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FUZZY SUPER RESOLVING NUMBER AND RESOLVING NUMBER OF SOME SPECIAL GRAPHS

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Abstract. Resolving number of a graph was introduced by Slater in the year 1975, which is used to navigate the position of the robot uniquely in a graph-structured framework. In this paper, we introduce fuzzy super resolving set, fuzzy super resolving number and certain crisp graph with $'2n'$ vertices and resolving number $'n'$, whose resolving set form a basis for R^n.

Keywords: Fuzzy set, Fuzzy graph, Resolving number of graph, Chromatic number of a graph, Isomorphic fuzzy graph.

1. Introduction

Fuzzy Mathematics was introduced by Lotfi Asker Zadeh in the year 1965. Later in the year, 1975 Rosenfield defined fuzzy graphs that have many applications in real life such as networking, colouring [7], cluster analysis [8], telecommunication [9], traffic network [6] etc. The concept of resolving number in graph theory is introduced independently by Slater, Harry and Melter [5]. Slater named it as locating set and locating the number, Harry and Melter used the word metric dimension.

Robot can navigate its position in Euclidean space using visual detection. Consider a robot moves in a graph-structured framework, each node is referred to as a landmark. By assuming that the robot can sense the distance between the landmarks, metric basis or resolving set is the minimum number of landmark required to determine the robot's position uniquely. In a weighted graph, the distance between two nodes is the length of the shortest path between them. But if the robot can sense each landmark in terms of less risk factor or dangerous situations or unsafe conditions like temperature, expose to the weather, radiation etc., motivates us to define a fuzzy resolving set. We assume the robot needs to identify a path which is high safety and minimum risk level, we define a fuzzy number indicating the safety level between each landmark having a path and the robot can move in a fuzzy graph-structured framework. And the fuzzy super resolving

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number indicates the minimum number of landmark required to navigate the robot's position uniquely from all landmark in terms of safety level or unsafe condition.

In this paper, instead of the length of the shortest path in graph theory, we consider the weight or strength of the connectedness between two nodes. We define fuzzy super resolving set and super resolving number.

2. Definitions

Definition 2.1. Let X be a nonempty set. A fuzzy subset A of X is an ordered pair $A = \{(x, \mu_A(x))/x \in X\}$ where $\mu_A : X \to [0,1]$ and $\mu_A(x)$ is interpreted as the membership value of element x in fuzzy set A for each $x \in X$.

Definition 2.2. Graph is an ordered pair $(V(G), E(G))$ consisting of a set $V(G)$ of vertices and a set $E(G)$, disjoint from $V(G)$, of edges, together with an incidence function $\Psi(G)$ that associates with each edge of G an unordered pair of (not necessarily distinct) vertices of G. [3]

Definition 2.3. Chromatic number of a graph is the minimum number of colours required to colour the vertices of a graph so that no two adjacent vertices can have the same colour and it is denoted as $\chi(G)$. [3]

Definition 2.4. Fuzzy Graph is an ordered triple $G(V, \sigma, \mu)$, where V is a nonempty set of vertices together with the functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ such that for all $a, b \in V$, $\mu(a,b) \leq \sigma(a) \wedge \sigma(b)$. The support of μ and σ are represented as $\mu^* = \{(a,b)/\mu(a,b) > 0\}$ and $\sigma^* = \{u/\sigma(u) > 0\}$. The fuzzy subgraph induced by S is defined as $H = (S, \tau, \nu)$ where $S \subseteq V$, $\tau(x) \subseteq \sigma(x) \forall x \in S$, $\nu(x,y) = \mu(x,y) \forall x, y \in S$ and it is denoted as $\langle S \rangle$. [2]

Definition 2.5. In a fuzzy graph $G(V, \sigma, \mu)$, the sequence of different vertices $v_1, v_2, ...v_n$ with $\mu(v_{i-1}, v_i) > 0$, $i = 1, 2, ...n$ is called a fuzzy path P of length $'n'$. The consecutive pairs are named as the edges of the path. The length of the longest path between v_1 to v_2 is called the diameter of $v_1 - v_2$ denoted as $\text{diam}(v_1, v_n)$. The edge with lowest membership value in the path is called the weakest edge of the path. The strength or weight of the path P is defined as the weight or the membership value of the weakest edge in the path.The weight of connectedness between v_1 to v_n is the maximum of the weight of all the paths between v_1 to v_n and is denoted by $\mu^\infty(v_1, v_n)$ [3], for our usage we represent it as $w(v_1, v_n)$. A strongest path joining v_1 and v_n has weight $\mu^\infty(v_1, v_n)$. The fuzzy path P is called a fuzzy cycle if $v_1 = v_n$ and $n \geq 3$.

Definition 2.6. Complete fuzzy graph (CFG) is a fuzzy graph $G(V, \sigma, \mu)$ such that $\mu(ab) = \sigma(a) \wedge \sigma(b)$ for all $a, b \in \sigma^*, \mu^\infty(a,b) = \mu(a,b)$ for all $a, b \in V$ and G has no cut vertices. [2]

Definition 2.7. The adjacency matrix A of a fuzzy graph $G(V, \sigma, \mu)$ is an $n \times n$ matrix defined as $X_{ij} = \mu(v_i, v_j)$ for $i \neq j$ and when $i = j, X_{ij} = \sigma(v_i)$.

Definition 2.8. An isomorphism $h : G \to G'$ is a bijective map $h : V \to V'$ which satisfies (i) $\sigma(v) = \sigma'(h(v)) \forall v \in V$, (ii) $\mu(u,v) = \mu'(h(u), h(v)) \forall u, v \in V$ and is denoted as $G \cong G'$. [10]

Definition 2.9. The co-weak isomorphism $h : G \to G'$ is a bijective map $h : V \to V'$, which satisfies $\mu(u,v) = \mu'(h(u), h(v)) \forall u, v \in V$. [10]

Definition 2.10. Let $G(V, \sigma, \mu)$ be a fuzzy graph, the complement of G is defined as $\bar{G}(V, \sigma, \bar{\mu})$ where $\bar{\mu}(u, v) = \sigma(u) \wedge \sigma(v) - \mu(u, v) \forall u, v \in V$. A fuzzy graph is said to be self complementary if $G \cong \bar{G}$. [10]

Result 1: Two fuzzy graphs are isomorphic if and only their complement are isomorphic. [10]

3. Fuzzy Super Resolving Number of a Fuzzy Graph

In this part, we introduce resolving number in graph theory into the fuzzy graph. We define for an ordered fuzzy subset $H = \{(x_1, \sigma(x_1)), (x_2, \sigma(x_2)), ...(x_k, \sigma(x_k))\}$, $|H| \geq 2$ of the fuzzy set σ in a fuzzy simple connected graph $G(V, \sigma, \mu)$ with the number of vertices $n \geq 3$, the representation of $(y, \sigma(y)) \in \sigma - H = \{(x_{k+1}, \sigma(x_{k+1})), (x_{k+2}, \sigma(x_{k+2})), \cdots (x_n, \sigma(x_n))\}$ with respect to H is an ordered K-tuple $\{w(y, x_1), w(y, x_2)...w(y, x_k)\}$, where $w(x, y)$ is the weight of the connectedness between x and y.The Fuzzy subset H is called a Fuzzy Resolving set of G, if every two element of $\sigma - H$ have distinct representation with respect to H. A fuzzy resolving set of minimum cardinality is the fuzzy resolving number of G denoted as $Fr(G)$. A fuzzy resolving set is called a Fuzzy Super Resolving set if any two element of σ have distinct representation with respect to H . We take $w(x, x) = \sigma(x)$ and the minimum cardinality of all super resolving set is the super resolving number denoted as $Sr(G)$.

The crisp set of the fuzzy resolving set H is denoted as $S = \{x_1, x_2, ...x_n\}$. If the induced fuzzy subgraph $\langle S \rangle$ is connected, then H is called the total fuzzy resolving set. The minimum cardinality of all fuzzy total resolving set of G is called the fuzzy total resolving number of G. The representation of the elements $(y_j, \sigma(y_j)) \in \sigma - H = \{(x_{k+1}, \sigma(x_{k+1})), (x_{k+2}, \sigma(x_{k+2})), ...(x_n, \sigma(x_n))\}$ with respect to the resolving set $H = \{(x_1, \sigma(x_1)), (x_2, \sigma(x_2)), ...(x_k, \sigma(x_k))\}$ is an ordered K-tuple $\{w(y_j, x_1), w(y, x_2)...w(y_j, x_k)\}$, $j = k+1, k+2, ...n$ which are arranged in a row form a matrix of order $n - k \times k$ called Fuzzy Resolving Matrix and is denoted as $R_{n-k \times k}$. and the representation of elements in σ with respect to the super resolving set are arranged in a row form a Fuzzy Super Resolving Matrix denoted as $S_{n \times k}$.

Note 1: $2 \leq Fr(G) \leq n - 1$.

Note 2: Though out this paper, we consider a connected graph G with number of vertices $n \geq 3$.

Example 3.1. Consider the following Fuzzy Graph G.

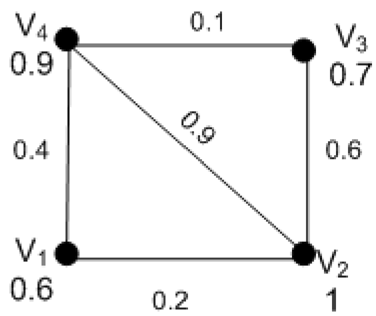

Fig.1 $G(V, \sigma, \mu)$

$V = \{v_1, v_2, v_3, v_4, \}$ $\sigma = \{(v_1, 0.6), (v_2, 1), (v_3, .7), (v_4, 0.9)\}$
$\mu(v_1v_2) = 0.2; \mu(v_2v_3) = 0.6; \mu(v_3v_4) = 0.1; \mu(v_4v_1) = 0.4; \mu(v_4v_2) = 0.9$

The adjacency matrix of $G =$ $\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} .6 & .2 & 0 & .4 \\ .2 & 1 & .6 & .9 \\ 0 & .6 & .7 & .6 \\ .4 & .9 & .1 & .9 \end{bmatrix} \end{array}$

The strength of connectedness between the vertices of $G =$ $\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} .6 & .4 & .4 & .4 \\ .4 & 1 & .6 & .9 \\ .4 & .6 & .7 & .6 \\ .4 & .9 & .6 & .9 \end{bmatrix} \end{array}$

The fuzzy super resolving number of G can be obtained from the connectedness matrix by manually searching the minimum subset of σ have distinct representation with respect to all vertices. That is, $Sr(G) = 2$.
We denote $(v_1, \sigma(v_1) = \sigma_1)$
The two element subset of σ are $H_1 = \{\sigma_1, \sigma_2\}$, $H_2 = \{\sigma_1, \sigma_3\}$, $H_3 = \{\sigma_1, \sigma_4\}$, $H_4 = \{\sigma_2, \sigma_3\}$, $H_5 = \{\sigma_2, \sigma_4\}$, $H_6 = \{\sigma_3, \sigma_4\}$.
$H_1 = \{\sigma_1, \sigma_2\}$, $H_4 = \{\sigma_2, \sigma_3\}$, $H_5 = \{\sigma_2, \sigma_4\}$ are fuzzy super resolving set. And their super resolving matrix are $\begin{bmatrix} .6 & .4 \\ .4 & 1 \\ .4 & .6 \\ .4 & .9 \end{bmatrix}$, $\begin{bmatrix} .4 & .4 \\ 1 & .6 \\ .6 & .7 \\ .9 & .6 \end{bmatrix}$ *and* $\begin{bmatrix} .4 & .4 \\ 1 & .9 \\ .6 & .6 \\ .9 & .9 \end{bmatrix}$ respectively.
$H_1 = \{\sigma_1, \sigma_2\}$, $H_3 = \{\sigma_1, \sigma_4\}$, $H_4 = \{\sigma_2, \sigma_3\}$, $H_5 = \{\sigma_2, \sigma_4\}$, $H_6 = \{\sigma_3, \sigma_4\}$ are fuzzy resolving set.
$\sigma_2/H_2 = (w(v_2, v_1), w(v_2, v_3)) = (.4, .6)$
$\sigma_4/H_2 = (w(v_4, v_1), w(v_4, v_3)) = (.4, .6)$.
Since σ_2/H_2 and σ_4/H_2 are having same representation. H_2 is not a fuzzy resolving set.

Theorem 3.1. *A fuzzy resolving set does not 2 need to be a fuzzy super resolving set. But a fuzzy super resolving set is always a fuzzy resolving set.*

Proof. Let G be a fuzzy connected graph with n vertices and let $H = \{\sigma_1, \sigma_2, \cdots \sigma_m\}$ be a resolving set of G, then the representation of σ_i/H for $i = m+1, m+2, ...n$ are all distinct. However σ_i/H may or may not be distinct for all $i = 1, 2, ...n$. That is, H does not need to be a fuzzy super resolving set.

Now let $H = \{\sigma_1, \sigma_2, \cdots \sigma_m\}$ is a super resolving set of G, then σ_i/H for $i = 1, 2, ...m+1, m+2, ...n$ are all distinct.which imply that, the representation of σ_i/H for $i = m+1, m+2, ...n$ are all distinct. Therefore, all fuzzy super resolving set of G is also a fuzzy resolving set of G. □

Theorem 3.2. *A fuzzy resolving set H of a connected graph G is a fuzzy super resolving set, if there exists at least one $u \in S$ such that $w(u, v_i)$ are distinct for all $v_i \in \sigma^*$.*

Proof. Let $G(V, \sigma, \mu)$ be a fuzzy connected graph, $V = \{v_1, v_2, ...v_n\}$ and the fuzzy resolving set $H = \{\sigma_1, \sigma_2, ...\sigma_k\}$.
The crisp set corresponding to H is $S = \{v_1, v_2, ...v_k\}$.
then, by the definition of resolving set, σ_{k+i}/H are all distinct for $i = 1, 2, ...n-k$.
Since there exists at least one $u \in S$ such that $w(u, v_i)$ are distinct for every $v_i \in \sigma^*$ $[i = 1, 2, ...k, k+1, ...n]$ then the representation of $\sigma_i/H = (w(v_i, v_1), w(v_i, v_2), ...w(v_i, v_k))$ for $i = 1, 2, ...k$ are all distinct (in every representation $w(u, v_i)$ will differ) and which is

also distinct for $i = 1, 2, ...k, k+1, ...n$. Which implies that H is a super resolving set of G. □

Theorem 3.3. *The super resolving number of any connected fuzzy graph G is '2', if $w(u,v) \neq w(u,w)[u \neq w]$ for any $v, w \in \sigma^*$ and some $u \in \sigma^*$.*

Proof. Let $G(V, \sigma, \mu)$ be a fuzzy connected graph, $\sigma^* = \{v_1, v_2, ...v_n\}$. Let $w(u,v) \neq w(u,w)$ for any $v, w \in \sigma^*$ and some $u \neq w \in \sigma^*$.
That is, for some $u \in \sigma^*$, say $u = v_1$, $w(v_1, v_1) = \sigma(v_1), w(v_1, v_2), w(v_1, v_3)...w(v_1, v_n)$ are all distinct.
Now any two element subset of σ^* of the form $H_i = \{v_1, v_{i+1}\}$, $i = 1, 2, ...n-1$ is a fuzzy super resolving set, since $\sigma_j/H_i = (w(v_j, v_1), w(v_j, v_{i+1}))$ are all distinct for $j = 1, 2, ...n$
Therefore, for any $H_i = \{v_1, v_{i+1}\}$, $i = 1, 2, ...n-1$ is a super resolving set of G.
Which implies that the resolving number of G is $'2'$. □

Example 3.2. Consider the following fuzzy graph in fig.2 with $V = \{v_1, v_2, v_3, v_4, v_5\}$
$\sigma(v_1) = 1; \sigma(v_2) = .5; \sigma(v_3) = .4; \sigma(v_4) = .5; \sigma(v_5) = .9$
$\mu(v_1v_2) = .2; \mu(v_2v_3) = .1; \mu(v_3v_4) = .4; \mu(v_4v_5) = .5; \mu(v_5v_1) = .9; \mu(v_1v_4) = .6; \mu(v_1v_3) = .5$

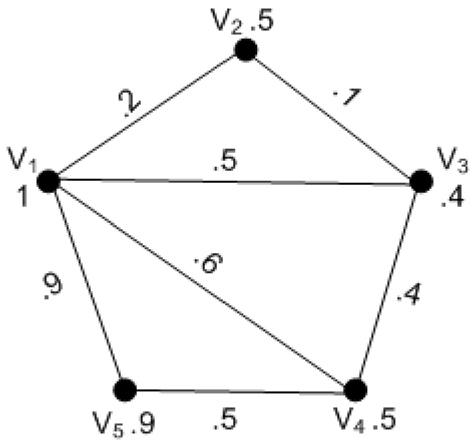

Fig.2

$w(v_1, v_1) = 1, w(v_1, v_2) = .2, w(v_1, v_3) = .5, w(v_1, v_4) = .6, w(v_1, v_5) = .9$
$w(v_1, v_i) \neq w(v_1, v_j)$ for $i \neq j$
$H_1 = \{\sigma_1, \sigma_2\}, H_2 = \{\sigma_1, \sigma_3\}, H_3 = \{\sigma_1, \sigma_4\}, H_4 = \{\sigma_1, \sigma_5\}$ are all have distinct representation with respect to σ, therefore these are all the super fuzzy resolving set of G.
$\sigma_1/H_1 = (1, .2)$ $\sigma_1/H_2 = (1, .5)$ $\sigma_1/H_3 = (1, .6)$ $\sigma_1/H_4 = (1, .9)$
$\sigma_2/H_1 = (.2, .5)$ $\sigma_2/H_2 = (.2, .2)$ $\sigma_2/H_3 = (.2, .2)$ $\sigma_2/H_4 = (.2, .2)$
$\sigma_3/H_1 = (.5, .2)$ $\sigma_3/H_2 = (.5, .4)$ $\sigma_3/H_3 = (.5, .5)$ $\sigma_3/H_4 = (.5, .5)$
$\sigma_4/H_1 = (.6, .2)$ $\sigma_4/H_2 = (.6, .5)$ $\sigma_4/H_3 = (.6, .5)$ $\sigma_4/H_4 = (.6, .6)$
$\sigma_5/H_1 = (.9, .2)$ $\sigma_5/H_2 = (.9, .5)$ $\sigma_5/H_3 = (.9, .6)$ $\sigma_5/H_4 = (.9, .9)$

Theorem 3.4. *If G and G' are isomorphic to each other then $Sr(G) = Sr(G')$.*

Proof. If G and G' are isomorphic to each other then there exists a bijective map $h : V \to V'$ which satisfies :-
(i) $\sigma(v) = \sigma'(h(v)) \forall v \in V$, (ii) $\mu(u,v) = \mu'(h(u), h(v)) \forall u, v \in V$
Let $V = \{v_1, v_2, ...v_n\}$ and $V' = \{h(v_1), h(v_2), ...h(v_n)\}$.

Assume that, $Sr(G) = k$ and $H = \{v_1, v_2, ...v_k\}$, $|H| = k \geq 2$, be the corresponding fuzzy super resolving set of G. Then the representation of σ_i/H are all distinct for $i = 1, 2, ...n$ in G.
Now let us define, $I = \{h(v_1), h(v_2), ...h(v_k)\}$,$|I| = k$
$\sigma'_i/I = (\mu'^{\infty}(h(v_i), h(v_1)), \mu'^{\infty}(h(v_i), h(v_2)), ...\mu'^{\infty}(h(v_i), h(v_k)))$

$= (\mu^{\infty}(v_i, v_1), \mu^{\infty}(v_i, v_2), ...\mu^{\infty}(v_i, v_k)) \quad [\mu(u, v) = \mu'(h(u), h(v)) \forall u, v \in V]$

Which are all distinct since, H is the fuzzy super resolving set of G . Therefore σ'_i/I are all distinct in G', which implies that, $I = \{h(v_1), h(v_2), ...h(v_k)\}$ is super fuzzy resolving set of G'.
To prove, I is the minimum resolving set of G'.

If there exist a super resolving set J with $|J| < k$, say $|J| = k - 1$
$J = \{h(v_1), h(v_2), ...h(v_{k-1})\}$ then take $K = \{v_1, v_2, ...v_{k-1}\}$
$\sigma_i/K = (\mu^{\infty}(v_i, v_1), \mu^{\infty}(v_i, v_2), ...\mu^{\infty}(v_i, v_{k-1}))$

$= (\mu'^{\infty}(h(v_i), h(v_1)), \mu'^{\infty}(h(v_i), h(v_2)), ...\mu'^{\infty}(h(v_i), h(v_{k-1})))$

are all distinct for $i = 1, 2, ...n - 1$, since $J = \{h(v_1), h(v_2), ...h(v_{k-1})\}$ is a super resolving set of G'.
$\Longrightarrow$ k is a resolving set of G and $Sr(G) = k - 1$.
Which is a contradiction to our assumption that $Sr(G) = k$
Therefore there does not exist a super resolving set J with $|J| < k$.
whcih implies that, $Sr(G) = Sr(G') = k$. □

Corollary 3.1. If G and G' are isomorphic to each other then $Fr(G) = Fr(G')$.

Theorem 3.5. *If G and G' are isomorphic to each other then $Fr(\bar{G}) = Fr(\bar{G}')$.*

Proof. Given G and G' are isomorphic to each other.
By result [1],Two fuzzy graphs are isomorphic if and only if their complement are isomorphic [13]. Therefore $\bar{G} \cong \bar{G}'$.
Now by corollary [3.1], If G and G' are isomorphic to each other then $Fr(G) = Fr(G')$.
Which will imply that, $Fr(\bar{G}) = Fr(\bar{G}')$.

□

4. Real Basis Generating Graphs $RG(n)$

For an ordered subset $W = \{w_1, w_2, ...w_k\}$ of vertices and a vertex v in a connected graph $G(V, E)$, the representation of v with respect to W is an ordered K-tuple $r(v/W) = \{d(v, w_1), d(v, w_2), ...d(v, w_k)\}$ with $d(u, v)$ is the shortest distance between the vertices u and v. The set W is called a resolving set of G, if every pair of vertices of G has distinct representation with respect to W. A resolving number of G is the cardinality of the minimum resolving set W and is denoted by $dim(G)$ [1].The representation of the elements in $V - W$ with respect to W is an ordered k-tuple which are arranged in a row form a matrix of order $n - k \times k$, we call it as resolving matrix $R_{n-k \times k}$. In addition to this, in certain Graphs, the representation of $V - W$ vertices with respect to W forms the basis for R^n with $'2n'$ $[n > 1]$ vertices and resolving number $'n'$. Some of the examples are as follows.

Example 4.1. In the following graph $G(V, E)$

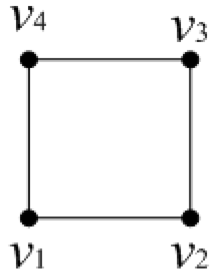

Fig.3 An example of $RG(2)$

$V = \{v_1, v_2, v_3, v_4\}$
The resolving set $W = \{v_1, v_2\}$
$r(v_3/W) = (2,1)$;$r(v_4/W) = (1,2)$
The vectors $(2,1)$ and $(1,2)$ forms the basis for R^2, since the rank of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ is equal to $'2'$.
The resolving number of $G = 2$.
The Chromatic number of $G = 2$.

Example 4.2. In the following Octahedral graph $G(V, E)$

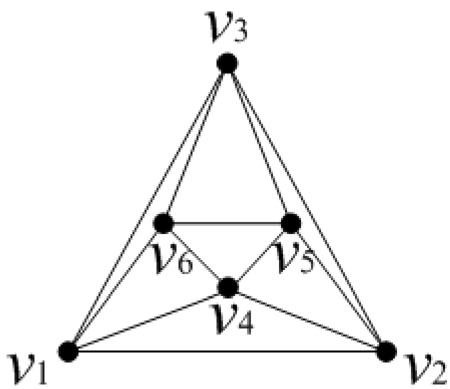

Fig.4 An example of $RG(3)$

$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$
The resolving set $W = \{v_1, v_2, v_3\}$
$r(v_4/W) = (1,1,2)$;$r(v_5/W) = (2,1,1)$;$r(v_6/W) = (1,2,1)\}$
The vectors $(1,1,2), (2,1,1)$ and $(1,2,1)$ form the basis for R^3.
The resolving number of $G = 3$.
The Chromatic number of $G = 3$.

Example 4.3. Consider the following graph $G(V, E)$ on $'8'$ vertices

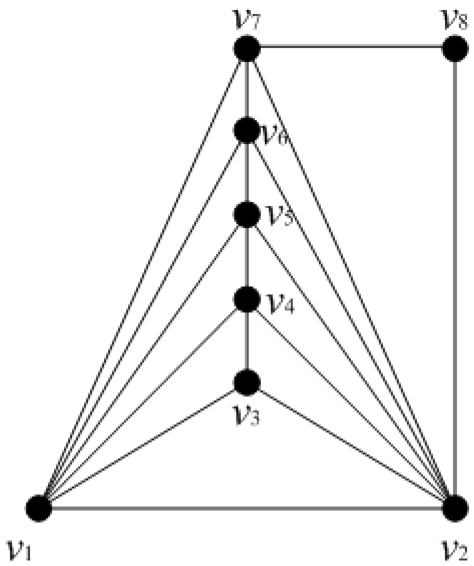

Fig.5 An example of $RG(4)$

$V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$
The resolving set $W = \{v_1, v_3, v_4, v_8\}$
$r(v_2/W) = (1, 1, 1, 1)$; $r(v_5/W) = (1, 2, 1, 2)$; $r(v_6/W) = (1, 2, 2, 2)$; $r(v_7/W) = (1, 2, 2, 1)\}$
The vectors $(1, 1, 1, 1), (1, 2, 1, 2), (1, 2, 2, 2)$ and $(1, 2, 2, 1)$ form the basis for R^4.
The resolving number of $G = 4$.
The Chromatic number of $G = 4$.
From the above examples [1, 2, and 3], we can see that, some graphs with $'2n'$ vertices, $n \geq 2$ and resolving number or metric dimension is $'n'$, the representation of $V - W$ vertices with respect to the resolving set W form the basis for R^n, we name such graphs as Real Basis generating Graphs denoted as $RG(n)$. The platonic solid and cube is an example of $RG(4)$.

Theorem 4.1. *For a Real basis generating graph $RG(n)$, The Chromatic number $\chi(RG(n)) \leq n + 1$ for $n \geq 2$.*

Proof. The number of vertices of $RG(n) = 2n$ and the resolving number of $RG(n) = n$. If the chromatic number $\chi(RG(n)) > n + 1$, say $n + 2$, then there exist some vertices $v_1, v_2, ...v_{n+2}$ which are adjacent to each other. Therefore, atleast two vertices of the resolving set W is from $v_1, v_2, ...v_{n+2}$ say v_{n+1} and v_{n+2} then the remaining vertices $v_1, v_2, ...v_n$ are all of distance $'1'$ from v_{n+1} and v_{n+2}. Therefore the two columns of the $n \times n$ resolving matrix are same [it is equat to $'1'$], and the rank of resolving matrix will not be equal to $'n'$. that is, for $\chi(RG(n)) > n+1$, the rank of resolving matrix will not be equal to $'n'$. Which is the contrary to the definition of $RG(n)$ that, the representation of $V - W$ vertices with respect to the resolving set W forms the basis for R^n. Therefore $\chi(RG(n)) \leq n + 1$. □

Theorem 4.2. *In $RG(n)$, $d(x, y) \neq cd(x, z)$ or $d(x, y) \neq d(x, z)$ $\forall x \in V - W$ and for any $u, v \in W$, where $c \in N$ is a constant.*

Proof. Let $G(V, E)$ be a real basis generating graph, $V = \{v_1, v_2, ...v_{2n}\}$ and let the resolving set of $RG(n)$, $W = \{v_1, v_2, ...v_n\}$, $|W| = n$
Assume that, there exist $u, v \in W \ni d(x, y) = d(x, z) = k$(say), $\forall x \in V - W$.

Then the resolving matrix will be of the form, $R_n = \begin{array}{c c} & \begin{array}{cccccc} v_1 & \dots & y & z & \dots & v_n \end{array} \\ \begin{array}{c} v_{n+1} \\ v_{n+2} \\ \vdots \\ v_{2n} \end{array} & \begin{bmatrix} & \dots & c_1 & c_1 & \dots & \\ & \dots & c_2 & c_2 & \dots & \\ & & \vdots & \vdots & \dots & \\ & \dots & c_n & c_n & \dots & \end{bmatrix} \end{array}$

We can see that in R_n two columns are linearly dependent[11].
$\implies \rho(R_n) \neq n$
Which is a contradiction to our assumption that, G is $RG(n)$.
Therefore, there does not exist some $u, v \in W$ and $\forall x \in V - W \ni d(x, y) = d(x, z) = k$. $\square$

Corollary 4.1. In $RG(n)$, $d(x, y) \neq cd(x, z)$ or $d(x, y) \neq d(x, z)$ $\forall x \in W$ and some $u, v \in V - W$, where $c \in N$ is a constant.

Corollary 4.2. There exist almost one vertex $u \in V - W$ in $RG(n)$ such that $d(u, x_1) = d(u, x_2) = \cdots = d(u, x_n) \quad \forall x_1, x_2, ...x_n \in W$.

Corollary 4.3. There exist almost one vertex $u \in V$ in $RG(n)$ such that $d(u, x_1) = d(u, x_2) = \cdots = d(u, x_n) \quad \forall x_1, x_2, ...x_n \in V - W$.

5. Conclusions

In this paper, we have introduced fuzzy resolving set and fuzzy super resolving set, which can be used to identify the set of landmark required to navigate the position of the robot uniquely in a fuzzy graph-structured framework. The minimum cardinality of fuzzy resolving set and fuzzy super resolving sets are named as fuzzy resolving number $Fr(G)$ and fuzzy super resolving number $Sr(G)$, respectively. We have also introduced a real basis generating crisp graph in this paper. And we would like to introduce real basis generating fuzzy graphs in our future work.

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