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INITIAL BOUNDS FOR CERTAIN CLASSES OF BI-UNIVALENT FUNCTIONS DEFINED BY THE (p, q)-LUCAS POLYNOMIALS

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ABSTRACT. Our present investigation is motivated essentially by the fact that, in Geometric Function Theory, one can find many interesting and fruitful usages of a wide variety of special functions and special polynomials. The main purpose of this article is to make use of the $(p, q)-$ Lucas polynomials $L_{p,q,n}(x)$ and the generating function $\mathcal{G}_{L_{p,\,q,\,n}(x)}(z)$, in order to introduce three new subclasses of the bi-univalent function class Σ. For functions belonging to the defined classes, we then derive coefficient inequalities and the Fekete-Szegö inequalities. Some interesting observations of the results presented here are also discussed. We also provide relevant connections of our results with those considered in earlier investigations.

Keywords: Univalent functions, bi-univalent functions, bi-Mocanu-convex functions, bi-$\alpha-$starlike functions, bi-starlike functions, bi-convex functions, Fekete-Szegö problem, Chebyshev polynomials, (p, q)-Lucas polynomials.

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1. Introduction

Let $\mathbb{R} = (-\infty, \infty)$ be the set of real numbers, $\mathbb{C}$ be the set of complex numbers and

$$\mathbb{N} := \{1, 2, 3, \ldots\} = \mathbb{N}_0 \setminus \{0\}$$

be the set of positive integers. Let also $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1}$$

which are analytic in the open unit disk $\Delta = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$. Further, by $\mathcal{S}$ we shall denote the class of all functions in $\mathcal{A}$ which are univalent in Δ.

It is well known that every function $f \in \mathcal{S}$ has an inverse f^{-1}, defined by

$$f^{-1}(f(z)) = z \qquad (z \in \Delta)$$

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and

$$f(f^{-1}(w)) = w(|w| < r_0(f);\ r_0(f) \geq \frac{1}{4}),$$

where

$$g(w) = f^{-1}(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \dots.$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in Δ if both a function f and it's inverse f^{-1} are univalent in Δ. Let Σ denote the class of bi-univalent functions in Δ given by (1).

In 2010, Srivastava *et al.* [24] revived the study of bi-univalent functions by their pioneering work on the study of coefficient problems. Various subclasses of the bi-univalent function class Σ were introduced and non-sharp estimates on the first two coefficients $|a_2|$ and $|a_3|$ in the Taylor-Maclaurin series expansion (1) were found in the recent investigations (see, for example, [1, 2, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27]) and including the references therein. The afore-cited all these papers on the subject were actually motivated by the work of Srivastava *et al.* [24]. However, the problem to find the coefficient bounds on $|a_n|$ $(n = 3, 4, \dots)$ for functions $f \in \Sigma$ is still an open problem.

For analytic functions f and g in Δ, f is said to be subordinate to g if there exists an analytic function w such that

$$w(0) = 0, \qquad |w(z)| < 1 \quad \text{and} \quad f(z) = g(w(z)) \qquad (z \in \Delta).$$

This subordination will be denoted here by

$$f \prec g \qquad (z \in \Delta)$$

or, conventionally, by

$$f(z) \prec g(z) \qquad (z \in \Delta).$$

In particular, when g is univalent in Δ,

$$f \prec g \qquad (z \in \Delta) \ \Leftrightarrow\ f(0) = g(0) \quad \text{and} \quad f(\Delta) \subset g(\Delta).$$

Let $p(x)$ and $q(x)$ be polynomials with real coefficients. The (p, q)-polynomials $L_{p,q,n}(x)$, or briefly $L_n(x)$ are given by the following recurrence relation (see [8, 9]):

$$L_n(x) = p(x)L_{n-1}(x) + q(x)L_{n-2}(x) \qquad (n \in \mathbb{N} \setminus \{1\}),$$

with

$$\begin{aligned} &L_0(x) = 2,\\ &L_1(x) = p(x),\\ &L_2(x) = p^2(x) + 2q(x),\\ &L_3(x) = p^3(x) + 3p(x)q(x),\\ &\vdots \end{aligned}$$

The generating function of the Lucas polynomials $L_n(x)$ (see [16]) is given by:

$$\mathcal{G}_{L_n(x)}(z) := \sum_{n=0}^{\infty} L_n(x)z^n = \frac{2 - p(x)z}{1 - p(x)z - q(x)z^2}\ . \tag{2}$$

Note that for particular values of p and q, the $(p, q)-$polynomial $L_n(x)$ leads to various polynomials, among those, we list few cases here (see, [16] for more details, also [3]):

(1) For $p(x) = x$ and $q(x) = 1$, we obtain the Lucas polynomials $L_n(x)$.
(2) For $p(x) = 2x$ and $q(x) = 1$, we attain the Pell-Lucas polynomials $Q_n(x)$.
(3) For $p(x) = 1$ and $q(x) = 2x$, we attain the Jacobsthal-Lucas polynomials $j_n(x)$.
(4) For $p(x) = 3x$ and $q(x) = -2$, we attain the Fermat-Lucas polynomials $f_n(x)$.
(5) For $p(x) = 2x$ and $q(x) = -1$, we have the Chebyshev polynomials $T_n(x)$ of the first kind.

We want to remark explicitly that, in [3] Altınkaya and S. Yalçin, first introduced a subclass of bi-univalent functions by using the $(p,q)-$Lucas polynomials. This methodology builds a bridge between the Theory of Geometric Functions and that of Special Functions, which are known as different areas. Thus, we aim to introduce several new classes of bi-univalent functions defined through the $(p,q)-$Lucas polynomials. Furthermore, we derive coefficient estimates and Fekete-Szegö inequalities for functions defined in those classes.

2. Coefficient Estimates and Fekete-Szegö Inequalities

In this section, we introduce three new subclasses $\mathcal{S}^*_\Sigma(\alpha,x), \mathcal{M}_\Sigma(\alpha,x), \mathcal{L}_\Sigma(\alpha,x)$ of the bi-univalent function class Σ.

A function $f\in\Sigma$ of the form (1) belongs to the class $\mathcal{S}^*_\Sigma(\alpha,x)$, $\alpha\geq 0$ and $z,w\in\Delta$, if the following conditions are satisfied:

$$\frac{zf'(z)}{f(z)}+\alpha\frac{z^2f''(z)}{f(z)}\prec \mathcal{G}_{L_n(x)}(z)-1$$

and for $g=f^{-1}$

$$\frac{wg'(w)}{g(w)}+\alpha\frac{w^2g''(w)}{g(w)}\prec \mathcal{G}_{L_n(x)}(w)-1.$$

Note that $S^*_\Sigma(x)\equiv\mathcal{S}^*_\Sigma(0,x)$ was introduced and studied by [3].

A function $f\in\Sigma$ of the form (1) belongs to the class $\mathcal{M}_\Sigma(\alpha,x)$, $0\leq\alpha\leq 1$ and $z,w\in\Delta$, if the following conditions are satisfied:

$$(1-\alpha)\frac{zf'(z)}{f(z)}+\alpha\left(1+\frac{zf''(z)}{f'(z)}\right)\prec \mathcal{G}_{L_n(x)}(z)-1$$

and for $g=f^{-1}$

$$(1-\alpha)\frac{wg'(w)}{g(w)}+\alpha\left(1+\frac{wg''(w)}{g'(w)}\right)\prec \mathcal{G}_{L_n(x)}(w)-1.$$

Note that the class $\mathcal{M}_\Sigma(\alpha,x)$, unifies the classes $S^*_\Sigma(x)$ and $K_\Sigma(x)$ like $\mathcal{M}_\Sigma(0,x)\equiv S^*_\Sigma(x)$ and $\mathcal{M}_\Sigma(1,x)\equiv K_\Sigma(x)$. For functions in the class $\mathcal{M}_\Sigma(\alpha,x)$, the following coefficient estimates and Fekete-Szegö inequality are obtained.

Next, a function $f\in\Sigma$ of the form (1) belongs to the class $\mathcal{L}_\Sigma(\alpha,x)$, $0\leq\alpha\leq 1$, and $z,\ w\in\Delta$, if the following conditions are satisfied:

$$\left(\frac{zf'(z)}{f(z)}\right)^{\alpha}\left(1+\frac{zf''(z)}{f'(z)}\right)^{1-\alpha}\prec \mathcal{G}_{L_n(x)}(z)-1$$

and for $g=f^{-1}$

$$\left(\frac{wg'(w)}{g(w)}\right)^{\alpha}\left(1+\frac{wg''(w)}{g'(w)}\right)^{1-\alpha}\prec \mathcal{G}_{L_n(x)}(w)-1.$$

Now, for functions in the classes $\mathcal{S}^*_\Sigma(\alpha,x), \mathcal{M}_\Sigma(0,x), \mathcal{L}_\Sigma(\alpha,x)$, the following coefficient estimates and Fekete-Szegö inequality are obtained.

Theorem 2.1. *Let $f(z)=z+\sum\limits_{n=2}^{\infty}a_nz^n$ be in the class $\mathcal{S}^*_\Sigma(\alpha,x)$. Then*

$$|a_2|\leq\frac{|p(x)|\sqrt{|p(x)|}}{\sqrt{|4\alpha^2p^2(x)+2q(x)(1+2\alpha)^2|}},\qquad |a_3|\leq\frac{|p(x)|}{2+6\alpha}+\frac{p^2(x)}{(1+2\alpha)^2}$$

and for $\nu \in \mathbb{R}$

$$\left|a_3 - \nu a_2^2\right| \leq \begin{cases} \dfrac{|p(x)|}{2+6\alpha}, & |\nu - 1| \leq \dfrac{\left|2\alpha^2 p^2(x) + q(x)(1+2\alpha)^2\right|}{2p^2(x)\,(1+3\alpha)} \\ \dfrac{|p(x)|^3\,|\nu-1|}{\left|4\alpha^2 p^2(x) + 2q(x)(1+2\alpha)^2\right|}, & |\nu - 1| \geq \dfrac{\left|2\alpha^2 p^2(x) + q(x)(1+2\alpha)^2\right|}{2p^2(x)\,(1+3\alpha)} \end{cases}.$$

Proof. Let $f \in \mathcal{S}^*_\Sigma(\alpha, x)$ be given by Taylor-Maclaurin expansion (1). Then, there are two analytic functions u and v such that

$$u(0) = 0, \quad v(0) = 0,$$

$$|u(z)| = \left|u_1 z + u_2 z^2 + \ldots\right| < 1, \quad |v(w)| = \left|v_1 w + v_2 w^2 + \ldots\right| < 1 \quad (\forall\, z,\ w \in \Delta).$$

Hence, we can write

$$\frac{zf'(z)}{f(z)} + \alpha \frac{z^2 f''(z)}{f(z)} = \mathcal{G}_{L_n(x)}(u(z)) - 1$$

and

$$\frac{wg'(w)}{g(w)} + \alpha \frac{w^2 g''(w)}{g(w)} = \mathcal{G}_{L_n(x)}(v(w)) - 1.$$

Or, equivalently,

$$\frac{zf'(z)}{f(z)} + \alpha \frac{z^2 f''(z)}{f(z)} = -1 + L_0(x) + L_1(x)u(z) + L_2(x)[u(z)]^2 + \ldots$$

and

$$\frac{wg'(w)}{g(w)} + \alpha \frac{w^2 g''(w)}{g(w)} = -1 + L_0(x) + L_1(x)v(w) + L_2(x)[v(w)]^2 + \ldots.$$

From the above equalities, we obtain

$$\frac{zf'(z)}{f(z)} + \alpha \frac{z^2 f''(z)}{f(z)} = 1 + L_1(x)u_1 z + [L_1(x)u_2 + L_2(x)u_1^2]z^2 + \ldots \tag{3}$$

and

$$\frac{wg'(w)}{g(w)} + \alpha \frac{w^2 g''(w)}{g(w)} = 1 + L_1(x)v_1 w + [L_1(x)v_2 + L_2(x)v_1^2]w^2 + \ldots. \tag{4}$$

Additionally, it is fairly well known that

$$|u_k| \leq 1, \qquad |v_k| \leq 1 \qquad (k \in \mathbb{N}). \tag{5}$$

Thus upon comparing the corresponding coefficients in (3) and (4), we have

$$(1+2\alpha)\,a_2 = L_1(x)u_1 \tag{6}$$

$$2\,(1+3\alpha)\,a_3 - (1+2\alpha)\,a_2^2 = L_1(x)u_2 + L_2(x)u_1^2 \tag{7}$$

$$-\,(1+2\alpha)\,a_2 = L_1(x)v_1 \tag{8}$$

and

$$(3+10\alpha)\,a_2^2 - 2\,(1+3\alpha)\,a_3 = L_1(x)v_2 + L_2(x)v_1^2. \tag{9}$$

From (6) and (8), we can easily see that

$$u_1 = -v_1 \tag{10}$$

and

$$2(1+2\alpha)^2 a_2^2 = [L_1(x)]^2(u_1^2 + v_1^2)$$

$$a_2^2 = \frac{[L_1(x)]^2(u_1^2+v_1^2)}{2(1+2\alpha)^2}\ . \tag{11}$$

If we add (7) to (11), we get

$$2\,(1+4\alpha)\,a_2^2 = L_1(x)(u_2+v_2) + L_2(x)(u_1^2+v_1^2). \tag{12}$$

By substituting (11) in (12), we reduce that

$$a_2^2 = \frac{[L_1(x)]^3\,(u_2+v_2)}{2\,(1+4\alpha)\,[L_1(x)]^2 - 2L_2(x)(1+2\alpha)^2} \tag{13}$$

which yields

$$|a_2| \le \frac{|p(x)|\,\sqrt{|p(x)|}}{\sqrt{|4\alpha^2 p^2(x) + 2q(x)(1+2\alpha)^2|}}.$$

By subtracting (9) from (7) and in view of (10), we obtain

$$\begin{aligned} 4(1+3\alpha)a_3 - 4(1+3\alpha)a_2^2 &= L_1(x)\,(u_2-v_2) + L_2(x)\left(u_1^2 - v_1^2\right) \\ a_3 &= \frac{L_1(x)\,(u_2-v_2)}{4(1+3\alpha)} + a_2^2. \end{aligned} \tag{14}$$

Then in view of (11), (14) becomes

$$a_3 = \frac{L_1(x)\,(u_2-v_2)}{4(1+3\alpha)} + \frac{[L_1(x)]^2(u_1^2+v_1^2)}{2(1+2\alpha)^2}.$$

Applying (5), we deduce that

$$|a_3| \le \frac{|p(x)|}{2+6\alpha} + \frac{p^2(x)}{(1+2\alpha)^2}.$$

From (14), for $\nu \in \mathbb{R}$, we write

$$a_3 - \nu a_2^2 = \frac{L_1(x)\,(u_2-v_2)}{4(1+3\alpha)} + (1-\nu)\,a_2^2. \tag{15}$$

By substituting (13) in (15), we have

$$\begin{aligned} a_3 - \nu a_2^2 &= \frac{L_1(x)\,(u_2-v_2)}{4(1+3\alpha)} + \left(\frac{(1-\nu)\,[L_1(x)]^3\,(u_2+v_2)}{2[(1+4\alpha)\,[L_1(x)]^2 - L_2(x)(1+2\alpha)^2]}\right) \\ &= L_1(x)\left\{\left(\Omega(\nu,\ x) + \frac{1}{4\,(1+3\alpha)}\right)u_2 + \left(\Omega(\nu,\ x) - \frac{1}{4\,(1+3\alpha)}\right)v_2\right\}, \end{aligned} \tag{16}$$

where

$$\Omega(\nu,\ x) = \frac{(1-\nu)\,[L_1(x)]^2}{2\,(1+4\alpha)\,[L_1(x)]^2 - 2L_2(x)(1+2\alpha)^2}.$$

Hence, in view of (5), we conclude that

$$\left|a_3 - \nu a_2^2\right| \le \begin{cases} \dfrac{|L_1(x)|}{2+6\alpha} & ; 0 \le |\Omega(\nu,\ x)| \le \dfrac{1}{4\,(1+3\alpha)} \\ 2\,|L_1(x)|\,|\Omega(\nu,\ x)| & ; |\Omega(\nu,\ x)| \ge \dfrac{1}{4\,(1+3\alpha)} \end{cases},$$

which evidently completes the proof of Theorem 2.1. □

Analysis similar to that in the proof of the previous Theorem shows that

Theorem 2.2. *Let* $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ *be in the class* $\mathcal{M}_\Sigma(\alpha, x)$. *Then*

$$|a_2| \leq \frac{|p(x)|\sqrt{|p(x)|}}{\sqrt{|\alpha(1+\alpha)p^2(x) + 2q(x)(1+\alpha)^2|}}, \qquad |a_3| \leq \frac{|p(x)|}{2+4\alpha} + \frac{p^2(x)}{(1+\alpha)^2}$$

and for $\nu \in \mathbb{R}$

$$\left|a_3 - \nu a_2^2\right| \leq \begin{cases} \dfrac{|p(x)|}{2+4\alpha}, & |\nu - 1| \leq \dfrac{\left|\alpha(1+\alpha)p^2(x) + 2q(x)(1+\alpha)^2\right|}{p^2(x)(2+4\alpha)} \\ \dfrac{|p(x)|^3 |\nu - 1|}{|\alpha(1+\alpha)p^2(x) + 2q(x)(1+\alpha)^2|}, & |\nu - 1| \geq \dfrac{\left|\alpha(1+\alpha)p^2(x) + 2q(x)(1+\alpha)^2\right|}{p^2(x)(2+4\alpha)} \end{cases}.$$

Theorem 2.3. *Let* $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ *be in the class* $\mathcal{L}_\Sigma(\alpha, x)$. *Then*

$$|a_2| \leq \frac{|p(x)|\sqrt{2|p(x)|}}{\sqrt{|(\alpha^2 - 5\alpha + 4)p^2(x) + 4q(x)(2-\alpha)^2|}}, \quad |a_3| \leq \frac{|p(x)|}{6-4\alpha} + \frac{p^2(x)}{(2-\alpha)^2}$$

and for $\nu \in \mathbb{R}$

$$\left|a_3 - \nu a_2^2\right| \leq \begin{cases} \dfrac{|p(x)|}{6-4\alpha}, & |\nu - 1| \leq \dfrac{\left|(\alpha^2 - 5\alpha + 4)p^2(x) + 4q(x)(2-\alpha)^2\right|}{4p^2(x)(3-2\alpha)} \\ \dfrac{2|p(x)|^3 |\nu - 1|}{|(\alpha^2 - 5\alpha + 4)p^2(x) + 4q(x)(2-\alpha)^2|}, & |\nu - 1| \geq \dfrac{\left|(\alpha^2 - 5\alpha + 4)p^2(x) + 4q(x)(2-\alpha)^2\right|}{4p^2(x)(3-2\alpha)} \end{cases}.$$

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Şahsene Altınkaya for the photography and short autobiography, see TWMS J. App. and Eng. Math. V.8, No.1a, 2018.