribution information at several different mass scales.

The plots also demonstrate some of the the undesirable properties of the variable M_{Eff}. There is, as has already been shown in [8], some correlation between M_{Eff} and the mass scale of particles being produced. However M_{Eff} has a considerable tail at higher values caused by production of sparticles above threshold.

The effects of Standard Model processes (and selection techniques required to reduce them) are beyond the scope of this paper. More detailed studies using a complete set of Standard Model backgrounds and detailed detector simulation will be a important component of future work. However we note that most SM events will have a small value of $m_{T\mathrm{Gen}}$ for the reasons discussed in section 3.3. Therefore, while these backgrounds might be expected to make it more difficult to extract information about lighter particles, we do not expect them to significantly affect the upper edge of the $m_{T\mathrm{Gen}}$ spectrum which contains the information about the heavier (here squark and/or gluino) particle masses.

The extent to which the approximations and assumption used in the calculation of $m_{T\mathrm{Gen}}$ can be justified is explored in figure 4. If particles from initial state radiation and the underlying event are allowed to "pollute" the

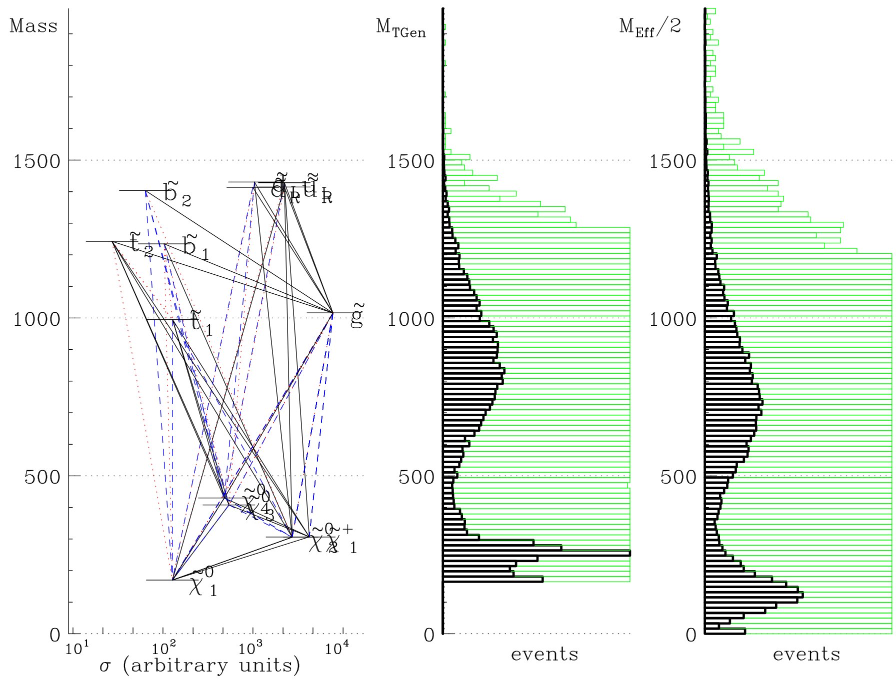

Figure 3: As for figure 1 but for a (non-Snowmass) point with a heavier sparticle spectrum, defined by the mSUGRA parameters: $\{m_0 = 1200$ GeV, $m_{\frac{1}{2}} = 420$ GeV, $\tan\beta = 10$, $m_t = 174$ GeV, $\mu < 0\}$ and with a spectrum generated using `Isajet`[15] version 7.58.

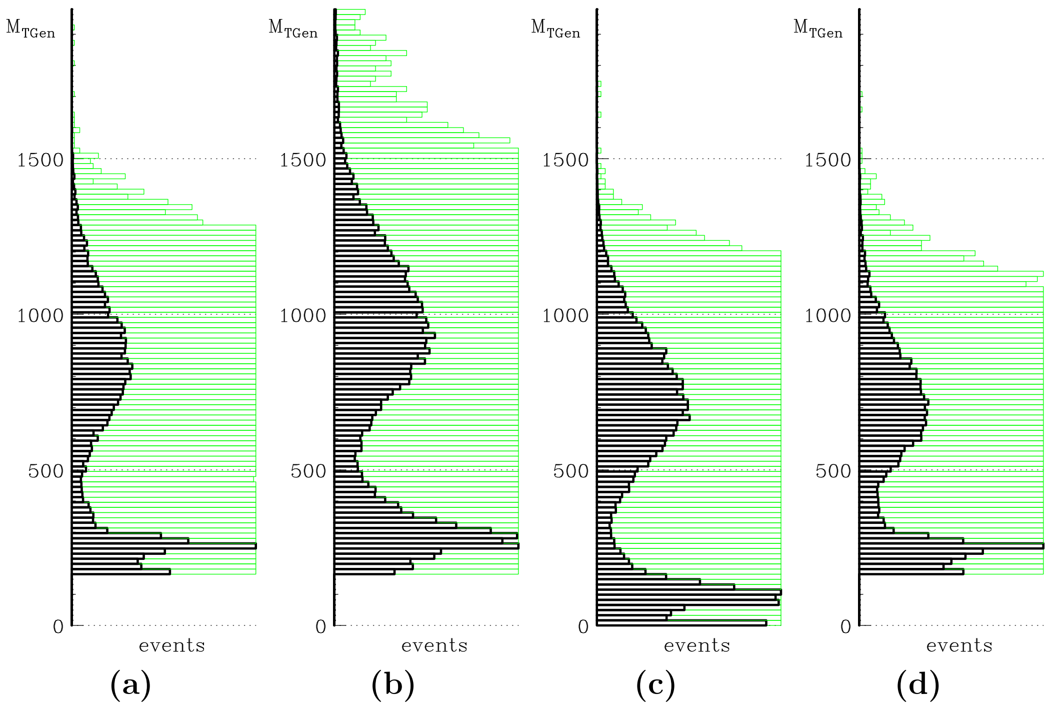

Figure 4: $m_{T\text{Gen}}$ distributions for the same spectrum as for figure 3, but with different assumptions. **(a)** The idealised case, as described in text. **(b)** As for (a) but now with initial state radiation and the underlying event, and including particles from both categories F and G ("interesting" and "ISR/underlying event") to form jets. **(c)** As for (a) but with the parameter, χ, (corresponding to the mass of the invisible particle) set to zero. **(d)** As for (a) but using the sum of the transverse momenta of the invisible particles for the missing transverse momentum.

final state (figure 4b) there is some smearing of the edges.[7] The effect of adding the ISR and underlying event is to shift the apparent position of the $m_{T\mathrm{Gen}}$ edge up by approximately 15-25% for our point (figure 4b). This value represents an upper limit to the uncertainty originating from these effects, which would be obtained in the unlikely case where the corrections from ISR and the underlying event were completely unknown. The extent to which this effect would be seen in experimental distributions will depend on the details of the event selection (for example on the rapidity cut on the jets). We note also that, as anticipated, if the same plots are generated for the "Truly Transverse" variant of the variable, $m_{TT\mathrm{Gen}}$, then the sensitivity of the endpoint to ISR is reduced (to around 10-15%) and there are proportionally fewer events at the endpoint. It remains to be seen whether the better strategy for the future will be to invest time in improving ISR rejection while focusing on $m_{T\mathrm{Gen}}$, or to use variants like $m_{TT\mathrm{Gen}}$ with less sensitivity to ISR but fewer statistics near the endpoint.

If the invisible particle mass is unknown, or if $m_{T\mathrm{Gen}}$ is being used as a selection variable, then the distribution with the mass parameter $\chi = 0$ is most appropriate (figure 4c). In that case the lower limit of the distribution (which cannot drop below χ) is pulled down toward the origin. The change in the position of the upper edge is much smaller, as one would expect in the case where the total energy in the final state is dominated by visible particles.

An efficient method of calculating $m_{T\mathrm{Gen}}$ which approximates the missing transverse momentum by the negative vector sum of the jet transverse momenta (see section 3.7) has been used for all these previous plots. This can be justified by its good agreement with the corresponding distribution obtained with the full numerical calculation of $m_{T\mathrm{Gen}}$ using the "true" missing momentum (figure 4d).

3.6 Cross section constraints

$m_{T\mathrm{Gen}}$ relies purely on the kinematics of four-momentum conservation in each event. It makes no use of cross section information, which will therefore always remain a vital tool, orthogonal to $m_{T\mathrm{Gen}}$, with which to constrain the overall mass scale.

[7] In the previous "unpolluted" plots, the assignment of final state particles to class F or G was determined from the kinematic history in the Monte Carlo event record.

3.7 Evaluating $m_{T\text{Gen}}$

Historically the main hurdle to the adoption of $m_{T\text{Gen}}$ has been the cost of evaluating it.[8] For each evaluation, m_{T2} typically requires a numerical minimisation to be performed which can take from a few thousandths to a few tenths of a second on most computers. The definition of $m_{T\text{Gen}}$ requires this minimisation to be repeated up-to $2^{(n_F)}$ times: once for each partition of F. Since typical jet definitions can lead to high jet multiplicities in supersymmetric events, up to about 20 jets in some parts of parameter space, a naive implementation can take many seconds, or even hours to calculate $m_{T\text{Gen}}$ for a single event.

For $m_{T\text{Gen}}$ to become usable, it is important to find less time-consuming ways in which the internal m_{T2} values can be calculated. Ideally, an analytic or closed form expression for m_{T2} is required to avoid time-consuming numerical minimisations.

The authors were fortunate to be contacted in November 2006 by Kyoungchul Kong and Konstantin Matchev (KKKM) [16]. KKKM informed the authors that they had derived an (undisclosed) analytic expression for m_{T2} valid for the special case in which the missing transverse momentum of the event was entirely balanced by the transverse momentum of the two "key visible particles" input to m_{T2}. In other words, this special case corresponds to having no net initial state radiation, or in the language of this paper $\left|\left(\sum_{i=1}^{(n_G)} g_i^\mu\right)_T\right| = 0$. It was not until June 2007 that the authors realised that this happens to be an interesting limit from the point of view of $m_{T\text{Gen}}$. If all reconstructed momenta are thrown into F, then G is empty by construction, and the special case is satisfied. The more complicated the event is, the more grounds one has for doing this, as the less sure one can be as to the provenance of any individual particle.

It is important to confirm the existence of the analytic expression claimed by KKKM and to perform timing tests using it in order to support the claim that $m_{T\text{Gen}}$ is now calculable in a reasonable time. The authors were not able to obtain the full expression for m_{T2} from KKKM.[9] It has therefore been

[8] The authors first proposed $m_{T\text{Gen}}$ for the analysis of the ATLAS Blind Data Challenge which concluded with the prize-giving at the ATLAS overview week in Prague, 2003, reported in New Scientist. See "Observations concerning the first ATLAS Blind Data Challenge" Barr, A J; Brochu, F M; Lester, C G; Palmer, M; Sabetfakhri, A; Aug 2003

[9] Though not releasing the full expression for m_{T2}, KKKM did release the answer for the special-special case where, in addition to $\left|\left(\sum_{i=1}^{(n_G)} g_i^\mu\right)_T\right| = 0$ one also has the masses of the two visible particles and the masses of the two hypothesised invisible particles all equal to zero. Though we did not make use of this information when deriving our own expression (it cannot be used for $m_{T\text{Gen}}$ as most partition create event "sides" which have

necessary to re-derive (what is hopefully) the same result independently in the Appendix. The authors understand that KKKM will release their own result in the near future [17].

Using the analytic form of m_{T2} derived in the Appendix, we find that even with a naive "try every partition of F" algorithm we can calculate $m_{T\text{Gen}}$ for a 20-particle event in order one-second on a typical personal computer.[10] The computation time scales as 2^N where N is the number of particles, so a 10-particle event can be processed in one thousandth of that time. The authors find this is more than fast enough to make $m_{T\text{Gen}}$ as usable as other standard event variables.[11]

4 Conclusion

In conclusion, we believe that $m_{T\text{Gen}}$ could be an invaluable variable for physicists working at the LHC, and other future colliders. We hope that if its usefulness is validated in subsequent dedicated detector studies, it will become a standard tool in the Swiss army knife for new physics searches.

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very large non-zero masses) we are grateful to have been able to use it as a check.

[10] This implementation is available from the authors on request.

[11] One would expect that, with more thought, it would be possible to create faster algorithms. Instead of trying all possible partitions when minimising over m_{T2} values, it might be possible to design an algorithm which flips momenta one-at-at-time from one side of the event to the other – hunting for the minimum. More work would be necessary to determine whether such algorithms could become stuck in local minima.

A Appendix

Here we derive an expression for m_{T2} for the special case in which the missing transverse momentum is *entirely* balanced by the two visible particles' transverse momentum – *i.e.* there must be no ISR. The particles themselves (and the hypothesised missing particles) are allowed to have arbitrary masses.

Within this appendix we adopt the same conventions and definitions of [5]. In the language of that paper, the assumptions of our "special case" can be phrased as "$\Sigma = \sigma$". We will begin, however, in the general case ($\Sigma \neq \sigma$) and only introduce the simplification of the special case when we can no longer make progress without it. Beginning with the general case, then, we have the following notation:

α^μ : Lorentz 1+2 momentum of key visible particle 1 (5)

β^μ : Lorentz 1+2 momentum of key visible particle 2 (6)

g^μ : Lorentz 1+2 momentum of junk or ISR (7)

p^μ : Lorentz 1+2 momentum of invisible particle (mass χ) produced with particle 1 (8)

q^μ : Lorentz 1+2 momentum of invisible particle (mass χ) produced with particle 2 (9)

Λ^μ : Lorentz 1+2 momentum of unit a mass particle which is stationary in the lab frame (10)

$\sqrt{s}$: real parameter (the reduced centre-of-mass energy from eq. 18 of [5]) (11)

which are related by

$$\alpha^\mu + \beta^\mu + g^\mu + p^\mu + q^\mu = \sqrt{s}\Lambda^\mu. \tag{12}$$

Note that there is a potential ambiguity here between *real* momenta, *measured* momenta, and *hypothesised* momenta. In this document, the quantities which are directly visible (α^μ, β^μ and g^μ) are taken to be be real momenta, or equivalently to be measured quantities with zero measurement error. Conversely p^μ, q^μ and $\sqrt{s}$ are quantities which cannot be measured. In this case these symbols refer to the *hypothesised* neutralino momenta and/or *hypothesised* centre of mass energies that are used throughout the process of describing the event while attempting to calculate m_{T2}.

For simplicity, some derived quantities are also defined:

$\sigma^\mu = \alpha^\mu + \beta^\mu$: Lorentz 1+2 momentum sum of the two key visible particles (13)

$\Delta^\mu = \alpha^\mu - \beta^\mu$: Lorentz 1+2 momentum difference of the two key visible particles (14)

$\Sigma^\mu = \sigma^\mu + g^\mu$: Lorentz 1+2 momentum sum of everything seen in the detector (15)

$B^\mu = p^\mu + q^\mu$: Lorentz 1+2 momentum sum of the two invisible particles. (16)

We already know that the particular momenta of p and q which need to be hypothesised to generate the value of m_{T2} fall into one of two categories.

Either they are in a "balanced" configuration in which $(\alpha+p)^2 = (\beta+q)^2$ or the value of m_{T2} is achieved for an "unbalanced" configuration in which this is not true. It is easy to determine whether a given set of momenta $\{\alpha^\mu, \beta^\mu, g^\mu\}$ generate m_{T2} from a balanced or an unbalanced configuration, and also easy to determine what m_{T2} is for the unbalanced cases. We concentrate first, therefore, on the harder case of how to calculate the value of m_{T2} if it has already been determined that it occurs in a "balanced" configuration.

A.1 Balanced configurations

In the "balanced configuration", the value of m_{T2} will be the minimum value of $(\alpha+p)^2$ over all allowed values of $\sqrt{s}$ provided that the following constraints are satisfied:

$$p^2 = \chi^2 \tag{17}$$
$$q^2 = \chi^2 \tag{18}$$
$$(\alpha+p)^2 = (\beta+q)^2. \tag{19}$$

The first two constraints just put p^μ and q^μ on mass shell. The final constraint is the one that makes the configuration mass balanced. When the constraint of equation (19) is satisfied we will refer to the both $(\alpha+p)^2$ and $(\beta+q)^2$ as M^2. The approach we will take will be to assume a fixed value of $\sqrt{s}$ and then solve the above three equations for p^μ and q^μ. We then explicitly minimise the resulting value of M by varying $\sqrt{s}$. The resulting minimum value of M is the value of m_{T2} we seek.

From equation (12) we can see that for fixed $\sqrt{s}$, the value of B^μ is fully determined:

$$B^\mu = \sqrt{s}\Lambda^\mu - \Sigma^\mu \tag{20}$$

and so the sum $p^\mu + q^\mu$ is fixed. We therefore choose to parametrise the three degrees of freedom which p^μ and q^μ have collectively by writing them in terms of an unknown Lorentz 1+2 vector γ^μ as follows:

$$p^\mu = \frac{1}{2}B^\mu + \gamma^\mu \tag{21}$$
$$q^\mu = \frac{1}{2}B^\mu - \gamma^\mu. \tag{22}$$

Our stated intention of determining p^μ and q^μ for fixed $\sqrt{s}$ is therefore really a requirement to determine the three components of γ^μ, from the three constraints in equations (17), (18) and (19). By substituting the two equations above into equations (17), (18) and (19) it is easy to show that the constraints on γ^μ are equivalent to the following:

$$\begin{aligned} \gamma.B &= 0 & (23)\\ \gamma.\sigma &= -\frac{1}{2}\Delta.(B+\sigma) & (24)\\ \gamma^2 &= -\frac{1}{4}(B^2-4\chi^2). & (25) \end{aligned}$$

The form of the above constraints motivates solving for γ^μ as a linear combination of the three linearly independent vectors B^μ, σ^μ and $w^\mu = \epsilon^{\mu\nu\tau}\sigma_\nu B_\tau$. Doing this, one finds two possible solutions:

$$\gamma^\mu = H^\mu \pm \hat{w}^\mu \sqrt{H^2 + \frac{1}{4}(B^2 - 4\chi^2)} \tag{26}$$

where

$$\hat{w}^\mu = \frac{w^\mu}{\sqrt{-w^2}} \tag{27}$$

and

$$H^\mu = \frac{-\frac{1}{2}\Delta.(B+\sigma)}{w^2}\left[(B^2)\sigma^\mu - (\sigma.B)B^\mu\right]. \tag{28}$$

Each solution corresponds to a kinematic configuration which is a valid realisation of original mass constraints, but the value of M will almost certainly be different in each case. Since our intention is to find $m_{\rm T2}$, we will eventually want to retain only the solution which gives the smaller value of M. As we do not yet know which solution that is, we retain both for the moment.

We have now accomplished what we set out to achieve in step one. For fixed $\sqrt{s}$ we have defined B^μ (with equation (20)). This value may be substituted into equations (26), (27) and (28) in order to find γ^μ. In terms of γ^μ we can then find the values of p^μ and q^μ (via equations (21) and (22)) which lead to the so called "balanced" kinematic structure in which both sides of the event have equal invariant mass M. All that now remains to do, is to *minimise* the value of M so-obtained over all allowed values of $\sqrt{s}$.

It is at this stage that we now move the "No ISR" special case that may be summarised as $\Sigma^\mu = \sigma^\mu$ or equivalently as $g^\mu = 0$. This change only affects terms with B^μ's in them as these are the only quantities containing Σ^μ. Having made the substitution $\Sigma^\mu \to \sigma^\mu$ there is a substantial amount of cancellation within the expressions in terms of which γ^μ is defined (equation (26)). The net effect of this cancellation leaves M's dependence on $\sqrt{s}$ in the relatively simple form:

$$M^2 = E + A\sqrt{s} \pm \lambda\sqrt{(\sqrt{s} - D)^2 - C^2} \tag{29}$$

for suitable values of the real quantities A, C, D, E and λ which do not depend on $\sqrt{s}$. It is straightforward to show that the minimum of this function occurs when $\sqrt{s}$ takes the value:[12]

$$\sqrt{s} = D + \frac{C}{\sqrt{1 - \frac{\lambda^2}{A^2}}} \tag{30}$$

All that is needed to complete the evaluation of m_{T2} in this special case, then, is to determine the quantities A, C, D, and λ. (E is not needed to calculate the value of $\sqrt{s}$ which minimises M^2.[13]) We can then evaluate $\sqrt{s}$ in terms of these quantities, allowing in turn B^μ, w^μ, H^μ, γ^μ, p^μ and finally m_{T2} to be calculated. It may be shown that the values needed are as follows:

$$A = \frac{1}{2}(\Lambda.\sigma) + \frac{1}{2}\frac{(\Lambda.\Delta)}{|\underline{\sigma}|^2}\left[(\sigma.\Delta) - (\Lambda.\sigma)(\Lambda.\Delta)\right], \tag{31}$$

$$C = \sqrt{|\underline{\sigma}|^2 + \frac{\chi^2}{J}}, \tag{32}$$

$$D = \Lambda.\sigma \tag{33}$$

and (34)

$$\lambda = \frac{\epsilon^{\mu\nu\tau}\Delta_\mu\sigma_\nu\Lambda_\tau\sqrt{J}}{|\underline{\sigma}|} \tag{35}$$

where (36)

$$J = \frac{|\underline{\sigma}|^2 - (\Lambda.\Delta)^2}{4|\underline{\sigma}|^2} \tag{37}$$

and the quantity $|\underline{\sigma}|^2$ is always evaluated in the lab frame.[14] Note that the condition expressed in equation (19) means that we can use a number of

[12] There is also a stationary point at $\sqrt{s} = D - C/\sqrt{1 - \frac{\lambda^2}{A^2}}$ but it can be shown that this is always unphysical.

[13] Added in 2009: For completeness we note that $E = \chi^2 + \frac{1}{2}(m_\alpha^2 + m_\beta^2) - \frac{1}{2}\sigma^2 + \frac{1}{2}\frac{(\Lambda.\Delta)}{|\underline{\sigma}|^2}\left[\sigma^2(\Lambda.\Delta) - (\Lambda.\sigma)(\sigma.\Delta)\right]$. We note further that once the value of $\sqrt{s}$ obtained in (30) is substituted into the expression for M in (29) one finds that $M^2_{\mathrm{min}} = E + AD + C\sqrt{A^2 - \lambda^2}$.

[14] In other words $|\underline{\sigma}|^2 = (\sigma.\Lambda)^2 - \sigma^2$.

different expressions to finally evaluate m_{T2}. The simplest would be $m_{\mathrm{T2}}^2 = (\alpha + p)^2$ or $m_{\mathrm{T2}}^2 = (\beta + q)^2$. However it is arguably nicer to preserve the explicit symmetry between the two sides of the event by instead evaluating m_{T2}^2 as the average of these two identical quantities. If this is done, one ends up with

$$m_{\mathrm{T2}}^2 = \frac{1}{2}(\alpha + p)^2 + \frac{1}{2}(\beta + q)^2 \tag{38}$$

$$= \chi^2 + \frac{1}{2}(m_\alpha^2 + m_\beta^2) + \frac{1}{2}(\sigma.B) + (\Delta.\gamma). \tag{39}$$

Note that it was the last of these forms which was used to generate the statement of equation (29).

A.2 Unbalanced solutions

As discussed in [5], the value of m_{T2} does not always arise from a configuration of hypothesised momenta in which both sides of the event have the same invariant mass. These unbalanced solutions arise if the momentum splitting which places one of the hypothesised neutralinos at the same transverse velocity, $\mathbf{v}_t = \mathbf{p}_T/E_T$, as its visible "partner" (thereby minimising the invariant mass of that side of the event) causes the invariant mass of the other side of the event (which is then fixed by momentum conservation) to be even lower. This statement is generally true, and does not require the move to the $\Sigma = \sigma$ special case considered in the section dealing with "balanced" solutions. Nevertheless, in order to write the full expression for the $\Sigma = \sigma$ case we need to take these possibilities into account.

A.3 Putting all cases together

We can now combine the two previous results into the following complete expression for m_{T2} valid for events in which the missing transverse momentum exactly balances the transverse momentum of the two important visible particles (*i.e.* valid for the case $\Sigma = \sigma$ also known as "no ISR").

$$m_{\mathrm{T2}}^2 = \begin{cases} (m_\alpha + \chi)^2 & \text{iff} \quad (m_\alpha + \chi)^2 \geq (\beta + \tilde{q})^2, \\ (m_\beta + \chi)^2 & \text{iff} \quad (m_\beta + \chi)^2 \geq (\alpha + \tilde{p})^2, \\ (\alpha + p)^2 & \text{or equivalently} \\ (\beta + q)^2 & \text{or equivalently} \\ \chi^2 + \frac{1}{2}(m_\alpha^2 + m_\beta^2) + \frac{1}{2}(\sigma.B) + (\Delta.\gamma) & \text{otherwise} \end{cases} \tag{40}$$

where $\tilde{q}^\mu = (\sqrt{\chi^2 + |\underline{\tilde{\mathbf{q}}}|^2}, \underline{\tilde{\mathbf{q}}})$ with $\underline{\tilde{\mathbf{q}}} = -\underline{\mathbf{\Sigma}} - \frac{\chi}{m_\alpha}\underline{\alpha}$ and $\tilde{p}^\mu = (\sqrt{\chi^2 + |\underline{\tilde{\mathbf{p}}}|^2}, \underline{\tilde{\mathbf{p}}})$ with $\underline{\tilde{\mathbf{p}}} = -\underline{\mathbf{\Sigma}} - \frac{\chi}{m_\beta}\underline{\beta}$ and in which p^μ and q^μ (which must not be confused with the entirely different quantities $\tilde{p}^\mu$ and $\tilde{q}^\mu$ just mentioned!) are defined by

$$p^\mu = \frac{1}{2}B^\mu + \gamma^\mu, \tag{41}$$

$$q^\mu = \frac{1}{2}B^\mu - \gamma^\mu, \tag{42}$$

$$\gamma^\mu = H^\mu \pm \hat{w}^\mu \sqrt{H^2 + \frac{1}{4}(B^2 - 4\chi^2)} \tag{43}$$

(choosing the sign which leads to the smaller value of m_{T2}) in which

$$\hat{w}^\mu = \frac{w^\mu}{\sqrt{-w^2}} \tag{44}$$

with

$$w^\mu = \epsilon^{\mu\nu\tau}\sigma_\nu B_\tau \tag{45}$$

and

$$H^\mu = \frac{-\frac{1}{2}\Delta.(B+\sigma)}{w^2}\big[(B^2)\sigma^\mu - (\sigma.B)B^\mu\big]. \tag{46}$$

and where we have taken in the "no ISR no junk" case

$$B^\mu = \sqrt{s}\Lambda^\mu - \sigma^\mu \tag{47}$$

having set

$$\sqrt{s} = D + \frac{C}{\sqrt{1 - \frac{\lambda^2}{A^2}}} \tag{48}$$

where

$$A = \frac{1}{2}(\Lambda.\sigma) + \frac{1}{2}\frac{(\Lambda.\Delta)}{|\underline{\sigma}|^2}\left[(\sigma.\Delta) - (\Lambda.\sigma)(\Lambda.\Delta)\right], \tag{49}$$

$$C = \sqrt{|\underline{\sigma}|^2 + \frac{\chi^2}{J}}, \tag{50}$$

$$D = \Lambda.\sigma \qquad \text{and} \tag{51}$$

$$\lambda = \frac{\epsilon^{\mu\nu\tau}\Delta_\mu\sigma_\nu\Lambda_\tau\sqrt{J}}{|\underline{\sigma}|} \tag{52}$$

in which

$$J = \frac{|\underline{\sigma}|^2 - (\Lambda.\Delta)^2}{4|\underline{\sigma}|^2} \qquad (53)$$

and where the quantity $|\underline{\sigma}| = \sqrt{(\sigma.\Lambda)^2 - \sigma^2}$ is the magnitude of the visible transverse momentum evaluated in the lab frame.

A.4 Addendum in light of arXiv:0711.4526 [18]

(This section was added in 2009. The purpose of the addition is to draw readers' attention to a nice result of relevance to this paper, published in a later paper by different authors [18]. It is hoped that by placing the forward reference to [18] here readers may be more likely to use of the simplification that the result affords.)

It was observed in [18] that, after some simplification, equation (40) may be written in the following equivalent but shorter form (still valid only for the case $\Sigma = \sigma$ also known as "no ISR") :

$$m_{\rm T2}^2 = \begin{cases} (m_\alpha + \chi)^2 & \text{iff} \quad (m_\alpha + \chi)^2 \geq (\beta + \tilde{q})^2, \\ (m_\beta + \chi)^2 & \text{iff} \quad (m_\beta + \chi)^2 \geq (\alpha + \tilde{p})^2, \\ \chi^2 + A_T + \sqrt{(1 + \frac{4\chi^2}{2A_T - m_\alpha^2 - m_\beta^2})(A_T^2 - m_\alpha^2 m_\beta^2)} & \text{otherwise} \end{cases} \qquad (54)$$

where the new quantity A_T is defined by $A_T = \sqrt{m_\alpha^2 + \underline{\alpha}^2}\sqrt{m_\beta^2 + \underline{\beta}^2} + \underline{\alpha}.\underline{\beta}$ and where all other quantities are defined as previously for (40). It will be noted that A_T is very closely related to the contransverse mass of [19]. We note that it is proved in [18] that the whole of (54) is invariant under simultaneous equal magnitude but anti-parallel boost of α and β in the transverse Lorentz 1+2 plane.

References

[1] **ATLAS** Collaboration, *ATLAS Detector and Physics Performance Technical Design Report 1.* No. CERN-LHCC-99-014 ATLAS-TDR-14. May, 1999.

[2] **ATLAS** Collaboration, *ATLAS Detector and Physics Performance Technical Design Report 2.* No. CERN-LHCC-99-015 ATLAS-TDR-15. May, 1999.

[3] **CMS** Collaboration, *CMS physics technical design report, volume II*, 2006. CERN-LHCC-2006-021, CMS-TDR-008-2.

[4] C. G. Lester and D. J. Summers, *Measuring masses of semi-invisibly decaying particle pairs produced at hadron colliders.*, *Phys. Lett.* **B463** (1999) [hep-ph/9906349].

[5] A. Barr, C. Lester, and P. Stephens, *A variable for measuring masses at hadron colliders when missing energy is expected; m_{T2}: the truth behind the glamour*, *J. Phys.* **G29** (2003) 2343–2363, [hep-ph/0304226].

[6] I. Hinchliffe, F. E. Paige, M. D. Shapiro, J. Soderqvist, and W. Yao, *Precision SUSY measurements at LHC*, *Phys. Rev.* **D55** (1997) 5520–5540, [hep-ph/9610544].

[7] **ATLAS** Collaboration, *Inclusive SUGRA measurements.* In [2], May, 1999. Sections 20.2.1.

[8] D. R. Tovey, *Measuring the SUSY mass scale at the LHC*, *Phys. Lett.* **B498** (2001) 1–10, [hep-ph/0006276].

[9] B. C. Allanach *et. al.*, *The Snowmass points and slopes: Benchmarks for SUSY searches*, hep-ph/0202233.

[10] G. Corcella *et. al.*, *HERWIG 6.5 release note*, hep-ph/0210213.

[11] S. Moretti, K. Odagiri, P. Richardson, M. H. Seymour, and B. R. Webber, *Implementation of supersymmetric processes in the HERWIG event generator*, *JHEP* **04** (2002) 028, [hep-ph/0204123].

[12] G. Marchesini *et. al.*, *HERWIG: A Monte Carlo event generator for simulating hadron emission reactions with interfering gluons. Version 5.1 - April 1991*, *Comput. Phys. Commun.* **67** (1992) 465–508.

[13] S. Catani, Y. L. Dokshitzer, M. H. Seymour, and B. R. Webber, *Longitudinally invariant K(t) clustering algorithms for hadron hadron collisions*, *Nucl. Phys.* **B406** (1993) 187–224.

[14] S. D. Ellis and D. E. Soper, *Successive combination jet algorithm for hadron collisions*, *Phys. Rev.* **D48** (1993) 3160–3166, [hep-ph/9305266].

[15] H. Baer, F. E. Paige, S. D. Protopopescu, and X. Tata, *ISAJET 7.48: A Monte Carlo event generator for p p, anti-p p, and e+ e- reactions*, hep-ph/0001086.

[16] K. Kong and K. Matchev, *Private Communication*, .

[17] K. Kong and K. Matchev, *To Appear*, .

[18] W. S. Cho, K. Choi, Y. G. Kim, and C. B. Park, *Measuring superparticle masses at hadron collider using the transverse mass kink*, *JHEP* **02** (2008) 035, [0711.4526].

[19] D. R. Tovey, *On measuring the masses of pair-produced semi-invisibly decaying particles at hadron colliders*, *JHEP* **04** (2008) 034, [0802.2879].