University of Reading

Modeling bidding competitiveness and position performance in multi-attribute construction auctions

Article

Published Version

Ballesteros-Pérez, P., del Campo-Hitschfeld, M. L., Mora-Melià, D. and Domínguez, D. (2015) Modeling bidding competitiveness and position performance in multi-attribute construction auctions. Operations Research Perspectives, 2. pp. 24-35. ISSN 2214-7160 doi: https://doi.org/10.1016/j.orp.2015.02.001 Available at http://centaur.reading.ac.uk/51062/

To link to this article DOI: http://dx.doi.org/10.1016/j.orp.2015.02.001

Publisher: Elsevier

Operations Research Perspectives 2 (2015) 24–35

Contents lists available at ScienceDirect

Operations Research Perspectives

journal homepage: www.elsevier.com/locate/orp

Modeling bidding competitiveness and position performance in multi-attribute construction auctions

Pablo Ballesteros-Pérez [a,*], Maria Luisa del Campo-Hitschfeld [b,1], Daniel Mora-Melià [a,2], David Domínguez [a,3]

[a] Dpto. de Ingeniería y Gestión de la Construcción, Facultad de Ingeniería. Universidad de Talca, Camino los Niches, km 1. Curicó, Chile

[b] Centro de Sistemas de Ingeniería KIPUS, Facultad de Ingeniería. Universidad de Talca, Camino los Niches, km 1. Curicó, Chile

ARTICLE INFO

Article history:
Received 17 October 2014
Received in revised form
17 December 2014
Accepted 11 February 2015
Available online 25 February 2015

Keywords:
Bidding
Tender
Auction
Competitiveness
Performance
Kumaraswamy

ABSTRACT

Currently, multi-attribute auctions are becoming widespread awarding mechanisms for contracts in construction, and in these auctions, criteria other than price are taken into account for ranking bidder proposals. Therefore, being the lowest-price bidder is no longer a guarantee of being awarded, thus increasing the importance of measuring any bidder's performance when not only the first position (lowest price) matters.

Modeling position performance allows a tender manager to calculate the probability curves related to the more likely positions to be occupied by any bidder who enters a competitive auction irrespective of the actual number of future participating bidders.

This paper details a practical methodology based on simple statistical calculations for modeling the performance of a single bidder or a group of bidders, constituting a useful resource for analyzing one's own success while benchmarking potential bidding competitors.

1. Introduction

The procurement process in the construction context is characterized by contractors that usually bid short-term project contracts rather than longer-term supply chain contracts [1]. In addition, the unique project delivery system constitutes another founding stone of this industry [2]; therefore, the supply chain in construction is disaggregated and distinguished by a collection of large and small firms, related bulk material suppliers, and many other support professionals [3]. In this context, the supply chain for a construction project generally encompasses architects and engineers, prime and specialty subcontractors, and material suppliers characterized by adversarial short-term relationships and driven by the competitive bidding process in which the "low bid wins" has been the dominant pricing model for many years [3].

In this sense, in 1974, Pim implied that any bidder who faces an auction against other $N-1$ competitors should expect a $1/N$ probability value of being the lowest bidder [4]. For obvious reasons, Pim's model was named the "equal probability model" [5]; however, this model did not take into consideration two major issues. First, there are usually bidders who outperform others, i.e., not all bidders can be equally successful when competing simultaneously under the same tender; otherwise, there would not be a winner (Pim's model therefore produces results that are only valid on average). Second, the number of bidders N is not generally known before the tender reaches its deadline, so the probability value $1/N$, despite being extremely simple, cannot be calculated either.

Of course, other bid tender forecasting models appeared (e.g., Carr (1982); Friedman (1956); Gates (1967); Skitmore (1991); and Wade and Harris (1976) [6–10] to cite some of the most representative) that solved, at least partially, these two major disadvantages of Pim's model at the expense of adding additional hypotheses and requiring more elaborated calculations. In fact, since then, Pim's model has always been used as a mere "control model".

Nevertheless, it remains unclear how well any company or bidder performs concerning its economic bids [11] and, in particular, how effective it is when compared to its competitors, especially in multi-attribute auctions in which other awarding criteria apart from the price are considered [12,13]. Hence, economic positions

* Corresponding author. Tel.: +56 75 2201734; fax: +56 75 325958.
E-mail addresses: pballesteros@utalca.cl, pabbalpe@hotmail.com (P. Ballesteros-Pérez), mdelcampo@utalca.cl, mdelcampo@kipus.cl (M.L. del Campo-Hitschfeld), damora@utalca.cl (D. Mora-Melià), ddominguez@utalca.cl (D. Domínguez).

1 Tel.: +56 75 2201767; fax: +56 75 325958.

2 Tel.: +56 75 2201786; fax: +56 75 325958.

3 Tel.: +56 75 2201797; fax: +56 75 325958.

http://dx.doi.org/10.1016/j.orp.2015.02.001

other than the first (lowest bid) can eventually win when taking into account the technical score. In this connection, multi-attribute auctions, due to their non-price criteria, have been proven to increase project success considering the whole project cycle [14,15], a fact that will undoubtedly encourage their use.

Initially, an alert reader might think that trying to model any bidder's performance would be as simple as calculating a relative frequency curve that would describe how often this bidder ends up being the first (lowest), second (second lowest), third… and so on, but in real-life situations, there are usually an insufficient number of previous encounters among bidders for these probability values to be calculated with any representativeness and/or accuracy [16]. Therefore, this straightforward approach is not generally feasible, and an alternative is required.

Concerning the importance of describing a bidder's position performance, it is worth highlighting that the term "performance" is far more complex than a Win/No Win ratio [17]. Instead, the concept of performance is directly related to how often any bidder reaches high positions. For example, a bidder that was repeatedly second when competing against 30 bidders would most likely have higher chances of being first in future tenders if it competes against only five bidders. However, assuming similarity between tenders, would it be able to beat another bidder that repeatedly occupied the first position competing against five bidders? These and other insights can be discovered by using a position performance approach without the need of complicated statistical procedures.

The importance of using the method suggested above lies in the fact that any bidder who needs to know how effective he or she is when competing against others will always need a framework with which to compare performances with those of rivals [18], and this can only be effectively achieved by using a quantitative and objective approach that, to the best of our knowledge, has not been proposed within the bidding literature to date.

This paper is organized as follows: Section 2 reviews the short literature on bidding performance, and then it presents the actual construction tender dataset that will be used as an example. Finally, it devotes the third subsection to outlining the methodology proposed for calculating a bidder's position performance. Section 3 develops calculations by means of a real case study, taking advantage of the tender dataset introduced in Section 2.2. Section 4 presents the major results including a validation subsection, and Sections 5 and 6 present the discussion addressing the mathematical limitations of the methodology and the conclusions, respectively.

2. Materials and methods

2.1. Literature review

Bidding performance concerns the relationship between bids submitted by different bidders in a competition [19]. Currently, as a likely consequence of the near global economic slowdown and construction demand shrinkage, the internationalization of construction companies has become of significant interest [20], and this jump into the international market forces firms to take part in foreign countries' bidding processes, multilateral funds and overseas tenders [21]. As a consequence, to beat other local and foreign competitors, a culture that enhances a company's competitiveness and performance becomes vital for success [22].

Similarly, predictive information concerning the competitiveness of contractors is a potentially valuable asset for many decision makers involved in the construction procurement process [23]. For instance, it is frequently stated that "the resulting fierce competition for jobs forces construction companies to look for more sophisticated analytical tools to analyze and improve their bidding strategies" [17]; this leads to the conclusion that "[construction] managers need statistical estimation techniques for effectively mining data generated by auctions to predict future behavior and to dynamically improve operational decisions" [24].

One approach to acquiring competitiveness information is by monitoring past bidding behavior [23], but this seems to be done rather subjectively in the construction setting [25], in contrast to other industries in which there seems to be a more structured monitoring [26], in particular in terms of innovative approaches to procurement (e.g., online auctions, dynamic bidding models, combinatorial auctions, sequential markets, e-marketplaces).

In a more general construction context, scattered efforts have been made to develop conceptual frameworks for assessing and comparing construction company's performance [27].

Obviously, this research gap also encompasses the lack of frameworks for bidding performance [28–30], for which only sporadic studies have appeared, most of them related to bidding accuracy, namely, cost estimating accuracy [31,32]. The scarce number of measurements for bidding competitiveness in the literature proposes indices that describe how close each bidder *i*'s bid (b_i) was to the lowest bidder's bid ($b_{\min}$) in a particular auction, for instance [19]:

$$C = \frac{b_i - b_{\min}}{b_{\min}} \tag{1}$$

where C is the measure of competitiveness and ranges from 0 (maximum competitiveness, when $b_i = b_{\min}$) to $+\infty$ (ideally, when $b_i = \infty$ or is infinitely expensive).

However, concerning competitiveness in bidding, paradoxically, a significant amount of research has been published linking the size of the bidder and the size of the contracts, i.e., proving that there are usually some affinities between them [19,33].

On the other hand, Data Envelopment Analysis (DEA), a non-parametric method for the estimation of production frontiers, has begun to be used to gain insight into bidders' comparative performances. This approach was first used in 2005 to develop a contractor prequalification system aiming to assist auctioneers in tenders to select the best contractors, as well as to inform contractors concerning their performance providing guidance for future improvement [34]. Five years later, another study stated that the best bids/candidates in the selection process are usually located on the DEA frontier, an outcome that has immediate applications regarding bid/no-bid decisions [35].

Particularly, the present work differs from these two studies on DEA in terms of how the concept "performance" is applied and to what end within a construction contract. Namely, "performance" in these works is conceived as how effectively the bidders carry out a contract when awarded, a measurement that can be used later by the auctioneer to rate future bidding proposals and to compare them, which definitively has nothing to do with analyzing how likely it is that each potential bidder will occupy a given position when competing against others, the main goal of the present study.

On the other hand, quite recently, Wang et al. [36] developed a Revenue/Cost Analysis Model for competitive bidding strategy planning. This approach used Price/Performance analysis models (P/PAM), marginal utility functions, and profit function to form a new method for planning the bidding strategy of maximum expected profit while trying to take into account that the auctioned item generally varies with the price.

Our study can be considered complementary to the one developed by Wang et al. [36], as the latter developed a tool for obtaining the maximum profit when bidding mid-term, but the method itself requires highly processed information that cannot always be derived solely from the application of marginal utility and profit functions, unlike the tool proposed here that can be applied when a short and recent tender dataset involving previous auctions from the same owner is available.

Finally, Ballesteros-Pérez et al. [18] devised a methodology for measuring bidding performance applicable for capped tenders (tenders with a maximum price threshold set by the auctioneer). This attempt is the most similar to the model proposed here, but it includes several improvements. First, the current method is valid for virtually any type of auction (capped and uncapped); second, it provides more robust statistical measures concerning the bidding performance and a series of new probability distributions that improve the fitness with construction tendering data; and third, it proposes closed mathematical expressions to calculate the position probability curves of any bidder. Nevertheless, it goes without saying that the first work by Ballesteros-Pérez et al. [18] actually paved the path for this study and even proposed a first expression to calculate group bidding performance, which will also be used here later.

Therefore, although recognition worthy, many of these previous publications did not take into account the way each bidder's performance can be easily described by means of its position probability curve, a calculation that is relatively simple, though not obvious.

In summary, due to the current competitive environment of the construction industry, "companies need to be aware of their performance status as well as their competitors' efficiency levels" [37]. There are multiple recent studies that indicate that past performance conditions not only predict future owner–contractor relationships [38] but also allow bidders who use reinforcement learning bidding strategies to consistently outperform those who do not [39]. This eventually means that bidders are still urged to implement systematic methods of performance assessment far beyond the common Bid/No bid decisions [40] to avoid subjective judgments based on past experience. This paper provides a fresh start when addressing this recurrent problem.

2.2. Tender dataset

The methodology presented later will take advantage of a bid tender dataset already published in 1994 (see Table 1 in Ref. [41]). This dataset contains 51 contracts and was originally donated by a construction company operating in the London area (United Kingdom) from April 1980 to June 1982. The number of bidders (N) for each tender ranges from 3 to 10 participants, and the identities of the actual 93 companies were replaced by a non-correlative random numerical code to preserve confidentiality.

Specifically, this bid tender dataset was chosen for several reasons. First, it is already published, so it does not need to be repeated here. Second, anyone can replicate the results without suspicion of authors' manipulation. Third, the size of the tender dataset is sufficiently large to allow representative performance calculations but not so large that these calculations occupy too much space.

The next logical step was to select the bidder performances to focus on. For illustrative purposes, three out of the 93 bidders were chosen, namely, 55, 152 and 24. They were selected because they reflect very different performance levels: The first exhibited poor performance; the second was an average bidder; and the third was a highly effective bidder. In addition, they also took part in a different set of contracts (each of the 51 contracts boasted a participation of one, two, three or none of these three bidders) and a different number of times (bidder 55 entered 20 contracts, 152 participated in 9 contracts, and 24 took part in 7 contracts).

2.3. Methodology outline

The methodology proposed is valid for nearly any type of bidding process that are currently used in the construction setting, that is, simultaneous and sequential bidding, first- and second-price auctions, sealed and open formats, etc., because the basic assumptions of the model are rather simple and shared among auction formats. Unfortunately, the model also has some drawbacks because to keep it as simple as possible, it was necessary to ignore other more complex (although likely) scenarios such as the existence of cover pricing and non-economic rational bidding. Finally, an additional limitation is the assumption that bidders will behave as they behaved in the past, an issue that can be partially minimized by using only a recent and short timespan when choosing historical bidding data.

Concerning implementation of the methodology, four steps must be sequentially followed:

1. Choose an adequate parameter for measuring bidders' positions. This parameter, which will be named P_{ik}, describes on a scale from 0 to 1 how close bidder i was to the first (lowest) and last (highest) bidders when taking part in auction k.
2. Adjust probability distributions for modeling both the bidder positions (by means of the parameter in step 1) and the number of bidders (N_k) who took part in previous encounters. That is, when modeling a number of auctions, we will have a series of P_{ik} values for each bidder and a series of N_k total participating bidders for all the auctions analyzed. We will fit a different Beta or Kumaraswamy distribution to the P_{ik} values of each bidder, whereas a single Poisson, Normal or Laplace distribution will constitute the best fit for the series of N_k values.
3. Obtain the joint probability distribution from the distributions calculated in step 2 to obtain the unique bidder's position performance probability curve. The position performance probability curve for bidder i describes the probabilities that this bidder i will occupy the first, second, third and so on positions in a future tender assuming that this bidder will perform as it did in the past but without the need of knowing how many bidders will participate in that future tender. In other words, we will cross the probabilities obtained by the distribution that represent the number of bidders N_k (Poisson, Normal or Laplace) with the probabilities of the distribution that models each bidder i's position performance parameter P_{ik} (Beta or Kumaraswamy).
4. Optionally, the three previous steps can be repeated or calculated simultaneously for several bidders, and then the curves are combined into a single group performance curve. This curve will allow knowing how likely it is that at least one of the bidders analyzed as a group occupies the first, second, and so on positions, a very useful result when trying to beat a specific group of key competitors.

These four steps will be progressively explained in detail in Sections 3 and 4.

3. Calculations

This section presents the necessary calculations for the methodology summarized in Section 2.3 to be implemented along with a case study taking advantage of the tender dataset introduced in Section 2.2.

3.1. Position performance parameter selection

A position performance parameter for bidder i in a tender k (P_{ik}) is defined as the coefficient calculated as a function of the position

Table 1
Position performance coefficient comparisons.

Position performance coefficient	Mathematical expression	$j_{ik}=$ 1 Lowest price	2	3 (example $N_k=5$)	4	5 Highest price
P_{ik} (*chosen*)	$(N_k-j_{ik}+0.5)/N_k$	0.90	0.70	0.50	0.30	0.10
P'_{ik}	$(N_k-j_{ik})/(N_k-1)$	1.00	0.75	0.50	0.25	0.00
P''_{ik}	$(N_k-j_{ik}+1)/N_k$	1.00	0.80	0.60	0.40	0.20
P'''_{ik}	$(N_k-j_{ik}+1)/(N_k+1)$	0.83	0.67	0.50	0.33	0.17

Position performance coefficient	Distances between bidders j and $j+1$	Central bidder $j=(N_k+1)/2$ at the midrange (0.5)	Bidder $j_{ik}=1$'s distance with upper bound (1)	Bidder $j_{ik}=N_k$'s distance with lower bound (0)
P_{ik} (*chosen*)	$\mathbf{1/N_k}$	**Yes**	$\mathbf{1/2N_k}$	$\mathbf{1/2N_k}$
P'_{ik}	$1/(N_k-1)$	**Yes**	0	0
P''_{ik}	$\mathbf{1/N_k}$	No	0	$1/N_k$
P'''_{ik}	$1/(N_k+1)$	**Yes**	$\mathbf{1/(N_{k+1})}$	$\mathbf{1/(N_{k+1})}$

that was achieved by the bidder i in tender k (j_{ik}) and the number of total participating bidders in that same tender k (N_k).

In the literature, very few examples of performance parameters can be found, examples being the following four displayed at the top of Table 1, and nearly the only exceptions: P_{ik} [30], P'_{ik} [18], P''_{ik} [42] and P'''_{ik} [43]. In addition, in the upper table, a numerical example for $N_k=5$ bidders is given to illustrate how each of these position performance coefficients calculates the values for a range of different positions j_{ik} from 1 to 5.

The four coefficients share one common feature: They always range from 0 to 1, and the closer this value is to 1, the higher the performance of bidder i was in tender k, i.e., the bidder occupied the first positions. However, they also exhibit important differences highlighted at the bottom of Table 1 (the lower table).

For example, P_{ik} and P''_{ik} keep the value $1/N_k$ as the constant distance that separates bidder j from $j-1$ and $j+1$. This is important because the magnitude $1/N_k$ also reflects the equivalent cumulative probability increment between bids in a tender with N_k bidders.

Second, not all of these position performance coefficients distribute the values symmetrically, i.e., assign a $P_{ik}=0.5$ value to the bidder who occupied the central position (when N_k is an odd number) or to the average of the couple of bidders who were right below and above the ideal intermediate position (when N_k is an even number). Again, this is also important because distributing coefficient values unevenly on the whole coefficient range causes problems when later fitting a probability distribution. In addition, an equivalency between bidders that occupied the same relative positions when competing against different N_k-1 bidders can also be established. For example, it is logical to attribute the same merit to a bidder that ended up being third when competing against five bidders than to a bidder that was second when competing against three. This works for any position j_{ik}, in addition to the central position when values are distributed symmetrically. Only P_{ik}, P'_{ik} and P'''_{ik} grant this condition.

Finally, it is recommended that the position performance coefficient avoids assigning the extreme values, that is, 1 and 0, to the best ($j_{ik}=1$) and worst ($j_{ik}=N_k$) bidders, respectively. This last condition, which is only fulfilled by coefficients P_{ik} and P'''_{ik}, is based upon the fact that any subset of a bidder's coefficient values that are exactly located at the extremes will noticeably worsen the p-value of the statistical distribution because that curve will necessarily have to work within the finite limits 0 and 1 in the X-axis, which are related to the probability values 0 and 1 in the Y-axis, respectively, as well, a fact that is not true when there are several coefficient values located at either $X=P_{ik}=0$ or $X=P_{ik}=1$.

Therefore, only one out of the four coefficients proposed meets the four conditions, the coefficient P_{ik}, and will henceforth represent any bidder's position performance irrespective of the number of bidders N_k.

Finally, it might be worth mentioning that working with the actual positions (j_{ik}) instead of with a from-0-to-1 coefficient is not generally possible because although it would be an option to use another probability distribution that fitted these positions directly from 0 to $+\infty$ (the Weibull distribution, for example), there would not be a sufficient number of previous encounters to fit that PDF accurately. In other words, converting the actual positions occupied into a coefficient as proposed here allows working with a substantially lower amount of data.

3.2. Position performance and number of bidders distribution selection

By using the tender dataset introduced in Section 2.2 with tenders $k=1,\ldots,51$ for bidders 55 ($i=1$), 152 ($i=2$) and 24 ($i=3$), whose respective positions (j_{1k}, j_{2k}, j_{3k}) in those 51 tenders are known, their respective position performance coefficients (P_{1k}, P_{2k}, P_{3k}) can be calculated according to the expression chosen in Table 1 for which the variables were already presented as well:

$$P_{ik}=\frac{N_k-j_{ik}+0.5}{N_k}. \tag{2}$$

If this expression is applied to the 20, 9 and 7 tender bidders that 55, 152 and 24 participated in (T_i values for $i=1,2,3$), respectively, the results can be observed in Table 2.

Furthermore, other calculations are performed at the bottom of Table 2. Namely, right below the N_k values, the total number of tenders (T) as well as their average (μ), standard deviation (σ), median (m) and Laplace's scale parameter (b, see Eq. (6)) values are calculated. These values will be useful for defining the best probability distribution for N_k later.

In addition, at the bottom, taking into account that the number of times each bidder entered a tender was known (T_i) and that the total number of tenders equals 51 (T), the three bidders' Participation Ratios (R_i) can also be calculated by dividing T_i by T.

Finally, the four rows at the bottom of Table 2 calculate the Maximum Likelihood Estimates (MLE) of the two shape parameters for both the Beta distribution (a_i and b_i) and the Kumaraswamy distribution (α_i and β_i) required to fit the series of the P_{ik} values for the three chosen bidders ($i=1,2$ and 3).

Concerning the best distribution to fit these series of P_{ik} values, it is known that the options in terms of the continuous statistical distributions that are supported on a bounded interval ([0, 1] in this case) are very short, the Beta distribution being the premier option [44]. However, the Kumaraswamy distribution is nearly as versatile as the Beta distribution and will also be considered.

The following are quotes from statistical researchers concerning these two options: "The Beta distribution is fairly tractable, but in some ways not fabulously so; in particular, its distribution function is an incomplete beta function ratio and its quantile function

Table 2
Bidders $i = 1, 2$ and 3's coefficients (P_{ik}) and participation ratios (R_i).

Tenders analyzed	Bidder ID ▸		55		152		24	
k	N_k	$\lvert N_k - m\rvert$	j_{1k}	P_{1k}	j_{2k}	P_{2k}	j_{3k}	P_{3k}
1	6	0.0	6	0.083				
2	4	2.0						
3	7	1.0						
4	6	0.0						
5	6	0.0	3	0.583				
6	9	3.0						
7	7	1.0	6	0.214				
8	4	2.0						
9	6	0.0						
10	4	2.0						
11	6	0.0						
12	6	0.0					4	0.417
13	4	2.0	3	0.375				
14	6	0.0						
15	6	0.0	6	0.083				
16	3	3.0						
17	10	4.0			4	0.650		
18	6	0.0	1	0.917				
19	9	3.0	9	0.056				
20	8	2.0	4	0.563	5	0.438		
21	7	1.0	5	0.357				
22	6	0.0						
23	5	1.0			5	0.100		
24	8	2.0			6	0.313		
25	6	0.0	4	0.417	6	0.083		
26	7	1.0	4	0.500			1	0.929
27	4	2.0			2	0.625		
28	7	1.0						
29	6	0.0	2	0.750				
30	6	0.0	5	0.250	2	0.750		
31	6	0.0						
32	6	0.0	6	0.083			1	0.917
33	6	0.0	4	0.417				
34	7	1.0			1	0.929		
35	6	0.0						
36	6	0.0					2	0.750
37	9	3.0						
38	5	1.0						
39	7	1.0						
40	8	2.0						
41	6	0.0					2	0.750
42	8	2.0	6	0.313				
43	6	0.0						
44	5	1.0	2	0.700			2	0.700
45	4	2.0	2	0.625				
46	7	1.0						
47	8	2.0	6	0.313	2	0.813		
48	7	1.0	7	0.071			2	0.786
49	5	1.0						
50	5	1.0						
51	6	0.0						
Total # *tenders* (T)	51	▾ $\sum \lvert N_k - m\rvert$ ▾	Bidder (i)	1		2		3
Average (μ)	6.235	52.000	T_i	20		9		7
Std. deviation (σ)	1.464		$R_i = T_i/T$	0.392		0.176		0.137
Median (m)	6.0		Beta distr. param. (a_i) (MLE)	0.970		0.849		3.847
Laplace scale param. (b)	1.020		Beta distr. param. (b_i) (MLE)	1.775		0.780		1.295
			Kumaraswamy's distribution parameter (α_i) (MLE)	0.859		0.840		3.701
			Kumaraswamy's distribution parameter (β_i) (MLE)	1.411		0.781		1.351

the inverse thereof" [44]. On the other hand: "The Kumaraswamy's distribution is a much simpler option to use especially in simulation studies due to the simple closed form of both its probability density function and cumulative distribution function" [45].

However, despite its use in the hydrological literature, in which it dates back to 1980 [45], "this distribution does not seem to be very familiar to statisticians", and it had not been investigated systematically until 2009, "nor had its relative interchangeability with the Beta distribution been widely appreciated" [44].

Particularly, in the case of this analysis, both the PDF and CDF of the distribution, which better fits the P_{ik} values, will have to be used, so, for the sake of mathematical simplicity, in this instance, the Kumaraswamy distribution will be the last standing option. Additionally, it is worth highlighting that this is possibly the first time that this distribution is applied in a construction management context.

Therefore, Kumaraswamy's distribution, which will henceforth be shortened to "Kum distribution", has the following density (PDF), cumulative (CDF) and quantile functions:

$$f(x) = f(x, \alpha, \beta) = \alpha\beta x^{\alpha-1}(1 - x^{\alpha})^{\beta-1} \quad (3)$$

$$F(x) = F(x, \alpha, \beta) = 1 - (1 - x^{\alpha})^{\beta} \quad (4)$$

$$Q(y) = F^{-1}(y) = \left(1 - (1 - y)^{1/\beta}\right)^{1/\alpha} \quad (5)$$

where $0 \le x = P_{ik} \le 1$ and $0 \le y = \text{Probability} \le 1$.

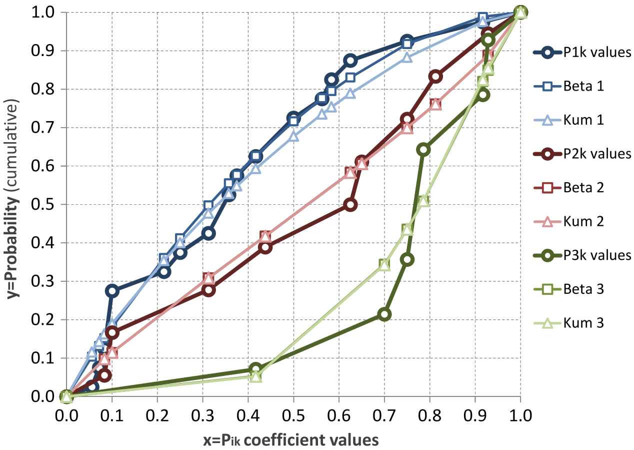

Fig. 1. Representation of the bidders $i = 1$, 2 and 3's P_{ik} curves goodness of fit for the Beta and Kumaraswamy distributions.

If the goodness of fit is obtained for both the Beta and Kum distributions (see Table 3), it can be verified that both provide very good results (p-values $\ll 0.05$ in all cases).

It must also be noted that the degrees of freedom (d.f.) for bidders $i = 1$ and $i = 3$ do not coincide with $T_i - 1$, that is, with 19 and 7, but with 15 and 5 instead, because bidder $i = 1$ had 4 repeated P_{1k} values, whereas bidder $i = 3$ had 2 repeated P_{3k} values, which account for that difference.

Finally, the fit of these two distributions is also displayed in Fig. 1, which shows that the adjustment is very similar for the three bidders' P_{ik} series of data.

Once the more suitable option for describing the three bidders' position performance coefficients (P_{ik}) is calculated, the study can be carried out on the distribution that best fits the variable "number of participating bidders" (N_k), for which statistics (μ, σ, m and b) were already calculated in Table 2.

Regarding this particular variable, there still seems to be no consensus among researchers concerning what probability distribution best fits N_k. One early study [6] suggested that the Poisson distribution might be a logical option; however, many researchers discredited this alternative some years later [46]. Other researchers have implemented the Normal distribution [18] despite the fact that this is indeed a continuous distribution, not a discrete one. The third option and the one chosen here will be the Laplace distribution, another continuous and non-parametric distribution not commonly used in construction management but paradoxically the only one that has the most adequate shape when the number of bidders decrease exponentially around the average N_k value m (this distribution has a "peak" exactly located at the median m and two concave tails at both sides) as displayed in Fig. 2.

The goodness of fit of these three distributions is shown in Table 4, taking into account that both the Normal and Laplace distributions have been converted into discrete distributions by calculating every x value as $F(N_k + 0.5) - F(N_k - 0.5)$.

It is also observed that to seek the best fit possible, Poisson's distribution parameter equaled μ; the Normal distribution used directly μ and σ; whereas the Laplace distribution parameters were the median m (location) and b (scale), all of which are calculated in Table 2. Particularly, the MLE of b equals:

$$b = \frac{1}{T}\sum_{k=1}^{T} |N_k - m| \tag{6}$$

whereas the Laplace Cumulative Distribution Function is:

$$F(x) = \frac{1}{2} + \frac{1}{2}\operatorname{sgn}(x - m)\left(1 - \exp\left(-\frac{|x - m|}{b}\right)\right) \tag{7}$$

where $x = N_k$ and $\operatorname{sgn}(\cdot)$ is the sign function.

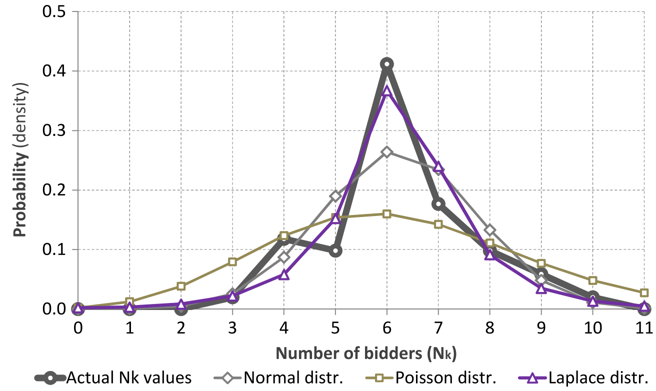

Fig. 2. Representation of the total number of participating bidders' N_k curves for the Normal, Poisson and Laplace distributions.

In all cases, the three distributions fulfilled the condition that their p-values were below 0.05, but the Laplace distribution constituted the best alternative in this occasion.

Finally, it must also be noted that both the Normal and Laplace distributions were discretized before calculating their p-values. As an example, the Discrete Laplace PDF used in Table 4 was calculated as follows:

$$\begin{aligned} &\textit{Discrete Laplace PDF}\ (x, m, b) = \\ &= F(x = N_k + 0.5) - F(x = N_k - 0.5) = \\ &= \frac{1}{2}\operatorname{sgn}(N_k + 0.5 - m)\left(1 - \exp\left(-\frac{|N_k + 0.5 - m|}{b}\right)\right) \\ &\quad - \frac{1}{2}\operatorname{sgn}(N_k - 0.5 - m)\left(1 - \exp\left(-\frac{|N_k - 0.5 - m|}{b}\right)\right) \end{aligned} \tag{8}$$

3.3. Position performance joint probability curve

The next step consists of calculating the position probability curve by taking into account simultaneously the probabilities of the best position performance coefficient and the number of participating bidders, i.e., the Kumaraswamy and Laplace distributions.

To obtain this particular curve, the outcome depends on the number of participating bidders N_k and on how competitive those bidders are. Specifically, if $N_k = 1$ (a situation that happens with a 0.004 probability, see Table 4), that bidder would have had a 100% probability of winning because there would not be another competitor. However, if $N_k = 2$ (a situation that happens with a 0.010 probability), the probability of occupying the first position would be equivalent to the probability that the position performance coefficient for this bidder i, that is, P_{ik}, was approximately located within the interval [0.5, 1] or, in a more general form, between, $\left[\frac{N_k - j_{ik}}{N_k} = \frac{2-1}{2} = 0.5,\ \frac{N_k - j_{ik} + 1}{N_k} = \frac{2-1+1}{2} = 1\right]$, which is indeed the range whose limits coincide with a distance $\frac{0.5}{N_k}$ above and below $P_{ik} = \frac{N_k - j_{ik} + 0.5}{N_k}$. This last probability would be calculated using Kumaraswamy's CDF (see Eq. (4)) and for the range above, as follows:

$$\begin{aligned} &\textit{Discrete Kumaraswamy PDF}\ (x = j_i, \alpha_i, \beta_i) = \\ &= F\left(x = \frac{N_k - j_i + 1}{N_k}, \alpha_i, \beta_i\right) - F\left(x = \frac{N_k - j_i}{N_k}, \alpha_i, \beta_i\right) = \\ &= \left(1 - \left(\frac{N_k - j_i}{N_k}\right)^{\alpha_i}\right)^{\beta_i} - \left(1 - \left(\frac{N_k - j_i + 1}{N_k}\right)^{\alpha_i}\right)^{\beta_i} \end{aligned} \tag{9}$$

Table 3
Bidders $i = 1, 2$ and 3's P_{ik} curves goodness of fit for the Beta and Kumaraswamy's distributions.

Bidder $i = 1$ P_{1k} values	Cumulative probability (*observed*)	Beta CDF (*expected*) $f(P_{1k}; a_1; b_1)$	Beta Chi^2 (*residuals*) (Obs. − Exp.)^2/Exp.	Kum. CDF (*expected*) $f(P_{1k}; \alpha_1; \beta_1)$	Kum Chi^2 (*residuals*) (Obs. − Exp.)^2/Exp.
0.056	0.025	0.104	0.060	0.116	0.071
0.071	0.075	0.132	0.024	0.143	0.032
0.083	0.125	0.152	0.005	0.163	0.009
0.100	0.275	0.181	0.049	0.189	0.039
0.214	0.325	0.361	0.004	0.354	0.002
0.250	0.375	0.412	0.003	0.400	0.002
0.313	0.425	0.498	0.011	0.477	0.006
0.357	0.525	0.555	0.002	0.528	0.000
0.375	0.575	0.577	0.000	0.548	0.001
0.417	0.625	0.626	0.000	0.593	0.002
0.500	0.725	0.717	0.000	0.677	0.003
0.563	0.775	0.777	0.000	0.735	0.002
0.583	0.825	0.795	0.001	0.754	0.007
0.625	0.875	0.830	0.002	0.789	0.009
0.750	0.925	0.918	0.000	0.883	0.002
0.917	0.975	0.988	0.000	0.976	0.000
			0.162	Chi^2(sum)	0.187
			15	d.f.	15
			4.26E−13	*p*-value	**1.26E−12**
Bidder $i = 2$ P_{2k} values	Cumulative probability (*observed*)	Beta CDF (*expected*) $f(P_{2k}; a_2; b_2)$	Beta Chi^2 (*residuals*) (Obs. − Exp.)^2/Exp.	Kum. CDF (*expected*) $f(P_{2k}; \alpha_2; \beta_2)$	Kum Chi^2 (*residuals*) (Obs. − Exp.)^2/Exp.
0.083	0.056	0.098	0.018	0.098	0.019
0.100	0.167	0.114	0.024	0.115	0.023
0.313	0.278	0.309	0.003	0.308	0.003
0.438	0.389	0.418	0.002	0.417	0.002
0.625	0.500	0.584	0.012	0.583	0.012
0.650	0.611	0.606	0.000	0.606	0.000
0.750	0.722	0.700	0.001	0.699	0.001
0.813	0.833	0.761	0.007	0.761	0.007
0.929	0.944	0.888	0.004	0.888	0.004
			0.070	Chi^2(sum)	0.070
			8	d.f.	8
			6.19E−08	*p*-value	**6.11E−08**
Bidder $i = 3$ P_{3k} values	Cumulative probability (*observed*)	Beta CDF (*expected*) $f(P_{3k}; a_3; b_3)$	Beta Chi^2 (*residuals*) (Obs. − Exp.)^2/Exp.	Kum. CDF (*expected*) $f(P_{3k}; \alpha_3; \beta_3)$	Kum Chi^2 (*residuals*) (Obs. − Exp.)^2/Exp.
0.417	0.071	0.053	0.006	0.053	0.007
0.700	0.214	0.344	0.049	0.343	0.048
0.750	0.357	0.436	0.014	0.435	0.014
0.786	0.643	0.509	0.035	0.509	0.035
0.917	0.786	0.820	0.001	0.825	0.002
0.929	0.929	0.850	0.007	0.855	0.006
			0.114	Chi^2(sum)	0.112
			5	d.f.	5
			2.23E−04	*p*-value	**2.16E−04**

Table 4
Total number of participating bidders' N_k probability curves and goodness of fit for the Normal, Poisson and Laplace distributions.

Number of bidders (N_k)	Number of tenders (T_N)	Relative freq. (*observed*) (FqN)	Normal PDF[a] (*expected*) $f(N_k; \mu; \sigma)$	Normal Chi^2 (residuals)	Poisson PDF (*expected*) $f(N_k; \mu)$	Poisson Chi^2 (residuals)	Laplace PDF[a] (*expected*) $f(N_k; m; b)$	Laplace Chi^2 (residuals)
0	0	0.000	0.000	0.000	0.002	0.002	0.002	0.002
1	0	0.000	0.001	0.001	0.012	0.012	0.004	0.004
2	0	0.000	0.005	0.005	0.038	0.038	0.010	0.010
3	1	0.020	0.025	0.001	0.079	0.045	0.027	0.002
4	6	0.118	0.087	0.011	0.123	0.000	0.072	0.029
5	5	0.098	0.190	0.044	0.154	0.020	0.191	0.046
6	21	0.412	0.264	0.083	0.160	0.397	0.388	0.002
7	9	0.176	0.234	0.014	0.142	0.008	0.191	0.001
8	5	0.098	0.133	0.009	0.111	0.002	0.072	0.010
9	3	0.059	0.048	0.002	0.077	0.004	0.027	0.038
10	1	0.020	0.011	0.007	0.048	0.017	0.010	0.009
11 *and so on*	0	0.000	0.002	0.002	0.027	0.027	0.004	0.004
$\sum N$ (*categ.*) 12	$T = \sum T_N$ 51	$\sum$ FqN 1.000	Chi^2 (sum)	0.179	Chi^2 (sum)	0.572	Chi^2 (sum)	0.156
			d.f. $= \sum N - 1$	11	d.f. $= \sum N - 1$	11	d.f. $= \sum N - 1$	11
			p-value	**5.46E−09**	*p*-value	**2.80E−06**	*p*-value	**2.61E−09**

[a] Since both the Normal and Laplace distributions are continuous, not discrete, the PDF for each $x = N_k$ value was calculated as the difference between their respective CDF evaluated at $x = N_k + 0.5$ and $x = N_k - 0.5$. Residuals are calculated as (Observed − Expected)^2/Expected.

Therefore, the probability that a bidder i in any tender k was first when $N_k = 2$ is:

$$\textit{Discrete Kum PDF}\ \ (x = j_i = 1, \alpha_i, \beta_i\ |N_k = 2) = \left(1 - \left(\frac{2-1}{2}\right)^{\alpha_i}\right)^{\beta_i} - \left(1 - \left(\frac{2-1+1}{2}\right)^{\alpha_i}\right)^{\beta_i}.$$

It is important to deduce that the probability of being first ($j_i = 1$) for a bidder i corresponds to the sum of probabilities from all possible scenarios that are calculated as the product of two probabilities, given by Eq. (8) (which allows calculating the probabilities that there were $N_k = 1, 2, \ldots + \infty$ bidders) multiplied by Eq. (9) (which calculates the probability that bidder i, according to its position performance curve defined by means of α_i and β_i, obtained a position performance coefficient located right within $\left[\frac{N_k - j_i}{N_k}, \frac{N_k - j_i + 1}{N_k}\right]$).

Logically, this result would only be valid for position $j_i = 1$; however, it is trivial to reach the general expression for the position performance joint probability curve for bidder i (J_i), which is:

$$J_i \text{ PDF}\ \ (x = j_i, \alpha_i, \beta_i, m, b) = \sum_{N_k = j_i}^{N_k = +\infty} \{\textit{Discrete Kum PDF}\ \ (x, \alpha_i, \beta_i) \cdot \textit{Discrete Laplace PDF}\ \ (x, m, b)\} = \frac{1}{2} \sum_{N_k = j_i}^{N_k = +\infty} \left\{ \begin{array}{l} \left\{ \left(1 - \left(\frac{N_k - j_i}{N_k}\right)^{\alpha_i}\right)^{\beta_i} - \left(1 - \left(\frac{N_k - j_i + 1}{N_k}\right)^{\alpha_i}\right)^{\beta_i} \right\} \\ \cdot \left\{ \begin{array}{l} \text{sgn}\ (N_k + 0.5 - m)\left(1 - \exp\left(-\frac{|N_k + 0.5 - m|}{b}\right)\right) \\ -\ \text{sgn}\ (N_k - 0.5 - m) \\ \times \left(1 - \exp\left(-\frac{|N_k - 0.5 - m|}{b}\right)\right). \end{array} \right\} \end{array} \right\} \quad (10)$$

Despite its disproportionate length, this equation is easy to handle.

It must also be noted that this expression has resulted in the equation above because two specific statistical distributions were chosen for P_{ik} (Kumaraswamy) and N_k (Laplace); however, the final result would have also been easy to obtain in case these distributions had been different.

4. Results

4.1. Single bidder's position performance joint probability curve

If Eq. (10) is applied to the data gathered so far for bidders $i = 1, 2$ and 3, Table 5 and Fig. 3 could be directly calculated where several considerations must be taken into account: J_i CDF curves were obtained directly by means of adding up the results displayed on the J_i PDF columns; and a correction is implemented over the *Discrete Laplace PDF* expression. Namely, instead of using Eq. (8) directly in Eq. (10), Eq. (8) was divided by the following correction factor $C_f = 1 - \textit{Discrete Laplace PDF}\ \ (x = N_k = 0.5, m, b)$, which allows not considering the probability that no bidder participated in a tender, subsequently enabling that the probabilities when $N_k = 1, 2, \ldots + \infty$ can be corrected (slightly increased) to equal 1 when added up. This correction factor C_f is a constant, so it can be left outside the sum $\sum_{N_k=j_i}^{N_k=+\infty} \{\cdot\}$, keeping the original and relative simplicity of Eq. (10).

Despite these minor details, the curves depicted in Fig. 3 constitute a valuable tool with a two-fold purpose. First, they identify the probabilities that any bidder, whose past performance was registered, ends up being the lowest, second lowest, etc., irrespective of the number of bidders. Second, these curves can be compared to each other, enabling an objective bidders' performance comparison, an issue that had not been solved in detail in the literature so far. For instance, as displayed in Fig. 3, Bidder 3 is the tougher competitor because it has higher chances of occupying the first positions compared to Bidders 2 and 3. At the same time, Bidder 2 statistically outperforms Bidder 1 in the first three positions ($j_i = 1, 2$ and 3). Hence, if a tender manager had to focus on beating a specific competitor, it would be Bidder 3.

Furthermore, a third use can be made of this methodology because it allows aggregating several bidders' probability curves as a group to calculate the probability that one of them occupied a given position. This is a remarkable feature for a tender manager whose concern is to beat several competitors at the same time and will be implemented below.

Table 5
Position performance joint probability curves for bidders $i = 1, 2$ and 3.

j_i	J_1 PDF	J_1 CDF	J_2 PDF	J_2 CDF	J_3 PDF	J_3 CDF
1	0.077	0.077	0.230	0.230	0.402	0.402
2	0.127	0.204	0.169	0.399	0.316	0.718
3	0.159	0.362	0.154	0.552	0.171	0.889
4	0.181	0.544	0.145	0.697	0.075	0.964
5	0.191	0.735	0.134	0.831	0.026	0.990
6	0.164	0.899	0.106	0.936	0.007	0.998
7	0.067	0.966	0.042	0.978	0.002	0.999
8	0.023	0.988	0.014	0.993	0.000	1.000
9	0.008	0.996	0.005	0.997	0.000	1.000
10	0.003	0.999	0.002	0.999	0.000	1.000
11	0.001	0.999	0.001	1.000	0.000	1.000

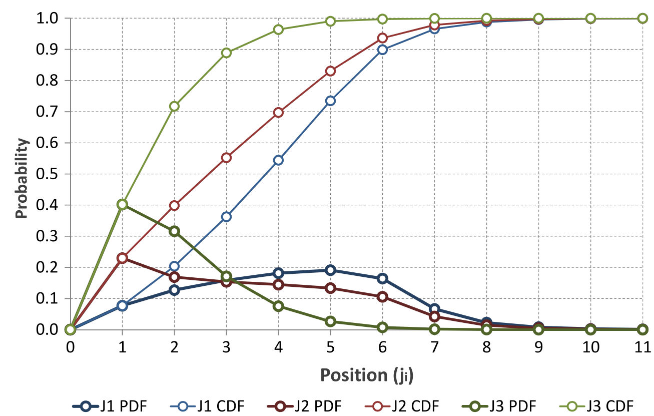

Fig. 3. Position performance joint probability curves for bidders $i = 1, 2$ and 3.

4.2. Group of bidders' position performance joint probability curve

A previous study, [18] proposed a way of calculating group performance curves for capped tenders based on the probability that several independent random events took place. This approach is equally valid here with hardly any changes:

$$\widehat{J_i} \text{ PDF}\ \ (x = j_i) = 1 - \prod_{i=1}^{i=n} (1 - R_i \cdot J_i \text{ PDF}) \quad (11)$$

where:

$\widehat{J_i}$ PDF $(\cdot)$ is a discrete probability density function that denotes the probability that one of the bidders ($i = 1, \ldots, n$) occupies the position $x = j_i$.

R_i are the bidders' participation ratios calculated at the bottom of Table 2, which represent how frequently each bidder participates in the set of tenders analyzed; therefore, $0 \leq R_i \leq 1$.

Finally, J_i PDF are the bidder i's position performance joint probability density function for $i = 1, \ldots, n$ calculated according to Eq. (10).

Table 6
Group (bidders $i = 1 + 2 + 3$) position performance probability curve calculations.

Bidders' participation ratios					
$i =$	1	2	3		
$R_i =$	0.392	0.176	0.137		
j_i	$R_1 * J_1$ PDF	$R_2 * J_2$ PDF	$R_3 * J_3$ PDF	PDF $1 - \Pi(1 - R_i * J_i)$	CDF $1 - \Pi(1 - R_i * J_i)$
1	0.030	0.041	0.055	**0.121**	**0.121**
2	0.050	0.030	0.043	**0.118**	**0.239**
3	0.062	0.027	0.023	**0.109**	**0.348**
4	0.071	0.026	0.010	**0.104**	**0.452**
5	0.075	0.024	0.004	**0.100**	**0.552**
6	0.064	0.019	0.001	**0.083**	**0.635**
7	0.026	0.007	0.000	**0.034**	**0.668**
8	0.009	0.002	0.000	**0.011**	**0.680**
9	0.003	0.001	0.000	**0.004**	**0.684**
10	0.001	0.000	0.000	**0.001**	**0.685**
11	0.000	0.000	0.000	**0.000**	**0.685**

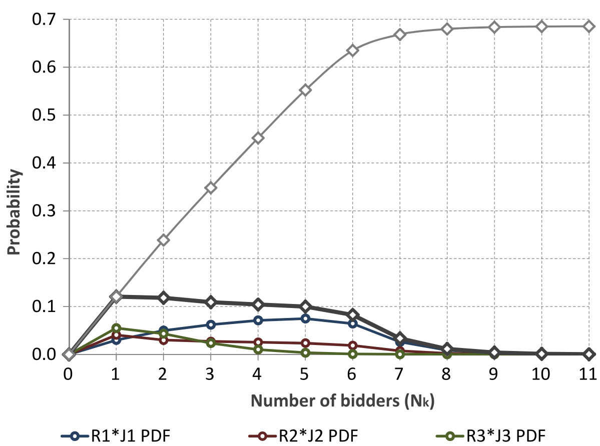

Fig. 4. Group (bidders $i = 1 + 2 + 3$) position performance probability curve.

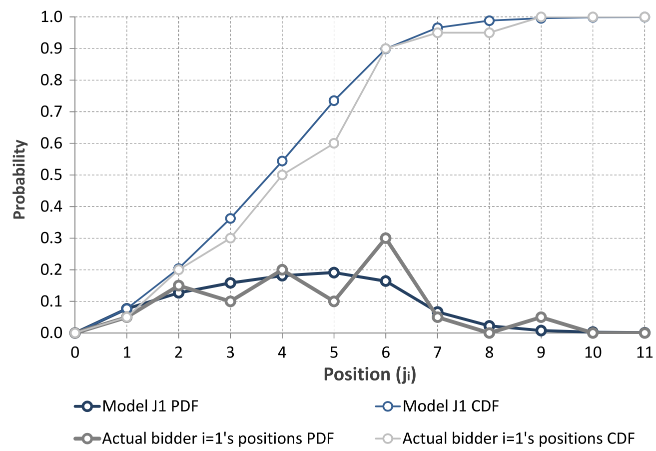

Fig. 5. Bidders $i = 1$'s actual and model position performance probability curves comparison.

Therefore, if Eq. (11) is applied to bidders $i = 1$, 2 and 3's position performance curves, Table 6 and Fig. 4 are easily obtained.

It is easy to note that the gray curve with thick line (obtained by applying Eq. (11)) will only reach the 100% probability value in the case that all the bidders from the tender dataset analyzed were incorporated into the calculations (93 bidders for the tender dataset presented in Section 2.2). Indeed, this is the reason for which these curves become flat when compared to Fig. 3.

Indeed, from Fig. 4, it is easy to determine that the probabilities that one bidder out of bidders $i = 1$, 2 and 3 occupies the first position (when no one knows how many or which bidders will participate) are approximately 12% and remain slightly decreasingly at approximately the 10% probability value up to the sixth position. Of course, none of these probability values seem to be too high to worry other competitors; however, this is due to the fact that these three bidders do not have high participation ratios, as shown at the top of Table 6. If any bidder knew for certain that one or several of these bidders is going to enter a bid in a forthcoming tender, these curves could be recalculated assigning a 100% participation ratio value to those specific competitors, as was considered in Fig. 3, and the group curve would raise their probabilities significantly, thus making the group of bidders 1, 2 and 3 much more competitive (for instance, bidder 1 alone reached 40% probability values of occupying the first position when its participation ratio was 100%, as Fig. 3 indicates).

With these last observations and calculations, every possible approach to study any bidder's position performance has been completed, and therefore, the case study is also complete.

4.3. Validation

As stated in the Introduction, the main reason to calculate the position performance curves by means of the method explained so far instead of directly calculating the relative frequency curves, which would describe how often a bidder ends up being the first, second, and so on, was that in real-life situations, there is not usually a sufficient number of previous encounters among construction bidders for these probability values to be calculated with accuracy.

Particularly, in the tender database used in this analysis, the first bidder ($i = 1$) was nearly the only one with sufficient participation (in addition to the bidder who gathered the complete database, which has not been displayed) to draw up its probability curves. To be exact, the first bidder participated 20 times out of the total 51, so if we represented its actual position performance density and cumulative curves, the result compared to the ones obtained in the model is depicted in Fig. 5.

Both couple of curves evidence quite similar trajectories, especially taking into account that the amount of data only allowed for 5%-probability steps, i.e., 1/20. This fact qualitatively validates that the results were quite close to the reality, although a deeper analysis might have been made if this bidder had participated in many more occasions. Nevertheless, from a 51-tender dataset, only two bidders out of 93 companies had sufficient data to barely represent their curves (the next most frequent bidder took part in 12 auctions), which means that in general, even with far bigger datasets, a significant percentage of the participating bidders could not be analyzed directly but for the method proposed here.

This is indeed the real advantage of the model and the evidence of the applicability in the construction industry: The proposed model allows for the analysis of the number of total participating bidders and the position performance of every bidder on a scale from 0 to 1, even when resorting to far shorter databases. Later, these results are aggregated into the single or group position probability curves, which are not expected to deviate much from the actual (but unknown, due to data scarcity) position performance curves.

Unfortunately, short tender databases along with the unique-project nature are common features of the construction context. Therefore, the method proposed, although not free of limitations, constitutes an approximate but useful tool.

5. Discussion

Early bid tender models (e.g., [6–10]) took into account the bid/cost ratio, the probability of winning and a few other variables under the condition that the product quality does not vary with the price [36]. This last condition does not usually hold in real-life situations [47], so the lowest bidder stops being identified with the most advantageous tenderer [11]. With this in mind, the methodology proposed above allows anyone analyzing a set of tender processes to calculate the position probability curves based on those bidders potentially involved in future tenders assuming that they will perform in accordance with their previous performance.

Specifically, by means of a four-step procedure, the best position performance parameter expression has been chosen for bidders, and subsequently, by gathering a series of these parameter values for different bidders, the Kumaraswamy distribution has been found to be a good candidate for modeling a single bidder's position performance when the number of participating bidders is known.

However, through modeling the number of bidders in parallel by means of the Laplace distribution, the position performance curve can be easily expressed irrespective of the number of bidders, a convenient feature because, generally, the number of participating bidders in future tenders is not known before a tender reaches its deadline in the Construction context.

Finally, the single bidder's position performance curves can be analyzed either independently or as a group, enhancing the applicability of the methodology because it is also quite common that one company who enters a bidding process is forced somehow to economically outperform several key competitors simultaneously. Nonetheless, the methodology also has two mathematical minor limitations. First, the Kumaraswamy distribution can be nearly (as a function of its two shape parameters α_i and β_i) but not totally symmetrical, which means that in case a bidder i's half number of parameters equaled P_{ik} and the other half equaled $1 - P_{ik}$, the Beta distribution would always have a slightly lower p-value because the Beta distribution can actually be totally symmetrical as long as $a_i = b_i$.

The second limitation is that both the Kumaraswamy and the Beta distribution require at least two previous non-equal P_{ik} values to be fitted, that is, when a tender dataset comprises bidders who only participated in one tender or when a bidder repeats the same position against the same number of competing bidders, it is advisable to resort to a simpler expression for which the supports are $[0,\ 1]$ in both axes (as in the Beta and Kumaraswamy distributions) instead of using Eq. (4), which is:

$$F(x) = F(x, \gamma_i) = x^{\gamma_i} \tag{12}$$

with $\gamma_i = LN\ 0.5/LN\ P_{ik}$ and P_{ik} being the unique (repeated or not) value position performance coefficient obtained by bidder i in the one or several tenders k (P_{ik} calculated according to Eq. (2)).

Furthermore, if a tender manager wanted to model a bidder from whom no previous encounters were available, it would be better to define its position performance curve parameters (either Beta's or Kumaraswamy's) as totally random, i.e., $a_i = b_i = \alpha_i = \beta_i = 1$.

Finally, an additional pending issue that justified the selection of the Kumaraswamy's over the Beta distribution was that the latter has less friendly cumulative and quantile distribution functions because they involve an Incomplete Beta function. The major finding of this paper, Eq. (10), made use of Kumaraswamy's CDF, but it is obvious that the quantile function can also be useful when, given a particular probability value y, it is recommended to calculate how many participating bidders N_k with which a particular bidder would be capable of ending first, second or in any specific position j_i (Eq. (13)); or, on the contrary, with a given number of bidders N_k, the best position that bidder i could achieve (Eq. (14)).

Both questions stem from the same expression, which takes advantage of Eq. (5):

$$\left(1 - (1-y)^{1/\beta}\right)^{1/\alpha} = \frac{N_k - j_i}{N_k}.$$

The expression above leads to Eqs. (13) and (14) when either variable's bidder i's position (j_i) or number of bidders (N_k) are worked out, respectively:

$$N_k = j_i / \left\{1 - \left(1 - (1-y)^{1/\beta}\right)^{1/\alpha}\right\} \tag{13}$$

$$j_i = N_k \left\{1 - \left(1 - (1-y)^{1/\beta}\right)^{1/\alpha}\right\}. \tag{14}$$

6. Conclusions

The "lowest bid method" has been dominant in the construction industry for many years and that is why the majority of bid tender forecasting models focused initially on the lowest bidder's bids. However, multi-attribute auctions give out a score to each bid, which is eventually added up to the technical score, allowing the auctioneer to determine the bidder that deserves to be the awardee. This paper focuses on analyzing the economic bidding performance by means of studying the bid order in a series of homogeneous auctions in which the awarding criterion might include more aspects in addition to the price. Obviously, if the only awarding criterion is the price, bidders' positions are not important except for the lowest bidder (first position); conversely, when other technical criteria are included to obtain the final bidders' ranking (such as in multi-attribute auctions), knowing the probabilities of occupying a given position as a consequence of the economic bid becomes more important.

The methodology proposed in this paper addresses the problem of quantitatively defining what level of bidding position performance should be expected when a bidder takes part in a future auction, that is, determining the more likely positions that the bidder will occupy when competing against a known or unknown number of bidders in a construction auction. In addition, the auction format was stated as not relevant when applying the methodology discussed above because all construction auction formats share the same variables the methodology makes use of, i.e., bidders' identities and positions as well as total number of participating bidders.

Additionally, the bidders' position performance can be treated individually or as a group, allowing a tender manager to enrich the analysis by calculating the probabilities that at least one bidder among several reaches any position in a competitive tender.

On the other hand, the methodology proposed involves the use of Kumaraswamy's distribution, possibly for the first time, in construction management, as well as the Laplace distribution for modeling the number of participating bidders. However, this last distribution might not be the best alternative for other tender databases in which the PDF describing the number of bidders were convex, being the other available non-parametric alternatives also analyzed here (the Poisson or the Normal distribution).

This paper has proposed an entirely different way of measuring bidding competitiveness in construction tenders, the applications of which, although not entirely flawless, as stated in the Discussions section, might go beyond the ones anticipated here when analyzing a single or a group of bidders' performance.

In particular, the main limitations of this research are inherent to the model hypotheses because it necessarily assumes that bidders will behave as they behaved in the past, whereas cover pricing and non-economic rational bidding are ignored. These three assumptions limit the practical use of the model developed while actually opening a future topic of research: achieving a more useful practical tool for modeling bidding position performance that is not dependent on these three previous assumptions.

Furthermore, there are other obvious shortcomings in the current experiment with regard to the case study database; the authors have made use of an old database that did not use electronic auctioning processes and from which multi-parameter bidding data were certainly not available, thereby limiting the analysis to the study of the economic bid positions. Therefore, the authors' acknowledge that there is certainly a room for improvement, for instance, in order to free future models from the limitations and hypotheses stated above, therefore, further research needs to be done in this area.

Acknowledgment

This research study was funded in Chile by CONICYT under the Programs Initiation into research 2013 (project number 11130666) and 2014 (project number 11140128).

References

[1] EC-Harris-LLP, BIS Research Paper No. 145. Supply Chain Analysis into the Construction Industry. A Report for the Construction Industrial Strategy, 2013, p. 35–8. https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/252026/bis-13-1168-supply-chain-analysis-into-the-construction-industry-report-for-the-construction-industrial-strategy.pdf.

[2] Ballesteros-Pérez P, González-Cruz MC, Pastor-Ferrando JP. Analysis of construction projects by means of value curves. Int J Proj Manage 2010;28: 719–31. http://dx.doi.org/10.1016/j.ijproman.2009.11.003.

[3] Benton WC, McHenry LF. Construction purchasing and supply chain management. McGraw-Hill; 2010. http://psbm.org/Ebooks/ConstructionSupply.pdf.

[4] Pim JC. Competitive tendering and bidding strategy. Natl Build 1974;55:541–5.

[5] Skitmore M. Predicting the probability of winning sealed bid auctions: the effect of outliers on bidding models. Constr Manag Econ 2004;22:101–9.

[6] Friedman L. A competitive bidding strategy. Oper Res 1956;1:104–12.

[7] Gates M. Bidding strategies and probabilities. J Constr Div Proc Am Soc Civ Eng 1967;93:75–107.

[8] Wade RL, Harris RB. LOMARK: A bidding strategy. J Constr Div 1976;102: 197–211. http://cedb.asce.org/cgi/WWWdisplay.cgi?6528 [accessed August 12, 2014].

[9] Carr RI. General bidding model. J Constr Div Proc Am Soc Civ Eng 1982;108: 639–50.

[10] Skitmore M. The contract bidder homogeneity assumption: an empirical analysis. Constr Manag Econ 1991;9:403–29.

[11] Perng Y-H, Juan Y-K, Chien S-F. Exploring the bidding situation for economically most advantageous tender projects using a bidding game. J Constr Eng Manage 2006;132:1037–42. http://dx.doi.org/10.1061/(ASCE)0733-9364(2006)132:10(1037).

[12] Bergman MA, Lundberg S. Tender evaluation and supplier selection methods in public procurement. J Purch Supply Manag 2013;19:73–83. http://dx.doi.org/10.1016/j.pursup.2013.02.003.

[13] Yu W, Wang K-W, Wang M-T. Pricing strategy for best value tender. J Constr Eng Manage 2013;139:675–84. http://dx.doi.org/10.1061/(ASCE)CO.1943-7862.0000635.

[14] Lo W, Yan MR. A system review of competitive bidding and qualification-based selection systems. In: C. 2007 conf.—constr. manag. econ. 'past, present futur., 2007. p. 131–41. http://www.scopus.com/inward/record.url?eid=2-s2.0-84877591303&partnerID=tZOtx3y1.

[15] Fuentes-Bargues JL, González-Cruz MC, González-Gaya C. La contratación pública de obras: situación actual y puntos de mejora. Inf Constr 2014; [in Spanish] in press. http://dx.doi.org/10.3989/ic.12.130.

[16] Ballesteros-Pérez P, González-Cruz MC, Cañavate-Grimal A. Mathematical relationships between scoring parameters in capped tendering. Int J Proj Manage 2012;30:850–62. http://dx.doi.org/10.1016/j.ijproman.2012.01.008.

[17] Mahdavi A, Hastak M. Quantitative analysis of bidding strategies: A hybrid agent based-system dynamics approach. In: Constr. res. congr. 2014 constr. a glob. netw.—proc. 2014 constr. res. congr. American Society of Civil Engineers (ASCE); 2014. p. 1129–38. http://dx.doi.org/10.1061/9780784413517.0116.

[18] Ballesteros-Pérez P, González-Cruz MC, Fernández-Diego M, Pellicer E. Estimating future bidding performance of competitor bidders in capped tenders. J Civ Eng Manag 2014;1–12. http://dx.doi.org/10.3846/13923730.2014.914096.

[19] Drew DS, Skitmore RM. Competitiveness in bidding: a consultant's perspective. Constr Manag Econ 1992;10:227–47. http://dx.doi.org/10.1080/01446199200000020.

[20] Jin Z, Deng F, Li H, Skitmore M. Practical framework for measuring performance of international construction firms. J Constr Eng Manage 2013;139:1154–67. http://dx.doi.org/10.1061/(ASCE)CO.1943-7862.0000718.

[21] Pellicer TM, Pellicer E, Eaton D. A macroeconomic regression analysis of the European construction industry. Eng Constr Archit Manag 2009;16:573–97. http://dx.doi.org/10.1108/09699980911002584.

[22] Sohn T-H, Kim H-R, Jang H-S. An MDB business competency assessment of Korean construction companies. KSCE J Civ Eng 2013;18:1314–21. http://dx.doi.org/10.1007/s12205-013-0478-7.

[23] Drew D.S., Skitmore M.. Analysing bidding performance; measuring the influence of contract size and type. In: CIB W-55/65. 1990. p. 129–39.

[24] Jap SD, Naik Pa. BidAnalyzer: a method for estimation and selection of dynamic bidding models. Mark Sci 2008;27:949–60. http://dx.doi.org/10.1287/mksc.1080.0363.

[25] Flanagan R, Norman G. An examination of the tendering pattern of individual building contractors. Build Technol Manag 1982;25–8. April.

[26] Pastor-Ferrando JP, Aragonés-Beltrán P, Hospitaler-Pérez A, García-Melón M. An ANP- and AHP-based approach for weighting criteria in public works bidding. J Oper Res Soc 2010;61:905–16. http://dx.doi.org/10.1057/jors.2010.13.

[27] Yu I, Kim K, Jung Y, Chin S. Comparable performance measurement system for construction companies. J Manage Eng 2007;23:131–9. http://dx.doi.org/10.1061/(ASCE)0742-597X(2007)23:3(131).

[28] Ballesteros-Pérez P, Skitmore M, Das R, del Campo-Hitschfeld ML. Quick abnormal-bid-detection method for construction contract auctions. J Constr Eng Manage 2015; http://dx.doi.org/10.1061/(ASCE)CO.1943-7862.0000978. In press.

[29] Fuentes-Bargues JL, González-Gaya C. Determination of disproportionate tenders in public procurement. J Invest Manag 2013;2:1–9. http://dx.doi.org/10.11648/j.jim.20130201.11.

[30] Ballesteros-Pérez P, González-Cruz MC, Cañavate-Grimal A, Pellicer E. Detecting abnormal and collusive bids in capped tendering. Autom Constr 2013;31:215–29. http://dx.doi.org/10.1016/j.autcon.2012.11.036.

[31] Harper CM, Molenaar KR, Anderson S, Schexnayder C. Synthesis of performance measures for highway cost estimating. J Manage Eng 2014;30: 04014005. http://dx.doi.org/10.1061/(ASCE)ME.1943-5479.0000244.

[32] Carr PG. Investigation of bid price competition measured through prebid project estimates, actual bid prices, and number of bidders. J Constr Eng Manage 2005;131:1165–72. http://dx.doi.org/10.1061/(ASCE)0733-9364(2005)131:11(1165).

[33] Schiereck D, Vogt J. Long-run M & A success of strategic bidders in the construction industry. Probl Perspect Manag 2013;11:46–67.

[34] McCabe B, Tran V, Ramani J. Construction prequalification using data envelopment analysis. Can J Civ Eng 2005;32:183–93. http://www.ingentaconnect.com/content/nrc/cjce/2005/00000032/00000001/art00017 [accessed August 13, 2014].

[35] El-Mashaleh MS. Decision to bid or not to bid: a data envelopment analysis approach. Can J Civ Eng 2010;37:37–44. http://dx.doi.org/10.1139/L09-119.

[36] Wang KW, Der Yu W, Wang MT. Revenue/cost Analysis Model for bidder's competitive bidding strategy planning. J Chin Inst Civ Hydraul Eng 2014;26:81–94. http://www.scopus.com/inward/record.url?eid=2-s2.0-84904458977&partnerID=tZOtx3y1.

[37] Horta IM, Camanho AS. Competitive positioning and performance assessment in the construction industry. Expert Syst Appl 2014;41:974–83. http://dx.doi.org/10.1016/j.eswa.2013.06.064.

[38] Abdallah AAN, Darayseh M, Waples E. Incomplete contract, agency theory and ethical performance: A synthesis of the factors affecting owners' and contractors' performance in the bidding construction process. J Gen Manag 2013;38:39–56. http://www.scopus.com/inward/record.url?eid=2-s2.0-84888404741&partnerID=tZOtx3y1.

[39] Kozan B, Zlatar I, Paravan D, Gubina AF. The advanced bidding strategy for power generators based on reinforcement learning. Energy Sources Part B 2014;9:79–86. http://dx.doi.org/10.1080/15567241003792358.

[40] Polat G, Bingol BN, Uysalol E. Modeling Bid/No bid decision using adaptive neuro fuzzy inference system (ANFIS): a case study. In: Constr. res. congr. 2014 constr. a glob. netw.—proc. 2014 constr. res. congr. American Society of Civil Engineers (ASCE); 2014. p. 1083–92. http://dx.doi.org/10.1061/9780784413517.0111.

[41] Skitmore M, Pemberton J. A multivariate approach to construction contract bidding mark-up strategies. J Oper Res Soc 1994;45:1263–72.

[42] Ballesteros-Pérez P, González-Cruz MC, Cañavate-Grimal A. On competitive bidding: scoring and position probability graphs. Int J Proj Manage 2013;31:434–48. http://dx.doi.org/10.1016/j.ijproman.2012.09.012.

[43] Ballesteros-Pérez P, González-Cruz MC, Pastor-Ferrando JP, Fernández-Diego M. The iso-score curve graph. A new tool for competitive bidding. Autom Constr 2012;22:481–90. http://dx.doi.org/10.1016/j.autcon.2011.11.007.

[44] Jones MC. Kumaraswamy's distribution: A beta-type distribution with some tractability advantages. Stat Methodol 2009;6:70–81. http://dx.doi.org/10.1016/j.stamet.2008.04.001.

[45] Kumaraswamy P. A generalized probability density function for double-bounded random processes. J Hydrol 1980;46:79–88. http://dx.doi.org/10.1016/0022-1694(80)90036-0.

[46] Engelbrecht-Wiggans R. A model for the distribution of the number of bidders in an auction. In: Cowles Found. discuss. pap. 495. 1978. pp. 1–13.

[47] Laryea S, Hughes W. Risk and price in the bidding process of contractors. J Constr Eng Manage 2011;137:248–58. http://cedb.asce.org/cgi/WWWdisplay.cgi?278030 [accessed September 01, 2014].