Feferman on Foundations. Logic, Mathematics, Philosophy. Edited by Gerhard Jäger and Wilfried Sieg. Outstanding Contributions to Logic, vol. 13. Springer, 2017.

Solomon Feferman (1928–2016) was one of the leading mathematical logicians in the second half of the 20th century. He belongs to a second generation of logicians who shaped logic as we know it today, following the generation of Alfred Tarski (his PhD advisor) and Kurt Gödel. The present volume, edited by Gerhard Jäger and Wilfried Sieg, contains a collection of articles by friends and colleagues of Feferman. It is devoted to his work which covers mathematical logic (from model theory to proof theory) as well as philosophy of mathematics. Published in the series *Outstanding Contributions to Logic*, it was the idea that Feferman could comment on the papers of the volume; an idea which didn't materialize as Feferman sadly passed away on July 25, 2016, just when the book was about to be finished. The papers give a good exposition of the areas to which Feferman contributed, as well as insights into his contributions.

As this review is directed toward a more philosophically minded audience, we will restrict ourselves to a more detailed analysis of those papers that have a philosophical purpose. However, a philosophical reader will also profit from the study of the more mathematical contributions, as they are not only written from an implicit philosophical standpoint, but they very often give technical arguments to sustain such a standpoint.

The book starts with an autobiography of Solomon Feferman; the contributions are separated into four parts: Part I. Mathematical Logic; Part II. Conceptual Expansions; Part III. Axiomatic Foundations; Part IV. From Logic to Philosophy.

From a certain perspective, the opening chapter with Feferman's autobiography can even be considered as the philosophically most interesting part of the volume. It does not only give a vivid picture of the intellectual life in the logic community in the 20th century, but it also demonstrates—quite detailed—how mathematical logic moved gradually from the bold philosophical questions behind Hilbert's Programme to more and more specialized technical investigations, which led to the separation of the different subareas of mathematical logic as we know them today. Feferman describes in detail how one or the other question, be it philosophical or mathematical, led him to take up different technical challenges which deepened his—and our—understanding of logic. The autobiography, which was not entirely finished by Feferman, is supplemented by a short CV including the list of Feferman's PhD students and an overview of his *active projects of 2016*, complied by the Wilfried Sieg und Rick Summer; it is followed by a complete (and impressive) list of his publications.

The first contribution of the volume is by Wilfrid Hodges (*From choosing elements to choosing concepts: The evolution of Feferman's work in model theory*, pp. 3–22). It elaborates on the emergence of model theory and Feferman's contribution to it, which started with the famous *Feferman-Vaught theorem.*

The bulk of the papers in Part II and III is devoted to the technical work of Feferman. Here, we like to highlight the contribution of Andrea Cantini, Kentaro Fujimoto, and Volker Halbach (*Feferman and the Truth*, pp. 287–314). Although it is mainly concerned with technical results about formalized truth theories, it contains an illuminating reflection on the history of formalized truth in modern logic, with special emphasis on the impact of Feferman's work.

Philosophy proper is addressed in Part IV. Solomon Feferman was an engaged, but not dogmatic advocate of *Predicativity* in Mathematics. In this framework, going back to Poincaré and Weyl, one strictly avoids impredicative concept formations. As a consequence, the methods of classical mathematics are severely restricted. It is, however, possible to save most—if not all—of "every day Mathematics", albeit for the price of sometimes much more complicated constructions. Laura Crosilla in her contribution *Predicativity and Feferman*, pp. 423–447, gives a historico-philosophical review of predicativity. Concerning Feferman, she reinforces the shift of emphasis of modern logic, writing (with reference to Gandy): "This once more clarifies the deep change in attitude between the early discussions on predicativity and its logical analysis, as the latter is an attempt at understanding predicativity rather than arguing for it as a foundational stance." In other words: Predicativism is not the base, but rather the (better: an) object of logical investigation in Mathematics. And the aim is not to restrict Mathematics, but to better understand the methods it is using.

The paper of Dag Westerståhl: *Sameness*, pp. 449–467, is a philosophical gem. In a discussion of the notion of *logicality*, Feferman had remarked: "No natural explanation is given by [the Tarski–Sher thesis on logicality] of what constitutes the *same* logical operation over arbitrary basic domains." Westerståhl accepts the challenge and provides an exemplary discussion on how formal tools should be deployed to capture an informal notion, in this case the one of sameness. Even if the given proposal might not convince everybody, the heuristic argumentation and the logical reflection is a methodological paragon.

Next, the Ernest Nagel Lecture *Gödel, Nagel, Minds, and Machines* which Feferman gave at Columbia University in 2007 is reprinted from the *Journal of Philosophy* CVI, 4, 2009. It starts with the history of the conflict between Gödel and Nagel over a possible reprint of Gödel's 1934 Princeton Lectures on his incompleteness results; the lectures were to be incorporated in the popular presentation of his results in the book of Nagel and Newman. But the main issue is a discussion of the "Minds Versus Machines Debate" where Feferman adds some new arguments, in particular that "there is an *equivocation* involved that lies in identifying *how* the mathematical mind works with the totality of *what* it can prove." Feferman suggests, "in order to straddle the mechanist/anti-mechanist divide at the level considered here, one will have to identify *finitely many basic forms of mathematical reasoning* which work in tandem to fully constrain and distinguish it. These would constitute the mechanist side of the picture, while the openness as to what counts as a mathematical concept would constitute the anti-mechanist side." He acknowledged that this is an ambitious program, for which he only had suggested some first steps; but we agree that it is "worthy of serious consideration."

The paper is complemented by a *A Brief Note on* Gödel, Nagel, Minds, and Machines by Wilfried Sieg. He recalls that "characterizing the 'logical structure of mathematics–what constitutes a proof' is a crucial task [for *proof theory*] that has not been fully addressed." In contrast to Feferman, Sieg argues that proof theory has with *(definitional) extensions* of PA and ZF the appropriate tools at hand.

Feferman has expressed skepticism about higher set theory, especially in the discussion concerning *new axioms*. The two last papers of the volume by Peter Koellner (*Feferman on Set Theory: Infinity up on Trial*, pp. 491–523) and by Charles Parsons (*Feferman's Skepticism About Set Theory*, pp. 525–543) are discussing this skepticism, in quite different manners.

The contribution of Koellner is probably the most controversial paper in the volume. It identifies five main arguments for "Feferman's reasons for maintaining that statements like the continuum hypothesis (CH) are not definite." These arguments are presented, one by one, first with arguments (Koellner finds) for Feferman's case; and then responded with detailed counter-arguments. As the author writes, "[t]he paper is really the continuation of a conversation that we [Feferman and Koellner] have been having for many years". It is apparent that it was written under the assumption that Feferman would have the opportunity to reply to it. In a personal note, expressing his feelings after he received the shocking news that Feferman had died, Koellner admits the he had his differences with Feferman and strongly disagreed with him, "but we are playful about it". And he cites Feferman as saying: "Peter, you are my favorite person to argue with." For the present paper, which remains a somewhat lonely voice without Feferman's reply, this is, of course, a problem. As much as the response to Feferman's arguments are elaborated, they are waiting for another reply.

We identified the following argument as the base of Koellner's rebuttal of Feferman's skepticism: "Feferman applies conceptual structuralism to number theory and, in conjunction with doing this, he embraces definiteness and realism in truth values. Why then does he not make the same move with set theory?" And, "the entire case *rests* on the claim that the concept of subsets of natural numbers is not completely clear." Koellner argues that Feferman is using the concept of *completely clear* somewhat incoherently in various cases concerning both, number theory and set theory, with the conclusion that "the concept of *being sufficiently clear to secure definiteness* is not sufficiently clear to secure definiteness." But this is only a negative result; it may serve to refute Feferman's realism concerning number theory; but it does not help to ensure the desired definiteness in set theory. In § 6, the formal result that CH is indefinite relative to the semi-constructive system SCS^+—conjectured by Feferman, and proven by Rathjen—is discussed. SCS^+ can be considered as a formal theory capturing Feferman's assumption that the concept of subset of natural numbers is indefinite. When Koellner questions this assumption, of course, there is no point in the formal theorem. The problem with his arguments was seen by Koellner himself: "In this paper my aim has been *negative* in that I have concentrated on rebutting Feferman's arguments to the effect that the concept of subsets of natural number—along with the richer concepts of set theory—are indefinite. But I have not advanced any *positive* arguments to the effect that this concept (or these richer concepts) *are* definite." Thus, before such positive arguments are given, Feferman still has his case that a looming indefiniteness may prevent us from applying conceptual structuralism to set theory. From this perspective, the paper should stimulate a more profound philosophical discussion of the notion of definiteness.

Koellner's contribution stands in contrast to Parsons's paper which provides an excellent exposition of Feferman's philosophy from a neutral perspective. Parsons recalls Feferman's anti-platonism, his sympathy for predicativity and predicativism, his engagement in constructivism and proof theory before going into philosophical considerations that characterize Feferman's position, in particular by comparing it with Gödel's.

In sum, this volume is an extraordinary tribute to the person Solomon Feferman and his work. It contains material which should inspire future generations to pursue his ideas, be it on the technical side or on the philosophical side. And as a particular lesson for the legacy of Feferman we like to cite Parsons: "He is first of all a mathematician. I find persuasive the conjecture that his philosophy has followed his mathematics."