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ON 2-ABSORBING FUZZY IDEALS OF COMMUTATIVE SEMIRINGS

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Abstract. The purpose of this paper is to introduce and study 2-absorbing fuzzy ideals of commutative semirings. Some basic operations on them are defined and some of its characterizations are obtained.

Keywords: 2-absorbing fuzzy ideals, intersection, homomorphism, cartesian product.

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1. Introduction

Semiring is a well known universal algebra. This is a generalization of an associative ring $(S, +, \cdot)$; If $(S, +)$ becomes a semigroup instead of a group then $(S, +, \cdot)$ reduces to a semiring. Semiring has been found very useful for solving problems in different areas of applied mathematics and information sciences, since the structure of a semiring provides an algebraic framework for modelling and studying the key factors in these applied areas. In fact, ideals of semiring play a central role in the structure theory and useful for many purposes.

As generalizations of prime and weakly prime ideals in commutative ring, the concept of 2-absorbing ideals of a commutative ring with non-zero unity was first introduced by Badawi [1] in 2007. After that so many authors, for example [2, 3], conducted research on this and developed this idea. In 2012, Darani [5] applied the concept in case of commutative semirings. Kumar et. al [8, 9, 10], Dubey et. al [6], Behzadipour et al [4] explored the concept in commutative semirings and characterized many results in terms of 2-absorbing ideals, weakly 2-absorbing ideals and 2-absorbing primary ideals.

In this paper, we have defined the concept of 2-absorbing fuzzy ideals in commutative semirings and investigated some of its basic results.

2. Preliminaries

We recall the following preliminaries for subsequent use.

Definition 2.1. [10] *A semiring is a non-empty set S on which operations addition and multiplication have been defined such that the following conditions are satisfied:*
(i) $(S, +)$ is a commutative monoid with identity 0

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(ii) $(S, \cdot)$ *is a commutative semigroup*
(iii) Multiplication distributes over addition from either side
(iv) $0s = 0 = s0$ *for all* $s \in S$.

Definition 2.2. [10] *A proper ideal* I *of a semiring* S *is said to be a 2-absorbing ideal of* S *if* $abc \in I$ *implies* $ab \in I$ *or* $bc \in I$ *or* $ac \in I$ *for all* $a, b, c \in S$.

Definition 2.3. [9] *Let* S *be a commutative semiring and* I *be a proper ideal of* S. *Then* I *is said to be a 2-absorbing primary ideal of* S *if whenever* $a, b, c \in S$ *and* $abc \in I$, *then* $ab \in I$ *or* $ac \in \sqrt{I}$ *or* $bc \in \sqrt{I}$

Definition 2.4. [12] *A fuzzy subset of a nonempty set* X *is defined as a function* $\mu : X \to [0,1]$. *For any fuzzy subset* μ *of* X *and* $t \in [0, 1]$, *the level subset of* μ *is denoted by* μ_t *and defined as* $\mu_t = \{x \in X | \mu(x) \geq t\}$.

Definition 2.5. *Let* μ *be a fuzzy subset of* S *and* $x \in S$. *Then radical of* μ *is denoted by* $\sqrt{\mu}$ *and defined as* $\sqrt{\mu}(x) = \sup\{\mu(x^n) | n \in N\}$.

Definition 2.6. *Let* μ *be a non empty fuzzy subset of a semiring* S *(i.e. anyone of* $\mu(x)$ *not equal to zero for some* $x \in S$*). Then* μ *is called a fuzzy left ideal of* S *if*

(i) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$,
(ii) $\mu(xy) \geq \mu(y)$.

for all $x, y \in S$.
Similarly we can define fuzzy right ideal of S.

Definition 2.7. *For* $A \subseteq S$ *the characteristic function* χ_A *of* A *is defined by*
$$\chi_A(x) = \begin{cases} 1 \text{ if } x \in A \\ 0 \text{ otherwise} \end{cases}.$$

Throughout this paper, unless otherwise mentioned S denotes a commutative semiring.

3. 2-absorbing fuzzy ideals

In this section, we have revisited the results which are true using the concept of fuzzy ideals. So, investing the results for 2-absorbing fuzzy ideals, we have omitted the proof for fuzzy ideals.

Definition 3.1. *A fuzzy ideal* μ *of* S *is called a 2-absorbing fuzzy ideal of* S *if for all* $x, y, z \in S$ *and* $t \in [0, 1]$, $\mu(xyz) \geq t$ *implies* $\mu(xy) \geq t$ *or* $\mu(yz) \geq t$ *or* $\mu(xz) \geq t$.

Example 3.1. *Let* $S = \{0, a, b, c\}$ *be a set with the operation addition* $+$ *and the multiplication* $\cdot$ *defined as follows:*

$+$	0	a	b	c
0	0	a	b	c
a	a	a	b	c
b	b	b	b	c
c	c	c	c	c

and

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	a	b	b
c	0	a	b	c

Then $(S, +, \cdot)$ *forms a commutative semiring.*
Define fuzzy subset μ *of* S *by* $\mu(0) = 1$, $\mu(a) = 0.2$, $\mu(b) = 0.4$, $\mu(c) = 0.7$. *Then* μ *is a 2-absorbing fuzzy ideal of* S.

Example 3.2. *Let* $S = \{0, a, b\}$ *be a set with the operation addition* $+$ *and the multiplication* $\cdot$ *defined as follows:*

$+$	0	a	b
0	0	a	b
a	a	0	b
b	b	b	0

and

$\cdot$	0	a	b
0	0	0	0
a	0	0	0
b	0	0	b

Then $(S, +, \cdot)$ forms a commutative semiring.
Define fuzzy subset μ of S by $\mu(0) = 1$, $\mu(a) = 0.5$, $\mu(b) = 0.8$. Then μ is a 2-absorbing fuzzy ideal of S.

Definition 3.2. *A fuzzy ideal μ of S is called a 2-absorbing fuzzy primary ideal of S if for $x, y, z \in S$ and $t \in [0, 1]$, $\mu(xyz) \geq t$ implies $\mu(xy) \geq t$ or $\sqrt{\mu}(yz) \geq t$ or $\sqrt{\mu}(xz) \geq t$.*

Theorem 3.1. *Let μ be a fuzzy subset of S. Then μ is a 2-absorbing fuzzy ideal of S if and only if μ_t is a 2-absorbing ideal of S.*

Proof. Let μ be a 2-absorbing fuzzy ideal of S and $abc \in \mu_t$ for $a,\ b,\ c \in S$ and $t \in [0, 1]$. Since μ is a 2-absorbing fuzzy ideal,

$$\begin{aligned} & abc \in \mu_t \Rightarrow \mu(abc) \geq t \\ & \Rightarrow \mu(ab) \geq t \text{ or } \mu(bc) \geq t \text{ or } \mu(ac) \geq t \\ & \Rightarrow ab \in \mu_t \text{ or } bc \in \mu_t \text{ or } ac \in \mu_t. \end{aligned}$$

Hence μ_t is a 2-absorbing ideal of S.
Conversely, suppose μ be a fuzzy subset of S such that μ_t is a 2-absorbing ideal of S for all $t \in [0, 1]$ but μ is not a 2-absorbing fuzzy ideal of S. So, for $a,\ b,\ c \in S$ and $t \in [0, 1]$, $\mu(abc) \geq t$ does not imply $\mu(ab) \geq t$ or $\mu(bc) \geq t$ or $\mu(ac) \geq t$ i.e. $abc \in \mu_t$ does not imply $ab \in \mu_t$ or $bc \in \mu_t$ or $ac \in \mu_t$ $-$ contradiction to the fact that μ_t is a 2-absorbing ideal of S. Therefore μ is a 2-absorbing fuzzy ideal of S. □

Theorem 3.2. *Let I be a 2-absorbing ideal of S and $\alpha \in [0, 1]$. Then a fuzzy subset μ of S, defined by*

$$\mu(x) = \begin{cases} 1 \text{ if } x \in I \\ \alpha \text{ otherwise} \end{cases}$$

Then μ is a 2-absorbing fuzzy ideal of S.

Proof. Let I be a 2-absorbing fuzzy ideal of S and $a, b, c \in S$. Suppose $\mu(abc) \geq t$.
Case I: $t = 1$
$abc \in \mu_1 \Rightarrow abc \in I$.
Now if $\mu(ab) < t = 1$ or $\mu(bc) < t = 1$ or $\mu(ac) < t = 1$, then $ab, bc, ac \notin I-$ contradiction to the fact that I is a 2-absorbing ideal of S.
Case II: $t = \alpha$
Here $\mu(abc) = \alpha$. If anyone of $\mu(ab)$ or $\mu(bc)$ or $\mu(ac)$ is equal to α, we are done. Otherwise all of $\mu(ab)$, $\mu(bc)$ and $\mu(ac)$ is equal to 1. In this case, $ab, bc, ca \in I$ but $abc \notin I$, $-$ contradiction to the fact that I is a 2-absorbing ideal of S.
Hence the proof. □

Theorem 3.3. *Let μ be a 2-absorbing fuzzy primary ideal of S. Then $\sqrt{\mu}$ is a 2-absorbing fuzzy ideal of S.*

Proof. Let μ be a 2-absorbing fuzzy primary ideal of S. Suppose for $x, y, z \in S$ and $t \in [0, 1]$, $\sqrt{\mu}(xyz) \geq t$. Then

$$\sqrt{\mu}(xyz) \geq t \Rightarrow \sup\{\mu(xyz)^n | n \in Z\} \geq t \Rightarrow \sup\{\mu(x^n y^n z^n) | n \in Z\} \geq t.$$

Suppose $yz,\ xz \notin \sqrt{\mu}$. So, $\mu(y^n z^n) \ngeq t$ and $\mu(x^n z^n) \ngeq t$, for $t \in [0, 1]$ and $n \in Z$. Since μ is a 2-absorbing fuzzy primary ideal of S, $\mu((x^n y^n)^k) \geq t$ for some $k \in Z$ and so $\sup\{\mu(x^m y^m) | m \in Z\} \geq t$ i.e. $xy \in \sqrt{\mu}$. Therefore $\sqrt{\mu}$ is a 2-absorbing ideal of S. □

Theorem 3.4. *Intersection of a non-empty collection of 2-absorbing fuzzy ideals of S is also a 2-absorbing fuzzy ideal of S.*

Proof. We know that intersection of a non-empty collection of fuzzy ideals of S is also a fuzzy ideal of S. Now for 2-absorbing ideals, let $\{\mu_i : i \in I\}$ be a non-empty family of 2-absorbing fuzzy ideals of S and $x, y, z \in S$.
Then

$$
\begin{aligned}
&(\bigcap_{i\in I}\mu_i)(xyz) \geq t \\
&\Rightarrow \inf_{i\in I}\{\mu_i(xyz)\} \geq t \\
&\Rightarrow \inf_{i\in I}\{\mu_i(xy)\} \geq t \text{ or } \inf_{i\in I}\{\mu_i(yz)\} \geq t \text{ or } \inf_{i\in I}\{\mu_i(xz)\} \geq t \\
&\Rightarrow (\bigcap_{i\in I}\mu_i)(xy) \geq t \text{ or } (\bigcap_{i\in I}\mu_i)(yz) \geq t \text{ or } (\bigcap_{i\in I}\mu_i)(xz) \geq t.
\end{aligned}
$$

Hence the result follows. □

Proposition 3.1. *Intersection of a non-empty collection of 2-absorbing fuzzy primary ideals of S is also a 2-absorbing fuzzy primary ideal of S.*

Proof. Similar as Theorem 3.4. □

Proposition 3.2. *Let $f : R \to S$ be a morphism of semirings and μ be a 2-absorbing fuzzy ideal of S. Then $f^{-1}(\mu)$* [11] *is also a 2-absorbing fuzzy ideal of R.*

Proof. Let $f : R \to S$ be a morphism of semirings and μ be a 2-absorbing fuzzy ideal of S. Now for $x, y, z \in S$ and $t \in [0, 1]$,

$$
\begin{aligned}
&f^{-1}(\mu)(xyz) \geq t \Rightarrow \mu(f(xyz)) \geq t \Rightarrow \mu(f(x)f(y)f(z)) \geq t \\
&\Rightarrow \mu(f(x)f(y)) \geq t \text{ or } \mu(f(y)f(z)) \geq t \text{ or } \mu(f(x)f(z)) \geq t \\
&\Rightarrow f^{-1}(\mu)(xy) \geq t \text{ or } f^{-1}(\mu)(yz) \geq t \text{ or } f^{-1}(\mu)(xz) \geq t
\end{aligned}
$$

Therefore $f^{-1}(\mu)$ is a 2-absorbing fuzzy ideal of R. □

Proposition 3.3. *Let $\mu_1,\ \mu_2$ be two fuzzy subsets of S such that μ_1 is a 2-absorbing fuzzy ideal of S and $\mu_1 \subseteq \mu_2$. If for $x, y, z \in S$ and $t \in [0, 1]$, $\mu_2(xyz) \geq \mu_1(xyz) \geq t$, then μ_2 is also a 2-absorbing fuzzy ideal of S.*

Proof. Suppose $\mu_1,\ \mu_2$ be two fuzzy subsets of S such that $\mu_1 \subseteq \mu_2$ and μ_1 is a 2-absorbing fuzzy ideal of S. Let $x, y, z \in S$ and $t \in [0, 1]$ be such that $\mu_2(xyz) \geq t$. Since $\mu_2(xyz) \geq \mu_1(xyz) \geq t$ and μ_1 is a 2-absorbing fuzzy ideal of S, $\mu_1(xy) \geq t$ or $\mu_1(yz) \geq t$ or $\mu_1(xz) \geq t$ which implies $\mu_2(xy) \geq \mu_1(xy) \geq t$ or $\mu_2(yz) \geq \mu_1(yz) \geq t$ or $\mu_2(xz) \geq \mu_1(xz) \geq t$. Hence μ_2 is a 2-absorbing fuzzy ideal of S. □

Definition 3.3. *Let μ and ν be two fuzzy subsets of S_1 and S_2, respectively. The cartesian product of μ and ν is defined by*

$$(\mu \times \nu)(x, y) = \min\{\mu(x), \nu(y)\}$$

for all $(x, y) \in S_1 \times S_2$.

Theorem 3.5. *Let $S_1,\ S_2$ be two commutative semiring and μ be a 2-absorbing fuzzy ideal of a semiring S_1. Then $\mu \times \chi_{S_2}$ is a 2-absorbing fuzzy ideal of $S_1 \times S_2$, where χ_{S_2} is the characteristic function of S_2.*

Proof. Suppose that $S_1,\ S_2$ be two commutative semiring and $\mu,\ \chi_{S_2}$ be two 2-absorbing fuzzy ideals of S_1, S_2 respectively. Now for $x, y, z \in S_1$, $p, q, r \in S_2$ and $t \in [0, 1]$

$$\begin{aligned}&(\mu \times \chi_{S_2})(xyz, pqr) \geq t\\ &\Rightarrow \min\{\mu(xyz), \chi_{S_2}(pqr)\} \geq t\\ &\Rightarrow \min\{\mu(xy), \chi_{S_2}(pqr)\} \geq t\\ &\text{ or } \min\{\mu(yz), \chi_{S_2}(pqr)\} \geq t\\ &\text{ or } \min\{\mu(xz), \chi_{S_2}(pqr)\} \geq t\end{aligned}$$

Since $p, q, r \in S_2$, $\chi_{S_2}(pqr) = 1$ and so $\min\{\mu(xy), \chi_{S_2}(mn)\} \geq t$ or $\min\{\mu(yz), \chi_{S_2}(mn)\} \geq t$ or $\min\{\mu(xz), \chi_{S_2}(mn)\} \geq t$ where $m, n \in \{p, q, r\}$. Therefore $\mu \times \chi_{S_2}$ is a 2-absorbing fuzzy ideal of $S_1 \times S_2$. □

4. Conclusions

2-absorbing fuzzy ideals of a commutative semirings are defined and studied with some operation on these ideals. Some characterizations of these ideals are obtained using level subset criteria, characteristic functions, homomorphisms and cartesian product. For future work, one could study other algebraic structures and extend the existing work to the framework of neutrosophic set and soft set.

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References

[1] Badawi, A., (2007), On 2-absorbing ideals of commutative rings, Bulletin of the Australian Mathematical Society, 75, pp. 417 - 429.

[2] Badawi, A., Darani, A.Y., (2013), On weakly 2-absorbing ideals of commutative rings, Houston Journal of Mathematics, 39(2), pp. 441 - 452.

[3] Badawi, A., Tekir, U., Yetkin, E., (2014), On 2-absorbing primary ideals in commutative ring, Bulletin of Korean Mathematical Society, 51(4), pp. 1163 - 1173.

[4] Behzadipour, H., Nasehpour, P., (2020), On 2-absorbing ideals of commutative semirings, Journal of Algebra and Its Applications, 19(2), 2050034.

[5] Darani, A.Y., (2012), On 2-absorbing and weakly 2-absorbing ideals of commutative semirings, Kyungpook Mathematical Journal, 52(1), pp. 91 - 97.

[6] Dubey, M.K., Sarohe, P., (2017), Generalizations of prime and primary Ideals in Commutative Semirings, Southeast Asian Bulletin of Mathematics, 41(1), pp. 9 - 20.

[7] Golan, J.S., (1999), Semirings and their applications, Kluwer Academic Publishers, Dordrecht.

[8] Kumar, P., Dubey, M.K., Sarohe, P., (2015), Some results on 2-absorbing ideals in commutative semirings, Journal of Mathematics and Applications, 38, pp. 77 - 84.

[9] Kumar, P., Dubey, M.K., Sarohe, P., (2016), On 2-absorbing primary ideals in a commutative semirings, Europian Journal of Pure and Applied Mathematics, 9(2), pp. 186 - 195.

[10] Kumar, P., Dubey, M.K., Sarohe, P., (2016), On 2-absorbing ideals in commutative semirings, Quasigroups and Related Systems, 24, pp. 67 - 74.

[11] Rosenfeld, A., (1971), Fuzzy groups, Journal of Mathematical Analysis and Applications, 35, pp. 512 - 517.

[12] Zadeh, L.A., (1965), Fuzzy Sets, Information and Control, 8, pp. 338 - 353.

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