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THE EFFECT OF (MIS-SPECIFIED) GARCH FILTERS ON THE FINITE SAMPLE DISTRIBUTION OF THE BDS TEST

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Abstract

This paper considers the effect of using a GARCH filter on the properties of the BDS test statistic as well as a number of other issues relating to the application of the test. It is found that, for certain values of the user-adjustable parameters, the finite sample distribution of the test is far-removed from asymptotic normality. In particular, when data generated from some completely different model class are filtered through a GARCH model, the frequency of rejection of iid falls, often substantially. The implication of this result is that it might be inappropriate to use non-rejection of iid of the standardised residuals of a GARCH model as evidence that the GARCH model "fits" the data.

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I. Introduction

Testing for nonlinearity in financial and economic time series has become a highly active area of research over the past decade. Interest in nonlinear models has developed in parallel with an expansion in the number of and understanding of the properties of tools for nonlinear data analysis. By far the most widely adopted test for non-linear structure has been the BDS test due to Brock, Dechert, Scheinkman and LeBaron (1987, revised in 1996). The base null hypothesis for this test is that the data are independently and identically distributed (iid), and any departure from iid should lead to rejection of this null in favour of an unspecified alternative hypothesis: the test is a pure hypothesis test. Hence the test can be considered a broad portmanteau test which has been shown to have reasonable power against a variety of nonlinear data generating processes (see Brock *et al.*, 1991 for an extensive Monte Carlo study).

The common finding among almost all researchers who apply the test is that the iid null is rejected, although this rejection could be the result of either linear or non-linear structure in the data. One way to turn the test into one against only nonlinear alternatives is to fit an autoregressive or ARMA model of sufficiently high order to ensure that the residuals are serially uncorrelated, and then to test the residuals for iid using the BDS test. If rejection occurs, and if the focus is limited to univariate time series[1], it must by definition imply that the data generating mechanism has inherent nonlinearities since linear dependence has been filtered out. Brock *et al.* (1991) show that the BDS test is asymptotically nuisance parameter free (NPF) when applied to the residuals of a linear model, implying that the same set of

[1] Another possibility is that the observed "nonlinearity" is simply picking up a linear conditional relationship between variables so that the evidence of nonlinearity would disappear once the appropriate conditioning model had been specified.

(standard normal) critical values can be used when the test is applied to residuals as to the raw data.

Unfortunately, evidence of nonlinearity *per se* does not give the researcher any clue as to the likely cause of the nonlinearity, and hence an appropriate functional form for the resultant nonlinear model. Thus the researcher is left to make an entirely separate decision on some other grounds as to which of a number of possible candidate models might best describe the data. In particular, the BDS test has reasonable power against the GARCH family of models, and it is often difficult to disentangle the nonlinearity generated by this form of dependence in the second moment from nonlinearities arising as a result of other causes (see Brooks, 1996, for a more detailed discussion of this issue). One solution to this potential problem is to estimate some form of GARCH model for the series (x_t) , such as

$$x_t = \mu + u_t \quad , \quad u_t \sim \mathrm{N}(0,h_t) \qquad (1)$$

$$h_t = \gamma + \alpha_1 u_{t-1}^2 + \beta_1 h_{t-1},$$

save the residuals ($\hat{u}_t$) and divide each residual by its corresponding conditional standard deviation estimate ($\sqrt{\hat{h}_t}$). These standardised residuals are themselves subjected to the BDS test and the null hypothesis then becomes one that the specified GARCH model is sufficient to model the nonlinear structure in the data against an unspecified alternative that it is not[2].

This procedure has been followed a number of times in the literature, and opinions are still mixed as to whether the GARCH model alone is sufficient to capture all of the nonlinearity in economic and financial data or not. Abhyankar *et al*. (1995) and Hsieh (1991), for example,

[2] It is, of course, possible that there may be further dependence in the conditional second moment which requires a higher order GARCH model to capture it, although this point is typically ignored in the testing literature which automatically assumes that the remaining structure must be of some other form.

find that a significant degree of nonlinearity remains in the standardised residuals of an EGARCH(1,1) model of intra-daily UK stock index futures and daily US stock index returns respectively. LeBaron (1988), on the other hand, finds that GARCH residuals do not reject iid on a sample of weekly value-weighted CRSP stock returns, a result echoed by Hsieh (1989) for a set of five daily US Dollar currency series.

The NPF result does not necessarily hold for nonlinear models, however. In particular, Brock and Potter (1992, p155) note that the BDS statistic only has the same distribution on the estimated residuals of a model as the true residuals when the data are generated by a member of the null model class. This, then, does not rule out the possibility that the use of inappropriate filter models could radically alter the null distribution of the test statistic, even with an infinite amount of data.

The problem is much greater still in the case of finite samples. Brock *et al*. (1991) themselves note:

"*...Our Monte Carlo experiments have found that the asymptotic distribution does not approximate very well the BDS statistic applied to standardised residuals of ARCH, GARCH and EGARCH models*." (p76).

This leads them to use Monte Carlo methods to derive a new set of critical values for comparison with BDS test statistics on the standardised residuals of ARCH and GARCH models for "small" sample sizes (100, 500, and 1000 observations). Hsieh (1991) extends the simulation to the case where the model under the null is an EGARCH. For certain values of the BDS test user-adjustable parameters, these studies find that the probability distribution for the test statistics can be markedly different from that of a standard normal distribution.

In this paper, we extend and generalise the recent Monte Carlo studies described above, and also that of Chappell *et al.* (1996a) to consider the effect of using (very) mis-specified GARCH filters on the finite sample distribution of the BDS test statistic. This line of enquiry appears highly relevant in the light of many recent studies that have followed this procedure (that is, linear and GARCH filtering and then a BDS application to "see what is left"). The remainder of the paper develops as follows. Section two gives a presentation of the BDS test and highlights a number of issues involved in its application to actual data. This treatment is kept deliberately brief since the formulation of the BDS test will probably be familiar to the vast majority of interested readers. Section three describes the Monte Carlo framework adopted, and section four displays and comments upon our results. Section five concludes and offers a number of practical suggestions which might guide future research in this area.

II. A Description of the BDS Test

The test of Brock, Dechert, Scheinkman and LeBaron (1996, hereafter, denoted BDS after the original paper which was written by only the first three authors) takes the concept of the correlation integral and transforms it into a formal test statistic which is asymptotically distributed as a standard normal variable under the null hypothesis of independent and identical distribution (iid) against an unspecified alternative. The BDS test statistic is calculated as follows. First, the "*m*-histories" of the data $x_t^m = (x_t, x_{t+1}, \ldots, x_{t-m+1})$ are calculated for $t = 1, 2, \ldots, T\text{-}m$ for some integer embedding dimension $m \geq 2$. The correlation integral is then computed, which counts the proportion of points in *m*-dimensional hyperspace that are within a distance ε of each other:

$$C_{m,T}(\varepsilon) = \frac{2}{(T-m+1)(T-m)} \sum_{\forall t<s} I_\varepsilon(x_t^m, x_s^m) \qquad (2)$$

where I_ε is an indicator function that equals one if $\left\| x_t^m - x_s^m \right\| < \varepsilon$ and zero otherwise, and $\|\bullet\|$ denotes the sup. norm. BDS show that under the null hypothesis that the observed x_t are iid, then

$C_{m,T}(\varepsilon,T) \xrightarrow{asymp.} C_1(\varepsilon)^m$ with probability one. The BDS test statistic, which therefore has a limiting standard normal distribution, then follows as:

$$W_{m,T}(\varepsilon) = T^{1/2} \frac{[C_{m,T}(\varepsilon) - C_{1,T}(\varepsilon)^m]}{\sigma_{m,T}(\varepsilon)} \qquad (3)$$

where $\sigma_{m,T}$ (ε) = 2 $[K^m + 2(\sum_{j=1}^{m-1} K^{m-j} C_{1,T}(\varepsilon)^{2j}) + (m-1)^2 C_{1,T}(\varepsilon)^{2m} - m^2 K C_{1,T}(\varepsilon)^{2m-2}\ ]^{1/2}$

and $K(\varepsilon)$ is estimated by $K(\varepsilon) = \dfrac{6 \sum_{\forall t<s<r} h_\varepsilon(x_t^m, x_s^m, x_r^m)}{[(T-m-+1)(T-m)(T-m-1)]}$

and $h_\varepsilon(i,j,k) = [I_\varepsilon(i,j)\ I_\varepsilon(j,k) + I_\varepsilon(i,k)\ I_\varepsilon(k,j) + I_\varepsilon(j,i)\ I_\varepsilon(i,k)]/3$

III. The Simulation Framework

We simulate samples of length 500 observations according to five models: an AR(1), an MA(1)[3], a GARCH(1,1), a SETAR(1,1) and a bilinear(0,0,1,1) model. The length 500 probably represents a lower bound on the sample sizes typically available in finance, although it is still much greater than would be available to macroeconomists. Then a GARCH(1,1) model is estimated for the data, and the standardised residuals are obtained. The BDS is calculated on these residuals for values of ε of 0.5, 1, 1.5 and 2 times the standard deviation of the data, for values of the embedding dimension, *m*, from 2 to 10.

[3] Although, as an anonymous referee has pointed out, a competent researcher would not blindly apply a GARCH(1,1) model before checking first for the presence of ARCH-type structure in the errors. In particular, an applied econometrician should therefore not fit a GARCH model to a series generated by a SETAR process unless it could be mistaken for ARCH, which seems unlikely given the results presented below. However, researchers do occasionally seem to use the BDS test exhaustively before applying any others, and for this reason and for completeness and ease of comparison, the results are presented for all 5 DGPs.

This procedure is repeated 5,000 times[4], and the proportion of rejections of iid at each significance level is recorded. Appropriate critical values for the GARCH filter are derived from the case where the data are generated by a GARCH(1,1) model and then an appropriately specified GARCH model is estimated on the data. The actual models estimated, including the necessary model parameters are as follows:

AR(1)

$$x_t = 0.5x_{t-1} + u_t \tag{4}$$

MA(1)

$$x_t = 0.5u_{t-1} + u_t \tag{5}$$

GARCH(1,1)

$$x_t = u_t \quad , \quad u_t \sim \mathrm{N}(0,h_t) \tag{6}$$

$$h_t = 0.01 + 0.1u_{t-1}^2 + 0.8h_{t-1}$$

SETAR(1,1)

$$x_t = \begin{cases} 0.5 - 0.5x_{t-1} + u_t & if \quad x_{t-1} < 2 \\ 0.5 + 0.5x_{t-1} + u_t & otherwise \end{cases} \tag{7}$$

Bilinear

$$x_t = 0.4x_{t-1}u_{t-1} + u_t \tag{8}$$

IV. Results

As a starting point for comparison with previous studies in this area, table 1 shows the percentage of rejections of iid when the BDS statistics are estimated on (unfiltered) data generated using the GARCH model in (6). The percentage of rejections is clearly very high - typically at least 80%, although a couple of further points are worth noting. First, this power is high, but far from 100%. Brock *et al*. (1991) show that the power of the test in detecting deviations from iid if the DGP is GARCH asymptotically approaches unity, but it is not clear at what rate. The sample size considered here is large by traditional econometric standards, but Brock *et al*. recommend 200 as a very minimum necessary; 500 observations is still considered a "small" sample for this test. Second, the power of the test appears to rise both as the ratio of ε / σ is increased, and also as the value of the embedding dimension is increased. Unfortunately, Brock *et al*. show that the test is considerably oversized in finite samples for large values of m relative to the number of observations (by "large", they mean $m > 5$ for $T = 500$); we will return to this point later.

Appropriate critical values for the BDS test applied to standardised residuals of a GARCH(1,1) model are given in table 2, along with corresponding critical values for a standard normal variate. The critical values are mostly similar to those of Brock *et al*. (1991, p277, table F2) and Hsieh (1991, p362, footnote 4), although the number of replications used in this study is more than twice as large, and hence should give more accurate probabilities in

[4] This value is severely constrained by CPU time. In spite of efficient code and fast machines, the steps involved (data generation, GARCH model estimation, and BDS statistic calculation) for a sample size of 500 takes a non-trivial amount of processing time.

the tails of the distribution[5]. Brock *et al*. (1991, p169) recommend the use of ε / σ = 0.5 to 1.5 and m = 2 to 5 for a sample of this size in order to achieve the best approximation to the asymptotic theory over the possible parameter space, while achieving satisfactory power. If the users of the test stay within the confines of these guidelines, then the critical values never stray far from the standard normal. However, many researchers do not follow this advice, and appear to use the test for higher values of m and ε / σ in spite of insufficiently long samples. For $\varepsilon / \sigma = 1$, the probability distribution under the null hypothesis is significantly skewed towards the left-hand tail, although the upper critical values remain approximately the same. The behaviour of the test is worst when ε / σ is small and m is large. For example, consider the case when $\varepsilon / \sigma = 0.5$ and $m = 10$. The null distribution is now skewed greatly towards the right-hand tail, although the left-hand tail is also fatter than that of a normal distribution. The upper critical value for a 2-sided test of size 1% is now 48.418 compared to 2.576 for a standard normal.

IV.1 The Effect of Mis-specified GARCH Filters

In order to assess the effect of mis-specified GARCH filers on the finite sample distribution of the BDS test statistic, it is first necessary to look at the rejection frequencies of the null of iid for the four models (AR, MA, SETAR, bilinear) in the absence of any filters. Results for the proportion of rejections of iid for each of these models are shown in tables 3 to 6. In all four cases, the power of the test is virtually 100%, although it is higher for the non-linear data generating processes than for the linear ones, and the proportion of rejections is reduced somewhat when $\varepsilon / \sigma = 0.5$ or $m > 5$; the power of the test is always greater than if the data are generated by a GARCH model, as Brock *et al*. (1991) note. Also, in all cases, rejection occurs

[5] For example, the 1% 2-sided critical values will be based on only 10 observations in each tail for the earlier studies and 25 observations here. This quantity is still rather small, and is prevented from further increases by the

in the right hand tail of the distribution, indicating more clustering of the *m*-histories in hyperspace than would be the case if the data were generated randomly (rather than less clustering which would be the case if the test statistics were significantly negative).

Tables 7 to 10 show the proportion of rejections when the data are generated by an AR(1), MA(1), SETAR and bilinear model respectively, but with a GARCH model then estimated on the data, and the BDS statistic calculated on the standardised residuals of the GARCH "fit". The null hypothesis is now that these standardised residuals are iid and thus that there is no further dependence left in the data. This null hypothesis should clearly be rejected, since in none of the four cases was the data generated by a model resembling one from the GARCH family. Thus one would expect that the proportion of rejections at a given significance level should remain close to 100%. On the whole, however, this is found not to be the case. For the linear DGP models (AR and MA), the fall in the proportion of rejections is mostly small when $\varepsilon / \sigma = 0.5$, 1, or 1.5 and $m \leq 5$. For the AR(1) DGP, for example, when $\varepsilon / \sigma = 1$ and $m = 3$, the fall for a 1% size of test is from 100% with no filter to 99.48% with the GARCH filter. The effect is very marked, however, for the cases when $\varepsilon / \sigma = 2$ or $m > 5$. The most extreme example is for the MA(1) DGP with $\varepsilon / \sigma = 2$ and $m = 10$; the rejection rate is around 73% with no filter using a 1% significance level, but this falls to just over 5% when the GARCH model is estimated and the BDS test is applied to its standardised residuals. It is important to note that, in this case, the linear model is not nested in the GARCH model since the latter was estimated under a specification of equation (1), which includes only a constant term in the mean equation.

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intense CPU requirements of the procedure discussed previously.

The effect of GARCH filtering is much more pronounced when the data generating process is nonlinear. First, when 500 observations are generated using a SETAR model, even for the "preferable" choices of ε / σ = (0.5 to 1.5) and m ($\leq$ 5) for a sample of this size, the number of times the null hypothesis that the series of interest is iid is rejected, falls significantly. For example, with $\varepsilon / \sigma = 1$ and $m = 5$, rejection of the iid null occurs virtually 100% of the time for the raw data, but only 83% of the time when the test is calculated on the residuals of a GARCH model of the data. Again, the problem becomes exacerbated with greater values of ε / σ and m.

Finally, the most interesting and extreme results are observed when samples are generated from a bilinear model, as tables 6 and 10 show. Rejection of the iid null occurs on the unfiltered data for almost every replication for almost all combinations of the user-adjustable parameters[6]. When the test statistic is re-computed on the standardised residuals of the GARCH model, however, evidence of nonlinear structure is significantly reduced. When $\varepsilon / \sigma = 2$ and m = 2 or 3, the two-sided rejection frequencies are virtually identical to those one would expect if the data were independent draws from a standard normal distribution. Other values of the user-adjustable parameters show a similar, although less marked picture. Weiss (1986) shows that bilinear and GARCH models can be mistaken for one another, and Bera and Higgins (1997) demonstrate that the two models have very similar unconditional moment structures. So perhaps one should not be surprised that the GARCH model kills the nonlinearities, making the resultant standardised residuals look like an iid process. The results presented here for the bilinear model nonetheless serve to show that the BDS test cannot reliably discriminate between the two models.

V. Conclusions

This paper has examined the use of BDS tests as a diagnostic for the adequacy of GARCH models as valid data descriptions. Our results have a number of implications for future research in this area. First, although Brock *et al*. (1991) recommend the use of $\varepsilon / \sigma = 0.5$ to 1.5 and $m \leq 5$ for sample sizes of 500 or less, this advice is often ignored by empirical researchers who presumably assume that more output is always better than less. Our finding is that the null distribution of the test statistic can depart substantially from its asymptotic normal distribution when $\varepsilon / \sigma = 2$ or $m \geq 5$. The conclusions of many recent papers rejecting iid appear to hinge on a body of evidence which includes results computed using these parameter values, although we suggest that the test is ill-behaved, unpredictable, and therefore should not be computed in these cases.

Second, many researchers have observed significant BDS statistics on linearly independent (i.e. ARMA-filtered) data, but insignificant BDS statistics on the standardised residuals of a GARCH model of that data (e.g. Hsieh, 1989). The researchers then typically conclude that the GARCH model can "explain" a large part of the nonlinearities or even stronger, that GARCH successfully "models" the series. The upshot of the results presented here is that one would probably observe similar results if the data were truly generated by a completely different model (e.g. plausibly a bilinear, or even a SETAR). This is important since, as Bera and Higgins (1997) note, a bilinear process has a potentially predictable conditional mean, whereas a GARCH process can only help to predict the conditional variance.

The researcher can never know the true data generating process, and hence any econometric study is essentially an exercise in attempting to find an acceptable approximation to it. In the

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[6] With the possible exception of $\varepsilon / \sigma = 0.5$ and $m \geq 7$.

case of the bilinear model, GARCH may provide such a reasonable approximation to the DGP; the same cannot be said of the SETAR model, however, where the conditional variance is typically assumed non-autocorrelated. In a number of recent applied studies, the two models (SETAR and GARCH) may both be plausible and competing data descriptions. For example, with regard to foreign exchange rates, the former may represent the behaviour of a series which is constrained to lie within two boundaries in the European exchange rate mechanism (see Chappell *et al.*, 1996b) and the latter can allow for volatility clustering effects (see Bollerslev, 1986).

Although the mis-specifications used as examples here are severe, and might be detected by a careful researcher jointly using other tests, our results should serve as a warning against placing too much emphasis on BDS statistics in this context. They can be fairly poor discriminators and inferences about the probable model class should be made jointly with tests which have power against only one class of alternatives, such as those proposed by Bera and Higgins (1997) or Tsay (1989). Singular use of the BDS portmanteau test could be misleading.

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Appendix: Tabulated Results

Table 1: Percentage of Rejections of the Null of iid at the 5% Level for Data Drawn from a GARCH-(1,1) Model

	m								
ε / σ	2	3	4	5	6	7	8	9	10
0.5	70.88	81.14	84.34	84.82	83.52	80.78	79.88	88.80	94.18
1	76.42	86.66	90.12	91.78	93.04	93.46	93.78	93.58	93.14
1.5	80.06	90.98	92.82	94.30	95.20	95.72	96.04	96.10	96.16
2	80.94	92.60	92.32	94.28	95.14	95.92	96.10	96.60	96.76

Table 2: The Null Distribution for the BDS Test for the Residuals of a GARCH Model with 500 Observations

		m									
%'age point	ε / σ	2	3	4	5	6	7	8	9	10	N(0,1)
0.5%	0.5	-2.583	-2.484	-2.956	-3.543	-4.553	-5.940	-7.016	-6.373	-5.410	-2.576
2.5%	0.5	-1.965	-2.071	-2.296	-2.755	-3.627	-4.670	-6.088	-5.810	-4.941	-1.960
5%	0.5	-1.719	-1.800	-1.976	-2.375	-3.078	-4.090	-5.363	-5.518	-4.726	-1.645
95%	0.5	1.515	1.579	1.773	2.173	3.008	4.532	7.139	11.799	19.783	1.645
97.5%	0.5	1.886	1.943	2.192	2.766	3.818	5.622	9.069	15.751	27.968	1.960
99.5%	0.5	2.467	2.623	3.228	4.122	5.258	7.984	13.192	24.197	48.418	2.576
0.5%	1	-2.495	-2.236	-2.067	-1.964	-1.940	-1.976	-2.055	-2.139	-2.327	-2.576
2.5%	1	-1.905	-1.743	-1.626	-1.569	-1.552	-1.570	-1.619	-1.727	-1.894	-1.960
5%	1	-1.660	-1.488	-1.411	-1.357	-1.351	-1.375	-1.420	-1.509	-1.634	-1.645
95%	1	1.175	1.078	1.007	1.004	1.026	1.099	1.204	1.358	1.569	1.645
97.5%	1	1.483	1.369	1.302	1.322	1.364	1.460	1.630	1.725	1.982	1.960
99.5%	1	2.155	2.109	2.122	2.101	2.269	2.481	2.840	3.020	3.282	2.576
0.5%	1.5	-3.890	-2.970	-2.620	-2.216	-2.033	-1.909	-1.913	-1.894	-1.887	-2.576
2.5%	1.5	-2.639	-2.075	-1.859	-1.695	-1.576	-1.545	-1.485	-1.467	-1.429	-1.960
5%	1.5	-2.208	-1.770	-1.546	-1.439	-1.359	-1.319	-1.272	-1.253	-1.245	-1.645
95%	1.5	1.112	1.006	0.913	0.862	0.846	0.826	0.838	0.837	0.876	1.645
97.5%	1.5	1.454	1.280	1.196	1.130	1.135	1.151	1.147	1.183	1.208	1.960
99.5%	1.5	2.013	2.013	1.933	1.981	2.402	2.494	2.709	2.985	3.156	2.576
0.5%	2	-13.96	-8.931	-7.197	-6.050	-5.500	-5.024	-4.566	-4.345	-4.148	-2.576
2.5%	2	-7.610	-5.171	-4.171	-3.594	-3.157	-2.894	-2.708	-2.523	-2.395	-1.960
5%	2	-5.778	-3.980	-3.170	2.725	-2.445	-2.234	-2.070	-1.957	-1.856	-1.645
95%	2	1.123	1.051	0.991	0.990	0.976	0.949	0.918	0.918	0.927	1.645
97.5%	2	1.559	1.458	1.389	1.296	1.276	1.290	1.310	1.343	1.313	1.960
99.5%	2	2.404	2.368	2.229	2.273	2.433	2.515	2.763	2.858	3.002	2.576

Table 3: Proportion of Rejections for the BDS Test for 500 Observations Simulated using an AR(1) Model

		m									
%'age point	ε / σ	2	3	4	5	6	7	8	9	10	N(0,1)
0.5%	0.5	0.00	0.00	0.00	0.00	0.00	0.12	2.96	12.50	49.46	0.5
2.5%	0.5	0.00	0.00	0.00	0.00	0.00	0.22	3.18	12.50	49.48	2.5
5%	0.5	0.00	0.00	0.00	0.00	0.00	0.34	3.66	12.50	49.48	5
95%	0.5	100.0	99.98	99.98	99.74	98.60	94.42	84.36	69.64	50.52	5
97.5%	0.5	100.0	99.96	99.92	99.62	98.24	93.46	82.76	68.88	50.50	2.5
99.5%	0.5	100.0	99.94	99.78	99.04	96.76	90.66	79.96	67.08	50.46	0.5
0.5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.5
2.5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.5
5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5
95%	1	100.0	100.0	100.0	100.0	100.0	99.92	99.88	99.70	99.50	5
97.5%	1	100.0	100.0	100.0	100.0	99.96	99.90	99.78	99.60	99.14	2.5
99.5%	1	100.0	100.0	100.0	99.92	99.78	99.70	99.40	98.80	98.28	0.5
0.5%	1.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.5
2.5%	1.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.5
5%	1.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5
95%	1.5	100.0	100.0	100.0	100.0	100.0	99.98	99.98	99.98	99.94	5
97.5%	1.5	99.98	99.98	99.98	99.98	99.96	99.98	99.96	99.90	99.72	2.5
99.5%	1.5	99.96	99.98	99.96	99.94	99.90	99.78	99.60	98.38	97.16	0.5
0.5%	2	0.20	0.06	0.06	0.06	0.04	0.00	0.00	0.00	0.02	0.5
2.5%	2	0.26	0.08	0.06	0.06	0.06	0.04	0.04	0.02	0.02	2.5
5%	2	0.26	0.10	0.06	0.06	0.06	0.06	0.04	0.04	0.02	5
95%	2	99.20	99.72	99.76	99.80	99.78	99.74	99.72	99.58	99.40	5
97.5%	2	99.08	99.68	99.74	99.70	99.64	99.58	99.50	99.28	99.06	2.5
99.5%	2	98.84	99.48	99.58	99.50	99.32	99.08	98.66	98.26	97.76	0.5

Table 4: Proportion of Rejections for the BDS Test for 500 Observations Simulated using an MA(1) Model

		m									
%'age point	ε / σ	2	3	4	5	6	7	8	9	10	N(0,1)
0.5%	0.5	0.00	0.00	0.00	0.00	0.02	0.72	6.78	19.70	61.06	0.5
2.5%	0.5	0.00	0.00	0.00	0.00	0.06	1.56	7.34	19.70	61.06	2.5
5%	0.5	0.00	0.00	0.00	0.00	0.12	1.88	8.52	19.70	61.06	5
95%	0.5	99.86	99.84	99.28	97.50	92.60	83.96	71.42	56.68	38.94	5
97.5%	0.5	99.70	99.64	98.84	96.28	90.66	81.76	69.04	55.94	38.94	2.5
99.5%	0.5	98.86	98.82	97.02	93.04	85.52	76.30	64.40	53.48	38.92	0.5
0.5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.5
2.5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.5
5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5
95%	1	99.94	100.0	99.94	99.78	99.58	99.04	97.94	96.28	94.20	5
97.5%	1	99.92	99.86	99.78	99.52	98.96	97.98	96.42	94.04	91.60	2.5
99.5%	1	99.42	99.50	99.12	98.00	96.02	93.68	90.70	87.44	83.98	0.5
0.5%	1.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.5
2.5%	1.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.5
5%	1.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5
95%	1.5	99.88	99.98	99.90	99.64	99.28	98.78	98.10	96.96	95.72	5
97.5%	1.5	99.66	99.86	99.64	99.26	98.58	97.62	96.34	94.46	92.64	2.5
99.5%	1.5	99.08	99.24	98.46	97.30	95.54	92.76	89.88	86.70	83.74	0.5
0.5%	2	0.88	0.18	0.12	0.06	0.02	0.02	0.00	0.00	0.00	0.5
2.5%	2	1.12	0.20	0.14	0.10	0.02	0.02	0.04	0.04	0.04	2.5
5%	2	1.44	0.30	0.18	0.10	0.04	0.06	0.04	0.06	0.06	5
95%	2	93.36	96.78	97.02	96.34	95.26	93.70	92.66	91.04	89.32	5
97.5%	2	92.12	95.88	95.72	94.72	93.06	91.62	89.48	87.30	85.40	2.5
99.5%	2	89.44	92.86	91.98	89.46	86.62	83.32	80.20	76.50	72.72	0.5

Table 5: Proportion of Rejections for the BDS Test for 500 Observations Simulated using a SETAR Model

		m									
%'age point	ε / σ	2	3	4	5	6	7	8	9	10	N(0,1)
0.5%	0.5	0.00	0.00	0.00	0.00	0.00	0.16	3.62	14.64	52.94	0.5
2.5%	0.5	0.00	0.00	0.00	0.00	0.04	0.48	3.90	14.64	52.94	2.5
5%	0.5	0.00	0.00	0.00	0.00	0.04	0.60	4.56	14.72	52.94	5
95%	0.5	100.0	100.0	99.94	99.66	98.56	93.32	83.06	67.02	47.06	5
97.5%	0.5	100.0	100.0	99.88	99.54	97.98	91.80	81.66	66.42	47.06	2.5
99.5%	0.5	99.98	99.98	99.76	99.00	96.16	89.18	77.66	64.72	47.06	0.5
0.5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.5
2.5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.5
5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5
95%	1	100.0	100.0	100.0	100.0	99.94	99.90	99.80	99.50	98.98	5
97.5%	1	100.0	100.0	99.98	99.96	99.94	99.82	99.56	99.18	98.48	2.5
99.5%	1	100.0	99.96	99.92	99.80	99.66	99.34	98.68	97.44	96.16	0.5
0.5%	1.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.5
2.5%	1.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.5
5%	1.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5
95%	1.5	100.0	100.0	100.0	99.98	99.98	99.86	99.82	99.58	99.26	5
97.5%	1.5	100.0	100.0	100.0	99.98	99.86	99.82	99.58	99.08	98.60	2.5
99.5%	1.5	100.0	100.0	99.92	99.76	99.52	99.02	98.28	97.70	96.94	0.5
0.5%	2	0.32	0.04	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.5
2.5%	2	0.44	0.08	0.04	0.04	0.02	0.02	0.00	0.00	0.00	2.5
5%	2	0.44	0.08	0.06	0.04	0.04	0.04	0.04	0.02	0.02	5
95%	2	98.08	99.36	99.34	99.20	99.08	98.54	97.98	97.08	96.46	5
97.5%	2	97.76	99.06	99.12	98.92	98.42	97.62	96.78	95.96	95.06	2.5
99.5%	2	96.94	98.54	98.24	97.56	96.42	95.08	93.72	91.86	89.66	0.5

Table 6: Proportion of Rejections for the BDS Test for 500 Observations Simulated using a Bilinear Model

		m									
%'age point	ε / σ	2	3	4	5	6	7	8	9	10	N(0,1)
0.5%	0.5	0.00	0.00	0.00	0.00	0.00	0.08	0.72	4.88	32.44	0.5
2.5%	0.5	0.00	0.00	0.00	0.00	0.00	0.12	1.08	5.98	32.56	2.5
5%	0.5	0.00	0.00	0.00	0.00	0.00	0.14	1.46	7.18	32.56	5
95%	0.5	100.0	100.0	100.0	99.94	99.46	96.94	90.30	77.90	63.46	5
97.5%	0.5	100.0	100.0	100.0	99.92	99.30	96.08	88.72	76.40	62.46	2.5
99.5%	0.5	100.0	99.98	99.98	99.74	98.40	94.08	85.52	73.14	60.48	0.5
0.5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.5
2.5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.5
5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5
95%	1	100.0	100.0	100.0	100.0	100.0	100.0	99.92	99.78	99.56	5
97.5%	1	100.0	100.0	100.0	100.0	100.0	99.98	99.82	99.64	99.26	2.5
99.5%	1	100.0	100.0	100.0	99.98	99.94	99.80	99.44	98.80	97.82	0.5
0.5%	1.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.5
2.5%	1.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.5
5%	1.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5
95%	1.5	100.0	100.0	100.0	100.0	100.0	99.98	99.94	99.84	99.76	5
97.5%	1.5	100.0	100.0	100.0	100.0	99.98	99.96	99.86	99.78	99.60	2.5
99.5%	1.5	100.0	100.0	100.0	100.0	99.84	99.72	99.48	99.10	98.72	0.5
0.5%	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.5
2.5%	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.5
5%	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5
95%	2	99.92	100.0	99.98	99.98	99.94	99.92	99.86	99.76	99.62	5
97.5%	2	99.88	99.96	99.98	99.94	99.92	99.86	99.76	99.62	99.38	2.5
99.5%	2	99.80	99.90	99.88	99.84	99.70	99.50	99.06	98.50	97.86	0.5

Table 7: Proportion of Rejections for the BDS Test for the Residuals of a GARCH Model with 500 Observations Simulated using an AR(1) Model

		m									
%'age point	ε / σ	2	3	4	5	6	7	8	9	10	N(0,1)
0.5%	0.5	0.00	0.00	0.00	0.00	0.02	0.08	0.40	2.00	3.24	0.5
2.5%	0.5	0.00	0.00	0.00	0.00	0.02	0.28	1.08	5.36	10.56	2.5
5%	0.5	0.00	0.00	0.00	0.00	0.02	0.40	1.70	9.14	16.56	5
95%	0.5	99.94	99.64	97.94	90.58	72.72	51.18	3.15	19.50	12.30	5
97.5%	0.5	99.84	99.18	95.70	85.38	63.70	40.18	23.04	13.16	8.86	2.5
99.5%	0.5	99.10	97.06	87.62	67.28	46.90	23.94	10.70	5.34	3.02	0.5
0.5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.5
2.5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.5
5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5
95%	1	100.00	100.00	99.94	99.88	99.56	98.94	97.74	94.68	89.84	5
97.5%	1	99.98	99.96	99.86	99.58	98.96	97.72	94.60	91.26	84.62	2.5
99.5%	1	99.74	99.48	98.58	97.14	94.04	88.04	77.98	70.94	61.78	0.5
0.5%	1.5	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.5
2.5%	1.5	0.02	0.02	0.00	0.00	0.00	0.00	0.02	0.02	0.02	2.5
5%	1.5	0.02	0.02	0.02	0.00	0.00	0.02	0.02	0.02	0.02	5
95%	1.5	99.82	99.94	99.90	99.86	99.74	99.38	99.08	98.48	97.78	5
97.5%	1.5	99.64	99.88	99.80	99.62	99.10	98.60	97.80	96.82	95.54	2.5
99.5%	1.5	98.94	99.20	98.36	96.78	89.78	84.64	75.42	63.08	54.28	0.5
0.5%	2	0.14	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.5
2.5%	2	0.82	0.30	0.20	0.20	0.20	0.18	0.18	0.22	0.22	2.5
5%	2	1.38	0.62	0.50	0.38	0.34	0.38	0.38	0.46	0.52	5
95%	2	83.40	91.36	92.68	92.52	91.74	90.86	90.08	88.74	87.06	5
97.5%	2	79.22	88.38	89.68	89.58	88.70	86.52	84.42	81.52	80.02	2.5
99.5%	2	68.36	76.84	78.00	72.86	63.30	55.58	42.74	35.14	27.30	0.5

Table 8: Proportion of Rejections for the BDS Test for the Residuals of a GARCH Model with 500 Observations Simulated using an MA(1) Model

		m									
%'age point	ε / σ	2	3	4	5	6	7	8	9	10	N(0,1)
0.5%	0.5	0.00	0.00	0.00	0.00	0.00	0.02	0.32	1.24	2.24	0.5
2.5%	0.5	0.00	0.00	0.00	0.00	0.00	0.14	0.90	4.52	7.76	2.5
5%	0.5	0.00	0.00	0.00	0.02	0.002	0.38	1.52	7.86	13.02	5
95%	0.5	99.66	99.00	95.24	84.94	65.34	41.70	26.78	17.48	11.70	5
97.5%	0.5	99.18	97.66	92.04	77.16	53.82	32.60	19.16	11.60	7.88	2.5
99.5%	0.5	97.62	93.82	79.78	55.74	34.72	17.42	8.94	4.52	2.78	0.5
0.5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.5
2.5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.5
5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5
95%	1	99.96	99.88	99.70	99.20	97.88	95.82	92.60	87.34	79.58	5
97.5%	1	99.82	99.66	99.30	97.84	95.82	92.42	86.52	80.60	71.90	2.5
99.5%	1	98.70	97.68	94.42	91.16	83.62	74.24	60.48	52.54	44.16	0.5
0.5%	1.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.5
2.5%	1.5	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.5
5%	1.5	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	5
95%	1.5	99.02	99.50	99.22	98.54	97.56	96.28	94.72	92.58	90.08	5
97.5%	1.5	98.16	99.10	98.26	97.20	95.12	92.84	89.72	86.34	82.78	2.5
99.5%	1.5	95.16	94.44	91.20	85.04	66.08	56.12	43.70	31.92	24.94	0.5
0.5%	2	0.40	0.28	0.24	0.22	0.22	0.22	0.22	0.22	0.26	0.5
2.5%	2	1.26	0.84	0.72	0.68	0.76	0.78	0.88	0.86	0.98	2.5
5%	2	2.00	1.30	1.24	1.24	1.32	1.40	1.52	1.58	1.68	5
95%	2	65.38	75.14	74.54	71.32	67.80	65.18	63.10	60.52	58.26	5
97.5%	2	56.42	65.68	63.18	61.54	58.08	53.36	49.56	45.84	44.38	2.5
99.5%	2	33.60	37.78	35.58	29.08	20.02	15.20	9.46	7.10	5.06	0.5

Table 9: Proportion of Rejections for the BDS Test for the Residuals of a GARCH Model with 500 Observations Simulated using a SETAR Model

		m									
%'age point	ε / σ	2	3	4	5	6	7	8	9	10	N(0,1)
0.5%	0.5	0.00	0.00	0.00	0.00	0.04	0.04	0.26	0.62	1.18	0.5
2.5%	0.5	0.00	0.00	0.00	0.04	0.08	0.36	1.04	3.00	4.78	2.5
5%	0.5	0.00	0.00	0.00	0.04	0.12	0.78	1.90	5.32	8.42	5
95%	0.5	96.72	96.90	91.98	78.10	56.82	34.74	21.46	14.68	10.64	5
97.5%	0.5	92.78	94.16	96.26	67.58	43.56	25.66	14.54	9.18	6.58	2.5
99.5%	0.5	84.42	84.12	65.08	40.78	25.48	11.92	5.96	2.86	1.90	0.5
0.5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.5
2.5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.02	2.5
5%	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.02	5
95%	1	99.28	99.86	99.66	99.04	97.82	95.62	91.38	84.42	76.14	5
97.5%	1	98.44	99.46	98.96	97.60	95.38	91.36	83.82	76.52	65.62	2.5
99.5%	1	92.24	95.14	91.40	88.02	78.80	67.02	50.78	41.96	33.26	0.5
0.5%	1.5	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.5
2.5%	1.5	0.04	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.5
5%	1.5	0.04	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5
95%	1.5	97.44	99.26	99.26	98.72	97.78	96.86	94.92	92.70	89.82	5
97.5%	1.5	95.50	98.62	98.04	97.18	95.52	92.96	90.14	86.16	82.68	2.5
99.5%	1.5	88.12	92.36	89.68	83.48	61.38	49.84	36.34	23.64	17.26	0.5
0.5%	2	0.24	0.18	0.12	0.12	0.08	0.08	0.08	0.08	0.10	0.5
2.5%	2	0.96	0.60	0.56	0.52	0.50	0.50	0.52	0.58	0.54	2.5
5%	2	1.74	0.84	0.70	0.78	0.74	0.78	0.80	0.98	1.06	5
95%	2	69.28	79.80	79.90	78.46	76.16	73.80	71.90	68.98	66.38	5
97.5%	2	62.30	72.28	71.24	70.38	67.18	62.88	57.92	53.46	51.46	2.5
99.5%	2	42.18	46.88	44.38	36.04	24.34	18.50	10.96	81.40	57.20	0.5

Table 10: Proportion of Rejections for the BDS Test for the Standardised Residuals of a GARCH Model Estimated on Data Created Using a Bilinear Model with 500 Observations

		m									
%'age point	ε / σ	2	3	4	5	6	7	8	9	10	N(0,1)
0.5%	0.5	0.00	0.00	0.00	0.02	0.04	0.08	0.18	0.40	0.32	0.5
2.5%	0.5	0.08	0.00	0.06	0.14	0.30	0.74	1.12	1.72	1.90	2.5
5%	0.5	0.12	0.08	0.08	0.34	0.72	1.54	2.74	3.54	3.80	5
95%	0.5	36.84	46.32	41.40	31.78	21.38	13.40	10.12	7.86	7.02	5
97.5%	0.5	26.70	35.82	31.26	21.62	13.30	8.34	5.50	4.30	3.56	2.5
99.5%	0.5	14.92	20.30	12.76	71.80	4.96	2.66	1.64	1.14	0.86	0.5
0.5%	1	0.02	0.00	0.00	0.00	0.02	0.02	0.02	0.00	0.00	0.5
2.5%	1	0.08	0.08	0.06	0.08	0.14	0.18	0.16	0.12	0.14	2.5
5%	1	0.22	0.16	0.16	0.16	0.28	0.32	0.46	0.58	0.62	5
95%	1	32.56	48.24	49.84	47.42	44.00	38.40	33.46	27.44	22.64	5
97.5%	1	22.76	36.74	37.48	34.84	30.42	26.36	20.92	18.24	13.96	2.5
99.5%	1	8.48	14.14	12.80	12.16	8.84	6.24	3.56	3.44	2.98	0.5
0.5%	1.5	0.36	0.22	0.18	0.20	0.24	0.36	0.36	0.38	0.40	0.5
2.5%	1.5	1.32	0.62	0.56	0.66	0.88	1.12	1.58	1.64	1.76	2.5
5%	1.5	2.34	1.26	1.32	1.56	1.70	1.98	2.46	2.98	3.38	5
95%	1.5	14.02	22.20	24.62	24.74	24.26	24.58	23.52	23.18	21.38	5
97.5%	1.5	7.20	13.64	14.66	15.56	15.16	14.32	14.64	13.76	13.04	2.5
99.5%	1.5	2.26	2.94	3.12	2.76	1.16	1.16	0.84	0.50	0.34	0.5
0.5%	2	0.86	1.00	1.04	1.10	1.04	0.98	1.08	1.12	1.20	0.5
2.5%	2	3.96	3.96	4.00	4.28	4.78	5.02	5.24	5.74	6.04	2.5
5%	2	6.80	6.68	7.28	7.90	8.38	9.08	9.86	10.74	11.58	5
95%	2	4.42	5.58	6.16	6.52	6.72	7.46	8.26	8.82	8.72	5
97.5%	2	2.00	2.94	3.26	3.84	4.16	4.20	4.24	4.16	4.48	2.5
99.5%	2	0.44	0.68	0.90	0.78	0.76	0.70	0.46	0.42	0.32	0.5