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SOME RESULTS ON VERTEX-EDGE NEIGHBORHOOD PRIME LABELING

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ABSTRACT. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. For $u \in V(G)$, $N_V(u) = \{w \in V(G)|uw \in E(G)\}$ and $N_E(u) = \{e \in E(G)|e = uv,$ for some $v \in V(G)\}$. A bijective function $f : V(G) \cup E(G) \to \{1, 2, 3, \ldots, |V(G) \cup E(G)|\}$ is said to be a vertex-edge neighborhood prime labeling, if for $u \in V(G)$ with $deg(u) = 1$, gcd $\{f(w), f(uw)|w \in N_V(u)\} = 1$; for $u \in V(G)$ with $deg(u) > 1$, $gcd\{f(w)|w \in N_V(u)\} = 1$ and $gcd\{f(e)|e \in N_E(u)\} = 1$. A graph which admits vertex-edge neighborhood prime labeling is called a vertex-edge neighborhood prime graph. In this paper we investigate vertex-edge neighborhood prime labeling for generalized web graph, generalized web graph without central vertex, splitting graph of path, splitting graph of star, graph obtained by switching of a vertex in path, graph obtained by switching of a vertex in cycle, middle graph of path.

Keywords: Neighborhood-prime labeling, vertex-edge neighborhood prime labeling.

AMS Subject Classification(2010): 05C78

1. Introduction and Definitions

In this paper we consider simple, finite, connected, undirected graph $G = (V(G), E(G))$ with $V(G)$ as vertex set and $E(G)$ as edge set of G respectively. For various notations and terminology of graph theory, we follow Gross and Yellen [3] and for some results of number theory, we follow Burton [2].

Let G be a graph with n vertices. A bijective function $f : V(G) \to \{1, 2, 3, \ldots, n\}$ is said to be a neighborhood-prime labeling if for every vertex u in $V(G)$ with $deg(u) > 1$, gcd $\{f(p)|p \in N(u)\} = 1$, where $N(u) = \{w \in V(G)|uw \in E(G)\}$. A graph which admits neighborhood-prime labeling is called a neighborhood-prime graph.

The notion of neighborhood-prime labeling was introduced by Patel and Shrimali [4]. In [5] they proved union of some grphs are neighborhood-prime graphs. They also proved that product of some graphs are neighborhood-prime [6]. For further list of results regarding neighborhood-prime graph reader may refer [1].

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For a graph G, a bijective function $f : V(G) \cup E(G) \to \{1, 2, 3, \ldots, |V(G) \cup E(G)|\}$ is said to be total neighborhood prime labeling, if for each vertex in G having degree greater than 1, the gcd of the labels of its neighbourhood vertices is 1 and the gcd of the labels of its incident edges is 1. A graph which admits total neighborhood prime labeling is called a total neighborhood prime graph.

Motivated by neighborhood-prime labeling, Rajesh and Methew[8] introduced the total neighborhood prime labeling. In the total neighborhood prime labeling conditions are applied on neighborhood vertices as well as incident edges of each vertex of degree greater than 1. They proved that path, cycle C_{4k} and comb graph admit total neighborhood prime labeling.

In the total neighborhood prime labeling vertex of degree 1 is not considered. Shrimali and pandya[7] extended the condition on vertex of degree 1 and they defined vertex-edge neighborhood prime labeling which is nothing but an extension of total neighborhood prime labeling.

Let G be a graph. For $u \in V(G)$, $N_V(u) = \{w \in V(G)|uw \in E(G)\}$ and $N_E(u) = \{e \in E(G)|e = uv, \text{ for some } v \in V(G)\}$. A bijective function $f : V(G) \cup E(G) \to \{1, 2, 3, \ldots, |V(G) \cup E(G)|\}$ is said to be a vertex-edge neighborhood prime labeling, if (1) for $u \in V(G)$ with $deg(u) = 1$, gcd $\{f(w), f(uw)|w \in N_V(u)\} = 1$; (2) for $u \in V(G)$ with $deg(u) > 1$, $gcd\{f(w)|w \in N_V(u)\} = 1$ and $gcd\{f(e)|e \in N_E(u)\} = 1$. A graph which admits vertex-edge neighborhood prime labeling is called a vertex-edge neighborhood prime graph.

In[7] Shrimali and Pandya proved that path, helm, sunlet, bistar, central edge subdivision of bistar, subdivision of edges of bistar admit vertex-edge neighborhood prime labeling.

A Helm H_n is the graph obtained from the wheel graph $W_n = C_n + K_1$ by attaching a pendent edge to each vertex of cycle in C_n. Apex vertex of helm graph is also known as central vertex.

The generalized web graph $W(t, n)$ is obtained from helm graph H_n by iterating the process of joining the pendent vertices to form a cycle and then adding pendent edges to new cycle, where t is number of copies of cycle C_n. $W_0(t, n)$ is web graph without central vertex, where central vertex of $W(t, n)$ is the same as central vertex of H_n.

For each vertex u of a graph G, take new vertex u'. Join u' to those vertices of G which are adjacent to u. The graph thus obtained is called splitting graph of G and it is denoted by $S'(G)$.

The switching of a vertex v in a graph G means removing all the edges incident to v and adding edges joining to all other vertices which are not adjacent to v in G.

The middle graph $M(G)$ of a connected graph G is the graph whose vertex set is $V(G) \cup E(G)$ where two vertices are adjacent, if

(i) they are adjacent edges of G or

(ii) one is a vertex of G and the other is an edge incident with it.

In this paper, we prove that generalized web graph, generalized web graph without central vertex, splitting graph of path, splitting graph of star, graph obtained by switching of a vertex in path, graph obtained by switching of a vertex in cycle and middle graph of path are vertex-edge neighborhood prime graphs.

2. Main Results

Theorem 2.1. *The generalized web graph $W(t, n)$ is a vertex-edge neighborhood prime graph for $t \geq 2$ and $n \geq 3$.*

Proof. Let $G = W(t,n)$ be a generalized web graph having t copies of cycle C_n. We denote the central vertex of G by u, vertices of j^{th} copy of cycle C_n in G by $u_{1,j}, u_{2,j}, u_{3,j}, \dots, u_{n,j}$ for $1 \le j \le t$. The pendent vertices are denoted by $u_{i,t+1}$ for $1 \le i \le n$. $u_{i,j}$ is adjacent to the vertices $u_{i,j-1}$, $u_{i,j+1}$, $u_{i-1,j}$ and $u_{i+1,j}$ for $2 \le i \le n$ and $2 \le j \le t$ where i is taken modulo n. u is adjacent to the vertices $u_{i,1}$ for $1 \le i \le n$. For each $i \in \{1, 2, ..., n\}$ and $j \in \{1, 2, \dots, t\}$, the edge between $u_{i,j}$ and $u_{i+1,j}$ is denoted by $e_{i,j}$, the edge between $u_{i,j}$ and $u_{i,j+1}$ is denoted by $e'_{i,j+1}$ and the edge between u and $u_{i,1}$ is denoted by $e'_{i,1}$.

Here, $V(G) = \{u_{i,j} / i = 1, 2, \dots, n; j = 1, 2, \dots, t+1\} \cup \{u\}$. So, $|V(G)| = (t+1)n + 1$ and $E(G) = \{e_{i,j} / i = 1, 2, \dots, n; j = 1, 2, .., t\} \cup \{e'_{i,j} / i = 1, 2, \dots, n; j = 1, 2, \dots, t+1\}$. So, $|E(G)| = (2t+1)n$. Therefore, $|V(G) \cup E(G)| = (3t+2)n + 1$.

We define $f : V(G) \cup E(G) \longrightarrow \{1, 2, 3, \dots, |V(G) \cup E(G)|\}$ as follows.
$f(u) = 1$,
$f(u_{1,2j}) = j + 1, \quad 1 \le j \le \lfloor \frac{t+1}{2} \rfloor$,
$f(u_{1,2j-1}) = t + 4 - j, \quad 1 \le j \le \lfloor \frac{t+2}{2} \rfloor$,
$f(u_{i,2j-1}) = (t+2)(i-1) + j + 1, \quad 2 \le i \le n$ and $1 \le j \le \lfloor \frac{t+2}{2} \rfloor$,
$f(u_{i,2j}) = (t+2)i + 2 - j, \quad 2 \le i \le n$ and $1 \le j \le \lfloor \frac{t+1}{2} \rfloor$,
$f(e_{i,j}) = (t+2)n + (n+1)j + i, \quad 1 \le i \le n$ and $1 \le j \le t$,
$f(e'_{i,1}) = (t+2)n + 1 + i, \quad 1 \le i \le n$,
$f(e'_{1,j}) = (t+2)n + (n+1)j, \quad 2 \le j \le t$,
$f(e'_{i,j}) = (2t+3)n + (t-1)(i-1) + j, \quad 2 \le i \le n$ and $2 \le j \le t$,

$$f(e'_{i,t+1}) = \begin{cases} \lfloor \frac{t+1}{2} \rfloor + 2, & i = 1 \\ \lfloor \frac{t}{2} \rfloor + 3 + (t+2)(i-1), & 2 \le i \le n. \end{cases}$$

We consider w as a vertex at each position in a graph G. We will show that each condition for vertex-edge neighborhood prime labeling is satisfied by f.

Case 1: $w = u_{i,t+1}$ for $i = 1, 2, .., n$ with $deg(w) = 1$.
We have to verify that gcd $\{f(u_{i,t}), f(e'_{i,t+1})\} = 1$, where $e'_{i,t+1} = u_{i,t}u_{i,t+1}$.
Take $i = 1$ and $t = 2k$. So we have $w = u_{1,2k+1}$ with $e'_{1,2k+1} = u_{1,2k}u_{1,2k+1}$.
Since $f(u_{1,2k}) = k + 1$ and $f(e'_{1,2k+1}) = k + 2$, gcd $\{f(u_{1,t}), f(e'_{1,t+1})\} = 1$.
If we take $i = 1$ and $t = 2k + 1$ then $f(u_{i,t}) = f(u_{1,2k+1}) = k + 4$ and
$f(e'_{i,t+1}) = f(e'_{1,2k+2}) = k + 3$. Therefore, gcd $\{f(u_{1,t}), f(e'_{1,t+1})\} = 1$.
Similarly we can prove for each i.

Case 2: $w = u_{i,j}$, $j \ne t+1$. We have $deg(w) \ge 2$.
For $w = u_{i,1}$, $u \in N_V(w)$. Since $f(u) = 1$, gcd $\{f(v)/v \in N_V(w)\} = 1$.
For $w = u_{i,j}$ where $j \ne 1$, $\{u_{i,j-1}, u_{i,j+1}\} \subseteq N_V(u_{i,j})$. Since $f(u_{i,j-1})$ and $f(u_{i,j+1})$ are consecutive numbers for every i and j, gcd $\{f(u_{i,j-1}), f(u_{i,j+1})\} = 1$.
For $w = u$, $N_V(u) = \{u_{1,1}, u_{2,1}, u_{3,1}, \dots, u_{n,1}\}$. since $f(u_{1,1})$ and $f(u_{2,1})$ are consecutive numbers, gcd $\{f(v)/v \in N_V(u)\} = 1$.
For $w = u_{i,j}$, $j \ne t+1$, $\{f(e)/e \in N_E(u_{i,j})\}$ contains at least two consecutive numbers. Therefore, gcd $\{f(e)/e \in N_E(u_{i,j})\} = 1$ for every i and j.

Hence, all the conditions are satisfied for vertex-edge neighborhood prime labeling. So, G is a vertex-edge neighborhood prime graph. □

Illustration 2.1 Vertex-edge neighborhood prime labeling of $W(6,6)$ is as shown in Figure 1.

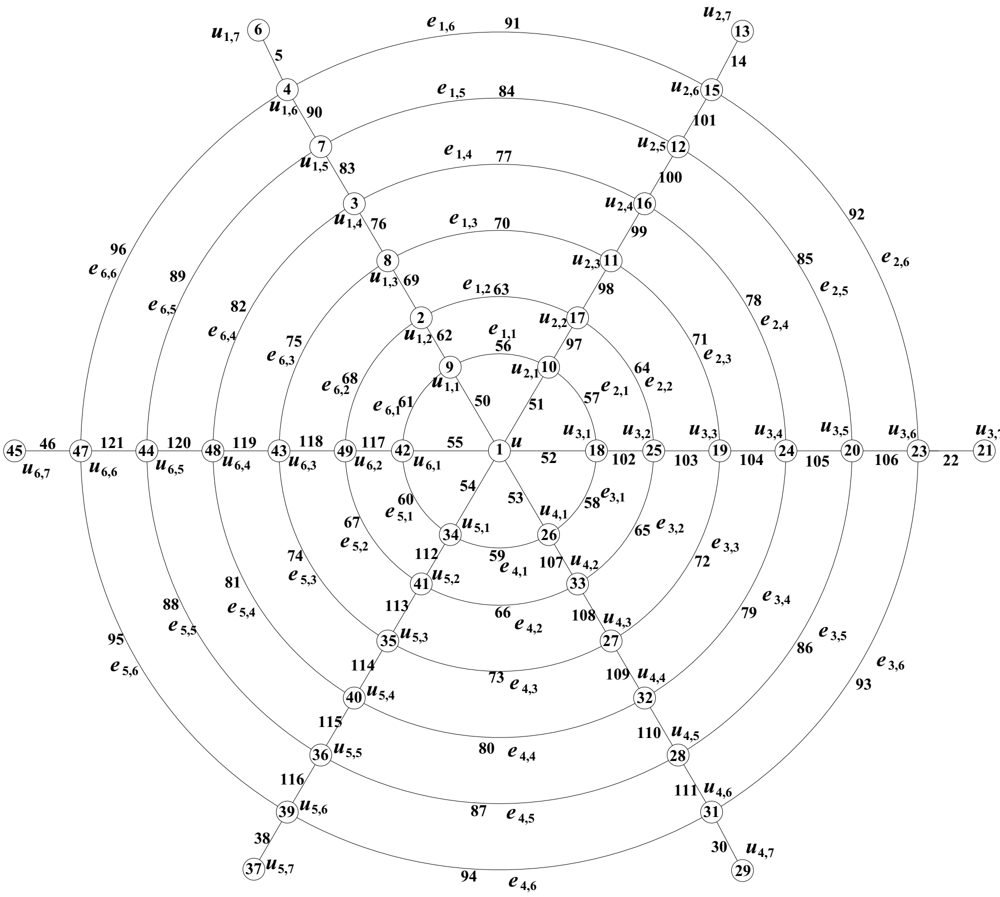

Figure 1: Vertex-edge neighborhood prime labeling of $W(6,6)$.

Theorem 2.2. *The generalized web graph without central vertex $W_0(t,n)$ is vertex-edge neighborhood prime graph for $t \geq 2$ and $n \geq 3$.*

Proof. Let $G = W_0(t,n)$ be a generalized web graph without central vertex having t copies of cycle C_n. We denote vertices of j^{th} copy of cycle C_n in G by $u_{1,j}, u_{2,j}, u_{3,j}, \ldots, u_{n,j}$ for $1 \leq j \leq t$. The pendent vertices are denoted by $u_{i,t+1}$ for $1 \leq i \leq n$. $u_{i,j}$ is adjacent to the vertices $u_{i,j-1}$, $u_{i,j+1}$, $u_{i-1,j}$ and $u_{i+1,j}$ for $2 \leq i \leq n$ and $2 \leq j \leq t$ where i is taken modulo n. For each $i \in \{1,2,...,n\}$ and $j \in \{1,2,\ldots,t\}$, the edge between $u_{i,j}$ and $u_{i+1,j}$ is denoted by $e_{i,j}$ and the edge between $u_{i,j}$ and $u_{i,j+1}$ is denoted by $e'_{i,j}$. Note that the vertices $u_{1,j}$ and $u_{n+1,j}$ are same.

Here, $V(G) = \{u_{i,j} \ / i = 1,2,\ldots,n; j = 1,2,\ldots,t+1\}$. So, $|V(G)| = (t+1)n$ and $E(G) = \{e_{i,j} \ / i = 1,2,\ldots,n; j = 1,2,\ldots,t\} \cup \{e'_{i,j} \ / i = 1,2,\ldots,n; j = 1,2,\ldots,t\}$. So, $|E(G)| = (2t)n$. Therefore, $|V(G) \cup E(G)| = (3t+1)n$.

Now, we define $f : V(G) \cup E(G) \longrightarrow \{1,2,3,\ldots,|V(G) \cup E(G)|\}$ as follows.
$f(u_{i,2j-1}) = (t+2)(i-1)+j, \quad 1 \leq i \leq n$ and $1 \leq j \leq \lfloor \frac{t+2}{2} \rfloor$,
$f(u_{i,2j}) = (t+2)i+1-j, \quad 1 \leq i \leq n$ and $1 \leq j \leq \lfloor \frac{t+1}{2} \rfloor$,
$f(e_{i,j}) = (t+2)n+(n+1)(j-1)+i, \quad 1 \leq i \leq n$ and $1 \leq j \leq t$,
$f(e'_{1,j}) = (t+2)n+(n+1)j, \quad 1 \leq j \leq t-1$,

$f(e'_{i,j}) = (2t+2)n + (t-1)(i-1) + j, \quad 2 \le i \le n$ and $1 \le j \le t-1$,
$f(e'_{i,t}) = -\lfloor \frac{t+1}{2} \rfloor + (t+2)i, \quad 1 \le i \le n.$

Let w be an arbitrary vertex of G. For a vertex w with degree 1, $f(v)$ and $f(vw)$ are either consecutive numbers or consecutive odd numbers, where vw is an incident edge of w. So, gcd $\{f(v), f(vw)\} = 1$. For a vertex w with degree grater than 1, $\{f(v)/v \in N_V(w)\}$ and $\{f(e)/e \in N_E(w)\}$ contain atleast two consecutive numbers or consecutive odd numbers or 1. So, gcd $\{f(v)/v \in N_V(w)\} = 1$ and gcd $\{f(e)/e \in N_E(w)\} = 1$.
Hence, G is vertex-edge neighborhood prime graph. □

Illustration 2.2 Vertex-edge neighborhood prime labeling of $W_0(4,5)$ is as shown in Figure 2.

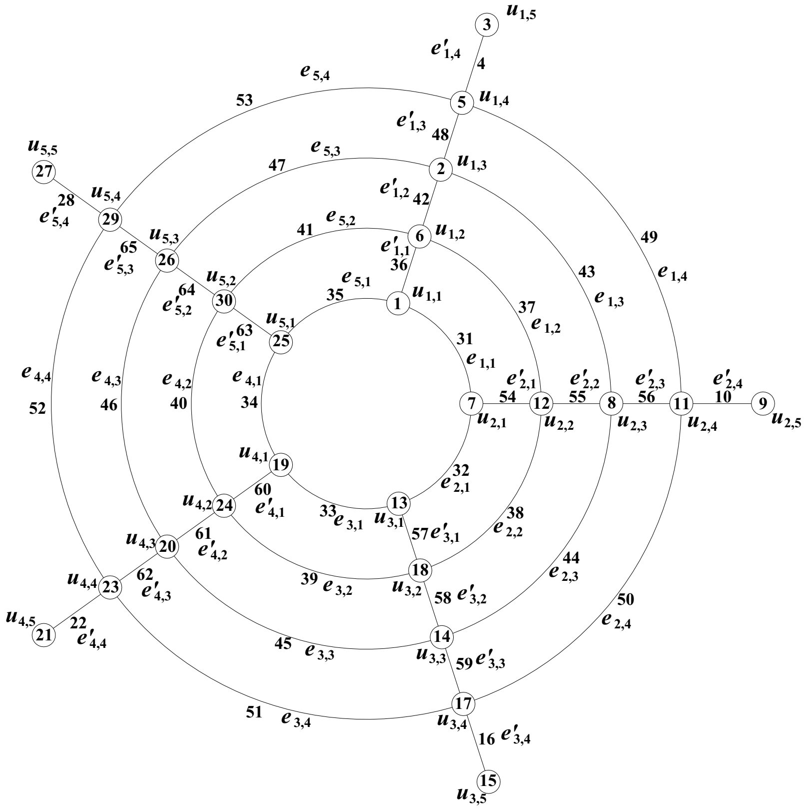

Figure 2: Vertex-edge neighborhood prime labeling of $W_0(4,5)$.

Theorem 2.3. *The splitting graph $S'(P_n)$ of path P_n is a vertex-edge neighborhood prime graph.*

Proof. Let $G = S'(P_n)$ be a splitting graph of P_n. We consider the following two cases.

Case 1: n is even.
Let $u_1, u_2, \ldots, u_n$ be the vertices of path P_n where u_i is adjacent to u_{i-1} and u_{i+1} for $i = 2, 3, \ldots, n-1$. For each $i \in \{1, 2, \ldots, n-1\}$, the edge between u_i and u_{i+1} is denoted by e_i . The duplicate vertex of u_i in a graph G is denoted by u'_i for each i. So, by definition of splitting graph, u'_{2i} is adjacent to u_{2i-1} and u_{2i+1} for $i = 1, 2, \ldots, \lfloor \frac{n-1}{2} \rfloor$. u'_{2i+1} is adjacent to u_{2i} and u_{2i+2} for $i = 1, 2, \ldots, \lceil \frac{n-3}{2} \rceil$. u'_n is adjacent to u_{n-1} and u'_1 is adjacent to u_2. For each i, the edge between u'_{2i-1} and u_{2i} , u'_{2i+1} and u_{2i} , u'_{2i} and u_{2i-1} , u'_{2i} and

u_{2i+1} is denoted by e'_{2i-1}, e'_{2i}, e''_{n-2i+1} and e''_{n-2i} respectively.

Here, $V(G) = \{u_1, u_2, .., u_n\} \cup \{u'_1, u'_2, \dots, u'_n\}$. So, $|V(G)| = 2n$ and
$E(G) = \{e_1, e_2, \dots, e_{n-1}\} \cup \{e'_1, e'_2, \dots, e'_{n-1}\} \cup \{e''_1, e''_2, \dots, e''_{n-1}\}$. So, $|E(G)| = 3(n-1)$.
Therefore, $|V(G) \cup E(G)| = 5n - 3$.

Now, we define $f : V(G) \cup E(G) \longrightarrow \{1, 2, 3, .., |V(G) \cup E(G)|\}$ as follows.

$$f(u_i) = \begin{cases} \dfrac{i}{2}, & i \text{ is even} \\ (n+3) - \left(\frac{i+1}{2}\right), & i \text{ is odd} \end{cases}$$

$$f(u'_i) = \begin{cases} n+2+i, & 1 \le i \le n-2 \\ \frac{n+2}{2}, & i = n-1 \\ 2n+1, & i = n \end{cases}$$

$$f(e_i) = \begin{cases} (4n-1)+i, & 1 \le i \le n-2 \\ 3n+1, & i = n-1. \end{cases}$$

$$f(e'_i) = 2n+1+i, \qquad 1 \le i \le n-1,$$

$$f(e''_i) = \begin{cases} \frac{n+4}{2}, & i = 1 \\ 3n+i, & 2 \le i \le n-1 \end{cases}$$

Case 2: n is odd.
We use same notations for vertices of path P_n, duplicate vertices and edge between u_i and u_{i+1} as in case 1. The adjacency between two vertices is also same as in case 1. The edges between u'_{2i-1} and u_{2i} , u'_{2i+1} and u_{2i} , u'_{2i} and u_{2i-1} , u'_{2i} and u_{2i+1} are denoted by e'_{2i-1}, e'_{2i}, e''_{2i-1} and e''_{2i} respectively.

Now, we define $f : V(G) \cup E(G) \longrightarrow \{1, 2, 3, .., |V(G) \cup E(G)|\}$ as follows.

$$f(u_i) = \begin{cases} n+3-\frac{i}{2}, & i \text{ is even}, i \neq n-1 \\ \frac{i+1}{2}, & i \text{ is odd} \end{cases}$$

$$f(u_{n-1}) = \begin{cases} \frac{n+5}{2}, & n \equiv 1 (mod 4) \\ \frac{n+7}{2}, & n \equiv 3 (mod 4) \end{cases}$$

$$f(u'_i) = \begin{cases} 2n+2, & i = 1 \\ n+2+i, & 2 \le i \le n-2 \\ \lfloor \frac{n+1}{2} \rfloor + 1, & i = n-1 \\ 2n+1, & i = n \end{cases}$$

$$f(e_i) = \begin{cases} 2n+3, & i = 1 \\ 4n+2-i, & 2 \le i \le n-1. \end{cases}$$

$$f(e'_i) = \begin{cases} n+3, & i = 1 \\ 4n-1+i, & 2 \le i \le n-2 \end{cases}$$

$$f(e'_{n-1}) = \begin{cases} \lfloor \frac{n+1}{2} \rfloor + 3, & n \equiv 1 (mod 4) \\ \lfloor \frac{n+1}{2} \rfloor + 2, & n \equiv 3 (mod 4) \end{cases}$$

$$f(e''_i) = 2n+3+i, \qquad 1 \le i \le n-1,$$

Let w be an arbitrary vertex of G. For a vertex w with degree 1, $f(v)$ and $f(vw)$ are either consecutive numbers or consecutive odd numbers or one of them is 1, where vw is an incident edge of w. So, gcd $\{f(v), f(vw)\} = 1$. For a vertex w with degree grater than 1, $\{f(v)/v \in N_V(w)\}$ and $\{f(e)/e \in N_E(w)\}$ contain at least two consecutive numbers or consecutive odd numbers or 1. So, gcd $\{f(v)/v \in N_V(w)\} = 1$ and gcd

$\{f(e)/e \in N_E(w)\} = 1$.
Hence, G is vertex-edge neighborhood prime graph. □

Illustration 2.3 Vertex-edge neighborhood prime labeling of $S'(P_8)$ is shown in Figure 3.

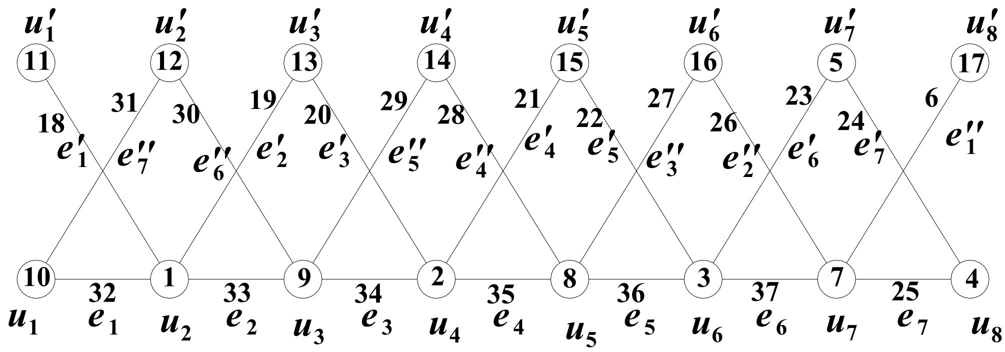

Figure 3: Vertex-edge neighborhood prime labeling of $S'(P_8)$.

Theorem 2.4. *The splitting graph $S'(K_{1,n})$ of star graph $K_{1,n}$ is a vertex-edge neighborhood prime graph.*

Proof. Let G be a splitting graph $S'(K_{1,n})$ of star graph $K_{1,n}$. We describe the graph G as follows.
In a graph G we denote the apex vertex of $K_{1,n}$ by u and pendent vertices of $K_{1,n}$ by $u_1, u_2, \ldots, u_n$. Duplicate vertices of apex and pendent vertices are denoted by u' and u'_i respectively for each i. So, by definition of splitting graph, u is adjacent to the vertices $u'_1, u'_2, \ldots, u'_n$ and u' is adjacent to $u_1, u_2, \ldots, u_n$. For each i, we denote edge between u and u'_i, u and u_i, u' and u_i, by e_i e'_i and e''_i respectively. Note that u'_1, u'_2,…,u'_n are the vertices with degree 1.

Here, $V(G) = \{u_1, u_2, \ldots, u_n\} \cup \{u'_1, u'_2, \ldots, u'_n\} \cup \{u, u'\}$ So, $|V(G)| = 2(n+1)$and
$E(G) = \{e_1, e_2, \ldots, e_n\} \cup \{e'_1, e'_2, \ldots, e'_n\} \cup \{e''_1, e''_2, \ldots, e''_n\}$. So, $|E(G)| = 3n$.
Therefore, $|V(G) \cup E(G)| = 5n + 2$.

Now, we define $f : V(G) \cup E(G) \longrightarrow \{1, 2, 3, \ldots, |V(G) \cup E(G)|\}$ as follows.
$f(u) = 1$,
$f(u') = 2$,
$f(u_i) = 2 + i, \quad 1 \leq i \leq n$,
$f(u'_i) = 3n + 2 + i, \quad 1 \leq i \leq n$,
$f(e_i) = 4n + 2 + i, \quad 1 \leq i \leq n$.

$$f(e'_i) = \begin{cases} n + 2 + 2i, & i \text{ is odd} \\ n + 1 + 2i, & i \text{ is even} \end{cases}$$

$$f(e''_i) = \begin{cases} n + 1 + 2i, & i \text{ is odd} \\ n + 2 + 2i, & i \text{ is even} \end{cases}$$

Let w be an arbitrary vertex of G. For every vertex w with degree 1, u is an adjacent vertex to w. Since $f(u) = 1$, gcd $\{f(u), f(uw)\} = 1$. For a vertex w with degree grater than 1, $\{f(v)/v \in N_V(w)\}$ and $\{f(e)/e \in N_E(w)\}$ contain atleast two consecutive numbers. So, gcd $\{f(v)/v \in N_V(w)\} = 1$ and gcd $\{f(e)/e \in N_E(w)\} = 1$.
Hence, G is vertex-edge neighborhood prime graph. □

Illustration 2.4 Vertex-edge neighborhood prime labeling of $S'(K_{1,4})$ is shown in Figure 4.

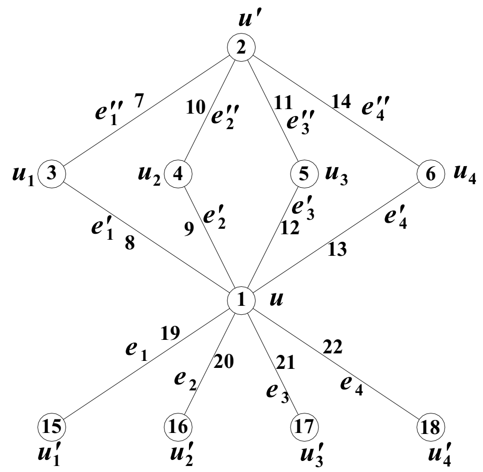

Figure 4: Vertex-edge neighborhood prime labeling of $S'(K_{1,4})$.

Theorem 2.5. *The graph G obtained by switching of an end vertex in path P_n is a vertex-edge neighborhood prime graph.*

Proof. Let u_1,u_2,...,u_n be the consecutive vertices of path P_n and G be a graph obtained by switching of the vertex u_1 in path P_n. So, by definition of switching of a vertex, u_1 is adjacent to $u_3, u_4, \ldots, u_n$. u_i is adjacent to u_{i-1} and u_{i+1} for $i = 3, 4, \ldots, n-1$. For each $i \in \{2, 3, \ldots, n-1\}$, the edge between u_i and u_{i+1} is denoted by e_{i-1} and for each $i \in \{3, 4, \ldots, n\}$, the edge between u_1 and u_i is denoted by e'_{i-2}. Note that u_2 is the only vertex with degree 1.

Here, $V(G) = \{u_1, u_2, \ldots, u_n\}$ So, $|V(G)| = n$ and
$E(G) = \{e_1, e_2, \ldots, e_{n-2}\} \cup \{e'_1, e'_2, \ldots, e'_{n-2}\}$. So, $|E(G)| = 2(n-2)$.
Therefore, $|V(G) \cup E(G)| = 3n - 4$.

Now, we define $f : V(G) \cup E(G) \longrightarrow \{1, 2, 3, \ldots, |V(G) \cup E(G)|\}$ as follows.

$$f(u_i) = \begin{cases} i, & i = 1, 2 \\ i+1, & 3 \leq i \leq n \end{cases}$$

$$f(e_i) = \begin{cases} 3, & i = 1 \\ 3n - 1 - 2i, & 2 \leq i \leq n-2 \end{cases}$$

$$f(e'_i) = 3n - 2 - 2i, \qquad 1 \leq i \leq n-2.$$

Let w be an arbitrary vertex of G. For a vertex w with degree 1, $f(v)$ and $f(vw)$ are consecutive numbers, where vw is an incident edge of w. So, gcd $\{f(v), f(vw)\} = 1$. For a vertex w with degree grater than 1, $\{f(v)/v \in N_V(w)\}$ and $\{f(e)/e \in N_E(w)\}$ contain atleast two consecutive numbers or consecutive odd numbers or 1. So, the conditions are satisfied. Hence, G is vertex-edge neighborhood prime graph. □

Illustration 2.5 Vertex-edge neighborhood prime labeling of the graph obtained by switching of an end vertex in a path P_7 is shown in Figure 5.

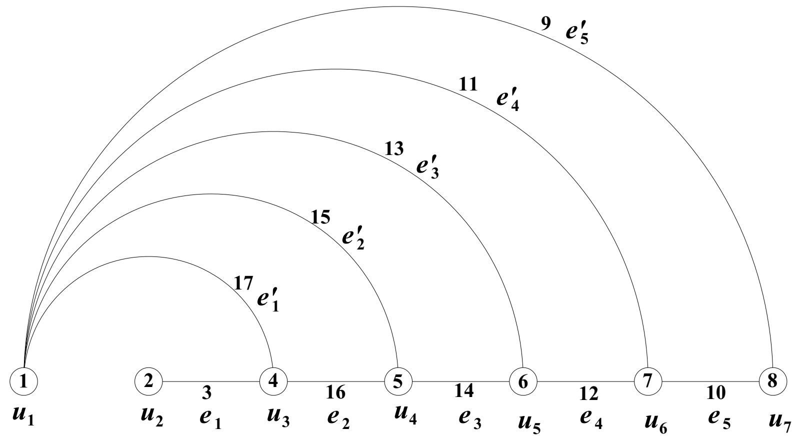

Figure 5: Vertex-edge neighborhood prime labeling of the graph obtained by switching of an end vertex in a path P_7.

Theorem 2.6. *The graph G obtained by switching of a vertex in cycle C_n is vertex-edge neighborhood prime graph.*

Proof. Let G be a graph obtained by switching of a vertex in cycle C_n. Denote the consecutive vertices of cycle C_n by u_1,u_2,...,u_n. Without loss of generality, we consider graph G by switching the vertex u_1 in cycle C_n. So by definition of switching of a vertex, u_1 adjacent to $u_3, u_4, \dots, u_{n-1}$. Also u_i is adjacent u_{i-1} and u_{i+1} for $i = 3, 4, \dots, n-1$. For each $i \in \{2, 3, \dots, n-1\}$, the edge between u_i and u_{i+1} by e_{i-1} and for each $i \in \{3, 4, \dots, n-1\}$, the edge between u_1 and u_i is denoted by e'_{i-2}.

Here, $V(G) = \{u_1, u_2, \dots, u_n\}$ So, $|V(G)| = n$ and $E(G) = \{e_1, e_2, \dots, e_{n-2}\} \cup \{e'_1, e'_2, \dots, e'_{n-3}\}$.
So, $|E(G)| = 2n - 5$.
Therefore, $|V(G) \cup E(G)| = 3n - 5$.

Now, we define $f : V(G) \cup E(G) \longrightarrow \{1, 2, 3, \dots, |V(G) \cup E(G)|\}$ as follows.

$$f(u_i) = \begin{cases} i, & i = 1, 2 \\ i + 1, & 3 \le i \le n - 1 \end{cases}$$

$$f(u_n) = \begin{cases} n + 1, & n \, is \, odd \\ n + 2, & n \, is \, even \end{cases}$$

$$f(e_i) = \begin{cases} 3, & i = 1 \\ 2n - i, & 2 \le i \le n - 3 \end{cases}$$

$$f(e_{n-2}) = \begin{cases} n + 2, & n \, is \, odd \\ n + 1, & n \, is \, even \end{cases}$$

$$f(e'_i) = 2n - 2 + i, \quad 1 \le i \le n - 3.$$

Let w be an arbitrary vertex of G. For a vertex w with degree 1, $f(v)$ and $f(vw)$ are consecutive numbers, where vw is an incident edge of w. So, gcd $\{f(v), f(vw)\} = 1$. For a vertex w with degree greater than 1, $\{f(v)/v \in N_V(w)\}$ and $\{f(e)/e \in N_E(w)\}$ contain atleast two consecutive numbers or consecutive odd numbers or 1.
So, gcd $\{f(v)/v \in N_V(w)\} = 1$ and gcd $\{f(e)/e \in N_E(w)\} = 1$.
Hence, G is vertex-edge neighborhood prime graph. □

Illustration 2.6 Vertex-edge neighborhood prime labeling of a graph obtained by switching of a vertex in C_8 is shown in Figure 6.

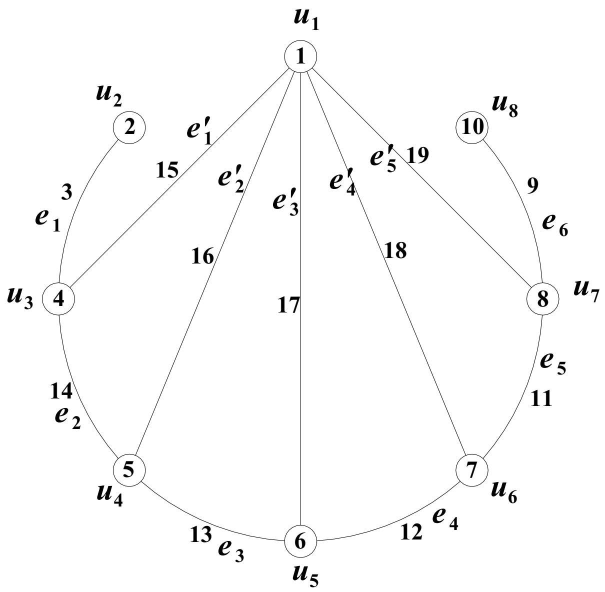

Figure 6: Vertex-edge neighborhood prime labeling of a graph obtained by switching of a vertex in C_8.

Theorem 2.7. *The middle graph $M(P_n)$ of path P_n is a vertex-edge neighborhood prime graph.*

Proof. Let G be a middle graph $M(P_n)$ of path P_n. Let $u_1, u_2, u_3, \ldots, u_n$ be the consecutive vertices of path P_n. Let $v_1, v_2, v_3, \ldots, v_{n-1}$ be added vertices corresponding to the edges $q_1, q_2, q_3, \ldots, q_{n-1}$ of path P_n to obtain middle graph G. So, by definition of middle graph u_i is adjacent to v_{i-1} and v_i for $i = 2, 3, \ldots, n-1$, u_1 is adjacent to v_1 and u_n is adjacent to v_{n-1}. Also v_i is adjacent to v_{i-1} and v_{i+1} for $i = 2, 3, \ldots, n-1$. For each $i \in \{1, 2, ..., n-2\}$, the edge between v_i and v_{i+1} is denoted by e_i. Also for each $i \in \{1, 2, ..., n-1\}$, the edge between v_i and u_{i+1} is denoted by e'_{2i} and the edge between v_i and u_i is denoted by e'_{2i-1}. u_1 and u_n are the only vertices with degree 1.

Here, $V(G) = \{u_1, u_2, u_3, \ldots, u_n\} \cup \{v_1, v_2, v_3, \ldots, v_{n-1}\}$ So, $|V(G)| = 2n - 1$ and $E(G) = \{e_1, e_2, \ldots, e_{n-2}\} \cup \{e'_1, e'_2, \ldots, e'_{2n-2}\}$. So, $|e(G)| = 3n - 4$.
Therefore $|V(G) \cup E(G)| = 5n - 5$.

We define $f : V(G) \cup E(G) \longrightarrow \{1, 2, 3, \ldots, |V(G) \cup E(G)|\}$ as follows.

$$f(u_i) = \begin{cases} 1, & i = 1 \\ 4n - 4 + i, & 2 \leq i \leq n-1 \end{cases}$$

$$f(u_n) = \begin{cases} n+1, & n \text{ is odd} \\ n+2, & n \text{ is even} \end{cases}$$

$$f(v_i) = 2 + i, \quad 1 \leq i \leq n-2,$$

$$f(v_{n-1}) = \begin{cases} n+2, & n \text{ is odd} \\ n+1, & n \text{ is even} \end{cases}$$

$$f(e_i) = 2n + 2 - i, \quad 1 \leq i \leq n-2.$$

$$f(e'_i) = \begin{cases} 2, & i = 1 \\ 2n + i, & 2 \leq i \leq 2n - 3 \end{cases}$$
$f(e'_{2n-2}) = n + 3$.

Let w be an arbitrary vertex of G. For a vertex w with degree 1, $f(v)$ and $f(vw)$ are either consecutive numbers or consecutive odd numbers, where vw is an incident edge of w. So, gcd $\{f(v), f(vw)\} = 1$. For a vertex $w \neq v_{n-1}$ with degree grater than 1, $\{f(v)/v \in N_V(w)\}$ contains atleast two consecutive numbers. For $w = v_{n-1}$, $\{u_n, u_{n-1}, v_{n-2}\} \subseteq N_V(w)$. Since $f(u_{n-1})$ is odd number, $f(v_{n-2})$ and $f(u_n)$ are either consecutive even numbers or consecutive numbers, gcd $\{f(v)/v \in N_V(v_{n-1})\} = 1$ So, gcd $\{f(v)/v \in N_V(w)\} = 1$. Since $\{f(e)/e \in N_E(w)\}$ contains atleast two consecutive numbers, gcd $\{f(e)/e \in N_E(w)\} = 1$.
Hence, G is vertex-edge neighborhood prime graph. □

Illustration 2.7 Vertex-edge neighborhood prime labeling of $M(P_{10})$ is as shown in Figure 7.

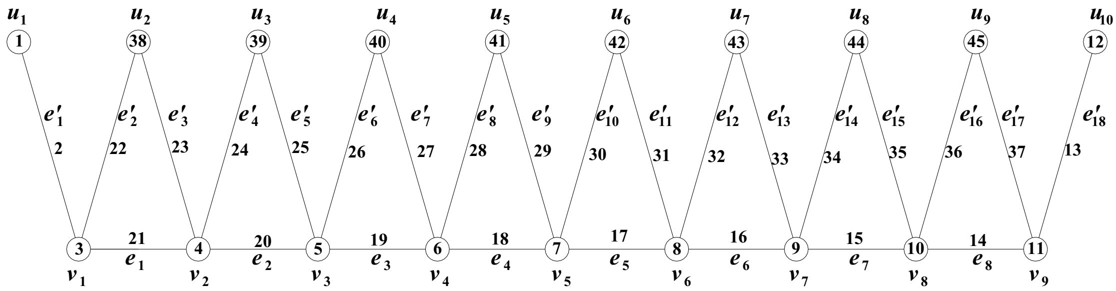

Figure 7: Vertex-edge neighborhood prime labeling of $M(P_{10})$.

3. Conclusions

In this paper we have shown that generalized web graph, generalized web graph without central vertex, splitting graph of path, splitting graph of star, graph obtained by switching of a vertex in path, graph obtained by switching of a vertex in cycle and middle graph of path are vertex-edge neighborhood prime graphs. Analogous results can be obtained for various graphs and graph operations in the context of vertex-edge neighborhood prime labeling.

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