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GENERALIZATION OF NON-COMMUTING GRAPH OF A FINITE GROUP

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ABSTRACT. In this paper we define the generalized non-commuting graph $\Gamma_{(H,K,L)}$ where H, K and L are three subgroups of a non-abelian group G. Take $(H \cup K \cup L) \setminus C_H(K \cup L) \cup C_{K \cup L}(H)$ as the vertices of the graph and two distinct vertices x and y join, whenever x or y is in H and $[x, y] \neq 1$. We obtain diameter and girth of this graph. Also, we discuss the dominating set and planarity of $\Gamma_{(H,K,L)}$. Moreover, we try to find a connection between $\Gamma_{(H,K,L)}$ and the relative commutativity degree of three subgroups $d(H, K \cup L)$.

Keywords: Commutativity degree, non-abelian group, non-commuting graph, dominating set.

AMS Subject Classification: 83-02, 99A00

1. Introduction

The study of algebraic structures by using the properties of graph is an exciting reserch topic in the last twenty years. This fact leads to many fascinating result and questions. There are many papers on assigning a graph to a ring or group and investigation of algebraic properties of ring or group using the associated graph, for instance see [1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 15, 18, 22, 24, 25, 26, 27, 29].

A simple graph Γ_G is associated with a group G, whose vertex set is $G \setminus Z(G)$ and the edge set is all pairs (x, y), where x and y are distinct non-central elements such that $[x, y] = x^{-1}y^{-1}xy \neq 1$. The non-commuting graph of G was introduced by Erdös. He asked whether there is a finite bound for the cardinalities of clique in Γ_G, if Γ_G has no infinite clique. This peoblem was posed by Neumann in [23] and a positive answer was given to Erdös's question. Later, many similar researches about this graph have been done by authors which some of them related to the work given by Neumann in [23]. Of course, there are some other ways to construct a graph associated with a given group or semigroup. We may refer to the works of Bertram et at. [9], Grunewald et at. [19], Moghadamfar et at. [21], Abdollahi et at. [1], and Williams [30], or recent papers on the relative non-commuting graph, Erfanian et at. [16].

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In this paper, we introduce the generalized non-commuting graph $\Gamma_{(H,K,L)}$. We state some of the basic graph theoretical properties of $\Gamma_{(H,K,L)}$ which are mostly new or a generalization of some results in [16]. For instance, determining diameter, dominating set, domination number and planarity of $\Gamma_{(H,K,L)}$. The third section aims to state a connection between the generalized non-commuting graph and the commutativity degree. We will present a formula for the number of edges of $\Gamma_{(H,K,L)}$ in terms of $d(H)$ and $d(H, K \cup L)$ which $d(H) = \frac{1}{|H|^2}|\{(x,y) \in H \times H : [x,y] = 1\}|$ and $d(H, K \cup L) = \frac{1}{|H||K \cup L|}|\{(h,z) \in H \times K \cup L : [h,z] = 1\}|$. Moreover, we observe that the generalized non-commuting star graph exists, although in [16] we see there is no relative non-commuting star graph. We also present some conditions that under it the generalized non-commuting $\Gamma_{(H,K,L)}$ is a complete bipartite graph or bipartite graph.

2. The generalized non-commuting graph

In this section, we introduce the generalized non-commuting graph for any non-abelian group G and subgroups H, K, L. Some graph theoretical propertices such as diameter, grith and dominating set will be presented.

Definition 2.1. *Let H, K and L be three subgroups of non-abelian group G. We associate a graph $\Gamma_{(H,K,L)}$ with the subgroups H, K and L as follows: Take $(H \cup K \cup L) \backslash C_H(K \cup L) \cup C_{K \cup L}(H)$ as the vertices of the graph and two distinct vertices x and y are adjacent, whenever x or y is in H and $[x,y] \neq 1$. Graph $\Gamma_{(H,K,L)}$ is called the generalized non-commuting graph of subgroups H, K and L of G.*
It is easy to see that if $L = K$, then generalized non-commuting graph $\Gamma_{(H,K,L)}$ coincides with the generalized non-commuting graph $\Gamma_{(H,K)}$ (see[17]).
If $K = L = G$, then the generalized non-commuting graph $\Gamma_{(H,K,L)}$ coincides with the relative non-commuting graph $\Gamma_{H,G}$ (see[16]).
If $H = K = L = G$, then the generalized non-commuting graph $\Gamma_{(H,K,L)}$ is the non-commuting graph Γ_G (see [1]). Thus we discuss the generalized non-commuting graph such that is dose not coincide with the relative non-commuting graph (see[14]), or non-commuting graph unless we mentioned it in the text. Let us start with the following result about the degree of the vertices. The proof is straightforward so we omit it.

Proposition 2.1. *Suppose $\Gamma_{(H,K,L)}$ is the generalized non-commuting graph of the non-abelian group G and its subgroups H, K and L. Then*
(i) If $x \in H \setminus K \cup L \cup C_H(K \cup L)$, then

$$deg(x) = |H \cup K \cup L| - |C_H(x) \cup C_{K \cup L}(x) \cup C_H(K \cup L)|$$

(ii) If $x \in H \cap (K \cap L)$ then

$$deg(x) = |H \cup K \cup L| - |C_H(x) \cup C_{K \cup L}(x)|$$

(iii) If $x \in K \cup L \setminus H \cup C_{K \cup L}(H)$, then

$$deg(x) = |H| - |C_H(x) \cup C_{H \cap (K \cup L)}(H)|$$

Recall that the diameter of a graph is a greatest distance between any pair of vertices and the grith of a graph is the length of the shortest cycle.

Theorem 2.1. *For given non-commuting group G and its subgroups H, K, L with trivial center, diam $(\Gamma_{(H,K,L)}) \leq 3$. Moreover, girth $(\Gamma_{(H,K,L)}) \leq 4$.*

Proof. Let x be a vertex of $\Gamma_{(H,K,L)}$. If $H \subseteq C_{K\cup L}(x)$, then $x \in C_{K\cup L}(H)$ which is a contradiction. Therefore, $H \nsubseteq C_{K\cup L}(x)$ and similarly $K\cup L \nsubseteq C_{K\cup L}(x)$. Suppose x and y are non-adjacent vertices. Now consider the following three cases:
Case 1. Suppose $x, y \in K\cup L$. There exist vertices $h_1, h_2 \in H$ such that $[x, h_1] \neq 1$ and $[y, h_2] \neq 1$. If x joins h_2 or y joins h_1, then $d(x, y) = 2$. Assume this dose not happen. Therefore, the non-central element h_1h_2 is adjacent to x and y.
Case 2. Let $x \in H$ and $y \in K\cup L$. Since $x \notin C_H(K\cup L)$ and $y \notin C_{K\cup L}(H)$, there exist $z \in K\cup L$ and $h \in H$ such that $[x, z] \neq 1$ and $[y, h] \neq 1$. If x joins h, then $d(x, y) = 2$. Assume they are not adjacent, and z joins to h, then $d(x, y) \leq 3$. If x is not adjacent to h and z dose not joins h, then exists $xh \in H$ such that $[xh, z] \neq 1$ and $[xh, y] \neq 1$. Thus $d(x, y) \leq 3$
Case 3.. If $x, y \in H$, then there exist $z_1, z_2 \in K\cup L$ such that $[x, z_1] \neq 1$ and $[y, z_2] \neq 1$. If x joins z_2 or y meets z_1, then $d(x, y) = 2$. Otherwise the non-central element z_1z_2 is adjacent to x and y, so $d(x, y) = 2$. Consequently, we can say that diam $\Gamma_{(H,K,L)} \leq 3$.
By similar argument, if $x \in K\cup L$ and $y \in H$, then there exist $h \in H$ and $z \in K\cup L$ such that $[x, h] \neq 1$ and $[y, z] \neq 1$. If y joins h, then there is a triangle of the form $\{x, h, y\}$. Assume y dose not join h and is adjacent with z. Thus there is cycle of the form $\{x, y, z, h\}$. Now suppose y and h are not adjacent and h dose not joins z. So there exist $xz \in K\cup L$, $[xz, h] \neq 1$ and $[xz, y] \neq 1$.
Hence, there is a cycle of the form $\{x, y, xz, h\}$ and girth $(\Gamma_{(H,K,L)}) \leq 4$. □

As a consequence of the above theorem we can say that the graph $\Gamma_{H,K,L}$ is connected. Now, let us start discussion about the dominating set of generalized non-commuting graphs. A subset S of a graph is a dominating set if very vertex which is not in S is adjacent to at least one member of S. We should note that the following three propositions are a generalization of some results in [16].

Proposition 2.2. *Let H, K and L be three subgroups of non-abelian group G. If $x \in H$ and $\{x\}$ is a dominating set for $\Gamma_{(H,K,L)}$, then $C_H(K\cup L) \cap C_{K\cup L}(H) = 1$, $x^2 = 1$ and $C_H(x) = \langle x\rangle$ or $\langle x, y\rangle$, where $y \in C_H(K\cup L)$ and $xy \in C_{K\cup L}(H)$.*

Proof. Suppose $1 \neq z \in C_{K\cup L}(H) \cap C_H(K\cup L)$. Thus $[z, h] = 1$ and $[z, m] = 1$ for all $h \in H$ and $m \in K\cup L$. It is clear that $zx \in H$ is a vertex and dose not join x, which is a contradiction. Now assume $x^2 \neq 1$. Then x^{-1} is a vertex which is not adjacent to x, which is a contradiction. If $t \in C_H(x)$ and $t \notin \{1, x\}$, then the vertex t is not adjacent to x, which is a contradiction. □

Proposition 2.3. *Let H, K and L be three subgroups of non-abelian group G and $S \subseteq V(\Gamma_{(H,K,L)})$. Then S is a dominating set for $\Gamma_{(H,K,L)}$ if and only if*

$$C_{K\cup L}(S) \cup C_H(S) \subseteq C_{K\cup L}(H) \cup C_H(K\cup L) \cup S.$$

Proof. Suppose that S is a dominating set. If t is a vertex such that $t \in C_{K\cup L}(S) \cup C_H(s)$, then $t \in C_{K\cup L}(S)$ or $t \in C_H(S)$ which implies $[t, S] = 1$.
As S is dominating set, $t \in S$. If t is not a vertex, then $t \in C_{K\cup L}(H) \cup C_H(K\cup L)$.
Conversely, we suppose on the contrary that S is not a dominating set. Thus there is a vertex $t \notin S$ which is not adjacent to any element of S. Therefore $[t, S] = 1$ and so

$$t \in C_{K\cup L}(S) \cup C_H(S) \subseteq C_{K\cup L}(H) \cup C_H(K\cup L) \cup S.$$

Hence $t \in S$, which is a contradiction. □

Proposition 2.4. *Let H, K, L be subgroups of non-ablian group G, $X = \{h_1, \cdots, h_n\}$ and $Y = \{y_1, \cdots, y_l\}$ are generating sets for H and $K\cup L$, respectively, such that h_ih_j,*

$y_sy_t \notin C_H(K\cup L)\cup C_{K\cup L}(H)$ for $1\le i<j\le n$ and $1\le s<t\le l$. If $X\cap C_H(K\cap L)=\{h_{m+1},\cdots,h_n\}$ and $Y\cap C_{K\cap L}(H)=\{y_{s+1},\cdots,y_t\}$, then

$$S=\{h_1,\cdots,h_m,y_1,\cdots,y_s\}\cup\{h_1h_{m+1},\cdots,h_1h_n,y_1y_{s+1},\cdots,y_1y_t\}$$

is a dominating set for $\Gamma_{(H,K,L)}$.

Proof. Let t be a vertex which dose not belong to S. Consider the following two cases:
Case 1. If $t\in H$, then there exists an element $g\in K\cup L$ such that $g=y_{i_1}^{\alpha_1}\cdots y_{i_m}^{\alpha_L}$, $y_{i_j}\in Y$, α_i are integers with $[t,g]\neq 1$. Thus $[t,y_{i_j}]\neq 1$ for some j, $1\le j\le m$. If $1\le i_j\le s$, then t joins $y_{i_j}\in S$ as required. If y_{i_j} is not a member of S, then $y_1y_{i_j}\in S$ is adjacent to t.
Case 2. If $t\in K\cup L$, then there exists an element $h\in H$ such that $h=h_{i_1}^{\beta_1}\cdots h_{i_m}^{\beta_n}$, $h_{i_j}\in X$, β_i are integers with $[t,h]\neq 1$. Thus $[t,h_{i_j}]\neq 1$ for some $1\le i_j\le n$. If $h_{i_j}\in S$, then result is clear. If h_{i_j} dose not belong to S, then $h_1h_{i_j}\in S$ joins t. □

Two graphs X and Y are isomorphic if there is a bijection, φ say, from $V(X)$ to $V(Y)$ such that x join y in X if and only if $\varphi(x)$ join $\varphi(y)$ in Y. We say that φ is an isomorphism from X to Y. Since φ is a bijection it has an inverse, which is an isomorphism from Y to X. If X and Y are isomorphic, then we write $X\cong Y$.

Theorem 2.2. *Suppose that H,K,L and N are four non-abelian subgroups of non-abelian group G. If $\Gamma_{H,K,L}\cong\Gamma_{N,K,L}$, then $\Gamma_{H\times A,K\times A,L\times A}\cong\Gamma_{N\times B,K\times B,L\times B}$, for any two ablian groups A and B with the same order.*

Proof. First we have $(H\cup K\cup L)\times A=H\times A\cup K\times A\cup L\times A$ and we show that $C_{H\times A}((K\cup L)\times A)\cup C_{(K\cup L)\times A}(H\times A)=(C_H(K\cup L)\ \cup C_{K\cup L}(H))\times A$. Suppose that $(m,a)\in C_{H\times A}((K\cup L)\times A)\cup C_{(K\cup L)\times A}(H\times A)$, then $(m,a)\in C_{H\times A}((K\cup L)\times A)$ or $C_{(K\cup L)\times A}(H\times A)$.
If $(m,a)\in C_{H\times A}((K\cup L)\times A)$, then $a\in A$, $m\in C_H(K\cup L)$, hence $(m,a)\in C_H(K\cup L)\times A$.
If $(m,a)\in C_{(K\cup L)\times A}(H\times A)$, then $m\in C_{(K\cup L)}(H)$, $a\in A$, hence $(m,a)\in C_{K\cup L}(H)\times A$.
So we have $(m,a)\in(C_H(K\cup L)\cup C_{K\cup L}(H))\times A$ and vice versa is also clear.
Now let $\phi:\Gamma_{H,K,L}\longrightarrow\Gamma_{N,K,L}$ be a graph isomorphism and $\psi:A\to B$ be a bijective map. Then it is easy to see that $\rho:\Gamma_{H\times A,K\times A,L\times A}\longrightarrow\Gamma_{N\times B,K\times B,L\times B}$ defined by $(m,a)\longrightarrow(\phi(m),\psi(a))$ is a graph isomorphism.
Let (m_1,a_1) and (m_2,a_2) be two vertices of graph $\Gamma_{H\times A,K\times A,L\times A}$ which are adjacent. Since $[(m_1,a_1),(m_2,a_2)]\neq 1$, so $[m_1,m_2]\neq 1$. Since the ϕ is graph isomorphism it follows that $[\phi(m_1),\phi(m_2)]\neq 1$ and therefore $[(\phi(m_1),\psi(a_1)),(\phi(m_2),\psi(a_2))]\neq 1$.
Hence $(\phi(m_1),\psi(a_1))$ is adjacent to $(\phi(m_2),\psi(a_2))$. □

3. The generalized non-commuting graphs and $d(H,K\cup L)$

For any finite group G, the commutativity degree of G, denoted by $d(G)$ is the probability that two randomly chosen elements of G commute with each other [20]. It can be defined as the following ratio:

$$d(G)=\frac{1}{|G|^2}\mid\{(x,y)\in G\times G:[x,y]=1\}$$

similarly, if H, K and L are three subgroups of G, then the generalized commutativity degree of H, $K\cup L$ in G is defined as follows:

$$d(H,K\cup L)=\frac{1}{|H||K\cup L|}|\{(h,z)\in H\times K\cup L;[h,z]=1\}|.$$

It is clear that if one of H or $K \cup L$ is a central subgroup of G, then $d(H, K \cup L) = 1$ (see[28]).
In this section, we present a formula for the number of edges of the generalized non-commuting graph $\Gamma_{(H,K,L)}$. Consequently we will give an upper bound for $|E(\Gamma_{(H,K,L)})|$.

Proposition 3.1. *Let H, K and L be subgroups of non-abelian group G. Then the number of edges for the generalized non-commuting graph is obtained by* (1)

$$|E(\Gamma_{(H,K,L)})| = |H||K|(1 - d(H, K \cup L)) + \frac{|H|^2}{2}(1 - d(H))$$

$$-\frac{|H \cap (K \cup L)|^2}{2}(1 - d(H \cap (K \cup L)))$$

Proof. It is clear that the number of edges with two ends in H is computed by $\frac{|H|^2}{2}(1 - d(H))$. Furthermore the number of edges with one end in H another in $K \cup L$ is $|H||K| - |H||K|d(H, K \cup L)$. Finally, we should eliminate the edges that have been calculated twice by $\frac{|H \cap (K \cup L)|^2}{2}(1 - d(H \cap (K \cup L)))$, which implies the assertion. □

Example 3.1. *In this example we compute the number of edges for some certain groups.*
(i) Suppose $D_8 = \langle a, b : a^4 = b^2 = 1, a^b = a^{-1}\rangle$ is the dihedral group of order 8, $H = \langle ab\rangle$, $K = \langle b\rangle$ and $L = \langle a\rangle$ are three subgroups of D_8.
Obviously, $V(\Gamma_{(H,K,L)}) = \{a, a^3, b, ab\}$, $d(H) = 1$, $d(H, K \cup L) = \frac{7}{10}$, $|E(\Gamma_{(H,K,L)})| = 3$ and $\Gamma_{(H,K,L)}$ is a bipartite graph.
(ii) Let $S_3 = \{e, (1\ 2), (1\ 3), (2\ 3), (1\ 2\ 3), (1\ 3\ 2)\}$ be the symmetric group of order 6, $H = \{e, (1\ 2)\}$, $K = \{e, (1\ 3)\}$ and $L = \{e, (1\ 2\ 3)\}$ be three subgroups of S_3. Obviously, $V(\Gamma_{H,K,L}) = \{(1\ 2), (1\ 3), (1\ 2\ 3)\}$, $d(H) = 1$, $d(H, K \cup L) = \dfrac{2}{3}$ and $|E(\Gamma_{(H,K,L)})| = 2$. It is clear that $\Gamma_{(H,K,L)}$ is a bipartitr graph.

Proposition 3.2. *Let $\Gamma_{(H,K,L)}$ be the generalized non-commuting graph. Then*

$$|E(\Gamma_{(H,K,L)})| \leq |H|\Big(|K \cup L| + \frac{3}{16}|H| - 1\Big) - |C_H(K \cup L)|(|K \cup L| - 1)$$

Proof. By using Theorem 2.1 in [16] and [20] the assertion is clear. □

Example 3.2. *Let $D_{2n} = \langle a, b : a^n = b^2 = 1, a^b = a^{-1}\rangle$ be the dihedral group of order $2n$, $H = \langle a\rangle$, $k = \langle b\rangle$ and $L = \langle ab\rangle$. If n is an even number, then $|V(\Gamma_{(H,K,L)})| = n$, $deg(a^i) = 2$, $i \neq \frac{n}{2}$, $1 \leq i \leq n-1$ and $deg(b) = deg(ab) = n-2$. Therefore $\Gamma_{(H,K,L)}$ is a bipartite graph.*
Moreover, $d(H, K \cup L) = \frac{n+4}{3n}$ and by proposition 3.1 or by the fact $\Gamma_{(H,K,L)}$ is a bipartite graph it follows that $|E(\Gamma_{(H,K,L)})| = n$.
If n is an odd number, then $|V(\Gamma_{(H,K,L)})| = n+1$. Furthermore $deg(a^i) = 2$, $1 \leq i \leq n-1$ and $deg(b) = deg(ab) = n-1$. Hence $\Gamma_{(H,K,L)}$ is a bipartitr graph. We deduce $d(H, K \cup L) = \frac{n+2}{3n}$ and so $|E(\Gamma_{(H,K,L)})| = n+1$.

Since there is no edge between vertices of $\Gamma_{(H,K,L)}$ which belongs to $K \cup L$, $\Gamma_{(H,K,L)}$ is not complete in general. $\Gamma_{(H,K,L)}$ is a complete graph if and only if $|H| = |K \cup L| = 2$ and generators of H and $K \cup L$ do not commute.
If H is an abelian group, obviously $\Gamma_{(H,K,L)}$ is bipartite. Clearly, if H is an abelion subgroup of G and $C_{K \cup L}(x) = 1$ for all $x \in H \backslash \{1\}$, then $\Gamma_{(H,K,L)}$ is complete bipartite.

Example 3.3. *In this example we present groups such that their associated generalized non-commuting graphs are complete bipartite. Let*

$$H = \{e, (1\ 3\ 4), (1\ 4\ 3)\}, K = \{e, (1\ 2\ 3), (1\ 3\ 2)\}$$

and

$$L = \{e, (1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3)\}$$

are three subgroups of alternating group A_4. It is clear that $V(\Gamma_{(H,K,L)}) = \{(1\ 3\ 4), (1\ 4\ 3), (1\ 2\ 3), (1\ 3\ 2), (1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3)\}$. *Moreover* $d(H) = 1$, $d(H, K \cup L) = \frac{4}{9}$ *and so by Proposition 3.1* $|E(\Gamma_{(H,K,L)})| = 10$. *Thus* $\Gamma_{(H,K,L)}$ *is a bipartite graph.*

Lemma 3.1. *Let H and L be two subgroups of group G. If $H \leq L$, then for every element $x \in L$,*

$$[H : C_H(x)] \leq [L : C_L(x)]$$

Proof. We know that $HC_L(x) \subseteq L$ for all x in L. So, $|HC_L(x)| \leq |L|$ and we have $(|H|\ |C_L(x)|)/(|H \cap C_L(x)|) \leq |L|$ or $|H|/(|H \cap C_L(x)|) \leq |L|/(|C_L(x)|)$. Thus $[H : C_H(x)] \leq [L : C_L(x)]$. □

Theorem 3.1. *Let H and L be two subgroups of group G. If H be subgroup of L, then*

$$d(L) \leq d(H, L) \leq d(H)$$

Proof. We can easily see that

$$\begin{aligned} d(H, L) &= \frac{1}{|H|\ |L|}|\{(h, l) \in H \times L : [h, l] = 1\}| \\ &= \frac{1}{|H|\ |L|}\sum_{l \in L}|\{h \in H : l \in C_L(h)\}| \\ &= \frac{1}{|H|\ |L|}\sum_{l \in L}|C_H(l)| \\ &\geq \frac{1}{|L|^2}\sum_{l \in L}|C_L(l)| = d(L). \end{aligned}$$

by Lemma 3.1 similarly,

$$\begin{aligned} d(H, L) &= \frac{1}{|H|\ |L|}|\{(h, l) \in H \times L : [h, l] = 1\}| \\ &= \frac{1}{|H|\ |L|}\sum_{h \in H}|\{l \in L : h \in C_L(l)\}| \\ &= \frac{1}{|H|\ |L|}\sum_{h \in H}|C_L(h)| \leq \frac{1}{|H|^2}\sum_{h \in H}|C_H(h)| = d(H). \end{aligned}$$

□

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